

## Pretend-but-Perform Contracts in Sharecropping†

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### 1. Introduction

We consider a sharecropping farm whose output depends on a single tenant's effort and the landlord's capital input. Tenants are each parametrized by a characteristic summarizing income-leisure preference and effect on output. The "Golden Rule" of Sharecropping Design for the landlord here prescribes, for each theoretically possible tenant, a share in output to be given to the tenant and a level of capital input by the landlord, leading the tenant to contribute an effort and deliver an output which ends up maximizing the landlord's income. This Golden Rule thus determines a map  $\Phi$  from the space  $T$  of tenant characteristics to the set of triplets consisting of tenant's share, landlord's capital contribution, and ensuing output.

Our purpose here is to examine what would happen if the landlord were in effect to let the tenant choose any triplet in  $\Phi(T)$  for them to jointly bring about. One may imagine the landlord as guaranteeing that the Golden Rule will be applied to any characteristic which the tenant may *pretend* to have but that he will be held responsible to *perform* and deliver the requisite output corresponding to his pretended identity (when provided the Golden Rule share and capital). The map  $\Phi(T)$  thus corresponds to a set of "pretend-but-perform" contracts.

Typically a tenant under this scheme will have an incentive to be pretensive, and this may be intuitively clear and unsurprising. What if any pattern there is to pretensions, however, is not obvious. Nor is it clear what welfare consequences may be in store for the landlord. In our model where tenant space is a two-parameter family, it turns out that much depends on whether the tenant is allowed to pretend in both parameters or in one alone, and then in which.

When both parameters are open to pretension, all tenants will emulate as closely as possible the same theoretically inadmissible

tenant, leaving the landlord with arbitrarily small positive income. On the other hand, a tenant restricted to pretend along a single parameter axis has a unique optimal pretension which is a one-to-one function of the tenant's characteristic on that axis. For pretensions along one of these axes, the landlord always earns more with his pretended than his true tenant. When pretensions are restricted to the other axis, for some tenants he will earn more with the true and for other tenants more with the pretended identity, the latter case being just as salient as the former.

To develop all this, Section 2 starts out by building the idea of a pretend—but—perform contract based on the Golden Rule of Kleindorfer and Sertel [1979] and checking, in Proposition 1, that the institution of such a contract actually operates in well-behaved fashion. Proposition 2 in this section measures the outcome under such a contract, in units suitable for the analysis to follow, as a function of the tenant's pretended identity. Section 3 presents the summary Proposition 3 concerning the existence and uniqueness of optimal pretended identity under three types of pretend—but—perform contract and the position of such identities relative to the true identity of the tenant. Section 4 makes welfare comparisons between the pretended identity outcome under three types of pretend—but—perform contract and the true identity outcome, collected in Proposition 4 and discussed thereafter. The paper ends with the summary and closing remarks of Section 5, followed by an Appendix devoted to a series of lemmata and a proof of technical nature.

## *2. Pretend—but—Perform Contracts*

### *2.1 The Golden Rule Solution*

We find ourselves on a single-tenant/single-landlord farm with

$$(1) \quad y = k^{\alpha}x \quad (0 \leq \alpha, 0 < \delta, 0 < \mu < 1 - \alpha),$$

$$(2) \quad u = w - \delta x^{\frac{1}{\mu}}$$

where  $y$  is the output produced through the landlord's input  $k$  of "capital goods" and the tenant's contribution  $x$  of his "effort", and  $u$  represents the tenant's preference between effort  $x$  and income  $w$  in units of  $y$ .<sup>1</sup> The landlord and tenant engage each other through a sharecropping contract. Thus

$$(3) \quad w = \lambda y \quad (0 \leq \lambda \leq 1),$$

where  $\lambda$  is the tenant's share in output. For  $k$  there is a market to which the landlord alone has access and in which  $k$  fetches a given rental  $\rho$ , so that the landlord's profit amounts to

$$(4) \quad p = z - \rho k \quad (0 < \rho),$$

where

$$(5) \quad z = (1 - \lambda)y$$

is the landlord's gross income before compensating for  $k$ .

We assume the functional forms (1) – (5) together with their accompanying restrictions on the parameters  $\alpha$ ,  $\mu$ ,  $\delta$  and  $\rho$ , and in fact the values of  $\alpha$  and  $\rho$ , to be known by all. We treat the characteristics  $\mu$  and  $\delta$  explicitly as private to the tenant, their values being known to himself but not necessarily to the landlord.

We consider an extensive form game where the landlord leads by setting the share  $\lambda$  and supplying an amount  $k \geq 0$  of capital, and then the tenant follows by contributing an amount  $x \geq 0$  of effort. When all parameter values are publicly known as in Kleindorfer and Sertel [1979], the Stackelberg solution  $(\lambda, \underline{k}, \underline{x})$  of this game and the consequent output are

$$(6) \quad \lambda = \mu$$

$$(7) \quad \underline{k} = \left[ \left( \frac{\alpha}{\rho} \right)^{1-\mu} \left( \frac{\mu^2}{\delta} \right)^\mu \right]^{\frac{1}{1-\alpha-\mu}}$$

$$(8) \quad \underline{x} = \left[ \left( \frac{\alpha}{\rho} \right)^{\alpha\mu} \left( \frac{\mu^2}{\delta} \right)^{(1-\alpha)\mu} \right]^{\frac{1}{1-\alpha-\mu}}$$

$$(9) \quad \underline{y} = \left[ \left( \frac{\alpha}{\rho} \right)^\alpha \left( \frac{\mu^2}{\delta} \right)^\mu \right]^{\frac{1}{1-\alpha-\mu}}.$$

(Maximize (2) subject to (1) and (3) to get the tenant's response function  $x(\lambda, k)$ . The consequent output  $y(\lambda, k)$  determines a profit  $p(\lambda, k)$  for the landlord, the maximum of which is obtained at (6) and (7) above. Then (8) and (9) follow.) Suppressing  $\underline{x}$  of (8) as in principle not directly observable, we will refer to the triplet  $(\lambda, \underline{k}, \underline{y})$  of

((6) (7), (9)) from here on as the (Kleindorfer and Sertel [1979]) *Golden Rule Solution*.

## 2.2 The Contract

The legal-economic institution of "pretend—but—perform" contracts we consider setting up in our present economic and informational environment is based on the simple idea that the tenant may be allowed to report his characteristics as any  $(\mu, \delta)$ , with the proviso that he will then join the landlord in enacting the Golden Rule Solution which his reported characteristics (and  $\alpha, \rho$ ) determine. Thus, with the "guarantee" that his share in output will be as in (6) and that the landlord will input capital as in (7), a tenant will *pretend* to be any  $(\mu, \delta)$  subject thereupon to *perform* the corresponding output (9).

