PRODUCTION AND MARKETING DECISIONS IN A DUOPOLISTIC MARKET

by

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ABSTRACT

In this thesis, the production – marketing decisions in a duopolistic market are studied. The defined problem consists of two competitor firms and is based on their pricing, advertising and production decisions. Objectives of both firms are assumed to be profit maximization.

The proposed solution procedure is based on a cyclic solution in which one of the firms solves its own problem at each step of the cycle. It is assumed that firms forecast their competitor's pricing and advertising decisions and use these data as an input for their own model. The termination condition of the cycle is defined as the equilibrium, where none of the firms changes its pricing and advertising policy significantly as a response to its competitor's decisions.

The problem is formulated as a non-linear mixed integer programming (NLMIP) model and two solution methods are employed. First, a commercial package to solve NLMIP problems (GAMS ®) is used. Second, a genetic algorithm (GA) developed to solve the problem is employed. Both of these methods are employed at each step of the solution cycle.

The problem is solved for different parametric conditions using both solution methods. Sixteen initial forecasts, combinations of low and high averages and standard deviations of pricing and advertising forecasts of the competitor, are analyzed. Under each of these forecast cases, 5 parameters*, price elasticity, cross-price elasticity, advertising lagged effect weights, ratio coefficient for competitor's advertising effect,* and *cross-moving demand* are analyzed. The impacts of these parameters on the pricing, advertising, and production decisions of the firms under the sixteen initial forecasts produce similar results. In addition, the demand volumes and the profit values are investigated for these parameters. Price elasticity is observed as the parameter having the greatest impact on decisions, followed by cross-moving demand, advertising lagged effect weights, cross-price elasticity, and ratio coefficient for competitor advertising effect, respectively.

Keeping the firm’s cost structure constant, competitor’s cost structure is changed so as to increase its production cost considerably compared to the previous experiments. Through a set of experiments, the impact of the change in the cost structure on the pricing, advertising and production decisions is investigated. A sensitivity analysis is applied to the logit demand function parameters over a wide range and the results are reported.

ÖZET

Bu tezde, duopolistik pazarda üretim – pazarlama kararları incelenmiştir. Problem, iki firmadan oluşmaktadır ve firmaların fiyat, reklam ve üretim kararlarına dayanmaktadır. Her iki firmanın amacı kâr maksimizasyonu olarak varsayılmıştır.

Önerilen çözüm prosedürü her bir adımında firmaların kendi modellerini çözdükleri döngüsel bir çözüme dayanmaktadır. Firmaların rakiplerinin fiyat ve reklam kararlarını tahmin ettikleri ve bu tahminleri kendi modellerine girdi olarak kullandıkları varsayılmıştır. Döngünün sona erme noktası; firmaların kararlarında, rakiplerinin kararlarına karşılık önemli bir değişiklik yapmadıkları durum olan denge noktası olarak tanımlanmıştır.

Problem, doğrusal olmayan tam sayılı programlama şeklinde formüle edilmiş; iki çözüm metodu uygulanmıştır. İlk olarak, ticari bir paket olan GAMS kullanılmıştır. Ardından, Genetik Algoritma ile çözüm metodu geliştirilmiştir. Her iki metot da, döngünün her adımında uygulanmıştır.

Problem, farklı parametrik koşullar için her iki metotla da çözülmüştür. Rakip firmanın fiyat ve reklam kararları tahminlerinde alçak ve yüksek ortalama ve standart sapmaları içeren on altı farklı başlangıç tahmini incelenmiştir. Bu tahminlerin her biri için beş parametre, *fiyat esnekliği, çapraz fiyat esnekliği, reklam geçmiş dönem etkisi ağırlıkları, rakibin reklam etkisi için oran katsayısı* ve *çapraz hareketli talep*, analiz edilmiştir. Bu analizlerin, on altı başlangıç tahmini için benzer sonuçlar verdiği görülmüştür. Ek olarak, talep ve kâr değerleri incelenmiştir. Fiyat esnekliğinin, kararlar üzerinde en etkili parametre olduğu, bunu sırasıyla çapraz hareketli talebin, reklam geçmiş dönem etkisi ağırlıklarının, çapraz fiyat esnekliğinin ve rakibin reklam etkisi için oran katsayısının izlediği görülmüştür.

Firmanın maliyet yapısı sabit tutularak, rakibin maliyet yapısı dana önceki analizlere oranla yüksek maliyet oluşturacak şekilde değiştirilmiştir. Bir dizi analizle, bu değişimin fiyat, reklam ve üretim kararı üzerindeki etkileri incelenmiştir. Ayrıca, logit talep fonksiyonu parametreleri de geniş bir aralıkta değiştirilerek hassasiyet analizi yapılmış, sonuçlar raporlanmıştır.

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1. INTRODUCTION AND PROBLEM DEFINITION

In today's competitive world, firms need to make their decisions involving more complex marketing and production issues. As the new technologies and new business types are developed, firms always have to keep up with these changes. These improvements and innovations in working methods enforce decision makers to take many issues into consideration. In decision-making, importance of coordination of various departmental actions is increasing with these changes in business methods. For instance, a manufacturing firm has to consider its marketing and production decisions by paying attention on the effects of each decision on the others.

Marketing and production actions have conflicting objectives in general. Marketing is interested in attracting customer's attention and mainly aims to increase total sales. On the other hand, production is mainly concerned with reducing production and inventory costs. The integration of these departmental actions in a cooperative manner has gained importance in last decades.

