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EXISTENCE AND COMPUTATION OF MATCHING EQUILIBRIA*

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We give a constructive proof of existence of equilibrium in two-sided matching markets and also show the set of equilibrium prices is pathwise connected. For piecewise linear preferences, an (optimal assignment type) algorithm based on these results can compute an equilibrium in a finite number of steps and likewise reach the buyer-optimal minimum equilibrium prices.

1. Introduction

In this paper we give a constructive proof of existence of competitive equilibrium in two-sided matching markets. The same approach shows that the set of equilibrium prices is pathwise connected. We then extract from these results an algorithm for finding equilibria which can be further specified to compute the minimum price equilibrium that is optimal for all buyers individually.

The two-sided matching markets we consider consist of indivisible objects on one side and buyers on the other. The indivisibles can be goods or bads, such as houses or tasks, and it is assumed that buyers are not interested in acquiring more than one object each. An allocation in such a market is given by a matching between the two sides and associated transfers of money.

This model has been shown to have remarkable properties. Notably among these, all core allocations can be achieved by price taking behaviour [Quinzii (1984)], and core payoffs have lattice structure so that there is in particular an outcome best for the buyers/worst for the sellers and a polar opposite one best for the sellers/worst for the buyers [Demange and Gale (1985)]. In fact, buyers canot gain by falsifying their preferences for any procedure selecting the buyers-optimal outcome, a generalized Vickrey-auction property raising hopes for mechanisms which could elicit truthful revelation [Demange and Gale (1985)]. The latter two properties hold also

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for 'ordinal' matching markets where divisible money has no explicit role [Roth (1982), Roth and Sotomayor (1988)].

Underlying these results have been the existence theorems: Quinzii (1984) showed that the matching game is balanced and invoked Scarf's theorem to prove the core is non-empty. Kaneko's (1982) method has been to convexify the model and apply a fixed-point theorem in standard manner. Crawford and Knoer (1981) discretized money to transform the model into the ordinal case, quoted the Gale–Shapley procedure for finding stable matchings, and asserted existence in the limit.

Our existence proof is elementary in that the independent fact we use is duality theorem for optimal assignments: a buyer in the model is represented by utility functions one for each object; given prices he chooses an object yielding maximal utility. (1) To begin, we artificially extend buyers' utility functions in a uniform way for prices below sellers' reservation levels. This we do for the purpose of obtaining a vector of prices which equilibriate (i.e., separate) buyers' choices. Dual solution of a maximal assignment problem (based on buyer utility levels at seller reservation prices) gives such a 'buyer-equilibrium' price vector in the artificial region. (2) The key step in the proof is: any buyer-equilibrium price vector too low for sellers can be made higher. We prove this fact by actually constructing a direction along which prices can be increased and a buyer-equilibrium maintained. (For this we assume utility functions are piecewise linear and lift this assumption later.) Such a direction is in fact supplied by dual solution of a 'multiplicative' optimal assignment problem involving buyers' marginal-utility-for-money evaluations at their most desired objects. Existence follows from (1) and (2).

All this can be turned into an algorithm for piecewise linear markets. Some caution in selecting directions turns out to be sufficient. Then computation takes a finite number of steps and an exact equilibrium is found. After setting up our model in section 2, and proving existence of equilibrium in section 3, we describe this algorithm in section 4.

In section 5 we prove that all equilibrium prices are connected to the minimum price vector by equilibrium price paths having everywhere a non-positive direction. Our interest in this fact apart from geometrical insight is computational in view of the fact we already mentioned that the minimum equilibrium price vector is 'incentive compatible' for buyers. Section 6 is devoted to an algorithm for finding minimum prices by tracing a path from any given equilibrium price vector.

In the last section we have concluding remarks on extensions and related research.

2. The market

A *matching market* consists of a set of buyers I and a set of objects J. No buyer has use for more than one object. Each buyer i has continuous decreasing *utility functions* u_{ij} from R to R where $u_{ij}(m)$ measures his utility in buying object j at price m. Each object j has a *reservation price* s_j.

A *matching* μ is any set of disjoint pairs (i,j). At any *price vector* $p \in R^J$ we define the *maximum utility vector* $q \in R^I$ by

$$q_i = \max u_{ij}(p_j)$$

and call $D(p) = \{(i,j) | u_{ij}(p_j) = q_i \geqq T0\}$ the *demand graph* at p.

