DATA ENVELOPMENT ANALYSIS: A TAXONOMY, A META REVIEW AND AN EXTENSION *(CONFIDENT-DEA)* WITH APPLICATION TO PREDICTING CROSS *(OECD)* COUNTRY BANKING SYSTEMS' EFFICIENCY

by
SAID GATTOUFI

Submitted to the Graduate School of Management
in partial fulfillment of
the requirements for the degree of

Doctor of Philosophy

Sabancı University
Fall 2002

DATA ENVELOPMENT ANALYSIS: A TAXONOMY, A META REVIEW AND AN EXTENSION *(CONFIDENT-DEA)* WITH APPLICATION TO PREDICTING CROSS *(OECD)* COUNTRY BANKING SYSTEMS' EFFICIENCY

APPROVED BY:

Professor Muhittin Oral ....
(Dissertation Supervisor)

Professor Ünver Çinar ....

Associate Professor Erdal Erel ....

Professor Arnold Reisman ....

Assistant Professor YunTong Wang ....

DATE OF APPROVAL: ...16...March 2002.

VERİ ZARFLAMA ANALİZİ:
SINIFLANDIRMA, META İNCELEME , GÜVENLİ VERİ ZARFLAMA YÖNTEMİ GELİŞTİRİLMESİ VE BUNUN OECD ÜLKELERİNDE BANKA SİSTEMLERİNİN ETKİNLİĞİNİ TAHMİN UYGULAMASI

Bu çalışma Veri Zarflama Analizi literatürüne üç konuda katkıda bulunmaktadır. Birinci olarak, Charnes- Cooper- Rhodes'in (1978) temel karı en fazlalaştırma zayıf aksiyomuna dayalı firma teorisi modeline analitik yaklaşımla veri zarflamanın köklerini geliştirir. Buna karşın yaklaşık zayıf aksiyom en fazlalaştırma modeli geliştirilmiştir. İlaveten, veri zarflama sonuçlarının örnek büyüklülüğüne duyarlılığı ile yaklaşık zayıf aksiyom en fazlalaştırma arasında direkt bağlantı bulunduğu saptanmıştır. İkinci olarak, bu çalışma bir sınıflandırma önererek veri zarflama literatürünü sistematik bir şekilde gruplandırmaktadır. Reisman (1988 ve 1992) tarafından geliştirilen bir sınıflandırma yöntemi uygulanarak 1995'den sonra makaleleri hakemler tarafından değerlendirilen dergilerde veri zarflama konusunda yayınlanan 989 makalenin içeriği incelenmiştir. Bu sınıflandırmanın sonuçları epistomolojiksel açıdan incelenmiştir. Son olarak, literatüre teorik katkısı bulunan,iki düzeyli konveks optimizasyon modeline dayalı , böylece NP –hard olan, güvenli veri zarflama yöntemi ve bunun çözümü metodu önerilmiştir. Güvenli veri zarflama yöntemi doğru olmayan verileri işleyerek veri zarflama yöntemine bir genelleme oluşturmakta ve böylece tahmin etmeye olanak sağlamaktadır. Cooper ve diğerlerinin (1999) önerdiği tek değerli etkinlik ölçme metodolojisini tamamlayıcı nitelikte olan güvenli veri zarflama yöntemi , örneğin verilerin doğru olmadığını yansıtan etkinlik güvenilirliği aralığı gibi, etkinlik ölçüleri değerleri aralığı sağlamaktadır. Sınırlandırılmış veri durumunda, güven aralığı etkinliği sınırlarını tanımlayan bir teorem verilmiştir. Doğru olmayan verilerin genel durumu için, etkinlik güven aralılığının üst ve alt sınırlarını tanımlamak için genetik algoritmaya dayalı metahüristic ile sağlanmaktadır. Her iki durumda, sınırlandırılmış doğru olmayan verilerine ilişkin aralıkların dağılımı göz önünde bulundurularak etkinlik ölçülerinin dağılımını elde etmek için Monte Carlo tipi simulasyon kullanılmaktadır. Doğru olmayan verileri inceleyen önceki veri zarflama analizi çalışmalarında sorgulamadan (uniform) dağılım varsayılmıştır. Öte yandan, güvenli veri zarflama analizinde her türlü dağılım kullanılabilinmekte ve böylece analizin kapsamı genişletilmektedir. Tanımlama için örneklerde kullanılan sınırlı veriler için (truncated) normal dağılım varsayılmıştır. Son zamanların yöneylem araştırması/yönetim bilimleri yayınlarının büyük bir kısmını karakterize eden "gerçek dünya için önemi" kaygısına kısmen bir tepki olarak, güvenli veri zarflama analizi OECD ülkelerindeki banka sistemlerinin etkinliklerinin tahmininde uygulanmıştır.

Anahtar kelimeler: güvenli veri zarflama analizi, etkinlik güven aralığı, veri zarflama analizi, doğru olmayan veri, sınıflandırma, içerik analizi, genetik algoritma, karı en fazlalaştırma zayıf aksiyomu, banka sistemleri, tahmin, OECD.

DATA ENVELOPMENT ANALYSIS: A TAXONOMY, A META REVIEW AND AN EXTENSION *(CONFIDENT-DEA)* WITH APPLICATION TO PREDICTING CROSS *(OECD)* COUNTRY BANKING SYSTEMS' EFFICIENCY

Abstract

This work contributes to the *Data Envelopment Analysis (DEA)* literature at three ways. **First**, it extends the roots of DEA by providing an analytical approach deriving the basic Charnes-Cooper-Rhodes (1978) model from the *Weak Axiom of Profit Maximization (WAPM)* of Firm Theory. This in turn, develops the *Approximate-Weak Axiom of Profit Maximization (A-WAPM)*. Additionaly, a direct connection is established between the sensitivity of DEA provided results to sample size and the *A-WAPM*. **Second,** this work provides a systematic way for classifying the existing DEA literature by offering a taxonomy. The contents of 989 post-1995 DEA articles in refereed journals are reviewed using a scheme developed by Reisman (1988 and 1992). This scheme analyzes the literature based on the nature of the articles (Theoretical, Application or both e.g. an advance in theory associated with a real world application) and on the basis of the research strategy used by the respective authors. Results of this classification are analyzed from an epistemological point of view. **Finally**, a theoretical contribution to the literature, *Confident-DEA* approach, is proposed involving a *bilevel convex* optimization model, and hence *NP-hard*, to which a solution method is suggested. *Confident-DEA* constitutes a generalization of DEA for dealing with imprecise data and hence allows prediction. Complementing the methodology proposed by Cooper et al (1999) which provides single valued efficiency measures, *Confident-DEA* provides a range of values for the efficiency measures, e.g. an efficiency confidence interval, reflecting the imprecision in data. For the case of bounded data, a theorem defining the bounds of the efficiency confidence interval is provided. For the general case of imprecise data, a Genetic-Algorithm-based metaheuristic is used to determine the upper and lower bounds defining the efficiency confidence interval. In both cases, a Monte-Carlo type simulation is used to determine the distribution of the efficiency measures, taking into account the distribution of the bounded imprecise data over their corresponding intervals. Previous DEA work dealing with imprecise data implicitly assumed a uniform distribution. *Confident-DEA*, on the other hand, allows for any type of distribution and hence expands the scope of the analysis. The bounded data used in illustative examples are assumed to have a truncated normal distribution. In partial reaction to the *anemia in relevance to the real world*, characterizing a large segment of recent OR/MS literature, *Confident-DEA* is applied to predict the efficiency of banking systems in OECD countries.

Keywords: Confident-DEA, Efficiency Confidence Interval, Data Envelopment Analysis, Imprecise Data, Taxonomy, Classification, Content Analysis, Genetic Algorithm, Weak Axiom of Profit Maximization, Banking Systems, Prediction, OECD.

This work is dedicated

First, to my parents Mohamed and Mabrouka Gattoufi who have tremendously supported and encouraged me in all my pursuits throughout life.

Second, to my lovely wife Aicha for her love and support; to Samah, Amira Salsabil and Mohamed Ali who have endured many years of my being away, in another land, to pursue this goal.

Last but certainly not least, to my friend-brother Said Angar for his encouragement and all forms of support.

May all who have helped me find here my gratefullness and recognition.

Acknowledgments

I am deeply indebted to Professors Arnold Reisman and Muhittin Oral. Words can never express the respect I have for them and the gratefullness that I owe them. Their high order of quality as human beings and their high skills as researchers will forever serve as role models for me.

May Professor Mohamed Ali Daouas, Governor of the Central Bank of Tunisia, find here my recognition and gratefulness for having strongly adviced me to go for a Ph.D. program in Turkey.

A special acknowledgement is expressed here to Professor Ünver Çinar for the help, the care, and the continuous encouragement.

My acknowledgments go to professor William Wager Cooper for his encouragement, his comments and for sharing his works, in-progress, and yet to be published; to Professor YunTong Wang who was always an inspiration and especially in the mathematical modeling merging Economic Firm Theory with DEA; to Professors Meltem Denizel, Arman Kosedağ acknowledged for the fruitful discussions and their constructive comments; to Dr Reha Osman Yolalan for his constructive comments, his special care and his repetitive encouragements; to Professor Erdal Erel for his valuable help and guidance in my early attempts at metaheuristics; to Professor Necmi Kemal Avkiran of Ipswich University for his help in banking literature and to Nuri Şendil of Northwestern University for his great help in the computer codes.

Professors Ahmet Fadhel Öncü, Arzu Wasti, Behlül Üsdiken, George Frankfurter, Yıldız Arıkan, Emine Batislam, Dilek Çetindamar, Mine Aksu and Hakan Orbay of The Graduate School of Management at Sabanci University; Professors Mohamed Romdhane, Chokri Ben Amara and Sonia Rebai of Université de Tunis; Professors Kurşat Aydoğan, Erdem Başçi, Savaş Alpay, Syed Mahmud Emre Berk and Yavuz Gunalay from Bilkent University; Professors Attilla Yaprak and Hugh Cannon from Wayne State University are all acknowledged for all kinds of support and help provided and their continuous encouragement.

Lastly, all the valuable help from Mrs Reyhan Doğan, Mrs Ülkü Köknel and Mrs Mine Mut is appreciated and acknowledged.

Any errors in this thesis are clearly mine.

TABLE OF CONTENTS

DATA ENVELOPMENT ANALYSIS: A TAXONOMY, A META REVIEW AND AN EXTENSION *(CONFIDENT-DEA)* WITH APPLICATION TO PREDICTING CROSS *(OECD)* COUNTRY BANKING SYSTEMS' EFFICIENCY

by
SAID GATTOUFI

Submitted to the Graduate School of Management
in partial fulfillment of
the requirements for the degree of

Doctor of Philosophy

Sabancı University
Fall 2002

DATA ENVELOPMENT ANALYSIS: A TAXONOMY, A META REVIEW AND AN EXTENSION *(CONFIDENT-DEA)* WITH APPLICATION TO PREDICTING CROSS *(OECD)* COUNTRY BANKING SYSTEMS' EFFICIENCY

APPROVED BY:

Professor Muhittin Oral (Dissertation Supervisor)	....
Professor Ünver Çinar	....
Associate Professor Erdal Erel	....
Professor Arnold Reisman	....
Assistant Professor YunTong Wang	....

DATE OF APPROVAL: ...16...March 2002.

VERİ ZARFLAMA ANALİZİ:
SINIFLANDIRMA, META İNCELEME , GÜVENLİ VERİ ZARFLAMA YÖNTEMİ GELİŞTİRİLMESİ VE BUNUN OECD ÜLKELERİNDE BANKA SİSTEMLERİNİN ETKİNLİĞİNİ TAHMİN UYGULAMASI

Bu çalışma Veri Zarflama Analizi literatürüne üç konuda katkıda bulunmaktadır. Birinci olarak, Charnes- Cooper- Rhodes'in (1978) temel karı en fazlalaştırma zayıf aksiyomuna dayalı firma teorisi modeline analitik yaklaşımla veri zarflamanın köklerini geliştirir. Buna karşın yaklaşık zayıf aksiyom en fazlalaştırma modeli geliştirilmiştir. İlaveten, veri zarflama sonuçlarının örnek büyüklülüğüne duyarlılığı ile yaklaşık zayıf aksiyom en fazlalaştırma arasında direkt bağlantı bulunduğu saptanmıştır. İkinci olarak, bu çalışma bir sınıflandırma önererek veri zarflama literatürünü sistematik bir şekilde gruplandırmaktadır. Reisman (1988 ve 1992) tarafından geliştirilen bir sınıflandırma yöntemi uygulanarak 1995'den sonra makaleleri hakemler tarafından değerlendirilen dergilerde veri zarflama konusunda yayınlanan 989 makalenin içeriği incelenmiştir. Bu sınıflandırmanın sonuçları epistomolojiksel açıdan incelenmiştir. Son olarak, literatüre teorik katkısı bulunan,iki düzeyli konveks optimizasyon modeline dayalı , böylece NP –hard olan, güvenli veri zarflama yöntemi ve bunun çözümü metodu önerilmiştir. Güvenli veri zarflama yöntemi doğru olmayan verileri işleyerek veri zarflama yöntemine bir genelleme oluşturmakta ve böylece tahmin etmeye olanak sağlamaktadır. Cooper ve diğerlerinin (1999) önerdiği tek değerli etkinlik ölçme metodolojisini tamamlayıcı nitelikte olan güvenli veri zarflama yöntemi , örneğin verilerin doğru olmadığını yansıtan etkinlik güvenilirliği aralığı gibi, etkinlik ölçüleri değerleri aralığı sağlamaktadır. Sınırlandırılmış veri durumunda, güven aralığı etkinliği sınırlarını tanımlayan bir teorem verilmiştir. Doğru olmayan verilerin genel durumu için, etkinlik güven aralılığının üst ve alt sınırlarını tanımlamak için genetik algoritmaya dayalı metahüristic ile sağlanmaktadır. Her iki durumda, sınırlandırılmış doğru olmayan verilerine ilişkin aralıkların dağılımı göz önünde bulundurularak etkinlik ölçülerinin dağılımını elde etmek için Monte Carlo tipi simulasyon kullanılmaktadır. Doğru olmayan verileri inceleyen önceki veri zarflama analizi çalışmalarında sorgulamadan (uniform) dağılım varsayılmıştır. Öte yandan, güvenli veri zarflama analizinde her türlü dağılım kullanılabilinmekte ve böylece analizin kapsamı genişletilmektedir. Tanımlama için örneklerde kullanılan sınırlı veriler için (truncated) normal dağılım varsayılmıştır. Son zamanların yöneylem araştırması/yönetim bilimleri yayınlarının büyük bir kısmını karakterize eden "gerçek dünya için önemi" kaygısına kısmen bir tepki olarak, güvenli veri zarflama analizi OECD ülkelerindeki banka sistemlerinin etkinliklerinin tahmininde uygulanmıştır.

Anahtar kelimeler: güvenli veri zarflama analizi, etkinlik güven aralığı, veri zarflama analizi, doğru olmayan veri, sınıflandırma, içerik analizi, genetik algoritma, karı en fazlalaştırma zayıf aksiyomu, banka sistemleri, tahmin, OECD.

DATA ENVELOPMENT ANALYSIS: A TAXONOMY, A META REVIEW AND AN EXTENSION *(CONFIDENT-DEA)* WITH APPLICATION TO PREDICTING CROSS *(OECD)* COUNTRY BANKING SYSTEMS' EFFICIENCY

Abstract

This work contributes to the *Data Envelopment Analysis (DEA)* literature at three ways. **First**, it extends the roots of DEA by providing an analytical approach deriving the basic Charnes-Cooper-Rhodes (1978) model from the *Weak Axiom of Profit Maximization (WAPM)* of Firm Theory. This in turn, develops the *Approximate-Weak Axiom of Profit Maximization (A-WAPM)*. Additionaly, a direct connection is established between the sensitivity of DEA provided results to sample size and the *A-WAPM*. **Second,** this work provides a systematic way for classifying the existing DEA literature by offering a taxonomy. The contents of 989 post-1995 DEA articles in refereed journals are reviewed using a scheme developed by Reisman (1988 and 1992). This scheme analyzes the literature based on the nature of the articles (Theoretical, Application or both e.g. an advance in theory associated with a real world application) and on the basis of the research strategy used by the respective authors. Results of this classification are analyzed from an epistemological point of view. **Finally**, a theoretical contribution to the literature, *Confident-DEA* approach, is proposed involving a *bilevel convex* optimization model, and hence *NP-hard*, to which a solution method is suggested. *Confident-DEA* constitutes a generalization of DEA for dealing with imprecise data and hence allows prediction. Complementing the methodology proposed by Cooper et al (1999) which provides single valued efficiency measures, *Confident-DEA* provides a range of values for the efficiency measures, e.g. an efficiency confidence interval, reflecting the imprecision in data. For the case of bounded data, a theorem defining the bounds of the efficiency confidence interval is provided. For the general case of imprecise data, a Genetic-Algorithm-based metaheuristic is used to determine the upper and lower bounds defining the efficiency confidence interval. In both cases, a Monte-Carlo type simulation is used to determine the distribution of the efficiency measures, taking into account the distribution of the bounded imprecise data over their corresponding intervals. Previous DEA work dealing with imprecise data implicitly assumed a uniform distribution. *Confident-DEA*, on the other hand, allows for any type of distribution and hence expands the scope of the analysis. The bounded data used in illustative examples are assumed to have a truncated normal distribution. In partial reaction to the *anemia in relevance to the real world*, characterizing a large segment of recent OR/MS literature, *Confident-DEA* is applied to predict the efficiency of banking systems in OECD countries.

Keywords: Confident-DEA, Efficiency Confidence Interval, Data Envelopment Analysis, Imprecise Data, Taxonomy, Classification, Content Analysis, Genetic Algorithm, Weak Axiom of Profit Maximization, Banking Systems, Prediction, OECD.

This work is dedicated

First, to my parents Mohamed and Mabrouka Gattoufi who have tremendously supported and encouraged me in all my pursuits throughout life.

Second, to my lovely wife Aicha for her love and support; to Samah, Amira Salsabil and Mohamed Ali who have endured many years of my being away, in another land, to pursue this goal.

Last but certainly not least, to my friend-brother Said Angar for his encouragement and all forms of support.

May all who have helped me find here my gratefullness and recognition.

Acknowledgments

I am deeply indebted to Professors Arnold Reisman and Muhittin Oral. Words can never express the respect I have for them and the gratefullness that I owe them. Their high order of quality as human beings and their high skills as researchers will forever serve as role models for me.

May Professor Mohamed Ali Daouas, Governor of the Central Bank of Tunisia, find here my recognition and gratefulness for having strongly adviced me to go for a Ph.D. program in Turkey.

A special acknowledgement is expressed here to Professor Ünver Çinar for the help, the care, and the continuous encouragement.

My acknowledgments go to professor William Wager Cooper for his encouragement, his comments and for sharing his works, in-progress, and yet to be published; to Professor YunTong Wang who was always an inspiration and especially in the mathematical modeling merging Economic Firm Theory with DEA; to Professors Meltem Denizel, Arman Kosedağ acknowledged for the fruitful discussions and their constructive comments; to Dr Reha Osman Yolalan for his constructive comments, his special care and his repetitive encouragements; to Professor Erdal Erel for his valuable help and guidance in my early attempts at metaheuristics; to Professor Necmi Kemal Avkiran of Ipswich University for his help in banking literature and to Nuri Şendil of Northwestern University for his great help in the computer codes.

Professors Ahmet Fadhel Öncü, Arzu Wasti, Behlül Üsdiken, George Frankfurter, Yıldız Arıkan, Emine Batislam, Dilek Çetindamar, Mine Aksu and Hakan Orbay of The Graduate School of Management at Sabanci University; Professors Mohamed Romdhane, Chokri Ben Amara and Sonia Rebai of Université de Tunis; Professors Kurşat Aydoğan, Erdem Başçi, Savaş Alpay, Syed Mahmud Emre Berk and Yavuz Gunalay from Bilkent University; Professors Attilla Yaprak and Hugh Cannon from Wayne State University are all acknowledged for all kinds of support and help provided and their continuous encouragement.

Lastly, all the valuable help from Mrs Reyhan Doğan, Mrs Ülkü Köknel and Mrs Mine Mut is appreciated and acknowledged.

Any errors in this thesis are clearly mine.

TABLE OF CONTENTS

CHAPTER 1: GENERAL INTRODUCTION

This work deals with the largely discussed concept of efficiency. There is an agreement in the existing literature, economic as well as OR/MS literature, that the modern measurement of economic efficiency was introduced by Farrell (1957) who drew upon the work of Debreu (1951) and Koopmans (1951) to define a simple measure of firm efficiency

> "Attempts have been made to construct "indices of efficiency", in which a weighted average of inputs is compared with output. These attempts have naturally run into all the usual index number problems. It is the purpose of this paper to provide a satisfactory measure of productive efficiency- one which takes account of all inputs and yet avoids index number problems". (Farrell 1957, pp. 253).

Although the original paper of Farrell "suffers from a central weakness: that he does not analyze the concept of efficiency" (Hall and Winsten 1959, pp. 85), it suggested that there are two interesting approaches to deal with efficiency measurement.

> "Although there are many possibilities two at once suggest themselves- a theoretical function specified by engineers and an empirical function based on the best results observed in practice". (Farrell 1957, pp. 255).

Charnes and Cooper (1985) elaborated on this idea and provide more precision about the concept of efficiency and how it can be dealt with in practice. In the following quotation, the authors provide a definition for the efficiency and justify the necessity for a *"relative"* rather than an *"absolute"* measure of efficiency.

> "Distinction between effectiveness and efficiency need not be emphasized in evaluating private enterprise activities.... We lay aside the more difficult problem of effectiveness and assume that this has been decided in the choice of inputs (resources) to be used and outputs (benefits) to be achieved, as well as the way in which the inputs and outputs are to be measured:
>
> 100% of efficiency is attained for any Decision Making Unit (DMU) only when:
>
> (a) None of its outputs can be increased without either
>   i. increasing one or more of its inputs or
>   ii. decreasing some of its other outputs
>
> (b) None of its inputs can be decreased without either
>   i. decreasing some of its outputs or
>   ii. increasing some of its other inputs
>
> Thus efficiency is represented by the attainment of Pareto (-Koopmans) optimality and conversely.

The above definition is formulated so that efficiency may be determined relative to prior theoretical knowledge. That is such knowledge may be available by reference to available theory as in parts of the natural sciences. It can also be arranged artificially, by design, to test Data Envelopment Analysis and other approaches to efficiency measurement... Such knowledge of true or theoretical efficiency is not available for other situations, however, as is in the Air Force applications reported in the first paper. For such uses, we need to extend the above definition to one which involves only *"relative"* efficiency as determined from the kind of data that are likely to be available.

> 100% relative efficiency is attained by any DMU only when comparisons with other relevant DMUs do not provide evidence of inefficiency in the use of any input or output.

With this characterization, the preceding definition is adjusted for immediate application to data we shall be considering. We should also note, however, that other combinations of the above definitions are also possible so that, in addition, pertinent aspects of any theoretically grounded norms or other types of available knowledge may also be used in common with other data when required." (Charnes and Cooper 1985, pp. 71-72)

An important factor in the popularity of Farrell's work is that it presented a fundamental and useful concept, the efficiency, in a consciously simplified exposition to attract a large audience, and he was successful in that sense.

> "It is hoped that the paper will be of interest to a wide range of economic staticians, businessman and civil servants, many of whom have little knowledge of economic theory or mathematics. .... Similarly, although the treatment of the efficient production function is largely inspired by activity analysis, no reference is made to this in the exposition. The professional economist can easily draw the necessary parallels for himself, as indeed, he can the note the similarity of the measure of *"technical efficiency"* and Debreu's coefficient of resource utilization." (Farrell 1957, pp. 253-254.

The major contribution of Charnes, Cooper and Rhodes (1978) in their CCR model, the publication of which is largely considered as the formal birth of *Data Envelopment Analysis (DEA)*, was that they presented the efficiency calculation problem "in a stringent mathematical form more readily understood and absorbed by the research community" (Forsund, 1999). More than twenty years after, the CCR model remains the central model in the DEA literature.

The state of the art in DEA has, after a relatively short incubation period, grown rapidly covering a wide range of fields and domains and appearing in a large number of scholar journals. Gattoufi et al (2001) documented that the number of DEA-articles in refereed journals stands, by August 2001, at 1809 appeared in 490 different journals. For the year 1999 only, 104 refereed journals are known to have published DEA article(s). This spectacular success of DEA indicates its large success in capturing new researcher, expanding to cover new fields and areas and producing continuously improved paradigms to deal with the efficiency in real world.

However, researchers in DEA should be aware of the danger and prevent *"the son from committing the father's sin"* by staying away from *"the swamp of relevance (Miser 1987)"* and not developing the *"anemia in relevance to real world"*.

This work provides contributions to the DEA literature at three levels: reviewing the state of the art, developing theoretical advances and applying those developments in real world context. The remainder of this document is organized in seven chapters.

In chapter 2, a brief presentation of the original CCR model is provided. The connection to economic theory is explained and the alternative methods to measure the efficiency are briefly discussed. After presenting an illustrative example, recent developments in DEA with dealing with imprecise data are presented and discussed. Finally, the application of DEA to analyze cross-country efficiency in banking is discussed emphasizing on its high relevance in the new globalized economy and conclusions about relevant directions for research are drawn.

Chapter 3 presents the results of the meta review of the DEA literature. It discusses epistemological issues in DEA, provides content analysis of the 989 articles representing the full census of post-1996 DEA articles that appeared in refereed journals. The content analysis is made using a scheme developed in Reisman (1989) and applied in previous meta reviews. A special focus was made on the US based OR/MS flagship journals. Because what is called here *"anemia in relevance to real world"*, DEA were less successful than Game Theory in terms of publication in these journals.

In chapter 4, a taxonomy for DEA literature is developed. The time is judged ripe for a general mapping of this literature in a manner that will provide a vivid and panoramic view of what exists and will clearly identify any existing gaps in the state of the art. Taxonomy is not only a tool for systematic storage of knowledge but it is also a neat way of pointing to knowledge expansion and building. It identifies voids, potential theoretical increments or developments and potential applications for the existing theory. Selected articles, judged representative of the DEA literature, are then classified based on the proposed taxonomy to show its ability to reflect the subtlety.

Chapter 5 starts the theoretical contribution section containing also Chapter 6 and Chapter 7. Efficiency measures for *Decision Making Units (DMU's)* provided by the original CCR-DEA model (1978), are derived in Chapter 5 as a natural extension of the

Weak Axiom of Profit Maximization (WAPM) (Varian, 1992) and the CCR is analytically justified based on this axiom. The *Approximate-Weak Axiom of Profit Maximization* is introduced as a *new concept* to analytically prove the derivation. Additionally, the *sensitivity* of the efficiency measures to increasing numbers of DMUs in the CCR model is treated analytically, a theorem is provided and an approach to define a *"parsimonious"* sample in an efficiency analysis study is suggested for additional research.

Chapter 6 and Chapter 7 develop a new methodology, it is given the name *Confident-DEA*, to deal with what is called in Cooper et al (1999) *imprecise data*. In Chapter 6, the case of cardinal data taking either the form of single valued or range of values is considered. In this case, believing that the imprecision in data should be reflected in the efficiency measure, the proof of a theorem that gives a rule for determining the upper and lower bound for the efficiency is provided, a kind of confidence interval for the efficiency coefficient of each DMU, hence the name *Confident-DEA*. The theorem invoked two new concepts called here the *"optimistic point of view"* and the *"pessimistic point of view"* of the DMU considered at each iteration of the efficiency measure computation. Once the range for the efficiency is determined, a Monte-Carlo simulation based method is suggested to determine the distribution of the efficiency coefficients over the confidence interval. It is important to emphasize that, unlike the method proposed in Cooper et al (1999) that provides single valued efficiency measure and implicitly assume uniform distribution over their range for the bounded data, the method proposed allows for any distribution for the bounded data. The simulation component proposes also benchmarks, in terms of inputs and outputs, for any DMU considered in the analysis and any desired level of efficiency included in the efficiency confidence interval. By doing so, stochastic component is added to the efficiency analysis with bounded data.

Chapter 7 goes further with imprecise data and extends Confident-DEA to allow for including *ordinal* data in the analysis. The basic model is formulated as a "bi-level linear program" hence an NP-hard problem. This justifies developing a heuristic solving method. The meta heuristic proposed uses "*genetic algorithm approach*". The standard genetic algorithm approach uses a single string of *genes*, a *chromosome*, to represent an *individual*. The meta heuristic developed in this chapter uses a panel of chromosomes to represent an individual. Unlike in the standard approach where a uni-dimensional crossover genetic modification is possible, the meta heuristic proposed here allows for bi-dimensional cutting for the crossover. This itself constitutes an important contribution to genetic algorithm theory, beside its utility in solving a bi-level linear program in general and the *Confident-DEA* model in particular.

After defining a *"satisfactory solution"* for the *Confident DEA* model that defines the bounds for the confidence efficiency interval, a Monte Carlo simulation is performed like the one described for Chapter 6 in order to define a distribution for the efficiency coefficients over their corresponding efficiency intervals. Illustrative example replicating the study in Cooper et al (forthcoming) is provided and the results obtained are found to be consistent with those determined in Cooper et al (forthcoming).

Aware enough of the *"anemia in relevance to real world"*, the study ends on an application to real world data by using *Confident-DEA* in Chapter 8 to forecast cross-country technical efficiency in OECD countries. This chapter provides forecasts and compares performance of banking systems of different countries for a single year, 1998, based on data describing the banking activities for the previous years. We proceed to assess the efficiency for 1998 in two ways: *Confident-DEA* and using observed realized data. For the former, the production factors data for the year 1998 are forecasted first using time series regression to define a forecasted confidence interval for each production factor. Considering those confidence interval as *imprecise* data (bounded factors more precisely), *Confident-DEA* is used to assess the efficiency of each banking system. The results obtained are compared to those obtained based on realizations using standard DEA. Ranking of countries based on the performance of their banking systems is provided and interesting conclusions are drawn.

In the final Chapter 9, general conclusions are drawn, limitations of what was presented are indicated and directions for future research are proposed.

CHAPTER 2: EFFICIENCY ANALYSIS USING DATA ENVELOPMENT ANALYSIS AND ITS APPLICATION IN BANKING

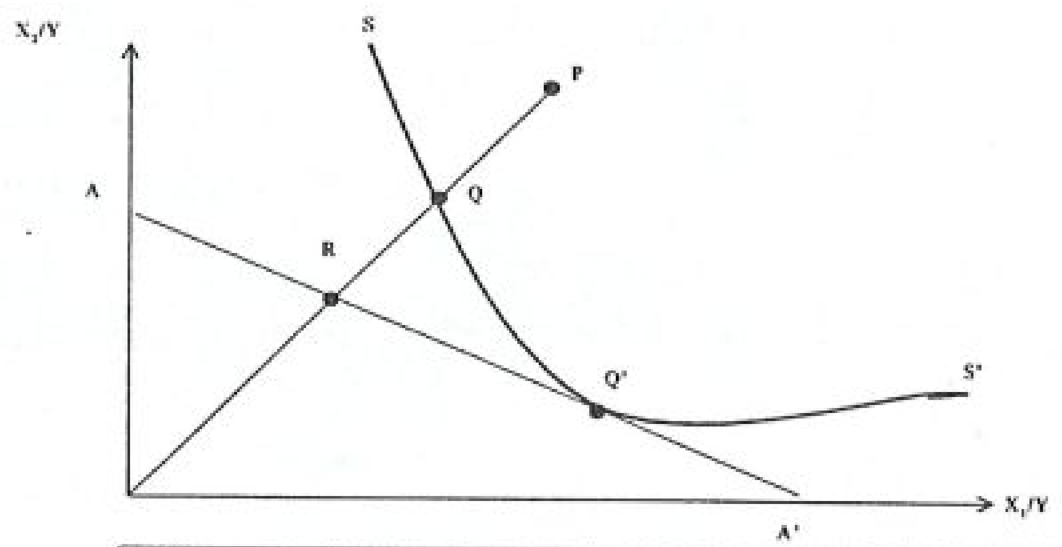
2.1. Efficiency of Economic Units as a Basic Concept for the Efficiency Analysis

The concept of efficiency is a central concept in economic theory and full efficiency is defined as the attainment of Pareto optimality (Koopmans 1951). The efficiency reflects the degree of goodness with which the economic units are performing their objectives. This raises issues of measuring efficiency and whether, indeed, an absolute measure of efficiency does exist. Charnes and Cooper (1985) suggested that efficiency measures are determined relative to prior theoretical knowledge about DMU performance or arranged artificially using DEA to produce relative efficiency measures rather than absolute ones.

These issues were widely discussed in the literature before the Charnes, Cooper and Rhodes (1978) paper. There is an agreement in the existing literature, see for example Coelli (1996), that the modern measurement of economic efficiency was introduced by Farrell (1957) who drew upon the work of Debreu (1951) and Koopmans (1951) to define a simple measure of firm efficiency. He proposed that the *economic efficiency* of a firm is a combination of its *technical efficiency*, which reflects its ability to obtain the maximal outputs from a given quantity of inputs, and its *allocative efficiency*, which reflects its ability to use inputs in optimal proportion given their respective prices.

In order to determine efficiency measures for the firms, Farrell (1957) proposes to first identify an assumed existing *efficient frontier* using the production function. Deviations from the efficient frontier have a natural interpretation as a measure of the *inefficiency* with which economic units, or firms, pursue their technical or behavioral objectives.

Figure 2.1 represents the efficient technical and allocative frontiers. The economic units are competing in the same market using two inputs to produce a single output. The isoquant SS', determined by a specific function, represents the best combinations of inputs to produce a single unit of output. This curve represents the technical efficient frontier. When the prices of inputs are considered, the line AA' represents the allocative efficient frontier. P, Q and Q' represent the level of inputs used by three different economic units to produce a single unit of output. Unit Q and unit Q' are two technically efficient units but the unit Q lacks allocative efficiency since it is not on the segment AA' defining the allocative efficient frontier and this means that unit Q can reduce its cost and any input reduction is impossible for both. The economic unit P is neither allocative nor technical efficient and this means that a single unit of output can be produced by the economic unit P cheaper and with less quantity of inputs. Q' is economically efficient since it is technically and allocatively efficient.



AA': Allocative efficient frontier
SS': Technical efficient frontier
X_i/Y: Quantity of input *i* needed to produce a unit of output.

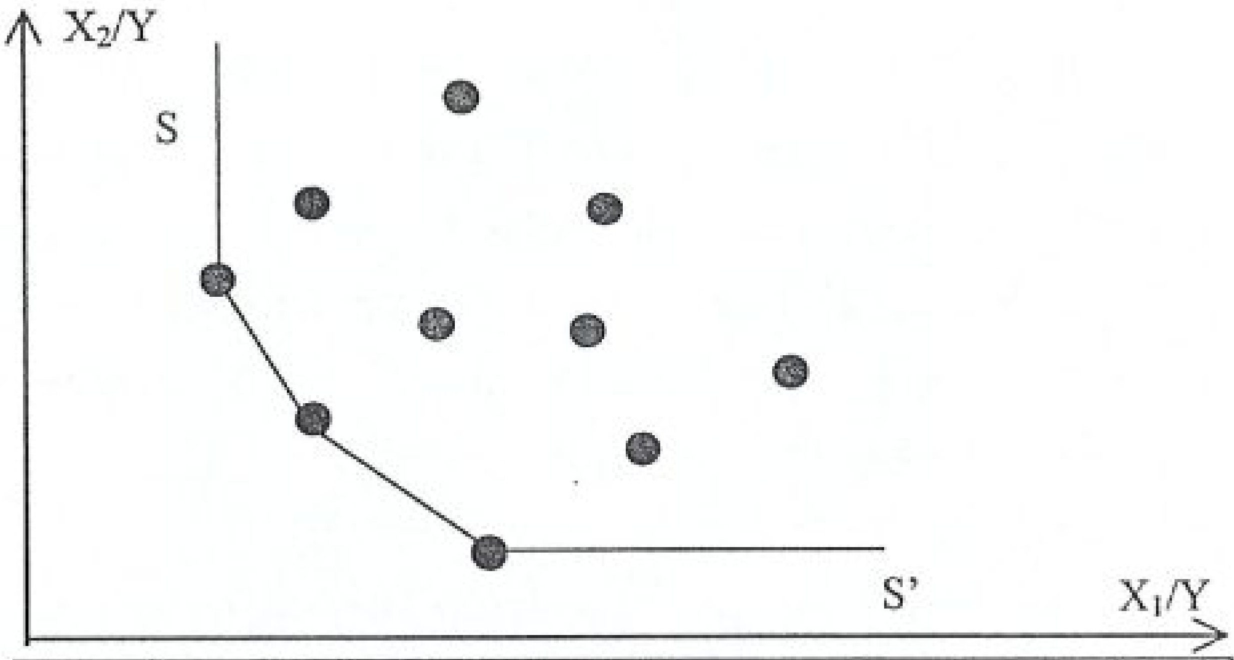
Figure 2.1: Technical and allocative input oriented efficient frontiers (Adapted form Farrell 1957)

In real life, the function defining the efficient frontier is often not known explicitly and has to be approximated in order to identify the efficient frontier. Although the concept of efficiency is often associated with production, cost and profit are also considered in the existing related literature.

Farrell (1957) suggested the use of either **(i)** a *non-parametric* piecewise linear convex form or **(ii)** a *parametric* function to determine the efficient frontier. DEA belongs to the first class of methods while mainly altered forms of Cobb-Douglas function constitute the second class.

The efficiency analysis can be either *output oriented* or *input oriented*. The first orientation determines, for each unit, the maximum quantity of output(s) that can be produced from a given quantity of input(s). In the second orientation, the quantity of output(s) is fixed and the minimum quantity of input(s) used to obtain the fixed level of output is determined. Deviation from the optimal situation is interpreted as inefficiency.

Figure 2.2 illustrates an input oriented piecewise linear efficient frontier. The economic units considered are competing in the same market to produce a single output using two inputs.



SS': Piecewise linear input oriented efficient frontier
X_i/Y: Quantity of input *i* needed to produce a unit of output

Figure 2.2: Piecewise linear input oriented efficient frontier

To sharpen the analysis, *returns to scale* can be taken into account. A production system is said to be under *constant return to scale* (CRS) if the proportional variations in outputs and outputs are the same. If the proportional variations are not equal, it is a situation of *variable return to scale*. Two situations are identified: **(i)** *an increasing return to scale* (IRS) corresponds to a situation where the proportional variation in inputs generates higher proportional variation in outputs and **(ii)** *a non increasing return to scale* (DRS) where at the opposite, the proportional variation in outputs is lower.

2.2. Parametric and Non-Parametric Approaches to Efficiency Analysis

Research on efficiency has developed, as suggested by Farrell (1957), mainly within two streams: parametric approaches and non-parametric approaches.

The parametric approaches specify an explicit functional form for the efficient frontier. This function is assumed to be reflecting the production process. This function, usually an altered form of Cobb-Douglas function, reflects the relationships between the inputs and the outputs involved in the production process. An econometric model is used to estimate the parameters of the underlying function. This model has the particularity of having an error term with two components, unlike the classical models. The first component of the error term, a white noise, reflects the classical randomness of econometric models. The second component reflects the one sided deviation from the efficient frontier. Different functional forms of the latter component of the error term are proposed in the literature. See for example Battese and Battese (1992), Battese and Coelli (1995) and Coelli (1996) for the software implementing their model.

The non-parametric approach, mainly Data Envelopment Analysis, its derivations and extensions, is a mathematical programming based methodology for efficiency analysis. The formulation of the problem leads to a linear program with an objective function reflecting the best efficiency level that the economic unit being evaluated can reach. The constraints of the linear program define a piecewise-linear frontier of a convex simplex that forms the *efficient frontier*.

2.2.1. Data Envelopment Analysis - Mathematical Model

The standard Data Envelopment Analysis was originally developed by Charnes, Cooper and Rhodes (1978) as a methodology to measure the relative efficiency of a homogeneous set of firms, called *Decision Making Units (DMU),* competing in the same market. The set of the best performers among them contributes to the definition of the *efficient frontier*. This frontier is defined as a convex combination of the best performers, considered as fully efficient since they are located on the efficient frontier. Deviation from the efficient frontier is interpreted as the measure of the inefficiency for the remaining DMUs. A virtual efficient target for each DMU is identified by projection on the efficient frontier. The ratio of the radial distance of the virtual efficient target to the radial distance of the corresponding DMU is defined to be the efficiency measure. In other words, less technical and more concrete, the efficiency of a given DMU is measured, in an input oriented DEA, by comparing the inputs it needs to those needed by the most efficient virtual *DMU* to produce an equivalent output. Conventionally, a fully efficient DMU is given *1* (unity) as measure of efficiency and all efficiency coefficients have non-zero value.

The original DEA model, the ratio form, defines for each DMU the best virtual target and the efficiency measure is defined as maximum ratio of the radial distance of the virtual target to the radial distance of the DMU being studied. The mathematical ratio form of the model is as follows:

$$\max_{u,v} h_o = \frac{\sum_{r=1}^{r=s} u_r y_{ro}}{\sum_{i=1}^{i=m} v_i x_{io}} \qquad (2.1.1)$$

subject to:

$$\frac{\sum_{r=1}^{r=s} u_r y_{rj}}{\sum_{i=1}^{i=m} v_i x_{ij}} \leq 1; j = 1,2,\ldots,n \qquad (2.1.2) \qquad\qquad (2.1)$$

$$v_i, u_r \geq \varepsilon \qquad (2.1.3)$$

Where $\varepsilon > 0$ *is a non-Archimedien value*

This model considers a set of ***n*** DMUs competing in the same market. The production process requires a set of ***m*** inputs to produce ***s*** outputs. The subscript '*o*' is generic taking any value 'o'=1,2,…,n. It refers to the DMU being evaluated, called the *base-DMU.*

The objective function described by Equation *(2.1.1)* measures the efficiency of the base-DMU via the maximum value of its ratio. The optimal value of the objective function is obtained through the generation of a virtual input, (a combination of all real inputs), and a virtual output, (a combination of all real outputs). Moreover, the relative weights given to the real factors (inputs and outputs) in the definition of the optimal virtual input and output are represented by the coefficients obtained at the optimum. The decision variables are in fact the coefficients ***(u,v)*** for the virtual factors.

The $m * n$ matrix x specifies the data regarding the inputs: x_{ij} is the quantity of input *(i)* used by DMU *(j)* in the production process. Correspondingly, the $s * n$ matrix y specifies the data regarding the outputs: y_{rj} is the quantity of output *(r)* produced by DMU *(j)* in the production process. The set of constraints in *(2.1.2)* states that all efficiency coefficients are constrained to be unity (1) or less, the value normalized for full efficiency. The classical non-negativity condition is replaced by condition *(2.1.3)* involving a *non-Archimedian* value. To ensure that the variables of the models, the weights for inputs and outputs, are accorded some worth, they are constrained to be not only positive but also greater than any positive real number. This is obtained by considering as lower bound for these variables a non-Archimedian infinitesimal smaller than any positive real number Ali and Seiford (1993a, 1993b) and Cooper (2000) provides further discussion about the subject.

The main drawback with this form is its unboundedness since if *(u,v)* is a solution, *(αu, αv)* are solutions as well. The fractional linear programming suggests considering a representative from each class of solutions. This can be obtained by normalizing and assess that $\sum_{i=1}^{i=m} \omega_i x_{io} = 1$, and though transforming the original model into a following linear program, called *multiplier form*:

$$\begin{aligned}
&\max_{\mu,\nu} h_o = \sum_{r=1}^{r=s} \mu_r y_{ro} && (2.2.1) \\
&s.t.: && \\
&\sum_{r=1}^{r=s} \mu_r y_{rj} - \sum_{i=1}^{i=m} \omega_i x_{ij} \le 0; j = 1,2,\ldots,n && (2.2.2) \\
&\sum_{i=1}^{i=m} \omega_i x_{io} = 1 && (2.2.3) \\
&\omega_i \ge \varepsilon \; \mu_r \ge \varepsilon && (2.2.4) \\
&\text{where } \varepsilon > 0 \text{ non-Archimedian} &&
\end{aligned} \qquad (2.2)$$

In this form, the shift in notation reflects nothing but the change in the form. Constraint *(2.3.3)* is a normalizing condition relative to the base-DMU.

2.2.2. An Illustrative Example:

Consider for illustrating the input oriented DEA approach to efficiency analysis of economic units an example adapted from Coelli (1996) consisting of a set of 5 DMUs competing in the same market using 2 inputs to produce a single output. *Table 2.1* contains the data about the production process.

Table 2.1: Data for an example illustrating DEA

DMU	Y	X1	X2	X1/Y	X2/Y
1	1	2	5	2	5
2	2	2	4	1	2
3	3	6	6	2	2
4	1	3	2	3	2
5	2	6	2	3	1

DMU3, for example, uses 6 units of input 1 and 6 units of input 2 to produce 3 units of output. That is three units of each input are needed for DMU3 to produce one unit of output. The multiplier form of the DEA model for DMU3 and its dual are as follows:

Multiplier Form

$$\max_{\mu\upsilon} h_3 = 3\mu$$

$s.t.$

$$\mu - 2\upsilon_1 - 5\upsilon_2 \le 0$$
$$2\mu - 2\upsilon_1 - 4\upsilon_2 \le 0$$
$$3\mu - 6\upsilon_1 - 6\upsilon_2 \le 0$$
$$\mu - 3\upsilon_1 - 3\upsilon_2 \le 0$$
$$2\mu - 6\upsilon_1 - 2\upsilon_2 \le 0$$
$$6\upsilon_1 + 6\upsilon_2 = 1$$
$$\upsilon_1, \upsilon_2 \ge \varepsilon$$

$\varepsilon > 0$ *Non Archi median*

Dual Form

$$\min_{\lambda,\theta} \theta$$
$$\lambda_1 + 2\lambda_2 + 3\lambda_3 + \lambda_4 + 2\lambda_5 \ge 3$$
$$6\theta - (2\lambda_1 + 2\lambda_2 + 6\lambda_3 + 3\lambda_4 + 6\lambda_5) \ge 0$$
$$6\theta - (5\lambda_1 + 4\lambda_2 + 6\lambda_3 + 2\lambda_4 + 2\lambda_5) \ge 0$$
$$\lambda \ge 0, \theta \ge 0$$

The solution for all the DMUs obtained from the DEA modeling is given in the following *Table 2 2:*

Table 2.2: Solution obtained from the DEA model

DMU	θ	λ1	λ2	λ3	λ4	λ5	IS1	IS2
1	0.5	.	0.5	.	.	.	.	0.5
2	1	.	1	.	.	.	.	.
3	0.833	.	1	.	.	0.5	.	.
4	0.714	.	0.214	.	.	0.286	.	.
5	1	.	.	.	.	1	.	.

The solution in *Table 2.2* suggests in the first column the efficiency coefficient for each DMU. The last two columns give the slacks. The remaining columns give the coefficient of the convex combination target for each one of the non-efficient DMUs. These results can be better seen by reference to *Figure 2.3* in which the efficient frontier and the location of the virtual targets for non-efficient DMUs are illustrated.

DMU2 and DMU5 are fully efficient and appear on the efficient frontier in *Figure2.3.* DMU1, DMU3 and DMU4 are not efficient and their virtual target are respectively DMU1', DMU3' and DMU4'. DMU3' and DMU4' are convex combination of efficient DMUs. The corresponding efficient DMUs in the convex combination of each DMU are called *peers*. The peers for both DMU3 and DMU4 are DMU2 and DMU5. DMU1 is an exception since it has only one peer and its target is located on the part of the efficient frontier parallel to one of the axis. This indicates the existence of a slack in the corresponding input. This means that an additional reduction of the corresponding input can be made. The concept of slack is one of the major difficulties of DEA. See for example Coelli (1996) and Norman and Stoker (1991) for further explanations.

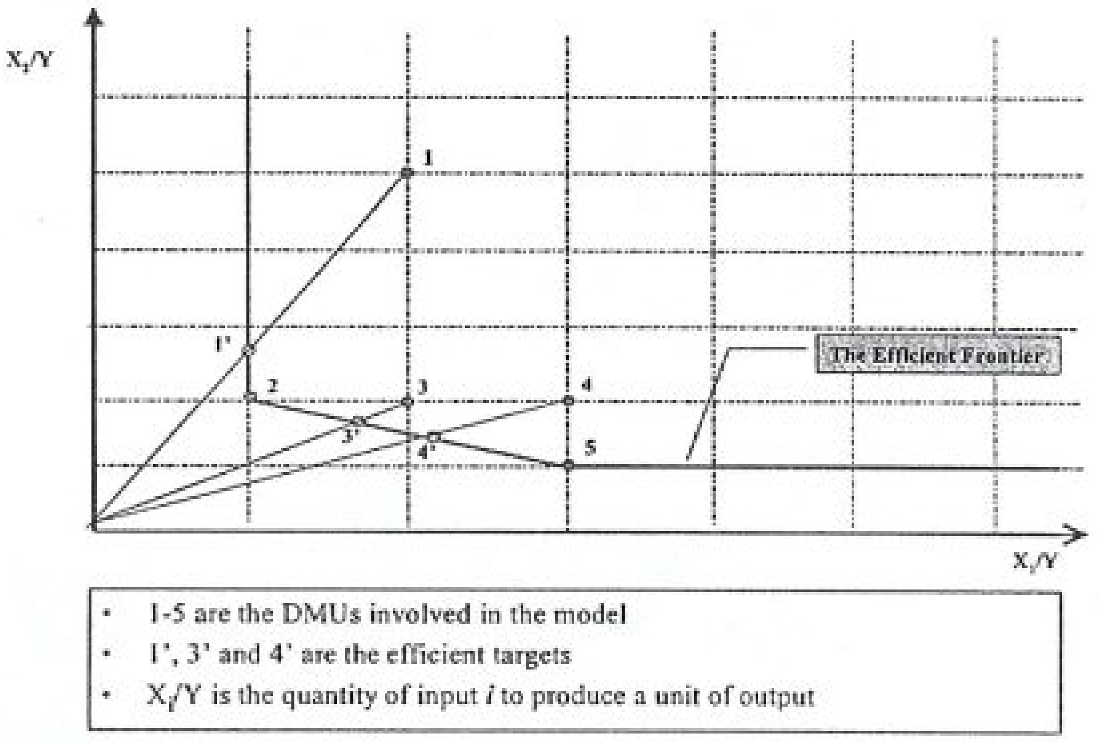


Figure 2.3: Efficient frontier and targets

2.2.3. DEA and its Relevance to the Real World:

It is important to emphasize that the non-parametric approach produces relative measures of efficiency. Any change in the composition of the set of DMUs studied affects these measures. However, adding or dropping some DMUs is often used to test the robustness of the ranking obtained based on the efficiency measures.

The DEA, non-parametric approach, has the advantage that no explicit functional form needs to be imposed. Also, the fact that the non-parametric approach is an extreme values method of analysis, although seen as disadvantage by some of the literature, we believe is an advantage. In a severe competition environment, managers who benchmark see the extreme values analysis as an advantage on the best performing rather than the average performing competitors.