Each department in a company has different objectives that are specific to its work area. The objectives of marketing are to increase sales or market share, to satisfy customers, and to create loyal customers as a result of customer satisfaction in general. Improvements in marketing aimed to find new methods to reach to customers, to persuade customers to select firm's products / services, to identify and satisfy customer needs etc. However, production's objective is totally different, which is providing simplified production processes and making production easy to manage. New production methods and technologies are being developed continuously to make manufacturing systems easier to control and to work with less failure. These differences in objectives of various departments lead to a non-cooperative working environment and cause sub-optimization of firm's profits. In most manufacturing firms, marketing and production are organizationally separate. Marketing department sets prices, decides advertising activities and other marketing mix variables. Marketing department aims to reach highest demand, highest market share, more profitable customers and similar goals for the firm while making these decisions. The market, by regarding these decisions, creates the demand. After the demand is generated, production department starts with production planning and then manufactures goods. In this case, production department aims to minimize its costs in given conditions. As seen from this procedure, departments usually act separately and try to achieve their individual goals. However, in such a case the most probable result is a sub-optimal firm profit. Conflicts between various departments are widely studied in the literature (see e.g., Kotler (1971), Shapiro (1977))

Shapiro (1977) lists eight general areas where potential conflicts are possible. He defines these areas as “necessary cooperation but potential conflict” and identifies them between marketing and production. He lists these eight areas as: (i) capacity planning and long-range sales forecasting; (ii) production scheduling and short-range sales forecasting; (iii) delivery and physical distribution; (iv) quality assurance; (v) breadth of product line; (vi) cost control; (vii) new product introduction; and (viii) adjunct services such as spare parts inventory support, installation, and repair. In the first two of these conflicting areas, he addresses problems in capacity planning, production scheduling, and sales forecasting. All of these areas are highly related with demand generation. These problems are faced frequently when marketing and production decisions are taken separately. If each department forecasts its demand according to its own decision variables, the possibility of inconsistency increases easily. In his third area, Shapiro points out the different requirements of departments in distribution and location areas. While marketing is interested in customer needs, production department is concerned with its raw material supply and distribution needs as well. In this case, distribution strategy and facility location problems arise. In fourth problem area, customer needs and quality assurance is stated. In today’s world, customer’s desires are to be satisfied with customized products. Marketing aims to supply such products to customers in order to satisfy their needs. On the other hand, this kind of production needs higher flexibility in manufacturing and leads to increasing quality problems because of “employee unfamiliarity and system errors”. For example, Swamidass (1987) points out the importance of including manufacturing managers in decision making procedure. He states that the role of manufacturing managers in strategic decision making affects positively the strategic results in environmental uncertainty, because the manufacturing managers are nearer to the production and more able to consider the abilities of their firms. Since the manufacturing managers can evaluate flexibility better, they should be involved in the strategic decision making in any case. Product variety is another problem area, since marketing department prefers to supply differentiated products to customers. However, a wide product line creates problems for production in raw material inventory planning, production setups etc. Order processing and transportation costs are also increasing with higher product variety. Cost control is the sixth problem area in Shapiro's list since marketing department's needs, such as rapid delivery, high quality, and wider product line; bring cost increases in production. New product introduction is one of key advantages for marketing department. However, production department faces many problems while introducing new products such as requirements of new equipments, new production processes, employee training etc. Some minor changes for marketing may cause major change in needs of manufacturing, and some objective conflicts arise in such cases. Eighth problem area is stated as adjunct services. Shapiro gives example of product installation and says that "factory people tend to view final installation as the final manufacturing operation while marketers view it as a customer service function". He states that these conflicting problems can be solved when both departments are convinced that they serve a higher objective. They have to understand that they "need for a balanced situation but still strongly represent their own interests".

Hausmann et al. (2002) discuss the importance of marketing and production to work together. They argue that both departments have to be involved in the strategic decision making process. They propose a research model which takes the relative importance of marketing and manufacturing within the business unit as inputs. They propose four possible outcomes of the model, namely marketing morale, manufacturing morale, competitive position, and profits. They conduct a survey and conclude that "a successful business strategy implementation will increasingly depend on the marketing / manufacturing ability to work together harmoniously". They also note as a result of empirical study that the business performance is increased when these departments work together for a common goal.

Decision making procedure of marketing and production has to be continued in coordination in order to overcome these conflict areas. Joint decision making is an important opportunity in this case.

Competition is another important point in today's business environment. As reaching and analyzing information becomes easier, the competition between companies increases. This situation makes harder to attract customers since it becomes easier for customers to compare many choices before giving decisions. Increasing competition enforces firms to pay more attention to their decisions and decision making strategies. Inconsistencies between departmental actions affect companies more in a competitive environment than it does in a monopoly or non-competitive market. External factors, such as competitor's decisions, also have to be considered in strategic decision making procedure.

Interest in competition is increasing in literature while the market conditions become more complex over time. Both marketing and production models representing competitive actions are studied. Competitive actions in marketing decisions are related with demand generation and can be evaluated by sales or revenue values. Furthermore, in production side of a company, competition is an important topic as it is in marketing side. Companies consider their production capabilities and cost in order to support their marketing department in competence. Stalk (1992) defines four basic principles of capabilities-based competition:

i) The building blocks of corporate strategy are not products and markets but business processes.

ii) Competitive success depends on transforming a company's key processes into strategic capabilities that consistently provide superior value to the customer.

iii) Companies create these capabilities by making strategic investments in a support infrastructure that links together and transcends traditional Strategic Business Units and functions.

iv) Because capabilities necessarily cut across functions, the champion of a capabilities-based strategy is the CEO.

As he points out, strategic actions are more important in a competitive business environment. He pays attention to business processes rather than products and markets. For various departments of a company, working in cooperation is very important to transform key processes into strategic capabilities. He also notes that strategic investments to install a supporting infrastructure linking strategic business units have to be done to be able to compete on capabilities. He states that "capabilities-based competitors identify their key business processes, manage them centrally, and invest in them heavily, looking for a long-term payback".

1.1. Problem Definition

Objectives of this study are to propose a model for firms competing in a duopolistic market and to investigate how this model gives results in different market conditions. Within the firms, the marketing and production decisions are considered simultaneously, based on a joint decision making model. Marketing decisions are represented in the model as pricing and advertising decisions that affect demand generation. On the other hand, production decisions are considered as production scheduling, inventory and backlog decisions. Since marketing actions are creating demand, these decisions are related with revenue generation. Production decisions are mainly related to cost reductions. However, since all decisions are taken simultaneously in the model, both departments have a common objective, which is profit maximization. In this setting, it is investigated how departments make their decisions jointly.