A competitive *equilibrium* is a pair (p, μ) where p is a price vector no smaller than s for any object, μ is a matching in $D(p)$, and $p_j = s_j$ for unwanted objects j, $q_i \leqq 0$ for unmatched buyers i.

2.1. Equivalent standard form

We shall assume reservation prices are equal to zero. There is no loss of generality in this because by defining new prices $p' = p - s$ and new utilities $v_{ij}(p'_j) = u_{ij}(p'_j + s_j)$ we would obtain a market with $s = 0$ which is equivalent to the original.

The highest utility buyer i may enjoy by being assigned to object j in any equilibrium is of course $u_{ij}(0)$. We define $a_{ij} = u_{ij}(0)$. It is necessary for our method of proof and computation that a market satisfies

$$a_{ij} \geqq 0, \tag{1}$$

$$u_{ij}(m) = a_{ij} - m \quad \text{for} \quad m \leqq 0, \tag{2}$$

$$I = \{1, \dots, n\} = J. \tag{3}$$

We will say that a market is in *standard form* if it satisfies (1)–(3). The next paragraph shows how any market can be put in standard form without impairing its set of equilibria.

For any $a_{ij} < 0$ we replace u_{ij} by $v_{ij}(m) = -m$. If i and j are matched with each other in an equilibrium (p, σ) of the new market, then p_j and $v_{ij}(p_j)$ are both 0. Hence (p, μ) where μ is σ minus all 'replacements' is also an equilibrium, indeed an equilibrium of the original market. For (2), we simply redefine utilities for negative prices as described. This will not affect equilibria because equilibrium prices are by definition non-negative. To bring about (3), if objects are fewer than buyers we add fictitious objects k defined by $u_{ik}(m) = -m$ for all i, and if buyers are fewer than objects we add fictitious buyers h defined by $u_{hj}(m) = -m$ for all j. Then again the restriction to real buyers/objects of an equilibrium of the extended market is an equilibrium of the original.

We shall distinguish a matching of size n by calling it an *assignment.*

Definition. A price vector p is *competitive* if the 'quasi-demand' graph $Q(p)=\{(i,j)|u_{ij}(p_j)=q_i\}$ contains an assignment and *buyers-rational* if the associated maximal utility vector q is non-negative.

Henceforth we assume markets are in standard form. It follows immediately from (1) and (3) that

Proposition. A price vector is an equilibrium price vector if and only if it is non-negative buyers-rational competitive.

3. Existence of equilibrium

We will prove existence of equilibrium for any matching market in standard form under a mild additional assumption that no buyer is willing to pay an infinite amount for any object, i.e.,

$$u_{ij}(m)<0 \quad \text{for} \quad m \text{ large.} \tag{4}$$

Theorem 1. There exists an equilibrium.

It is clearly sufficient to prove the theorem for the special case where utility functions are (decreasing) piecewise linear because the general statement then follows by uniform approximation. We shall therefore assume that u_{ij} are *piecewise linear.*

Here is our proof in outline: The set of buyers-rational competitive prices P is non-empty (Lemma 1), closed, and bounded above. Furthermore, all competitive prices can be increased in a suitable direction (Lemma 3). Therefore any vector in P whose minimum component is as large as possible is necessarily non-negative and so an equilibrium price vector.

Lemma 1. There exist buyers-rational competitive prices.

Proof. Let μ be a maximal assignment for the assignment matrix $A=(a_{ij})$. By Duality Theorem, there exist dual vectors $q\in R^I$, $p\in R^J$ satisfying

$$q_i+p_j=a_{ij} \qquad \text{for} \quad (i,j)\in\mu,$$

$$q_i+p_j\geqq a_{ij} \qquad \text{for all} \quad (i,j), \quad \text{hence}$$

$$q_i=a_{i\mu(i)}-p_{\mu(i)}\geqq a_{ij}-p_j \qquad \text{for all} \quad i \quad \text{and} \quad j.$$

By adding a suitable constant to all q_i and subtracting it from all p_j, if necessary, we ensure p is non-positive. Then, from (1) q is buyers-rational, and from (2) p is competitive.

We will say that a non-zero vector r is a *competitive direction* for p if $p+\lambda r$

is competitive for all sufficiently small $\lambda \geqq 0$. Our next lemma characterizes non-negative competitive directions.