However, the main weaknesses of the non-parametric approach are *(i)* its deterministic nature and *(ii)* its high sensitivity to the number of DMUs, as observations, and the number of factors (inputs and outputs) as parameters of the model. The larger the difference is in favour of DMUs, the more reliable is the efficiency measure evaluation.

Despite its highly appreciated stochastic nature, the parametric approach has a constraint of imposing an explicit form for the function defining the efficient frontier. This implies that an adequate sample size is needed to derive reliable statistical inference for the parameters.

Regardless of the approach, the objective is to define an accurate frontier. However, since the two approaches use significantly different techniques, having their own strengths and weaknesses, it is most unlikely that they produce consistent results.

As a final remark, we believe that non-parametric approaches, and parametric as well, for efficiency analysis are not only techniques. Oral et al (1992) wrote about these approaches:

> "they represent new approaches of analysis designed to evaluate the operational performances of economic units. They can be used as complementary to the financial ratios analysis for tactical and strategic purposes. The use of both types of measures, those obtained from financial ratios and those obtained from non-parametric approaches, makes the evaluation process of organizational performance more comprehensive and complete." Oral et al 1992, pp.

Particularly, Oral et al (1990) stressed the importance of these alternative tools and their considerable relevance for the understanding of the operating performance of banking institutions and by means of consequence their high potential benefits in improving the performance of banks.

2.3. Data Envelopment Analysis with Imprecise Data

In the DEA approach, DMUs are considered as economic units using inputs to produce outputs. Although the inputs and outputs are usually assumed to be observable and measurable, in many real life situations these factors are not precisely known except *(i)* to the extent that the true values lie within prescribed bounds, and *(ii)* to satisfy certain ordinal relations. Cooper et al (1999) refers to such kind of data as *imprecise data*. We henceforth use the same terminology.

In a situation where data are imprecise, the application of the standard DEA leads to a non-linear program and the piecewise linear efficient frontier defined by the standard DEA approach is not guaranteed. Also, ordinal data cannot be considered in the standard model.

The early literature dealing with imprecise data was simply devoted to extend the standard DEA for coping with ordinal data. Golany (1988) presented a model incorporating ordinal relations among the weights of the DEA model. Cook et al (1993) presented a framework for incorporating a single input within the standard DEA framework. In a follow-up work, Cook et al (1996) extended their framework to the case where more than one factor is ordinal. Kim et al (1999) developed a procedure for handling both ordinal data and weights preferences.

Recently, Cooper et al (1999) developed a unified approach to treating mixtures involving bounded data in addition to ordinal data and ordinal relations among the weights. Their approach, the *Imprecise Data Envelopment Analysis* (*IDEA*), extends the standard DEA to cope with imprecise data. In a following-up work, Cooper et al (forthcoming) presented an illustrative application of their unified approach. Formulating the basic DEA model using imprecise data leads to a *non-linear optimization problem*. For the linearization, IDEA proceeds in two steps, scale transformations followed by variable alterations. The transformed model has the form of a standard DEA model. The solution for the original model is obtained from that of the transformed model using the reverse variable alterations and scale transformations.

A common criticism in this respect is that these approaches do not reflect explicitly the imprecision of the data within the of efficiency coefficients provided. That is, the efficiency measure obtained for each DMU is single-valued regardless the data are single-valued or imprecise.

Three major critics can be addressed as regard to the *IDEA*. First, the authors, in order to linearize the model obtained from the application of standard DEA to imprecise data, transformed the status of data to variables. That is, the authors consider the factors of data not precisely known as variables. This leads to an optimization problem where they decide about data as well as about variables. The basic Operations Research methodology requires a clear identification and separation between the decision variables, object of decision for the optimal level they should have, and the parameters represented by the coefficient defined by the data of the problem.

Second, for a variable defined as having bounded data, the *IDEA* approach requires that for the DMU used as anchor for the scale transformation and variable alteration, that is the DMU with the highest range for the corresponding bounded variable, the range is transformed into a single-valued. If this "*approximation*" is not made, the reverse transformations to retrieve the solution for the original problem can not be performed. This reduces some of the generality of the *IDEA* approach. However, this was corrected in Cooper et al (2000) by introducing dummy DMUs in the analysis.

Finally, the major critic is conceptual in nature. The problem with the existing literature dealing with imprecise data is *the derivation of **single-valued** measures from **imprecise multi-valued** data*. The efficiency measure should reflect the imprecision in data and a range of values for the efficiency measure is more appropriate than a single-value. This range can be considered as a confidence interval for the efficiency measure. Later in this study, a new methodology, called *Confident-DEA,* is provided. It extends the standard DEA to the case of imprecise data and produce efficiency confidence intervals.

2.4. DEA and the Efficiency Analysis in Banking Industry

As reported by several studies (See Berger et al (2000), Berger and Mester (1997), Berger and Mester (1999), Leahy et al (2001) among others), during the last two decades the financial industry experienced a continuous and sustainable overall growth. However, the same decade was characterized by a high rate of failures and of mergers. Financial crises became common rather than rare events. This can be interpreted as a fact that the financial industry is crossing a cycle of deep mutation with high turbulence. This trend was exacerbated by developments in information technology and the new environment created by the globalized international economy.

These evolutions forced financial institutions to develop new tools of analysis and improvement of managerial performance. Classical tools for analysis are insufficient for strategic insight. Hence, according to Oral et al (1992) and Dietsch and Lozano-Vivas (2000), complementary methods are needed, for a comprehensive and complete analysis of organizational performance.

In an extensive literature survey paper, Berger and Humphrey (1997) reviewed and analysed 130 studies that apply frontier efficiency analysis to financial institutions in 21 countries. The authors reported that branch-efficiency and overall bank efficiency are the most studied but only five studies have compared efficiency levels across countries. They reported also a lack of correspondence among the efficiency levels and rankings for different measurement approaches. They analyzed the findings of the studies and described how such approaches can help financial institutions improve managerial performance. However, the authors concluded that it is not possible to determine which of the two approaches, parametric or non-parametric, dominates the other and suggested that more research comparing these techniques is needed. About cross-country, they wrote:

> "clearly, this is an area where more work is needed, and especially the proper specification of country–specific environmental influences that will justify using a common frontier for cross-country comparisons of efficiency" Berger and Humphrey 1997, pp.

This literature survey leads to the conclusion that, in recent years, cross-country efficiency comparison using different approaches is a promising and important area of research. Though few researchers studied the cross-country efficiency, most of the studies were not performed from a macro-level point of view. Often, data about banking institutions from different countries are used rather than aggregate data although the use of aggregate data guarantees the adjustment to the mean and avoids the systematic sampling bias.

On the other hand and despite its potential relevance, the issue of imprecise data is not addressed in the literature devoted to the efficiency analysis of financial institutions. Thus explicit consideration of inexact or imprecise inputs leading to a distribution of output values in the context of banking efficiency studies will be an important contribution to the literature. The existing DEA-based approaches addressing banking performance used in fact always single-valued data.

However, in economic and financial studies, proxies are often used instead of the non-available real single-valued data for some factors involved. The use of a range of values for some or all of these factors will then be more appropriate in order to make the proxy more reliable by reflecting the uncertainty about the right real values. One way of defining those ranges is assuming that the real values of each factor lie within prescribed bounds defined by percentages of those proxies. The use of *Confident-DEA* as alternative to the classical DEA-based approaches gives an additional credibility to the use of proxies in financial data.

Rankings and ratings of countries obtained from international financial agencies as well as confidence intervals obtained from econometric or time series forecasting models can be considered as imprecise data and considered for the efficiency analysis and the evaluation of performance. Cross-Country studies of financial institutions, and particularly banking systems, combining econometric and/or time series modeling with

Confident-DEA can be promising tools helping strategic decision makers, for descriptive as well as prescriptive purposes. As a potential application, *Confident-DEA* can be of high relevance for the early warning of financial crisis, crisis that can lead to the failure of financial systems, or financial institutions, if not detected early enough and dammed up. Üçer et al (1999) used DEA, among other methods, to generate a composite indicator they use as leading indicator of currency crises in Turkey. Avkiran and Gattoufi (2001) established, using DEA, an empirical linkage between financial crises in some OECD countries and deterioration of the cost-efficiency of their commercial banking systems.

CHAPTER 3: EPISTEMOLOGY OF DATA ENVELOPMENT ANALYSIS: CLASSIFICATION AND CONTENT ANALYSIS OF THE LITERATURE

3.1. Introduction

The main objective of this chapter was investigation of the Data Envelopment Analysis (DEA) literature over its entire life span in order to (a) Obtain a succinct, quantitative, yet operationally meaningful summary of its contributions; (b) Draw some conclusions regarding its historical impact on society in general and on the OR/MS profession in particular; and (c) Derive some insights and hence to suggest some future directions for course correction. The secondary objective sought findings that may shed light on the course of research in other OR/MS subdisciplines. Obviously, "there is a need and relevance of good survey articles ... two of the top ten most frequently cited articles in Interfaces ... were survey papers" (Gupta, 1997).

This chapter presents an epistemological review of the Data Envelopment Analysis sub-discipline. Specifically, using the Gattoufi et al (2001a) bibliography as raw data, a number of graphs and charts are created and presented statistically analyze the literature from inception through to August 2001. Classification based on Reisman (1994, 1995) as represented in Gattoufi et al (2001d) are used to discuss the incidence and the mix of theoretical as opposed to application based articles published over time. Lastly, results of this content analysis are also invoked to discuss research strategy patterns used by contributors to the DEA literature as compared to their counterparts in other OR/MS fields such as Flowshop Scheduling (Reisman et al, 1997), Cellular Manufacturing (Reisman et al, 1997) and Game Theory (Reisman et al, 2001)

In a larger context, this chapter contributes to the ongoing debate regarding the status and future of OR/MS as a management discipline due to what we call *"anemia in relevance to real world"* in recent OR/MS literature. In this debate, there are arguments that OR/MS in general has undergone what Ackoff (1987) called a "devolution" and Corbett et al, (1993) called "natural drift" toward what Abott, (1988) called "professional regression" from its roots in the real world to a preoccupation with mathematical constructs. DEA represents a counter-example to this claim.

Abott (1988) documented the extinction of several professions as the result of trends similar to those he observes in OR/MS. On the other hand, Blumstein (1987) argues that the future would be bright were we to do some more "missionary" work in addressing and solving real problems of consequence in the world that have never been addressed. DEA represents an example confirming this call or challenge.

Seiford (1996) traced the evolution of DEA from the initial publication e.g. CCR, to the [then] current state of the art (SOA). Additionally, he summarized the State of the Art circa 1980, 1985, 1990 and 1995. At his last milestone year, e.g. 1995, the literature base consisted of roughly 700 (Sarafoğlu 1998) articles published in refereed journals. An updated Bibliography, made available in Gattoufi et al (2001d), report that the DEA literature has grown to more than 1800 articles published in refereed journals circa August 2001. This represents an increase of more than 150%. As much it was imperative to survey the literature at that point in time, it is even more so now.

This chapter discusses the statistical trends of the Data Envelopment Analysis (DEA) literature: (1) In terms of objective findings such as the number of articles, their authorship and publishing outlets appearing in refereed journals over the entire lifespan of the field. (2) In terms of article content classifications based on a classification scheme developed in Reisman (1989) and previously applied in Reisman et al (1994, 1997 and 2001) to other OR/MS subdisciplines. The scheme differentiates contributions to theory as opposed to application or a joint contribution of theory followed by an application for validation. Also, the differentiation is made based on the basis of several well-defined research strategies (Reisman 1989) invoked from the OR/MS literature. The statistical analysis covers all the articles reported in Gattoufi et al (2001 a) while the classification covers a vast bulk of the recent literature (Post-1995).

3.2. Statistical Analysis of the DEA-Literature

3.2.1. Statistics About the Articles and the Journals:

Figure 3.1 shows the accumulation of DEA journals articles on a semi-logarithmic scale. If one eliminates the first three years of data e.g., the gestation period, as well as the incomplete year 2001 the result is close to being a straight line time-trend line described by the equation:

$$Number\ of\ Publications = ce^{0.2549\,t}$$

In other words the literature growth is perfectly exponential with a 25.5% annual growth rate. This fact alone demonstrates the vitality of the field. This is especially poignant when compared to other contemporaneous OR/MS growth disciplines e.g. Flowshop Scheduling with a rate of 0.0528 and Cell Manufacturing with a rate of 0.0498 as reported in Reisman et al (2002).

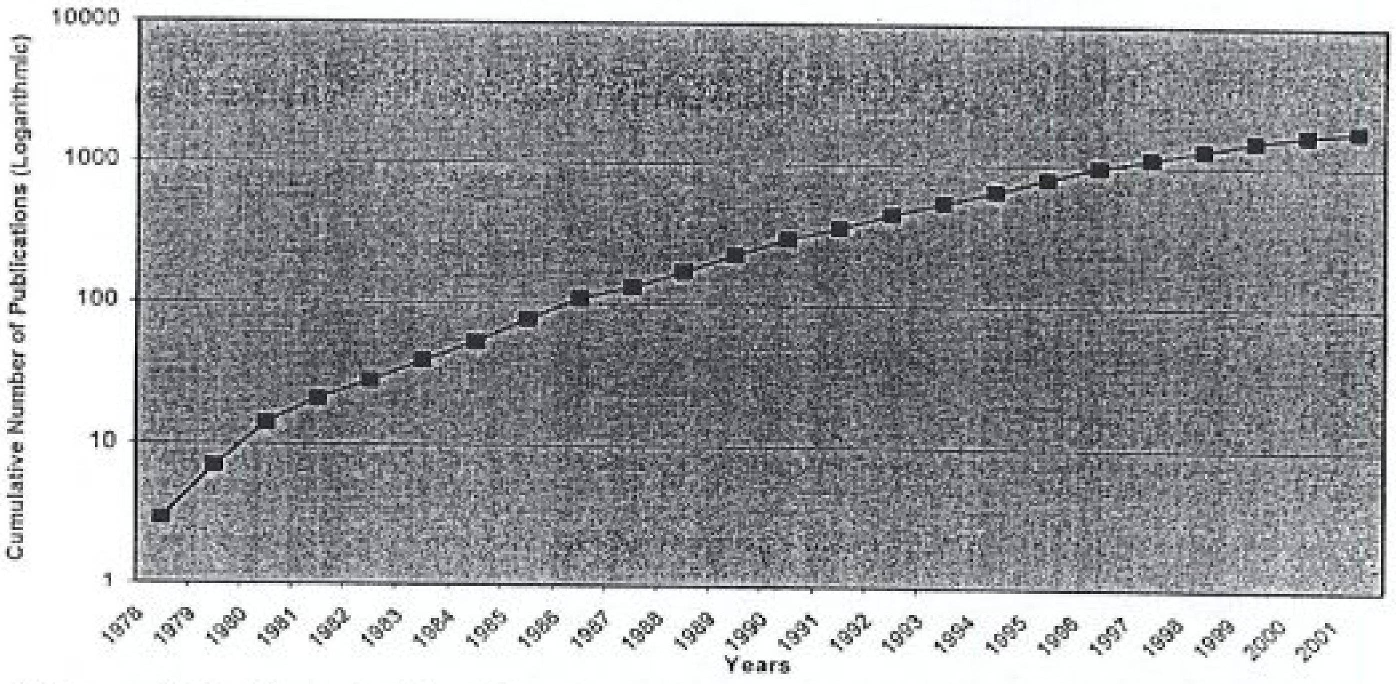


Figure 3.1: Cumulative Number of DEA Articles for the Period 1978-2001

Table 3.1 in its right-most column shows that 55% of all articles published have appeared since 1996 and 64% since 1995. The online availability of most of the post-1995 published articles, (two third of the total), allowed for detailed content analysis and classification. The results obtained are discussed later in this chapter based on systematic way of classification.

Table 3.1: Total Number of DEA Articles per Year for the Period 1978-2001

Year	Nbr Of Pub	% over all	Cumulative (Ascending)	% over all	Cumulative (Descending)	% over all
1978	3	% 0	3	% 0	1797	% 100
1979	4	% 0	7	% 0	1794	% 100
1980	7	% 0	14	% 1	1790	% 100
1981	7	% 0	21	% 1	1783	% 99
1982	7	% 0	28	% 2	1776	% 99
1983	11	% 1	39	% 2	1769	% 98
1984	14	% 1	53	% 3	1758	% 98
1985	24	% 1	77	% 4	1744	% 97
1986	31	% 2	108	% 6	1720	% 96
1987	23	% 1	131	% 7	1689	% 94
1988	36	% 2	167	% 9	1666	% 93
1989	55	% 3	222	% 12	1630	% 91
1990	69	% 4	291	% 16	1575	% 88
1991	59	% 3	350	% 19	1506	% 84
1992	90	% 5	440	% 24	1447	% 81
1993	96	% 5	536	% 30	1357	% 76
1994	118	% 7	654	% 36	1261	% 70
1995	154	% 9	808	% 45	1143	% 64
1996	165	% 9	973	% 54	989	% 55
1997	136	% 8	1109	% 62	824	% 46
1998	171	% 10	1280	% 71	688	% 38
1999	208	% 12	1488	% 83	517	% 29
2000	167	% 9	1655	% 92	309	% 17
2001	142	% 8	1797	% 100	142	% 8
	1797					

The top 20 journals accounts for roughly 50% of the publications while the top 100 accounts for roughly 70%. Alternately stated, *Figure 3.2* shows that 5% of the journals have published 50% of the articles while 30% of journals account for 80% of the publications. This means that although a small number of journals have published the majority of the publications, there was a large number of journals having each published a small number of DEA based articles, reflecting the large diffusion of DEA beyond the field of birth of DEA. In fact, Gattoufi et al, (2001b, 2001c) report that there are 490 refereed journals, including some non-Anglophone journals, known to have published at least one DEA based article.

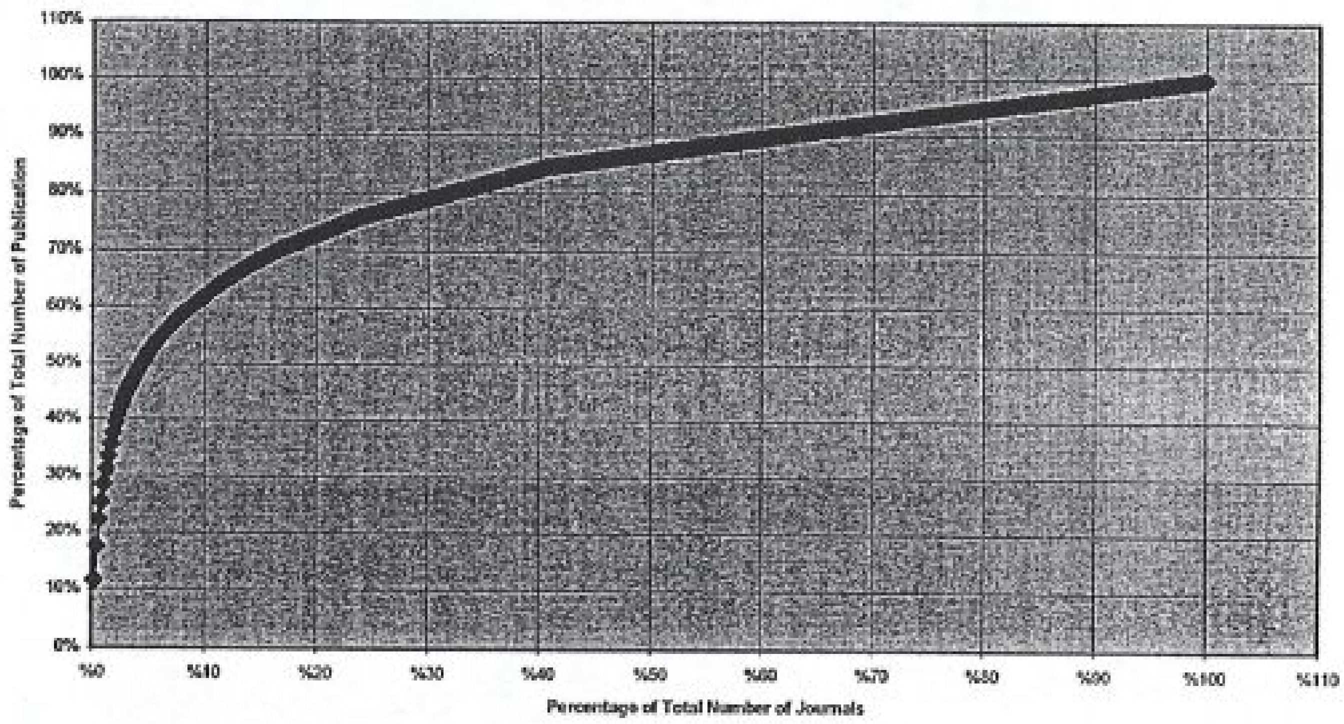


Figure 3.2: Cumulative Number of Articles per Journal

Table 1 in the Appendix, provides the list of refereed journals known to have publish at least one article explicitly related to DEA. The journals are arranged in descending order on the number of articles DEA-Articles they published. The *European Journal of Operational Research (EJOR)* leads the park with 204 publications as of August 2001. It is followed by the *Journal of Productivity Analysis (JPA)* with 109 publications in the same period. The *Journal of the Operational Research Society (JORS)* comes up third with 77. Thus the top three, among 490 journals, have published 22% of the DEA articles thus far.

As can be seen from *Figure 3.3*, among the three top DEA publishing journals, the *European Journal of Operational Research* (EJOR) has been capturing the lion's share of 390 DEA publications during most years in the lifetime of this discipline. The original DEA article was also published in the EJOR.

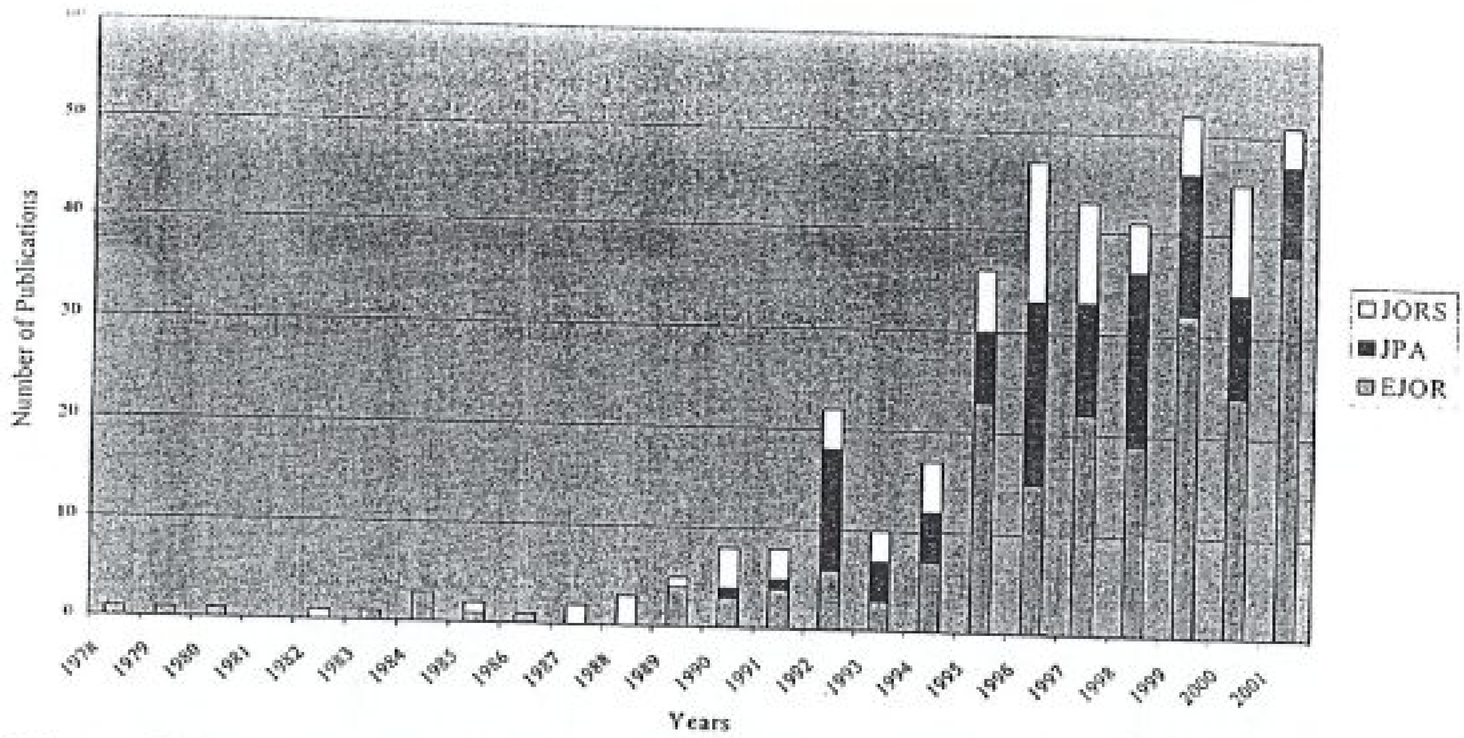


Figure 3.3: DEA Publications in Top 3 Journals: EJOR, JPA, and JORS

3.2.2. Facts About the Major DEA Articles Producers

In this section, we will examine the journals-of-choice for the top five producers (authors) of the DEA based literature.

William Wager Cooper, one of the founders of DEA, is still the leading contributor to its literature base. To the date, we found that he has authored or co-authored 84 articles in refereed journals. Of these, fifteen (15) appeared in the *European Journal of Operational Research*, nine (9) each in the journal of *Socio-Economic Planning Sciences* and in the *Annals of Operations Research*. The *Journal of Productivity Analysis* is next with eight (8) articles followed by five (5) in *Management Science*. Significantly, as of the date of this writing[1], none of his contributions appeared in *Operations Research*, although his CCR (1978) contribution represents the number one most cited DEA article (Sarafoglu 1998).

Jati Sengupta is the next most prolific writer on DEA based subjects. The vast bulk of his publications appeared in the *International Journal of Systems Science*. Significantly, his name does not appear among the top 20 most influential contributors either in Seiford (1996) an interview-based (subjective ranking) nor in Sarafoglu (1998) a citation (objective ranking) analysis.

Rolf Fare is next with 60 publications. His work in DEA has been widely distributed among 29 different journals worldwide with at most five (5) publications in *Management Science* and the *European Journal of Operational Research*. In terms of citations, his articles rank 4 (Fare, Grosskopf and Lovell 1985) and 12 (Fare, Grosskopf and Lovell 1985) respectively.

Abraham Charnes comes in fourth in the number of publications e.g. 57 due to his untimely death in 1992. The volume 73 of the *Annals of Operations Research* (1997): *'From Efficiency calculations to a new approach for organizing and analyzing: DEA fifteen years later'* was dedicated to Abraham Charnes. As one of the founders of the field, his contribution to CCR is the most respected, based on both interviews (Seiford 1996) and on citations (Sarafoglu 1998).

Like Fare, *Shawna Grosskopf*, with 53 publications and often a co-author with Fare, including the articles appearing among the 20 most cited has spread his contribution among as many as 29 journals. *Socio-Economic Planning Sciences* and the *Journal of Public Economics* are first with five (5) articles each. It is worth noticing his recent shift to the journal of *Socio-Economic Planning Sciences* as a preferred outlet by Fare and Grosskopf. Their first article in *Socio-Economic Planning Sciences* was in fact published in 1998. Most of these articles were a kind of forum between them and Cooper et al., about the congestion component of inefficiency.

The bottom line is that the most influential of the top five most prolific contributors have not utilized any one journal for the bulk of their writings. Surprisingly, none of these have yet published a DEA based article in *Operations Research* one of the premiere OR/MS U.S. based Journals. Banker and Morey did publish in *Operations Research* (Banker and Morey 1986). Banker comes in 8th in our classification for the most prolific authors. He co-authored the BCC model (Banker, Charnes and Cooper 1984) article published in *Management Science.* This article was reported as the third most influential article by Seiford (1996) and by Sarafoglu (1998).

3.3. Diffusion of DEA in Terms of Outlets, Fields, and Geography

An important indicator of the degree of diffusion of any field is the number of "good" journals publishing the output of its researchers. An even better indicator of rapidity of diffusion to new disciplines is the yearly additional number of journals that serves as outlets for the field. The large spread over different domains and through an increasingly higher number of outlets in different regions and different languages reflects the high relevance of the field to the scientific community.

---

[1]I am grateful to W.W. Cooper who provided me kindly in a personal communication with a forthcoming paper in *Operations Research* by Cooper, Park and Yu entitled: "An Illustrative application of IDEA (Imprecise Data Envelopment Analysis) to a Korean Mobile Telecommunication Company".

Table 3.2: Number of Journals Publishing DEA-Articles per Year

Year	Number of Journals	Cumulative Number of Journals
1978	3	11
1979	4	14
1980	6	17
1981	6	22
1982	6	26
1983	10	32
1984	11	38
1985	17	48
1986	24	61
1987	19	74
1988	26	85
1989	32	105
1990	40	123
1991	42	151
1992	59	177
1993	61	206
1994	78	250
1995	84	284
1996	64	312
1997	66	342
1998	81	380
1999	108	425
2000	98	466
2001	73	490

Table 3.2 reports the number of journals publishing DEA articles for each year. The cumulative column reports, the cumulative number of journals that published DEA articles for all the elapsed lifetime of DEA. The period covered in the Cumulative column starts in 1951.

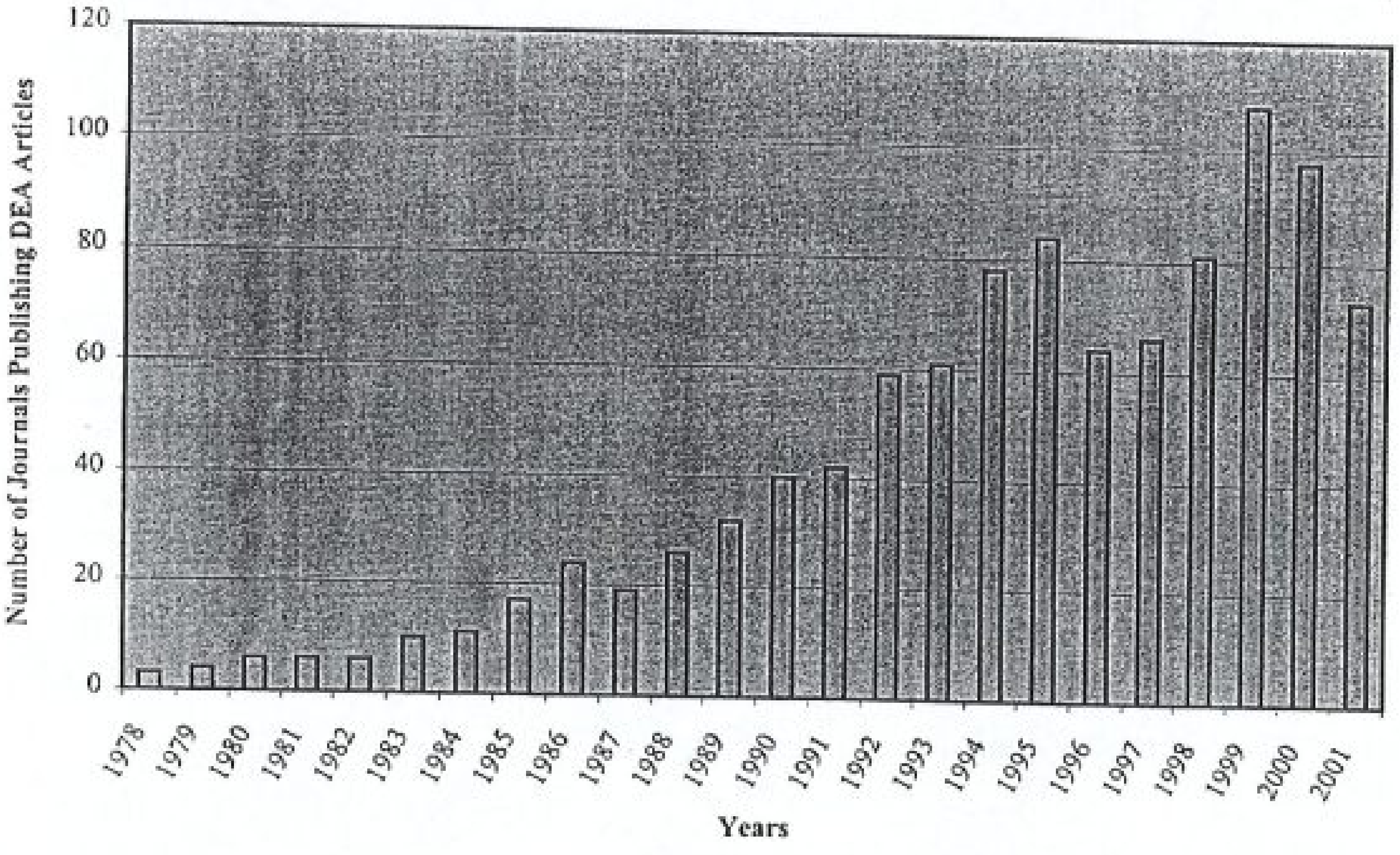


Figure 3.4: Number of Journals Publishing DEA Articles Per Year

The number of journals serving as outlets for DEA-literature varied from 3 in 1978 to a top 108 during 1999. As can be seen in *Figure 3.4*, the number of journals serving as outlets has shown a decrease starting 1995 followed by a big jump during 1999. It is worth to notice that the examination of the number of publications in the top 10 journals showed some decrease that was compensated by new journals from new fields that publish for the first time. However, all the top 10 journals kept publishing DEA-articles during recent years (post-1996).

Also, it is worth to mention that several journals edited special issues totally or partially devoted DEA in recent years. It is the case of Annals of Operations Research (Vol 66, 1996 and Vol 73, 1997), Computers and Operations Research (Vol 23, 1996), European Journal of Operational Research (Vol 98, 1997) and Journal of Productivity Analysis (Vol 7, 1997). Earlier special issues were edited by and Journal of Banking and Finance (Vol 17, 1993) and Journal of Econometrics (Vol 46, 1990) and Annals of Operations Research (Vol 2, 1985).

Figure 3.5 provides a semi logarithmic curve with a linear time-trend of the number of new outlets included every year. This translates the fact that this number has increased exponentially during the time period studied. However, it should be noticed that there is a trend of slowing during the three last years. This may indicates saturation in the potential number of outlets for which DEA can be relevant rather than by a decrease in popularity of DEA.

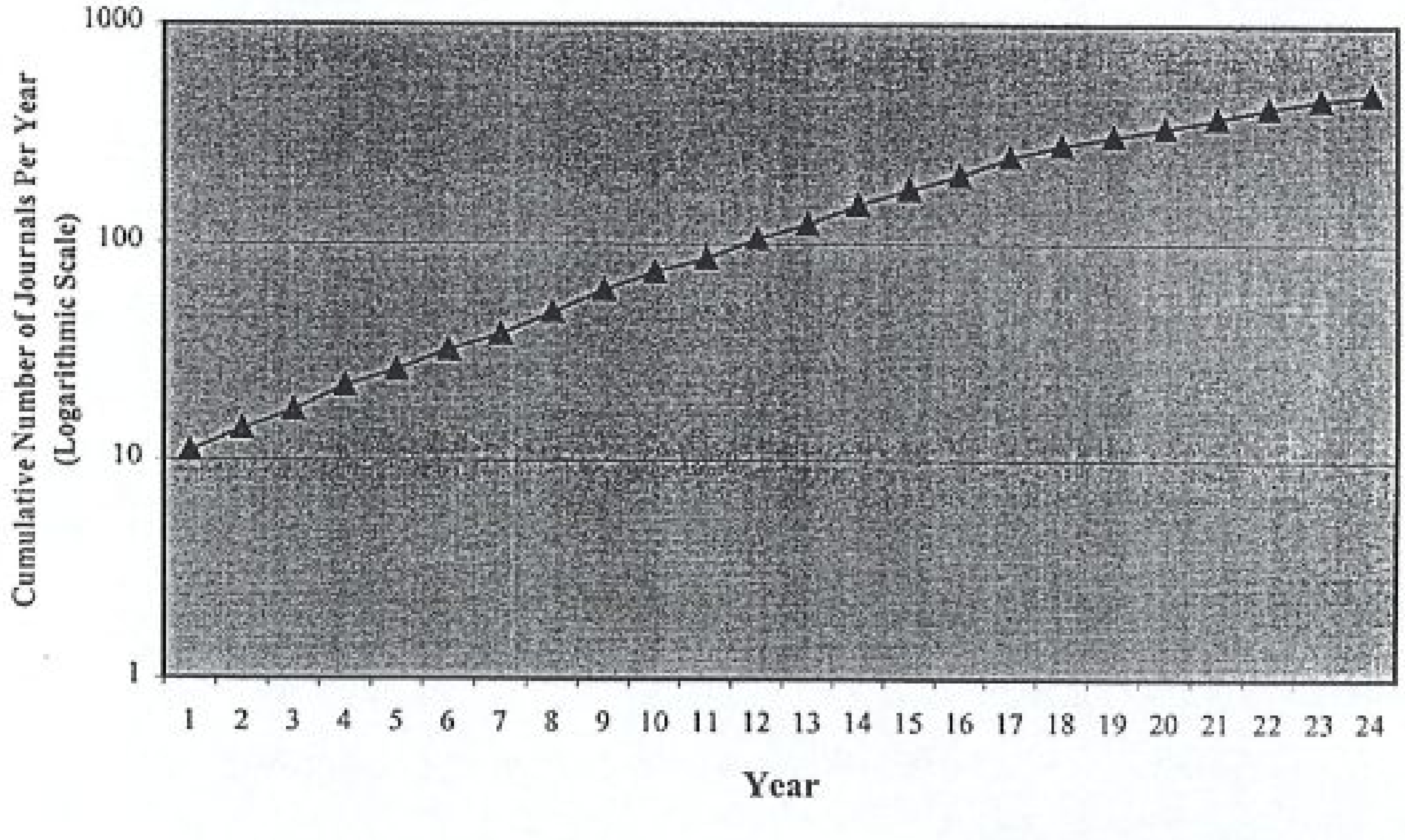


Figure 3.5: Cumulative Number of New Journals Publishing DEA-Articles Per Year (Semi-Logarithmic Scale)

3.4. DEA Literature in the U.S. Based Flagship OR/MS Journals: An Enigma?

It is interesting to note that even though the founders of the field, Charnes Cooper and Rhodes, are all U.S. based, the first article in the field was published in a European journal. Moreover, European journals continue to provide publishing outlets of choice for DEA authors. *Management Science*, one of the flagship U.S. based OR/MS journals is ranked fifth on this list. The other U.S. based flagship OR/MS journal, *Operations Research*, having published a total of only five (5) DEA articles as of August 2001 is, as shown in *Table 3.3*, tied in the 55th ranking with 8 other journals. The first *Operations Research* DEA publication appeared in 1986 and the last in 2001, this is roughly one article every three years.

Table 3.3: Journals with the Same Total of Articles with DEA Content as in *Operations Research*

Ranking	Journal	Number of Papers
55	Accounting Review	5
56	Computers & Industrial Engineering	5
57	Economics Letters	5
58	Forest Science	5
59	Health Services Management Research	5
60	IEEE Transactions on Engineering Management	5
61	Journal of Economic Theory	5
62	Operations Research	5
63	Scientometrics	5

In as much as Seiford (1996) shows that the CCR article has been widely cited by econometricians, this lack of interest on the part of *Operations Research* clearly can not be taken as a poor reflection on DEA. However, when coupled with the findings by Reisman and Kirschnick (1994) it does reflect on the editorial practices if not policies of *Operations Research.*

As is shown in *Table 3.4*, reproduced from Gattoufi et al (2001 c) no DEA articles appeared in any of the three U.S. based flagship journals[2] circa 1980. However, circa 1985 these three journals accounted for 7.78% of all articles published in refereed journals and 7.57%, circa 1990; but the percentage steadily decreased to 4.36% circa 1995 and 3.72% circa 2001.

[2] *Operations Research, Management Science,* and *Interfaces*

Table 3.3: Number of DEA-Articles in Three U.S. based Flagship OR/MS Journals

	Interfaces (Cumul)	Mgt Sc (Cumul)	OR (Cumul)	Total (Cumul)	Total DEA Publications (Cumul)	Percentage from to date (%)
circa 1980	0	0	0	0	28	0.00
circa 1985	1	6	0	7	90	7.78
circa 1990	4	17	2	23	304	7.57
circa 1995	6	28	2	36	825	4.36
circa 2001	11	51	5	67	1801	3.72

The ever lower "market share" by these three journals can be explained by at least two non-mutually exclusive hypotheses.

The first hypothesis is that the U.S. based OR/MS community/establishment was "asleep at the wheel" so to speak or worse yet was operating on a paradigm that was out of sync with the field's original paradigm. Instead of working "in the swamps of relevance" applying "whatever tools they had" (Miser (1987)) in order to solve problems of "significance to society" (Blumstein (1987)), the establishment became preoccupied with improvements, no matter how incremental or minute, to the extant theoretical base (Reisman and Kirschnick (1994 and 1995)). In fairness it must be recognized that even in its original paradigm OR/MS was and is concerned with "planning decisions" and with "forecasting" while DEA is essentially an *ex post facto* evaluation for the "control aspects of management" (Cooper (1999)). Consequently, DEA is out of sync with the existing OR/MS establishment's paradigm on two counts.

This hypothesis is further supported by the fact that nine out of the ten most prolific contributors to the DEA literature are associated primarily with North American institutions – one is a Canadian, and, it is supported by the fact that the *European Journal of Operational Research* alone, with a count of 204, published more DEA articles than the total of those published in the US based flagship journals. The latter currently stands at 67. This was true in the early DEA years and has been true ever since. Significantly, even though the founders of the field, Charnes, Cooper and Rhodes, were all U.S. based, the first article in the field was published in a European journal.

The other hypothesis is that significant diffusion into other disciplines and professions has taken place. Supporting this is the fact that currently DEA articles can be found in 490 refereed journals worldwide. Many of them publish in languages other than English. The list of refereed journals having DEA content includes those publishing in French, German, Italian, Spanish as well as a number of languages from East Europe, Scandinavia, Asia/Pacific, and the Near East.

There is yet another way to measure the diffusion of DEA out of OR/MS. It may well be argued that the vitality of a field of knowledge is not independent of its diffusion into other disciplines and professions. Of the 490 refereed journals having DEA articles only ten (10) (slightly over 0.2%) can be considered to be hard core OR/MS journals[3]. As is shown in Gattoufi et al. (2001X) they account for 28 % of the DEA literature extant August 2001. Thus, almost three-quarters or 72% of the DEA literature was published in archival journals outside its birth discipline. Additional indicators of DEA's diffusion to other disciplines is the fact that the *Journal of Productivity Analysis,* ranked second in terms of DEA publications, is not an OR/MS journal but a microeconomic oriented journal and that *Applied Economics* ranks 7th, *Socio-Economic Planning Sciences* ranks 8th,the *Journal of Banking and Finance* ranks 12th and *Journal of* Econometrics ranks 13th.

3.5. The DEA Compared to Other OR/MS Subdisciplines

This section compares the results reported here to those reported in existing Meta review of OR/MS subdisciplines. Reisman et al (2001) reviewed the Game Theory (GT) appearing in the U.S-based OR/MS flagship journals. Reisman et al (2002) provides a 3-disciplines analysis of vitality based on previous individual meta review of each discipline. In addition to DEA, Cellular Manufacturing and Flowshop Scheduling were included in the analysis. Beside considerations of availability of other meta reviews, there is yet another justification for the validity of such comparisons.

GT is like DEA in that it is not only a technique but also an approach to solve real world problem that can be fairly described by the quartet of "conceptualisation - formal modeling - obtaining formal solution - implementation" (Oral and Kettani, 1993). While GT failed at the "implementation" level, as documented in Reisman (2001), DEA registered spectacular success. These similarities generated periodically high quality articles viewing DEA from the GT point of view making a joint use of the fame and prestige of GT and the success of DEA in real world application. DEA can in fact be of high contribution for the GT to overcome its failure in implementing real world

[3] *Cited in the order of DEA article count: European Journal of Operational Research, Journal of the Operational Research Society, Annals of Operations Research, Management Science, OMEGA, Computers and Operations Research, INFOR, Interfaces, Operations Research Letters, Operations Research, International Transactions on Operations Research*

application. The abstract of the most recent of these articles (Hao and Yan, 2000) is reproduced and discussed later.

Flowshop Scheduling and Cellular manufacturing on the other hand, although they have an *"appealing brand name"* announcing a high relevance to real world problems (industry) by promising practical rules and schemes, they largely failed in realising this *"raison d'etre"* and still in these fields *"more than three machines is considered as hard problem"*.

3.5.1. DEA Compared to Game Theory

In several respects, DEA appears to be a counter-point to Game Theory (GT). Starting with its first article (CCR, 1978) as shown earlier, DEA had real-world grounding. This real world grounding has richly permeated the field's entire life span while its theoretical base evolved both in depth and in scope.

GT was not very well received among economists in the 1940s. "In the immediate post-war period GT was viewed with some suspicion: it was not really economics. It received more attention in other disciplines" (Weintraub, 1992). The Nobel committee and the media subsequent to the announcement spoke glowingly about the various applications of GT. Its literature claims many "applications" to a diversity of fields as varied as economics, military science/war gaming, political science, marketing, pricing, industrial relations, negotiations, bidding, sports, and a broad range of other business problems. To which one can add a number of the biological/behavioral sciences, such as *"evolutionary competition", "adaptations", "parental investment in child rising and why some animals desert their mates", "animal's fighting behavior"*, etc... and hence the Nobel Prize.

However, in a meta review of GT publications in the flagship US-based OR/MS journals (*Operations Research, Management Science and Interfaces*), Reisman et al (2001) find that over time there were indeed very few applications that were directly grounded in the real world. There were only 30, out of 144 or 21% reviewed, articles deemed to fall into the Applications categories (type A4, A5 as defined in Reisman (1989) and presented in the next section). On average, this translates into three such articles every four years, i.e. less than one article of real-world application per year within the entire lifetimes of the three US-based flagship OR/MS journals! Although this indeed is a higher percentage and/or frequency than the authors have found to be the case for the life-cycle (all articles in all journals) of some other, albeit younger, OR/MS subdisciplines, e.g. Flowshop Scheduling and Sequencing (Reisman et al 1997a) and in Cellular Manufacturing (Reisman et al 1997b), it is much less than the three journals surveyed for Reisman et al (2001) have published for all of OR/MS.

The review of all the 67 DEA articles published in U.S-based flagship OR/MS journals shows that 14 out of the 67 articles or 21% were pure application (A4 and A5 type) and 24 or 36% presented advances in theory followed by application with real world data. In total, 67% of the articles presented real world application, and this to compare with 21% in the case of GT.

3.5.2. DEA Flow Shop Scheduling (FSS) and Cellular Manufacturing (CM):

As another meaningful comparison between DEA and other older OR/MS subdisciplines, the following statistics, supported by *Figure 3.5* reproduced from Reisman et al (2002), provide indications of DEA's vitality vis a vis Flow Shop Scheduling (FSS), and Cellular Manufacturing (CM).

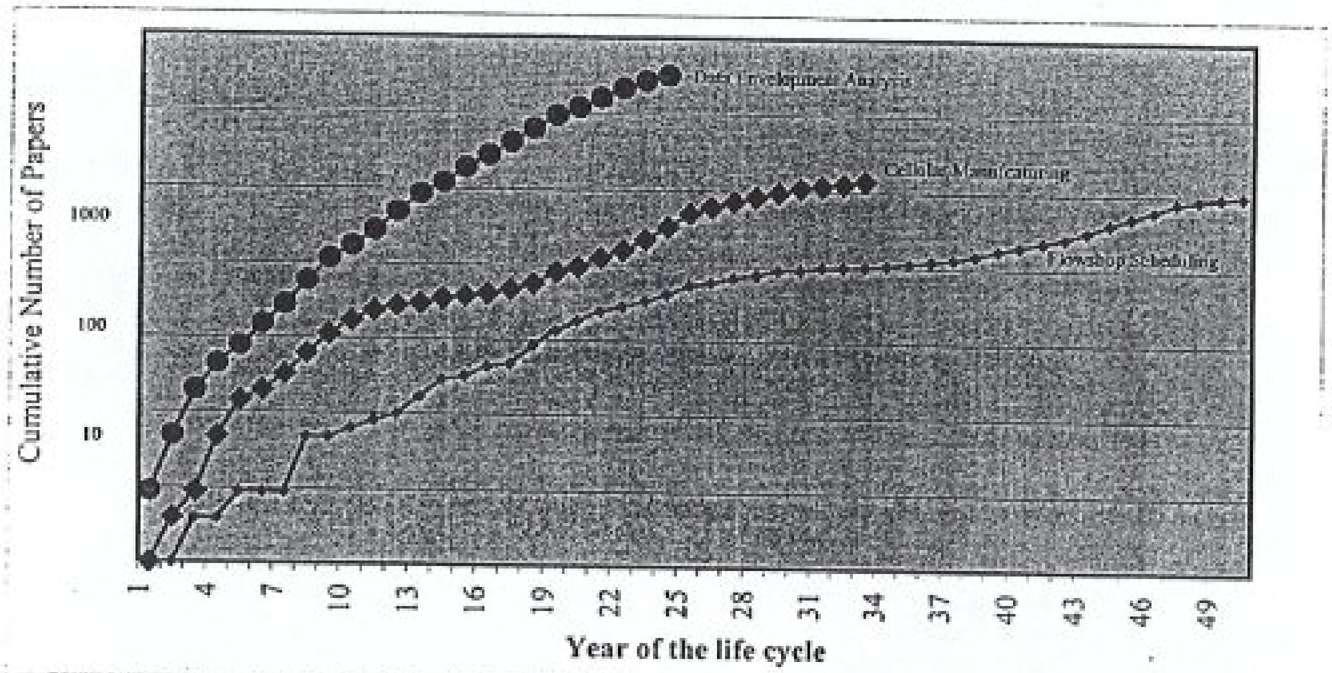


Figure 3.5: 3-Discipline Comparative Curves

1. The accumulation of the literature in each of the three disciplines is shown (at a high precision of fit) to be exponential with DEA's growth parameter being 0.255 while that of FSS and CM respectively being 0.151 and 0.106.
2. This is analogous to a 25.5% rate of interest, compounded annually, on a money deposit versus one of 15.1% or 10.6%.
3. The official year of birth for FSS is 1952, for CM it is 1969 and for DEA, it is 1978. Thus DEA is by far the youngest of the three disciplines.

4. The total number of FSS, CM and DEA articles published in refereed journals circa August 2001, is respectively 316, 374, and 1797.
5. Thus the average (over the discipline's lifetime) number of articles published per year is respectively 316/49 = **6.4**; 374/32 = **11.7**; and 1797/23 = **78.2**

The above comparative vitality numbers speak for themselves

3.6. Classification of Recent DEA-Literature and its Content Analysis

The life cycle of Data Envelopment Analysis (DEA) was reviewed and classified on a scale ranging from pure theory to bona fide application. The articles were classified in terms of seven types of research processes used by authors. Next, statistical correlations were performed relating data from the above classifications. The findings show that the literature is dominated by applications to real world.