To "implement" the Golden Rule Solution in this way, we present to the tenant and landlord a contract, which once signed, renders straying away from (7) and (9) against either party's interest. Observe that (9) can be written as

$$(10) \quad y = \left[ \left( \frac{\mu}{\delta} \right)^2 \mu \underline{k}^\alpha \right]^{\frac{1}{1-\mu}}$$

with  $\underline{k}$  as in (7). This gives us the functional form

$$(11) \quad \hat{y}(k) = \left[ \left( \frac{\mu}{\delta} \right)^2 \mu k^\alpha \right]^{\frac{1}{1-\mu}}.$$

Letting now  $\Upsilon = \nabla \times \Delta$  be the space of *tenant identities*, where  $\nabla = \{\mu \mid 0 < \mu < 1 - \alpha\}$  and  $\Delta = \{\delta \mid 0 < \delta\}$ , and noting that the function  $\hat{y}$  from  $K = \{k \mid k \geq 0\}$  to the real line  $\mathbb{R}$  is nothing but the evaluation of a unique *commitment*  $y: \Upsilon \rightarrow \mathbb{R}^K$  at an identity  $\tau \in \Upsilon$  so that

$$(12) \quad \hat{y}(k) = y[\tau](k),$$

we state the

*Pretend-but-Perform Contract:*

The landlord and tenant agree that, upon the declaration of an identity  $\tau = (\mu, \delta)$  by the tenant, the landlord will supply capital  $k$ , which the tenant will use to produce an output  $y$ , on the condition that their respective incomes will be

$$(13) \quad z = \begin{cases} (1 - \mu)y & \text{if } k = \underline{k}(\tau) \\ 0 & \text{otherwise} \end{cases} \quad (\text{see (7)})$$

$$(14) \quad w = \begin{cases} \mu y & \text{if } y = \underline{y}[\tau](k) \\ 0 & \text{otherwise.} \end{cases} \quad (\text{see (11)})$$

Generally the tenant's claims of identity may be restricted to a subset  $\Sigma$  of  $\Upsilon$ , as delineated by economically sensible boundaries. If a tenant's true characteristic  $\bar{\mu}$  is public knowledge, for example, he may be allowed to declare an identity only from  $\Sigma = \bar{\mu} \times \Delta$ . For any  $\Sigma \subset \Upsilon$ ,  $\text{PPC}(\Sigma)$  will denote the Pretend-but-Perform contract with tenant identities restricted to  $\Sigma$ . We will assume that *the true identity*  $\bar{\tau}$  of a tenant facing a  $\text{PPC}(\Sigma)$  belongs to  $\Sigma$ .

Before going any further we check in Proposition 1 below that the sanctions built into a PPC indeed induce landlord and tenant both to act "correctly". To that end, note that under a  $\text{PPC}(\Sigma)$ , if a tenant  $\bar{\tau} = (\bar{\mu}, \bar{\delta})$  claims an identity  $\tau = (\mu, \delta)$ , the landlord chooses  $k = \underline{k}(\tau)$  and tenant performs the output  $y = \underline{y}(\tau)$ , then the profit of the landlord is

$$(15) \quad \underline{p} = (1 - \alpha - \mu) \left[ \left( \frac{\alpha}{\rho} \right)^\alpha \left( \frac{\mu^2}{\delta} \right)^\mu \right]^{\frac{1}{1-\alpha-\mu}},$$

and the utility of the tenant is

$$(16) \quad \underline{u} = \mu \left[ \left( \frac{\alpha}{\rho} \right)^\alpha \left( \frac{\mu^2}{\delta} \right)^\mu \right]^{\frac{1}{1-\alpha-\mu}} - \bar{\delta} \left[ \left( \frac{\alpha}{\rho} \right)^\alpha \left( \frac{\mu^2}{\delta} \right)^{1-\alpha} \right]^{\frac{\mu}{\bar{\mu}(1-\alpha-\mu)}}.$$

(Observe that  $y[\tau](\underline{k}(\tau)) = \underline{y}(\tau)$ , so that (15) follows from (4), (13), (7) and (9). Note further that the disutility of effort of the tenant who has produced  $\underline{y}(\tau)$  with  $\underline{k}(\tau)$  units of capital amounts to

$$\frac{1}{\delta[\underline{k}(\tau)^{-\alpha} \underline{y}(\tau)]^{\frac{1}{\mu}}},$$

which yields (14) through (2), (14), (7) and (9).)

**Proposition 1:** Under a PPC( $\Sigma$ ), the tenant so declares an identity  $\tau \in \Sigma$  and the landlord and tenant both behave in such a way that  $k = \underline{k}(\tau)$  and  $y = \underline{y}(\tau)$ .

**Proof:** Assume that a tenant (of true identity  $\bar{\tau}$ ) claims an identity  $\tau \in \Sigma$  under a PPC( $\Sigma$ ).

(i) If the landlord contributes a positive  $k \neq \underline{k}(\tau)$ , then from (13), he receives no income and incurs a loss of  $\rho k > 0$ , whereas he can guarantee himself a zero loss by choosing  $k = 0$ . One concludes that the landlord chooses  $k \in \{0, \underline{k}(\tau)\}$ .

(ii) Define  $\Theta = \{\theta \in \Sigma \mid \underline{u}(\theta) > 0\}$ . Check that  $\bar{\tau} \in \Theta$  so that  $\Theta \neq \emptyset$ . Now let  $u_0(\tau)$  (resp.,  $u^0(\tau)$ ) be the maximal utility the tenant can attain if the landlord chooses  $k = 0$  (resp.,  $k = \underline{k}(\tau)$ ). Check that  $(u_0(\tau'), u^0(\tau')) = (0, u')$  for some positive number  $u'$  if  $\tau' \in \Theta$ , and  $(u_0(\tau''), u^0(\tau'')) = (0, 0)$  if  $\tau'' \notin \Theta$ , so that in view of (i) the payoff to the tenant from any claim of identity  $\tau' \in \Theta$  dominates that from any  $\tau'' \notin \Theta$ . One concludes that  $\tau \in \Theta$ .

(iii) Since  $\tau \in \Theta$  as one (including the landlord) now knows, the landlord knows that, if he chooses  $k = \underline{k}(\tau)$ , then the tenant will produce the output  $y = \underline{y}[\tau](\underline{k}(\tau))$  so as to attain positive utility (as opposed to producing any other output and consequently receiving non-positive utility). The landlord therefore chooses  $k = \underline{k}(\tau)$  so as himself to attain positive profit (as opposed to choosing  $k = 0$  and receiving zero profit).

(iv) In view of (iii), the tenant produces  $y = \underline{y}[\tau](\underline{k}(\tau)) = \underline{y}(\tau)$  so as to attain positive (and maximal) utility.

### 2.3 Measuring the Outcome

As a consequence of Proposition 1, expressions (15) and (16) give us the payoffs accruing to landlord and tenant depending on the tenant's declared identity under a PPC. In the next two sections we will investigate the tenant's optimal choice of identity and compare the consequent payoffs with those which obtain when the Golden Rule Solution is applied to the tenant's true identity. To simplify the analysis and final comparisons, it will be useful to change from the units which have served us so far to new units for tenant characteristics and a new scale along which to measure output.