Given a price vector p, we let c_{ij} denote $-u'_{ij}(p_j)$ where u'_{ij} stands for the right derivative of u_{ij}.

Lemma 2. A non-negative direction r is competitive for p if and only if there is an assignment μ in $Q(p)$ such that

$$c_{i\mu(i)} r_{\mu(i)} \leqq c_{ij} r_j \quad \text{for all} \quad (i,j) \in Q(p). \tag{5}$$

Proof. By piecewise linearity

$$u_{ij}(p_j + \lambda r_j) = u_{ij}(p_j) - \lambda c_{ij} r_j \tag{6}$$

for all (i,j) and $\lambda \geqq 0$ small. For the 'if' part, simply by definition

$$u_{i\mu(i)}(p_{\mu(i)}) = u_{ij}(p_j) \quad \text{for} \quad (i,j) \in Q(p), \tag{7}$$

$$u_{i\mu(i)}(p_{\mu(i)}) > u_{ij}(p_j) \quad \text{for} \quad (i,j) \notin Q(p). \tag{8}$$

Substitution of (7), (8), (5) in (6) gives

$$u_{i\mu(i)}(p_{\mu(i)} + \lambda r_{\mu(i)}) \geqq u_{ij}(p_j + \lambda r_j)$$

for all (i,j) and (so μ belongs to $Q(p+\lambda r)$) for all $\lambda \geqq 0$ small enough. The 'only if' part is similar.

Before going further let us note that any assignment μ for which the inequalities (5) hold (for a direction r) has minimum $\prod c_{i\mu(i)}$, or equivalently minimum $\sum \log c_{i\mu(i)}$, among all assignments in $Q(p)$. [This is easily verified by considering any assignment μ' in $Q(p)$, taking those inequalities in (5) which correspond to pairs (i,j) in μ', and comparing the products on each side.] We now state and prove our key lemma by showing that to any assignment which is minimal in this sense among all assignments in $Q(p)$ there correspond competitive direction(s) for p:

Lemma 3. There exist positive competitive directions for competitive prices.

Proof. Let p be any competitive price vector and define the matrix C_p with entries equal to $\log c_{ij}$ for $(i,j) \in Q(p)$ and a very large number for $(i,j) \notin Q(p)$. Let μ be a minimal assignment for C_p.

By Duality Theorem, there exist dual variables ω_i, ρ_j satisfying

$$\omega_i + \rho_j \leqq \log c_{ij} \quad \text{for} \quad (i,j) \in Q(p),$$

$$\omega_i + \rho_j = \log c_{ij} \quad \text{for} \quad (i,j) \in \mu.$$

Taking exponents, then

$$e_i^{\omega} \leqq c_{ij} e_j^{-p}$$

for all $(i,j) \in Q(p)$ with equality for $(i,j) \in \mu$. Letting $r_j = e_j^{-p}$ and rearranging yields (5). Hence r is competitive for p by Lemma 2.

Proof (*Theorem 1*). The set of buyers-rational competitive prices P is non-empty by Lemma 1, bounded above by (4), and of course closed. So there exists a p in P maximal in the sense that $\min p_j \geqq \min p'_j$ for all p' in P. Let p be such a vector.

If p_j is negative for any j then, from (1) and (2) the corresponding maximal utility vector q is positive, and so by Lemma 3 there exist prices in P strictly greater than p. This is not possible since p is maximal. Hence p is non-negative and so an equilibrium price vector.

4. Computing an equilibrium

For a matching market in standard form with piecewise linear utility functions, our existence proof suggests the following procedure for finding an equilibrium:

Algorithm

(Step 0) Compute p' a non-positive competitive price vector as dual solution to finding a maximal assignment in A.

(Step t) If p^t is non-negative stop, for p^t is an equilibrium price vector.

Otherwise, compute r^t a positive competitive direction for p^t as dual solution to finding a minimal assignment in C_P^t.

Find λ^t the maximum step-size $\lambda \geqq 0$ such that $p^t + \lambda r^t$ is competitive, and call $p^{t+1} = p^t + \lambda^t r^t$.