1.2. Solution Strategy

Since the market structure is assumed to be duopolistic, the solution methodology aims to reach solutions for both of the firms. The solution strategy presented here is based on a cyclic solution method rather than a simultaneous solution procedure. Each firm is assumed to use its competitor's decisions as input for its own problem and then takes decisions in order to reach its own goal. Here, each firm uses its competitor's pricing and advertising expense decisions as inputs for its own model. Both of the firms are assumed to have same objective, which is maximizing their own profits. Since each firm aims to maximize its profits, it interacts with its competitor in demand generation. Production decisions of the competitor do not have any impact on the other firm's objective, so only price and advertising expense information of the other firm are used as inputs. Cycling procedure is followed until both firms do not make any changes in their policies as a response to the other firm's actions. The proposed general solution procedure aims to show whether both of the firm's decisions reach equilibrium in terms of price, advertising expenses and production. This cycle is generated until both of the firm's decisions reach equilibrium. Of course, this cycle is interrupted even if no equilibrium occurs after a certain number of steps.

The model is solved by two different solution methods, and the results are compared. First, a non-linear mixed integer programming (NLMIP) model is formulated. The package program GAMS® is used to solve the NLMIP model and will be referred to from here on as a standard solution tool for solving the NLMIP model as a mathematical programming model. Each output of a solution (in terms of price and advertising expenses) is included as an input for the other firm's problem. Demand function and objective function constitute the non-linear part of the problem. The constraints which are included to deal with the piecewise linear nature of the production cost curve involve binary variables. At each step of the cycle, NLMIP model is solved and results are included into the other firm's model. An initial feasible solution for NLMIP is given at the first step of the cycle. Although inputs of the model about competitor's decisions change, this point is used without any modification at each step in order to provide a consistent solution procedure. Since the model is non-linear, the initial solution affects the results. Thus, the initial solution to be employed is decided upon after a certain number of trail solutions consisting of the first few steps of the cycle.

Second, a solution methodology based on genetic algorithm (GA) is employed. Since solution space of the problem is large and includes local optima, GA is used in addition to GAMS®. Search methodology of GA is defined for the problem, and a cycling solution methodology is applied using GA. This approach is designed to reach optimal or near-optimal solutions of the problem within the general cyclic solution methodology proposed.

GA is used to solve the model of the firms at each step of the cycle. Here, price and advertising expense decisions of a firm's competitor are used as an input for the firm's model as it is used in the solution methodology using GAMS®. In the general procedure, outputs of this solution (price and advertising expense decisions) are included into the competitor firm's model in addition to its own parameters and the model thus obtained is again solved using GA. In conclusion, GA is used to solve individual models of each firm, so it is executed at each step of the cycle. The roles of GAMS® and GA in the general solution procedure are represented in Figure 1.1.

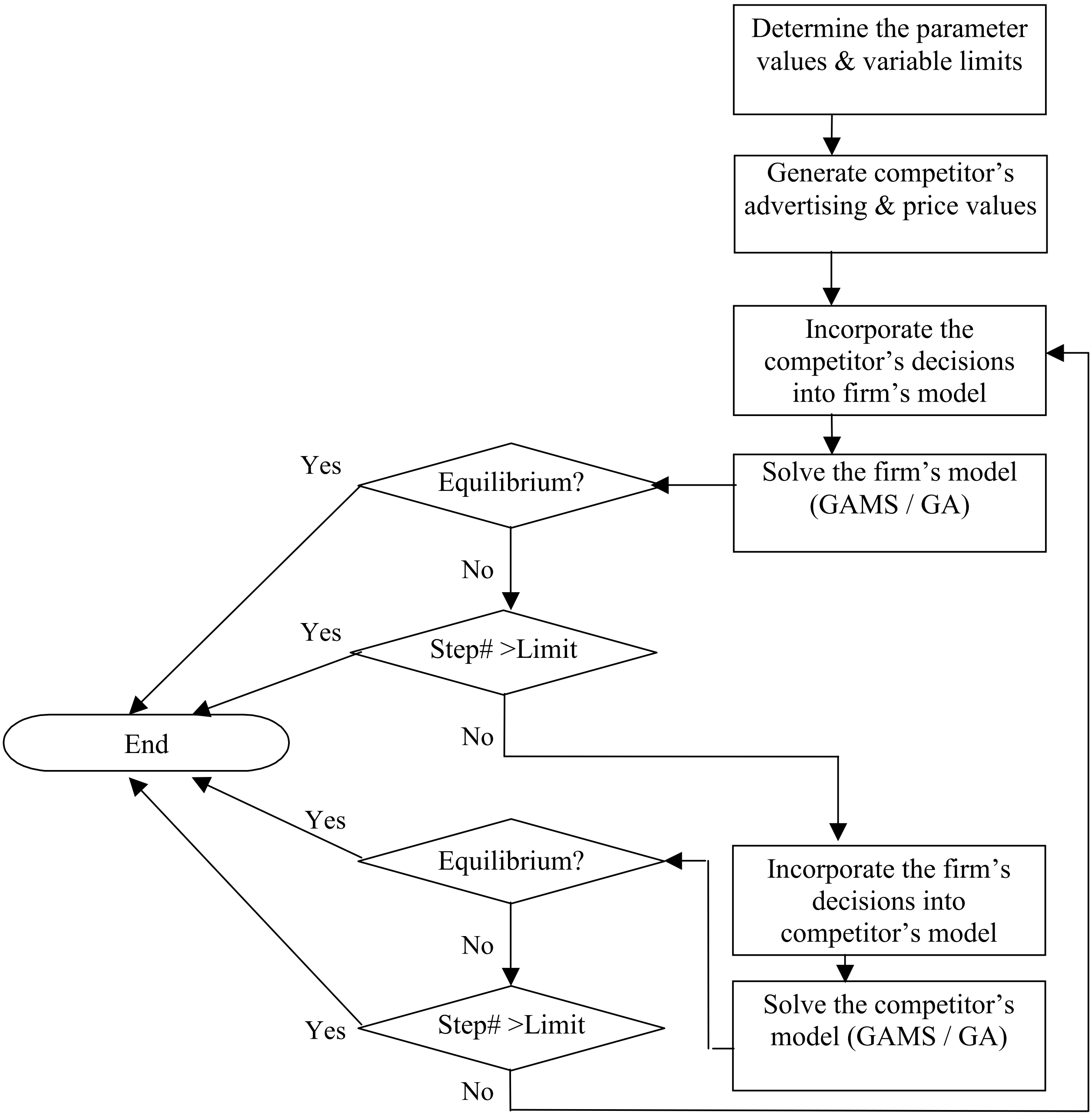


Figure 1.1 The design of the general solution procedure

Chapter 2 includes a comprehensive literature review on the marketing – production models. In this chapter, major studies on marketing and production are explained as well as joint decision making studies.