3.6.1. Research Strategies used in OR/MS:

To complement the qualitative review of DEA research by Seiford (1996), we provide in this section a statistical review of the entire life cycle of DEA using the same categories as those used in Reisman and Kirschnick (1994, 1995). Specifically, all articles were classified according to their respective authors' use of the following research strategies, reproduced from Reisman (1988, 1992 and 1995):

1. **Ripple:** an extension of previous theoretical or applied type of research in a given discipline or subdiscipline.
2. **Embedding:** the development of a more generalized formulation or a more global theory by embedding several known models or theories.
3. **Transfer of technology:** the use of what is known in one discipline to model problem domain falling in some other, perhaps disparate, discipline.
4. **Bridging:** the bridging of known models or of known theories resulting in the growth of the contributing and/or some initially unrelated field of knowledge.
5. **Creative application:** the direct (not by analogous) application of a known methodology to a problem or research question that was not previously so addressed.
6. **Structuring:** the process of organization and documentation of the organizational phenomena in the form of models.

7. **Statistical modeling:** models that arise from analyses performed on empirically obtained data. These models arise from statistical manipulations such as regression or cluster analysis rather than from logical derivations based on various assumptions.

The above strategies are graphically depicted in *Figure 3.6.*

Although the seven research processes just described cover much of what is done by OR/MS workers, it cannot be claimed that they are either exclusive or complete. For example, consider the case where the research worker turns into what appears to be a stonewall; that is, she or he can see a desirable objective but is methodologically stymied by her/his discipline, particularly if it is conceived to constitute a ripple process. The impasse can perhaps be overcome by transferring technology from another discipline, or a creative application suggested by other work, but it must also be recognized that it may call for a fundamentally new intellectual approach. It must also be recognized that some research approaches may reach dead ends, either in prompting useful further model development or the ability to represent actuality adequately. New vistas may be opened up by bridging with other disciplines or by an embedding process, but again a fundamentally new approach may be called for (Reisman 1988, 1992).

W. W. Cooper, who saw an early draft of Reisman and Kirschnick (1994) wrote to the authors a recollection from his own experience, reported in there:.

> "This paper stimulated my thinking and also brought back many memories. One of the possibilities to be considered is the reinforcing effects which may occur when several of the strategies you describe are employed simultaneously. A case in point from my own experience is the original article which Abe Charnes and I wrote with Bob Mellon and published in the April 1952 issue of *Econometrica* (a really abstract methodology oriented journal entitled "Blending Aviation Gasolines- A Study in Programming Interdependent Activities in an Integrated Oil Company" (Charnes, Cooper and Mellon 1952). This was the first reported actual application of linear programming and the effect was enormous both on industrial practice more than one industry and theoretical-methodological research (in more than one discipline). Many things were involved---a new application, new methodologies and new substantive theory. Perhaps this was due to the mix of disciplines in our team which included chemical engineering and refinery experience (Mellon), mathematics and engineering (Charnes) and economics, management and accounting" (Cooper).

At this point, Cooper inserted a footnote that, "We only discovered at a later date that this was to be called "*operations research*" or still later, "*management science*".

He then continued:

> "These wide ranging and continuing effects, or at least the speed which these occurred, may also have been due to the times and the psychological aftermath (of euphoria) resulting from the great historical divide we now refer to as 'World War II'..." (Cooper 1994).

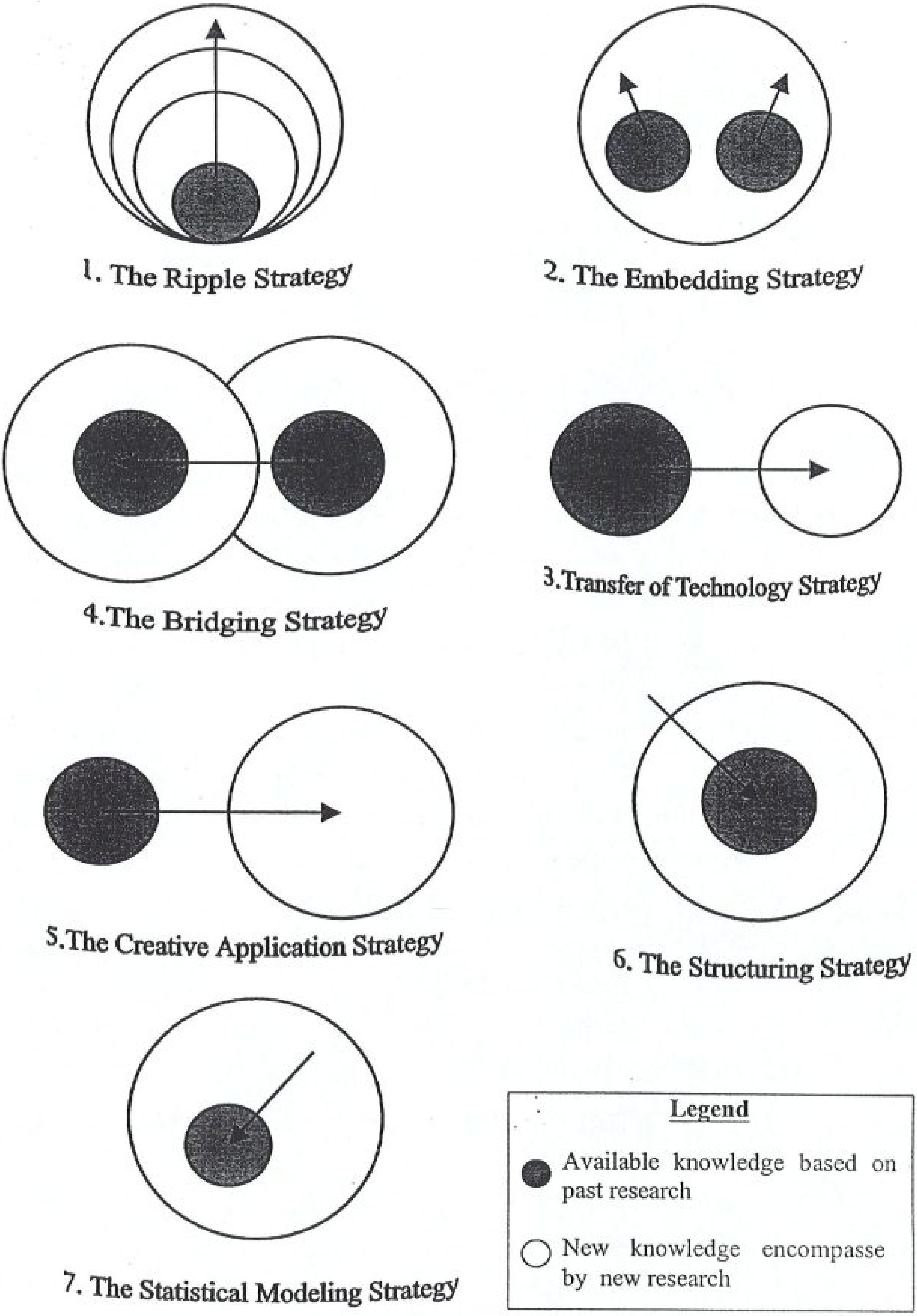


Figure 3.6: The seven categories of research strategy used in OR/MS literature

(Adapted from Reisman, 1995)

In Cooper's example, a *creative application* of linear programming followed the *structural process* of blending a gasoline, and the result was validated *empirically* in industrial practice. The work involved new models, new methods, and substantive theory as a result of *bridging process* between the state of knowledge in linear programming and that of chemical engineering. This example exhibits the importance of not limiting one's research to the *ripple* process.

3.6.2. Nature of Research Articles in OR/MS:

The literature of OR/MS in general, and that of DEA in particular, uses the word "application" to imply anything from a bona fide solution of a real-world problem to an interesting model that is but a figment of the author(s)' imagination. Moreover, the word "data" is often used in referring either to real-world application or to numbers extracted from a random numbers generator simulating a real-world process. Consequently, the articles in this database were also classified using a scale for classifying theoretical and applied articles developed in Reisman (1994 & 1995).

Accordingly, each article first was judged to be part of either theory literature or the applications literature. Articles falling into the first group are formal constructs that are theoretical in nature. They may be motivated by or even based on real-world problems and offer a wide range of potential applications. Each theory article was sub-classified to distinguish those that used synthetic numbers for various tests or example (labeled **T2**) from those that did not (labeled **T1**). On the other hand, if the article was judged to fall in the application area, it was then classified according five-point scale:

A1. *A figment of the modeler' s imagination, a result of logico-deductive reasoning;*

A2. *A figment of the modeler' s imagination that uses synthetic data;*

A3. *A grounding in the real-world;*

A4. *A grounding in the real-world data and a demonstrated application that made a difference;*

A5. *Either category three or four above with the additional use of synthetic data to test sensitivity, conduct an error analysis, and/or explore behavior boundaries.*

A typical characteristic for DEA literature is the large bulk of articles having theoretical developments followed by the application of these developments to real-world problems. This led us to add a third class (labeled **TxAx**) which combines these classifications. The following section illustrates the classification scheme.

3.6.3. A Sample of Selected Articles and their Classification

We consider in this section to classify selected articles from the literature. Although there is always a subjective side to selecting illustrative articles, the sample of articles selected here is supposed to be representing different periods, different journals, different domains of application and different research strategies. Articles included in the classification sample are among the most cited and most often refereed as pioneering in the topic or the application domain they are devoted to.

Each article is given a three domain code that identifes the research strategy(ies) it uses appearing in the importance order as primary, secondary and tertiary. In some articles classified, two or three strategies were invoked while in others only a primary strategy was adopted.

Presented next are the abstracts of the articles selected for the classification and their corresponding three-domain code.

A Game Theoretical Model of DEA Efficiency
G. Hao, Q.L. Wei and H. Yan
Journal of The Operational Research Society-2000

Motivated by the inherent competitive nature of the DEA efficiency assessment process, some effort has been made to relate DEA models to game theory. Game theory is considered not only a more natural source of representing competitive situations, but also beneficial in revealing additional insights into practical efficiency analysis. Past studies are limited to connecting efficiency games to some particular versions of DEA models. The generalised DEA model considered in this study unifies various important DEA models and presents a basic formulation for the DEA family. By introducing a generalised convex cone constrained efficiency game model in assembling the generalised DEA model, a rigorous connection between game theory and the DEA family is established. We prove the existence of optimal strategies in the generalised efficiency game. We show the equivalence between game efficiency and DEA efficiency. We also provide convex programming models for determination of the optimal strategies of the proposed games, and show that the game efficiency unit corresponds to the non-dominated solution in its corresponding multi-objective programming problem. Our study largely extends the latest developments in this area. The significance of such an extension is for research and applications of both game theory and DEA.

Keywords: convex cone, data envelopment analysis, game theory

[T1] : [2 , - , -]

Computational Accuracy and Infinitesimals in Data Envelopment Analysis
A.I. Ali and L.M. Seiford
INFOR-1993

An analysis clarifies the role of the non-Archimedean infinitesimal in the Charnes, Cooper, and Rhodes (1978) and Banker, Charnes, and Cooper (1984) models, used in data envelopment analysis (DEA). The analysis establishes that the associated dual linear programs can be infeasible for the multiplier side and unbounded for the associated dual envelopment side program. Sufficient conditions are established for feasibility and boundedness. Computational testing indicates that the improper selection of a value for the non-Archimedean infinitesimal can result in serious errors. While there exists a threshold value for the infinitesimal that yields finite solutions, smaller values may disguise equally serious errors. When a numerical value is used to represent the non-Archimedean infinitesimal, results are sensitive not only to the specific value for the infinitesimal but also to the pricing tolerances that are employed in standard linear programming software.

[T1A4] : [3 , 1 , -]

Thus the article "A Game Theoretical Model of DEA Efficiency" by Hao, Wei and Yan (2000), no matter how significant can clearly be classified as [T1] for it does not even attempt to illustrate with real or synthetic data its theoretical developments.

On the other hand, the articles classified as [T1A4] differ from the above in that they discuss a "demonstrated application that made a difference". Hence the inclusion of A4 next to T1.

The next two articles, e.g., "Measuring the Efficiency of Decision Making Units" by A. Charnes, W.W. Cooper and E. Rhodes and "IDEA and AR-IDEA: Models for Dealing with Imprecise Data in DEA" by W.W. Cooper, K.S Park and G. Yu; are classified as T2 because they represent theoretical "advancements with synthetic numbers" used for illustration purposes.

Measuring the Efficiency of Decision Making Units
A. Charnes, W.W. Cooper and E. Rhodes
European Journal of Operational Research-1978

A nonlinear (nonconvex) programming model provides a new definition of efficiency for use in evaluating activities of not-for-profit entities participating in public programs. A scalar measure of the efficiency of each participating unit is thereby provided, along with methods for objectively deter- mining weights by reference to the observational data for the multiple outputs and multiple inputs that characterize such programs. Equivalences are established to ordinary linear programming models for effecting computations. The duals to these linear programming models provide a new way for estimating extremal relations from observational data. Connections between engineering and economic approaches to efficiency are delineated along with new interpretations and ways of using them in evaluating and controlling managerial behavior in public programs.

[T2] : [6 , 3 , -]

IDEA and AR-IDEA: Models for Dealing with Imprecise Data in DEA
W.W. Cooper, K.S Park and G. Yu
Management Science-1999

Data Envelopment Analysis (DEA) is a nonparametric approach to evaluating the relative efficiency of decision making units (DMUs) that use multiple inputs to produce multiple outputs. An assumption underlying DEA is that all the data assume the form of specific numerical values. In some applications, however, the data may be imprecise. For instance, some of the data may be known only within specified bounds, while other data may be known only in terms of ordinal relations. DEA with imprecise data or, more compactly, the Imprecise Data Envelopment Analysis (IDEA) method developed in this paper permits

mixtures of imprecisely- and exactly-known data, which the IDEA models transform into ordinary linear programming forms. This is carried even further in the present paper to comprehend the now extensively employed Assurance Region (AR) concepts in which bounds are placed on the variables rather than the data. We refer to this approach as AR-IDEA, because it replaces conditions on the variables with transformations of the data and thus also aligns the developments we describe in this paper with what are known as cone-ratio envelopments in DEA. As a result, one unified approach, referred to as the AR-IDEA model, is achieved which includes not only imprecise data capabilities but also assurance region and cone-ratio envelopment concepts.
Keywords: DEA Efficiency; Imprecise Data; Assurance Regions

[T2] : [1 , - , -]

The article "An Empirical Study on Measuring Operating Efficiency and Profitability of Bank Branches" by M. Oral and R. Yolalan is a direct application of existing DEA theory to real-world problem of consequence while the last two abstracts, e.g. "Evaluating Program and Managerial Efficiency: An Application of Data Envelopment Analysis to Program Follow Through" by A. Charnes, W.W. Cooper and E. Rhodes and "DEA and the Discriminant Analysis of Ratios for Ranking Units" by Z. Sinuany-Stern and L. Friedman respectively as [T2A5] and [T1A] differ from above in that they both conducted sensitivity and/or error analyses in their theory enhancing real-world applications.

An Empirical Study on Measuring Operating Efficiency and Profitability of Bank Branches

M. Oral and R. Yolalan

European Journal of Operational Research-1992

This paper discusses the methodology of an empirical study that was employed to measure the operating efficiencies of a set of 20 bank branches of a major Turkish Commercial Bank offering relatively homogeneous products in a multi-market business environment. The methodology was based on the concepts and principles of Data Envelopment Analysis (DEA). The results of the study have indicated that this kind of approach is not only complementary to traditionally used financial ratios but also a useful bank management tool in reallocating resources between the branches in order to achieve higher efficiencies. It has been also observed that the service-efficient bank branches were also the most profitable ones, suggesting the existence of a relationship between service efficiency and profitability.
Keywords: Efficiency, productivity, performance evaluation, banking, mathematical programming

[A4] : [5 , - , -]

Evaluating Program and Managerial Efficiency: An Application of Data Envelopment Analysis to Program Follow Through

A. Charnes, W.W. Cooper and E. Rhodes

Management Science-1981

A model for measuring the efficiency of Decision Making Units (= DMU's) is presented along with related methods of implementation and interpretation. The term DMU is intended to emphasize an orientation toward managed entities in the public and/or not-for-profit sectors. The proposed approach is applicable to the multiple outputs and designated inputs, which are common for such DMU's. A priori weights, or imputations of a market-price-value character are not required.
A mathematical programming model applied to observational data provides a new way of

obtaining empirical estimates of extremal relations-such as the production functions and/or efficient production possibility surfaces that are a cornerstone of modern economics. The resulting extremal relations are used to envelop the observations in order to obtain the efficiency measures that form a focus of the present paper.
An illustrative application utilizes data from Program Follow Through (= PFT). A large scale social experiment in public school education, it was designed to test the advantages of PFT relative to designated NFT (= Non-Follow Through) counterparts in various parts of the U.S. It is possible that the resulting observations are contaminated with inefficiencies due to the way DMU's were managed en route to assessing whether PFT (as a program) is superior to its NFT alternative. A further mathematical programming development is therefore under- taken to distinguish between "management efficiency" and "program efficiency." This is done via procedures referred to as Data Envelopment Analysis (= DEA) in which one first obtains boundaries or envelopes from the data for PFT and NFT, respectively. These boundaries provide a basis for estimating the relative efficiency of the DMU's operating under these programs. These DMU's are then adjusted up to their program boundaries, after which a new inter-program envelope is obtained for evaluating the PFT and NFT programs with the estimated managerial inefficiencies eliminated.
The claimed superiority of PFT fails to be validated in this illustrative application. Our DEA approach, however, suggests the additional possibility of new approaches obtained from PFT-NFT combinations, which may be superior to either of them alone. Validating such possibilities cannot be done only by statistical or other modelings. It requires recourse to field studies, including audits (e.g., of a U.S. General Accounting Office variety) and therefore ways in which the results of a DEA approach may be used to guide such further studies (or audits) are also indicated.
(PROGRAM EFFICIENCY; MANAGERIAL EFFICIENCY; EFFICIENCY FRONTIERS)

[T2A5] : [6 , 3 , 5]

DEA and the Discriminant Analysis of Ratios for Ranking Units
Z. Sinuany-Stern and L. Friedman
European Journal of Operational Research-1998

The purpose of this study is to develop a new method which provides for given inputs and outputs the best common weights for all the units that discriminate optimally between the efficient and inefficient units as pregiven by the Data Envelopment Analysis (DEA), in order to rank all the units on the same scale. This new method, Discriminant Data Envelopment Analysis of Ratios (DR/DEA), presents a further post-optimality analysis of DEA for organizational units when their multiple inputs and outputs are given. We construct the ratio between the composite output and the composite input, where their common weights are computed by a new non-linear optimization of goodness of separation between the two pregiven groups. A practical use of DR/DEA is that the common weights may be utilized for ranking the units on a unified scale. DR/DEA is a new use of a two-group discriminant criterion that has been presented here for ratios, rather than the traditional discriminant analysis, which applies to a linear function. Moreover, non-parametric statistical tests are employed to verify the consistency between the classification from DEA (efficient and inefficient units) and the post-classification as generated by DR/DEA.
Keywords: Data envelopment analysis; Discriminant analysis; Ranking; Scaling; Multicriteria decision analysis

[T1A5] : [4 , 5 , 7]

Although there is always a subjective side to selecting illustrative articles, those selected here are intended to be representing different periods and by the same meaning different generations of researchers, different journals, different domains of application and different research strategies. In all cases the papers included are among the most cited and the most often refereed as pioneering in the topic they discuss.

3.7. Content Analysis:

All post-1996 articles listed in the Gattoufi et al (2001a) updated bibliography were classified using the classification scheme described in the previous section. The number of the articles totaled 989 published in 297 journals. Two sets of results follow:

- Results of classification taking into account the nature of the article resulting in three categories. These are: Strictly theory, strictly application, and a contribution to theory followed by an application with real-world data used validate the theory. This set of results is provided in *Table 3.4*.
- The results based on the research strategy used in the article. As described in an earlier subsection, seven strategies are considered in this classification. This set of results is provided in *Table 3.5*.

3.7.1. Nature of Paper: Classification Results

It is important to emphasize that while a primary strategy is required for each article, a secondary or tertiary are not. Thus in some articles classified, two or three strategies were invoked while in others only a primary strategy was adopted.

This section is devoted to provide more in depth analysis of the results reported in *Table 3.4* and to draw some conclusions about the current *"state of the art"* in DEA. Additionally, some speculations about future trends in the literature are provided.

Table 3.4: Classification based on the nature of the Articles

	1996	1997	1998	1999	2000	2001*	Total		
T1	32	20	11	19	14	12	108	T	
T2	17	7	13	21	20	23	101	209	21%
A1	0	0	1	0	0	0	1		
A2	0	0	1	0	0	0	1		
A3	3	2	3	5	1	1	15		
A4	20	18	43	72	58	23	234	A	
A5	49	47	57	34	35	42	264	515	52%
T1A2	1	1	5	0	3	1	11		
T1A3	9	4	5	3	3	2	26		
T1A4	17	10	19	22	13	11	90		
T1A5	[illegible]	6	8	15	2	9	60		
T2A2	0	0	3	2	3	3	11		
T2A3	1	2	1	4	5	1	14		
T2A4	3	1	5	9	7	11	36	T+A	
T2A5	2	2	5	3	3	2	17	265	27%
Tot Per Year	165	136	171	209	167	141	989		

* As of August 2001

Starting with results shown in *Table 3.4*, this classification shows remarkable results. Articles discussing an *"application to real world"*, dominated the DEA-literature for the period 1996-2001. This is confirmed by the fact that among the 989 articles classified, 209 or 21% are *Theoretical*, T-type in the classification scheme, 515 or 52% as *Application*, A-type in the classification scheme, and 265 or 27% provide advances to theory followed by an application in the real world using either real data or data grounded in the real world. The latter were classified as A+T-type in terms of the classification scheme.

Furthermore, among the 515 A-type articles mentioned above, 498 were pure real-world applications using data from realization of real world processes, classified as sub-types A4 or A5 in the classification scheme, and only 17 as a total for A1, A2 and A3 categories. Among the articles presenting both advances in theory and in application, the dominance of pure real world applications was less contrasted yet it is still significant. Among the 265 T+A-type articles, 203 were classified as T+A4 or T+A5. In total, 701 out of 989 or 71% of the articles used real world data.

As for the T-type theoretical articles, it is interesting to notice the even distribution between the two sub-classes. Among the 209 T-type articles, 108 or 52% were purely theoretical in nature and classified as T1-type. Thus 108 out of 989 or 11% is the proportion of articles that are pure theoretical in nature.

As a final remark, it is important to note that even in the theoretical articles, the share of T2-type is steadily increasing over the years at the opposite of that of T1-type. This reflects the tendency of DEA researchers to get away from solely providing models having high level of abstraction. They provide an illustration of the theory using data at least synthetic.

3.7.2. Research Strategies Used in DEA literature: Classification Results

We turn now to an analysis of the results dealing with the research strategies used in the DEA-literature for the period 1996-2001, as reported in *Table 3.5*.

The research-strategy-based classification results complement and reinforce those provided by the nature-based classification regarding the high involvement of real world data in the DEA-literature. Among the 989 articles reviewed, 394 or 40% were identified as primarily using the *"creative application"*, 521 or 53% used this strategy either as primary or as secondary one and 574 or 58% used this strategy as primary, secondary or tertiary. The *"structuring"* strategy, highly involving real world, is used 143 or 14% at the primary level and 161 or 16% at either the primary or the secondary level. As primary strategy, 54% uses either *"structuring"* or *"creative application"*.

Table 3.5: Classification based on the research strategy(ies) used

		1996	1997	1998	1999	2000	2001	Total				
1*	P	65	41	40	54	37	43	280	28%			
	S	1	2	1	3	2	2	11	1%	291	29%	
	T	0	0	0	0	0	0	0	0%	291	29%	17%***
2*	P	12	10	13	25	22	15	97	10%			
	S	13	22	45	25	28	21	154	16%	251	25%	
	T	0	1	1	0	0	1	3	0%	254	26%	15%***
3*	P	1	0	2	0	3	0	6	1%			
	S	15	15	8	20	15	17	90	9%	96	10%	
	T	2	0	1	5	1	1	10	1%	106	11%	6%***
4*	P	11	9	14	15	9	6	64	6%			
	S	8	8	8	14	14	12	64	6%	128	13%	
	T	2	3	2	8	4	4	23	2%	151	15%	9%***
5*	P	60	52	86	84	69	43	394	40%			
	S	22	24	18	28	21	14	127	13%	521	53%	
	T	10	8	9	10	8	8	53	5%	574	58%	33%***
6*	P	15	24	15	30	27	32	143	14%			
	S	2	2	2	8	2	2	18	2%	161	16%	
	T	0	0	0	0	0	2	2	0%	163	16%	10%***
7*	P	1	0	1	1	0	2	5	1%			
	S	10	[illegible]	[illegible]	18	11	11	75	8%	80	8%	
	T	9	15	25	14	15	18	96	10%	176	18%	10%***
								1715**				

*** 1=Ripple; 2=Embedding; 3=Transfer of Technology; 4=Bridging; 5=Creative Application; 6=Structuring; 7=Statistical Modeling)**

**** The total number of hits for all the classification (number of strategies' uses in the classification, regardless their type and their level.**

***** Percentage of the number of hits per strategy out of the total number of hits in (**)**

However, 280 articles (28%) invoked the *"ripple"* as a primary process and in 291 articles (29%) as primary secondary or tertiary. The *"ripple"* constitutes the second most invoked research strategy in DEA literature, particularly as primary strategy. In fact, it was rarely used as secondary strategy and never used as tertiary one.

"Anemia in relevance to real world" an important symptom of a "natural drift" (Corbett, and Van Wassenhove (1993)) away from the "swamp of relevance" (Miser (1987)) would be characterized by a trend toward an excessive use of the *"ripple"*

strategy especially in T-type, A1-type and A2-type articles. Another symptom of the above problems would be a decrease in the use of *"embedding"* and *"bridging"* strategies. This would reflect on the *"introversion"* (Ackoff (1987)) of the field and its *"impotence"* to develop new horizons and establish bridges with new fruitful worlds. This is fortunately not yet the case with the DEA-literature, at least not for the period studied. However, researchers in DEA should be aware of the danger and prevent *"the son from committing the father's sin"*.

Although the use of *"embedding"* and *"bridging"* strategies in the recent DEA literature did not register a spectacular increase over time, it never decreased. These two were often used as secondary strategies with the former used at the primary level in 97 articles (10%) and in 154 articles (16%) at either primary or secondary level. The latter strategy was invoked at the primary or the secondary levels in 64 articles (6%).

Finally, *"statistical modeling"* was particularly invoked as a tertiary strategy and rarely at the primary level. The *"transfer of technology"* strategy was the least used in this literature particularly not as a primary strategy. The high use of "*bridging*" and "*embedding*" strategies compared to the use of *"transfer of technology"* can be considered as an indicator of the degree of *"expansionism"* that characterizes the recent DEA-literature. This notion can be justified by the fact that DEA did not merely develop its own *"technology"*, it *"exported"* to an increasing number of fields and domains.

3.8.Concluding Remarks:

Having said all this, is DEA heading the way of U.S. based academic OR/MS in general? With due consideration to the fact that the DEA theory advancing articles are becoming more and more mathematically sophisticated there are no other indications of such being the case. On the contrary the ratio of articles strictly reporting advances to theory compared to those applying it to an ever wider and disparate arena of real world problems is 21%.

However, does the decline in the number of journals per year, after the pick of 104 reached in 1999, associated with the decline in the number of articles published may be explained by a saturation? Or does it indicate that the most glorious days of DEA are past? The ongoing updating and analysis for the full census of the year 2001 may provide further insight.

As a final note, one can say that DEA committed the *"sin of the father"* by acquiring, like *"Operations Research"* a misleading and not a very appealing *"brand name"*. Will the son share the father's destiny? This is presumes agreement about who is *"the fatherhood"*. Does such agreement exist? Can it even be questioned or does it belong to *"taboo"* issues?

CHAPTER 4: A TAXONOMY FOR DATA ENVELOPMENT ANALYSIS

4.1. Introduction

Data Envelopment Analysis (DEA) has enjoyed a high number and a high incidence of real-world applications. This can be observed via a casual perusal of the DEA based literature. However it happens to be a fact that is documented via *content analysis*, as in Reisman and Kirschnick (1994 and 2000), of all DEA articles appearing in refereed journals between 1951 and 2001 Gattoufi (2001b)]. Moreover, the real world acceptance of DEA is in stark contrast to many other and older OR/MS sub-disciplines over their entire life-times (Reisman et al., 1997a & b and 2001). Presented here is a taxonomic framework for classifying this literature.

4.2. Relevance of a Taxonomy for DEA: a Discussion

Like in any new sub-discipline of Management Science, the Data Envelopment Analysis literature is growing at an exponential rate (See Figure 3.1). This literature is recording advancements in theory and in solution methodology while at the same time expanding the universe of its applications.

Seiford traced the evolution of Data Envelopment Analysis from the early articles of Koopmans (1951), Debreu (1951) and Farrell (1957), largely considered as providing the seminal idea for DEA and the pioneer paper published by Charnes et al (1978)[1], judged as the one announcing birth to DEA, until 1995. He provides an analysis of the typical features of each five-years-period. He reports the topics rankings for the most influential papers and what he reported about the most influential papers was partially confirmed by a citation analysis realized by Sarafoglou (1998). Seiford (1996) reports also the novel applications and the advances gained in the theory of DEA. Finally, he claims that "stochastic DEA is the most critical and difficult future issue in DEA."

[1] Although there is a large agreement about 1978 as the "official birth" of DEA, articles that are considered as providing the seminal idea of DEA are often included in the DEA bibliography. The list includes Koopmans (1951) Debreu (1951) and Farrell (1957).

(Seiford 1996, pp: 106-107). Although the topic was not new to the DEA literature, the period 1996-2001 saw a jump in the number of articles attempting to provide DEA with the well appreciated and appealing stochastic nature. However, one can question if it was a natural trend or a "bias" in the trend made by an "influential paper" produced by an "influential Seiford"? Was DEA in need to develop into that direction or it was "the aftermaths of the natural drift" experienced by some OR/MS researchers? How much success does these developments realized in terms of implementation to real world? One fact however is to mention: the gap between "parametric methods" and DEA was significantly reduced, but was it the right direction? More debate and more insights are needed to understand the ongoing of the field by having a larger view through a more systematic analysis.

The time is now ripe for a general mapping of this literature in a manner that will provide a vivid and panoramic view of what exists and will clearly identify any existing gaps in the state of the art. Such a taxonomic approach was suggested by Reisman (1993), yet, Seiford (1996) DEA literature survey of record and Seiford (1997) bibliography as well as Berger & Humphrey (1997) survey of record of efficiency in financial institutions lack taxonomic features. Hence there is a need for a taxonomy of the DEA literature.

> "Graphically, symbolically or both, they vividly display the similarities and the differences among the various contributions, thus demonstrating the relationship of all contributions and the practical applications of each to other. They provide a framework by which all of the existing knowledge can be systematically filed and therefore recalled efficiently and effectively. By providing what amounts to an aerial view- a picture of the "territory"- they often identify the voids in the literature." Emphasizing the role of taxonomy as knowledge consolidation means, the author writes "knowledge consolidation is a means to various ends, and it is also an end itself. It is a means toward the end of more efficient and more effective teaching and learning of new or existing knowledge. It is a means toward the end of more efficient storage and more effective recall and/or retention of knowledge. It is a means toward a more efficient and more effective processes of research leading to the yet unknown, to the design of the yet unavailable, and it is means toward more efficient problem solving..." (Reisman 1993, pp. 29)

However, for usefulness and effectiveness in the validation of taxonomy, several features are required.

> "The key to taxonomy effectiveness rests on criteria of comprehensiveness, parsimony and usefulness. Obviously, to be effective, a taxonomy must represent the full spectrum of the research chosen for categorization. Thus, comprehensiveness is a necessary condition for effectiveness. It is, however, not sufficient. To further be effective, a taxonomy should be parsimonious. It should not include unnecessary categories. Finally, to be considered effective, the taxonomy should be robust and generally useful. The categories should be reasonably if not mutually exclusive, i.e., non-overlapping, reasonably distinct, meaningful, commonplace, and descriptive to allow utilization by a wide variety of interested persons". (Vogel and Weterbe 1984, pp.)

Hence a taxonomy is not only a tool for systematic storage of knowledge but it is also a neat way of pointing to knowledge expansion and building. It identifies voids, potential theoretical increments or developments and potential applications for the existing theory. DEA has generated a large enough amount of knowledge allowing it to be considered as a separate field of knowledge. The increasing interest in DEA as an alternative tool for performance measurement makes the elaboration of this field's taxonomy more crucial in helping the already-in researchers as well as in attracting potential newcomers to the field.

This attempt to define a taxonomy for DEA proceeds in an arborescent way adopting the "attribute vector description" (Reisman 1992) as illustrated in *Figure 4.1*.

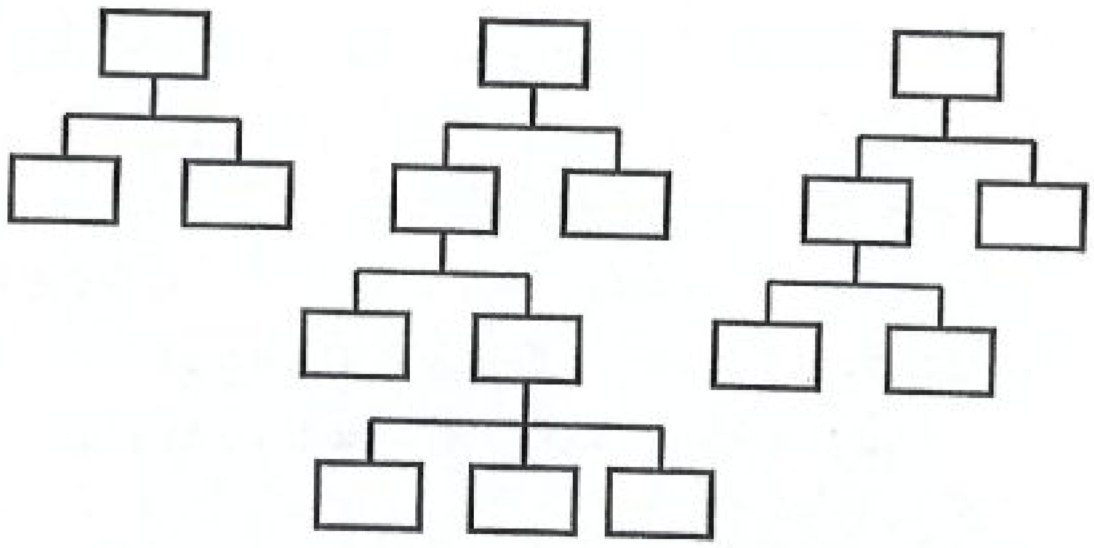


Figure 4.1: Attribute vector description

(Adapted from Reisman, 1992)

As is the case in one of the greatest, and best-known taxonomies of all time, the Periodic Table of Elements, Mendeleyev (1889), what is presented here is open for incremental evolutions. A taxonomy is very much dependent on the definition of the boundaries of the universe it classifies. Hence, the classification developed in this study is open to expansion as the scope of DEA enlarges.

4.3. A Taxonomy for Data Envelopment Analysis literature

As indicated above the DEA related literature is first classified on four basic factors D, E, A. and N. Under each of these factors, the most discriminating attributes are listed. The full taxonomy is illustrated in *Table 4.1*. Using the proposed taxonomy, each study can be given an identification code with sixteen domains grouped in four classes or keys for classification:

Class 1/ Key 1: Data. Is subdivided into two domains, the first domain describes the source of the data and the second describing the degree of imprecision in the data.

Class 2/ Key 2: Envelopment. Is subdivided into six domains characterising the frontier, the mathematical model used for the envelopment of the data and the nature of the measures provided by the analysis.

Class 3/ Key 3: Analysis. Is subdivided into six domains identifying the different options in the analysis related with the sample or with the model.

Class 4/ Key 4: Nature of the study and the methodology it uses. For the nature of paper, it is either a pure theoretical paper or a paper having hypothetical data not grounded in real world or an application of an already developed approach to a real world problem with real or simulated data. The study can be both a theoretical development and its empirical validation to a real world problem using either observed or simulated data. The methodology indicates if the study uses a pure DEA approach or combining DEA to other methodologies. Several combinations are listed.

Table 4.1: DEAN-A Taxonomy for DEA literature

(. . . /. . .) : (. . . /. . . / . . . /. . . / . . . /. . .) : (. . . /. . . /. . . /. . . /. . . /. . .) : (. . . /. . .)

Data — Envelopment — Analysis — Nature/Meth

Key 1: Data.

First level (Field 1): Source of the data.

- 0: No data.
- 1: Hypothetical data.
- 2: Simulated data.
  - 2.1: Monte Carlo simulation
  - 2.2: Production process simulation
- 3: Real world data.
  - 3.1: Not-for-profit organization.
    - 3.1.1: Public.
      - 3.1.1.1: Agriculture.
      - 3.1.1.2: Defence.
      - 3.1.1.3: Education.
      - 3.1.1.4: Energy.
      - 3.1.1.5: Environment.
      - 3.1.1.6: Health care.
      - 3.1.1.7: Public Administartion and its organization.
      - 3.1.1.8: Public finance.
      - 3.1.1.9: Public services other then health care and education.
      - 3.1.1.10: Macro economic aggregated data.
    - 3.1.2: Private.
      - 3.1.2.1: Education.
      - 3.1.2.2: Health care.
      - 3.1.2.3: Social programs and charities.
    - 3.1.3: Public and private
      - 3.1.3.1: Health care

3.2: For-profit organization.
    3.2.1: Agriculture.
    3.2.2: Industry:
        3.2.2.1: Mining industry
        3.2.2.2: Food industry
        3.2.2.3: Energy
        3.2.2.4: Textile
        3.2.2.5: Manufacturing
        3.2.2.6: High technology industry
        3.2.2.7: Roads,buildings and related industry
        3.2.2.8: Metallurgy indusry (Iron and steel,...)
    3.2.3: Services.
        3.2.3.1: Banking.
            3.2.3.1.1: Branch banking
            3.2.3.1.2: Overall activity
            3.2.3.1.3: Cross-country
        3.2.3.2: Transportation.
            3.2.3.2.1: Air
            3.2.3.2.2: Roads
            3.2.3.2.3: Sea and fluvial
        3.2.3.3: Telecommunication and postal services
        3.2.3.4: Computer related services (software,...)
        3.2.3.5: Insurance
        3.2.3.6: Retailing (Pharmacy, Restaurants, ...)
3.3: For-profit and Not-for-profit:
    3.3.1: Industry
    3.3.2: Services
    3.3.3: Agriculture
    3.3.4: Health care
    3.3.5: Education

Second level (Field 2): Degree of imprecision in the data.

0: No data.
1: Cardinal data.
    1.1: Single-valued.
    1.2: Multi-valued bounded.
        1.2.1: Bounded uniformly distributed.
        1.2.2: Bounded non-uniformly distributed
        1.2.3: Fuzzy data
2: Ordinal data.
    2.1: Individual ranking.
    2.2: Categorizing data (clusters of DMUs).
3: Imprecise data.
    3.1: Mixture of individual ordinal and single-valued cardinal factors.
    3.2: Mixture of categorizing ordinal and single-valued cardinal factors.
    3.3: Imprecise data-all forms
4: Missing data

Key 2: Envelopment

First level (Field 3): Stochasticity of the frontier.

1: Deterministic frontier.
    1.1: Single deterministic
    1.2: Multiple deterministic
2: Stochatic frontier.
    2.1: Stochastic factors only.
    2.2: Stochastic factors.
    2.3: Stochastic multipliers and/or weights.
    2.4: More than one form of stochasticity.
3: Deterministic and stochastic.

Second level (Field 4): Special Restrictions:

0: No special restrictions.
1: Cone ratio.
2: Assurance region.
3: Non-discretionary variables.
4: Free disposal hull.
5: Stochastic restrictions and/or relaxations.
6: Other restrictions and/or relaxations.
7:Many simultaneous restrictions and/relaxations
8: Many restrictions/relaxations considered separately

Third level (Field 5): Orientation and Return to scale.

1: Oriented:
  1.1: Input oriented
    1.1.1: Constant return to scale.
    1.1.2: Variable return to scale.
    1.1.3: Both Constant and Variable return to scale are considered.
  1.2: Output oriented
    1.2.1: Constant return to scale.
    1.2.2: Variable return to scale.
    1.2.3: Both Constant and Variable return to scale are considered.
  1.3: Both Input oriented and Output oriented cases are considered.
2: Additive Modellling (combines both input and output orientations):
  2.1: Constant return to scale.
  2.2: Variable return to scale.
  2.3: Both Constant and Variable return to scale are considered.
3: Both oriented and additive modelling are considered separately.
4: Multiplicative modelling

Fourth level (Field 6): Convexity of the mathematical model.

1: Convex Linear model.
  1.1: Continuous linear programming model.
  1.2: Discrete or Mixed linear programming model.
  1.3: Fuzzy linear programming model.
  1.4: Chance constrained or stochastic linear programming model.
  1.5: Graphical presentation
2: Convex non-linear model.
  2.1: Polynomial hard problem.
  2.2: Non-polynomial hard problem
  2.3: Ratio Form.
3: Non-convex linear model.
  3.1: Graphical presentation
4: Non-convex non-linear model.

Fifth level (Field 7): Solving method.

0: No solving method proposed or standard method is used (without mention).
1: Single stage solving method.
  1.1: Exact method explicitely presented.
  1.2: Approximated method.
  1.3: Heuristic method.
  1.4: Meta-heuristic method.
  1.5: Graphical
2: Multi-stage.
  2.1: Exact method explicitely presented.
  2.2: Approximated method.
  2.3: Heuristic method.
  2.4: Meta-heuristic method.

Sixth level (Field 8): Efficiency measures provided by the solution.

1: Single-valued measures.
2: Multi-valued measures.
  2.1: Exact multi-valued measures
  2.2: Fuzzy multi-valued measures
3: Stochastic measures
  3.1: Stochastic single-valued measures
  3.2: Stochastic multi-valued measures

Key 3: Analysis.

First level (Field 9): Purpose

1: Prescriptive
2: Descriptive
3: Planning
4: Predicting and/or forecasting
5: Multi-purposes.

Second level (Field 10): Time horizon.

1: Single period analysis.
2: Multi-period analysis.
  2.1: Time windows.

2.2: Malmquist index.
2.3: Dynamic DEA modelling
2.4: Total factor productivity analysis type.
2.5: Other forms

Third Level (Field 11): Efficiency

1: Technical efficiency
    1.1: Technical and scale efficiencies
    1.2: Technical and scale efficiencies and congestion
2: Cost efficiency (Allocative and technical efficiencies)
3: Incentive efficiency (adapted from agency theory in game theory)
4: Different types of efficiency studied.

Fourth level (Field 12): Level of aggregation in the analysis for real world applications.

0: Not real world problem.
1: Unit level
2: Organization level.
3: System level.
4: Multi-level.
5: Cross-systems analysis
    5.1: National or Local level.
    5.2: Cross-country level.

Fifth level (Field 13): Sensitivity Analysis and Robustness:

0: No sensitivity analysis
1: Pre-optimal sensitivity analysis
2: Post-optimal Sensitivity Analysis
    2.1: Sensitivity of the measures to data charateristics:
        2.1.1: Sensitivity to model specifications (Factors, orientation, type of efficiency, ...)
        2.1.2: Sensitivity to sample size
        2.1.3: Sensitivity to sample spread (outliers, homogeneity, ...)
    2.2: Sensitivity of the measures to variations in factors values.
    2.3: Sensitivity to stochasticity in data.
    2.4: Sensitivity to restrictions/relaxations.
    2.5: Sensitivity of stochasticity in the frontier.
3: Robustness and stability of the results analysis (rankings, measures, ...).
4: More than one type of sensitivity analysis.
5: Other forms of sensitivity analysis

Sixth level (Field 14): Technique(s) used for sensitivity and robustness analysis:

0: No sensitivity analysis
1: Analytical analysis
2: Empirical analysis
3: Simulation:
    3.1: Monte Carlo.
    3.2: Bootstrapping.
    3.3: Least trimmed squares.
4: Statistical tests.
5: Econometric modelling.
6: Other techniques.
7: Combination of several techniques.

Key 4: Nature of the study and the methodology it uses:

First level (Field 15): Nature

1: Theoretical.
2: Application in:
    2.1: Finance
    2.2: Production and operations management
    2.3: Industrial organization
    2.4: Marketing
    2.5: Human resources management
3: Theory and its emperical validation
    3.1: Finance
    3.2: Production and operations management
    3.3: Industrial organization
    3.5: Marketing
    3.6: Human resources management

Second level (Field 16): **Methodology**

1: Paper having the form of general analysis (without mathematical modelling), literature survey, book or software review, comments, short reply and erratum.
2: DEA and/or its extensions.
3: Study comparing DEA with other methodologies used in the same context.
4: Methodology combining DEA with Economic Theory
- 4.1: DEA and Micro-Economics/Firm Theory
  - 4.1.1: DEA and Production Theory
  - 4.1.2: DEA and Firm Theory
- 4.2: DEA and Game Theory
  - 4.2.1: DEA and Agency Theory
- 4.3: DEA and Econometrics Theory

5: Methodology combining DEA with other OR/MS techniques
- 5.1: DEA and Goal Programming
- 5.2: DEA and Multi Criteria Decision-Making
- 5.3: DEA and Fuzzy Sets Theory
- 5.4: DEA and Supply Chain Management
- 5.5: DEA and Total Quality Management
- 5.6: DEA and Production Management Techniques (Line Balancing, MRP
- 5.7: DEA and Inventory Theory
- 5.8: DEA and Queuing Theory
- 5.9: DEA and Scheduling
- 5.10: DEA and Stochastic Programming
- 5.11: DEA and general forms of Mathematical Programming
- 5.12: DEA and Multi Objective Linear Programming
- 5.13: DEA and Network Theory

6: Methodology combining DEA with Statistic:
- 6.1: DEA and Statistical Inference Theory
- 6.2: DEA and Discriminant Analysis
- 6.3: DEA and Cluster Analysis
- 6.4: DEA and Statistical Testing
- 6.5: DEA and Bootstrapping technique

7: Methodology combining DEA with Organization Theory
8: Methodology combining DEA with several other techniques or approaches
9: Comparative study for different combinations
10: Methodology combining DEA with Marketing Theory

4.4. Illustrative Examples

In this section, the above taxonomy will be illustrated by several examples chosen to demonstrate the descriptive power, comprehensiveness, and the parsimony of the taxonomy. Starting with two theoretical articles, the first one presented the initial formulation and the second a more recent generalizing the first formulation of DEA. It was published in Management Science, one of the US-based OR/MS flagship journals. A follow up article (Cooper et al, 2000) provided solution for the minor shortcomings the latter presented. Thus the classification of these papers will portray the degree of changes and elaboration in the more recent IDEA model.

Measuring the Efficiency of Decision Making Units
A. Charnes, W.W. Cooper and E. Rhodes
European Journal of Operational Research-1978

A nonlinear (nonconvex) programming model provides a new definition of efficiency for use in evaluating activities of not-for-profit entities participating in public programs. A scalar measure of the efficiency of each participating unit is thereby provided, along with methods for objectively deter- mining weights by reference to the observational data for the multiple outputs and multiple inputs that characterize such programs. Equivalences are established to ordinary linear programming models for effecting computations. The duals to these linear programming models provide a new way for estimating extremal relations from observational data. Connections between engineering and economic approaches to efficiency are delineated along with new interpretations and ways of using them in evaluating and controlling managerial behavior in public programs.

As subcase of the DEAN taxonomy it has therefore the following classification:

(0 / 0) : (11 / 0 / 121 / 11 / 11 / 1) : (3 / 1 / 1 / 0 / 0 / 0) : (1 / 2)

IDEA and AR-IDEA: Models for Dealing with Imprecise Data in DEA
W.W. Cooper, K.S Park and G. Yu
Management Science-1999

Data Envelopment Analysis (DEA) is a nonparametric approach to evaluating the relative efficiency of decision making units (DMUs) that use multiple inputs to produce multiple outputs. An assumption underlying DEA is that all the data assume the form of specific numerical values. In some applications, however, the data may be imprecise. For instance, some of the data may be known only within specified bounds, while other data may be known only in terms of ordinal relations. DEA with imprecise data or, more compactly, the Imprecise Data Envelopment Analysis (IDEA) method developed in this paper permits mixtures of imprecisely- and exactly-known data, which the IDEA models transform into ordinary linear programming forms. This is carried even further in the present paper to comprehend the now extensively employed Assurance Region (AR) concepts in which bounds are placed on the variables rather than the data. We refer to this approach as AR-IDEA, because it replaces conditions on the variables with transformations of the data and thus also aligns the developments we describe in this paper with what are known as cone-ratio envelopments in DEA. As a result, one unified approach, referred to as the AR-IDEA model, is achieved which includes not only imprecise data capabilities but also assurance region and cone-ratio envelopment concepts.
Keywords: DEA Efficiency; Imprecise Data; Assurance Regions

As subcase of the DEAN taxonomy this paper has the following classification:

(1 / 33) : (11 / 2 / 111 / 22 / 21 / 1) : (1 / 1 / 1 / 0 / 0 / 0) : (1 / 2)

The original paper did not mention data. Implicitly, it assumed the classical single-valued data points to pertain. The latter identifies two types of *"imprecision"* in data: bounded data and ordinal data. It goes on to provide an approach for dealing with each of these forms of imprecision. Also, while the original paper imposes no restrictions on the weights, the latter allows for an *"Assurance Region"* to be imposed to the weights. Finally, while the original model is easily transformed into a standard linear program, the latter requires elaborate multi-stage transformations to bring the non-linear model into the standard DEA format. These differences, and more, are made clear and easily noticed by the DEAN-classification.

The paper by Hao et al (2000) involves no real-world or synthetic data. It illustrates the *"embedding research strategy"* of research (Reisman 1988) by providing a game theory framework for the Generalized DEA (GDEA). A generalized convex cone constrained efficiency game model to assemble GDEA is proposed and the equivalence between the efficiency game and DEA efficiency is established.