We have already encountered all ingredients of this algorithm: Computation of p^t, r^t, λ^t is straightforward as in the proofs of Lemmas 1, 3, 2 respectively. Thus Algorithm generates prices which are competitive and buyers-rational [from standardness conditions (1)–(2) and the fact that the price of at least one object is non-positive throughout]. Hence stopping rule holds as asserted. Yet, even though prices are monotonically increasing, the path so generated may actually converge to a point short of the non-negative orthant, as shown for an ordinary 2×2 market in Alkan (1983). In the remainder of this section we show how to slightly strengthen Algorithm so it terminates at an equilibrium in a finite number of steps.

A dual solution for an optimal assignment problem is called *basic* if pairs (i,j) for which dual equations hold form a connected graph. Likewise we shall call competitive prices p *basic* if $Q(p)$ is connected. We define *Basic-price Algorithm* as a refinement of Algorithm in which

(i) prices p^t as well as direction r^t correspond to basic dual solutions,

(ii) step-size λ^t is conditioned on all prices in the linear segment $[p^t, p^{t+1}]$ being basic competitive.

Theorem 2. Basic-price Algorithm finds equilibrium prices in a finite number of steps.

Proof. The algorithm generates a piecewise linear path of basic buyers-rational competitive prices L.

Call a maximal subset of R^n on which all u_{ij} are linear a *linear domain* (a rectangular box of dimension n). Since L is monotone increasing and [from (4)] the set of buyers-rational prices is bounded above, L goes through a finite number of distinct linear domains. To prove the theorem, it therefore suffices to show that L takes a finite number of steps in a linear domain.

For any basic competitive price vector p, there exists a *minimal* connected subgraph S in $Q(p)$ containing an assignment μ which we shall refer to as an *assignment tree*. Thus

$$u_{i\mu(i)}(p_{\mu(i)}) = u_{ij}(p_j) \quad \text{for} \quad (i,j) \in S, \tag{9}$$

$$u_{i\mu(i)}(p_{\mu(i)}) \geqq u_{ij}(p_j) \quad \text{for} \quad (i,j) \notin S. \tag{10}$$

In any linear domain and for any assignment tree, since the system of eqs. (9) has rank $n-1$ and (10) defines a convex set, the set of prices satisfying (9)–(10) is a linear segment. Furthermore, for any step t of L, we can identify a linear segment, say L^t in $[p^t, p^{t+1}]$ and an assignment tree, say S^t, such that (9)–(10) hold for S^t and all $p \in L^t$. We conclude that, for any such (S^t, L^t) and $(S^{t'}, L^{t'})$ associated with two steps $t \neq t'$ in a linear domain, the assignment trees S^t and $S^{t'}$ are distinct (for otherwise L^t and $L^{t'}$ are collinear, so all p intermediate to L^t and $L^{t'}$ also satisfy (9)–(10), contradicting step-size maximality). Thus the number of steps L can take in a linear domain is bounded by the number of assignment trees and so is finite.

5. Equilibrium prices are connected

It is well known that the set of equilibrium prices E has the structure of a lattice [Demange and Gale (1985)]. In this section we show E is pathwise connected.

By the lattice property (and since E is closed and 'non-negative'), there exists a *minimum equilibrium price vector* p^*. At any $p \in E$, we define the set of *devaluable* objects

$$J^* = \{j \mid p_j > p_j^*\}, \quad \text{and let}$$

$$I^* = \{i \mid (i,j) \in D(p) \quad \text{and} \quad j \in J^*\}.$$

We shall denote $J_* = \{j \mid p_j = p_j^*\}$. A key observation is the following.

Lemma 4. For any $p \in E$, $|I^*| = |J^*|$.

Proof. For any $i \in I^*$ there exists by definition $(i,j) \in D(p)$ with $j \in J^*$, so that $u_{ij}(p_j^*) > u_{ij}(p_j) \geqq u_{ik}(p_k) = u_{ik}(p_k^*)$ for all $k \in J_*$, implying $\mu^*(i) \in J^*$ for any assignment μ^* in $D(p^*)$. Hence $|I^*| \leqq |J^*|$. Of course $|J^*| \leqq |I^*|$.

Definition. A non-zero vector r is an *equilibrium direction* for p if $p + \lambda r$ is an equilibrium price vector for all sufficiently small $\lambda \geqq 0$.

Lemma 5. There exist non-positive equilibrium directions for all equilibrium prices other than p^*.