First we define the new output

$$Y = cy,$$

where

$$c = [(1 - \alpha) \left( \frac{\alpha}{\rho} \right)^{\frac{\alpha}{1-\alpha-\mu}}]^{-1},$$

so that the profit of the landlord and the utility of the tenant now become

$$P = c\underline{p}, \quad U = c\underline{u},$$

respectively. The rental of capital in terms of this new unit of output now is  $r = c\rho$ . We will refer to the quantity  $G = Y - rk$  as the *value added* by labor. Next, denoting

$$m = \frac{\mu}{1 - \alpha} \quad d = \frac{c\delta}{1 - \alpha}$$

$$M = \{m \mid \mu \in \mathbb{V}\} = (0, 1), \quad D = \{d \mid \delta \in \Delta\} = (0, \infty),$$

we define

$$T = M \times D$$

as our new space of tenants.

Denoting  $S = \{(m, d) \mid (\mu, \delta) \in \Sigma\}$  for any  $\Sigma \subset \Upsilon$ , since the transformation  $\Upsilon \rightarrow T$  is an isomorphism, there is no harm in henceforth speaking of a PPC(S) instead of a PPC( $\Sigma$ ). This transformation

enables us to present the following concise expression of the outcome under a PPC(S) as a function of the tenant's chosen identity.

**Proposition 2:** Under a PPC(S), if a tenant  $\bar{t} = (\bar{m}, \bar{d})$  were to declare an identity  $t = (m, d) \in S$ , then the output

$$(17) \quad Y = \left( \frac{1}{1 - \alpha} \right) G$$

would be produced, where

$$(18) \quad G = \left( \frac{m^2}{d} \right)^{\frac{1}{1 - m}}$$

is the value added by labor. The consequent payoffs to the landlord and tenant would be, respectively.

$$(19) \quad P = (1 - m)G,$$

$$(20) \quad U = mG - \bar{d}(1 - \alpha)G^{\frac{1}{\bar{m}}}.$$

**Proof:** In view of Proposition 1, simply apply the transformation above to (9), (13), and (14) to obtain (17), (19), and (20), respectively. Check from (7) and (9) that  $\alpha \underline{y} = r \underline{k}$ , so that the value added  $G = Y - r \underline{k}$  is indeed given by (18).

**Note:** Our change in units has placed us in a world of value added sharing, as it were, in the sense that  $m$  is the tenant's share in  $G$ , the value added by labor (cf. Kleindorfer and Sertel [1978, 1980]).

In the light of Proposition 2, a tenant  $(\bar{m}, \bar{d})$  who has cosigned a PPC(S) will declare an identity  $t^* = (m^*, d^*)$  which maximizes his (continuous) utility  $U(m, d)$  given in (20), if indeed such a maximum exists. Note that the tenant identity space  $T$  is open and unbounded, so that such a  $t^*$  need not exist for the PPC(T) or for just any PPC(S), unless for example  $S$  is compact. We will restrict our analysis to the study of pretend—but—perform contracts PPC(S) in the three rectangular cases

- (i)  $S = T$ ,
- (ii)  $S = \bar{m} \times D$ ,
- (iii)  $S = M \times \bar{d}$ ,

where  $(\bar{m}, \bar{d}) \in S$  stands for the tenant's true identity. Here (i) is the case where the tenant has total freedom in declaring a chosen identity, while (ii) and (iii) restrict his declaration to be truthful in the  $m$  and  $d$  dimensions, respectively.

### 3. Optimal Pretensions

We collect in Proposition 3 a statement about the existence and uniqueness of a tenant's optimal pretension and what we are able to assert about its position in relation to the tenant's true identity.

#### Proposition 3:

- (I) In neither the case  $S = T$  nor the case  $S = M \times 1$  does a tenant have an optimal pretension under the  $PPC(S)$ . In fact, in both cases, were  $(1, 1)$  appended to  $S$ , every tenant under such a contract would achieve maximal utility by pretending this limiting "identity".
- (II) In the case  $S = \bar{m} \times D$  for any  $\bar{m} \in M$ , under the  $PPC(S)$  every tenant  $(\bar{m}, \bar{d})$  in  $S$  has a unique optimal pretension, namely that with  $d^* = (1 - \alpha)\bar{d}$ .
- (III) In the case  $S = M \times \bar{d}$  for any  $\bar{d} \in D \setminus \{1\}$ , under the  $PPC(S)$  every tenant  $(\bar{m}, \bar{d})$  in  $S$  has a unique optimal pretension, namely with  $m^*$  given implicitly as the proper solution to equation (21) of Lemma 1 in the Appendix. Unfortunately, we cannot present  $m^*$  explicitly, but offer the functions  $m^1$  and  $m_1$  defined in Lemma 2 through equations (27) and (28), respectively, to serve as lower bound for  $m^*$  in cases  $\bar{d} > 1$  and  $\bar{d} < 1$ , respectively. (See Figure 1 below.) Moreover, at every  $\bar{d} \in D \setminus \{1\}$ ,  $m^*$  is an increasing and differentiable function of  $\bar{m}$  (Lemma 4). Finally, the equation of Lemma 5(1) defines a convex function (from  $M$  to  $D$ ) whose graph  $C$  (see Figure 1) is the set of "unpretentious"

tenants and separates the (convex) set  $C_u$  (above) of "underclaimers" from the set  $C_o$  (below) of "overclaimers".

*Proof:* See Lemmas 1 – 5 of the Appendix of which our proposition is simply a summary.