A Game Theoretical Model of DEA Efficiency
G. Hao, Q.L. Wei and H. Yan
Journal of the Operational Research Society-2000

Motivated by the inherent competitive nature of the DEA efficiency assessment process, some effort has been made to relate DEA models to game theory. Game theory is considered not only a more natural source of representing competitive situations, but also beneficial in revealing additional insights into practical efficiency analysis. Past studies are limited to connecting efficiency games to some particular versions of DEA models. The generalised DEA model considered in this study unifies various important DEA models and presents a basic formulation for the DEA family. By introducing a generalised convex cone constrained efficiency game model in assembling the generalised DEA model, a rigorous connection between game theory and the DEA family is established. We prove the existence of optimal strategies in the generalised efficiency game. We show the equivalence between game efficiency and DEA efficiency. We also provide convex programming models for determination of the optimal strategies of the proposed games, and show that the game efficiency unit corresponds to the non-dominated solution in its corresponding multi-objective programming problem. Our study largely extends the latest developments in this area. The significance of such an extension is for research and applications of both game theory and DEA.
Keywords: convex cone, data envelopment analysis, game theory

This paper is a contribution to theory. It makes no pretence of describing an application. This subcase of the DEAN taxonomy is established as:

(0 / 0) : (11 / 1 / 113 / 11 / 11 / 1) : (1 / 1 / 1 / 0 / 0 / 0) : (1 / 42)

The following two abstracts represent purely technical papers, although illustrated with real world data. They deal with the well-known non-Archimedian epsilon involved in formulating the linear program used in DEA. To ensure that the variables of the models, the weights for inputs and outputs, are accorded some worth, they are constrained to be not only positive but also greater than any positive real number. This is obtained by considering, as lower bound for these variables, a non-Archimedian infinitesimal smaller than any positive real number (Related ref. cited in chapter 2).

Computational Accuracy and Infinitesimals in Data Envelopment Analysis
A.I. Ali and L.M. Seiford
INFOR-1993

An analysis clarifies the role of the non-Archimedean infinitesimal in the Charnes, Cooper, and Rhodes (1978) and Banker, Charnes, and Cooper (1984) models, used in data envelopment analysis (DEA). The analysis establishes that the associated dual linear programs can be infeasible for the multiplier side and unbounded for the associated dual envelopment side program. Sufficient conditions are established for feasibility and boundedness. Computational testing indicates that the improper selection of a value for the non-Archimedean infinitesimal can result in serious errors. While there exists a threshold value for the infinitesimal that yields finite solutions, smaller values may disguise equally serious errors. When a numerical value is used to represent the non-Archimedean infinitesimal, results are sensitive not only to the specific value for the infinitesimal but also to the pricing tolerances that are employed in standard linear programming software.

This is the: it has therefore the following classification:

(3117 / 11) : (11/ 0 / 113 / 11/ 11/ 1) : (1/ 1/ 1/ 3 / 0 / 0) : (1/ 2)

subcase of the DEAN taxonomy.

The next paper provides a counterexample for the above and provides an alternative way to define an assurance interval for epsilon. The paper uses is illustrated by the same set of data. Both use the same basic DEA models. The two studies are so close that the DEAN taxonomy cannot distinguish between them. It is worth to mention that this article was among those rare ones published in *Operations Research*, the flagship of the *"neoclassic orthodoxy"* of OR/MS scientific community.

An Assurance Interval for the Non-Archimedian Epsilon in DEA Models
S. Mehrabian, G.R. Jahanshahloo, M.R. Alirezaee and G.R. Amin
Operations Research-2000

This paper clarifies the role of non-Archimedean infinitesimal ε in DEA models so that the associated linear programs may be infeasible (for the multiplier side) and unbounded (for the envelopment side) for certain values of e. It is shown that the bound of ε proposed by Ali and Seiford (1993) is invalid for feasibility and boundedness of the linear programs. A procedure is presented for determining an assurance interval of ε. It is also shown that an assurance value for ε can be found using a single linear program.

The above is the

(3117 / 11) : (11/ 0 / 113 / 11/ 11/ 1) : (1/ 1/ 1/ 3 / 0 / 0) : (1/ 2)

subcase of the DEAN taxonomy.

At the other extreme, the papers by Oral and Yolalan (1990) and Avkiran (2001) involve real-world data. They represent a contribution to practice and make no even pretence of contributing to DEA theory. The formulation and the modelling of the real-world problems is itself one of the research strategies identified in Reisman et al. (1994, 1995, 1997 and 2001).

Banking performance analysis and particularly branch performance in banking was the subject of many applications of DEA. Berger & Humphrey (1997), Camanho & Dyson (199) and Thanassoulis (1999) provide good reviews of the literature applying DEA, pure or embedded in a general methodology with other techniques, to analyse the performance of financial institutions in general, and banks in particular.

Reported next is a pioneering, and often cited, application of DEA to the analysis of branch efficiency in a private bank in Turkey.

An Empirical Study on Measuring Operating Efficiency and Profitability of Bank Branches
M. Oral and R. O Yolalan
European Journal of Operational Research-1992

This paper discusses the methodology of an empirical study that was employed to measure the operating efficiencies of a set of 20 bank branches of a major Turkish Commercial Bank offering relatively homogeneous products in a multi-market business environment. The methodology was based on the concepts and principles of Data Envelopment Analysis (DEA). The results of the study have indicated that this kind of approach is not only complementary to traditionally used financial ratios but also a useful bank management tool in reallocating resources between the branches in order to achieve higher efficiencies. It has been also observed that the service-efficient bank branches were also the most profitable ones, suggesting the existence of a relationship between service efficiency and profitability.
Keywords: Efficiency, productivity, performance evaluation, banking, mathematical programming

It is the: **(32311/11) : (11/0/111/11/11/1) : (2/1/1/2/0/0) : (22/2)**
subcase of the DEAN taxonomy.

The next application analyzes performances of the Australian higher education systems. Although a large number of DEA applications have been devoted to analyze the performance of education systems, components of education systems or even related fields, researchers appear to be attracted to this fructuous field. This permanent attractiveness and permanent interest of DEA researchers, or "bias" in the trend of DEA applications, can be related to the initial DEA application, which was actually devoted to the evaluation of an education program.

Investigating Technical and Scale Efficiencies of Australian Universities Through Data Envelopment Analysis
N.K. Avkiran
Socio-Economic Planning Sciences-2001

Performance indicators in the public sector have often been criticised for being inadequate and not conducive to analysing efficiency. The main objective of this study is to use data envelopment analysis (DEA) to examine the relative efficiency of Australian universities. Three performance models are developed, namely, overall performance, performance on delivery of educational services, and performance on fee-paying enrolments. The findings based on 1995 data show that the university sector was performing well on technical and scale efficiency but there was room for improving performance on fee-paying enrolments. There were also small slacks in input utilisation. More universities were operating at decreasing returns to scale, indicating a potential to downsize. DEA helps in identifying the reference sets for inefficient institutions and objectively determines productivity improvements. As such, it can be a valuable benchmarking tool for educational administrators and assist in more efficient allocation of scarce resources. In the absence of market mechanisms to price educational outputs, which renders traditional production or cost functions inappropriate, universities are particularly obliged to seek alternative efficiency analysis methods such as DEA.
Keywords: DEA; University performance; Technical efficiency; Scale efficiency

This is the
(3113/11) : (11/0/123/11/11/1) : (2/1/1/3/0/0) : (23/2)
subcase of the DEAN taxonomy.

In between the two extremes of pure theoretical or technical papers and pure empirical application, there is a large range of DEA publications. We provide next a selection of papers with different degrees of theoretical elaboration while involving empirical validation with a real-world application.

The paper, which first introduced DEA theory to the world e.g., Charnes, Cooper, and Rhodes, E. (1978) described a methodology developed by the authors and used in a real world application. A follow-up paper by the same authors (A. Charnes, W.W. Cooper, and Rhodes, E. (1981)) provides further discussions and details about the real world application implementing the new methodology. The authors were concerned with the evaluation of a nation-wide large-scale social experiment in public school education. DEA was used to test the advantages of the schools involved in the social program as compared to their counterparts not adopting it in various parts of the U.S.

Evaluating Program and Managerial Efficiency: An Application of Data Envelopment Analysis to Program Follow Through

A Charnes, W.W. Cooper and E. Rhodes

Management Science-1981

A model for measuring the efficiency of Decision Making Units (= DMU's) is presented along with related methods of implementation and interpretation. The term DMU is intended to emphasize an orientation toward managed entities in the public and/or not-for-profit sectors. The proposed approach is applicable to the multiple outputs and designated inputs, which are common for such DMU's. A priori weights, or imputations of a market-price-value character are not required.

A mathematical programming model applied to observational data provides a new way of obtaining empirical estimates of extremal relations-such as the production functions and/or efficient production possibility surfaces that are a cornerstone of modern economics. The resulting extremal relations are used to envelop the observations in order to obtain the efficiency measures that form a focus of the present paper.

An illustrative application utilizes data from Program Follow Through (= PFT). A large scale social experiment in public school education, it was designed to test the advantages of PFT relative to designated NFT (= Non-Follow Through) counterparts in various parts of the U.S. It is possible that the resulting observations are contaminated with inefficiencies due to the way DMU's were managed en route to assessing whether PFT (as a program) is superior to its NFT alternative. A further mathematical programming development is therefore under- taken to distinguish between "management efficiency" and "program efficiency." This is done via procedures referred to as Data Envelopment Analysis (= DEA) in which one first obtains boundaries or envelopes from the data for PFT and NFT, respectively. These boundaries provide a basis for estimating the relative efficiency of the DMU's operating under these programs. These DMU's are then adjusted up to their program boundaries, after which a new inter-program envelope is obtained for evaluating the PFT and NFT programs with the estimated managerial inefficiencies eliminated.

The claimed superiority of PFT fails to be validated in this illustrative application. Our DEA approach, however, suggests the additional possibility of new approaches obtained from PFT-NFT combinations, which may be superior to either of them alone. Validating such possibilities cannot be done only by statistical or other modelings. It requires recourse to field studies, including audits (e.g., of a U.S. General Accounting Office variety) and therefore ways in which the results of a DEA approach may be used to guide such further studies (or audits) are also indicated.

(PROGRAM EFFICIENCY; MANAGERIAL EFFICIENCY; EFFICIENCY FRONTIERS)

This is the: **(3113/11) : (11/0/113/11/11/1) : (3/1/1/3/0/0) : (33/2)**
subcase of the DEAN taxonomy

The following recent publications are shown as examples to further illustrate differences in more specific sub-attributes of the DEAN taxonomy. These papers are characterized by both a contribution to the DEA theory and applying the advances they provide to real world situations.

The first paper deals with a facet of inefficiency: congestion. Although early DEA studies showed little interest in the congestion component of inefficiency, there is recently an increasing interest in analyzing congestion in production processes and its effects on efficiency. As reported in Cooper et al (2001), while the interest in this topic were reawakened by Fare & Svenson (1980), Fare & Groskopf (1983) gave it an implementable format. Recently, congestion was the topic of several papers. These provided both theoretical developments and some applications. The most recent of these, Cooper et al (2001), is classified next.

Using DEA to Improve the Management of Congestion in Chinese Industries (1981-1997)
W.W. Cooper, H. Deng, B. Gu, S. Li and R.M. Thrall
Socio-Economic Planning Sciences-2001

Congestion is said to be present when increases in inputs result in output reductions. An ''iron rice bowl'' policy instituted in China shortly after the revolution led by Mao Tze Tung resulted in congestion that ultimately led to bankruptcy in the textile industry, and near bankruptcy in other industries. A major policy shift away from the ''iron rice bowl policy'' in 1990 led to massive layoffs and increasing social tensions. Were these massive layouts necessary? Extensions of data envelopment analysis models effected in the present paper identified inefficiencies in the management of congestion. Using textiles and automobiles for illustration, t is shown how elimination of such managerial inefficiencies could have led to output augmentation without reducing employment. Thus, even in the presence of congestion, it proved to be possible to identify additional (managerial) inefficiencies that provided opportunities for improvement. In the heavily congested textile industry, these output augmentations could have been accompanied by reductions in the amounts of capital used (as an added bonus).In any case, we show how to identify and evaluate new types of efficiency-viz., the efficiency with which needed (or desired) inefficiencies are managed.
Keywords: Efficiency; Congestion; Employment; Data envelopment analysis.

This is:

(3225 / 11) : (11/ 0 / 122 /11/ 11/ 1) : (3 / 1/ 1/ 3 / 0 / 0) : (22 / 2)

subcase of the DEAN taxonomy

The second paper is another illustration of the *"bridging"* research strategy identified by Reisman et al. (1994). DEA is embedded with Discriminant Analysis (DA) in a clever way making the latter complementary to the former. While DEA provides a binary discrimination of DMUs involved, DA proceeds to a second step in order to generate a ranking for the DMUs consistent with the results provided by DEA in the first step. This paper partially deals then with the problem of classifying efficient DMUs to which several studies were devoted, see for example Sinuany-Stern et al (1994) and Cooper and Tone (1997) among others.

DEA and the Discriminant Analysis of Ratios for Ranking Units
Z. Sinuany-Stern and L. Friedman
European Journal of Operational Research-1998

The purpose of this study is to develop a new method which provides for given inputs and outputs the best common weights for all the units that discriminate optimally between the efficient and inefficient units as pregiven by the Data Envelopment Analysis (DEA), in order to rank all the units on the same scale. This new method, Discriminant Data Envelopment Analysis of Ratios (DR/DEA), presents a further post-optimality analysis of DEA for organizational units when their multiple inputs and outputs are given. We construct the ratio between the composite output and the composite input, where their common weights are computed by a new non-linear optimization of goodness of separation between the two pregiven groups. A practical use of DR/DEA is that the common weights may be utilized for ranking the units on a unified scale. DR/DEA is a new use of a two-group discriminant criterion that has been presented here for ratios, rather than the traditional discriminant analysis, which applies to a linear function. Moreover, non-parametric statistical tests are employed to verify the consistency between the classification from DEA (efficient and inefficient units) and the post-classification as generated by DR/DEA.

Keywords: Data envelopment analysis; Discriminant analysis; Ranking; Scaling; Multicriteria decision analysis

This is:

(3117 / 11) : (11/ 0 / 1 /11/ 11/ 1) : (2 / 1/ 1/ 3 / 5 / 4) : (33 / 62)

subcase of the DEAN taxonomy.

Although there is always a subjective side to selecting illustrative papers, the set of papers selected in this classification are intended to be representing different periods, different journals, different domains of application and different research strategies. In all cases, the papers included were among the most cited and the most often refereed as pioneering in the topic they discuss.

4.5. Concluding Remarks

Because our major objective was to identify the voids in the literature we erred on the side what might appear to be an excess of detail. The DEAN taxonomy of *Table 4.1* is perhaps too detailed than called for by common usage. However, it is easier to aggregate data than not to have collected it in sufficient detail. The following facts however confirm and consolidate the discussion provided in Chapter 3.

- The large range of real-world domains to which were applied the DEA.
- The large range of scientific fields with which DEA established bridges.
- Surprisingly enough, although like that there is a large variety of DEA-models developed in the literature, the DEA-literature was mainly based on a single basic model: the original CCR model. There is always reference to this model to indicate the improvements achieved or the changes made on the basic assumptions. None of the subsequent models developed could become a substitute benchmark.
- Surprisingly enough, still many developments can be done. Mention here a basic assumption always adopted in the DEA literature: linearity of the costs. Relaxing this assumption constitutes a new and promising large void for research. None of the articles classified either in this chapter or the previous one did such relaxation.
- A noticeable effort is the "struggle" of DEA researcher to fill the gap between DEA and other stochastic methods, basically regression methods. A large variety of methods were proposed, though not with much success, to provide DEA with the stochastic nature. This work presents yet a new attempt, hopefully with success. However, the beauty of DEA remains, we believe, its simplicity and ability to explain such a developed concept, the efficiency, in a so simple deterministic way.

CHAPTER 5: DATA ENVELOPMENT ANALYSIS: A JUSTIFICATION BASED ON THE WEAK AXIOM OF PROFIT MAXIMIZATION (WAPM) IN THE THEORY OF THE FIRM

5.1. Introduction

From its very outset, the Charnes, Cooper and Rhodes (1978), CCR-model, Data Envelopment Analysis (DEA) has enjoyed a high number and a high incidence of real-world applications. This can be observed via a casual perusal of the DEA based literature. However it happens to be a fact that is documented via *content analysis*, as in Reisman and Kirschnick (1994) and (2000), of all DEA articles appearing in refereed journals between 1978 and 2001 in Gattoufi et al (2001). Moreover, the real world acceptance of DEA is in stark contrast to many other and older OR/MS sub-disciplines over their entire life-times, Reisman et. al. (1997a & b), and (2001). Though the firmness of DEAs roots in the theory of *mathematical programming* is beyond question, this chapter extends these roots into the fertile soil of *economic theory*.

Efficiency measures for *Decision Making Units (DMU's)* provided by the original CCR-DEA model (1978), are herein derived as a natural extension of the *Weak Axiom of Profit Maximization (WAPM)* (Varian, 1992). Additionally, the *sensitivity* of the efficiency measures to increasing numbers of DMUs in the CCR-DEA model is treated analytically[1].

The remainder of this chapter is organized into four sections. Section 2 presents the CCR-DEA model. In section 3, the CCR is analytically justified based on the *Weak Axiom of Profit Maximization.* Section 4 discusses the sensitivity of results obtained from a DEA model to changes in the number of units considered within a given analysis. Section 5 is devoted to drawing some conclusions and discussing future research expanding the two results presented in this chapter.

[1] Although they have theoretical assessment, sensitivity analyses in the OR/MS literature are predominantly experimental in nature. Typically, they involve different form of simulation.

5.2. Data Envelopment Analysis: the Charnes – Cooper - Rhodes (CCR) Model

Productivity measurement and efficiency evaluation methods in economics, business and engineering are ratio-based approaches to assess the performance of economic units, e.g., firms, products, production systems or, in the parlance of DEA, DMUs. Output-to-input ratio measures are commonly used in these fields to evaluate performance of such units. However, most of these approaches are used to provide absolute measures of performance. DEA, although it is also a ratio-based approach, has the distinguished characteristic of always providing *relative* measures of performance for each DMU in a set of such DMUs. The DMUs involved in the analysis are assumed to be homogeneous and competing in the same market while utilizing the same set of inputs to produce the same set of outputs.

The best performers among the DMUs considered are used to define what is called the *efficient frontier*. Specifically this frontier is defined as a convex combination of the best performers, considered to be fully efficient. Deviations from the efficient frontier are interpreted as measures of inefficiency for the remaining DMUs. A *virtually* efficient target, belonging to the efficient frontier, is identified for each inefficient DMU. The radial deviation from the efficient virtual target is interpreted as a measure of inefficiency. Thus the ratio of the radial distance of the virtual efficient target to the radial distance of the corresponding DMU defines the efficiency measure. Its complement is the unit measure of its inefficiency. In less technical and more concrete terms, the efficiency of a given DMU is measured, (in an input oriented DEA), by comparing the inputs it needs to those needed by the most efficient virtual DMU in order to produce an equivalent amount of output. Conventionally, a fully efficient DMU is given *1* (unity) as a measure of efficiency and all efficiency coefficients have non-zero values. The mathematical form of the original DEA model, the ratio form known as the CCR model, is as follows:

$$
\begin{aligned}
&\underset{u,v}{Max}\ \theta^o = \frac{\sum_{l=1}^{l=s} u_l y_l^o}{\sum_{k=1}^{k=m} v_k x_k^o} && (5.1.1) \\
&Subject\ to: && \\
&\frac{\sum_{l=1}^{l=s} u_l y_l^i}{\sum_{k=1}^{k=m} v_k x_k^i} \le 1\ for\ i = 1,2,\dots,n. && (5.1.2) \\
&u_l, v_k \ge \varepsilon; && (5.1.3). \\
&\varepsilon\ \text{is a non}-\text{Archimedian positive value} &&
\end{aligned}
\qquad (5.1)
$$

As reported in previous chapter, the main drawback with this form is its unboundedness of optimal solutions since if *(u,v)* represents a solution then for any positive real value λ, $(\lambda u, \lambda v)$ as well is a solution. Fractional linear programming suggests considering a representative from each class of solutions. This can be obtained by normalizing and transforming the original model into the following linear program, called *the multiplier form*:

$$
\begin{aligned}
&\underset{u,v}{Max}\ \theta^o = \sum_{l=1}^{l=s} u_l y_l^o && (5.2.1) \\
&Subject\ to: && \\
&\sum_{l=1}^{l=s} \mu_l y_l^i - \sum_{k=1}^{k=m} v_k x_k^i \le 1\ for\ i = 1,2,\dots,n. && (5.2.2) \\
&\sum_{k=1}^{k=m} v_k x_k^o = 1 && (5.2.3) \\
&\mu_l, v_k \ge \varepsilon; && (5.2.4) \\
&\varepsilon\ \text{is a non}-\text{Archimedian positive value} &&
\end{aligned}
\qquad (5.2)
$$

In practice, it is preferable to solve the dual of the multiplier form for several reasons as detailed in Cooper et al (2000).

5.3. Further Grounding of the (CCR) DEA Model in the Economic Theory

In the classical firm theory, a firm is characterised by the set of all possible production plans it can realize, namely the *Possible Production Set (PPS)*. A firm's performance, under given economic conditions, is evaluated solely on the choices it makes from its PPS.

In contrast, DEA focuses on *relative* measures of performance. These are fully based on the firm's individual performance as compared to that of all of its relevant competitors. These differing perspectives reflect the differences between economic theory and management science. In economic theory, firms are treated similar to consumers. Any firm-specific internal details are ignored in the competitive market. In management science, on the other hand, firms are treated as live organizations, within which management makes ongoing decisions at various levels.

This chapter extends the roots of DEA into the soil of classical firm theory [Varian (1992)]. The cross-disciplinary bridging (Reisman, 1988) is established via behavioral-economic rationalization for the ratio used in the CCR-DEA model. The rationale is based on the *Weak Axiom of Profit Maximization.*

5.3.1. The Weak Axiom of Profit Maximization (WAPM)

Consider a finite set of n homogeneous DMUs, regarded as a set of n firms competing in the same environment. Assume that all firms have the same *PPS,* using a common set of m inputs to produce a common set of s outputs. This means that any DMU is able to attain any production plan belonging to the common *PPS.* However, every DMU is characterized by a single (chosen and attained) production plan.

To be specific, each DMU (i), for i=1,2,...,n, is characterized by a production plan $(-x_1^i,-x_2^i,...,-x_m^i, y_1^i, y_2^i,..., y_s^i)$ which belongs to the common set of all realizable production plans:

$$Y = \{(-x_1^i,-x_2^i,...,-x_m^i, y_1^i, y_2^i,..., y_s^i), i = 0,1,2,..., n\}. \quad (5.3)$$

The negative sign is used to distinguish inputs from outputs. Given the required data for all DMUs, the objective next is to measure their respective relative efficiencies.

A basic assumption underlying economic theory of firm behavior is that a firm acts so as to maximize its profit. Accepting this assumption leads to accepting all its implications. The *Weak Axiom of Profit Maximization (WAPM)* happens to be a well established (see Varian, 1992) restriction that is imposed by the *Profit Maximization* hypothesis.

To describe the *WAPM,* consider a given DMU (or firm) and its list of observed price vectors $p^t = (p_x^t, p_y^t); t = 1,2,...,n$ and its respective production plans $(-x^t, y^t); t = 1,2,..., n$ chosen by the firm under the above price vectors. The collection of $\{ p^t, (-x^t, y^t); t = 1,2,...,n.\}$ is called *data.*

If the firm is maximizing its profit, the observed chosen production vector $(-x^t, y^t)$ for a given price vector p^t must generate a profit at least as much as, or greater than, the profit generated by any alternative production plan $(-x^s, y^s); s \neq t$ available to the firm using the given price vector. Although the alternative production plans are not all identified, some of them are described by the set of vectors: $(-x^s, y^s); s = 1,2,\ldots,n$. Hence, a *necessary condition* for profit maximization is:

$$p^t(-x^t, y^t) \geq p^t(-x^s, y^s);\ \textit{for any}\quad s = 1,2,\ldots, n. \qquad (5.4)$$

The data restriction condition (5.4) is called the *Weak Axiom of Profit Maximization.*

5.3.2. Derivation of the CCR-DEA Model Using the WAPM

Consider a specific DMU, which is to be comparatively evaluated against the remaining DMUs in a given set. Assume that DMU(o) chooses the production plan described by the vector:

$$(-x_1^o, -x_2^o, \ldots, -x_m^o, y_1^o, y_2^o, \ldots, y_s^o) \in PPS \qquad (5.5)$$

Let $p^o = (p_x^o, p_y^o)$ be the observed price vector for DMU(o) with $p_x^o = (p_{x_1}^o, p_{x_2}^o, \ldots, p_{x_m}^o) \geq 0$ and $p_y^o = (p_{y_1}^o, p_{y_2}^o, \ldots, p_{y_s}^o) \geq 0$, where $p_{x_k}^o$ is the unit price for input x_k; $k = 1,2,\ldots, m$ and $p_{y_l}^o$ is the unit price for output $y_l; l = 1,2,\ldots,s$. Assuming that DMU(o) satisfies the *WAPM* leads to:

$$-\sum_{k=1}^{k=m} p_{x_k}^o x_k^o + \sum_{l=1}^{l=s} p_{y_l}^o y_l^o \geq -\sum_{k=1}^{k=m} p_{x_k}^o x_k^i + \sum_{l=1}^{l=s} p_{y_l}^o y_l^i \ \textit{for}\ i = 1,2,\ldots, n. \qquad (5.6)$$

This means that the base DMU(o) chooses the production plan defined by *(5.3)* because it generates, for the given price vector, the highest profit when compared with the profit generated by any of the remaining production plans belonging to the production set. Clearly, DMU(o) could have chosen any one of the remaining production plans. Yet it chose not to, thereby leaving them for the other DMUs.

Without loss of generality, we normalize the profit of DMU(o) generated from the chosen production plan $(-x_1^o, -x_2^o, \ldots, -x_m^o, y_1^o, y_2^o, \ldots, y_s^o)$ to be zero. That is, we assume:

$$-\sum_{k=1}^{k=m} p_{x_k}^o x_k^o + \sum_{l=1}^{l=s} p_{y_l}^o y_l^o = 0\ , \qquad (5.7)$$

By *WAPM*, this implies that the profits generated by other realizable production plans $(-x_1^i, -x_2^i, ..., -x_m^i, y_1^i, y_2^i, ..., y_s^i); i = 1,2,...,n;$ are less than or equal to zero. Therefore

$$-\sum_{k=1}^{k=m} p_{x_k}^o x_k^i + \sum_{l=1}^{l=s} p_{y_l}^o y_l^i \leq 0 \quad (5.8)$$

or

$$\frac{\sum_{l=1}^{l=s} p_{y_l}^o y_l^i}{\sum_{k=1}^{k=m} p_{x_k}^o x_k^i} \leq 1 \text{ for } i = 1,2,..., n \quad (5.9)$$

and particularly

$$\frac{\sum_{l=1}^{l=s} p_{y_l}^o y_l^o}{\sum_{k=1}^{k=m} p_{x_k}^o x_k^0} = 1 . \quad (5.10)$$

However, for the given price vector $p^o = (p_x^o, p_y^o)$ condition *(5.6)* may not hold or such price vector where condition *(5.6)* holds may not even exist. In the first case, it might be because the real production plan $(-x_1^o, -x_2^o, ..., -x_m^o, y_1^o, y_2^o, ..., y_s^o)$ adopted by the base DMU(o) was not based solely on profit maximization (yet the production plan could be technically efficient). In the second case, the firm simply (mistakenly) chose an inefficient plan[2]. Additionally, in this case, the real data represented by the current production plan may not be optimal in any economic environment (characterized by price vectors). Referring to Figure 2.1, unit R illustrates the first case while unit P illustrates the second case wherein the firm is hopeless of being fully efficient irrespective of the price vector.

Since economic theory of the firm presumes firms (DMUs) to be profit-maximizers, the *WAPM* must be satisfied by the firms' production choices. The converse holds as well, namely, the firm must be a profit-maximizer if it always satisfies the *WAPM*. This equivalence between the *WAPM* and the assumption of profit maximization is well known in firm theory literature (see Varian, 1992).

[2] In the terminology of Farrell (1957), it is both technically and allocatively inefficient due to either incomplete information and/or bounded rationality.

What can be said about if the firm's behavior does not conform to the WAPM in such a way that the observed data violates the WAPM? Certainly, the profit-maximization hypothesis or assumption is not going to be rejected. The profit maximization hypothesis has been very successful as a theoretical model for approximating or describing the firm's behavior. The deviation or violation of the WAPM can be interpreted in many ways. A firm that violates the *WAPM* may have other objectives in addition to profit maximization (indeed this is often the case). The violation of the *WAPM* can also be interpreted as a consequence of the bounded rationality characterized by incomplete information about technical and/or economic environments the firm is dealing with. And more.

Whatever may the case be, any firm violating the *WAPM*[3] must try to reduce, as much as is possible, its deviation from the WAPM described situation, namely the optimal situation as dictated by the profit maximization assumption.

Definition 5.1: Approximate-Weak Axiom of Profit Maximization (A-WAPM).

Under the postulate of profit maximization, a firm is said to conform to the *A-WAPM* if and only if it is willing to reduce as much as possible its deviation from the optimal conditions prescribed by the *WAPM*

Theorem 5.1:

Under the profit maximization postulate of the firm, the CCR-DEA model and the A-WAPM are equivalent.

Proof:

If firms violate the WAPM, the assumption (A-WAPM) that firms reduce as much as is possible their deviation from the conditions defined by the *WAPM* is equivalent to the assumption that, given all the technical or economic information in hand, they are willing to maximize their profit.

The reduction, as much as possible, of the deviation from the optimal conditions defined by the *WAPM* defines the above so-called *Approximate Weak Axiom of Profit Maximization.* The *A-WAPM* is then equivalent to saying that the present choice of the

[3] Here we mean the firm DMU(o) did not choose the "apparent'" optimal production plan from among the hypothetically possible plans, namely from the plans chosen by other DMUs

production plan generates the highest realizable profit level under the constraints that all other production plans by the other DMUs are *at most* at *their* optimal levels (efficient production plans).

To properly formulate and apply the *A-WAPM* in the context of DEA, let the maximum realizable profit be normalized to zero. More specifically, the maximum profit that any DMU can achieve *for any given price vector*[4] is normalized to be equal to zero. With no loss of generality it is clear as far as the WAPM is concerned, that one can assume all DMUs to have at most zero profits. For the base DMU(o), if its profit is zero at that given price vector it is classified as a fully efficient DMU. Obviously in this case the *WAPM* is satisfied. Alternatively, if DMU(o)'s profit is strictly less than zero (it can not be positive by the above assumption), we will try to find a best price vector p^* (economic environment) such that DMU(o) maximizes its profit from the chosen production plan $(-x_1^o, -x_2^o, \ldots, -x_m^o, y_1^o, y_2^o, \ldots, y_s^o)$ under the condition that the profit from any other production plan by other DMUs is at most zero, which is condition *(5.8)*. Since input and output price variables are separable this is equivalent to saying

$$\frac{\sum_{l=1}^{l=s} p_{y_l}^* y_l^i}{\sum_{k=1}^{k=m} p_{x_k}^* x_k^i} \leq 1 \ \textit{for} \ i = 1,2,\ldots, n \text{ and } \theta = \frac{\sum_{l=1}^{l=s} p_{y_l}^* y_l^o}{\sum_{k=1}^{k=m} p_{x_k}^* x_k^o} \text{ is maximized.}$$

This is exactly what the CCR-DEA model states.

$$\underset{p}{Max}\,\theta^o = \frac{\sum_{l=1}^{l=s} p_{y_l}^* y_l^o}{\sum_{k=1}^{k=m} p_{x_k}^* x_k^o} \qquad (5.11.1)$$

subject to: *(5.11)*

$$\frac{\sum_{l=1}^{l=s} p_{y_l}^* y_l^i}{\sum_{k=1}^{k=m} p_{x_k}^* x_k^i} \leq 1 \ \textit{for} \ i = 1,2,\ldots,n. \qquad (5.11.2)$$

[4] It will be shown corresponding to set a unit efficiency coefficient, the maximum achievable level of

5.3.3. Related Literature

In the previous section, the *CCR-DEA* model was derived from the classical economic theory of the firm. Specifically, since the *WAPM* characterizes the firm's profit maximization behavior assumption, any departure from the *WAPM* indicates the firm's departure from the profit optimization assumption (not necessarily being the firm's intention), which further indicates that the firm is operating inefficiently. The relative departure from the *WAPM* therefore can be used to measure the relative inefficiency of the firm, based on the firm's own production plan as well as other counterparts or competing firms' plans. This naturally induces the *CCR-DEA* model to evaluate the relative efficiency between firms under the Profit Maximization postulate.

In fact, Varian (1990) proposed *goodness-of-fit* measures for the violations of relevant conditions defined by the optimization models on firms, consumers, and so on. However, in testing the optimization model of a firm, he focuses on a single firm. He suggests to first test, based on the observed data, if the firm violates the *WAPM*, and if it does, define a measure to evaluate the significance of the deviation from conditions defined by the *WAPM*.

However, the purpose of this work is to measure the relative efficiency of a number of firms given that each firm has a single set of data. For a given firm, the departure (if there is any) from the *WAPM* measures its relative efficiency with regard to other firms (or their production plans since here a firm is uniquely characterized by a production plan). Therefore, the perspective of this work differs significantly from the one proposed by Varian (1990).

5. 4: Sensitivity Analysis

It is important to once again emphasize that the DEA produces relative efficiency measures reflecting the performance level of each one of the firms, or DMUs, involved in the analysis as compared to its competitors. However, the reliability of the analysis strongly depends on the number of factors, inputs and outputs, as parameters involved and on the number of firms or DMUs considered for the comparative analysis as

---

fficiency for any DMU.

observations. The larger is the difference in favor of the number of DMUs, the more reliable is the analysis.

In practice, as suggested by Cooper et al (2000), the *"rule of thumb"* which provides guidance for the number of DMUs required for a reliable DEA analysis is:

$$n \geq Max\{m \cdot s, 3(m+s)\} \qquad (5.12)$$

where n, m, and s are respectively the number of DMUs, the number of inputs and the number of outputs involved in the analysis.

The rule described by *(5.12)* relates the minimum suitable number of DMUs to the appropriate number of inputs and outputs involved in the analysis. From a particular point of view, it offers two *"degrees of freedom"* for balancing a DEA model: the first one is the number of DMUs included in the analysis and the second one is the number of factors (inputs and outputs) involved.

While theory is often used to identify the potential inputs and outputs to include in the model, there is often no helpful theoretical guide for deciding the number of DMUs. Thus the use of *(5.12)* may create a tendency to include as many DMUs as possible in the analysis. This however affects the *parsimony principle*, implicitly and commonly used in modeling real world problems, especially in statistics and econometrics, which argues for the use of just the necessary and needed (not more) information. The adoption of the parsimony principle leads to models made as simple as possible but not simpler. [See for example Poirier (1995) and Griliches and Intriligator (1997)]

The effect of sample size on the efficiency measure has been studied in the literature. By means of Monte Carlo simulation, Zhang and Bartels (1998) analyzed the effect of sample size on the mean DEA based efficiency. They pointed out that the average technical efficiency tends to decrease when the sample size increases. They conclude that comparing average measures of inefficiency, also known as structural inefficiency, between samples of different sizes leads to biased results. Staat (2001) concludes that the sample size bias explains the inconsistency between measures obtained from some types of DEA models, like Free Disposal Hall (FDH) and DEA with non-discretionary variables, which generate efficiency scores on differently sized samples. These studies show the central importance of the sample size in efficiency analysis.

The following result provides guidance in the above and counter balances the *"rule of thumb"* provided by *(5.12)*.

Theorem 5.2 (Sensitivity to Additional DMUs)

Let n and n' be two positive integer values such that $n' > n$. Consider an initial set of n DMUs and an extended set of n' DMUs, obtained by adding $(n'-n)$ DMUs to the initial set.

Let $\theta_i(n)$ and $\theta_i(n')$ be the efficiency coefficients for the i^{th} DMU obtained by considering respectively the initial and the extended sets of DMUs for a DEA model. Then:

$$\begin{aligned} &i)\, 0 \le \theta_i(n) \le 1 \;\; for \;\; i = 1,...,n \\ &ii)\, 0 \le \theta_i(n') \le 1 \; for \;\; i = 1,...,n' \\ &iii)\, \theta_i(n') \le \theta_i(n) \; for \; i = 1,...,n \end{aligned} \qquad (5.13)$$

Proof:

The proof straightforwardly follows from the optimality assumptions. Any additional DMU will imply additional constraints in the DEA model. These constraints will reduce the feasibility domain. This means that the maximum value of efficiency for any given DMU already considered will never increase if additional DMUs are included in the analysis.

Alternately stated, when more DMUs are added in the evaluation no DMU's efficiency will strictly increase. Any DMU identified as inefficient will remain so when it is evaluated in a larger pool of DMUs. However, an efficient DMU can in fact become inefficient in a larger pool of DMUs.

Hence the Theorem is thus proven.

Remarks:

- The dependence of the efficiency measures on sample size as observed from the sensitivity theorem could be regarded as one of the reasons why the DEA model can be concerned with only the relative measures rather than the absolute measures of efficiency for the firms considered in the analysis.
- There is an interesting link between A-WAPM and the sensitivity theorem. In fact, A-WAPM implies the sensitivity theorem. This is easily seen from the following

observation: the more DMUs are included, i.e., the more data are considered, the more likely that the WAPM is violated. Therefore, as a measure of the violation of WAPM, the relative efficiency cannot increase. More formally:

Let $\theta_i(n')$ *and* $\theta_i(n)$ be the efficiency measures for DMU$_i$ calculated by the CCR-DEA model for two samples of DMUs with sizes $n' \geq n$. As shown in previous section, $1-\theta_i(n)$ is the best measure of the violation of the WAPM postulated by the A-WAPM or equivalently by the CCR-DEA. Therefore, when more DMUs are included, which means that additional data is taken into consideration, the WAPM is more likely to be violated. Hence, $1-\theta_i(n') \geq 1-\theta_i(n)$ thus $\theta_i(n) \geq \theta_i(n')$.

5.5: Conclusions and Directions for Future Research

The classical domain for applications of the WAPM involves, as stated in Varian (1992) *"the activity analysis of a single firm"*. This work represents an extension to multi-firm analysis. Also, one can say that DEA is a practical, empirical, real world interpretation of a theoretical artefact that is believed to conform to common sense and the rationality principle expressed by the WAPM (Varian, 1992) in Firm theory.

The Sensitivity Theorem helps in the implementation of the parsimony principle, commonly known in econometrics and statistics (see for example Poirier (1995) and Griliches and Intriligator (1997)), into the context of DEA. It counterbalances the trend to put more observations in the model. The parsimonious sample can be an alternative way for dealing with outliers in DEA.

Based on this theorem, one can define a *"parsimonious"* sample for DEA. In a situation where it is not required to include all observations (DMUs) in the sample, one can think about those DMUs that are most relevant for the analysis. The others can be considered as "redundant" observations. If the study focuses on the reference set, the set of DMUs defining the efficient frontier, one can use either forward selection or backword elimination to define a "parsimonious sample" for the analysis.

The forward selection proceeds by first chosing an initial sample, preferably satisfying (5.12), so as to identify the reference set. This initial sample is chosen either by "judgement" or randomly. An extra DMU is then added to the initial sample: it is a "relevant" observation if it has an effect on the reference set and this reference set is updated, otherwise it is not relevant and by the same meaning it will not be considered to be in the "parsimonious" sample. The iterations end when all DMUs are tested.

The backward elimination proceeds by first considering all observations to define an initial reference set. One DMU is then eliminated: if this alters the reference set, it means that the eliminated DMU is "relevant" and should not be eliminated. Otherwise, the eliminated DMU is a redundant observation and it is not required for the definition of the *"parsimonious sample"*. The iterations end when all DMUs are tested, but (5.12) should be preferably respected.

Although the two extensions of theory presented herein do not represent paradigm shifts for DEA nor for Firm Theory, they do open a number of directions for future research in each discipline and at the important bi-disciplinary interface herein created between them.

CHAPTER 6: *CONFIDENT-DEA* METHODOLOGY FOR BOUNDED DATA

6.1. The Mathematical Modeling

Given the inputs and outputs to be considered, the general ratio-form of *Data Envelopment Analysis* (DEA) model is used to determine the relative efficiency coefficient and perform efficiency analyses. The following general model is considered in this chapter:

$$
\left.
\begin{array}{ll}
\underset{u,w}{Max}\ h_0 = \dfrac{\sum_{r=1}^{r=s} u_r y_{ro}}{\sum_{i=1}^{i=m} w_i x_{io}} & (6.1.1) \\
Subject\ \ to: & \\
\dfrac{\sum_{r=1}^{r=s} u_r y_{rj}}{\sum_{i=1}^{i=m} w_i x_{ij}} \le 1;\ j = 1,2,\ldots,\ n & (6.1.2) \\
y_r = (y_{rj}) \in RD_r^+ & (6.1.3) \\
x_i = (x_{ij}) \in RD_i^- & (6.1.4) \\
u = (u_r) \in RA^+ \ ,u_r \ge 0 & (6.1.5) \\
w = (w_j) \in RA^- \ , w_i \ge 0 & (6.1.6)
\end{array}
\right\} \quad (6.1)
$$

where ***x*** represents the matrix of input values for each DMU. It specifies the values of inputs used in the production process. ***y*** on the other hand represents the output matrix. It specifies, for each DMU, the values of the different outputs that results from the production process. ***u*** and ***w*** are the coefficient vectors to be determined by solving the model. $\boldsymbol{RD_r^+}$, $\boldsymbol{RD_r^-}$ $\boldsymbol{RA_r^+}$ and $\boldsymbol{RA_r^-}$ respectively represent domains for the outputs, inputs, output multipliers and input multipliers.

The standard DEA model only allows for single-valued data. However, the real world situations often dictate data the values of which lye within some prescribed bounds. Moreover, the data may be ordinal rather than cardinal in form. In Cooper et al (1999) these are labelled "*imprecise data*". Lastly, the "*data*" may represent the decision-maker's judgemental restrictions on the relative weights allowed to each or some of the factors and/or their multipliers. This is known in the DEA literature as the *Assurance-Region*. A specific domain for the solution search can be imposed and this is known in the DEA literature as the *Cone-Ratio*.

The general form presented above allows for all forms of data as well as all forms of restrictions on multipliers.

By imposing a normalizing constraint, the above model is transformed to the multiplier form of DEA model presented in Cooper et al (1999):

$$\left.\begin{array}{ll}
\underset{\mu,\omega}{Max}\ h_o = \sum_{r=1}^{r=s} \mu_r y_{ro} & (6.2.1)\\
Subject\ \ to: & \\
\sum_{r=1}^{r=s} \mu_r y_{rj} - \sum_{i=1}^{i=m} \omega_i x_{ij} \le 0;\ j = 1,2,\ldots,n & (6.2.2)\\
\sum_{i=1}^{i=m} \omega_i x_{io} = 1 & (6.2.3)\\
y_r = (y_{rj}) \in D_r^+ & (6.2.4)\\
x_i = (x_{ij}) \in D_i^- & (6.2.5)\\
\mu = (\mu_r) \in A^+,\ \mu_r \ge 0 & (6.2.6)\\
\omega = (\omega_j) \in A^-,\ \omega_i \ge 0 & (6.2.7)
\end{array}\right\} \quad (6.2)$$

This form is obtained by the restriction: $\sum_{i=1}^{i=m} \omega_i x_{io} = 1$. The change in notation reflects nothing but the shift from the ratio to the multiplier form of the model.

In the case of imprecise data, the model presented above is not linear any longer and the standard DEA approach cannot be applied. Cooper et al. (1999) proposes a unified approach to treating mixtures involving bounded data in addition to ordinal data and ordinal relations among the weights. Their approach, the *Imprecise Data Envelopment Analysis* (*IDEA*), extends the standard DEA to cope with imprecise data. In a following-up work, Cooper et al (forthcoming) presented an illustrative application of their unified approach. Formulating the basic DEA model using imprecise data leads to a *non-linear optimization problem*. For the linearization, IDEA proceeds in two steps, scale transformations followed by variable alterations. The transformed model has the form of a standard DEA model. The solution for the original model is obtained from that of the transformed model using the reverse variable alterations and scale transformations.

The purpose of this chapter is to develop an alternative method, given the name *Confident-DEA*, to the one presented in Cooper et al. (1999). It is based on the belief that imprecision in data should be reflected in the efficiency measures provided by the model. This is achieved by providing a range for the efficiency measures, an en efficiency confidence interval, for each DMU instead of the single valued measure.

The upper bound for each DMU is obtained by solving the following model:

$$
\left.
\begin{array}{ll}
\underset{x,y}{Max}\ \underset{\mu,\omega}{Max}\ h_o = \sum_{r=1}^{r=s} \mu_r y_{ro} & (6.3.1) \\
subject\ to: & \\
\sum_{r=1}^{r=s} \mu_r y_{rj} - \sum_{i=1}^{i=m} \omega_i x_{ij} \le 0;\ j = 1,2,\ldots,n & (6.3.2) \\
\sum_{i=1}^{i=m} \omega_i x_{io} = 1 & (6.3.3) \\
y_r = (y_{rj}) \in D_r^+ & (6.3.4) \\
x_i = (x_{ij}) \in D_i^- & (6.3.5) \\
\mu = (\mu_r) \in A^+, \mu_r \ge 0 & (6.3.6) \\
\omega = (\omega_j) \in A^-, \omega_i \ge 0 & (6.3.7)
\end{array}
\right\} \quad (6.3)
$$

The lower bound on the other hand is determined by considering the following minimization model:

$$
\left.
\begin{array}{ll}
\underset{x,y}{Min}\ \underset{\mu,\omega}{Max}\ h_o = \sum_{r=1}^{r=s} \mu_r y_{ro} & (6.4.1) \\
subject\ to: & \\
\sum_{r=1}^{r=s} \mu_r y_{rj} - \sum_{i=1}^{i=m} \omega_i x_{ij} \le 0,\ j = 1,2,\ldots,n & (6.4.2) \\
\sum_{i=1}^{i=m} \omega_i x_{io} = 1 & (6.4.3) \\
y_r = (y_{rj}) \in D_r^+ & (6.4.4) \\
x_i = (x_{ij}) \in D_i^- & (6.4.5) \\
\mu = (\mu_r) \in A^+, \mu_r \ge 0 & (6.4.6) \\
\omega = (\omega_j) \in A^-, \omega_i \ge 0 & (6.4.7)
\end{array}
\right\} \quad (6.4)
$$

Both models used in the *Confident-DEA* are *non-linear convex* problems and can be written in the general form of a *bilevel convex* model discussed in greater detail by Bard (1999). There are two levels of optimization: multipliers are subjects at the lower level while the factors are subjects at the upper level. The model proceeds by determining the optimal multipliers for a given level of the factors. The general mathematical form of a b*ilevel convex* problem is:

$$\begin{array}{llr} \underset{x\in X}{Max}\, F(x,\omega(x)) & (6.5.1) & \\ Subject\ to \quad G(x,\omega(x)) \leq 0 & (6.5.2) & \\ \qquad \omega(x) = \underset{y\in Y}{Max}\, f(x,y) & (6.5.3) & (6.5) \\ \qquad\quad Subject\ to \ \ g(x,y) \leq 0 & (6.5.4) & \end{array}$$

where F, G, f and g are convex functions.

The better known Max-Min problem is a particular case of the general bilevel convex problem. Jeroslow (1985) proved that the Max-Min problem is NP-hard and this result was confirmed by Hansen et al (1992) who proved that the linear bilevel programming problem is strongly NP-hard. This represents a higher order of difficulty in solving the general form of convex bilevel optimization problems and justifies the use of heuristics.

In this chapter considers the case of bounded data and a theorem for defining the efficiency confidence intervals is provided. The more general case of imprecise data is considered in Chapter 7.

6.2. Confident-DEA and Optimistic/Pessimistic Point of View

This section considers a situation where data for each DMU is either single valued or bounded (values known to be varying within an interval having fixed and known bounds). For this situation, we define what we call the *"optimistic point of view"* and *"pessimistic point of view"* of each DMU. The *Confident-DEA* model's basic theorem is then provided with its proof.

Definition 6.1: Optimistic point of view for a given DMU:

The *"optimistic point of view* for each DMU considered for the relative efficiency evaluation among a set of DMUs is defined by the situation where the base-DMU is using the minimum allowed quantities of inputs to produce the maximum permitted quantities of outputs. Conversely, its competitors use maximum of inputs in producing the minimum of outputs.

Definition 6.2: Pessimistic point of view for a given DMU:

The *"pessimistic point of view"* for each DMU considered for the relative efficiency evaluation among a set of DMUs is defined by the situation where the base-DMU is using the maximum quantities of inputs, allowed by boundaries on the data, in producing the minimum quantities of outputs. Conversely, its competitors use minimum allowed quantities of inputs to produce the maximum quantities of outputs.

Theorem 6.1:

- The maximum level of efficiency for each DMU is reached under conditions represented by its optimistic point of view.
- The minimum level of efficiency for each DMU is reached under conditions represented by its pessimistic point of view.

Proof:

The proof of the first claim in the theorem is provided here. The second claim can be easily derived by analogy.

The proof proceeds in a recurrent way and considers first the case of two DMUs (DMU_1) and (DMU_2), both using the same single input (X_1) for producing the same single output (Y_1). We use the original CCR ratio form of DEA for the proof and for this case, the CCR formulation considering DMU_1 as base one is as follows:

$$
\left.
\begin{array}{ll}
\underset{u,v}{Max}\ \theta = \dfrac{u_1 y_{11}}{v_1 x_{11}} & (6.6.1) \\
\textit{subject to:} & \\
\dfrac{u_1 y_{11}}{v_1 x_{11}} \leq 1 & (6.6.2) \\
\dfrac{u_1 y_{12}}{v_1 x_{12}} \leq 1 & (6.6.3) \\
u_1, v_1 \geq \varepsilon & (6.6.4) \\
\varepsilon\ Non-Archimedian. & (6.6.5)
\end{array}
\right\} \quad (6.6)
$$

where $\boldsymbol{u}$ and $\boldsymbol{v}$ are the weights to be determined.

Without loss of generality, we can assume $v_1=1$ and use u as a single variable. This leads to following form derived from (6.1):

$$
\begin{aligned}
&\underset{u,v}{Max} \;\; \theta = \frac{uy_{11}}{x_{11}} && (6.7.1) \\
&\textit{subject to :} && \\
&u \le \frac{x_{11}}{y_{11}} && (6.7.2) \\
&u \le \frac{x_{12}}{y_{12}} && (6.7.3) \\
&u \ge \varepsilon && (6.7.4) \\
&\varepsilon \;\; Non-Archimedia\;\; n. && (6.7.5)
\end{aligned}
\qquad (6.7)
$$

The maximum is then obtained as

$$
u_* = Min\left\{\frac{x_{11}}{y_{11}}, \frac{x_{12}}{y_{12}}\right\} \qquad (6.8)
$$
$$
\text{thus } u_* = u(x_{11}, x_{12}, y_{11}, y_{12})
$$

Assume that the bounds for the inputs and outputs are defined as follows:

$$
x_{11} \in \left[\underline{x_{11}}, \overline{x_{11}}\right]; x_{12} \in \left[\underline{x_{12}}, \overline{x_{12}}\right] and\; y_{11} \in \left[\underline{y_{11}}, \overline{y_{11}}\right]; y_{11} \in \left[\underline{y_{12}}, \overline{y_{12}}\right] \qquad (6.9)
$$

then from (6.8) we have: $u^* \le Min\left\{\frac{x_{11}}{y_{11}}, \frac{\overline{x_{12}}}{\underline{y_{12}}}\right\}$

$$
\text{Let: } \overline{u(x_{11}, y_{11})} = Min\left\{\frac{x_{11}}{y_{11}}, \frac{\overline{x_{12}}}{\underline{y_{12}}}\right\} \qquad (6.10)
$$

__Case 1:__ If $\frac{x_{11}}{y_{11}} \le \frac{\overline{x_{12}}}{\underline{y_{12}}}$ then $\overline{u(x_{11}, y_{11})} = \frac{x_{11}}{y_{11}}$ which gives, by plugging in (6.7), an efficiency coefficient for DMU$_1$: $\theta_1 = 1$. This means that DMU$_1$ is already efficient compared to DMU$_2$. Neither a decrease in the input nor an increase in the output will improve the efficiency, which is already at its maximum level.