Proof. Let (p, μ) be an equilibrium and u'_{ij} stand for the left derivative of u_{ij}. Define the (square) matrix B_P with an entry b_{ij} for each $(i,j) \in I^* \times J^*$ equalling $\log(-u'_{ij}(p_j))$ if $(i,j) \in D(p)$ and a very negative number otherwise.

Let σ be a maximal matching in $I^* \times J^*$ for the matrix B_P. Then by Duality Theorem, analogously to Lemma 3, there exist positive r_j for every $j \in J^*$ such that

$$b_{i\sigma(i)} r_{\sigma(i)} \geqq b_{ij} r_j \quad \text{for all} \quad (i,j) \in D(p) \cap I^* \times J^*,$$

and so, analogously to Lemma 2,

$$u_{i\sigma(i)}(p_{\sigma(i)} - \lambda r_{\sigma(i)}) \geqq u_{ij}(p_{ij} - \lambda r_{ij}) \tag{11}$$

for all $(i,j) \in I \times J^*$ and sufficiently small $\lambda \geqq 0$.

Extend r to a direction in R^n by defining $r_j = 0$ for all $j \in J_*$. By Lemma 4, $\mu(j) \in I - I^*$ for all $j \in J_*$. So we can extend σ to an assignment by defining $\sigma(j) = \mu(j)$ for all $j \in J_*$. Noting Lemma 4, one easily verifies that the inequalities (11) hold for all (i,j) for $\lambda \geqq 0$ small enough. Thus $(p - \lambda r, \sigma)$ is an equilibrium for all sufficiently small $\lambda \geqq 0$.

Theorem 3. Equilibrium prices are pathwise connected.

Proof. For markets with piecewise linear utility functions, by Lemma 5 all equilibrium prices are connected by piecewise linear paths of equilibrium prices to p^*, hence to each other. The result for general utility functions follows by uniform approximation.

6. Computing minimum equilibrium prices

The minimum equilibrium prices p^* are 'incentive compatible' in the sense that no group of buyers could benefit by misreporting their preferences for a mechanism implementing p^* [Demange and Gale (1985)]. Motivated by this powerful fact and using the geometry in the previous section, we next spell out an algorithm which computes p^* given any equilibrium. The procedure is essentially an adaptation of Basic-price Algorithm for equilibrium-preserving descent in subspaces of the price region.

Our first result is a characterization of devaluable objects by which one can identify J^* by a simple routine in the demand graph $D(p)$. Of course if $p_j=0$ then $j \in J_*$.

Definition. A path of edges (i,j) in $D(p)$ is *alternating* if there is an assignment in $D(p)$ which contains every even-numbered edge on the path. At any $p \in E$, we designate J_0 the set of objects which are connected by an alternating path in $D(p)$ to an object j with $p_j=0$ and write J^0 for its complement $J-J_0$. (Remark: Objects whose prices are zero belong to J_0.)

Lemma 6. At any equilibrium price vector p, $J^*=J^0$.

Proof. Suppose an object in J^* is connected by an alternating path in $D(p)$ to an object whose price is zero. Then there exist two (adjacent) objects on the path, say j and k, such that $j \in J^*$, $k \in J_*$, and $(i,j) \in D(p)$ where $i=\mu(k)$ for an assignment μ in $D(p)$. But this contradicts Lemma 4 so J^* is in J^0.

Conversely, let μ be an assignment in $D(p)$. By definition (trivially), there exists no $(i,j) \in D(p)$ with $j \in J^0$ and $\mu(i) \in J_0$. Hence Lemma 4 holds with J^0 in place of J^* and so does the proof of Lemma 5. Therefore there exist equilibrium prices p' such that $p'_j < p_j$ for $j \in J^0$, establishing J^0 is in J^*.

Upon this characterization Lemma 5 suggests the following procedure for computing p^* starting from any equilibrium prices p: identify J_0 at p and stop if $J_0=J$. Otherwise, find a non-positive equilibrium direction whose negative components correspond to objects in J^0, devalue p in this direction so long as an equilibrium assignment exists and the set J_0 remains unchanged, and repeat until $J_0=J$. As in the case of Algorithm in section 4, this procedure could in general fail to terminate. We show below that it does terminate provided direction selections are 'basic'.