Chapter 3 introduces the modeling environment, which defines the market structure, the pricing, the advertising, and the structure of firms. The proposed demand function and the mathematical model for the problem are reported in this chapter.

Chapter 4 explains the solution procedure and describes the basic theory and concepts of GA. In addition, it includes the design of experiments.

Chapter 5 reports the results of experiments and includes conclusions based on these results.

2. MARKETING – PRODUCTION MODELS

Modeling the marketing – production decisions has been studied for a long time both in marketing and production literatures. Since these two different operations have a strong relationship, models combining both have received increasing attention. However, models representing each of them separately are developed before joint models.

During development of marketing models, the main aim was to analyse the market, sales and revenue. Market share can also be seen as an issue in these models (see e.g., Karnani (1985), Monahan (1987)). Several decision variables are considered in marketing models, e.g. price, promotion, distribution, quality and so on. Profit or revenue maximization are usually considered as model objectives.

Game theoretic model of Nash states basic concepts for these models. There are three widely used models of duopoly:

1. *Cournot* (based on symmetric quantity competition),
2. *Bertrand* (based on symmetric price competition), and
3. *Stackelberg* (based on asymmetric quantity competition with a first and a second mover).

In the Cournot model (some times called Cournot / Nash duopoly), two firms produce identical goods and make their output decisions independently. Each takes the other's output, and selects its own best output with given the downward sloping market demand curve for the product in question. Total output and market price are represented as equilibria to the "non-cooperative" production game between the two firms.

The Bertrand model assumes that each firm competes on price. In effect, each firm bids for the business of consumers in a market with homogeneous products. In the Stackelburg model, the first mover (leader) tries to take account of the likely output decision of the other firm (the second mover or follower) when making its output decision. In effect, it chooses its profit maximizing output given the "reaction function" of its competitor.

More complicated models of marketing are developed after these basic models. Models defining sales as market share are named as market share attraction models. Kotler (1965) defines a market share attraction model for a duopolistic market. He takes price, advertising expenses and distribution expenses as decision variables of each firm, and makes a simulation study. He also considers growth, seasonality, merchandising and competitive characteristics of the market. He classifies marketing strategy in nine classes as: *(i)* non-adaptive strategy in that firm saves marketing mix levels through the product life cycle; *(ii)* time dependent strategy in which marketing mix decisions are functions of time; *(iii)* competitively adaptive strategy which follows the changes in the marketing mix of competitor and adapts it into firm's decisions; *(iv)* sales-responsive strategy that leads firm to adjust its marketing mix decisions according to past sales; *(v)* profit-responsive strategy where the firm changes its marketing mix decisions "in response to inter-period changes in profits"; *(vi)* completely adaptive strategy in which firm takes changes in its own and its competitor's sales and profits, in time passage, and in its competitor's marketing mix into consideration; *(vii)* diagnostic strategy in which firm first distinguishes possible reasons of changes and then acts; *(viii)* adaptive profit-maximizing strategy where firm uses (or if not actually available predicts) its competitor's marketing mix decisions to take its own decisions aiming profit maximization; and *(ix)* joint profit-maximizing strategy in which the firm convinces its competitor to maximize sum of their profits.

Krishnan and Gupta (1967) study a mathematical model for a duopolistic market with two control variables, price and promotional effort. In their model, each firm knows the other firm's manufacturing costs and the total market size is constant for a given period. Each firm's market share depends on its relative effective promotional effort and the difference between its own and its competitor's price. They also add different effectiveness of promotional effort for each firm. According to these assumptions, they suggested the following model:

$$f_i = \frac{\alpha_i s_i}{(\alpha_1 s_1 + \alpha_2 s_2)} + k(p_1 + p_2 - 2p_i) \quad \text{for } i=1,\ 2; \tag{2.1}$$

where, s_i is the expenditure on promotional effort of firm i and α_i is the coefficient of effectiveness for firm i. In this model, p_i is the price of i^{th} firm, where k is a positive constant and f_i is the market share for firm i. They consider a constant unit cost and formulate the revenue function as:

$$R_i = A\left[\frac{\alpha_i s_i}{(\alpha_1 s_1 + \alpha_2 s_2)} + k(p_1 + p_2 - 2p_i)\right](p_i - c_i) - s_i \tag{2.2}$$

where R_i is revenue for firm i, A is the total constant market in units of product and c_i is constant unit production cost.

Karnani (1985) aims to deduce the practical implications of market share attraction models. He presents a game theoretic analysis of market share attraction models using the solution concept of Nash equilibrium. In an oligopolistic market, he defines market share in his model as:

$$s_i = \frac{a_i p_i^{-\alpha} (\prod_{k=1}^{m} e_{ik}^{\in_k})}{\sum_{j=1}^{n} a_j p_j^{-\alpha} (\prod_{k=1}^{m} e_{jk}^{\in_k})} \tag{2.3}$$

where,

e_{ik} : expenditure on marketing activity k by firm i,

p_i : product price for firm i,

a_i : a measure of consumer preference for firm i,

s_i : market share in terms of revenue for firm i,

n : number of firms in the market,

R : total market size in terms of revenue, and

$\alpha, \theta, \in_1, \ldots, \in_m$: industry specific parameters.