__Case 2:__ If $\frac{x_{11}}{y_{11}} > \frac{\overline{x_{12}}}{\underline{y_{12}}}$ then $\overline{u(x_{11}, y_{11})} = \frac{\overline{x_{12}}}{\underline{y_{12}}}$ which gives, by plugging in (6.7), that: $\theta = \overline{u(x_{11}, y_{11})}\frac{y_{11}}{x_{11}} = \frac{\overline{x_{12}}}{\underline{y_{12}}}\frac{y_{11}}{x_{11}} \le \frac{\overline{x_{12}}}{\underline{y_{12}}}\frac{\overline{y_{11}}}{\underline{x_{11}}} = \overline{\theta}$ which means that DMU$_1$, considered as base DMU, is not fully efficient and the maximum level of efficiency it can reach is defined by the conditions prescribed by its optimistic point of view e.g.: minimum input and maximum output.

This ends the proof for the case of two DMUs using a single input to produce a single output.

Consider now the general case of n DMUs using the same set of m inputs to produce the same set of s outputs. The CCR ratio form for a generic DMU indexed *"o"* in this general case is:

$$\left.\begin{aligned}
&\underset{u,v}{Max}\ \theta_o = \frac{\sum_{r=1}^{r=s} u_r y_{ro}}{\sum_{i=1}^{i=m} w_i x_{io}} && (6.10.1)\\
&Subject\ to:\\
&\frac{\sum_{r=1}^{r=s} u_r y_{rj}}{\sum_{i=1}^{i=m} w_i x_{ij}} \leq 1;\ j=1,2,\ldots,n && (6.10.2)\\
&u, v \geq \varepsilon && (6.10.3)\\
&\varepsilon\ Non-Archimedia\ n && (6.10.4)
\end{aligned}\right\}\quad (6.11)$$

Suppose that:

$$x_{ij} \in \left[\overline{x_{ij}}, \underline{x_{ij}}\right] \text{and } y_{rj} \in \left[\overline{y_{rj}}, \underline{y_{rj}}\right] \text{for i =1,2,…,m; j =1,2,…,n; r =1,2,…,s} \quad (6.12)$$

Let:

$$\left.\begin{aligned}
&\theta_o = \frac{\sum_{r=1}^{r=s} u_r y_{ro}}{\sum_{i=1}^{i=m} w_i x_{io}};\theta_o = \theta_o(u,v,x_o,y_o) && (6.13.1)\\
&and:\\
&\theta_j = \frac{\sum_{r=1}^{r=s} u_r y_{rj}}{\sum_{i=1}^{i=m} w_i x_{ij}};\theta_j = \theta_j(u,v,x_j,y_j) && (6.13.2)
\end{aligned}\right\}\quad (6.13)$$

The problem in (6.11) becomes:

$$\left.\begin{aligned}
&\underset{u,v}{Max}\,\theta_o && (6.14.1)\\
&Subject\ to:\\
&\theta_j \leq 1; j=1,2,\ldots,n && (6.14.2)\\
&u,v \geq \varepsilon && (6.14.3)\\
&\varepsilon\ Non-Archimedian && (6.14.4)
\end{aligned}\right\}\quad (6.14)$$

Let θ_o^* be the maximum value for θ_o and $\theta_o^* = \theta_o(u^*,v^*,x_o,y_o)$ where (u^*,v^*) is the optimal solution of (6.14). Two cases are then to be considered:

Case 1: $\theta_o^* = \theta_o(u^*, v^*, x_o, y_o) = 1$ and $\theta_j^* = \theta_j(u^*, v^*, x_j, y_j) \le 1; j = 1,2,...,n$:

In this case, the base DMU is already efficient and neither a decrease in input nor an increase in output will improve its efficiency because the base DMU is already fully efficient. Its efficiency coefficient is at its maximum level. Thus the theorem holds trivially in this case.

Case 2: $\theta_o^* = \theta_o(u^*, v^*, x_o, y_o) < 1$ and $\theta_j^* = \theta_j(u^*, v^*, x_j, y_j) \le 1; j = 1,2,...,n$:

In this case, there must exist at least one DMU, other than the base-DMU, which is fully efficient.

Otherwise, $\theta_j^* = \theta_j(u^*, v^*, x_j, y_j) < 1; j = 1,2,...,n$. That is: $\dfrac{\sum_{r=1}^{r=s} u_r^* y_{rj}}{\sum_{i=1}^{i=m} w_i^* x_{ij}} < 1; j = 1,2,...,n$

which contradicts with the fact that (u^*, v^*) is the optimal solution.

Let $J = \{j' : DMU_{j'} \text{ is inefficient}$ and $J^c = \{j'' : DMU_{j''} \text{ is efficient}$

$$\text{That is: } \theta_{j'}^* < 1; j' \in J \text{ and } \theta_{j''}^* = 1; j'' \in J^c \qquad (6.15)$$

Let $\Omega = \left\{ \begin{array}{l} (u^*, v^*) : (u^*, v^*) \text{ is the optimal solution of} \\ \text{the CCR model with } DMU_o \text{ as base } DMU \end{array} \right\}$

If each DMU, except the base one, choses its values for the inputs and outputs the vector: $(\overline{x_j}, \underline{y_j}); j \ne o$ where $\overline{x_j} = (\overline{x_{1j}}, ..., \overline{x_{mj}}); j \ne o$ and $\underline{y_j} = (\underline{y_{1j}}, ..., \underline{y_{sj}}); j \ne o$ then we still have:

$$\begin{cases} \theta_{j'}(u^*, v^*, \overline{x_{j'}}, \underline{y_{j'}}) < 1; j' \in J & (6.16.1) \\ \theta_{j''}(u^*, v^*, \overline{x_{j''}}, \underline{y_{j''}}) \le 1; j'' \in J^c & (6.16.2) \\ \theta_o(u^*, v^*, x_o, y_o) < 1 & (6.16.3) \end{cases} \qquad (6.16)$$

Therefore, $(u^*, v^*) \in \Omega$ will remain a feasible solution, although it may not be an optimal one, for the CCR model at $(\overline{x_j}, \underline{y_j}); j \ne o$ and $(x_0, y_o); j = o$.

If DMU_o decreases its inputs and increases its outputs, the efficiency coefficient value $\theta_o^* = \theta_o^*(u^*, v^*, x_o, y_o) = \dfrac{\sum_{r=1}^{r=s} u_r^* y_{ro}}{\sum_{i=1}^{i=m} v_r^* x_{io}}$ will increase as well. Therefore, the maximum is reached at the conditions defined by the optimistic point of view of DMU_o considered as the base DMU.

Thus far proof was provided that the upper level of efficiency for each DMU, considered for the relative efficiency in a set of competing DMUs, is reached under conditions prescribed by its optimistic point of view. The proof for the claim that the lower level of efficiency is reached under conditions prescribed by its pessimistic point of view is similarly obtained. The theorem is proved.

The theorem shows precisely what Charnes and Cooper (1985) describe as natural *"desired directions"* for production factors in efficiency analysis. They explain that *augmentation* is the desired direction for *outputs* and *diminution* is the desired direction for *inputs*.

6.3. An Illustrative Example

An example illustrating the *Confident-DEA* approach in the case of bounded cardinal data is provided next. The results obtained with *Confident-DEA* are compared to those from *IDEA* approach developed in Cooper et al (1999).

Consider eight (8) decision making units (DMUs) competing in the same market using two (2) inputs to produce two (2) outputs. DMU1 uses 124 units of input 1 and an imprecise quantity, at least 40 units and at most 50 units, of input 2 to produce 89.8 units of output 1 and an imprecise quantity, more than 55 units but less than 65 units, of output 2. *Table 6.1* contains the data for the example.

Table 6.1: Data for an illustrative example with imprecise data

DMUs	X1	X2	Y1	Y2
1	124	[40 , 50]	89.8	[55 , 65]
2	95	[20 , 30]	99.6	[60 , 70]
3	92	[60 , 70]	87	[70 , 80]
4	61	[90 , 100]	99.4	[95 , 100]
5	63	[70 , 80]	96.4	[65 , 75]
6	50	[30 , 40]	86	[85 , 95]
7	40	[50 , 60]	71	[50 , 60]
8	16	[10 , 20]	98	[95 , 100]

The data corresponding to the optimistic point of view of DMU 1 is given in *Table 6.2* and data for the pessimistic point of view can be obtained by analogy.

Table 6.2: Optimistic point of view of DMU 1

DMUs	X1	X2	Y1	Y2
1	124	40	89.8	65
2	95	30	99.6	60
3	92	70	87	70
4	61	100	99.4	95
5	63	80	96.4	65
6	50	40	86	85
7	40	60	71	50
8	16	20	98	95

For each DMU, two DEA models are considered and solved. These models represent for each DMU the *optimistic point of view* and the *pessimistic point of view.* The solution from the *optimistic point of view* model defines the upper bound for the efficiency measure while the solution from the *pessimistic point of view* model determines its lower bound. The DEA model shown in (6.18) represents the optimistic point of view of DMU1.

$$\begin{aligned}
&\underset{\mu,\omega}{Max}\, h_0 = 89.8\mu_1 + 65\mu_2 \\
&Subject\ to: \\
&89.8\mu_1 + 65\mu_2 - 124\omega_1 - 40\omega_2 \le 0 \\
&99.6\mu_1 + 60\mu_2 - 95\omega_1 - 30\omega_2 \le 0 \\
&87\mu_1 + 70\mu_2 - 92\omega_1 - 70\omega_2 \le 0 \\
&99.4\mu_1 + 95\mu_2 - 61\omega_1 - 100\omega_2 \le 0 \qquad\qquad (6.17) \\
&96.4\mu_1 + 65\mu_2 - 63\omega_1 - 80\omega_2 \le 0 \\
&86\mu_1 + 85\mu_2 - 50\omega_1 - 40\omega_2 \le 0 \\
&71\mu_1 + 50\mu_2 - 40\omega_1 - 60\omega_2 \le 0 \\
&98\mu_1 + 95\mu_2 - 16\omega_1 - 20\omega_2 \le 0 \\
&124\omega_1 + 40\omega_2 = 1 \\
&\mu_1, \mu_2, \omega_1, \omega_2 \ge \varepsilon \\
&\varepsilon\ Non-Archimedian
\end{aligned}$$

Table 3: Pessimistic/Optimistic efficiency Coefficients

DMUs	Pessimistic	Optimistic
1	0.1832	0.4581
2	0.3387	1
3	0.1544	0.2959
4	0.2660	0.2761
5	0.2498	0.2810
6	0.2808	0.6667
7	0.2898	0.2898
8	1	1

In order to compare the results obtained from *Confident-DEA* with those obtained from IDEA, Table 1 data were adjusted as shown in *Table 4*. Single-valued data for the anchoring DMU are required in both steps within *the IDEA* computations. Once again these involve scale transformation and variable alteration. For DMU 4, selected as the anchor for both input X2 and output Y2, the corresponding values of data-points ([90,100] and [95,100]) are changed to be single-valued (100). For DMU 8, which could be selected as anchor for output Y2, the corresponding value is also altered to be 100.

Significantly, *IDEA* based measures coincide with the upper bound of efficiency confidence interval the *Confident-DEA* provides for each DMU. That is, *IDEA* is precisely the optimistic point of view case in *Confident-DEA*. Consequently, *Confident-DEA* constitutes a generalization of *IDEA*.

Table 4: Adjusted Data for comparison of *IDEA* and *Confident-DEA*

DMUs	X1	X2	Y1	Y2
1	124	[40 , 50]	89.8	[55 , 65]
2	95	[20 , 30]	99.6	[60 , 70]
3	92	[60 , 70]	87	[70 , 80]
4	61	100	99.4	100
5	63	[70 , 80]	96.4	[65 , 75]
6	50	[30 , 40]	86	[85 , 95]
7	40	[50 , 60]	71	[50 , 60]
8	16	[10 , 20]	98	100

Regarding the benchmarking problem in IDEA, Cooper et al. (2001a) suggested introducing dummy DMUs to overcome *IDEA's* shortcoming requiring single-valued data for the DMU used as benchmark in scale transformations and variable alterations. To derive the efficiency measures defined by the initially non-linear problem, the back transformation are made

Table 5: Comparative Results for Confident-DEA and IDEA

DMUs	Pessimistic	Optimistic	IDEA
1	0.1835	0.4587	0.4581
2	0.3388	1	1
3	0.1544	0.2959	0.2959
4	0.2660	0.2660	0.2660
5	0.2498	0.2810	0.2811
6	0.2808	0.6333	0.6333
7	0.2898	0.2898	0.2898
8	1	1	1

6.4. Confident-DEA and the Stochastic Imprecise Data: a Simulation Approach

6.4.1. Background for Simulation

Simulation is one of the most widely used operations research/management science (OR/MS) techniques. Gupta (1997) analyzed 1294 articles appearing in INTERFACES from 1970 through 1992. She found that simulation was second only to the "mathematical programming" among 13 techniques considered.

Simulation modeling has been used extensively in operations research to reproduce or to mimic the behavior of complex system. It was an alternative to analytic modeling and often both are used simultaneously for purpose of validation. Banks et al (1996) and Law and Kelton (2000) provides more details.

The literature related with simulation provides three binary criteria for the classification of a simulation model: *discrete* versus *continuous* event, *dynamic* versus *static* and *deterministic* versus *stochastic* model. Discrete simulation models reproduce the behaviour of systems for which the state variables change instantaneously at separated points in time. A bank desk is an example of such a system since the state variables, the number of customers in the bank, change only when a customer arrives or when a customer finishes being served and departs. Continuous simulation models consider reproducing the behaviour of systems for which the state variables change continuously with respect to time. An airplane moving through the air is an example of a continuous system. Dynamic versus static distinguish whether the system behaviour changes over time or is time-independent. The last distinction recognizes that the state variables are described by probability distributions.

Monte Carlo Simulation is the most well-known simulation method. It can be defined as a simulation scheme employing random numbers, which are used for solving certain stochastic or deterministic problems where the passage of time plays no substansive role. Thus, *Monte Carlo* simulations are generally static rather than dynamic. The *uniform* distribution or any other well-defined statistical distribution can describe the randomness of numbers. However, *uniform* and *normal* distributions are the most commonly used in *Monte Carlo* simulations to provide approximate solutions to mathematical problems by performing statistical computer-based sampling experiments. The name "*Monte Carlo*" simulation or method originated during World War II, when it was applied to problems related to the development of the atomic bomb.

Simulation methods generated a large literature, particularly Banks et al (1996), Law and Kelton (2000), Fishman (1996) and Hillier and Lieberman (1995) are recommended books for a more in depth review. For the joint use of DEA and simulation McMullen and Frazier (1998) and Bardhan et al (1998), are some examples.

6.4.2. Monte-Carlo Simulation for DEA with Imprecise Stochastic Data

The case of stochastic cardinal data is addressed in this section and a methodology to conduct *Confident-DEA* in such environment is proposed. It assumes that the distribution of the values of the bounded factors over the interval is known. When this distribution is ignored, as it is the case in the illustrative example presented in section (6.3), and in Cooper et al (1999 and Forthcoming), this means that the uniform distribution is implicitly assumed.

For illustration, assume, in the section (6.3) example, that the bounded factor values have a *two-tail truncated normal distribution*. An efficiency histogram is obtained for each DMU by simulating different values for bounded factors. The [0,1] interval is divided into 20 equally large sub-intervals. The 21st corresponds to the value one of full efficiency. The histogram represents the number of times each one of the efficiency sub-intervals is hit during the total number of simulations, which in this case are 100,000. Benchmarks for each intermediary level of efficiency are provided as well.

It should be emphasized that in the simulation approach, there is no unique solution. The benchmarks provided may change if when the simulation is run again. The histogram can be used to define an approximate parametric distribution of the efficiency coefficients. However, is beyond the limits of this work.

Table 6.4 provides the efficiency confidence intervals for the eight DMUs obtained by running the simulation 100,000 times. Although a slight difference can be noticed in the boundaries of some confidence intervals compared with those provided in *Table 6.3*, the two sets of efficiency intervals are mutually compatible. The failure of the simulation in capturing the exact values provided in *Table 6.3* is a consequence of the truncated normal distribution assumption. This distribution implies that low probabilities are associated with the extreme values, and those values define the exact solutions corresponding to both the optimistic and pessimistic points of view.

Table 6.4: Efficiency Confidence Intervals Obtained from Simulations

	DMU1	DMU2	DMU3	DMU4	DMU5	DMU6	DMU7	DMU8
Highest Efficiency	0.4543	1	0.2938	0.2759	0.2810	0.6149	0.2898	1
Lowest Efficiency	0.1882	0.3445	0.1544	0.2660	0.2498	0.2867	0.2898	1

The efficiency histogram for DMU 1 is shown in *Figure 6.1*. This histogram shows that the efficiency level of [0.30,0.35[was realised 45,463 times out of 100,000. This means that this efficiency level is the most likely to be realized, taking into account the normal distribution of values over the predefined interval for Y2, the factor with bounded data. Hence the probability of 0.45463 for the realization of this level of efficiency for DMU1. For each DMU, the efficiency confidence intervals and the distributions of the efficiency values over this interval are defined.

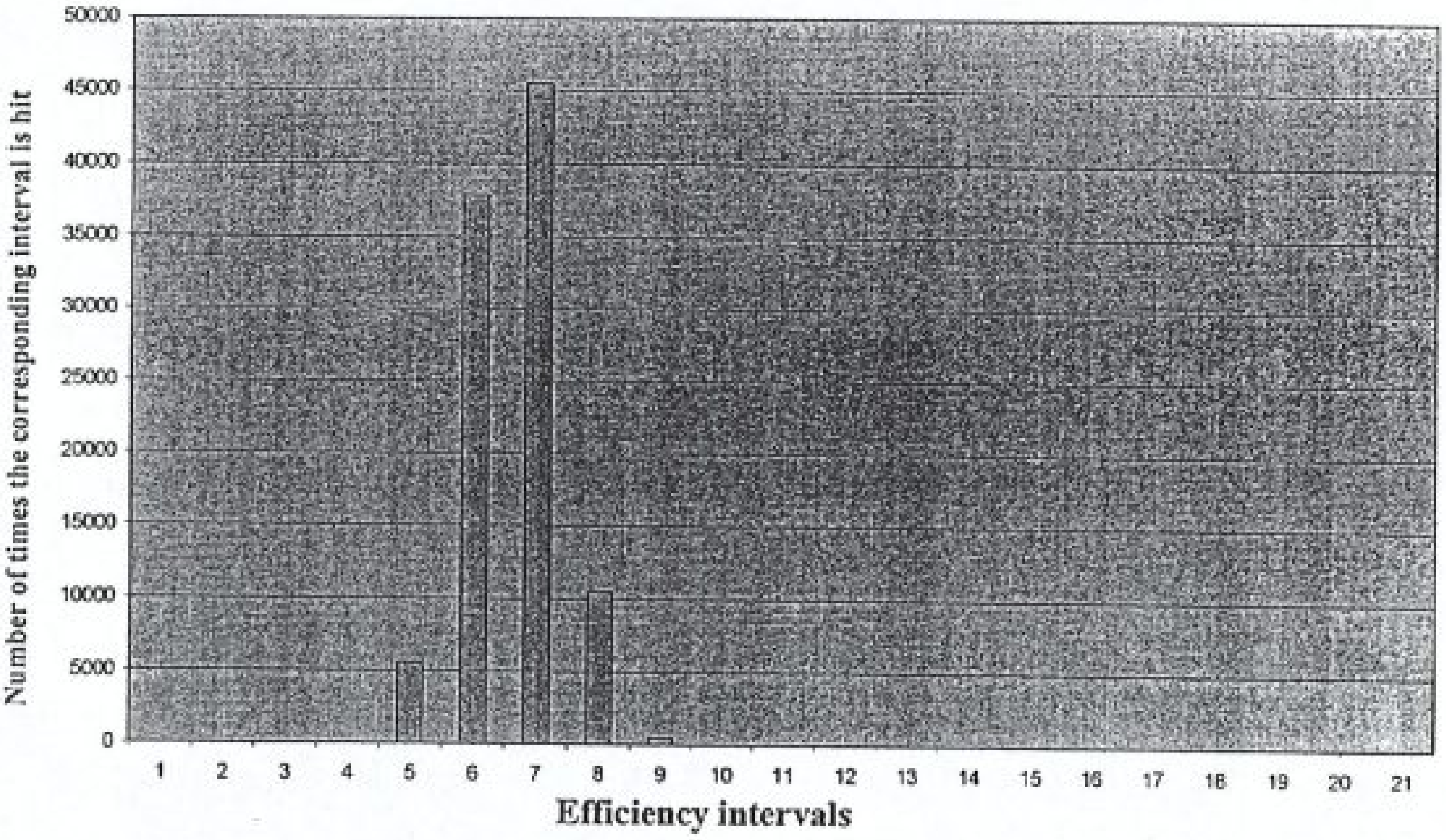


Figure 6.1: Efficiency histogram for DMU 1

Results for all DMUs are provided in *Table 6 5*. The DMUs are listed in columns and the efficiency bounding intervals are shown in rows. Each entry in the table indicates the likelihood of the corresponding DMU having a relative *Confident-DEA* efficiency in the corresponding efficiency interval.

Table 5: Distribution of the efficiency measures over the *efficiency confidence interval* of each DMU

	DMU1	DMU2	DMU3	DMU4	DMU5	DMU6	DMU7	DMU8
[0.00 , 0.05[	0	0	0	0	0	0	0	0
[0.05 , 0.10[	0	0	0	0	0	0	0	0
[0.10 , 0.15[	0	0	0	0	0	0	0	0
[0.15 , 0.20[	0.00067	0	0.41545	0	0	0	0	0
[0.20 , 0.25[	0.05596	0	0.55693	0	0.99085	0	0	0
[0.25 , 0.30[	0.37858	0	0.02762	1	0.00915	0.01757	1	0
[0.30 , 0.35[	0.45463	0.00001	0	0	0	0.14994	0	0
[0.35 , 0.40[	0.10525	0.00161	0	0	0	0.36844	0	0
[0.40 , 0.45[	0.00490	0.01387	0	0	0	0.32703	0	0
[0.45 , 0.50[	0.00001	0.05742	0	0	0	0.11781	0	0
[0.50 , 0.55[	0	0.14625	0	0	0	0.01805	0	0
[0.55 , 0.60[	0	0.23110	0	0	0	0.00110	0	0
[0.60 , 0.65[	0	0.24098	0	0	0	0.00006	0	0
[0.65 , 0.70[	0	0.17154	0	0	0	0	0	0
[0.70 , 0.75[	0	0.08977	0	0	0	0	0	0
[0.75 , 0.80[	0	0.03454	0	0	0	0	0	0
[0.80 , 0.85[	0	0.01008	0	0	0	0	0	0
[0.85 , 0.90[	0	0.00229	0	0	0	0	0	0
[0.90 , 0.95[	0	0.00043	0	0	0	0	0	0
[0.95 , 1.00[	0	0.00009	0	0	0	0	0	0
1.00	0	0.00002	0	0	0	0	0	1

6.5. Conclusions

Confident-DEA, allows *imprecise cardinal data* e.g., a mixture of single-valued and bounded data, to be reflected in the efficiency measures. The resulting range for measures may be considered an *efficiency confidence interval*. Hence, the name *Confident-DEA* is prosed. A rule for determining the upper and lower bounds for the efficiency is provided via a proven theorem. The spread of the efficiency confidence interval in any application may be considered as a measure of the "risk" attached to the corresponding DMU: the larger the spread of the interval, the higher the uncertainty in the level of the corresponding DMU's efficiency and therefore the higher is the risk attached to the corresponding DMU.

Once the range for the efficiency is determined, a Monte-Carlo simulation based method is suggested to determine the distribution of the efficiency coefficients over the confidence interval. Significantly, IDEA (Cooper et al., 1999) always results in a single valued efficiency measure and implicitly assumes a uniform distribution for the bounded data. *Confident-DEA* on the other hand allows use of any distribution for the bounded data. Additionally, the simulation component proposes benchmarks, in terms of inputs and outputs, for any DMU considered and for any desired level of efficiency included in the its confidence interval. Moreover, the variance of the distribution of the efficiency measures over the efficiency confidence interval is yet another indicator of the risk, volatility, attached to the corresponding DMU.

A potential application of Confident DEA is in predicting efficiency. Given the relative nature of these measures this can not be done directly using a time-series of efficiency measures for the DMUs. By predicting the production factors, one can generate prediction confidence intervals. These intervals are then considered as data in Confident-DEA to provide the efficiency confidence interval for each DMU. The results obtained from the simulation component can be used to define a parametric approximation for the distribution of efficiency measures over their corresponding confidence intervals.

CHAPTER.7: *CONFIDENT-DEA* FOR IMPRECISE STOCHASTIC DATA (BOUNDED AND ORDINAL): A METHODOLOGY COMBINING GENETIC ALGORITHM AND MONTE CARLO SIMULATION

7.1. Introduction

Early research in optimization was concerned with finding the optimal solution to problems, or rather to a model of a real-world problem. That is finding an optimal (exact) solution to an approximate representation of the real world was the concern. Much has been written about the modeling process and its importance and different issues it raises. Oral and Kettani (1993) defines what they call the *"quartet of modeling-validation process"*. They identify four facets for their process: *conceptualisation* leading to conceptual model of the real world problem, *formal modeling* leading to formal model, a decision is made based on the *formal solution* obtained from the formal model and the final step is the *implementation* in a managerial situation. Landry et al (1996) introduced the *model legitimisation* as an additional concept to the quartet. The new concept invoked in the implementation facet, they claim, has to be distinguished from the *model validation*. Much more has been written about the modeling process and further discussions are provided in Reisman (1994), Reisman et al (1997a) and Reisman et al (1997b).

This chapter focuses on the facet *"obtaining formal solution"* in the Oral-Kettani's quartet mentioned above. This assumes that the conceptualisation of the problem was achieved and a formal model was developed.

7.2. Optimization Theory

Various structured solving methods were advised in the literature to solve optimization problems. These methods called *algorithms,* present manners to search for optimal solution more efficiently than by complete enumeration. The most famous example is the *Simplex* algorithm for linear programming problems, developed by Dantzig (1953).

However, these algorithms were capable of solving small size problems. As computing power increased, it became possible to solve larger problems and researchers became interested in how the solution times varied with the size of a problem. The computing effort for some problems could be shown to grow as a low-order polynomial in the size of the problem. For many other problems, the computational effort required was an exponential function of problem size. A new classification of optimization problems is then considered: *easy problems* for which a polynomial solving algorithm exists and *hard problems* for which such an algorithm does not exist. Technically they are called *P-Problem* and *NP-Problems* respectively. Further characterisation defines *NP-hard* and *NP-complete*. Many attempts were made to show that these classes are identical. However, there is a circumstantial evidence that the two classes are different. This evidence argues in favor of finding alternative ways of solving hard problems.

Research subsequent to the first stream of optimization methods and algorithms, which seek exact optimal solution, developed into two streams: *approximations* and *heuristics*. The first stream concerned itself with developing methods to find good approximation of exact solutions. These methods are based on *Lagrangean relaxation.* The second stream, the heuristics, was concerned with developing methods of finding solutions *good enough* to be accepted instead of the optimal ones. Meta-heuristics constitute a sub-class of heuristics that gained success in more recent applications. The basic feature of this approach consists of first coding the real problem in a special standard form then solving the coded form of the problem rather than the original one. Among meta-heuristics that gained particular success are *Simulated Annealing, Artificial Neural networks, Tabu Search* and *Genetic Algorithms.*

An obvious problem with heuristics is to know "*how good is the solution considered*" when the optimal solution is unknown. Further, heuristics cannot guarantee absolute optimality of the solution derived as they may find only a local rather than global optimum. Another serious shortcoming of heuristics is their high sensitivity to initial conditions. Starting with different initial conditions, a different local optimum can be reached and the new local optimum can be either an improvement or deterioration of the precedent one. An obvious gain from using heuristics is, however, their flexibility and ability to cope with more complicated, hence more realistic models.

This new stream represented a shift in the concern from finding an exact solution for an approximate model to finding an approximate solution to a more representative model. The cost of more representativeness was less exactness in the solutions.

A common feature to all the methods is to pick an initial solution and then check the neighbourhood; gradually an acceptable solution is defined. How to evaluate the current solution? How to move to the next alternative solution? How to search? When to stop? Those are the issues that characterize different heuristic methods.

The concern in this chapter goes to *Genetic Algorithms (GA)* as a heuristic method for solving optimization problems. The nature of the *Confident-DEA* makes it perfectly fitting in the special coding standard of Genetic Algorithms approach.

7.3. Genetic Algorithm Approach for Optimization Problems:

Holland (1992) and his associates suggested initially in the sixties and seventies the basic principles of Genetic Algorithms. They are inspired by the mechanism of natural selection where stronger individuals are likely to be the winners in a competing environment. Through the genetic evolution method, an optimal, or a satisfactory, solution can be found and represented by the final winner of the genetic game. The name Genetic Algorithm originates from the analogy between the representation of a complex structure by means of a vector of components, and the idea of the genetic structure of *chromosomes* familiar to biologists. A vector, generally a sequence of 0-1 components, represents a chromosome and each component represents a *gene* that reflects a specific elementary characteristic. Manipulations made on chromosomes are called *genetic operators* and the most common, *crossover and mutation,* are illustrated in *Figure 7.1*.

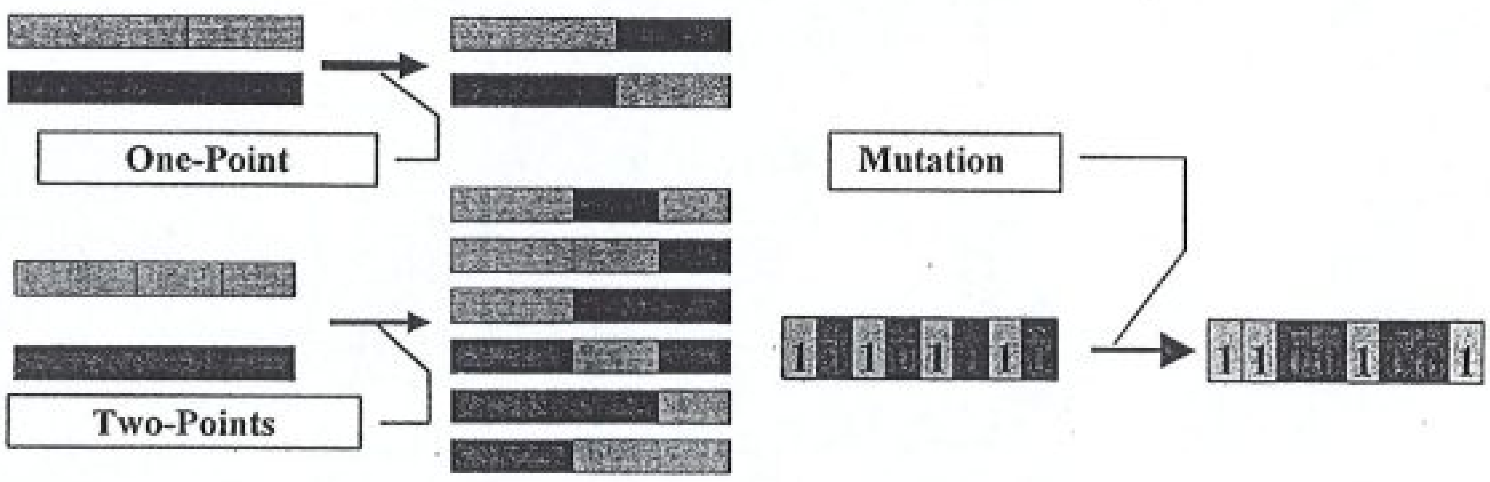
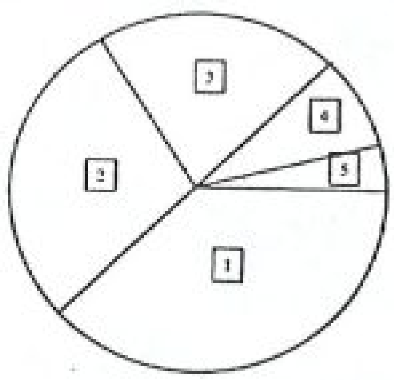


Figure 7.1: The genetic operators: crossover and permutation

The idea of Genetic Algorithm in optimization can be understood as an intelligent neighbouring random search method. While several methods using random sampling have been used, the *Genetic Algorithm* approach is more flexible and provides a new framework for a variety of problems.

The original version, Holland's version, of the Genetic Algorithm works by maintaining a population of M chromosomes considered as potential *parents*. Each chromosome is evaluated using a given function, and assigned a *fitness value*. Each chromosome encodes a solution to the problem and its fitness value is related to the objective function value for that solution. One parent, a chromosome, is selected on a fitness basis (the better the fitness value, the higher the chance of being chosen), while the other parent is chosen randomly. They are then *mated* by choosing a crossover point X at random, the *offspring* consists of the pre-X section from one parent followed by the post-X section of the other.

The Genetic Algorithm in general allows a *population* composed of many individuals to *evolve* under specified *selection rules* to a state that *maximizes* the *fitness*, a measure of goodness of individuals. It emulates the *survival-of-the-fittest* mechanism in nature. A mating pool is extracted from the original population of individuals or *chromosomes*. The Genetic Algorithm presumes that each chromosome, a potential candidate, can be represented by a set of parameters called *genes* and can be structured by a string of values in binary form. These selected chromosomes constitute the original set of parents. The most used scheme for the selection mechanism at this level is the *Roulette Wheel Selection*, illustrated in *Figure 7.2*.



- Sum the fitness of all the population members,
- Generate a random number (n) between 0 and the total fitness value,
- Return the first population member whose fitness added to the preceding population members greater than or equal to "n".

• Figure 7.2: Roulette Wheel selection method

The *genes* of the parents are mixed and recombined for the production of *offspring* in the next *generation*. The Genetic Algorithm cycle is illustrated in *Figure7.3*, which gives details about different steps for Genetic Algorithm in practice.

A large literature is available reflecting the success of Genetic Algorithms in different optimization problems. The most cited reference books are Holland (1992), Goldberg (1989), Back (1996) and Man, Tang and Kwong (1999).

The following diagram, *Figure 7.3*, presents the different steps for a genetic algorithm and shows the feedback used by the approach in order to renew the original population and generate new individuals in order to create the next generation.

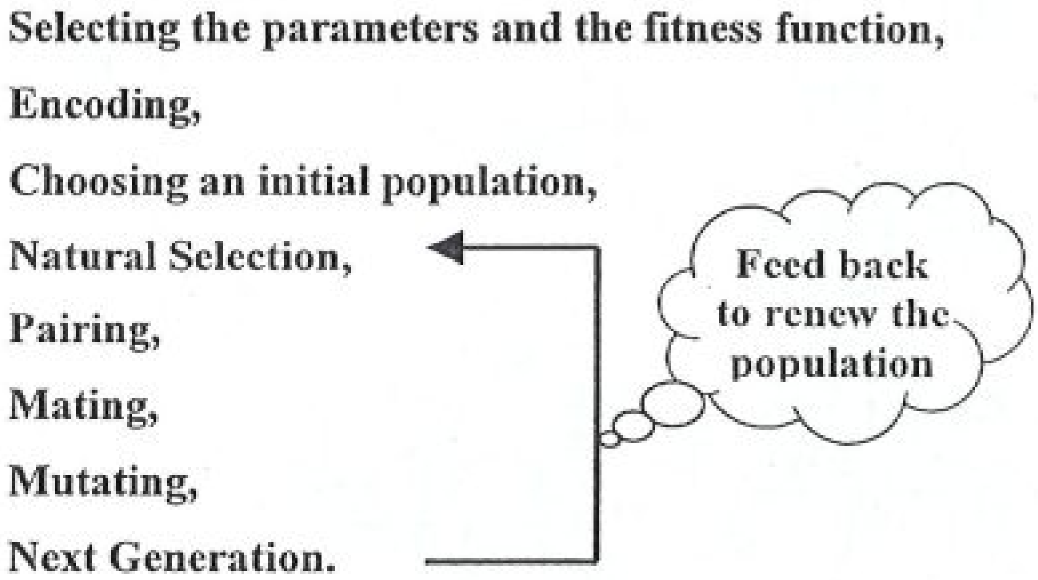


Figure 7.3: The genetic Algorithm Process

7.4. *Confident-DEA* with Imprecise Data: A Genetic-Algorithm-Based Meta-Heuristic Solving Method

In Chapter 6, the *Confident-DEA* was introduced in a context of cardinal data having either the form of single valued or bounded data. This chapter generalizes the *Confident-DEA* approach to general situation of imprecise data. As mentioned in Chapter 6, when imprecision in data is considered, the standard DEA model is not a linear program any longer. Furthermore, it can be seen as *bi-level convex* model, an *NP-hard* problem. This justifies finding heuristics solving methods. The choice goes to genetic algorithm because the high predisposition of the model to this meta heuristic.

The more general case of *Confident-DEA* proposed in this section uses Genetic-Algorithm to handle a mixture of data involving ordinal, single-valued and bounded. The steps of the meta-heuristic are described in *Figure 7.4, Figure 7.5* and *Figure 7.6*.

As any meta-heuristic, the first step is the encoding process that enables representing DMUs in the standard form for Genetic Algorithm use. For each DMU, a string of numbers is defined (continuous or discrete) representing the values of factors. For the factors presumed to be known exactly (single valued), there will be a single-value substring for each. For the bounded factors, each will be represented by a substring containing all possible values obtained from the discretization of the corresponding range. That is, the final string of numbers representing the DMU will be composed of substrings each one representing the possible value(s) for one factor. The key idea in the *Confident-DEA* approach is to represent each DMU by a set of *chromosomes*, binary strings, in which each *gene*, 1 or 0, refers to whether or not the corresponding value is assigned to the corresponding factor.

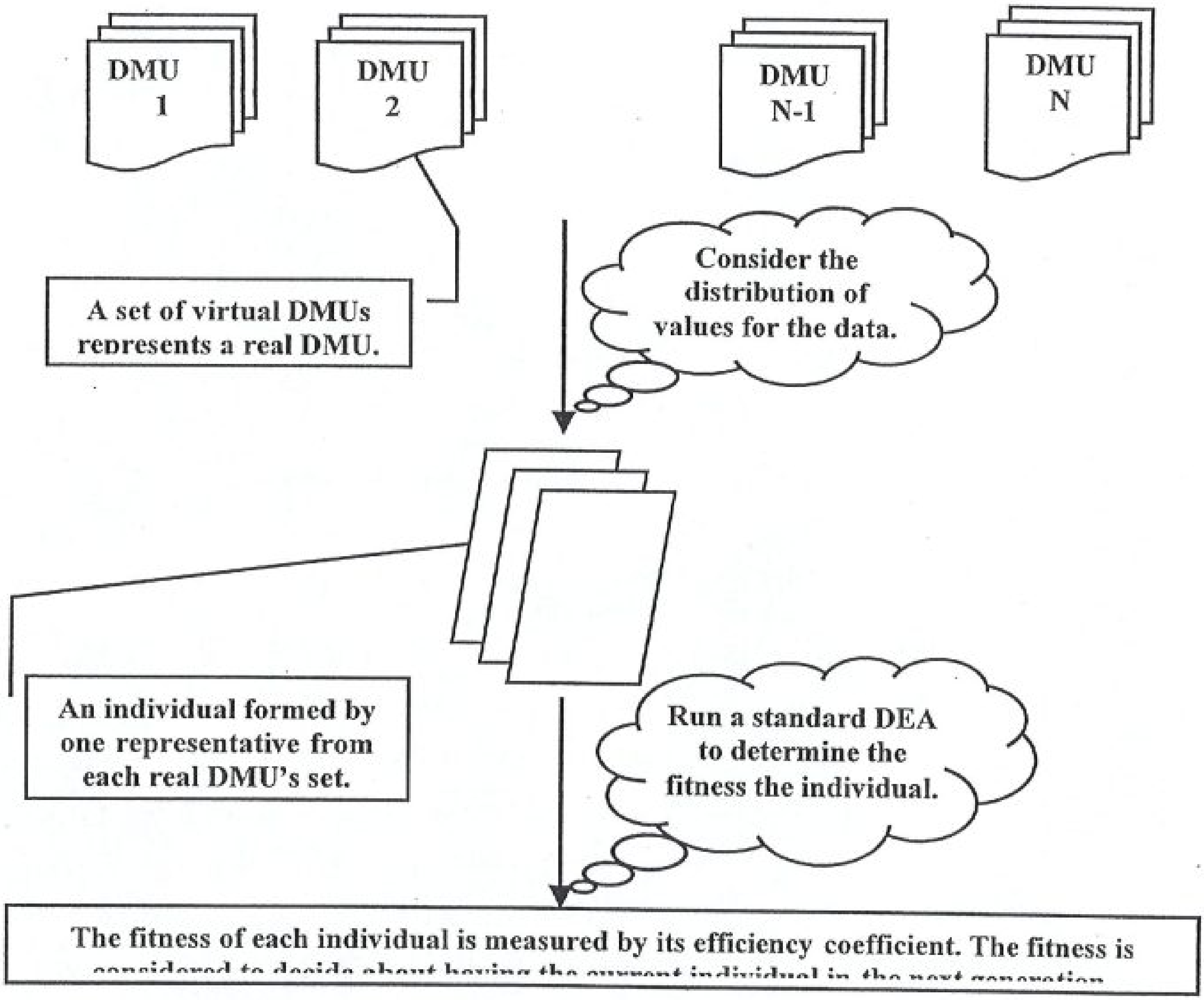


Figure 7.4: Splitting-up process and definition of an *"individual"*

Each DMU is split into a set of chromosomes, each one representing a *virtual single-valued* alternative for the real imprecise DMU.

For the illustration of the splitting-up process and generation of virtual DMUs, let a DMU using two inputs, $\mathbf{X}_1$ and $\mathbf{X}_2$ to produce two outputs $\mathbf{Y}_1$ and $\mathbf{Y}_2$. Suppose that $\mathbf{X}_1$ and $\mathbf{Y}_1$ are presumed to be described by exact data while $\mathbf{X}_2$ and $\mathbf{Y}_2$ are described by bounded data.

Let $X_1 = 20$ and $Y_1 = 30$ while $1 < X_2 < 5$ and $11 < Y_2 < 15$.

The factor's order is arbitrarily chosen as $\{X_1 ; Y_1 ; X_2 ; Y_2\}$. The semi-columns are used only for the purpose of explanation .The string of numbers representing this DMU will then be: {20 ; 30 ; 2 3 4 ; 12 13 14 }. The set of chromosomes representing this DMU will then be:

$K_1 = \{1 ; 1 ; 1\,0\,0 ; 1\,0\,0\}$, $K_2 = \{1 ; 1 ; 1\,0\,0 ; 0\,1\,0\}$, $K_3 = \{1 ; 1 ; 1\,0\,0 ; 0\,0\,1\}$,

$K_4 = \{1 ; 1 ; 0\,1\,0 ; 1\,0\,0\}$, $K_5 = \{1 ; 1 ; 0\,1\,0 ; 0\,1\,0\}$, $K_6 = \{1 ; 1 ; 0\,1\,0 ; 0\,0\,1\}$,

$K_8 = \{1 ; 1 ; 0\,0\,1 ; 1\,0\,0\}$, $K_8 = \{1 ; 1 ; 0\,0\,1 ; 0\,1\,0\}$, $K_9 = \{1 ; 1 ; 0\,0\,1 ; 0\,0\,1\}$.

The factors' value of the virtual DMU represented by K_1 are:

$$X_1 = 20;\ Y_1 = 30;\ X_2 = 2;\ Y_2 = 12.$$

By doing so for all DMUs, a set of virtual DMUs is obtained for each DMU. An *individual* is defined by a set of chromosomes determined by choosing, taking into account the distribution of imprecise values to make the approach stochastic, a representative from each set of virtual DMUs with exact data representing a real DMU. It is important to remark that, unlike the standard Genetic Algorithm procedure, an individual here is represented by a binary matrix rather then a binary string.

Once the encoding is realised, the Genetic Algorithm heuristic for *Confident-DEA* proceeds basically in two phases:

(i) selection of the initial population using the *Roulette Wheel* method, and

(ii) creation of the offspring using genetic modifications, to define the next generation. Multi-point crossover with high probability, around 0.9, and mutation with low probability, in the range 0.001-0.1 are the genetic modification used in the meta-heuristic. The cutting of the matrix-individual to define the crossover points is both vertical and horizontal. The size of the initial population as well as the number of iterations is set up arbitrarily at the beginning.

An initial population using the *Roulette Wheel* selection mechanisms is generated, and it constitutes the mating pool. An individual is a set of chromosomes, each one representing a DMU. All DMUs are represented in each individual and there is a single representative, a chromosome, of each DMU in each individual. The fitness function is the efficiency coefficient of the base-DMU.

Once the initial population is determined, the next phase is the creation of the next generation. This phase proceeds in three steps illustrated in *Figure 7.5*: ***(i)*** the mating of two selected individuals, considered as future parents ***(ii)*** make crossover with high probability and ***(iii)*** make mutation with low probability. All genetic modifications are decided based on the fitness of the individual determined by running a standard DEA model. The fitness measure is the efficiency coefficient of the base-DMU and it is computed for the selected individual at each step. Considering the binary matrix representing the individual, the corresponding virtual DMUs are identified. By solving the corresponding DEA model, the fitness, that is the efficiency coefficient of the base-DMU, is determined.

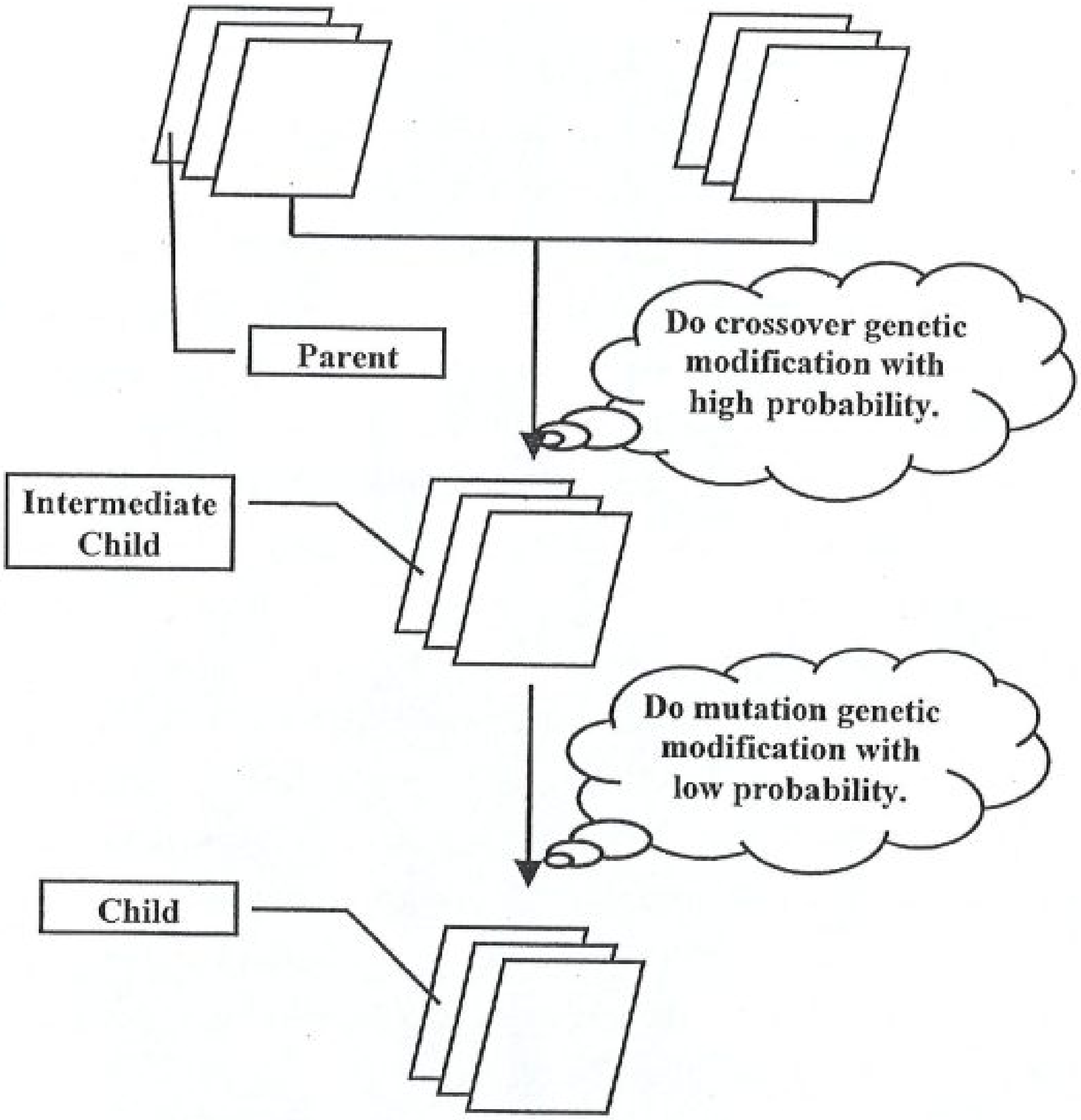


Figure 7.5: Genetic modifications: Crossover and Mutation

The process continues until a new generation is obtained. This new generation replaces the former generation and the process in initiated again. Iteration stops when the number of generations reaches the predetermined number.

The meta-heuristic proceeds in depth first, which means that all iterations are run for the first base-DMU to determine the lowest level of efficiency, then the iterations are run to determine the highest level. Once done with the first DMU, the process is iterated for the second base-DMU and so on.

Using this Genetic Algorithm based approach, summarized in *Figure 7.6* and *Figure 7.7*, an upper bound and a lower bound for the efficiency coefficient of each DMU are defined. Like any heuristic or meta-heuristic, obtaining an optimal solution is not guaranteed.

7.5. A Simulation-Based Component of *Confident-DEA*

The third component of *Confident-DEA* is a simulation based heuristic. It proceeds in three phases:

(i) define the individuals in the same way described for the Genetic Algorithm based heuristic,

(ii) run a standard DEA for each individual in order to determine its efficiency coefficient and

(iii) determine the confidence interval and the distribution of efficiency coefficient for each DMU by using a Monte Carlo type simulation.