Definition. For any $p \in E$, let $D(p; J^0)$ be the restriction of the demand graph $D(p)$ to objects in J^0. We shall refer to the set of all objects in a maximal connected subgraph of $D(p; J^0)$ as a *devaluable block*. For any devaluable block H at p, we define the set of buyers $I^H = \{i | (i,j) \in D(p) \text{ and } j \in H\}$. [Note from Lemma 4 that $|I^H| = |H|$.]

Minimum Price Algorithm

(Step 0) Let p be any equilibrium price vector and call $p' = p$.

(Step t) If $J_0 = J$ stop, for $p^t = p^*$.

Otherwise, identify a devaluable block of objects H. Define matrix B_p^t with entries b_{ij} (as in the proof of Lemma 5) for $(i,j) \in I^H \times H$ and call β the corresponding maximal matching problem. Construct a non-positive equilibrium direction $-r^t$, with negative components for j iff $j \in H$, by computing a basic dual solution of β.

Find λ^t the maximum step-size $\lambda \geqq 0$ such that $D(p^t - \lambda r^t)$ contains an assignment and H is a devaluable block at $p^t - \lambda r^t$. Call $p^{t+1} = p^t - \lambda^t r^t$.

Remarks. The stopping rule follows from Lemma 6 and it is routine to identify devaluable blocks in a demand graph. For the second paragraph in Step t we refer to the proof of Lemma 5.

For step-size determination: We need to find the smallest $\lambda > 0$ for which a pair (i,j) not in $D(p^t)$ joins $D(p^t - \lambda r^t)$ (then prices for higher λ might not be in E) and for this it is clearly sufficient to check $(i,j) \in I^H \times H$ only. We need also to determine the smallest $\lambda > 0$ such that H is no longer a devaluable block, that is to say the smallest $\lambda > 0$ such that a buyer not in I^H demands an object in H. (The step-size is the smaller of the two λ's.)

Theorem 4. Minimum Price Algorithm finds p^* in a finite number of steps.

Proof. The algorithm generates a piecewise linear path of equilibrium prices each segment of which corresponds to a step and, by monotonicity, descends through a finite sequence of linear domains. In consecutive steps the set of devaluable objects either stays constant or shrinks. Moreover, because equilibrium directions correspond to basic dual solutions, throughout steps in which J^0 is constant, the number of devaluable blocks is non-increasing. Therefore we need only prove that in a linear domain the number of steps in which the devaluable blocks stay constant is bounded above.

But as long as devaluable blocks stay constant in a linear domain, no spanning tree could recur, for otherwise step-size maximality is contradicted by the same argument we gave for Basic-Price Algorithm. So again the number of spanning trees supplies the bound we are after.

7. Concluding remarks

A more general two-sided matching market than ours is the symmetric one in which sellers price discriminate and are characterized by utility functions one for each buyer on the opposite side. Our method of proof easily extends to this general case [Alkan and Gale (1988)] and so would computation.

A related problem is fair allocation of indivisibles: A set of objects and a quantity of money, say X, are to be partitioned among a number of individuals in efficient envy-free manner and such that no individual gets more than one object. But envy-free allocations are none other than competitive equilibria in which the sum of prices equal X and, at least for those cases where there are as many individuals as objects, envy-free implies efficient. By the same constructive approach as in this paper, Alkan, Demange and Gale (1988) show that fair allocations exist and a number of interesting structural properties hold.

We have repeatedly referred to the 'incentive compatibility' property associated with minimum equilibrium prices. Thus buyers may be invited to submit their utility functions for an implementation of this outcome and theory suggests they will without falsifying. Hence computation has a role to play. However one problem associated with this program is the informational burden it would impose on buyers. Perhaps for the case of quasi-linear preferences (i.e., where all utility functions are linear) this may be practicable. But otherwise how does an individual assess his family of utility functions and in what form report it?

In a companion paper [Alkan (1988)], we describe an 'auction-hall' procedure which terminates at the buyers-optimal outcome and which has the advantage that buyers need not make any global assessment regarding their preferences, nor report any quantitative data, but only indicate the objects they want at current prices (and directions) posted by an auctioneer. This procedure involves a (finite) sequence of applications of the Hungarian algorithm. Associated with this approach is an alternate elementary proof of existence for matching equilibria based on Hall's (1935) Theorem alone.

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