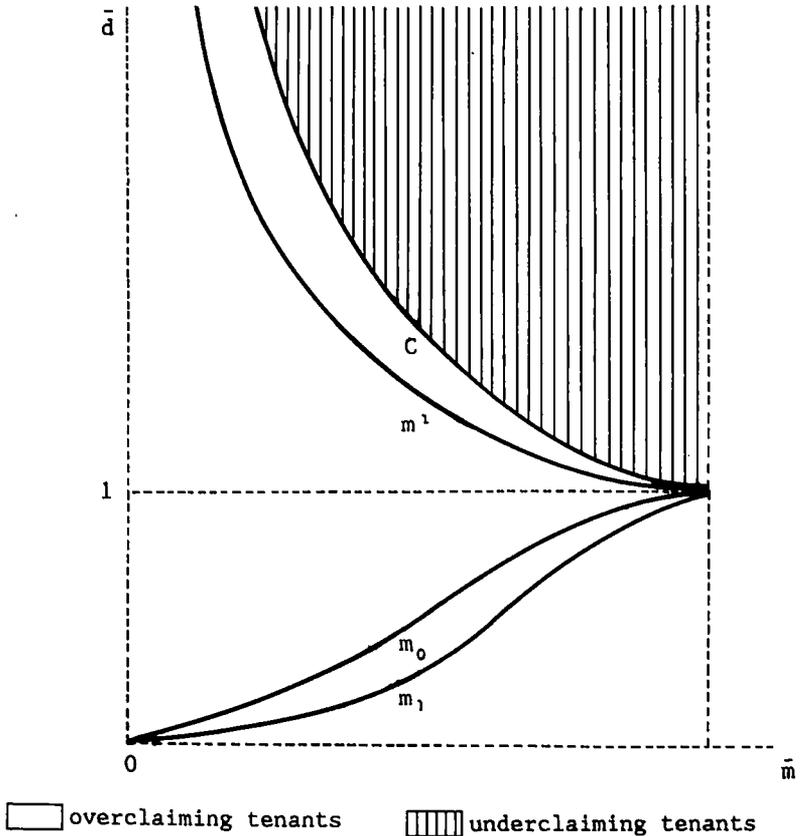


Figure 1: Optimal pretensions under  $PPC(\bar{d} \times M)$

**4. Welfare Comparisons**

This section aims to measure and assess the outcome under a pretend—but—perform contract in comparison with the straightforward outcome where there are no pretensions. This can be stated more precisely: Proposition 2 established an outcome pair  $(P, U)(\bar{t}, t)$ , measuring the landlord's profit and tenant's utility for each  $\bar{t} \in T$  and any pretension  $t \in T$ . Proposition 3 described an optimal pretension map  $\pi_s : S \rightarrow \text{Closure}(S)$  under a PPC(S) for each of the three cases (i) – (iii) of  $S \subset T$ . Defining  $W = P + U$ , we now compare for our three cases (i) – (iii) of permissible pretensions the *pretended identity outcome*  $(P^*, U^*, W^*)(\bar{t}) = (P, U, W)(\bar{t}, \pi_s(\bar{t}))$  with the *true identity outcome*  $(\bar{P}, \bar{U}, \bar{W})(\bar{t}) = (P; U, W)(\bar{t}, \bar{t})$ .<sup>2</sup>

**Proposition 4:**

(I) For any tenant  $(\bar{m}, \bar{d})$  under the PPC(T)

$$\frac{U^*}{\bar{U}} = \frac{1}{\bar{m}} \frac{1 - \bar{m}}{1 - (1-\alpha)\bar{m}} \left( \frac{1}{(1 - \alpha)\bar{m}} \right)^{\frac{\bar{m}}{1-\bar{m}}} > 1,$$

$$\frac{P^*}{\bar{P}} = 0, \text{ and}$$

$$\frac{W^*}{\bar{W}} = \frac{1 - \bar{m}}{1 - (1-\alpha)\bar{m}} \left( \frac{1}{(1 - \alpha)\bar{m}} \right)^{\frac{\bar{m}}{1-\bar{m}}}$$

which is an increasing function of  $\bar{m}$  onto  $(0, \infty)$  equalling unity at an  $m(T) \in (0, \frac{1}{2})$ .

(II) For any tenant  $(\bar{m}, \bar{d})$  under the PPC( $\bar{m} \times D$ )

$$\frac{U^*}{\bar{U}} = \frac{1 - \bar{m}}{1 - (1-\alpha)\bar{m}} \left( \frac{1}{(1 - \alpha)} \right)^{\frac{\bar{m}}{1-\bar{m}}},$$

$$\frac{P^*}{P} = \left( \frac{1}{1 - \alpha} \right)^{\frac{\bar{m}}{1 - \bar{m}}},$$

$$\frac{W^*}{W} = \frac{1 - \bar{m}^2}{1 - (1 - \alpha)\bar{m}^2} \left( \frac{1}{1 - \alpha} \right)^{\frac{\bar{m}}{1 - \bar{m}}},$$

and these ratios are all strictly greater than unity, except in the case of  $\alpha = 0$  where they equal unity.

(III) For any tenant  $(\bar{m}, \bar{d})$  under the PPC( $M \times \bar{d}$ )

$$\frac{U^*}{U} > 1,$$

for all but the unpretentious tenants mentioned in Proposition 3 (III) in which case  $U^*/U = 1$ , and

(1) in the case  $\bar{d} > 1$ ,

$$\frac{P^*}{P} (\geq) 1 \text{ and } \frac{W^*}{W} (\geq) 1 \text{ iff } m^* (\geq) \bar{m},$$

(2) in the case  $\bar{d} = 1$ ,

$$\frac{P^*}{P} = 0 \text{ and } \frac{W^*}{W} < 1, \text{ and}$$

(3) in the case  $\bar{d} < 1$ ,

$$\frac{P^*}{P} > 1 \text{ if } \bar{m} > m_1(\bar{d}), \text{ and } \frac{W^*}{W} > 1 \text{ if } \bar{m} > m_0(\bar{d}).$$

(See Figure 1).

**Proof:** *ad* (I): With reference to Proposition 3(i), the identities here follow upon substituting the first order condition in the statement of

Lemma 1(1) in the expressions for  $U$ ,  $P$ ,  $W$ , dividing each by  $\bar{U}$ ,  $\bar{P}$ ,  $\bar{W}$  respectively, and then taking the limit as  $m$  approaches 1. The assertions concerning the sizes of the identities are straightforward to check.

*ad* (II): Follows from Lemma 1 (2), (19) and (20).

*ad* (III): See the end of the Appendix.

Thus, instituting pretend—but—perform contracts in sharecropping based on the Golden Rule, as compared with the straightforward use of the same Rule, while unsurprisingly improving the lot of the tenant, may bring higher or lower profit to the landlord, and more or less welfare for the society of two depending on the range ( $S$ ) of pretensions permitted and the kind of tenant ( $\bar{t}$ ). When all pretensions are allowed ( $S = T$ ), the landlord loses virtually all profit and the utilitarian welfare  $W$  of our landlord—tenant pair may fall if the tenant's (true) parameter is low enough. If  $\bar{m}$  is not too low, e.g. at least a half, then the relative exaggeration  $m^*/\bar{m}$  is less pronounced and the landlord can be compensated for loss in profit with still enough output left to leave the tenant better off as compared with ordinary sharecropping under the Golden Rule.

When pretensions are to be allowed in the  $d$  characteristic and this alone ( $S = \bar{m} \times D$ ), there is a clear case for the PPC institution, as both tenant and landlord become better off living out the world of the tenant's pretension than that of the tenant's true identity. This is so, moreover, regardless of which tenant  $\bar{t} \in T$  we happen to have.

The most variegated welfare consequences of instituting pretend—but—perform contracts arise when the tenant is permitted to pretend along the  $m$ -axis ( $S = M \times \bar{d}$ ), and here everything depends first on the tenant's characteristic  $\bar{d}$ . If  $\bar{d} = 1$ , the PPC virtually erases all profit and this without sufficient improvement of the tenant's lot to compensate for the decline in the landlord's welfare, so social welfare falls. If  $\bar{d} > 1$ , however, both profit and social welfare are higher with the PPC than without precisely in the case where the tenant ( $\bar{t} \in C_0$ ) exaggerates his  $m$  characteristic in his pretension. (See Figure 1, recalling Proposition 3 (III).) Finally, if  $\bar{d} < 1$  all we can assert is that the landlord will be better off under the PPC( $M \times \bar{d}$ ) if the

tenant has  $\bar{m}$  above  $m_1(\bar{d})$  and that for smaller  $\bar{m}$  the landlord can be compensated for any decline in profit with the tenant still better off after the compensating transfer, so long as  $\bar{m}$  exceeds  $m_0(\bar{d})$ .