Once an individual is chosen, the efficiency coefficient of each one of its virtual DMUs is computed by solving the corresponding standard DEA model. These values are stored for future comparison. In the next iteration, the coefficients obtained are compared with previous results in order to determine the minimum and the maximum efficiency level for each DMU. Once the predetermined number of iterations is reached, the output of the heuristic has three components. First, a confidence efficiency interval for each DMU is determined. Second, benchmarks for different level of efficiency are identified. Finally, the distribution for the efficiency coefficient is defined based on the frequency histogram number of hits for each predefined sub-interval of [0-1]. The interval [0-1] is in fact pre-divided in a set of sub-intervals with the equal length. This predetermined length reflects the degree of precision in efficiency measure fixed by the modeler. A counter is placed in each sub-interval to record the frequency of efficiency coefficient corresponding to this sub-interval. A histogram is obtained for each DMU and the corresponding efficiency distribution is determined by smoothing the histogram.

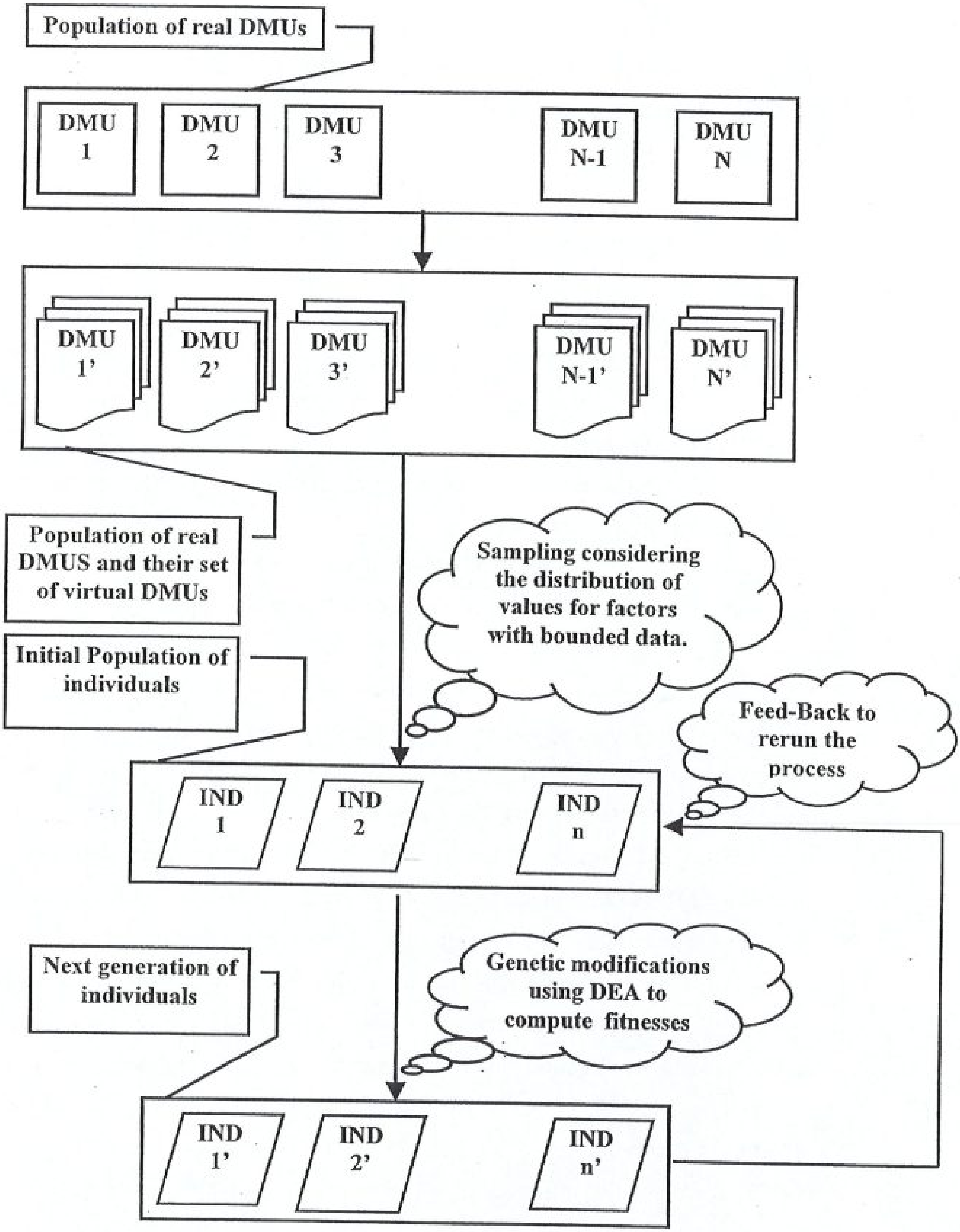


Figure 7.6: Methodological Contribution: Marriage of DEA with Genetic Algorithm Procedure

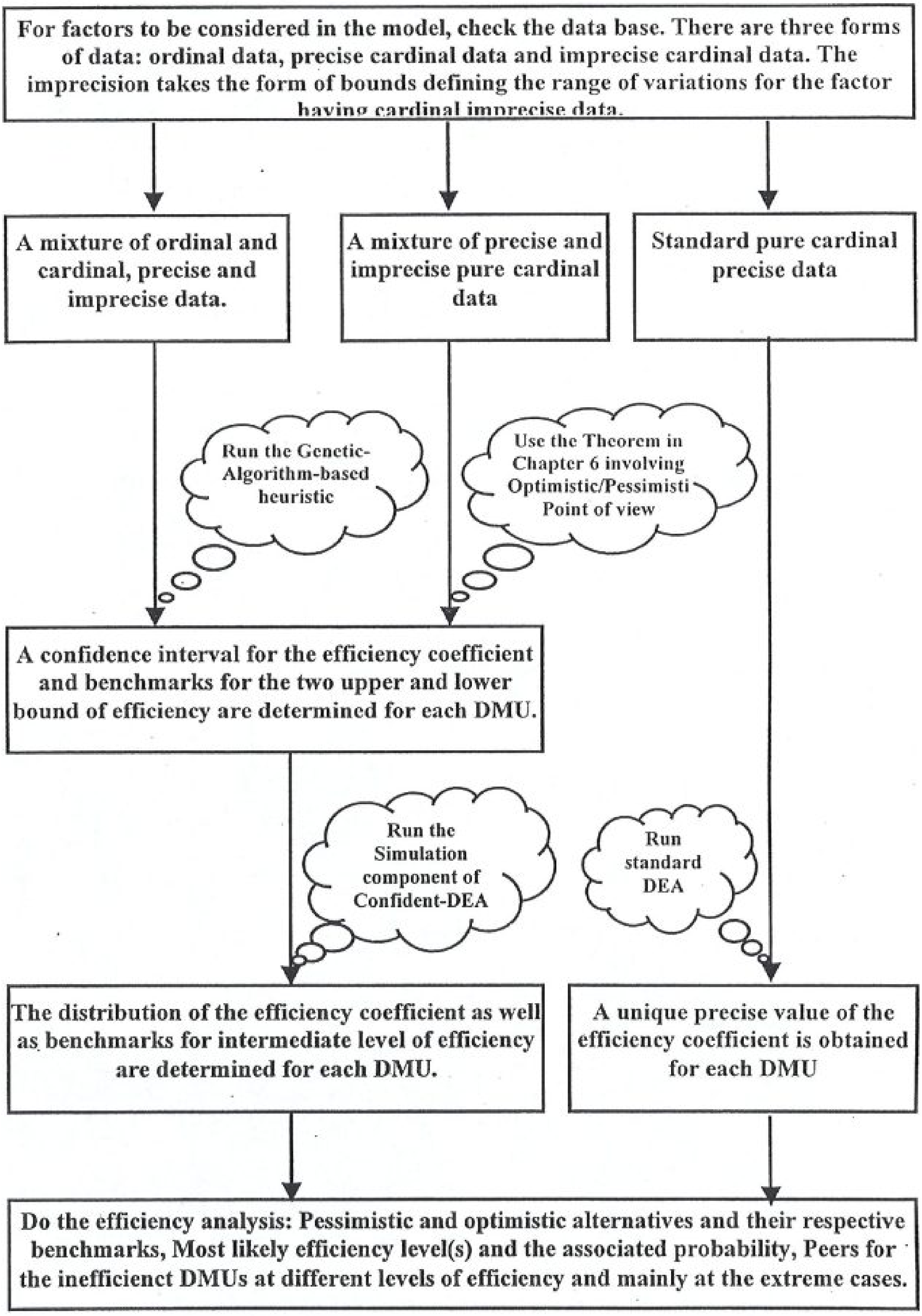


Figure 7.8: The unified *Confident-DEA* approach to efficiency analysis with single-valued and imprecise data

7.6 An Illustrative Example

To illustrate the methodology and for comparative purposes, consider the data contained in Cooper et al (forthcoming) suumarized in *Table 7.1.* The description of the real-world case is reproduced here from the above mentioned reference:

"...

2.Problem and Data Detail

We now turn to an example in order to provide a concrete illustration. In particular, we use this example to show how ordinal and bounded data, as well as exact data, can be combined into the one unified approach provided by IDEA.

As noted in the Introduction, our example involves efficiency evaluations of the branch offices of a mobile telecommunications corporation in Korea. We refer to this company as TELCOM and note that it has eight branch offices which are located in: Seoul (the biggest of these cities), Kangnung, Taejun, Taegu, Jeonju, Pusan, Kwangju and, finally, Jeju (the smallest city). See Figure 1.

All branch offices perform a number of common tasks with given manpower and operating costs. For simplicity, we confine our attention to the following key tasks: (a) operation and management of mobile telecommunication facilities, which includes maintenance and repair of facilities (such as exchanges) and the bases in their respective areas, and (b) handling customer relations and securing subscribers and other potential customers in a satisfactory manner — including anticipating and responding to customer needs and wants.

Figure 1. A map of South Korea showing the branch offices and their control areas in TELCOM.

The data to be used, as exhibited in Table I, are summarized as follows:

INPUTS

(X1)*Manpower*: This represents the number of regular employees and excludes personnel used for specialized tasks such as undertaking the development of new technology and research on satellite services. These manpower data are exact as given under X1 in column 1 under Inputs in Table I.

(X2)*Operating cost*: This consists of variable costs relevant to providing services of mobile telephones and pagers. It excludes interest cost, depreciation and cost of equipment investment (because these costs are handled by headquarters). Labor costs are also excluded, because they are implicitly included in X1, manpower. These operating cost data, also exact, are given under X2 in column 2.

(X3)*Level of management for facilities and customers*: Here we turn to ordinal relations represented by the rankings under X3 in column 3. This input reflects the experience (assumed to be correlated with skill) of the team used in facility maintenance and repair, as well as experience (and skill) in dealing with customers and securing new business. Headquarter management evaluates this factor for all branch offices once a year and uses the results of these rankings to guide salary evaluations, effect promotions, award bonuses, etc. The results of these evaluations are reported in the ordinal rankings of the eight offices shown under X3 in Table I. These rankings are shown for 1996 in the form of numerical values so that, for instance, the levels of experience and effort associated with Kwangju are given the highest rank, and Jeju, the lowest. However, these numerical values are only intended to represent the rankings accorded to these aspects of management performance.

Using the above three inputs, each branch office produces three outputs which are summarized under the columns headed Y1, Y2 and Y3 in Table I as follows:

OUTPUTS

(Y1)*Revenue*: These are the receipts from providing services of mobile telephones and pagers. See column for Y1 under Outputs in Table I. These data are exact. Note that all eight branch offices adhere to the same price schemes for services of mobile telephones and pagers.

(Y2)*Rate of facility failures*: This is the number of failures for exchanges and bases in 1996. These data, also exact, are given under Y2.

(Y3)*Rate of call completion*: This is the number of successful calls per total number of calls initiated by customers in 1996. Referred to as call completion rates (under Y3 in Table I), these data are also exact.

Table I: Data for Efficiency Evaluation of the Branch Offices in TELCOM†

DMU (j)	Inputs			Outputs			
	X1: Man-power (num.)	X2: Operating cost (mill. $)	X3††: Managem-ent level (rank)	Y1: Revenue (mill. $)	Y2†††: Facility success rate (%)	Y3: Call com-pletion Rate (%)	Y3' (ratio %)
Seoul (1)	124 (1.000)	18.22 (1.000)	4	25.53 (1.000)	89.8 (0.902)	64.5 (0.661)	[80, 85]
Pusan (2)	95 (0.766)	9.23 (0.507)	2	18.43 (0.722)	99.6 (1.000)	62.9 (0.644)	[85, 90]
Taegu (3)	92 (0.742)	8.07 (0.443)	6	10.29 (0.403)	87.0 (0.873)	74.0 (0.758)	[75, 80]
Kwangju (4)	61 (0.492)	5.62 (0.308)	8	8.32 (0.326)	99.4 (0.998)	97.6 (1.000)	100
Taejun (5)	63 (0.508)	5.33 (0.293)	7	7.04 (0.276)	96.4 (0.968)	71.0 (0.727)	[70, 75]
Jeonju (6)	50 (0.403)	3.53 (0.194)	3	6.42 (0.251)	86.0 (0.863)	94.2 (0.965)	[90, 95]
Kangnung (7)	40 (0.333)	3.50 (0.192)	5	2.20 (0.086)	71.0 (0.713)	60.0 (0.615)	[80, 85]
Jeju (8)	16 (0.129)	1.17 (0.064)	1	2.87 (0.112)	98.0 (0.984)	97.1 (0.995)	[95, 100]

† The data in parentheses represent transformed data obtained by dividing the values in each column by the column maximum.

†† The data in this column only reflect the relations $x_{34} \geq x_{35} \geq \ldots \geq x_{32} \geq x_{38}$.

††† The data in this column represent "the rate of successes = 100 − rate of failures," which are represented in this manner for convenience in handling output data.

Now we introduce another variable, Y3′, to reflect the fact that the rate of call completion is highly dependent on regional characteristics of each branch office. For example, there are many high mountains in the Kangnung region, and this seriously influences the call completion rate in a negative manner. As shown under Y3 in Table I, the Kangnung office has the lowest call completion rate at 60%. In contrast, the topography of the Kwangju region is almost flat. Thus, its call completion rate, which is highest at 97.62%, is partly attributable to this fact. High buildings and subways can also affect the call completion rate, so big cities like Seoul (population of about 12 million) and Pusan (7 million) exhibit similar negative effects, with Pusan being doubly affected because it is also located in mountainous territory.

For reasons like these, the efficiency evaluation team at company headquarters believes that the original data on rate of call completion (Y3) need to be adjusted to obtain a fairer evaluation. Failing agreement on a suitable method of weighting (or scaling), it was decided to replace the original data in the column under Y3 with the bounds given under Y3′ as established by an evaluation team appointed by company management. To see what was done we note that the call completion rate of the Seoul office (64.5%) in column Y3 was restated to [80%, 85%] in column Y3′. The meaning of this replacement is that the call completion rate in Seoul should be accorded some value between these limits. Because comparisons are related to the rate for Kwangju, its 97.6% rate was adjusted upwards to 100% in accordance with the anchoring used in IDEA. All comparisons are then effected relative to this highest score. (Note that $64.5 \div 97.6 \approx 66.1\%$, which represents the rating of Seoul relative to Kwangju in terms of the original call completion rate, is not acceptable for use in our evaluations, because it falls outside the 80 – 85% completion rate prescribed as bounds for the "correct" rate.)[1,2]…"

Table 7.1: Imprecise Data for an Illustrative Example (Adapted from Cooper et al. (forthcoming))

	X_1	X_2	X_3	Y_1	Y_2	Y_3
DMU 1	124	18.22	4	25.53	89.8	[80;85]
DMU 2	95	9.23	2	18.43	99.6	[85;90]
DMU 3	92	8.07	6	10.29	87	[75;80]
DMU 4	61	5.62	8	8.32	99.4	100
DMU 5	63	5.33	7	7.04	96.4	[70;75]
DMU 6	50	3.53	3	6.42	86	[90;95]
DMU 7	40	3.5	5	2.2	71	[80;85]
DMU 8	16	1.17	1	2.87	98	[95;100]

The GA based heuristics is used to determine the bounds for the efficiency confidence interval. The results are presented in *Table 7.2*. This table also contains the efficiency measures obtained by Cooper et al (forthcoming).

"[1] We have only described the results of data scaling from Y3 to Y3′ since the scaling process is very complicated and its discussion here would entail a long and involve diversion from the mainstream of this paper. We note, however, that in this scaling the expert (i.e., evaluation team) considered a variety of negative influences in arriving at the bounds used under Y3′ for the call completion rates. Examples were the number and height of mountains, the number and size of high buildings, and areas occupied by subways (including underpasses)."

"[2] The call completion rates (Y3) can be viewed as "non-discretionary (or exogenously fixed)" outputs because these are attributed to regional characteristics as mentioned in this paper. For the treatment of exogenously fixed inputs and outputs, see Banker and Morey (1986). Extensions of IDEA to non-discretionary variables can be done, but we do not attempt such an extension in the present paper. Instead, we only call attention to the fact that the call completion rates as discretionary outputs must be within the limits prescribed by the scaled data under Y3′."

Table 7.2: Comparative results of *IDEA* and Simulated *Confident-DEA* with Imprecise Data

	DMU 1	DMU 2	DMU 3	DMU 4	DMU 5	DMU 6	DMU 7	DMU 8
Results of IDEA	1	1	0.894	1	0.976	1	0.895	1
Highest Efficiency	1	1	0.8363	0.8723	0.8746	0.9842	0.8113	1
Lowest Efficiency	1	1	0.5941	0.7159	0.2018	0.7414	0.2546	1

One can notice a small deviation from the exact optimal solution as determined by *IDEA*. This is due to the assumption of normality, which gives small weights to extreme values where the highest and lowest values of efficiency coefficients are most likely to be reached.

To determine the distribution of the efficiency measures over their corresponding efficiency confidence interval, the simulation component of Confident-DEA was run 100,000 times. The results are provided in the appendix. The efficiency histogram for DMU6 is in *Figure 7.8*.

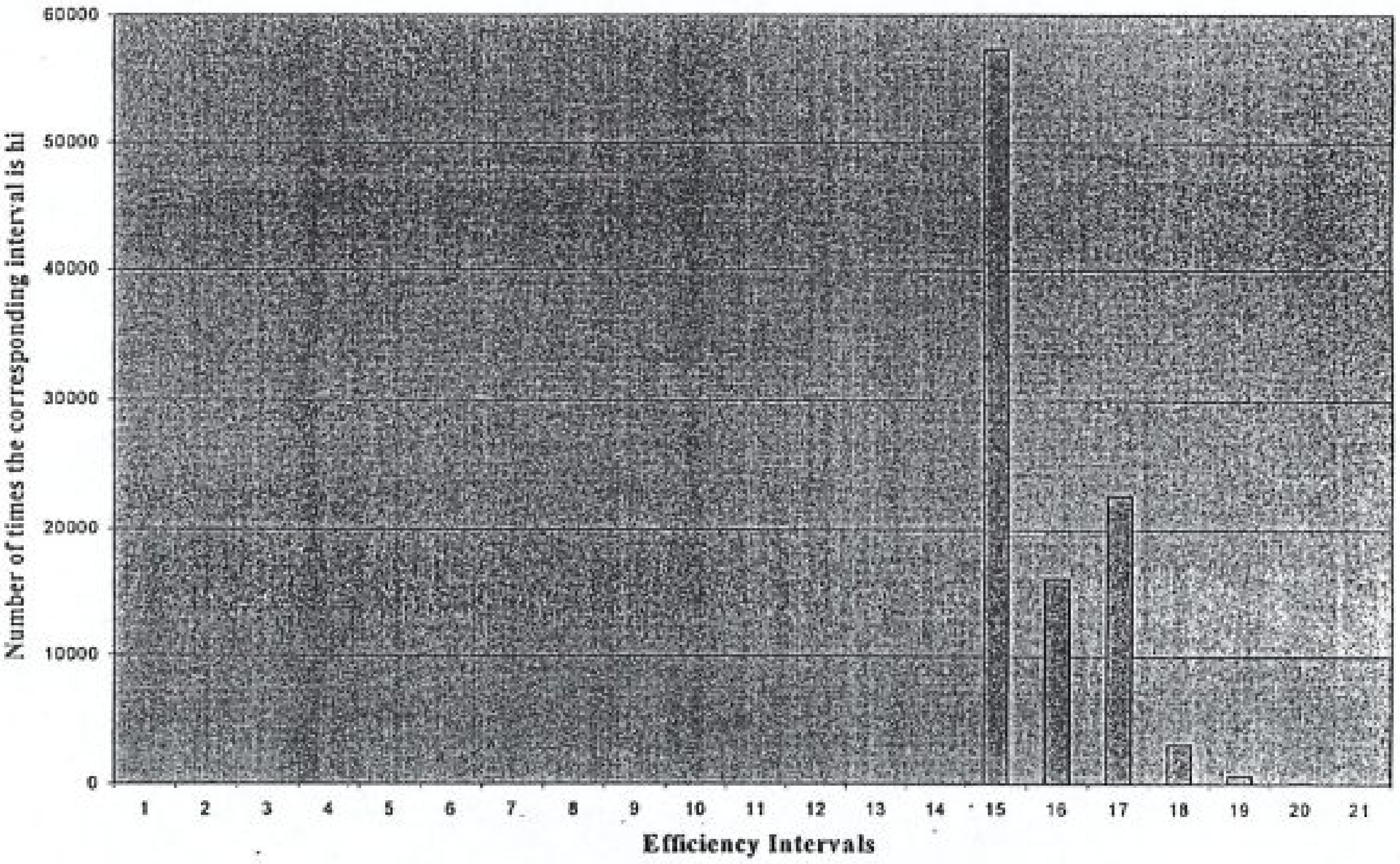


Figure 7.8: Efficiency Histogram for DMU 6

As in the case of cardinal bounded data, the Monte Carlo simulation component in for *Confident-DEA* permits in the case of imprecise data the approximation of the distribution of efficiency values over the efficiency confidence interval.

7.7. Concluding Remarks

In conclusion it can be affirmed that *Confident-DEA* generalizes *IDEA* in the sense that the efficiency levels identified by *IDEA* for each DMU coincide with the *optimistic point of view* in the *Confident-DEA* approach, in the case of bounded cardinal data. However, although the consistency between the two approaches is confirmed in the general case of imprecise data, more analytical and conceptual work is needed to establish the exact correspondence. An equivalent to the optimistic and pessimistic point of view in the case of ordinal data needs in fact to be defined.

While the Simulation-based component provides distribution for the efficiency coefficients as well as benchmarks for each intermediary level of efficiency, it is not highly efficient in finding the extreme solution. The Genetic-Algorithm-based component provides a better solution for the bounds of the efficiency interval. Combining results from both components will provide more complete and reliable information about the efficiency interval. *Figure 7.7* contains a presentation of the unified approach.

CHAPTER 8: PREDICTING TECHNICAL EFFICIENCY OF COMMERCIAL BANKING SYSTEMS IN OECD COUNTRIES USING *CONFIDENT-DEA*

8.1. The Research Subject and its Context

The ongoing globalization of the international economy, and as part of it the gradual deregulation and liberalization of financial services, has created new market realities marked by a progressive harmonization of regulatory environments. Financial institutions from different countries representing different global regions are brought into closer international competition and an increasing cross-border activity of financial institutions. This process was enhanced by the high rate of introduction of information technologies. The increasing activity of financial institutions makes them exposed to both domestic and foreign environments and cross-border influences impact, the performance of these institutions in both a positive and a negative manner.

Because of these changes, financial services industry structures are rapidly changing. Deep mutation is still ongoing in this industry. A continuing wave of mergers, acquisitions, and in many cases, collapses of giant financial institutions at local, regional and international levels confirm this deep mutation. Also, the development of network companies consisting of a teaming up of companies for better competitiveness, as analyzed in Lakhal et al (1999), was a new characteristic phenomenon of the globalization era in the finance industry.

The dynamics and vitality in the financial industry, however, increased the vulnerability of financial systems and crises are not rare events any more. Also, the liberalization of the international economy facilitates the cross-border contamination of financial crises. The Mexican crisis and its impact on the American financial system, the Asian, Russian and the Japanese banking crises and their respective regional and international impacts on financial systems exemplify the vulnerability of the current financial system worldwide.

Such an era of financial turbulence created an extreme necessity for countries to continuously observe the performance levels of their financial institutions and be vigilant of any deterioration in such performance relative to international counterparts.

A central component of any national financial system is its banking system. It constitutes the main channel for in-border and cross-border circulation of funds. The level of performance of any national banking system, seen mainly through its productivity level, indicates the healthiness and the competitiveness level of the national financial system. Competitiveness and productivity levels are in fact strongly related to one another, as proved analytically in Oral et al (1999) at the firm level. A similarly strong relation is expected to hold at system level as well. The banking system is a determinant parameter for the performance of the national financial system and as national economy overall. Placing the performance of banking system as a national strategic issue is now a fact. It is becoming more significant with the liberalization of financial services as directed by the World Trade Organization. This gradual liberalization makes national banking systems face a sharpening competition in the local market, from both local competitors and from the cross-border activities of international counterparts. Often, the cross-border Banks outperform the local institutions as confirmed by Mercan et al (forthcoming) for the case of Turkey.

Necessity for vigilance and prudential considerations demand extensive insight into the relative performance of any national banking system vis a vis international counterparts. Benchmarks are needed for policymakers and regulatory institutions for the definition of targets in terms of effectiveness and performance. Additional factors need to be considered to provide more accurate forecasts because of the rapidity of change in the environment. This is confirmed by the fact that although the Mexican and the Asian crises were predicted by national and by international financial institutions, the extent they had and the continuing effect at regional and international scale were largely surprising for all institutions and agencies.

Cross-country efficiency analysis of banking systems can be made either by studying and comparing individual banking institutions belonging to different countries or by considering the overall banking systems of the respective countries utilizing aggregate data. This study deals with the latter and countries considered in the sample are assumed to constitute a homogeneous set of countries competing in the same global market. Moreover, this study is oriented toward the overall performance of banking systems as opposed to the performance of their components.

By improving relative efficiency, banking institutions at the micro-economic level and banking systems at the macro-economic level can gain important comparative advantages. It is worth mentioning that Berger (1993) found that X-inefficiency, also known as managerial inefficiency, of banks on the average consumes 20% of the total cost in the US banking system. Such high cost levels due to inefficiency indicate the substantial improvements that can be realized. However, in terms of profit, banks in the United States are confirmed to be among the best performers. This *paradox* suggests that analyses based on both revenues and costs while considering environmental, in-border and cross-border, factors will better reflect the performance of national banking systems.

Evaluation of a national banking system performance can be made using either the methods of classical accounting and hence financial ratios, or the concept of efficiency developed in recent economic theory. Berger et al (1993) and Berger et al (1997) provide a more in depth discussion and a large literature survey.

Efficiency analysis was more attractive to researchers and different approaches were developed. The most distinguished are the parametric methods based on microeconomic theory and econometric techniques, and the non-parametric methods based on linear programming. Oral et al (1992) stressed the complementarity between the classical financial methods and the more recent non-parametric methods in a sense that the former deals with purely technical performance while the latter evaluates the operational performance. The use of both types of measures makes the evaluation process of performance more comprehensive and complete.

This study forecasts and compares performance of banking systems of different countries for a single year based on data describing the banking activities for the previous years. The sample retained for this study is a set of countries belonging to the Organization for Economic Cooperation and Development (OECD). The importance of these countries as major economies and the availability of accurate, harmonized and reliable data about their banking systems justify the choice. Using the unified and official OECD database (1997 and 2000), this study assesses the relative performance of the commercial banking systems of each country in comparison to the other members included in the sample.

8.2. Cross-Country Efficiency Analysis in Banking: A Literature Survey

In recent years, cross-country efficiency of financial institutions and systems has enjoyed increasing interest among researchers. This interest persists despite the intrinsic difficulty of identifying and modelling environmental considerations across countries. It is related to the increasing cross-border activity of financial institutions seeking more insights into the notions of comparative advantage.

Berger et al. (2000) studied the globalisation of financial institutions through the analysis of cross-border bank performance. The authors reviewed several hundred studies covering topics related to cross border activities. They raise some doubt about the accuracy of the results obtained for cross-country analysis. They report that although some institutions of certain nations are substantially more efficient than the institutions of other nations, the ranking among nations differed across the studies.

Dietsch and Lozano-Vivas (2000) compared the cost efficiency of banking systems in France and Spain. They used a Distribution Free approach, a variant of Stochastic Frontier approach, to evaluate the cost efficiency of commercial banks. Two models are estimated, where one model takes into account the environmental conditions and the other ignores environmental conditions. The results suggest that, without environmental conditions, French commercial banks are substantially more cost efficient than their Spanish counterparts. When environmental variables are included, this gap is drastically reduced.

Berger and Humphrey (1997) presented a survey of 130 studies that apply frontier efficiency analysis to financial institutions in 21 countries. The authors report that only five studies, reported here later, have compared efficiency levels across countries. Regarding cross-country studies, the authors wrote "Clearly, this is an area where more work is needed, and especially the proper specification of country-specific environmental influences that will justify using a common frontier for cross-country comparisons of efficiency" (Berger and Humphrey 1997, p.188).

Rutenberg and Elias (1996) used the Thick-Frontier Approach to determine the average efficiency of banks in 15 developed countries. Pastor et al. (1997) used DEA to determine the average efficiency of banks in 8 developed countries among the 15 covered by the previous study. Comparative results about the relative performance of banking systems in those countries do not differ dramatically. Berg et al. (1993) performed DEA of banks in Norway, Sweden and Finland to compare banking performance in Nordic countries. In a follow-up study, Bukh et al. (1995) added Denmark. Bergendahl (1995) used data relative to the banks of the same four countries to develop a *virtual ideal composite* bank. The components of this ideal bank are the most efficient parts of the banks in the sample. These three studies lead to consistent conclusions about the relative performance of banks in Nordic countries.

Molyneux et al. (1996) use in their parametric approach an altered Cobb-Douglas production function to compare efficiency in the major European banking markets. Their results indicate noticeable differences in cost characteristics across the French, German, Spanish and Italian banking markets. Cost savings appear to occur mainly through the increased average size of bank branches measured by total assets rather than through the size of the banking firm.

Allen and Rai (1996) use the Stochastic Cost Frontier Approach and Distribution-Free Model to estimate a global cost function of the banking industry in 15 developed countries to test for both input and output inefficiencies. In addition to conclusions about relative performance, their main result suggests that large banks are much more inefficient than other banks.

From the above literature survey, we conclude that although some studies have compared efficiency in different countries, none of them considered predicting of technical efficiency. Banking units Data from different countries were often used in these studies to determine an average measure of performance for each country in periods past. However, none of the studies cited used aggregate data for banking systems of the countries studied.

Coming to the factors used in economic and financial studies, it happens often that proxies are used instead of the non-available single-valued data. The use of ranges obtained by assuming that the single-valued value of each factor lies within prescribed bounds defined by percentages of the corresponding proxy is more appropriate. Also, there is an abundant information about financial institutions that takes the form of ordinal data representing rankings or other judgemental evaluations. These are imprecise data and non traditional efficiency analysis models are needed if one wants to include them as factors in the analysis.

The use of *Confident-DEA* as alternative to the classical DEA-based approaches gives an additional credibility to the use of proxies in financial data. Also, forecasts from econometric models for the factors can be included in order to produce forecasts about the future performance of the DMUs being studied. Rankings and ratings of countries obtained from international financial agencies as well as confidence intervals obtained from econometric or time series forecasting models can be considered as imprecise data and considered for the efficiency analysis and the evaluation of performance.

Despite its potential relevance, the issue of imprecise data is not addressed in the literature devoted to the efficiency analysis of financial institutions and hence constitutes a promising subject for research. Furthermore, cross-country studies of financial institutions, and particularly banking systems, combining econometric and/or time series modelling with *Confident-DEA* can be promising tools helping strategic decision makers, for descriptive as well as prescriptive purposes. As a potential application, *Confident-DEA* can be of high relevance for the early warning of financial crisis, crisis that can lead to the failure of financial systems, or financial institutions, if not detected enough early and dammed up. Üçer et al (1999) used DEA, among other methods, to generate a composite index as a leading indicator of currency crises in Turkey.

8.3. Measuring Bank Efficiency: Theoretical Background

The banking efficiency literature is dominated by studies in the USA, where the larger market and number of banks have traditionally facilitated econometric modelling. Most of the efficiency literature is on the cost effects of economies of scale (size) and scope (product mix) (Hunter and Timme 1986; Berger et al. 1987; Elyasiani and Mehdian 1990; Ferrier and Lovell 1990; Noulas et al. 1990; Hunter and Timme 1991; Berger and Humphrey 1991; Fields et al. 1993; McAllister and McManus 1993; Rhodes 1993).

Unfortunately, the conventional scale and scope economies studies are beset with a number of problems. For example, the translog cost or production function, traditionally used in parametric efficiency analysis approaches, generates a poor approximation when banks of assorted sizes are used. Another potential problem is that scope economies can be confounded with X-efficiency differences when applied to banks *off* the efficient frontier. In response to these problems, alternative research designs have emerged. For example, in some studies the translog function has been replaced by non-parametric estimation procedures such as the kernel regression technique (McAllister and McManus 1993). Other researchers have moved away from the cost or production functions and focused on the profit function instead, in an effort to estimate optimal scope economies (Berger et al. 1993). The theoretical appeal of working with the profit function is that it accounts for the revenue effects as well as the cost effects of operating at incorrect levels or mixes of inputs and outputs (Akhavein et al. 1997).

More recently, focus has shifted to X-efficiencies; e.g., the ability of management to control costs and generate revenues (Elyasiani and Mehdian 1990; Ferrier and Lovell 1990; English et al. 1993; Allen and Rai 1996; Mester 1996). X-efficiency comprises allocative and technical efficiencies of banks, where allocative inefficiency is defined as a decline in performance from selecting an ineffective production plan, and technical inefficiency is defined as the poor implementation of this production plan (Berger et al. 1993). Existing studies indicate that X-inefficiencies constitute 20% or more of costs, while scale and scope inefficiencies account for less than 5% of costs in banking (Berger et al. 1993).

There is no consensus on the best procedure for measuring X-efficiencies. The principal measurement problem is distinguishing variations in X-efficiency from random error. Examples of different procedures are the econometric frontier approach (EFA), the thick frontier approach (TFA), the distribution-free approach (DFA), and the DEA. Assumptions in each of these four approaches differ on the distribution of X-efficiency and random error (Berger et al. 1993). DEA, the procedure adopted in this study, usually assumes no random error, thus implying that all deviations from the estimated efficient frontier actually constitute X-inefficiencies. Other researchers who have recently used DEA in measuring relative bank efficiency include Berg et al. (1992), Berg et al. (1993), Drake and Howcroft (1994), Elyasiani and Mehdian (1995), Favero and Papi (1995), Fukuyama (1995), Haag and Jaska (1995), Sherman and Ladino (1995), Wheelock and Wilson (1995), Zaim (1995), Grifell-Tatje and Lovell (1996), Miller and Noulas (1996), Bhattacharyya et al. (1997), Resti (1997), and Avkiran (1999).

Though it is common to use parametric methods to study macro-financial systems, very few studies have used the DEA approach. One of the objectives of this study is to establish the utility of DEA as an alternative method in the context of macro level analysis.

While there is no consensus among researchers about the inputs and outputs of a bank, there are two principal schools of thought on bank behaviour. One of these is the *intermediation* approach to modelling bank behaviour in which deposits are regarded as being converted into loans (Mester, 1987). The intermediation approach is preferable since it normally includes interest expense, a large proportion of any bank's total costs (Elyasiani and Mehdian, 1990; Berger and Humphrey, 1991). The other principal approach is the *production* approach where banks are regarded as using labour and capital to generate deposits and loans (outputs are usually measured in number of accounts rather than dollars).

Two other approaches are used in the banking literature to modelling bank behaviour. One approach is that of *value-added* (Berger and Humphrey, 1992). Under this approach high value creating activities such as the making of loans and taking deposits are classified as outputs and measured in dollar terms, whereas labour, physical capital and purchased funds are classified as inputs (Wheelock and Wilson, 1995). The other approach is referred to as *user-cost*. The user-cost approach assigns an asset as an output if the financial returns are greater than the opportunity cost of funds. Similarly, a liability item is regarded as an output if the financial costs are less than the opportunity cost. If neither of these conditions is satisfied, the asset or the liability is classified as input (Berger and Humphrey, 1992). The user-cost approach is usually attributed to Hancock (1986). According to Hancock, user costs can be calculated for all the assets and liabilities on the balance sheet. The assignment of assets and liability items as inputs or outputs may change with movements in interest rates and service charges.

8.4. Predicting Technical Efficiency: An Application of *Confident-DEA*

Predicting technical efficiency of the banking systems in a subset of countries belonging to OECD using the *Confident-DEA* methodology developed in earlier chapters is considered next. The application consists of three steps:

1. a. Predicting factors identified for the efficiency analysis based on the relevant theory and **b.** Defining the forecasted confidence interval for each factor for each country. Toward this end, time-series regression is used and two sets of data are identified: one defining the confidence interval as a two sided one standard deviation around the mean and the other as a half standard deviation around the mean. It should be noticed that this approach is stochastic in nature since it takes into account the assumption that the forecasted factors are normally distributed around the mean.

The first step generates confidence intervals for predicting the production factors of each banking system considered in the sample. These intervals are considered as a set of imprecise cardinal data.

2. Determining the confidence interval of efficiency for each country using *Confident-DEA* for the imprecise data generated in the first step. The lower bound of the efficiency confidence interval represents the *pessimistic point of view* and the upper bound represents the *optimistic point of view* of the corresponding country. As defined in *Chapter 5*, the *"pessimistic point of view"* for a given banking system considered for the relative efficiency evaluation, among a set of banking systems, is defined by the situation where the given banking system is using the maximum quantities of inputs, allowed by the boundaries on the data, for producing the minimum quantities of outputs. Its competitors are, at the opposite extreme, using the minimum allowed quantities of inputs for producing the maximum quantities of outputs. The *"optimistic point of view"* for a given banking system is of course the opposite of the above situation.

Finally, a Monte Carlo simulation is used to determine the distribution of the efficiency measures for each banking system over its corresponding efficiency confidence interval. This distribution is defined based on the frequency histogram obtained by the number of hits for each predefined sub-interval of [0-1]. The interval [0-1] is in fact pre-divided in a set of sub-intervals with the same chosen length. A counter is placed in each sub-interval to record the frequency of efficiency coefficient corresponding to this sub-interval and an efficiency histogram is obtained for each banking system. A corresponding parametric distribution can be determined by smoothing the histogram. Developing such distribution and the appropriate way to define it is a projected extension for this study. The proportion of hits in each interval from the total number of simulation gives the degree of likelihood of the corresponding efficiency level for the corresponding country. The Monte-Carlo simulation also provides benchmarks for each level of efficiency for each country by identifying a representative value, if it exists, for each level of efficiency.

To run the regressions, *Excel* is used. A specific computer code is developed and written in *MATLAB* for *Confident-DEA*. The data and the results are summarized and documented in Appendix **A3**.

8.5. Data and the Sample of Countries Included in the Analysis:

Following the intermediation approach, the three inputs used in the calculation of input technical efficiencies are *fixed assets*[1], *deposits* (non-bank) and *labour*. The fixed assets and deposits are measured in US dollars and labour is measured in terms of number of staff.[2] The two outputs are *loans* and *securities* and measured in US dollars for all countries.

The OECD database on bank profitability, OECD (2000), was used in this study. This database provides aggregate data on financial statements of banks in the 28 member countries for the period 1979-2000; however, the annual membership fluctuates from year to year due to missing data. Furthermore, the coverage of banks in the database is not the same for each country. Sample homogeneity is raised by including all those institutions that conduct ordinary banking business, namely institutions that primarily take deposits from the public at large and lend for a wide range of purposes. The focus is on commercial banks. The end-of-period exchange rates of the International Monetary Fund, reported in International Financial Statistics (IFS), are used to convert all the amounts from local currencies to the US dollar.

After elimination of countries due to incomplete data, the resulted data base includes the commercial banking system of 17 countries. They are Denmark, Finland, France, Germany, Iceland, Japan, Korea, Luxembourg, Mexico, Norway, Portugal, Spain, Sweden, Switzerland, Turkey, the United Kingdom and the United States. Due to small sample size as well as the "big size" of the commercial banking systems of the United States, Japan and Switzerland, each one of them were split into two sub-systems, namely commercial banks and large commercial banks. The number of "banking systems" included in the original sample is brought to twenty.[3] The year 1998 is the common most recent year for which data are available for all the countries considered in the sample. The first step in this procedure is to forecast the production factors level for each country for the year 1998 based on data from all previous years and to define forecasted efficiency confidence interval each banking system using *Confident-DEA*. In a second step, the efficiency of each banking system is calculated

[1] Fixed assets was proxied by the balance sheet item 'Other assets' in the OECD data base.
[2] A better measure of labour input is full-time equivalent, which captures differences due to different proportions of part-time versus full-time employees. Unfortunately, this was not available in the OECD data base.

based on real data and using standard DEA. Results from the two steps are compared to test the robustness of the predicting approach.

One should emphasize that the relative nature of the efficiency measures provided by DEA make it meaningless to use time series reporting the efficiency measures in previous years to forecast the efficiency.

8.6. Results and Cross-Country Efficiency Analysis

8.6.1. Predicting the Production Factors:

The time series of each production factor involved in the efficiency analysis is presumed useful to forecast (extrapolate) the factor levels for the year 1998. Simple regression is used to estimate the parameters defining the first second and third order time trend of each factor. A confidence interval is built around each forecasted value using the corresponding standard deviation. Two sets of data are generated: one using half standard deviation and the other one standard deviation around the forecasted value. *Exhibit 8.1* provides a sample of results obtained from regressions to define the trend of the factors.

Exhibit 8.1: Results Obtained by Regressing *"loans"* of Swiss Commercial Banks

SUMMARY OUTPUT

Regression Statistics	
Multiple R	0.958207
R Square	0.918161
Adjusted R Square	0.913347
Standard Error	5547.745
Observations	19

ANOVA

	df	*SS*	*MS*	*F*	*ignificance F*
Regression	1	5.87E+09	5.87E+09	190.7255	1.14E-10
Residual	17	5.23E+08	30777471		
Total	18	6.39E+09			

	Coefficient	*andard Err*	*t Stat*	*P-value*	*Lower 95%*	*pper 95%*	*ower 95.0*	*pper 95.0*
Intercept	7273.949	2649.419	2.745488	0.013798	1684.1551	12863.74	1684.155	12863.74
X Variable 1	3209.101	232.3695	13.81034	1.14E-10	2718.8438	3699.359	2718.844	3699.359
Forecasted Value	71455.97							
Standard Deviation	5547.745							

The two sets of data providing forecasted confidence intervals for different production factors are provided in **A3** It is noticed that some countries present higher variances for their factors than others resulting in excessively large confidence intervals for some factors in some countries. This can be partially explained by the shortness of the time series. However, the high variability reflects a high volatility and

---

[3] For these three countries in our sample, the OECD data base divides commercial banks into

variability in the production factors levels in the banking industry, especially during crisis periods. As example, we cite the number of workers in the Korean banking industry jumping from 103,900 during 1996 to 114,000 during 1997 and then falling to 76,000 during 1998. This was a direct effect of the 1997 financial crisis in Korea.

8.6.2. Predicting the Technical Efficiency

The confidence intervals of the factors for each country considered as imprecise (bounded data) and *Confident-DEA* are used to determine the efficiency confidence interval as well as the efficiency histogram for each country. *Table 8.1* provides the results obtained for the case where the data set is constructed using just a half standard deviation to define the predicting confidence interval, appearing and the columns *pessimistic* and *optimistic*. The same table provides the efficiency coefficients obtained using real production factors levels data as realized by the commercial banking systems during 1998, appearing under the column *realization*.

Table 8.1: Predicted confidence interval of efficiency and realized efficiency of the commercial banking systems in selected OECD countries for the year 1998

Country	Pessimistic	Realization	Optimistic
dnk	0.5468	0.7852	0.8658
fin	0.5912	0.7422	1
fra	1	1	1
ger	1	1	1
ice	0.7183	1	0.9831
japcb	0.8128	1	1
japlcb	1	1	1
kor	0.4648	0.5672	0.7129
lux	1	1	1
mex	0.695	0.6294	1
nor	0.9102	1	1
por	0.5601	0.669	0.8341
spa	0.5725	0.7897	0.8143
swe	0.9252	1	1
swicb	0.5831	0.6049	0.7332
swilcb	0.8115	0.9833	1
tur	0.3858	0.4164	0.4944
uk	0.6424	0.7194	0.9308
uscb	0.6336	0.6736	0.7677
uslcb	0.6959	0.708	0.8651

The ranking of the countries based on the performance of their commercial banking systems is reported in *Table 8.2*.

'commercial banks' and 'large commercial banks'.

Table 8.2: Ranking of the OECD countries considered in the sample based on the efficiency of their commercial banking systems

Pessimistic		Realization		Optimistic	
1	fra	1	fra	1	fin
2	ger	2	ger	2	fra
3	japlcb	3	ice	3	ger
4	lux	4	japcb	4	japcb
5	swe	5	japlcb	5	japlcb
6	nor	6	lux	6	lux
7	japcb	7	nor	7	mex
8	swilcb	8	swe	8	nor
9	ice	9	swilcb	9	swe
10	uslcb	10	spa	10	swilcb
11	mex	11	dnk	11	ice
12	uk	12	fin	12	uk
13	uscb	13	uk	13	dnk
14	fin	14	uslcb	14	uslcb
15	swicb	15	uscb	15	por
16	spa	16	por	16	spa
17	por	17	mex	17	uscb
18	dnk	18	swicb	18	swicb
19	kor	19	kor	19	kor
20	tur	20	tur	20	tur

The consistency between the forecasted efficiency and realized ones is obvious and not at all surprising in light of the method used to forecast. All the realized efficiency coefficients had fallen in the corresponding efficiency confidence interval. Furthermore, country rankings are not surprising. The Turkish banking system, as expected is at the bottom along with the Korean and the Portuguese ones. Among the best performers are France, Germany and Luxembourg. These results confirm previous findings in Avkiran and Gattoufi (2001) as well as the findings of the works reported there. Compared with those findings, the results reported here show that Scandinavian banking systems do not perform as well as they were in previous years. good performer as they were. Is also noticed, compared to previous years, the net amelioration in performance of the Japanese banking system as well as banking system in UK and the deterioration in the performance of Swiss commercial banks in contrast with their large compatriot.

8.6.3. Conclusions, Relevance of the Results and Directions for Research:

An important interpretation can be given to the variability in the efficiency. The spread of the efficiency confidence interval can be used as a measure of the riskiness related of the corresponding banking system: the narrower is the interval, the more stable is the banking system. This is the case of the Swedish and the Norwegian banking systems compared with their counterparts in Korea, Mexico, Portugal, Spain and the UK. From this point of view, one should notice that the relative stability of the banking systems in Turkey and the United States indicates a tendency to a standstill at the same current level of performance.

The consistency between prediction and realization for 1998 confirms the validity of the model developed and the reliability of the results it provides. Predicting efficiency can be used to establish a ranking of banking systems based on their future relative technical efficiency. This, in turn, can be used as an early warning of financial crises in countries for which a substantial deterioration in the future relative performance of their banking system. Although theoretical models intended to develop leading indicators failed in providing reliable results when tested on past financial crises as reported in Berg and Patillo (1999) and in Goldfajn and Valdes (1998), empirical studies, like Kaminsky and Reinhart (2000), Kaminsky et al (1998) and Avkiran and Gattoufi (2001), established connections between financial crises and the deterioration of banking system. Such early warnings help authorities in intervening at an early time to prevent financial crises from happening or at least reduce their effects. This can be also useful in making investment decisions, and keeping authorities vigilant about the performance of the local systems as compared to their global peers, and identifying trends for future performance. It also can be a useful tool for international financial agencies and institutions concerned with the stability of the international financial system in an increasing globalized environment.

However, one should be cautious about the fact that the measures resulting from DEA are not absolute. They are relative, highly sensitive to extreme values as well as being deterministic in nature. Also, a larger sample would provide more reliable and accurate conclusions. Finally, one should notice that the standard deviations of the factors are artificial, resulting from the regression, rather than real. Real variability of the factors, if available, can enforce the stochastic nature of the present approach.

The analysis could be significantly improved in different ways. First, by including environmental factors, if data are available, like those used in Hasan et al. (2000), and Dietsch and Lozano-Vivas (2000). Second, by using more sophisticated time series methods to forecast the production factors. Third, by increasing the sample size and including more factors, alternatively ordinal factors and use the *Confident-DEA* version devoted to the efficiency analysis with imprecise data and developed in an earlier chapter.

CHAPTER 9: CONCLUSIONS, LIMITATIONS AND DIRECTIONS FOR FUTURE RESEARCH

This chapter summarizes the contributions made to DEA literature in this work, discusses their limitations and suggests agenda for fututure research.

Contributions at the **first** level are to the DEA methodology and to its roots. The DEA roots in ecconomic theory are extended by providing an anlytical derivation of the basic Charnes-Cooper-Rhodes (1978) (CCR) model from the *Weak Axiom of Profit Maximization (WAPM)* in the Firm Theory. This in term, invoked suggesting a new axiom, the *Approximate-Weak Axiom of Profit Maximization (A-WAPM)*.

Although the connection between economics theory and DEA is not new, the analytical connection with the *WAPM* is however not established in previous studies. Futhermore, the *A-WAPM* introduced so far constitutes a new concept in the Firm Theory. The interface created between DEA and the *WAPM* represents a promising domain for research. This interface particualrily extends the *WAPM* from the classical single firm case to cases involving a multiplicity of firms.

Regarding the DEA methodology itself, the issue of DMU sample size to be used in the analysis is discussed. A direct connection is established between the sensitivity of results obtained from DEA to the sample size used and the *A-WAPM*.

Discussion of the sensitivity of obtained results of DEA to sample size is intended to counter balance the traditional tendency to include more DMUs in the analysis in order to better satisfy the *"rule of thumb"*, suggested and largely adopted in the literature, for the minimum sample size required for a reliable results and conclusions derived from DEA. It is assumed that the far from the limit conditions is the sample size, the more credible is the analysis. efficiency analysis using DEA. The *"persimonious sample"* and the methods proposed (forward selection or backward elimination) for determining such a sample in a very particular case is an attempt to initiate a debate about a largely ignored issue.

Contributions on the **second** level of of this work provide a meta analysis of the "state of the art in DEA". These in turn comprise three parts: gathering the data and providing a large bibliography of over 1800 DEA-articles published in refreed journals, suggesting a taxonomy for a systematic and hence easier classification of the

DEA-litterature and providing a content anlysis of the recent DEA-literature (post 1995) using the scheme suggested in Reisman (1988, 1992) and applied to meta reviews in other sub-disciplines of the OR/MS literature (Reisman 1997a, 1997b and 2001). The content analysis is based on the nature of each contribution and the research stretegy(ies) used to derive the underlying results.