Unlike Karnani (1985), Monahan (1987) assumes a duopolistic market rather than an oligopolistic one and he also excludes effect of price on market share. In addition, he assumes a fixed market potential. His study derives equilibrium allocations and establishes their dependence on factors such as gross profit margins, attraction elasticity, resource budgets, and relative effectiveness effort. He uses

$$S_i(x_i, y_i) = \frac{V_i a_i x_i^{\beta_i}}{a_i x_i^{\beta_i} + b_i y_i^{\beta_i}} \quad (2.4)$$

and

$$\beta_i = \frac{d(a_i x_i^{\beta_i})}{dx_i} \frac{x_i}{a_i x_i^{\beta_i}} \quad (2.5)$$

where $S_i(x_i,y_i)$ is market share of firm X in the i^{th} market when V_i is total market potential and x_i and y_i are marketing efforts for firms X and Y, respectively. a_i and b_i are parameters. β_i is attraction elasticity of effort and it is assumed to be equal for both of the firms. Then, he studies optimal resource allocations of firms in different markets maximizing their profits given limited budget constraint.

Nakanishi and Cooper (1974) suggest a theory for parameter estimations for such big problems.

Vidale and Wolfe (1957) study sales response to advertising and base their study on an empirical study. They suggest three parameters describing the interaction of sales and advertising:

a) Sales Decay Constant: Sales of a product, *S(t),* decreases in a constant rate, if it is not promoted. They represent this situation by

$$S(t) = S(0)e^{-\lambda t} \quad (2.6)$$

b) Saturation Level: This constant, *M*, is described as practical limit of sales that can be generated. It depends not only on the product being promoted but also on the advertising medium used.
c) Response Constant: They define a response constant, *r*, as the sales generated per advertising dollar, when $S = 0$. In general, it is defined as $r(M-S) / M$ when sales are at level *S*.

By using these parameters, the model they developed is as follows:

$$\frac{\delta S}{\delta t} = \frac{rA(t)(M - S)}{M - \lambda S} \quad (2.7)$$

where $A(t)$ is the rate of advertising expenditure. They study the sales response to advertising efforts, the impact of advertising pulse, and the total advertising budget allocation.

Deal (1979) extends Vidale and Wolfe (1957)'s monopolistic model by using the following duopolistic model:

$$\begin{aligned}\dot{x}_1(t) &= -a_1 x_1(t) + b_1 u_1(t)[M - x_1(t) - x_2(t)]/M \\ \dot{x}_2(t) &= -a_2 x_2(t) + b_2 u_2(t)[M - x_1(t) - x_2(t)]/M\end{aligned} \tag{2.8}$$

where, $x_i(t)$ is sales for brand i at time t, $\dot{x}_i(t)$ is the first derivative of sales with respect to time for brand i at time t, $u_i(t)$ is the advertising expenditure for brand i at time t, a_i is the sales decay parameter, b_i is the sales response parameter and M is the total market potential size.

He lets firms to be able to set different objectives and suggest the following model:

$$\max_{u_i} J_i = w_i x_i(t_f)/[x_i(t_f) + x_j(t_f)] + \int_{t_0}^{t_f} \{c_i x_i(t) - u_i^2(t)\} dt \tag{2.9}$$

where i and j are the firms, c_i is the net revenue coefficient, w_i is the weighting factor for the performance index, t_o and t_f are initial and termination times of the planning horizon, respectively. Deal uses the same model for both firms while allowing them to give different importance to the market share and profit maximization.

Lilien and Rangaswamy (1998) state benefits of using decision models as: i) improve consistency of decisions, ii) explore more decision points, iii) assess the relative impact of variables, iv) facilitate group decision making, and v) update subjective mental models.

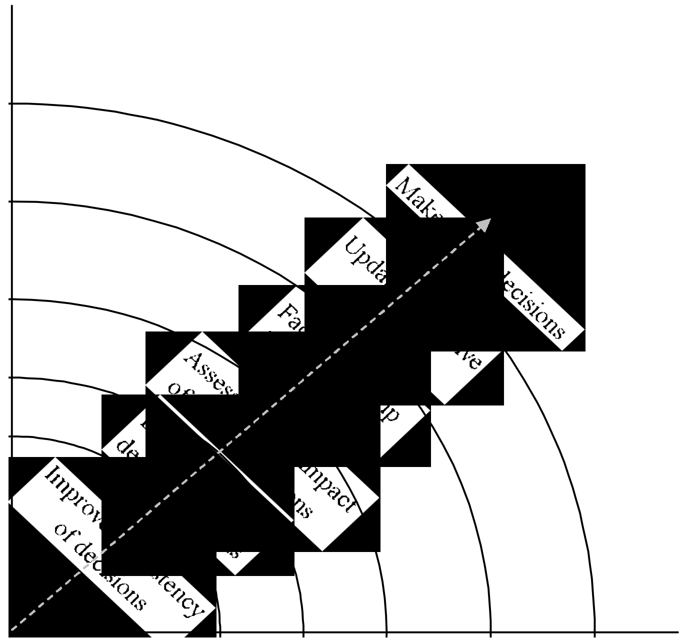


Figure 2.1. Benefits of using decision models

Before they define types of market response models, they address Saunders' (1987) eight phenomena to show the abilities of models. These phenomena are:

P1) Output is zero when input is zero.

P2) The relationship between input and output is linear.

P3) Returns decrease as the scale of input increases (every additional unit of input gives less output than the previous one gave).

P4) Output cannot exceed some level (saturation).

P5) Returns increase as scale of input increases (every additional unit of input gives more output than the previous one gave).

P6) Returns first increase then decrease as input increases (S-shaped return).

P7) Input must exceed some level before it produces any output (threshold).

P8) Beyond some level of input, output declines (supersaturation point).