### 5. Summary and Concluding Remarks.

We have considered, in the setting of a sharecropping farm, two individuals facing each other as sole available partners in a joint production, namely a landlord, who leads by setting the share and supplying input of capital, and a tenant, who follows by contributing effort. The tenant maximizes a utility characterized by two parameters identifying his income/leisure preferences. A *straightforward* landlord knowing his tenant's characteristics in this setting would simply reckon the tenant's utility-maximizing response to all pairs of share and capital he could allot, and choose among these that pair leading to an output which maximizes residual profit. This "straightforward" account thus completely describes what takes place on a "Golden Farm", golden in the sense of being profit-maximizing for our landlord equipped with so-called complete information (Kleindorfer and Sertel (1979)). Thus we have two formulae, which we dub the Golden Rule of Sharecropping, specifying for each theoretically possible tenant a share and a quantity of capital input which the landlord would allot, and a third formula recording the (Golden) output arising through the consequent effort which the tenant contributes. These formulae provide the basis for pretend—but—perform contracts, our subject matter in this study. Pretend—but—perform contracts permit the tenant to assume any chosen identity with the understanding that he and the landlord will then act out their parts in realizing the Golden Farm outcome. The tenant is in effect allowed to receive any share and capital input fitting the Golden Rule, so long as he "guarantees" to deliver the requisite Golden Output.

We investigated what happens under pretend—but—perform contracts of three canonical types, accordingly as one or the other tenant parameter or the pair of these is freely declared. Section 4 detailed the comparison of welfare achieved by both participants under pretend—but—perform versus straightforward sharecropping for each contract type. Contracts allowing just  $d$  declarations stood out as creating a blanket Pareto improvement regardless of the tenant the landlord is dealing with. Allowing pretensions in both parameters  $m$  and  $d$  took us to the other extreme, resulting in a total loss for the land-

lord, though for more than half the abstract tenant space tenant gained more than enough to compensate for this loss. Contracts allowing declarations of just m fell in between, yielding Pareto improvements for a good half of the tenant space but a loss to the landlord for the other half (with always a gain for the tenant).

All the cases where the landlord's lot improves exposes a problematic naiveté that might stigmatize optimal incentive schemes based on agents' true characteristics. For, after all, we are led to the paradoxical conclusion that a landlord fully cognizant of his tenant's identity would be wiser to apply the Golden Rule of Sharecropping with contracts permitting pretended characteristics, knowing in advance that not truth but falsity will ensue, bringing about an improvement in the welfare of both. We cannot overemphasize the fact that this invitation to a pretensive order proves a blessing to each inhabitant of the Golden Farm for all theoretically possible tenant types. Landlord and economist alike would be well advised to give up holding truth-telling and truth-acting sacrosanct and to acknowledge the potential that might lie in a world of impersonating agents.

Of course, it is not difficult to see through this paradox of straightforward rationality, as we hope this paper has done. Naturally, for it to arise one has to be constrained to an inefficient institution, where the capabilities of the participants can be so utilized and the consequent output so divided in alternative ways as to dominate the inefficient outcome. If it is no wonder that improvements get scored when pretensions are allowed, it still seems remarkable that such occurrences turn up in our tale, not as an exception, but in an impressively robust manner.

Inefficient modes of interaction among noncooperating agents few in number abound in economic reality. And, caricature as it may be of "real" sharecropping, our Golden Farm offers a genuine parable questioning what sense straightforward rationality has in inefficient settings. Our results bear out that, given the reality of one or another institution which — with its definition of rights and rules for interaction — leads its agents to inefficient outcomes, some may well find it in their interests to allow others to pretend, not to escape whatever costs they would otherwise incur in unearthing the truth but even under complete information, and the (public) welfare economist may well be forced to praise their wisdom in so doing by comparison with the straightforward outcome. All this has inspired us to consider "games of pretension" in an abstract setting (Koray and Sertel [1983]).<sup>3</sup>

One can of course conceive of other Farms (i.e., triplets of formulae as described in the opening paragraph of this section) which could serve the landlord (or tenant or both jointly) better were they

taken as fulcrums for pretend—but—perform drama. One could in fact devise an "optimal" Farm which maximizes landlord's expected profits with respect to a prior distribution he may hold on possible tenant types, and do so subject even to eliciting the truth, as in the usual principal-agent theory in the fashion of Myerson [1982]. Our purpose here has not been to look for an optimal institution to replace the sharecropping farm, although in a literature stemming from an early [1981] version of this paper we have proposed economic design based on the pretend—but—perform idea (Koray and Sertel [1986, 1987]). Our interest here has rather been to look at overall behavior when some agents are allowed to pretend within the confines of a given institution. What we have seen suggests that real life under inefficient institutions may well abound with identity revelation equilibria characterized by misrepresentation. It would seem interesting to investigate what prospects avail in such circumstances to learn the truth behind the play, allowing agents may anticipate how recovered truth may be put to use, particularly since the verity of a principal's prior beliefs appears a matter unquestioned in "optimal mechanism design".

## APPENDIX

*Lemma 1:* Let  $\bar{t} = (\bar{m}, \bar{d})$  be a tenant.

(1) For any identity  $t = (m, d) \in T$  claimed by the tenant under the PPC(T) a necessary (first order) condition of optimality is that

$$d = m^2 \left( \frac{\bar{m}m}{1 - \alpha \bar{d}} \right)^{\frac{\bar{m}(1-m)}{\bar{m}(1-\bar{m})}},$$

so that the utility of such a claim is strictly increasing in  $m \in M = (0, 1)$ , affording  $\bar{t}$  no optimal claim of identity.

(2) Under the PPC( $\bar{m} \times D$ ), the  $d \in D$  claimed by  $\bar{t}$  is

$$d^* = (1 - \alpha)\bar{d}.$$

(3) Under the PPC( $M \times \bar{d}$ ), the  $m \in M$  claimed by  $\bar{t}$

(i) is the unique solution  $m^* \in M$  of

$$(21) \quad \frac{(1 - \alpha)\bar{d}}{\bar{m}} \left(\frac{m^2}{\bar{d}}\right)^{\frac{\bar{m}(1-m)}{m(1-\bar{m})}} = m + \frac{(1 - m)^2}{2(1 - m) + \ln\left(\frac{m^2}{\bar{d}}\right)}$$

if  $\bar{d} > 1$ ,

(ii) yields a higher utility the greater  $m \in M$  is, so that  $\bar{t}$  has no optimal claim of identity, if  $\bar{d} = 1$ ,

(iii) is the unique solution  $m^*$  of (21) in the interval  $(n, 1)$ , where  $n = \max(\bar{m}, m_o(\bar{d}))$  and  $m_o(\bar{d})$  is the (unique) solution in  $M$  of

$$m^2 e^{2(1-m)} = \bar{d},$$

if  $\bar{d} < 1$  (See Figure 1).

(Generally now, for any function  $f: A \times B \rightarrow C$  and any  $a \in A$ , define  $f_a: B \rightarrow C$  by  $f_a(b) = f(a,b)$ . Similarly for any  $b \in B$ , define  $f_b: A \rightarrow C$  by  $f_b(a) = f(a,b)$ . Note that the utility function of a tenant  $\bar{t} = (\bar{m}, \bar{d})$  is  $U: T \rightarrow R$  (see (20)),  $U_{\bar{d}}: M \rightarrow R$ , and  $U_{\bar{m}}: D \rightarrow R$  under the  $PPC(T)$ ,  $PPC(M \times \bar{d})$ , and  $PPC(\bar{m} \times D)$ , respectively.)