Regarding the data collection, a list of 1809 articles published in 490 refereed journals is compiled. This compares with 1259 such articles reported by the Tavares (2002) database and described as the largest and most comprehensive database recording the DEA literature. This part of the study addressed the practices of U.S-based OR/MS flagship journals to explain the enigmatic reticence, staistically confirmed, toward publication of articles with DEA content. This can be understood as an *aftermath* of "the natural drift" (Corbett and Wassenhove, 1993) that characterized the *neoclassic* U.S-based OR/MS literature and particularly its flagship journals for the last three decades.

Regarding their nature, articles are classified either as theoretical or as an application. A third class is formed by those articles that provide advances in theorey followed by an application to a real world problem illustrating the importance and relevance of the theoretical advances proposed. The strategies considered in the content analysis are ripple, embedding, transfer of technology, bridging, creative application, structuring and statistical modeling.

The main conclusion drawn from reviewing the full set of the post-1996 DEA refereed articles is the high usage mode of real world data. 71% of articles provide real world applications. Another important fact is the perfectly exponential increase of literature growth over the DEA lifespan. Least but very significant is the statistical documentation of the diffusion of DEA to other disciplines and professions as confirmed by the variety of the 490 journals known as have published DEA articles. Despite a decline in number of publications in the last two years that can be explained by a saturation effect, the DEA literature does not show any symptoms of the chronical illnesses that have characterized the OR/MS literature especially during the last two decades. A low proportion of ripple research coupled with a high proportions of bridging and embedding research indicates that DEA is not falling into the inbreeding trap. Furthermore, the high proportion of articles using real world data leave no doubt about the absence of aversion to real world problems types of research. When compared to other subdisciplines (Game Theory, Cellular Manufacturing and

Flowshop Scheduling) the DEA literature was found to have a much higher vitality. Finally, the increasing number of DEA articles with a stochastic nature is noticeable. Was this a "natural drift" guiding the literature or it was a bias created by an influential article written by an influential person (Seiford, 1996)? The fact that this article had a large circulation, as reported by the author, before publication strengthens the biased trend assumption.

Having said that, a larger review is needed to better analyze the trends in the literature, so as to identify the main streams and especially identify the gaps that constitute voids in the literature and hence directions for future research. Because a full census of the literature is not a realistic task, different sampling methods can be used such as chosing a set of journal for full census analysis, tracking the trends in the articles of the most productive researchers in the field. Another interesting research might be the analysis of the DEA literature in a specific field such as agriculture, financial institutions or services in general. These three fields combined comprise lion share of DEA-literature.

At the **third** level, contributions took the form of an extension in DEA theoretical base by proposing the *Confident-DEA*. This generalized the existing DEA approach for dealing with imprecise data in efficiency analyses. Unlike the methodology proposed by Cooper et al (1999) which provides single-valued efficiency measures, *Confident-DEA* provides a range of values for the efficiency measures. Specifically, an efficiency confidence interval reflects imprecision in data.

For the case of bounded data, a theorem defining the bounds of the efficiency confidence interval is provided. For the general case of imprecise data, a Genetic-Algorithm-based metaheuristic is used to determine the upper and lower bounds defining the efficiency confidence interval. In both cases, a Monte-Carlo type simulation is used to determine the distribution of the efficiency measures, taking into account the distribution of the bounded data over their corresponding intervals. Unlike previous DEA work dealing with imprecise data which implicitly assumes a uniform distribution, *Confident-DEA* allows for any type of distribution. The bounded data used in the illustative examples are assumed to have a truncated normal distribution.

In reaction, if but in part, to the *anemia in relevance to the real world* that characterized a large amount of recent OR/MS literature, *Confident-DEA* is applied to predict the efficiency of banking systems in OECD countries. A sample of 17 countries belonging to the OECD are considered for cross-country efficiency analysis.

Specifically, using the intermediation approach as a theoretical banking theory background, the respective countries commercial banking systems performances are analyzed for their respective relative efficiency. The mutual consistency between the predicted results and those realized for the year 1998 confirms the validity of the model for predicting the efficiency.

Although the *Confident-DEA* provides an exact solution for the bounded data case, the results it provided for the general case of imprecise data (a mixture of single-valued, bounded and ordinal data) are obtained using heuristic methods. Improvements can of course be made particularly to the Genetic Algorithm used. In this study, only a single level horizontal cutting is used for the crossover genetic modification. Results can be improved if both horizontal and vertical multi level cuttings are considered. On the other hand, the Monte Carlo method can be followed by a smoothing method to determine the distribution of efficiency coefficients for each DMU over the efficiency confidence interval. Parametric and non-parametric methods are potential methods.

The predicting model, although simple, provided interesting results. A more sophisticated modeling, both in terms of production factors (inputs and outputs) considered and the statistical techniques used for prediction those factors' level, would provide better and more reliable results.

REFERENCES

Abott, A., 1988. The system of professions: an essay on the expert division of labor. University of Chicago Press, Chicago, IL.

Ackoff, R. 1979. The future of Operations Research is past. Journal of the Operational Research Society 30, 93-104.

Ackoff, R. 1987. OR: A post mortem. Operations Research 35, 471-474.

Akhavein J.D., Berger, A.N., Humphrey, D.B., 1997. The effects of megamergers on efficiency and prices: evidence from a bank profit function. Review of Industrial Organization 12, 95-139.

Ali, A.I., Seiford, L.M., 1993. Computational accuracy and infinitesimals in Data Envelopment Analysis. INFOR 31 (4), 290-297.

Ali, A.I., Seiford, L.M., 1993. The mathematical programming approach to efficiency analysis. in The Measurement of Productive Efficiency, Oxford University Press, New York.

Allen, L., Rai, A., 1996. Operational efficiency in banking: an international comparison. Journal of Banking and Finance 20, 655-672.

Avkiran, N.K., 1999. The evidence on efficiency gains: the role of mergers and the benefits to the public. Journal of Banking and Finance 23, 991-1013.

Avkiran, N.K., 2001. Investigating technical and scale efficiencies of Australian universities through Data Envelopment Analysis. Socio-Economic Planning Sciences 35 (1), 57-80.

Avkiran, N.K., Gattoufi, S., 2001. Do Cost Inefficiencies Precede Financial Crisis? Paper presented at The 14th Australasian Finance & Banking Conference, Sydney

Back, T. 1996. Evolutionary algorithms in theory and practice. Oxford University Press, New York.

Banker, R. D., Charnes, A., Cooper, W.W., 1984. Some Models for Estimating Technical and Scale Inefficiencies in Data Envelopment Analysis. Management Science 30 (9), 1078-92.

Banker, R. D., Morey, R.C., 1986. Efficiency Analysis for Exogenously Fixed Inputs and Outputs. Operations Research 34 (4), 513-521.

Banks, J., Carson, J.S., Nelson, B.L., 1996. Discrete event system simulation. Second Edition, Prentice-Hall, Upper Saddle River, New Jersey.

Bard, J.F. 1998. Practical bilevel optimization: algorithms and applications, Kluwer Academic Publishers, Dordrecht, The Netherlands.

Bardhan, I.R., Cooper, W.W., Kumbhakar, S.C., 1998. A simulation study of joint uses of Data Envelopment Analysis and statistical regressions for production function estimation and efficiency evaluation. The Journal of Productivity Analysis, 9, 249-278.

Basset, G.Jr., R. Koenker, R, 1978. Asymptotic theory of least absolute value error regressions. Journal of the American Statistical Association 73, pp. 618-622.

Battese, G.E., Coelli, T.J., 1992. Frontier production functions, technical efficiency and panel data: with application to paddy farmers in India. Journal of Productivity Analysis, 3, 153-169.

Battese, G.E., Coelli, T.J., 1995. A model for technical inefficiency effects in a stochastic frontier production function for panel data. Empirical Economics, 20, 325-332.

Berg, S.A., Forsund, F., Hjalmarsson, L., Suominen, M., 1993. Banking efficiency in the Nordic countries. Journal of Banking and Finance 17, 371-388.

Berg, S.A., Patillo, C., 1999. Are currency crises predictable? A test. IMF Staff Papers, International Monetary Fund, Washington DC, USA.

Bergendahl, G., 1995. DEA and benchmarks for Nordic banks, Working Paper, Gothenburg University, Gothenburg, Sweden.

Berger A.N., Hanweck, G.A., Humphrey, D.B., 1987. Competitive viability in banking: scale, scope and product mix economies. Journal of Monetary Economics 20, 501-520.

Berger, A.N., 1993, Distribution-Free estimates of efficiency in the US banking industry and tests of the standard distributional assumptions, Journal of Productivity Analysis 4, 261-292.

Berger, A.N., DeYoung, R., Genay, H., Udell, F.G., 2000. Globalisation of financial institutions: evidence from cross-border performance. Forthcoming in Brookings-Wharton Papers on Financial Services.

Berger, A.N., Hancock, D., Humphrey, D.B., 1993. Bank efficiency derived from the profit function. Journal of Banking and Finance 17, 317-347.

Berger, A.N., Humphrey, D.B., 1991. The dominance of inefficiencies over scale and product mix economics in banking. Journal of Monetary Economics 28, 117-148.

Berger, A.N., Humphrey, D.B., 1992. Measurement and efficiency issues in commercial banking. In Output Measurement in the Service Sectors Ed. Griliches Z., The University of Chicago Press, Chicago, 245-300.

Berger, A.N., Humphrey, D.B., 1997. Efficiency of financial institutions: international survey and directions for future research. European Journal of Operational Research 98 (2), 175-212.

Berger, A.N., Hunter, W.C., Timme, S.G., 1993. The efficiency of financial institutions: a review and preview of research past, present and future. Journal of Banking and Finance 17, 221-249.

Berger, A.N., Mester, L.J, 1997. What explains the dramatic changes in cost and profit performance of the US banking industry? Working Paper, Financial Institutions Center, Wharton School, University of Pennsylvania.

Berger, A.N., Mester, L.J., 1997. Inside the black box: What explains differences in the efficiencies of financial institutions? Working Paper, Financial Institutions Center, Wharton School, University of Pennsylvania.

Bhattacharyya, A., Lovell, C.A.K., Sahay, P., 1997. The impact of liberalisation on the productive efficiency of Indian commercial banks. European Journal of Operational Research 98, 332-345.

Blumstein, A., 1987. The current missionary role of OR/MS. Operations Research 35, 926-929.

Bukh, P.N.D., Berg, S.A., Forsund, F.R., 1995. Banking efficiency in the Nordic countries: A four-country Malmquist index analysis. Working Paper, University of Arhus, Denmark.

Camanho, A.S., Dyson, R.G., 1999. Efficiency, size, benchmarks and targets for bank branches: an application of data envelopment analysis. Journal of the Operational Research Society 50 (9), 903-915.

Charnes, A., and Cooper, W.W., 1985. Preface to topics in Data Envelopment Analysis. Annals of Operations Research 2 (1), 59-94.

Charnes, A., Cooper WW, Golany, B., Seiford, L.M., Stutz, J., 1985. Foundations of Data Envelopment Analysis for Pareto-Koopmans efficient empirical production functions. Journal of Econometrics 30, 91-107.

Charnes, A., Cooper, W.W., Mellon, B., 1952. Blending aviation gasolines: a study in programming interdependent activities in an integrated oil company. Econometrica 20 (2), 135-159.

Charnes, A., Cooper, W.W., Rhodes, E., 1978. Evaluating program and managerial efficiency: an application of Data Envelopment Analysis to Program Follow Through. Management Science 27 (6), 668-697.

Charnes, A., Cooper, W.W., Rhodes, E., 1981. Measuring the efficiency of Decision Making Units. European Journal of Operational Research 2, 429-444.

Cheng, C.H., Kumar, A., Motwani, J., Reisman, A., Madan, M., 1999. An Objective Rank-Order Analysis of the Technology Innovation Management Journals. IEEE Transactions on Engineering Management, February

Coelli, T., 1996. A Guide to DEAP2.1: A Data Envelopment Analysis (Computer) program. Working Paper, Centre for Efficiency and Productivity Analysis, University of New England, Australia.

Cook, W.D., Kress, M., Seiford, L.M., 1993. On the use of ordinal data in Data Envelopment Analysis. Journal of The Operational Research Society 44, 2, 133-140.

Cook, W.D., Kress, M., Seiford, L.M., 1996. Data Envelopment Analysis in the presence of both quantitative and qualitative Factors. Journal of The Operational Research Society 47, 945-953.

Cooper, W.W., 1994. Personal communication.

Cooper, W.W., Tone, K., 1997. Measures of inefficiency in data envelopment analysis and stochastic frontier estimation. European Journal of Operational Research 99 (1), 72-88.

Cooper, W.W., 1999. Operations Research/Management Science: where it's been. where it should be going?, Journal of The Operational Research Society 50, 3-11.

Cooper, W.W., Park, K.S., Yu, G., 1999. IDEA and AR-IDEA: Models for dealing with imprecise data in DEA. Management Science 45 (4), 597-607.

Cooper, W.W., Deng, H., Gu, B., Li, S., Thrall, R.M., 2001. Using DEA to improve the management of congestion in Chinese industries (1981-1997). Socio-Economic Planning Science 35, 227-242.

Cooper, W.W., Park, K.S. and Yu, G., 2001. IDEA (Imprecise Data Envelopment Analysis) with CMDS (Column Maximum Decision Making Units). Journal of The Operational Research Society 52 (2), 178-181.

Cooper, W.W., Park, K.S., Yu, G., (forthcoming). An illustrative application of IDEA (Imprecise Data Envelopment Analysis) to a Korean Mobile Telecommunication Company. Operations Research.

Corbett, C.J., Van Wassenhove, L.N., 1993. The natural drift: what happened to Operations Research? Operations Research 41, 625-640

Cummins, J., D., Zi, H., 1998. Comparison of frontier efficiency methods: an application to the U.S. life insurance industry. Journal of Productivity Analysis, 10, 131-152.

Dantzig, G.B., 1953. Computational algorithms of the revised simplex method. Report, RM 1266, The Rand Corporation, Santa Monica, CA.

Debreu, G., 1951. The coefficient of resource utilisation. Econometrica 19, 273-292.

Dietsch, M., Lozano-Vivas, A., 2000. How the environment determines banking efficiency: A comparison between French and Spanish industries. Journal of Banking and Finance 24 (6), 985-1004.

Elyasiani, E., Mehdian, S., 1990. Efficiency in the commercial banking industry, a production frontier approach. Applied Economics 22, 539-551.

English, M., Grosskopf, S., Hayes, K., Yaisawarng, S., 1993. Output allocative and technical efficiency of banks. Journal of Banking and Finance 17, 349-366.

Fare, R., Grosskopf, S., Norris, M., Zhang, Z., 1994. Productivity growth, technical progress, and efficiency change in industrialized countries. American Economic Review 84 (1), 66-83.

Farrell, M.J., 1957. The measurement of productive efficiency. Journal of Royal Statistical Society, Series A, CXX, 253-281.

Ferrier G.D., Lovell, C.A.K., 1990. Measuring cost fficiency in banking. Journal of Econometrics 46, 229-245.

Fields J.A., Murphy, N.B., Tirtiroglu, D., 1993. An International comparison of scale economies in banking: evidence from Turkey. Journal of Financial Services Research 7, 111-125.

Forsund, F.R., 1992. A comparison of parametric and non-parametric efficiency measures: the case of Norwegian ferries. The Journal of Productivity Analysis 3, 25-43.

Forsund, F.R., 1999. The evolution of DEA-the economics perspective. Working Paper, Departement of Economics, University of Oslo.

Gattoufi, S., Oral, M., Reisman, A., 2001a. Data Envelopment Analysis literature: a bibliography update (1951-2001). Working Paper. Sabanci University, Istanbul, Turkey. Submitted to the Journal of Socio Economic Planning Sciences.

Gattoufi, S., Oral, M., Reisman, A. 2001b. A taxonomy for Data Envelopment Analysis. Working Paper. Sabanci University, Istanbul, Turkey. Submitted to the Journal of Socio Economic Planning Sciences.

Gattoufi, S., Oral, M., Reisman, A., 2001c. Content Analysis of Data Envelopment Analysis literature. Working Paper. Sabanci University, Istanbul, Turkey.

Gattoufi, S., Oral, M., Reisman, A., 2001d. Epistemology of Data Envelopment Analysis. Working Paper. Sabanci University, Istanbul, Turkey

Gattoufi, S., Oral, M., Reisman, A., 2001e. Data Envelopment Analysis literature: classification, bibliography and a systematic identification of voids. Working Paper. Sabanci University, Istanbul, Turkey.

Golany, B., 1988. A note on including ordinal relations among multipliers in Data Envelopment Analysis. Management Science 34, 8, 1029-1033.

Goldberg, D. E., 1989. Genetic algorithms in search, optimization and machine learning Addison-Wesley Publishing Company, Reading, Massachusetts.

Goldfajn, I., Valdes, R.O., 1998. Are currency crises predictable?. European Economic Review 42, 873-885.

Grifell-Tatje, E., Lovell, C.A.K., 1996. Deregulation and productivity decline: the case of Spanish savings banks. European Economic Review 40, 1281-1303.

Griliches, Z., Intriligator, M.D., 1997. Handbook of econometrics, Elsevier Science B.V., Amsterdam, Fourth Edition.

Haag, S.E., Jaska, P.V., 1995. Interpreting inefficiency ratings: an application of bank branch operating efficiencies. Managerial and Decision Economics 16, 7-14.

Hall, M. and Winsten, C., 1959. The ambiguous notion of efficiency. Economic Journal LXIX, 71-86.

Hancock, D., 1986. A Model of the financial firm with imperfect asset and deposit elasticities. Journal of Banking and Finance 10, 37-54.

Hansen, P., Jaumard, B., Savard, G.,1992. New Branch-and-Bound rules for linear bilevel programming. SIAM Journal of Scientific and Statistical Computing, 13, 5, 1194-1217.

Hao, G., Wei, Q.L., Yan, H., 2000. A game theoretical model of DEA efficiency. Journal of the Operational Research Society 2000; 51(11): 1319-29.

Hasan, I., Lozano-Vivas, A., Pastor, J.T., 2000. Cross border performance in European banking. Manuscript- Personnel Communication.

Holland, J. H., 1992. Adaptation in Natural and Artificial Systems. Second Edition, The MIT Press, Cambridge, Massachusetts.

Hunter, W.C., Timme, S.G., 1986. Technical change, organizational form, and the structure of bank productivity. Journal of Money, Credit and Banking 18, 152-166.

Hunter, W.C., Timme, S.G., 1991. Technological change in large U.S. commercial banks. Journal of Business 64, 339-362.

Jeroslow, R.G., 1985. The polynomial hierarchy and a simple model for competitive analysis. Mathematical Programming, 32, 146-164.

Kaminsky, G., Lizondo, S., Reinhart, C.M., 1998. Leading Indicators of Currency Crises. IMF Staff Papers, International Monetary Fund, Washington DC, USA.

Kim, H.S., Park, C.G., Park, K.S., 1999. An application of Data Envelopment Analysis in telephone offices evaluation with partial data. Computers & Operations Research 26, 59-72.

Koopmans, T.C., 1951. An analysis of production as an efficient combination of activities. in T.C. Koopmans, Ed., Activity Analysis of Production and Allocation, Cowles Commission for Research in Economics, Monograph13, Wiley, New York.

Kumar, A., Motwani, J., Reisman, A., 1996. Transfer of Technologies: A Classification of Motivations. The Journal of Technology Transfer 21 (1-2), 34-42.

Lakhal, S., Martel, A., Oral, M., Montreuil, B., 1999. Network companies and competitiveness: a framework for analysis. European Journal of Operational Research 118, 278-294.

Landry, M., Banville, C., Oral, M., 1996. Model legitimisation in operational research. European Journal of Operational Research 92, 443-457.

Leahy, M., Schich, S., Wehinger, G., Pelgrin, F., Thorgeirsson, T., 2001. Contribution of financial systems to growth in OECD countries. Working Paper, Economic Department, OECD, 280

Man, K. F., Tang, K. S., Kwong, S., 1999. Genetic algorithms: concepts and designs. Springer-Verlag, London, England.

McAllister, P.H., McManus, D., 1993. Resolving the scale efficiency puzzle in banking. Journal of Banking and Finance 17, 389-405.

Mehrabian, S., Jahanshahloo, G.R., Alirezaee, M.R., Amin, G.R., 2000. An assurance interval for the non-Archimedean epsilon in DEA models. Operations Research 48(2), 344-347.

Mendeleyev, D.I., 1889. The periodic law of chemical elements (Faraday lectures). Journal of the Chemical Society 1889; 55: 643-56.

Mercan, M., Reisman, A., Emel, A.B., Yolalan, R., (Forthcoming). The Effect of scale and Mode of Ownership on the Financial Performance of the Turkish Banking Sector: Results of DEA. Socio-Economic Planning Sciences.

Mester, L. (1987) Efficient Production of Financial Services: Scale and Scope Economies, Business Review of Federal Reserve Bank of Philadelphia, January/February, 15-25.

Mester, L.J., 1996. A study of bank efficiency taking into account risk-preferences. Journal of Banking and Finance 20, 1025-1045.

Miller, S.M., Noulas, A.G., 1996. The technical efficiency of large bank production. Journal of Banking and Finance 20, 495-509.

Miser, H., 1987. Science and professionalism in Operations Research. Operations Research 35, 314-319.

Molyneux, P., Altunbas, Y., Gardener, E., 1996. Efficiency in European banking, John Wiley & Sons Press, Chikester, England.

Norman, M., Stoker, B., 1991. Data Envelopment Analysis: the assessment of performance. John Wiley & Sons, Chichester, UK.

Noulas, A.G., Ray, S.C., Miller, S.M., 1990. Returns to scale and input substitution for large U.S. banks. Journal of Money, Credit and Banking 22, 94-108.

OECD, 1997. Bank profitability: financial statements of banks.

OECD, 2000. Bank profitability: financial statements of banks.

OECD, 2000. Bank profitability: methodological country notes.

Oral, M , Yolalan, R., 1990. An empirical study on measuring operating efficiency and profitability of bank branches. European Journal of Operational Research 46 (3), 282-94.

Oral, M., Kettani, O., 1993. The facets of the modelling and validation process in operations research. European Journal of Operational Research 66 (2), 216-234.

Oral, M., Çinar, U., Chabchoub, H., 1999. Linking industrial competitiveness and productivity at the firm level. European Journal of Operational Research 118, 271-277.

Oral, M., Kettani, O., Yolalan, R., 1992. An empirical study on analyzing the productivity of bank branches. IIE Transactions 24, 5, 166-176.

Pastor, J., Perez, F., Quesada, J., 1997. Efficiency analysis in banking firms: An international comparison. European Journal of Operational Research 98, 396-408.

Poirier, D.J., 1995. Intermediate statistics and econometrics, MIT Press, Cambridge, Massachusetts.

Reisman, A., 1988a. Finding researchable topics via a taxonomy of a field of knowledge. Operations Research Letters 7 (6).

Reisman, A., 1988b. On alternative strategies for doing research in the management and social sciences. IEEE Transactions on Engineering Management 35 (4), 215-221.

Reisman, A., 1989. A system approach to identifying knowledge voids in problem solving disciplines and professions: a focus on the management sciences. Knowledge in Society: An International Journal of Knowledge Transfer 1 (4), 67-86.

Reisman, A., Xu, X., 1992. On stages of knowledge growth in the management sciences, IEEE Transactions on Management. 39 (2), 119-128.

Reisman, A., 1993. Operations Management: time for meta research. in Prospectives in Operations Management: Essays in Honor of Elwood S. Buffa, Kluwer Academic Publishers, Boston, MA, pp.471-482, January

Reisman, A., 1994. Creativity in MS/OR: Expanding Knowledge by Consolidating Knowledge. Interfaces 24 (3), 91-99.

Reisman, A., Kirschnick, F., 1994. The devolution of OR/MS: implications from a statistical content analysis of papers in flagship journals. Operations Research 42 (4), 577-588.

Reisman, A., Kirschnick, F., 1995. Research strategies used by OR/MS workers as shown by an analysis of papers in flagship journals. Operations Research 43 (5), 731-740.

Reisman, A., Kumar, A., Motwani, J., 1997. Flowshop Scheduling/Sequencing Research: a statistical review of the literature 1952-1994. IEEE Transactions on Engineering Management 44 (3), 316-329.

Reisman, A., Kumar, A., Motwani, J., Cheng, C.G., 1997. Cellular Manufacturing: a statistical review of the literature (1965-1995). Operations Research 45 (4), 508-520

Reisman, A., Kumar, A., Motwani, J., 1999. A Meta Review of game theory publications in the flagship OR/MS journals: 1952-1995. The Journal of Management History.

Reisman, A., Kirschnick, F., 2000. A role for content analysis in research on OR/MS research literature. An invited paper, Omega 28, 247-248.

Reisman, A., Kumar, A., Motwani, J., 2001. A Meta review of game theory publications in the flagship OR/MS journals. Management Decision 39 (2), 147-155.

Reisman, A., Oral, M., Gattoufi, S., 2001. Absolutely positively Operations Research: 50 years of contributions by William Wager Cooper. Working Paper, Sabanci University, Istanbul, Turkey. Submitted to Operations Research.

Reisman, A., Kumar, A., Oral, M., Gattoufi, S., 2002. A statistical comparison of research strategies pursued within three OR/MS subdisciplines and in OR/MS as a whole. Working paper, Sabanci University, Istanbul, Turkey

Reisman, A., Kumar, A., Oral, M., Gattoufi, S., 2002. Comparative epistemology of three OR/MS sub-disciplines: Data Envelopment Analysis, Flow Shop Scheduling, and Cellular Manufacturing. Working Paper, Sabanci University, Istanbul, Turkey.

Resti, A., 1997. Evaluating the cost-efficiency of the Italian banking system: what can be learned from the joint application of parametric and non-parametric techniques. Journal of Banking and Finance 21, 221-250.

Rhoades, S.A., 1993. Efficiency effects of horizontal in-market bank mergers. Journal of Banking and Finance 17, 411-422.

Sarafoglou, N., 1998. The most influential DEA publications: a comment on Seiford. The Journal of Productivity Analysis 9 (3), 279-281.

Seiford, L.M., Thrall, R.M., 1990. Recent development in DEA: the mathematical programming approach to frontier analysis. Journal of Econometrics 46, 7-38.

Seiford, L.M., 1997. A bibliography for Data Envelopment Analysis (1978-1996). Annals of Operations Research 73, 393-438.

Seiford, L.M., 1996. Data Envelopment Analysis: The Evolution of the State of the Art (1978-1995). The Journal of Productivity Analysis 7, 99-137.

Sinuany-Stern, Z., Friedman, L., 1998. DEA and the discriminant analysis of ratios for ranking units. European Journal of Operational Research111 (3), 470-478.

Sinuany-Stern, Z., Mehrez, A., Barboy, A., 1994. Academic departments and efficiency via DEA. Computers and Operations Research 21 (5), 543-546.

Staat, M., 2001. The effect of size on the mean efficiency in DEA: comment. Journal of Productivity Analysis 15, 129-137.

Thanassoulis, E., 1999. Data Envelopment Analysis and its use in banking. INTERFACES 29 (3), 1-13

Thanassoulis, E., 1993. A comparison of regression analysis and data envelopment analysis as alternative methods for performance assessment. Journal of The Operational Research Society 44 (11), 1129-1144.

Üçer, M., Rijkeghem, C.V., Yolalan, R.O., 1999. Leading indicators of currency crises: a brief literature survey and an application to turkey. Yapi Kredi Economic Review 9 (2), 3-24.

Varian, H.R., 1990. Goodness-of-fit in optimizing models. Journal of Econometrics 46 125-140.

Varian, H.R., 1990. Microeconomic analysis, Third Edition, W. W. Norton & Company, New York, 1992.

Vogel, D.R., Weterbe, J.C., 1984. MIS research: a profile of leading journals and universities. Data Base Fall, 3-14.

Wang, Y., Gattoufi, S., Oral , M., Reisman, A., 2001. Data Envelopment Analysis: a justification based on the Weak Axiom of Profit Maximization in the theory of the firm. Working Paper, Sabanci University, Istanbul, Turkey. Submitted to the European Journal of Operational Research.

Weintraub, E.R., 1992. Toward a history of game theory. Duke University Press, Durham, NC.

Wheelock, D.C., Wilson, P.W., 1995. Evaluating the efficiency of commercial banks: does our view of what banks do matter?. Federal Reserve Bank of St.Louis Review 77, 39-52.

Zhang, Y., Bartels, R., 1998. The effect of size on the mean efficiency in DEA with an application to electricity distribution in Australia Sweden and New Zealand. Journal of Productivity Analysis 9, 187-204.

A.1. List of Journal Known to have published DEA-Articles as of August 2001 (Listed by Decreasing Number of Articles they Published)

1-European Journal of Operational Research-204
2-Journal of Productivity Analysis -109
3-Journal of The Operational Research Society-77
4-Annals of Operations Research-53
5-Management Science-51
6-OMEGA-50
7-Applied Economics-42
8-Socio-Economic Planning Sciences-39
9-International Journal of Systems Science-37
10-International Journal of Production Economics-33
11-Computers & Operations Research-25
12-Journal of Banking & Finance-25
13-Journal of Econometrics-19
14-Journal of The Operational Research Society of Japan-18
15-Applied Economics Letters-17
16-Managerial & Decision Economics-16
17-Review of Economics and Statistics-16
18-American Journal of Agricultural Economics-15
19-INFOR-14
20-Journal of Medical Systems-14
21-Health Care Management Science-13
22-Research in Governmental & Nonprofit Accounting-12
23-INTERFACES-11
24-Medical Care -11
25-Transportation Research-11
26-Health Services Research-10
27-Computers, Environment & Urban Systems -9
28-Operations Research Letters-9
29-Operations Research: Communication of the Operational Research Society of Japan-9
30-Decision Sciencess-8
31-Economic Journal: The Journal of the Royal Economic Society-8
32-Economics of Education Review-8
33-Financial Accountability & Management-8
34-OR Insight-8
35-American Economic Review -7
36-Econometrica-7
37-International Journal of Production Research-7
38-Journal of Agricultural Economics-7
39-Scandinavian Journal of Economics-7
40-Zeitschrift Fur Nationalokonomie / Journal of Economics-7
41-Annals of Public & Cooperative Economics-6
42-Applied Financial Economics-6
43-Education Economics-6
44-Glasnik Mathematick-6
45-Health Economics-6
46-IIE Transactions-6
47-Journal of Accounting & Public Policy-6
48-Journal of Comparative Economics-6
49-Journal of Economics & Business-6
50-Journal of Health & Human Services Administration-6
51-Journal of Health Economics-6
52-Journal of Operations Management-6
53-Journal of Public Economics-6
54-Public Productivity & Management Review-6
55-Accounting Review-5
56-Computers & Industrial Engineering-5
57-Economics Letters-5

58-Forest Science-5
59-Health Services Management Research-5
60-IEEE Transactions on Engineering Management-5
61-Journal of Economic Theory-5
62-Operations Research-5
63-Scientometrics-5
64-Applied Stochastic Models & Data Analysis-4
65-Cornell Hotel & Restaurant Administration Quarterly-4
66-Econometric Theory-4
67-Electricity Journal-4
68-Energy Economics-4
69-International Economic Review-4
70-International Journal of Hospitality Management-4
71-International Journal of Operations & Production Management-4
72-International Journal of Transport Economics-4
73-International Transactions on Operational Research-4
74-Journal of Agricultural & Resource Economics-4
75-Journal of Business & Economic Statistics -4
76-Journal of Monetary Economics-4
77-Journal of Money, Credit & Banking-4
78-Journal of Retailing-4
79-Journal of Transport Economics & Policy-4
80-Public Choice-4
81-Review of Industrial Organization-4
82-Revista de Economia Aplicada-4
83-Transportation-4
84-Agricultural Economics-3
85-Agricultural Systems-3
86-Aquaculture-3
87-Cahiers d'Economie et Sociologie Rurales-3
88-Central European Journal for Operations Research & Economics-3
89-Contemporary Economic Policy-3
90-Economic Modelling-3
91-Educational Administration Quarterly-3
92-Fuzzy Sets & Systems-3
93-Health Policy-3
94-Hospital & Health Services Administration-3
95-Ifo Studien-3
96-Internal Auditing-3
97-Investigaciones Economicas-3
98-Journal of Business Logistics-3
99-Journal of Business Research-3
100-Journal of Enterprise Management -3
101-Journal of Environmental Management-3
102-Journal of International Financial Markets, Institutions & Money-3
103-Journal of Quantitative Economics-3
104-Manchester School of Economic and Social Studies-3
105-Oxford Economic Papers-3
106-Oxford Review of Education-3
107-Recherches Economiques de Louvain-3
108-Resources & Energy-3
109-Revista Brasileira de Economia-3
110-Schweizerische Zeitschrift fur Volkswirtschaft undStatistik / Swiss Journal of Economics & Statistics-3
111-Service Industries Journal-3
112-Southern Economic Journal-3
113-Technological Forecasting & Social Change-3
114-Telecommunications Policy-3
115-Utilities Policy-3
116-Zeitshrift für Operations Research-3

117-Administration in Social Work-2
118-Agricultural & Resource Economics Review-2
119-American Business Review-2
120-Annals of Regional Science-2
121-Applied Mathematical Modelling-2
122-Asian Economic Journal-2
123-Australian Economic Review-2
124-Australian Journal of Agricultural & Resource Economics-2
125-Canadian Journal of Agricultural Economics / Revue Canadienne D' Economie Rurale -2
126-China Economic Review-2
127-Computers & Mathematics with Applications-2
128-Contemporary Accounting Research-2
129-Cybernetics & Systems-2
130-Development Southern Africa-2
131-Economic and Social Review-2
132-Economic Papers-2
133-Economie et Prévision-2
134-Ekonomicky Casopis-2
135-El Trimestre Economico-2
136-Empirical Economics-2
137-Energy-2
138-Energy Policy-2
139-Engineering Costs & Production Economics-2
140-Engineering Management Journal-2
141-Environment & Planning-2
142-Evaluation & Research In Education-2
143-Expert Systems With Applications-2
144-Federal Reserve Bank of Dallas Financial Industry Studies-2
145-Federal Reserve Bank of St Louis Review-2
146-Forest Products Journal-2
147-Health Care Supervisor-2
148-IEEE Transactions on Power Systems-2
149-Information & Software Technology-2
150-Information Economics & Policy-2
151-Information Systems Research-2
152-International Journal of Production Management-2
153-International Journal of Public Administration-2
154-International Journal of Research in Marketing-2
155-Jahrbucher Fur Nationalokonomie Und Statistik-2
156-Japan & the World Economy-2
157-Journal of Accounting, Auditing & Finance-2
158-Journal of Agricultural & Applied Economics-2
159-Journal of Air Transport Management-2
160-Journal of Applied Business Research-2
161-Journal of Business-2
162-Journal of Economic Behavior & Organization-2
163-Journal of Evolutionary Economics-2
164-Journal of Financial Services Research-2
165-Journal of Management Information Systems-2
166-Journal of Multi-Criteria Decision Analysis-2
167-Journal of Optimization Theory & Applications-2
168-Journal of Personal Selling & Sales Management-2
169-Journal of Regional Science-2
170-Journal of Systems Science & Mathematical Sciences-2
171-Journal of the Acoustical Society of America-2
172-Logistics & Transportation Review-2
173-Management International Review-2
174-Optimization-2
175-OR Spektrum-2
176-Organizational Behavior & Human Decision Processes-2

177-Pacific & Asian Journal of Energy-2
178-Papers of the Regional Science Association-2
179-Politica Economica-2
180-Production & Operations Management-2
181-Production Planning & Control
-2
182-Public Money & Management-2
183-Public Productivity Review -2
184-Public Utilities Fortnightly-2
185-Quarterly Journal of Economics-2
186-Regional Science & Urban Economics-2
187-Resources & Energy Economics-2
188-Revista Espanola de Economia-2
189-Strategic Management Journal-2
190-Supply Chain Management-2
191-Total Quality Management-2
192-Transportation Planning & Technology-2
193-Academia Economic Papers-1
194-Academy of Medecine-1
195-Academy of Management Journal-1
196-Accounting & Finance-1
197-Accounting Auditing & Accountability Journal-1
198-Accounting, Organisations & Society-1
199-Acta Hospitalia-1
200-Administration & Society-1
201-Advances in Health Economics & Health Services Research-1
202-Advances in Mathematical Programming & Financial Planning-1
203-Agrarwirtschaft-1
204-Air Force Journal of Logistics-1
205-American Real Estate & Urban Economics Association Journal-1
206-American Sociological Review-1
207-Anatolia-1
208-Antitrust Bulletin-1
209-Applications of Management Science-1
210-Arabian Journal For Science & Engineering -1
211-Asia Pacific Journal of Economics & Business-1
212-Asia Pacific Journal of Management-1
213-Asia-Pacific Economic Review-1
214-Asia-Pacific Journal of Operational Research-1
215-Assessment Journal-1
216-Association of Energy Engineering Journal-1
217-Auditing: A Journal of Practice & Theory-1
218-Australian Journal of Agricultural Economics-1
219-Australian Journal of Management-1
220-Australian Journal of Regional Studies-1
221-Benchmarking for Quality Management & Technology-1
222-Biotechnology & Bioengineering-1
223-Board of Governors of the Federal Reserve System-1
224-Bulletin of Economic Research-1
225-Business & Society-1
226-Business Strategy Review-1
227-Canadian Business-1
228-Canadian Journal of Economics-Revue Canadienne D Economie-1
229-Canadian Journal of Forest Research-Revue Canadienne De Recherche Forestiere-1
230-Canadian Public Policy-1
231-Chartered Accountants Journal of New Zealand-1
232-Chinese Journal of Public Health-1
233-Chinese Science Bulletin-1
234-Communications in Statistics-Simulation and Computation-1
235-Community Dentistry and Oral Epidemology Journal-1

236-Community Mental Health Journal-1
237-Computational Optimization & Applications-1
238-Computational Statistics &Data Analysis-1
239-Computer Science in Economics & Management -1
240-Computerworld-1
241-Contemporary Policy Issues-1
242-Control & Information Engineers-1
243-Cowles Commission for Research in Economics-1
244-Criminal Justice Review-1
245-Critical Care Medicine-1
246-Decision Line-1
247-Decision Support Systems-1
248-Die Gesamte Staatswissenschaft / Journal of Institutional & Theoretical Economics-1
249-Eastern Economic Journal-1
250-Eastern European Economics-1
251-Ecological Economics-1
252-Economia Aplicada-1
253-Economic Development Quarterly-1
254-Economic Issues-1
255-Economic Notes-1
256-Economic Perspectives-1
257-Economic Record -1
258-Economica-1
259-Economics of Innovation & New Technology-1
260-Economics of Transition-1
261-Ekonomicko-Matematicky Obzor-1
262-Ekonomiska Samfundets Tidskrift-1
263-Electoral Studies-1
264-Empirica-1
265-Energy Journal-1
266-Energy Systems & Policy-1
267-Engineering Economist-1
268-Environmental Professional-1
269-European Economic Review-1
270-European Journal of Cancer-1
271-European Journal of Purchasing & Supply Management-1
272-European Review of Agricultural Economics-1
273-European Transactions On Telecommunications -1
274-Evaluation of Health Professions-1
275-Federal Reserve Bank of Atlanta Economic Review-1
276-Federal Reserve Bank of Dalls Economic Review -1
277-Federal Reserve Bank of Philadelphia Business Review -1
278-Finances Publiques / Public Finance-1
279-Fiscal Studies-1
280-Fondazione Eni Enrico Mattei Note di Lavoro-1
281-Fortune-1
282-Fuel & Energy Abstracts-1
283-Gerontologist -1
284-Gesundheitsokonomie und Qualitatsmanagement-1
285-Government Accountants Journal-1
286-Government Accounting & Auditing Update-1
287-Hacienda Publica Espanola-1
288-Harvard Business Review-1
289-Health Care Strategic Management-1
290-Higher Education-1
291-Higher Education Quarterly-1
292-IEEE Transactions On Medical Imaging -1
293-IEEE Transactions on Software Engineering-1
294-IEEE Transactions on Systems Management & Cybernitics-1
295-IMA Journal of Mathematics Applied in Business & Industry-1

296-IMA Journal of Mathematics Applied In Medicine & Biology-1
297-Industrial Marketing Management-1
298-Information Fusion-1
299-Information Systems Journal-1
300-Intelligent Data Analysis-1
301-International Journal of Bank Marketing-1
302-International Journal of Business Performance Management-1
303-International Journal of Contemporary Hospitality Management-1
304-International Journal of Electronic Commerce-1
305-International Journal of Flexible Manufacturing Systems-1
306-International Journal of Health Management-1
307-International Journal of Industrial Organization-1
308-International Journal of Knowledge & Policy-1
309-International Journal of Manpower-1
310-International Journal of Physical Distribution & Materials Management-1
311-International Journal of Policy Analysis & Information Systems-1
312-International Journal of Public Sector Management-1
313-International Journal of Services Industry Management-1
314-International Journal of Services Technology & Management-1
315-International Journal of Technology Management-1
316-International Journal of the Economics of Business-1
317-International Journal on Policy & Information-1
318-International Review of Economics & Finance-1
319-International Review of Law & Economics-1
320-International Review of Retail, Distribution & Consumer Research-1
321-Investigação Operacional-1
322-Istanbul Stock Exchange Review-1
323-Journal of American Statistic Association-1
324-Journal of Applied Econometrics-1
325-Journal of Applied Mathematics & Decision Sciences-1
326-Journal of Applied Statistics-1
327-Journal of Behavioral Health Services & Research-1
328-Journal of Business & Economic Studies-1
329-Journal of Business Venturing-1
330-Journal of Cleaner Production-1
331-Journal of Computer Information System-1
332-Journal of Consumer Research-1
333-Journal of Contemporary Hospitality Management-1
334-Journal of Cost Analysis & Management-1
335-Journal of Development Economics-1
336-Journal of Economic Education-1
337-Journal of Economics-1
338-Journal of Economics and Finance -1
339-Journal of Educational Research-1
340-Journal of Emerging Markets-1
341-Journal of Engineering Design-1
342-Journal of Environmental Economics & Management-1
343-Journal of Financial Economics-1
344-Journal of General Systems-1
345-Journal of High Technology Management Research-1
346-Journal of Hospitality & Tourism Research-1
347-Journal of Human Resources-1
348-Journal of Information Technology -1
349-Journal of Institutional & Theoretical Economics-1
350-Journal of Intelligent Manufacturing-1
351-Journal of International Development-1
352-Journal of Investing-1
353-Journal of Labor Research -1
354-Journal of Luminescence-1
355-Journal of Management-1

356-Journal of Management Accounting Research-1
357-Journal of Management Governance-1
358-Journal of Management in Medicine-1
359-Journal of Management Science & Policy Analysis-1
360-Journal of Marketing-1
361-Journal of Marketing Research-1
362-Journal of Mathematical Analysts & Applications-1
363-Journal of Mathematical Economics -1
364-Journal of Multinational Financial Management-1
365-Journal of Multivariate Analysis-1
366-Journal of Operations & Quantitative Management-1
367-Journal of Policy Analysis Research & Theory-1
368-Journal of Policy Modeling-1
369-Journal of Professional Services Marketing-1
370-Journal of Propulsion & Power-1
371-Journal of Public Budgeting, Accounting & Financial Management-1
372-Journal of Public Health Medicine-1
373-Journal of Real Estate Finance & Economics-1
374-Journal of Real Estate Literature-1
375-Journal of Real Estate Research-1
376-Journal of Regional Analysis & Policy -1
377-Journal of Retail Banking-1
378-Journal of Royal Statistical Society-1
379-Journal of Scientific & Industrial Research-1
380-Journal of Shanghai Institute of Mechanical Engineering-1
381-Journal of Small Business Management-1
382-Journal of Socio-Economics-1
383-Journal of Speech & Hearing Research-1
384-Journal of Sport Management-1
385-Journal of Supply Chain Management-1
386-Journal of Sustainable Agriculture-1
387-Journal of Systems & Software-1
388-Journal of Systems Science & Systems Engineering-1
389-Journal of the American Society for Information Science-1
390-Journal of The American Statistical Association-1
391-Journal of the Royal Statistical Society-1
392-Journal of Transport Economics & Policy-1
393-Journal of Transport Engineering-1
394-Journal of Transportation Engineering-1
395-Journal of Urban Economics-1
396-KIELER MILCHW FORSCH-1
397-Korean Journal of Chemical Engineering-1
398-Kyklos-1
399-Lecture notes in Economics & Mathmatics-1
400-Library-1
401-Local Government Studies-1
402-Managed Care Quarterly-1
403-Management Decision-1
404-Management Review-1
405-Management System: Communications of Japan Industrial Management Association-1
406-Maritime Policy & Management-1
407-Marketing in the Service Industries-1
408-Marketing Science-1
409-Mathematica Japonica-1
410-Mathematical & Computer Modelling-1
411-Mathematical Programming-1
412-Mathematical Social Sciences-1
413-Medical Decision Making-1
414-Methods of Information in Medecine-1
415-Middle East Technical University Studies in Development-1

416-MIS Quarterly-1
417-Mitigation & Adaptation Strategies for Global Change-1
418-Modelling & Simulation-1
419-MOST-1
420-National Forum of Educational Administration & Supervision Journal-1
421-National Tax Journal-1
422-Naval Research Logistics Quarterly-1
423-Network World-1
424-New England Economic Review-1
425-New Zealand Economic Papers-1
426-Non-Profit & Voluntary Sector Quarterly-1
427-Non-Profit Management & Leadership-1
428-Oil & Gas Journal -1
429-Organisation Science -1
430-Pacific Economic Review-1
431-Pacific-Basin Finance Journal-1
432-Policy & Politics-1
433-Policy Study Journal-1
434-Politicka Ekonomie-1
435-Proceedings of National Academy of Science-1
436-Production Animale-1
437-Progress in Tourism and Hospitality Research-1
438-Psychometrika-1
439-Public Administration-1
440-Public Budgeting & Financial Management-1
441-Public Finance Quarterly-1
442-Public Money-1
443-Quality Progress-1
444-Quarterly Journal of International Agriculture-1
445-Quarterly Review of Economics & Finance-1
446-R & D Management-1
447-Regional Science Review-1
448-Regional Studies-1
449-Remote Sensing Review-1
450-Research Evaluation-1
451-Research Policy-1
452-Review of Development Economics-1
453-Review of Economic Studies-1
454-Review of Financial Economies-1
455-Review of Income & Wealth-1
456-Revue D Epidemiologie Et De Sante Publique-1
457-Revue d'Economie du Developpement-1
458-Revue D'Economie Industrielle-1
459-Revue Economique-1
460-Rivista Internazionale di Scienze Economiche e Commerciali-1
461-School Library Media Quarterly-1
462-Scottish Journal of Political Economy-1
463-Singapore Economic Review-1
464-Sloan Management Review-1
465-Social Indicators Research -1
466-Society of Chartered Property and Casualty Underwriters-CPCU Journal-1
467-Solar Energy -1
468-South African Journal of Economics-1
469-Southern Business & Economics Journal-1
470-State & Local Government Review-1
471-Structural Change & Economic Dynamics-1
472-Systems Science & Mathematical Sciences-1
473-Technology Knowledge Activities-1
474-Telecommunications Review-1

475-Telematics & Informatics: An International Journal on Telecommunications and Information Technology-1
476-The American Economic Review-1
477-The Engineering Economist-1
478-Transactions on Operational Research-1
479-Transport Policy-1
480-Transportation Journal-1
481-Transportation Research Record-1
482-Trimestre Economico-1
483-Vestnik Moskovskogo Universiteta-1
484-Water Resources Research-1
485-Weltwirttschftliches Archiv / Review of World Economics-1
486-World Development-1
487-Xibei Fangzhi Gongxueyuan Xuebao-1
488-Yapi Kredi Economic Review-1
489-Zeitschnft Fur Bffentliche Und Gemeinwirtschaftliche Un Terneh Men-1
490-Zeitschrift fur Betriebswirtschaft-1

A.2.1: Computer Code For *Confident-DEA* with Bounded Data

```
%This code identifies the pessimistic and optimistic point of view for
%each DMU and computes the corresponding efficiency coefficient.
□
clear;
□
%For a given cardinal input output data at Per_Tech_Imp_1.dat wihch
%have ranges for each factor with fixed lower and upper bounds
%For constants lower and upper bounds are the same. Outputs first
%inputs follow.
load halfstd.txt;
%Parameters
NoDMU=20;     %Number of DMUs
NoCOL=5;     %Number of inputs+outputs
NoOUT=2;     %Number of outputs
NoINP=NoCOL-NoOUT;  %Number of inputs
beql =[1];  %right side of the equality constraint
b=zeros(NoDMU,1); %right side of the inequality constraints
lb = zeros(NoCOL,1) ; %Positivity constraints
fvalarrpess=ones(1,NoDMU);%[1 1 1 1 1];
fvalarropts=zeros(1,NoDMU);%[0 0 0 0 0];
for ii=1:1:NoDMU
for w=1:1:2 %1 is optimistic 2 is pessimistic
for i=1:1:NoDMU
    %Outputs
    for j=1:1:NoOUT
        %Optimistic
        if w==1
            if i==ii    %Current DMU
                input(i,j)=halfstd(i,2*j);
            else        %other DMUs
                input(i,j)=halfstd(i,2*j-1);
            end
        %Pessimistic
        else
            if i==ii    %Current DMU
                input(i,j)=halfstd(i,2*j-1);
            else        %other DMUs
                input(i,j)=halfstd(i,2*j);
            end
        end
    end
    %Inputs
    for j=NoOUT+1:1:NoCOL
        %Optimistic
        if w==1
            if i==ii    %Current DMU
                input(i,j)=halfstd(i,2*j-1);
            else        %other DMUs
                input(i,j)=halfstd(i,2*j);
            end
        %Pessimistic
        else
            if i==ii    %Current DMU
                input(i,j)=halfstd(i,2*j);
            else        %other DMUs
                input(i,j)=halfstd(i,2*j-1);
            end
        end
```

```
        end
    end
    input;%the corresponding data matrix used for the DEA model.
    %Make the A matrix for inequalities positive for outputs negative for
    inputs
    if NoOUT>=1
        A=input(:,1);
    else
        A=-input(:,1);
    end
    for j=2:NoCOL;
        if j <= NoOUT
            A=[A input(:,j)];
        else
            A=[A -input(:,j)];
        end
    end
    %Aeqarr has in its rows corresponding coefficients for each DMU for
    equality constraint
    Aeqlarr = input;
    %Substitute zeros to columns for outputs
    for i=1:NoOUT
        Aeqlarr(:,i) = zeros(NoDMU,1) ;
    end
    %carr has in its rows corresponding coefficients for each DMU for
    objective function
    carr=input;
    %Substitute zeros to columns for inputs
    for i=NoOUT+1:NoCOL
        carr(:,i)=zeros(NoDMU,1);
    end
    [x,fval,exitflag, output, lambda]=linprog(-carr(ii,:), A, b,
    Aeqlarr(ii,:), beql, lb);
    eff=-fval;
    %Store the optimistic efficiency for iith DMU
    if w==1
        fvalarropts(ii)=eff;
        inputopts(:,:,ii)=input;
    else
    %Store the pessimistic efficiency for iith DMU
        fvalarrpess(ii)=eff;
        inputpess(:,:,ii)=input;
    end
    end
    end
    inputopts;
    inputpess;
    fvalarropts
    fvalarrpess
```