The following are basic response models and their abilities in handling these phenomena.

i) The linear model: With its form of

$$Y = a + bX \tag{2.10}$$

this model can handle P1, P2, and also P4 and P7 if it is constrained to lie in a range. It does not accommodate P3, P5, and P6.

ii) The power series model: If parameters are selected appropriately, this model can handle P1, P2, P3, P5, P6, and P8 with its form of

$$Y = a + bX + cX^2 + dX^3 + ... \quad (2.11)$$

iii) The fractional root model: It is defined as

$$Y = a + bX^C \quad (2.12)$$

It can handle P1, P2, P3, P4, and P5, depending on the parameters. If *c=1/2*, it is called the square root model. It is called the reciprocal model when $c = 1$. If $a = 0$, then c also means the elasticity of where it is called the reciprocal model when $c = 1$. If $a = 0$, then c also means the elasticity of X on Y.

iv) The semilog model: In the

$$Y = a + b\ln(X) \quad (2.13)$$

form, this model handles P3 and P7 and can represent advertising activities showing decreasing returns when increasing efforts.

v) The exponential model: It has a form of

$$Y = ae^{bX}, \quad (2.14)$$

where $X > 0$. It handles P5 and, if b is negative, P4. It is widely used in price response functions.

vi) The modified exponential model: It is formulated as

$$Y = a(1 - e^{-bX}) + c. \quad (2.15)$$

It has a saturation level of $a+c$ and lower bound of c, and it can handle P3 and P4.

vii) The logistic model: With its form of

$$Y = \frac{a}{(1 + e^{-b+cX})} + d, \quad (2.16)$$

it reaches $a+d$ as saturation level and has a S-shape first with increasing and then decreasing returns.

viii) The Gompertz model: This is less widely used S-shaped function as

$$Y = ab^{cX} + d, \quad (2.17)$$

where $a > 0$, $1 > b > 0$, and $c < 1$. This model handles P1, P4, and P6.

ix) The ADVBUDG model: this model is suggested by Little (1970) as

$$Y = b + (b - a)\frac{X^c}{d + X^c}. \quad (2.18)$$

It is S-shaped for $c > 1$ and concave for $0 < c < 1$. Its lower bound is b where upper bound is a. This model handles P1, P3, P4, and P6 and it is widely used in modeling response to advertising and selling effort.

Lilien and Rangaswamy (1998) refer also to Saunders (1987) when describing three kinds of interactions between marketing mix elements: i) No interaction exists, ii) they are multiplicative, or iii) they are both multiplicative and additive. With separate response functions $f(X_1)$ and $g(X_2)$ for two different marketing mix elements, X_1 and X_2, these can be represented as:

$$Y = af(X_1) + bg(X_2), \tag{2.19}$$

$$Y = af(X_1)g(X_2), \tag{2.20}$$

$$Y = af(X_1) + bg(X_2) + cf(X_1)g(X_2), \tag{2.21}$$

respectively. a, b, and c are parameters in these equations.

Of many issues in marketing decision models, competition plays one of the most important roles. Reaction matrices are used as a modeling approach to competition. In case of duopoly and competence on price and advertising, these matrices are described by Lilien and Rangaswamy (1998). The following is an example, where η's are constant elasticities and a multiplicative function is employed to represent the interaction.

		Firm 2	
		P_2	A_2
Firm 1	P_1	η_{P1P2}	η_{P1A2}
	A_1	η_{A1P2}^{*}	η_{A1A2}

*η_{A1P2} = percentage change in A_1 with a 1% change in P_2.

Figure 2.2. Reaction matrix: two firms; two marketing variables

Elasticities can be estimated with the equations,

$$\log P_1(t) = a_1 + b_1 \log P_2(t) + b_2 \log A_2(t) \tag{2.22}$$

$$\log A_1(t) = a_2 + b_3 \log P_2(t) + b_4 \log A_2(t) \tag{2.23}$$

where b_1 is estimation of η_{P1P2}, b_2 is an estimate of η_{P1A2}. Lambin, Naert, and Bultez (1975) make an application for this matrix and show that firm 2 significantly reacts to the actions of firm 1. They also show that indirect responses based on price-advertising relations are important, and that reaction behaviour is complex.

Lilien and Rangaswamy (1998) describe four different advertising budget decision methods on which Patti and Blasko (1981) and Blasko and Patti (1984) worked and reported statistics.

a) Affordable method: This method represents the idea of a firm should spend whatever funds it has available for advertising. It leads to fluctuating advertising budget and this makes harder to make long-run plans.
b) Percentage of sales method: This method is based on setting the following term's advertising budget as a percentage of sales (either current or anticipated). It provides an affordable budget and encourages the managers to think relationships between advertising cost, selling price, and profit per unit. However, there is logical basis on determining the percentage. What is generally applied when setting percentage is to consider past applications, competitor's applications and costs.
c) Competitive parity method: In this method, firms set their advertising budgets to match competitor's outlays. This may prevent advertising wars but it is illogical when considering advertising reputations, resources, and objectives.
d) Objective and task method: The budget is developed by *i)* defining advertising objectives as specifically as possible; *ii*) determining the tasks needed to achieve these objectives, and *iii*) estimating the costs of these tasks in this method. Its major limitation is that it does not indicate how to choose the objectives and how to evaluate them and to decide whether they are worth the cost of attaining them.

Rao and Miller (1975) define a model for sales response to advertising expenses. They assumed that advertising has an immediate effect and a lagged effect. The lagged effect decays exponentially. Their model is,

$$S_t = c_0 + c_1 A_t + c_1 \lambda A_{t-1} + c_1 \lambda^2 A_{t-2} + \ldots + \eta_t \qquad (2.24)$$

where S_t is market share at time t, A_t is advertising spending at time t, c_0, c_1, λ are constants ($0 < \lambda < 1$), and η_t is random disturbance. With this model, the short run effect of advertising is

$$\frac{dS_t}{dA_t} = c_1 . \tag{2.25}$$

The long run effect is

$$c_1 + c_1\lambda + c_1\lambda^2 + ... = \frac{c_1}{1-\lambda} . \tag{2.26}$$

Little (1970) introduces a model, ADBUDG, for advertising effects. His assumptions are the following:

i) If advertising expenses are zero, sales will decrease to a minimum level.

ii) Independently from how much spent on advertising, the sales are limited by a maximum level, called as saturation point.

iii) There is some advertising rate that will maintain current sales.

iv) An estimate can be made on data analysis or managerial judgment of the effect on share by the end of one period of a 50% increase in advertising over the maintenance level.

Figure 2.3 represents the assumptions stated above.