*Proof:* *ad* (1): Compute that the derivative  $DU = \left(\frac{\partial U}{\partial m}, \frac{\partial U}{\partial d}\right): T \rightarrow R$  is given by

$$(22) \quad \frac{\partial U}{\partial m} = \frac{G}{(1 - m)^2} (B - AH) ,$$

$$\frac{\partial U}{\partial d} = \frac{G}{(1 - m)d} (H - m) ,$$

where

$$A = 2(1 - m) + \ln\left(\frac{m^2}{d}\right),$$

$$B = mA + (1 - m)^2,$$

$$H = \frac{(1 - \alpha)\bar{d}}{\bar{m}} G \frac{(1 - \bar{m})}{\bar{m}}$$

To check that, for any  $m \in M$ ,  $U(m, \cdot)$  attains its maximum at  $d(m)$  given in Lemma 1(1), set  $\partial U / \partial d = 0$  (i.e.,  $H = m$ ) to obtain  $d(m)$  and observe  $\frac{\partial^2 U}{\partial d^2}(m, d(m)) = -\left(\frac{m}{(1 - m)d}\right)^2 GH < 0$ . Check further that, for any  $m \in M$ ;  $\frac{\partial U}{\partial m}(m, d(m)) = G > 0$ , from which (1) follows.

*ad* (2): The derivative  $U'_m = \frac{\partial U}{\partial d}(\bar{m}, \cdot)$  vanishes at  $d^* = (1 - \alpha)\bar{d}$ , at which point it is strictly decreasing in  $d$ , so that  $U_{\bar{m}}$  attains its unique maximum on  $D$  at  $d^*$ , proving (2).

*ad* (3): For the functions  $G_{\bar{d}}$ ,  $B_{\bar{d}}$ ,  $A_{\bar{d}}$ ,  $H_{\bar{d}}$  denote  $g$ ,  $a$ ,  $b$ ,  $h$ , respectively. Now the derivative  $U'_{\bar{d}} = \frac{\partial U}{\partial m}(\cdot, \bar{d})$  amounts to

$$(23) \quad U_d = \frac{g}{(1 - m)^2} (b - ah).$$

(i) Assume  $d > 1$ . Check that  $\lim_{m \rightarrow 0} U_{\bar{d}}(m) = -(1 - \alpha)\bar{d} < 0$ ,

$\lim_{m \rightarrow 1} U_{\bar{d}}(m) = 0$ , and that

$$(24) \quad U_{\bar{d}}(\bar{m}) = (\bar{m} - (1 - \alpha)\bar{m}^2)g(\bar{m}),$$

which is positive. Thus, the set  $I = \{m \in M \mid U_{\bar{d}}(m) \geq U_{\bar{d}}(\bar{m})\}$  is not only nonempty and (from the continuity of  $U_{\bar{d}}$ ) closed in  $M$ , but also compact in the real line as well as in  $M$ . Hence,  $U_{\bar{d}}$  attains its maximum on  $I$ , i.e.  $\bar{m}$  has an optimal claim of identity with  $m^* \in I$ .

Now check that  $a < 0 < h$  on  $M$ , so that  $U'_d(m) = 0$  if  $m \in \hat{M} = \{m \in M \mid b(m) < 0\}$  and  $h(m) = s(m)$  where the ratio

$$s(m) = \frac{b(m)}{a(m)}$$

is understood to define a function  $s: M \rightarrow R$ . Note that (21) is nothing but the equation  $h = s$ . We have  $b' = a$ ,  $a' = \frac{2(1 - m^2)}{m} > 0$ , and  $s' = \frac{ab' - a'b}{a^2} = 1 - \frac{a'b}{a^2}$ , so that  $s' > 0$  on  $\hat{M}$ . On the other hand,

$$(25) \quad h'(m) = \frac{1 - \bar{m}}{\bar{m}} \left(\frac{1}{1 - m}\right)^2 ah,$$

so that  $h' < 0$  on  $\hat{M}$ . It follows now from the continuity of both  $h$  and  $s$  on  $\hat{M}$  that  $h = s$  at most once on  $\hat{M}$ . Consequently,  $U_d = 0$  at most once on  $M$ , and this completes the proof of (3) (i).

(ii) Assuming  $\bar{d} = 1$ , check that  $a < 0 < b, h$  on  $M$ , so that  $U_d > 0$  on  $M$ .

(iii) Finally, assume  $\bar{d} < 1$ . The existence of an optimal claim  $m^*$  in this case follows as in (i) above, for  $\lim_{m \rightarrow 0} U_d(m) < 0$ ,  $\lim_{m \rightarrow 1} U_d(m) =$

$-\infty$  and  $U_d(\bar{m}) > 0$ , so that the set  $I = \{m \in M \mid U_d(m) \geq 0\}$  is nonempty and compact in both the real line and  $M$ .

Check that  $b, h > 0$  on  $M$ , so that, from (23),  $U'_d(m) = 0$  on  $M$  iff  $m \in \bar{M} = \{m \in M \mid a(m) > 0\}$  and  $h(m) = s(m)$ , where the ratio  $s$  of (i) is understood to define a function  $s: \bar{M} \rightarrow R$ . From the previous paragraph, therefore,  $m^* \in N = I \cap \bar{M}$ . Now since  $U_d = (m - \bar{m}h)g$ , we have  $I = \{m \in M \mid \bar{m}h(m) \leq m\}$ . From (25),  $h'' = \left(\frac{1 - \bar{m}}{\bar{m}}\right)\left(\frac{1}{1 - m}\right)^2 \left[\left(\frac{1 - \bar{m}}{\bar{m}}\right)\left(\frac{1}{1 - m}\right)^2 a^2 + a'\right]h > 0$  on  $M$ , i.e.  $h$

is convex on  $M$ , so that  $I$  is an interval. Furthermore, checking that  $M = (m_0(d), 1)$  shows  $N$  also to be an interval. To show that  $m^* \in N$  is unique, we will demonstrate that  $h = s$ , i.e.  $U_d^\perp = 0$ , at most once in  $N$ .

To this end we first claim that  $s' < 1$  on  $N$ . To verify this claim, note that  $\lim_{m \rightarrow 1} s'(m) = 1$ , and now using the facts  $b' = a$  and  $b > ma$ , obtain

$$s'' = \frac{2(a')^2b - a(aa' + a''b)}{a^3} > -\frac{a''b + aa'}{a^2} > \frac{2b - m^2aa'}{m^2a^2} > \frac{2ma - m^2aa'}{ma^2} = \frac{2m^2}{a} > 0$$

on  $\bar{M}$ , i.e.  $s'$  is increasing on  $\bar{M} \supset N$ . Next we claim that  $h'(\tilde{m}) > 1$  at any  $\tilde{m} \in N$  for which  $h(\tilde{m}) = s(\tilde{m})$ . For, since  $h \leq m/\bar{m}$  on  $I$ , we have  $\frac{1 - \bar{m}}{\bar{m}} \geq \frac{h - m}{m}$  on  $I$ , so that, with substitutions, (25)

yields  $h'(\tilde{m}) > \frac{b(\tilde{m})}{\tilde{m}a(\tilde{m})} = 1 + \frac{(1 - \tilde{m})^2}{\tilde{m}a(\tilde{m})} > 1$ . Now since both  $h$

and  $s$  are continuous on the interval  $N$  and  $s'(\tilde{m}) < 1 < h'(\tilde{m})$ , it follows that there exists at most one  $\tilde{m} \in N$ .