A.2.2: Computer Code For the Simulation Component of *Confident-DEA* with Bounded Data

```
%This code simulates the case of DEA with cardinal bounded data.
□
%It provides minimum and max efficiency as well as histogram for each
DMU.
□
load thesis.dat;
□
%Parameters
□
NoDMU=8;      %Number of DMUs
NoCOL=4;      %Number of inputs+outputs
NoOUT=2;      %Number of outputs
NoINP=NoCOL-NoOUT;  %Number of inputs
NoRuns=10000;
beql =[1];  %right side of the equality constraint
b=zeros(NoDMU,1); %right side of the inequality constraints
lb = zeros(NoCOL,1) ; %Positivity constraints
fvalarrmin=ones(1,NoDMU);%[1 1 1 1 1];
fvalarrmax=zeros(1,NoDMU);%[0 0 0 0 0];
fvalarrfreq=zeros(NoDMU,21);
%Array to hold frequencies 20 intervals for each DMU between 0 and 1
%The 21st box is for full efficent DMU.
%Calculates mu sigma values for a 6 sigma interval equating to
%variable interval from inputvar
for i=1:NoDMU
    for j=1:2:2*NoCOL
        musigma(i,j)=(thesis(i,j)+thesis(i,j+1))/2;      %mu
        musigma(i,j+1)=(thesis(i,j+1)-thesis(i,j))/6;    %sigma
    end
end
for q=1:NoRuns           %number of runs
% Randomly assigns an input data from normal distribution
%It truncates if value comes out of bounds
for i=1:NoDMU
    k=1;
    for j=1:NoCOL
        ival=musigma(i,k)+randn*musigma(i,k+1);
        if ival<thesis(i,k)
            ival=thesis(i,k);
        end
        if ival>thesis(i,k+1)
            ival=thesis(i,k+1);
        end
        input(i,j)=ival;
        k=k+2;
    end
end
%Make the A matrix for inequalities positive for outputs negative for
inputs
if NoOUT>=1
    A=input(:,1);
else
    A=-input(:,1);
end
for j=2:NoCOL;
```

```
    if j <= NoOUT
        A=[A input(:,j)];
    else
        A=[A -input(:,j)];
    end
end
%Aeqarr has in its rows corresponding coefficients for each DMU for
equality constraint
Aeqlarr = input;
%Substitute zeros to columns for outputs
for i=1:NoOUT
    Aeqlarr(:,i) = zeros(NoDMU,1) ;
end
%carr has in its rows corresponding coefficients for each DMU for
objective function
carr=input ;
%Substitute zeros to columns for inputs
for i=NoOUT+1:NoCOL
    carr(:,i)=zeros(NoDMU,1);
end
for i = 1:NoDMU
[x,fval,exitflag, output, lambda]=linprog(-carr(i,:), A, b,
Aeqlarr(i,:), beql, lb);
eff=-fval;
if eff >=0.9999
    eff=1;
end
bucket=floor(20*eff)+1;
if fvalarrfreq(i,bucket) == 0
    inputinter(:,:,i,bucket)=input;
end
fvalarrfreq(i,bucket)=fvalarrfreq(i,bucket)+1;
if fvalarrmin(i)>=eff
    fvalarrmin(i)=eff;
    inputmin(:,:,i)=input;
end
if fvalarrmax(i)<=eff
    fvalarrmax(i)=eff;
    inputmax(:,:,i)=input;
end
end
end
fvalarrmax
fvalarrmin
fvalarrfreq
inputmin
inputmax
inputinter;
```

A.2.3: Computer Code For Genetic-Algorithm-Based Solving Method for the *Confident-DEA* With Imprecise Data

```
%This code applies GA to DEA with Imprecise data (Cardinal+ordinal
data)assuming uniform distribution for the initial population
□
%and applying 1 horizantal cut to the output input data matrice for
the crossover.
□
load ordinal.dat;
%Format of the data file ordinal.dat
%Cardinal Output, Cardinal Input, Ordinal Output, Ordinal Input
%The format of an individuals in the population pop
%Cardinal Output, Ordinal Output, Cardinal Input, Ordinal Input
%Parameters
NoDMU=8;                    %Number of DMUs
NoCOL=6;                    %Number of inputs+outputs
NoOUT=3;                    %Number of outputs
NoINP=NoCOL-NoOUT;          %Number of inputs
NoCarINP=2;                 %Number of cardinal inputs
NoOrdINP=NoINP-NoCarINP;%Number of ordinal inputs
NoCarOUT=3;                 %Number of cardinal outputs
NoOrdOUT=NoOUT-NoCarOUT;%Number of ordinal outputs
NoORD=NoOrdINP+NoOrdOUT;%Number of ordinal factors
NoCAR=NoCarINP+NoCarOUT;%Number of cardinal factors
Popsize=10; %Population size divisible by 2  100
NoGen=10;   %Number of generations           200
CROSSLIM=0.9; %Crossover probability  0.75-0.99
MUTLIM=0.01;  %Mutation probability   0.001-0.25
beql =[1];  %right side of the equality constraint
b=zeros(NoDMU,1); %right side of the inequality constraints
lb = zeros(NoCOL,1) ; %Positivity constraints
fvalarrmin=ones(1,NoDMU);   %[1 1 1 1 1];
fvalarrmax=zeros(1,NoDMU);  %[0 0 0 0 0];
%Normalization of cardinal factors
%Search for maximum values
for j=2:2:2*NoCAR
    maxfact(j/2)=max(ordinal(:,j));
end
%Division by the highest value just found
for i=1:NoDMU
     for j=2:2:2*NoCAR
          normdata(i,j-1)= ordinal(i,j-1)/maxfact(j/2);
          normdata(i,j)  = ordinal(i,j)  /maxfact(j/2);
     end
end
%INITIAL Population
for q=1:Popsize
% Randomly assigns an input data from uniform distribution
%Cardinal
for i=1:NoDMU
     k=1;
     for j=1:NoCOL
          if j<=NoCarOUT | (j>NoOUT & j<=NoOUT+NoCarINP)
```

```
            ival= normdata(i,k) + rand* (normdata(i,k+1)-
normdata(i,k));
            pop(i,j,q)=ival;
        else
            pop(i,j,q)=0;
        end
        k=k+2;
    end
end
%Ordinal
for oo=1:NoOrdOUT
    randomm=rand(NoDMU-1,1);
    randoms=sort(randomm);
    randoms=[randoms;1];
    for i=1:NoDMU
        pop(i,NoCarOUT+oo,q)=randoms( ordinal(i,2*NoCAR+oo) );
    end
end
for oi=1:NoOrdINP
    randomm=rand(NoDMU-1,1);
    randoms=sort(randomm);
    randoms=[randoms;1];
    for i=1:NoDMU
        pop(i,NoOUT+NoCarINP+oi,q)=randoms(
ordinal(i,2*NoCAR+NoOrdOUT+oi) );
    end
end
end%Initial population
for t=1:NoDMU %DMU
for w=1:2 %1 is max 2 is min
for g=1:NoGen %For each generation
%Calculate objective values for each input in pop
for q=1:Popsize
%Make the A matrix for inequalities positive for outputs negative for
inputs
if NoOUT>=1
    A=pop(:,1,q);
else
    A=-pop(:,1,q);
end
for j=2:NoCOL;
    if j <= NoOUT
        A=[A pop(:,j,q)];
    else
        A=[A -pop(:,j,q)];
    end
end
%Aeqarr has in its rows corresponding coefficients for each DMU for
equality constraint
Aeqlarr = pop(:,:,q);
%Substitute zeros to columns for outputs
for i=1:NoOUT
    Aeqlarr(:,i) = zeros(NoDMU,1) ;
end
%carr has in its rows corresponding coefficients for each DMU for
objective function
carr=pop(:,:,q);
%Substitute zeros to columns for inputs
for i=NoOUT+1:NoCOL
    carr(:,i)=zeros(NoDMU,1);
end
```

```
%Calculate the efficiency for the tth DMU and qth instance
[x,fval,exitflag, output, lambda]=linprog(-carr(t,:), A, b,
Aeqlarr(t,:), beql, lb);
eff=-fval;
if eff >=0.9999
    eff=1;
end
%Store the efficiency of qth instance of the tth DMU for roulette
wheel
if w==1
    popobj(q)=eff^2;
else
    popobj(q)=1/eff;
end
%Store the minumum efficiency for tth DMU met in the process
if fvalarrmin(t)>=eff
    fvalarrmin(t)=eff;
    inputmin(:,:,t)=pop(:,:,q);
end
%Store the maximum efficiency for tth DMU met in the process
if fvalarrmax(t)<=eff
    fvalarrmax(t)=eff;
    inputmax(:,:,t)=pop(:,:,q);
end
end%for Calculate objective values for each input in pop q 1:Popsize
%If maximum objective value reaches to 1 stop maximization as there
cannot be a higher value
if fvalarrmax(t)==1
elseif w==1
    break;
end
popobjsum=sum(popobj);
for g=1:2:Popsize %New generation
%selecting a parent randomly from the population using roulette wheel
select=rand*popobjsum;
popsum=0;
q1=0;%first parent
while popsum<=select
    q1=q1+1;
    popsum=popsum+popobj(q1);
end
select=rand*popobjsum;
popsum=0;
q2=0;%second parent
while popsum<=select
    q2=q2+1;
    popsum=popsum+popobj(q2);
end
crossprob=rand;
if crossprob<=CROSSLIM
    %Crossover
    crosspoint=ceil(rand*(NoDMU-1));
    %Crossover first half
    for i=1:crosspoint
        newpop(i,:,g)=pop(i,:,q1);
        newpop(i,:,g+1)=pop(i,:,q2);
    end
    %Crossover second half
    for i=crosspoint+1:NoDMU
        newpop(i,:,g)=pop(i,:,q2);
        newpop(i,:,g+1)=pop(i,:,q1);
```

```
    end
else
    %No crossover pass to new generation without change
    newpop(:,:,g)=pop(:,:,q1);
    newpop(:,:,g+1)=pop(:,:,q2);
end
%Mutation
for mm=0:1
    mutprob=rand;
    if mutprob<=MUTLIM
        mutrow=ceil(rand*NoDMU);
        mutcol=ceil(rand*NoCOL);
        if mutcol<=NoCarOUT
            ival=normdata(mutrow,2*mutcol-1) + rand*
(normdata(mutrow,2*mutcol)-normdata(mutrow,2*mutcol-1));
            if mutcol>NoOUT & mutcol<=NoOUT+NoCarINP
                %Mutation for cardinal data
                ival=normdata(mutrow,2*mutcol-NoOrdOUT-1) + ...
                        rand* (normdata(mutrow,2*mutcol-NoOrdOUT)-
normdata(mutrow,2*mutcol-NoOrdOUT-1));
            else
                %Mutation for ordinal data
            end
        end
        newpop(mutrow,mutcol,g+mm)=ival;
    end
end
end %for new generation
pop=newpop;
end %while
end %w=1:2 %1 is max 2 is min
end %t=1:NoDMU %DMU
fvalarrmax
fvalarrmin
inputmax;
inputmax;
```

A.2.4: Computer Code For the Simulation Component of *Confident-DEA* with Imprecise Data

```
%This code simulates the case of DEA with Imprecise data. The normal
%distribution is assumed for cardinal bounded data, the uniform
%distribution is used for the ordinal data . The code provides the
%minimum and maximum efficiency values
and the histogram of the %efficiency coefficients.
load ordinal.dat;
%Format of the data file ordinal.dat
%Cardinal Output, Cardinal Input, Ordinal Output, Ordinal Input
%The format of an individuals in the population pop Cardinal Output,
%Ordinal Output, Cardinal Input, Ordinal Input Parameters
NoDMU=8;                  %Number of DMUs
NoCOL=6;                  %Number of inputs+outputs
NoOUT=3;                  %Number of outputs
NoINP=NoCOL-NoOUT;        %Number of inputs
NoCarINP=2;               %Number of cardinal inputs
NoOrdINP=NoINP-NoCarINP;%Number of ordinal inputs
NoCarOUT=3;               %Number of cardinal outputs
NoOrdOUT=NoOUT-NoCarOUT;%Number of ordinal outputs
NoORD=NoOrdINP+NoOrdOUT;%Number of ordinal factors
NoCAR=NoCarINP+NoCarOUT;%Number of cardinal factors
NoRuns=100000;
modstep=100;
NoBuckets=20;       %is equal to (1/stepsize) To them a Last bucket is
added for fully efficient DMUs
beql =[1];  %right side of the equality constraint
b=zeros(NoDMU,1); %right side of the inequality constraints
lb = zeros(NoCOL,1) ; %Positivity constraints
fvalarrmin=ones(1,NoDMU);%[1 1 1 1 1];
fvalarrmax=zeros(1,NoDMU);%[0 0 0 0 0];
fvalarrfreq=zeros(NoDMU,NoBuckets+1);    %Array to hold frequencies 20
intervals for each DMU between 0 and 1
%The 21st box is for full efficent DMU.
%Normalization of cardinal factors
%Search for maximum values
for j=2:2:2*NoCAR
    maxfact(j/2)=max(ordinal(:,j));
end
%Division by the highest value just found
for i=1:NoDMU
    for j=2:2:2*NoCAR
        normdata(i,j-1)= ordinal(i,j-1)/maxfact(j/2);
        normdata(i,j)  = ordinal(i,j)  /maxfact(j/2);
    end
end
% Calculates mu sigma values for a 6 sigma interval equating to
variable interval from data file
for i=1;NoDMU
    for j=1:2:2*NoCAR
        musigma(i,j)  =(normdata(i,j)  +normdata(i,j+1))/2;    %mu
        musigma(i,j+1)=(normdata(i,j+1)-normdata(i,j))  /6;    %sigma
    end
end
for q=1:NoRuns           %number of runs.
if mod(q,modstep) == 0
q
end
```

```
%CARDINAL:Randomly assigns an input data for cardinals from normal
%distribution.It truncates if value comes out of bounds
for i=1:NoDMU
     k=1;
     for j=1:NoCOL
          if j<=NoCarOUT | (j>NoOUT & j<=NoOUT+NoCarINP)
               ival=musigma(i,k)+randn*musigma(i,k+1);
               if ival<normdata(i,k)
                    ival=normdata(i,k);
               end
               if ival>normdata(i,k+1)
                    ival=normdata(i,k+1);
               end
               input(i,j)=ival;
          else
               input(i,j)=0;
          end
          k=k+2;
     end
end
%ORDINAL
for oo=1:NoOrdOUT
     randomm=rand(NoDMU-1,1);
     randoms=sort(randomm);
     randoms=[randoms;1];
     for i=1:NoDMU
          input(i,NoCarOUT+oo)=randoms( ordinal(i,2*NoCAR+oo) );
     end
end
for oi=1:NoOrdINP
     randomm=rand(NoDMU-1,1);
     randoms=sort(randomm);
     randoms=[randoms;1];
     for i=1:NoDMU
          input(i,NoOUT+NoCarINP+oi)=randoms(
ordinal(i,2*NoCAR+NoOrdOUT+oi) );
     end
end
%Make the A matrix for inequalities positive for outputs negative for
inputs
if NoOUT>=1
    A=input(:,1);
else
    A=-input(:,1);
end
for j=2:NoCOL;
    if j <= NoOUT
        A=[A input(:,j)];
    else
        A=[A -input(:,j)];
    end
end
%Aeqarr has in its rows corresponding coefficients for each DMU for
equality constraint
Aeqlarr = input;
%Substitute zeros to columns for outputs
for i=1:NoOUT
    Aeqlarr(:,i) = zeros(NoDMU,1) ;
end
%carr has in its rows corresponding coefficients for each DMU for
objective function
```

```
carr=input ;
%Substitute zeros to columns for inputs
for i=NoOUT+1:NoCOL
    carr(:,i)=zeros(NoDMU,1);
end
for i = 1:NoDMU
[x,fval,exitflag, output, lambda]=linprog(-carr(i,:), A, b,
Aeqlarr(i,:), beql, lb);
eff=-fval;
if eff >=0.9999
    eff=1;
end
bucket=floor(NoBuckets*eff)+1;
if fvalarrfreq(i,bucket) == 0
    inputinter(:,:,i,bucket)=input;
end
fvalarrfreq(i,bucket)=fvalarrfreq(i,bucket)+1;
if fvalarrmin(i)>=eff
    fvalarrmin(i)=eff;
    inputmin(:,:,i)=input;
end
if fvalarrmax(i)<=eff
    fvalarrmax(i)=eff;
    inputmax(:,:,i)=input;
end
end
end
ord=1;
for j=1:NoCOL
    if j<=NoCarOUT | (j>NoOUT & j<=NoOUT+NoCarINP)
        outmin(:,j,:)=inputmin(:,j,:)*maxfact(j);
        outmax(:,j,:)=inputmax(:,j,:)*maxfact(j);
    else
        for i=1:NoDMU
            outmin(:,j,i)=( ordinal(:,2*NoCAR+ord) );
            outmax(:,j,i)=( ordinal(:,2*NoCAR+ord) );
        end
        ord=ord+1;
    end
end
for bucket=1:NoBuckets+1
ord=1;
for j=1:NoCOL
    if j<=NoCarOUT | (j>NoOUT & j<=NoOUT+NoCarINP)
        outinter(:,j,:,bucket)=inputinter(:,j,:,bucket)*maxfact(j);
    else
        for i=1:NoDMU
            if fvalarrfreq(i,bucket) > 0
                outinter(:,j,i,bucket)=( ordinal(:,2*NoCAR+ord) );
            end
        end
        ord=ord+1;
    end
end
end
outmin;
inputmin;
outmax;
inputmax;
inputinter;
```

A.2.5: Exhibit from the Solution Provided by the Simulation Component of *Confident-DEA* with Bounded Data Using the Computer Code in A.2.2

```
%Solution obtained from running the simulation for DEA with imprecise
data.Number of runs= 100000.Results are to compare with Cooper et al
(forthcoming in OR)

fvalarrmax =
1.0000  1.0000  0.8363  0.8723  0.8746  0.9842  0.8113  1.0000

fvalarrmin =
1.0000  1.0000  0.5941  0.7159  0.2018  0.7414  0.2546  1.0000

fvalarrfreq =

 Columns 1 through 8
0        0        0        0        0        0        0        0
0        0        0        0        0        0        0        0
0        0        0        0        0        0        0        0
0        0        0        0        0        0        0        0
0        0        0        0        4        0        0        0
0        0        0        0        0        0        0        0
0        0        0        0        0        1        87537    4694
0        0        0        0        0        0        0        0

 Columns 9 through 16
0        0        0        0        0        0        0        0
0        0        0        0        0        0        0        0
0        0        0        88744    10982    180      72       19
0        0        0        0        0        0        99971    22
0        0        0        95864    3934     118      62       14
0        0        0        0        0        0        57348    16156
2998     2013     1313     766      402      192      69       14
0        0        0        0        0        0        0        0

 Columns 17 through 21
0        0        0        0        100000
0        0        0        0        100000
3        0        0        0        0
5        2        0        0        0
3        1        0        0        0
22683    3064     666      83       0
1        0        0        0        0
0        0        0        0        100000
```

A.3.1. Data About Swiss Commercial Banks

Years	Loans	Securities	Fixed Assets	Deposits	Number of Employees	End-of-Period Exchange rate	Loans	Securities	Fixed Assets	Deposits	Number of Employees
1979	25334	7290	2333	22497	13800	0.632911	16034.17	4613.92119	1476.581	14238.599	13800
1980	30319	7827	2835	24399	14600	0.567054	17192.51	4438.331658	1607.598	13835.551	14600
1981	32470	8460	4362	28619	15900	0.556019	18053.94	4703.92074	2425.355	15912.708	15900
1982	36512	9844	5442	35348	17200	0.501379	18306.35	4935.574876	2728.505	17722.745	17200
1983	40055	11628	6261	42053	18000	0.458821	18378.08	5335.170588	2872.678	19294.8	18000
1984	43684	12001	6495	47400	19100	0.386847	16899.02	4642.550847	2512.571	18336.548	19100
1985	46251	13945	5788	47128	20200	0.48158	22273.56	6715.6331	2787.385	22695.902	20200
1986	47683	16729	6091	46924	22400	0.615953	29370.49	10304.27774	3751.77	28902.979	22400
1987	49673	18828	6693	49903	25000	0.782473	38867.78	14732.40164	5237.092	39047.75	25000
1988	59906	20728	6918	55965	25900	0.664894	39831.14	13781.92283	4599.737	37210.793	25900
1989	67353	24400	7795	59633	27000	0.646621	43551.86	15777.5524	5040.411	38559.95	27000
1990	74007	24266	8947	63942	28900	0.771903	57126.23	18730.9982	6906.216	49357.022	28900
1991	76536	25062	9262	67664	28100	0.737735	56463.29	18489.11457	6832.902	49918.101	28100
1992	77108	26448	8532	69333	26800	0.686813	52958.78	18164.83022	5859.889	47618.806	26800
1993	80898	29945	8455	75551	26700	0.675904	54679.28	20239.94528	5714.768	51065.223	26700
1994	78090	31201	8486	76069	26600	0.762486	59542.53	23790.32569	6470.456	58001.548	26600
1995	76700	32428	9833	77913	26500	0.869187	66666.64	28185.99604	8546.716	67720.967	26500
1996	81899	36581	12526	90913	26000	0.742721	60828.11	27169.4769	9303.323	67522.994	26000
1997	88643	39463	15127	106380	26000	0.687144	60910.51	27116.76367	10394.43	73098.379	26000
1998	85328	43760	14699	115314	27000	0.72648	61989.09	31790.7648	10678.53	83773.315	27000

A.3.2. 1998 Forecasted Confidence Intervals for the Production Factors of Commercial Banks in OECD Countries Obtained Using Half Standard Deviation

		Outputs				Inputs					
	Country	Loans (min)	Loans (max)	Securities (min)	Securities (max)	Fixed Assets (min)	Fixed Assets (max)	Deposits (min)	Deposits (max)	Number of Employees (min)	Number of Employees (max)
1	dnk	83646	94563	54240	58108	14884	26685	109282	118323	38781	40531
2	fin	45735	61811	27785	32206	19747	22528	49936	59377	15268	17975
3	fra	653912	706449	476317	485926	161592	186898	495835	517757	197601	200804
4	ger	1013250	1060654	346633	356271	41702	43681	777420	817818	208838	213762
5	ice	3248	3570	648	725	157	194	2997	3288	2159	2298
6	japcb	4633313	5098665	1019545	1128052	652845	792092	4874319	5483541	391096	417649
7	japlcb	2576420	2856931	503799	559102	456336	557739	2588480	2939968	135614	141460
8	kor	150230	175898	63877	73727	63718	71487	195778	225558	84376	88020
9	lux	118255	127205	154454	163804	20128	25262	228168	257574	20235	20905
10	mex	106711	120012	22156	28174	10062	15309	87169	106651	113997	117945
11	nor	61519	67381	9815	11946	4293	4970	46773	51318	10505	11766
12	por	87832	91567	49393	54952	12605	18597	116542	123440	59612	60717
13	spa	251713	282472	114008	124443	108584	123922	274654	307008	138249	142906
14	swe	93749	111212	95783	102783	19852	25825	110129	120926	37713	40771
15	swicb	68632	74230	29768	31683	9908	10676	81364	85560	24795	26030
16	swilcb	324839	351513	157631	167999	109592	126268	331739	348242	61911	63570
17	tur	38235	40957	14562	15858	23854	25022	65592	69188	146369	153729
18	uk	1002414	1097245	386317	406916	274219	291064	937355	1088124	409751	431367
19	uscb	3189532	3294533	1048821	1098264	495609	516913	3444124	3530374	1509887	1524281
20	uslcb	2159318	2251727	645490	674782	421343	441785	2081310	2160007	875726	898302

A.3.3. 1998 Forecasted Confidence Intervals for the Production Factors of Commercial Banks in OECD Countries Obtained Using 1 Standard Deviation

	Country	Loans (min)	Loans (max)	Securities (min)	Securities (max)	Fixed Assets (min)	Fixed Assets (max)	Deposits (min)	Deposits (max)	Number of Employees (min)	Number of Employees (max)
1	dnk	78188	100022	52305	60042	8983	32586	104761	122844	37906	41406
2	fin	37787	69819	25574	34417	18357	23919	45216	64097	13914	19329
3	fra	627644	732718	471512	490731	148938	199552	484875	528717	195999	202406
4	ger	989563	1084351	341815	361090	40713	44670	757221	838017	206376	216224
5	ice	3087	3731	610	764	138	213	2852	3433	2089	2368
6	japcb	4400637	5331341	965291	1182306	583222	861715	4569709	5788152	377819	430925
7	japlcb	2436164	2997187	476148	586753	405635	608441	2412736	3115712	132690	144383
8	kor	65908	77004	28811	32640	9524	11059	79265	87659	24178	26647
9	lux	113780	131681	149779	168479	17560	27829	213465	272277	19900	21240
10	mex	100046	126668	19146	31183	7439	17932	77429	116391	112023	119920
11	nor	58573	70316	8749	13012	3955	5309	44501	53591	9874	12397
12	por	85950	93440	46614	57731	9608	21594	113093	126889	59060	61269
13	spa	236319	297856	108791	129661	100916	131591	258477	323185	135920	145234
14	swe	85017	119943	92283	106283	16866	28811	104730	126325	36184	42300
15	swicb	65908	77004	28811	32640	9524	11059	79265	87659	24178	26647
16	swilcb	311501	364850	152446	173183	101254	134606	323487	356493	61081	64399
17	tur	36949	42293	13915	16505	23271	25605	63794	70986	142689	157410
18	uk	955104	1144625	376018	417215	265797	299486	861971	1163509	398943	442175
19	uscb	3137076	3347018	1024099	1122986	484957	527565	3400999	3573499	1502690	1531478
20	uslcb	2113893	2297671	630844	689429	411122	452007	2041961	2199356	864438	909590

A.3.4. 1998 Forecasted Confidence Intervals for the Technical Efficiency of Commercial Banks in OECD Countries Obtained Using *Confident-DEA* Using Data Provided in A.3.2

	Pessimistic		Realization		Optimistic	
1	ger	1	fra	1	dnk	1
2	japlcb	1	ger	1	fin	1
3	lux	1	ice	1	fra	1
4	fra	0.9447	japcb	1	ger	1
5	swe	0.7824	japlcb	1	ice	1
6	nor	0.78	lux	1	japcb	1
7	swilcb	0.7047	nor	1	japlcb	1
8	japcb	0.6287	swe	1	lux	1
9	ice	0.6107	swilcb	0.9833	mex	1
10	uslcb	0.6103	spa	0.7897	nor	1
11	uscb	0.562	dnk	0.7852	por	1
12	mex	0.5442	fin	0.7422	swe	1
13	uk	0.5268	uk	0.7194	swilcb	1
14	kor	0.522	uslcb	0.708	uk	1
15	swicb	0.522	uscb	0.6736	spa	0.9707
16	por	0.4765	por	0.669	uslcb	0.9506
17	spa	0.4701	mex	0.6294	uscb	0.8322
18	dnk	0.4625	swicb	0.6049	kor	0.8218
19	fin	0.4536	kor	0.5672	swicb	0.8218
20	tur	0.3263	tur	0.4164	tur	0.5586

REFERENCES

Abott, A., 1988. The system of professions: an essay on the expert division of labor. University of Chicago Press, Chicago, IL.

Ackoff, R. 1979. The future of Operations Research is past. Journal of the Operational Research Society 30, 93-104.

Ackoff, R. 1987. OR: A post mortem. Operations Research 35, 471-474.

Akhavein J.D., Berger, A.N., Humphrey, D.B., 1997. The effects of megamergers on efficiency and prices: evidence from a bank profit function. Review of Industrial Organization 12, 95-139.

Ali, A.I., Seiford, L.M., 1993. Computational accuracy and infinitesimals in Data Envelopment Analysis. INFOR 31 (4), 290-297.

Ali, A.I., Seiford, L.M., 1993. The mathematical programming approach to efficiency analysis. in The Measurement of Productive Efficiency, Oxford University Press, New York.

Allen, L., Rai, A., 1996. Operational efficiency in banking: an international comparison. Journal of Banking and Finance 20, 655-672.

Avkiran, N.K., 1999. The evidence on efficiency gains: the role of mergers and the benefits to the public. Journal of Banking and Finance 23, 991-1013.

Avkiran, N.K., 2001. Investigating technical and scale efficiencies of Australian universities through Data Envelopment Analysis. Socio-Economic Planning Sciences 35 (1), 57-80.

Avkiran, N.K., Gattoufi, S., 2001. Do Cost Inefficiencies Precede Financial Crisis? Paper presented at The 14th Australasian Finance & Banking Conference, Sydney

Back, T. 1996. Evolutionary algorithms in theory and practice. Oxford University Press, New York.

Banker, R. D., Charnes, A., Cooper, W.W., 1984. Some Models for Estimating Technical and Scale Inefficiencies in Data Envelopment Analysis. Management Science 30 (9), 1078-92.

Banker, R. D., Morey, R.C., 1986. Efficiency Analysis for Exogenously Fixed Inputs and Outputs. Operations Research 34 (4), 513-521.

Banks, J., Carson, J.S., Nelson, B.L., 1996. Discrete event system simulation. Second Edition, Prentice-Hall, Upper Saddle River, New Jersey.

Bard, J.F. 1998. Practical bilevel optimization: algorithms and applications, Kluwer Academic Publishers, Dordrecht, The Netherlands.

Bardhan, I.R., Cooper, W.W., Kumbhakar, S.C., 1998. A simulation study of joint uses of Data Envelopment Analysis and statistical regressions for production function estimation and efficiency evaluation. The Journal of Productivity Analysis, 9, 249-278.

Basset, G.Jr., R. Koenker, R, 1978. Asymptotic theory of least absolute value error regressions. Journal of the American Statistical Association 73, pp. 618-622.

Battese, G.E., Coelli, T.J., 1992. Frontier production functions, technical efficiency and panel data: with application to paddy farmers in India. Journal of Productivity Analysis, 3, 153-169.

Battese, G.E., Coelli, T.J., 1995. A model for technical inefficiency effects in a stochastic frontier production function for panel data. Empirical Economics, 20, 325-332.

Berg, S.A., Forsund, F., Hjalmarsson, L., Suominen, M., 1993. Banking efficiency in the Nordic countries. Journal of Banking and Finance 17, 371-388.

Berg, S.A., Patillo, C., 1999. Are currency crises predictable? A test. IMF Staff Papers, International Monetary Fund, Washington DC, USA.

Bergendahl, G., 1995. DEA and benchmarks for Nordic banks, Working Paper, Gothenburg University, Gothenburg, Sweden.

Berger A.N., Hanweck, G.A., Humphrey, D.B., 1987. Competitive viability in banking: scale, scope and product mix economies. Journal of Monetary Economics 20, 501-520.

Berger, A.N., 1993, Distribution-Free estimates of efficiency in the US banking industry and tests of the standard distributional assumptions, Journal of Productivity Analysis 4, 261-292.

Berger, A.N., DeYoung, R., Genay, H., Udell, F.G., 2000. Globalisation of financial institutions: evidence from cross-border performance. Forthcoming in Brookings-Wharton Papers on Financial Services.

Berger, A.N., Hancock, D., Humphrey, D.B., 1993. Bank efficiency derived from the profit function. Journal of Banking and Finance 17, 317-347.

Berger, A.N., Humphrey, D.B., 1991. The dominance of inefficiencies over scale and product mix economics in banking. Journal of Monetary Economics 28, 117-148.

Berger, A.N., Humphrey, D.B., 1992. Measurement and efficiency issues in commercial banking. In Output Measurement in the Service Sectors Ed. Griliches Z., The University of Chicago Press, Chicago, 245-300.

Berger, A.N., Humphrey, D.B., 1997. Efficiency of financial institutions: international survey and directions for future research. European Journal of Operational Research 98 (2), 175-212.

Berger, A.N., Hunter, W.C., Timme, S.G., 1993. The efficiency of financial institutions: a review and preview of research past, present and future. Journal of Banking and Finance 17, 221-249.

Berger, A.N., Mester, L.J, 1997. What explains the dramatic changes in cost and profit performance of the US banking industry? Working Paper, Financial Institutions Center, Wharton School, University of Pennsylvania.

Berger, A.N., Mester, L.J., 1997. Inside the black box: What explains differences in the efficiencies of financial institutions? Working Paper, Financial Institutions Center, Wharton School, University of Pennsylvania.

Bhattacharyya, A., Lovell, C.A.K., Sahay, P., 1997. The impact of liberalisation on the productive efficiency of Indian commercial banks. European Journal of Operational Research 98, 332-345.

Blumstein, A., 1987. The current missionary role of OR/MS. Operations Research 35, 926-929.

Bukh, P.N.D., Berg, S.A., Forsund, F.R., 1995. Banking efficiency in the Nordic countries: A four-country Malmquist index analysis. Working Paper, University of Arhus, Denmark.

Camanho, A.S., Dyson, R.G., 1999. Efficiency, size, benchmarks and targets for bank branches: an application of data envelopment analysis. Journal of the Operational Research Society 50 (9), 903-915.

Charnes, A., and Cooper, W.W., 1985. Preface to topics in Data Envelopment Analysis. Annals of Operations Research 2 (1), 59-94.

Charnes, A., Cooper WW, Golany, B., Seiford, L.M., Stutz, J., 1985. Foundations of Data Envelopment Analysis for Pareto-Koopmans efficient empirical production functions. Journal of Econometrics 30, 91-107.

Charnes, A., Cooper, W.W., Mellon, B., 1952. Blending aviation gasolines: a study in programming interdependent activities in an integrated oil company. Econometrica 20 (2), 135-159.

Charnes, A., Cooper, W.W., Rhodes, E., 1978. Evaluating program and managerial efficiency: an application of Data Envelopment Analysis to Program Follow Through. Management Science 27 (6), 668-697.

Charnes, A., Cooper, W.W., Rhodes, E., 1981. Measuring the efficiency of Decision Making Units. European Journal of Operational Research 2, 429-444.

Cheng, C.H., Kumar, A., Motwani, J., Reisman, A., Madan, M., 1999. An Objective Rank-Order Analysis of the Technology Innovation Management Journals. IEEE Transactions on Engineering Management, February

Coelli, T., 1996. A Guide to DEAP2.1: A Data Envelopment Analysis (Computer) program. Working Paper, Centre for Efficiency and Productivity Analysis, University of New England, Australia.

Cook, W.D., Kress, M., Seiford, L.M., 1993. On the use of ordinal data in Data Envelopment Analysis. Journal of The Operational Research Society 44, 2, 133-140.

Cook, W.D., Kress, M., Seiford, L.M., 1996. Data Envelopment Analysis in the presence of both quantitative and qualitative Factors. Journal of The Operational Research Society 47, 945-953.

Cooper, W.W., 1994. Personal communication.

Cooper, W.W., Tone, K., 1997. Measures of inefficiency in data envelopment analysis and stochastic frontier estimation. European Journal of Operational Research 99 (1), 72-88.

Cooper, W.W., 1999. Operations Research/Management Science: where it's been. where it should be going?, Journal of The Operational Research Society 50, 3-11.

Cooper, W.W., Park, K.S., Yu, G., 1999. IDEA and AR-IDEA: Models for dealing with imprecise data in DEA. Management Science 45 (4), 597-607.

Cooper, W.W., Deng, H., Gu, B., Li, S., Thrall, R.M., 2001. Using DEA to improve the management of congestion in Chinese industries (1981-1997). Socio-Economic Planning Science 35, 227-242.

Cooper, W.W., Park, K.S. and Yu, G., 2001. IDEA (Imprecise Data Envelopment Analysis) with CMDS (Column Maximum Decision Making Units). Journal of The Operational Research Society 52 (2), 178-181.

Cooper, W.W., Park, K.S., Yu, G., (forthcoming). An illustrative application of IDEA (Imprecise Data Envelopment Analysis) to a Korean Mobile Telecommunication Company. Operations Research.

Corbett, C.J., Van Wassenhove, L.N., 1993. The natural drift: what happened to Operations Research? Operations Research 41, 625-640

Cummins, J., D., Zi, H., 1998. Comparison of frontier efficiency methods: an application to the U.S. life insurance industry. Journal of Productivity Analysis, 10, 131-152.

Dantzig, G.B., 1953. Computational algorithms of the revised simplex method. Report, RM 1266, The Rand Corporation, Santa Monica, CA.

Debreu, G., 1951. The coefficient of resource utilisation. Econometrica 19, 273-292.

Dietsch, M., Lozano-Vivas, A., 2000. How the environment determines banking efficiency: A comparison between French and Spanish industries. Journal of Banking and Finance 24 (6), 985-1004.

Elyasiani, E., Mehdian, S., 1990. Efficiency in the commercial banking industry, a production frontier approach. Applied Economics 22, 539-551.

English, M., Grosskopf, S., Hayes, K., Yaisawarng, S., 1993. Output allocative and technical efficiency of banks. Journal of Banking and Finance 17, 349-366.

Fare, R., Grosskopf, S., Norris, M., Zhang, Z., 1994. Productivity growth, technical progress, and efficiency change in industrialized countries. American Economic Review 84 (1), 66-83.

Farrell, M.J., 1957. The measurement of productive efficiency. Journal of Royal Statistical Society, Series A, CXX, 253-281.

Ferrier G.D., Lovell, C.A.K., 1990. Measuring cost fficiency in banking. Journal of Econometrics 46, 229-245.

Fields J.A., Murphy, N.B., Tirtiroglu, D., 1993. An International comparison of scale economies in banking: evidence from Turkey. Journal of Financial Services Research 7, 111-125.

Forsund, F.R., 1992. A comparison of parametric and non-parametric efficiency measures: the case of Norwegian ferries. The Journal of Productivity Analysis 3, 25-43.

Forsund, F.R., 1999. The evolution of DEA-the economics perspective. Working Paper, Departement of Economics, University of Oslo.

Gattoufi, S., Oral, M., Reisman, A., 2001a. Data Envelopment Analysis literature: a bibliography update (1951-2001). Working Paper. Sabanci University, Istanbul, Turkey. Submitted to the Journal of Socio Economic Planning Sciences.

Gattoufi, S., Oral, M., Reisman, A. 2001b. A taxonomy for Data Envelopment Analysis. Working Paper. Sabanci University, Istanbul, Turkey. Submitted to the Journal of Socio Economic Planning Sciences.

Gattoufi, S., Oral, M., Reisman, A., 2001c. Content Analysis of Data Envelopment Analysis literature. Working Paper. Sabanci University, Istanbul, Turkey.

Gattoufi, S., Oral, M., Reisman, A., 2001d. Epistemology of Data Envelopment Analysis. Working Paper. Sabanci University, Istanbul, Turkey

Gattoufi, S., Oral, M., Reisman, A., 2001e. Data Envelopment Analysis literature: classification, bibliography and a systematic identification of voids. Working Paper. Sabanci University, Istanbul, Turkey.

Golany, B., 1988. A note on including ordinal relations among multipliers in Data Envelopment Analysis. Management Science 34, 8, 1029-1033.

Goldberg, D. E., 1989. Genetic algorithms in search, optimization and machine learning Addison-Wesley Publishing Company, Reading, Massachusetts.

Goldfajn, I., Valdes, R.O., 1998. Are currency crises predictable?. European Economic Review 42, 873-885.

Grifell-Tatje, E., Lovell, C.A.K., 1996. Deregulation and productivity decline: the case of Spanish savings banks. European Economic Review 40, 1281-1303.

Griliches, Z., Intriligator, M.D., 1997. Handbook of econometrics, Elsevier Science B.V., Amsterdam, Fourth Edition.

Haag, S.E., Jaska, P.V., 1995. Interpreting inefficiency ratings: an application of bank branch operating efficiencies. Managerial and Decision Economics 16, 7-14.

Hall, M. and Winsten, C., 1959. The ambiguous notion of efficiency. Economic Journal LXIX, 71-86.

Hancock, D., 1986. A Model of the financial firm with imperfect asset and deposit elasticities. Journal of Banking and Finance 10, 37-54.

Hansen, P., Jaumard, B., Savard, G.,1992. New Branch-and-Bound rules for linear bilevel programming. SIAM Journal of Scientific and Statistical Computing, 13, 5, 1194-1217.

Hao, G., Wei, Q.L., Yan, H., 2000. A game theoretical model of DEA efficiency. Journal of the Operational Research Society 2000; 51(11): 1319-29.

Hasan, I., Lozano-Vivas, A., Pastor, J.T., 2000. Cross border performance in European banking. Manuscript- Personnel Communication.

Holland, J. H., 1992. Adaptation in Natural and Artificial Systems. Second Edition, The MIT Press, Cambridge, Massachusetts.

Hunter, W.C., Timme, S.G., 1986. Technical change, organizational form, and the structure of bank productivity. Journal of Money, Credit and Banking 18, 152-166.

Hunter, W.C., Timme, S.G., 1991. Technological change in large U.S. commercial banks. Journal of Business 64, 339-362.

Jeroslow, R.G., 1985. The polynomial hierarchy and a simple model for competitive analysis. Mathematical Programming, 32, 146-164.

Kaminsky, G., Lizondo, S., Reinhart, C.M., 1998. Leading Indicators of Currency Crises. IMF Staff Papers, International Monetary Fund, Washington DC, USA.

Kim, H.S., Park, C.G., Park, K.S., 1999. An application of Data Envelopment Analysis in telephone offices evaluation with partial data. Computers & Operations Research 26, 59-72.

Koopmans, T.C., 1951. An analysis of production as an efficient combination of activities. in T.C. Koopmans, Ed., Activity Analysis of Production and Allocation, Cowles Commission for Research in Economics, Monograph13, Wiley, New York.

Kumar, A., Motwani, J., Reisman, A., 1996. Transfer of Technologies: A Classification of Motivations. The Journal of Technology Transfer 21 (1-2), 34-42.

Lakhal, S., Martel, A., Oral, M., Montreuil, B., 1999. Network companies and competitiveness: a framework for analysis. European Journal of Operational Research 118, 278-294.

Landry, M., Banville, C., Oral, M., 1996. Model legitimisation in operational research. European Journal of Operational Research 92, 443-457.

Leahy, M., Schich, S., Wehinger, G., Pelgrin, F., Thorgeirsson, T., 2001. Contribution of financial systems to growth in OECD countries. Working Paper, Economic Department, OECD, 280

Man, K. F., Tang, K. S., Kwong, S., 1999. Genetic algorithms: concepts and designs. Springer-Verlag, London, England.

McAllister, P.H., McManus, D., 1993. Resolving the scale efficiency puzzle in banking. Journal of Banking and Finance 17, 389-405.

Mehrabian, S., Jahanshahloo, G.R., Alirezaee, M.R., Amin, G.R., 2000. An assurance interval for the non-Archimedean epsilon in DEA models. Operations Research 48(2), 344-347.

Mendeleyev, D.I., 1889. The periodic law of chemical elements (Faraday lectures). Journal of the Chemical Society 1889; 55: 643-56.

Mercan, M., Reisman, A., Emel, A.B., Yolalan, R., (Forthcoming). The Effect of scale and Mode of Ownership on the Financial Performance of the Turkish Banking Sector: Results of DEA. Socio-Economic Planning Sciences.

Mester, L. (1987) Efficient Production of Financial Services: Scale and Scope Economies, Business Review of Federal Reserve Bank of Philadelphia, January/February, 15-25.

Mester, L.J., 1996. A study of bank efficiency taking into account risk-preferences. Journal of Banking and Finance 20, 1025-1045.

Miller, S.M., Noulas, A.G., 1996. The technical efficiency of large bank production. Journal of Banking and Finance 20, 495-509.

Miser, H., 1987. Science and professionalism in Operations Research. Operations Research 35, 314-319.

Molyneux, P., Altunbas, Y., Gardener, E., 1996. Efficiency in European banking, John Wiley & Sons Press, Chikester, England.

Norman, M., Stoker, B., 1991. Data Envelopment Analysis: the assessment of performance. John Wiley & Sons, Chichester, UK.

Noulas, A.G., Ray, S.C., Miller, S.M., 1990. Returns to scale and input substitution for large U.S. banks. Journal of Money, Credit and Banking 22, 94-108.

OECD, 1997. Bank profitability: financial statements of banks.

OECD, 2000. Bank profitability: financial statements of banks.

OECD, 2000. Bank profitability: methodological country notes.

Oral, M , Yolalan, R., 1990. An empirical study on measuring operating efficiency and profitability of bank branches. European Journal of Operational Research 46 (3), 282-94.

Oral, M., Kettani, O., 1993. The facets of the modelling and validation process in operations research. European Journal of Operational Research 66 (2), 216-234.

Oral, M., Çinar, U., Chabchoub, H., 1999. Linking industrial competitiveness and productivity at the firm level. European Journal of Operational Research 118, 271-277.

Oral, M., Kettani, O., Yolalan, R., 1992. An empirical study on analyzing the productivity of bank branches. IIE Transactions 24, 5, 166-176.

Pastor, J., Perez, F., Quesada, J., 1997. Efficiency analysis in banking firms: An international comparison. European Journal of Operational Research 98, 396-408.

Poirier, D.J., 1995. Intermediate statistics and econometrics, MIT Press, Cambridge, Massachusetts.

Reisman, A., 1988a. Finding researchable topics via a taxonomy of a field of knowledge. Operations Research Letters 7 (6).

Reisman, A., 1988b. On alternative strategies for doing research in the management and social sciences. IEEE Transactions on Engineering Management 35 (4), 215-221.

Reisman, A., 1989. A system approach to identifying knowledge voids in problem solving disciplines and professions: a focus on the management sciences. Knowledge in Society: An International Journal of Knowledge Transfer 1 (4), 67-86.

Reisman, A., Xu, X., 1992. On stages of knowledge growth in the management sciences, IEEE Transactions on Management. 39 (2), 119-128.

Reisman, A., 1993. Operations Management: time for meta research. in Prospectives in Operations Management: Essays in Honor of Elwood S. Buffa, Kluwer Academic Publishers, Boston, MA, pp.471-482, January

Reisman, A., 1994. Creativity in MS/OR: Expanding Knowledge by Consolidating Knowledge. Interfaces 24 (3), 91-99.

Reisman, A., Kirschnick, F., 1994. The devolution of OR/MS: implications from a statistical content analysis of papers in flagship journals. Operations Research 42 (4), 577-588.

Reisman, A., Kirschnick, F., 1995. Research strategies used by OR/MS workers as shown by an analysis of papers in flagship journals. Operations Research 43 (5), 731-740.

Reisman, A., Kumar, A., Motwani, J., 1997. Flowshop Scheduling/Sequencing Research: a statistical review of the literature 1952-1994. IEEE Transactions on Engineering Management 44 (3), 316-329.

Reisman, A., Kumar, A., Motwani, J., Cheng, C.G., 1997. Cellular Manufacturing: a statistical review of the literature (1965-1995). Operations Research 45 (4), 508-520

Reisman, A., Kumar, A., Motwani, J., 1999. A Meta Review of game theory publications in the flagship OR/MS journals: 1952-1995. The Journal of Management History.

Reisman, A., Kirschnick, F., 2000. A role for content analysis in research on OR/MS research literature. An invited paper, Omega 28, 247-248.

Reisman, A., Kumar, A., Motwani, J., 2001. A Meta review of game theory publications in the flagship OR/MS journals. Management Decision 39 (2), 147-155.

Reisman, A., Oral, M., Gattoufi, S., 2001. Absolutely positively Operations Research: 50 years of contributions by William Wager Cooper. Working Paper, Sabanci University, Istanbul, Turkey. Submitted to Operations Research.

Reisman, A., Kumar, A., Oral, M., Gattoufi, S., 2002. A statistical comparison of research strategies pursued within three OR/MS subdisciplines and in OR/MS as a whole. Working paper, Sabanci University, Istanbul, Turkey

Reisman, A., Kumar, A., Oral, M., Gattoufi, S., 2002. Comparative epistemology of three OR/MS sub-disciplines: Data Envelopment Analysis, Flow Shop Scheduling, and Cellular Manufacturing. Working Paper, Sabanci University, Istanbul, Turkey.

Resti, A., 1997. Evaluating the cost-efficiency of the Italian banking system: what can be learned from the joint application of parametric and non-parametric techniques. Journal of Banking and Finance 21, 221-250.

Rhoades, S.A., 1993. Efficiency effects of horizontal in-market bank mergers. Journal of Banking and Finance 17, 411-422.

Sarafoglou, N., 1998. The most influential DEA publications: a comment on Seiford. The Journal of Productivity Analysis 9 (3), 279-281.

Seiford, L.M., Thrall, R.M., 1990. Recent development in DEA: the mathematical programming approach to frontier analysis. Journal of Econometrics 46, 7-38.

Seiford, L.M., 1997. A bibliography for Data Envelopment Analysis (1978-1996). Annals of Operations Research 73, 393-438.

Seiford, L.M., 1996. Data Envelopment Analysis: The Evolution of the State of the Art (1978-1995). The Journal of Productivity Analysis 7, 99-137.

Sinuany-Stern, Z., Friedman, L., 1998. DEA and the discriminant analysis of ratios for ranking units. European Journal of Operational Research111 (3), 470-478.

Sinuany-Stern, Z., Mehrez, A., Barboy, A., 1994. Academic departments and efficiency via DEA. Computers and Operations Research 21 (5), 543-546.

Staat, M., 2001. The effect of size on the mean efficiency in DEA: comment. Journal of Productivity Analysis 15, 129-137.

Thanassoulis, E., 1999. Data Envelopment Analysis and its use in banking. INTERFACES 29 (3), 1-13

Thanassoulis, E., 1993. A comparison of regression analysis and data envelopment analysis as alternative methods for performance assessment. Journal of The Operational Research Society 44 (11), 1129-1144.

Üçer, M., Rijkeghem, C.V., Yolalan, R.O., 1999. Leading indicators of currency crises: a brief literature survey and an application to turkey. Yapi Kredi Economic Review 9 (2), 3-24.

Varian, H.R., 1990. Goodness-of-fit in optimizing models. Journal of Econometrics 46 125-140.

Varian, H.R., 1990. Microeconomic analysis, Third Edition, W. W. Norton & Company, New York, 1992.

Vogel, D.R., Weterbe, J.C., 1984. MIS research: a profile of leading journals and universities. Data Base Fall, 3-14.

Wang, Y., Gattoufi, S., Oral , M., Reisman, A., 2001. Data Envelopment Analysis: a justification based on the Weak Axiom of Profit Maximization in the theory of the firm. Working Paper, Sabanci University, Istanbul, Turkey. Submitted to the European Journal of Operational Research.

Weintraub, E.R., 1992. Toward a history of game theory. Duke University Press, Durham, NC.

Wheelock, D.C., Wilson, P.W., 1995. Evaluating the efficiency of commercial banks: does our view of what banks do matter?. Federal Reserve Bank of St.Louis Review 77, 39-52.

Zhang, Y., Bartels, R., 1998. The effect of size on the mean efficiency in DEA with an application to electricity distribution in Australia Sweden and New Zealand. Journal of Productivity Analysis 9, 187-204.