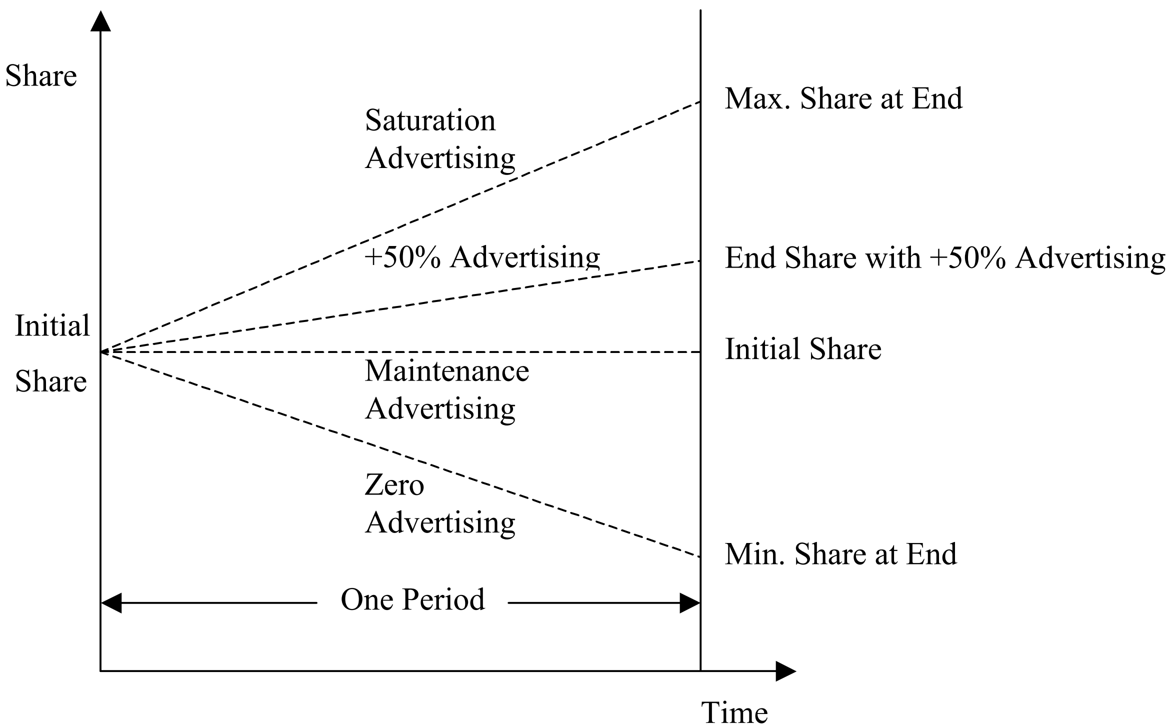


Figure 2.3. Little's assumptions for advertising

He sets

$$Sales = b + (a-b)\frac{adv^c}{d + adv^c} \tag{2.27}$$

where *a, b, c*, and *d* are constants. The curve will be S-shaped if $c > 1$ as shown in Figure 2.4, and concave when $0 < c < 1$.

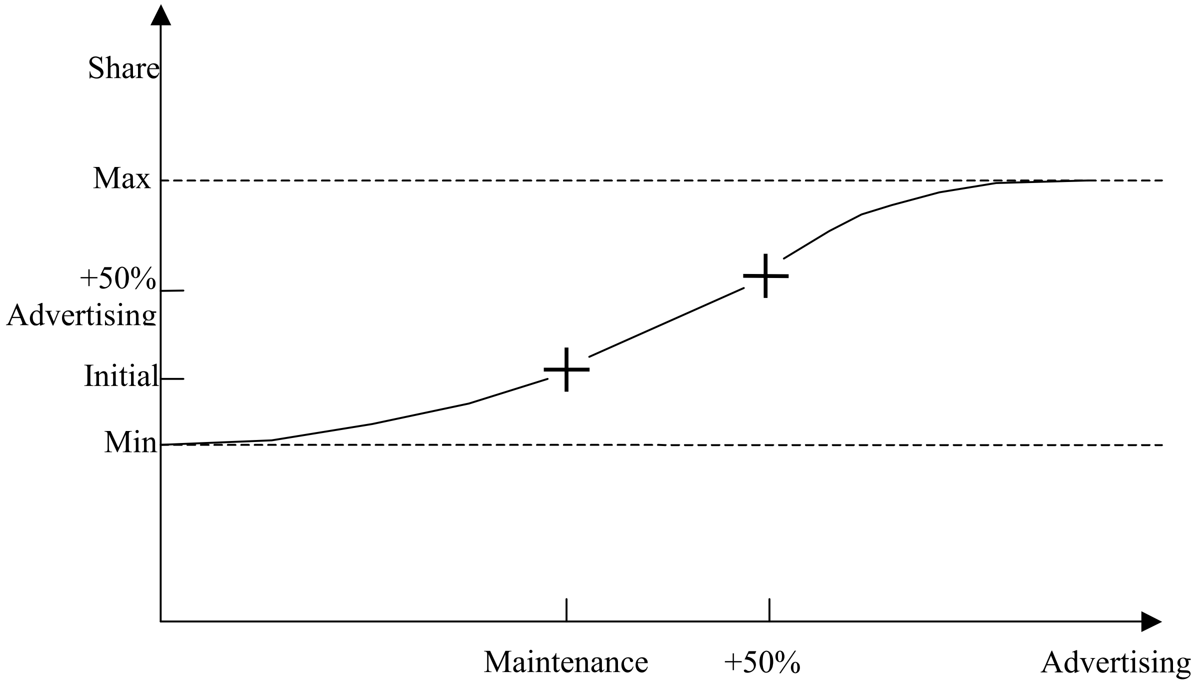


Figure 2.4. Advertising vs. sales graph for Little's model (ADBUDG)

Batra *et al.* (1996) point out that logit or logistic regression models are widely used by direct marketers. They state that direct marketers estimate their logit models on previous campaigns to forecast responds to upcoming marketing action.