Now denote the lower and upper endpoints of the interval  $I$  by  $\underline{n}$  and  $\bar{n}$ , respectively. Recalling that  $s' < 1$  on  $\bar{M} = (m_0(\bar{d}), 1)$ ,  $h'$  is increasing on  $\bar{M}$  and  $h'(m^*) > 1$ , we can thus conclude that  $h \neq s$  in  $(\bar{n}, 1)$ , i.e. that  $h = s$  exactly once (at  $m^*$ ) in the interval  $(\underline{n}, \bar{n})$ . Recalling further that  $\bar{m} \in I$ , note that if  $m_0(\bar{d}) \geq \bar{m}$  then  $m_0(\bar{d}) = \underline{n} = n$ , leaving nothing further to prove. On the other hand, if  $m_0(\bar{d}) < \bar{m}$ , then  $n > \underline{n}$  implying that  $m^* \in (n, 1) \subset (\underline{n}, 1)$ , for

$$(26) \quad U_d^\perp(\bar{m}) = \frac{g(\bar{m})}{(1 - \bar{m})^2} (\alpha b(m) + (1 - \alpha)(1 - \bar{m})^2)$$

and so  $U_d^\perp(\bar{m}) > 0$ . This completes the proof.

Now, let  $h_{\bar{d}}: M \times M \rightarrow R$  and  $s_{\bar{d}}: M \rightarrow R$  be the expressions on the left- and right-hand side of equation (21), respectively, and define  $f_{\bar{d}} = h_{\bar{d}} - s_{\bar{d}}$ . We will feel free from here on to suppress the subscript  $\bar{d}$ .

*Lemma 2:* For any tenant  $\bar{t} = (\bar{m}, \bar{d})$ , the optimal pretension

$$m_{\bar{d}}(\bar{m}) > \begin{cases} m^1(\bar{d}) & \text{if } \bar{d} > 1 \\ m_1(\bar{d}) & \text{if } \bar{d} < 1, \end{cases}$$

where  $m^1(\bar{d})$  and  $m_1(\bar{d})$  are the (unique) solutions in  $M$  of, respectively, the equations

$$(27) \quad m^2 e^{\frac{1-m^2}{m}} = \bar{d},$$

$$(28) \quad m^2 e^{1-m} = \bar{d}.$$

(see Figure 1.)

*Proof:* Let  $\bar{t} = (\bar{m}, \bar{d})$  be a tenant. First check that of the functions, say  $k(m)$  and  $\bar{k}(m)$ , respectively, on the left-hand side of (27) and (28),  $k$  is decreasing and  $\bar{k}$  increasing on  $M$ . Furthermore,  $\lim_{m \rightarrow 0} k(m) = \infty$ ,  $\lim_{m \rightarrow 0} \bar{k}(m) = 0$  and  $\lim_{m \rightarrow 1} k(m) = \lim_{m \rightarrow 1} \bar{k}(m) = 1$ . Thus,  $m^1$  and  $m_1$  are uniquely defined.

Assume  $\bar{d} > 1$ . Check further in reference to the proof of Lemma 1(3)(i) that  $b(m) < 0$  iff  $m \in (m^1, 1)$  so that  $m^* = m(\bar{m}) > m^1$ .

Now assume  $\bar{d} < 1$ . Check in reference to Lemma 1(3)(iii) that  $m_1 > m_0$ . Since  $m^* \in (m_0, 1)$ , it suffices to show that  $m^* \notin (m_0, m_1]$ . To this end, first check that  $h(\bar{m}, m_1) =$

$$\left(\frac{1-\alpha}{\bar{m}}\right) m_1 e^{\frac{1-m_1}{\bar{m}}}$$

and that  $\frac{\partial h}{\partial \bar{m}}(\bar{m}, m_1) = \frac{m_1 - m}{\bar{m}^2} h(\bar{m}, m_1)$ .

Indeed,  $h(\cdot, m_1)$  attains its maximum at  $\bar{m} = m_1$ , i.e.  $h(\bar{m}, m_1) \leq h(m_1, m_1) = (1 - \alpha)m_1 < 1$ . Furthermore, recalling from the proof of Lemma 1(3)(iii) that  $h(\bar{m}, \cdot)$  is an increasing function of  $m$  for  $m > m_0$ , we infer that  $h(\bar{m}, m) < 1$  for any  $m \in (m_0, m_1]$ . On the other hand,  $s(m_1) = 1$ , and checking now that  $s$  is decreasing on  $(m_0, m_1]$ , we see that  $s(m) > 1$  on  $(m_0, m_1]$ , i.e. that  $f(\bar{m}, m) \neq 0$  for any  $m \in (m_0, m_1]$ . Since  $f(\bar{m}, m^*) = 0$ , we conclude that  $m^* \notin (m_0, m_1]$ , completing the proof.

*Lemma 3:* For any tenant  $\bar{t} = (\bar{m}, \bar{d})$  with  $\bar{d} \neq 1$ ,

$$h_{\bar{d}}(\bar{m}, m_{\bar{d}}(\bar{m})) < 1.$$

*Proof:* Take any tenant  $\bar{t} = (\bar{m}, \bar{d})$  and denote  $m_{\bar{d}}(\bar{m}) = m^*$ .

Assuming  $\bar{d} > 1$ , recall that  $a < 0$  on  $M$ . Since  $b(m^*) = m^*a(m^*) + (1 - m^*)^2 > m^*a(m^*)$ , we have  $h(\bar{m}, m^*) = \frac{b(m^*)}{a(m^*)} < m^* < 1$ .

On the other hand, if  $\bar{d} < 1$ , check in reference to Lemma 2 ( $\bar{d} < 1$ ) above that  $s(m) < 1$  for any  $m \in (m_1, 1)$ .

*Lemma 4:* For any tenant  $\bar{t} = (\bar{m}, \bar{d})$  with  $\bar{d} \neq 1$ , the optimal preten-  
sion  $m_{\bar{d}}: M \rightarrow M$  is differentiable and increasing.

*Proof:* Let  $\bar{t} = (\bar{m}, \bar{d})$  be a tenant with  $\bar{d} \neq 1$  and write  $m(\bar{m}) = m^*$ . Recall from the proofs of (i) and (iii) of Lemma 1(3) that

$$\frac{\partial f}{\partial \bar{m}}(\bar{m}, m^*) = \frac{\partial h}{\partial \bar{m}}(\bar{m}, m^*) - \frac{ds}{d\bar{m}}(m^*) \neq 0,$$

so that

$$\frac{dm}{d\bar{m}}(\bar{m}) = - \frac{\frac{\partial f}{\partial m}(m, m^*)}{\frac{\partial f}{\partial \bar{m}}(\bar{m}, m^*)} \in \mathbb{R},$$

i.e.  $m$  is differentiable at  $\bar{m}$ .

In case  $\bar{d} > 1$ , recall further that  $\frac{\partial f}{\partial \bar{m}}(\bar{m}, m^*) < 0$ , so that  $\frac{dm}{d\bar{m}}(\bar{m})$  has the same sign as  $\frac{\partial f}{\partial m}(\bar{m}, m^*) = \frac{\partial h}{\partial m}(\bar{m}, m^*)$ . Now

$$\frac{\partial h}{\partial m}(\bar{m}, m^*) = - \frac{\text{lng}(m) + \bar{m}}{\bar{m}^2} h(\bar{m}, m^*),$$

so that  $\frac{\partial h}{\partial m}(\bar{m}, m^*) > 0$  for any  $m$  with  $\text{lng}(m) + \bar{m} < 0$ . Check

$\text{lng}(m) + \bar{m} < 0$  for any  $m \in (m^1, 1)$ , e.g. for  $m = m^*$ . Thus,  $\frac{\partial h}{\partial m}(\bar{m}, m^*) > 0$  and therefore  $\frac{dm}{d\bar{m}}(\bar{m}) > 0$ .

On the other hand, if  $\bar{d} < 1$ , recall that in this case  $\frac{\partial f}{\partial \bar{m}}(\bar{m}, m^*) > 0$ , so that  $\frac{dm}{d\bar{m}}(\bar{m})$  now disagrees in sign with  $\frac{\partial h}{\partial m}(\bar{m}, m^*)$ . Check that  $\text{lng}(m) + \bar{m} > 0$  for any  $m \in (m^1, 1)$ , e.g. for  $m = m^*$ , so that  $\frac{\partial h}{\partial m}(\bar{m}, m^*) < 0$  and  $\frac{dm}{d\bar{m}}(\bar{m}) > 0$ . This completes the proof.

*Lemma 5:* Let  $\bar{t} = (\bar{m}, \bar{d})$  be a tenant.

(1) If  $\bar{d} > 1$ , then

$$m_{\bar{d}}(\bar{m}) (\cong) \bar{m} \text{ iff } \bar{m}^2 e^{\frac{(1-m)(1+(2\alpha-1)m)}{\alpha m}} (\cong) \bar{d}.$$

(2) If  $\bar{d} < 1$ , then  $m_{\bar{d}}(\bar{m}) > \bar{m}$ .

*Proof:* *ad* (1): It follows from the proof of Lemma 1(3)(i) that  $m_{\bar{d}}(\bar{m}) \geq (\bar{m})$  when  $U'_{\bar{d}}(\bar{m}) \leq 0$ . The proof now follows by exploiting (26).

*ad* (2): See Lemma 1(3)(iii).

*Proof of Proposition 4* (III):  $U^*/U \geq 1$  in any case. From uniqueness of optimal pretensions (see Proposition 3 (III)), it follows that  $U^*/U = 1$  only for the unpretentious tenants.

(1): Assume  $d > 1$ . Now check that  $W = g - (1 - \alpha)\bar{d}g^{\frac{1}{m}}$ , and

$$(29) \quad \frac{\partial W}{\partial m}(\bar{m}, m) = \frac{g(m)a(m)}{(1 - m)^2} (1 - h(\bar{m}, m)).$$

Recall  $g > 0 > a$ , so  $\frac{\partial h}{\partial m}(m, \bar{m}) < 0$  (see (25)) and  $1 - h(\bar{m}, m)$  is increasing on  $M$ . Hence the set  $\{m \in M \mid \frac{\partial W}{\partial m}(\bar{m}, m) < 0\}$  is an interval. Finally, note from Lemma 3 that  $\frac{\partial W}{\partial m}(\bar{m}, m^*) < 0$ , and (from the fact that  $1 - h(\bar{m}, \bar{m}) = (1 - \alpha)\bar{m} > 0$ ) that  $\frac{\partial W}{\partial m}(\bar{m}, \bar{m}) < 0$ . We therefore have  $W^*(\bar{m}) < W$  iff  $m^*(\bar{m}) < \bar{m}$ . Also, from the fact that  $P^* = (1 - m^*)g(m^*)$  is decreasing on  $M$ , we have  $P^*(\bar{m}) < P$  iff  $m^*(\bar{m}) < \bar{m}$ .

(2): Check that if  $d = 1$  the limiting identity  $(1, 1)$  in this case would yield a value added amounting to  $e^{-2} - (1 - \alpha)e^{-2/\bar{m}}$ . The assertions now follow routinely.

(3): Assume  $\bar{d} < 1$  now. Recalling that  $a > 0$  on  $(m_o(\bar{d}), 1)$  and that  $m^* \in (m_o(\bar{d}), 1)$  (Lemma (3)(iii)), from (29) above we have  $\frac{\partial W}{\partial m}(\bar{m}, m^*) > 0$ . Since  $\bar{m} > m_o(\bar{d})$  by hypothesis, furthermore,

$\frac{\partial W}{\partial m}(\bar{m}, \bar{m}) > 0$  also. On the other hand, check that  $h$  is increasing ( $1 - h$  is decreasing) on  $m_0(\bar{d}, 1)$ , so that  $\{m \in M \mid m > m_0(\bar{d}), W(m, m) > 0\}$  is an interval. Our proposition now follows from the fact that  $m^* > \bar{m}$ .

Finally, check that  $P = (1 - m)g$  is increasing on  $(m_1(\bar{d}), 1)$  and imitate the argument above (end of the proof for (1)) to see our assertion regarding  $P^*/\bar{P}$ .

### Notes

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<sup>1</sup> Equivalently, we could have taken  $y = k^{\alpha}\ell^{\beta}$  and  $u = w - \delta\ell^{\gamma}$  with  $0 < \alpha, \beta < 1 < \gamma$  and  $\delta, \gamma(1 - \alpha) - \beta > 0$ , as in the standard model of Kleindorfer and Sertel [1979] and Sertel [1982]. Transforming this with  $x = \ell^{\beta}$  we obtain our present

$y = k^{\alpha}x$  and  $u = w - \delta x^{\frac{1}{\mu}}$  where  $\mu = \beta/\gamma$ .

<sup>2</sup>  $(\bar{P}, \bar{U}, \bar{W})(\bar{t})$  is the ordinary sharecropping outcome described by Kleindorfer and Sertel [1979].

<sup>3</sup> These bear at least some resemblance to the "metagames" proposed by Howard [1971] and briefly regarded again by Moulin [1981].

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*Abstract:* This paper is an exercise in the design of contracts within a world of two persons, namely a landlord and a tenant, restricted to sharecropping. The landlord sets the shares according to the profit-maximizing "golden rule" ([4]), which depends on certain characteristics of the tenant. The tenant declares these characteristics, and is thus allowed to pretend to be someone else, but must then perform consistently with his pretended identity. This is the idea of a "pretend—but—perform contract". It turns out that legalizing such contracts improves the lot, not only for the tenant, but in certain salient classes of cases, that also of the landlord, as compared to using the true identity of the tenant.