Lilien and Little (1976) and Lilien (1979) investigate the factors which condition the marketing of industrial products, in the ADVISOR project. The model defines marketing or advertising spending and based on last year's sales and number of customers the marketing program must reach. The proposed model is,

$$marketing_t = \beta_0 sales_{t-1}^{\beta_1} users^{\beta_2} \prod_i \mathrm{var}_i^{\beta i} \tag{2.28}$$

where marketing is spending on marketing in dollars, sales is sales in dollars (lagged one year), users is number of individuals the marketing program must reach, $\beta_0, \ldots, \beta_I$ are regression coefficients, and var_i represents other variables (stage in life cycle, product plans, etc.).

Gatignon (1993) states the traditional response function generally as

$$y = f_1(X, Z; \beta, \gamma, \varepsilon) \tag{2.29}$$

where

y = a measure of market performance such as product sales,

X = a set of marketing variables (possibly with lagged effects) hypothesized to influence y,

Z = a set of environmental variables (which could contain the marketing activities of competitors) hypothesized to influence y,

β = the response parameters of the marketing variables,

γ = the response parameters of the environmental variables,

ε = disturbance term (possibly with a specific covariance structure due to time dynamics or due to competitive model specification).

He also defines a function, the process function, which explains the parameter vector β of response function as the following.

$$\beta = f_2(X, Z; \alpha, \delta, \nu) \tag{2.30}$$

This equation is developed by three main considerations: *i*) marketing variables, *ii*) environmental conditions, and *iii*) stochastic element. Marketing variables generally affect each other positively. Chances in environmental conditions also affect the response equation, such as changes in effectiveness of marketing variables. The introduction of stochastic component represents the unexpected portion of the market-response parameter.

He defines advertising effect with its both direct and indirect effects on response function. He assumes that advertising affects response indirectly over distribution while also affecting directly. So the distribution and response functions are formulated as the following.

$$D_t = \alpha_0 + \alpha_1 D_{t-1} + \alpha_2 A_t + \varepsilon_t \tag{2.31}$$

$$S_t = \beta_0 + \beta_1 D_t + \beta_2 A_t + \nu_t \tag{2.32}$$

and when they are combined,

$$S_t = \beta_0 + \beta_1\alpha_0 + \beta_1\alpha_1 D_{t-1} + \beta_1\alpha_2 A_t + \beta_2 A_t + \beta_1\varepsilon_t + \nu_t \tag{2.33}$$

where D_t is distribution at time t.

Rao (1993) lays out a framework for reviewing carious pricing models. He analyses static and dynamic models for single products. In dynamic models, he focuses on pricing of product over a time path under different market conditions. He concludes that a skimming policy is optimal when customers expect decreases in price in the future, costs are declining by the time, the word-of-mouth effects are weak; and market is becoming saturated. He also works on multiple-product pricing models and points out the opportunities that bundling creates.

Batra, *et al.* (1996) point out the long-run impact of advertising activities as follows:

> "If we believe that advertising generates a substantial lagged effect on sales, then the impact of advertising campaign may not be known for certain until an unacceptable length of time has passed. For example, an important contribution of a six-month campaign might be its impact twelve months hence. Research has estimated that, at least for frequently purchased nondurable goods, the effect of an advertising exposure can take up to nine months to get dissipated."

As they define, the lagged effect of advertising on sales is crucial. However, they also point out that it is very hard to evaluate this lagged effect, since there may be two problems. First, there might be changes in other factors affecting demand in this lagged effect period and it might be hard to isolate sales change affected by advertising activities. Second, for more timely and accurate information, variables that respond more quickly to advertising input must be sought. The long-run impact of advertising is shown in Figure 2.5.

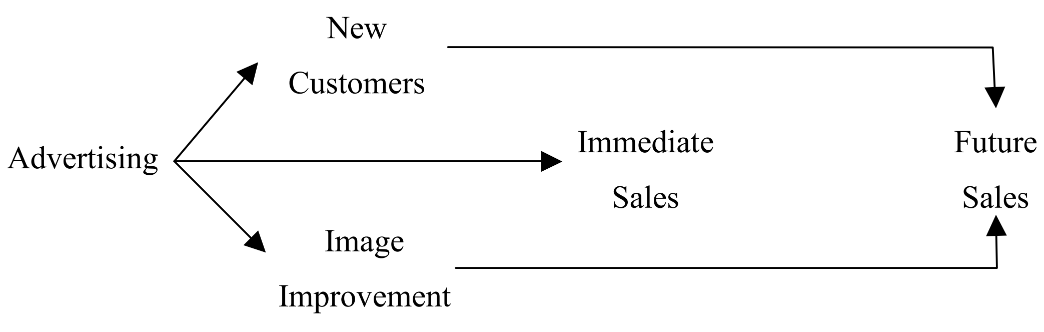


Figure 2.5 Long-run impact of advertising

Eliashberg and Steinberg (1993) work on joint decision making models for marketing-production with convex and concave cost functions. They analyze decentralized versus coordinated decision making. They show that in decentralized case firms lead to sub-optimal profits. They also notice that prices are higher and lot size is smaller in coordinated case than they are in decentralized case. After studying various possible formulations of decentralized versus joint decision making problems, they reach the conclusion that coordinated system performs better.

Ulusoy and Yazgac (1995) develop a marketing-production joint decision making model; dealing with pricing, advertising and production decision over a time period for a multi-product supplying company. They include advertising lagged effects into demand generation function and employ a piece-wise linear cost curve. Golden section search and linear mixed integer programming is used to solve the problem. In advertising budget allocation, dynamic programming is used and results for both fluctuating and smooth demand cases are reported.

Most of the studies about marketing and production decisions are interested in models that are considering only a firm's problems. Competition effects and market conditions about environmental effects are mostly discarded in these studies. On the other hand, when the competition is included into the models, the models are based on marketing decisions. In brief, they do not discuss marketing – production joint decision making within a firm in a competitive market and the models represent marketing decisions' effects on the demand generation. These models exclude the production decisions of a firm.

The absence of models discussing marketing – production joint decision making in a competitive fashion is the main motivation point of this study. The objective is to formulate a model and a solution procedure for this problem.

3. A MATHEMATICAL MODEL FOR THE PROBLEM

In this study, a mathematical model representing both marketing and manufacturing decisions is developed aiming to maximize the firm's profits while its competitor operates under the same objective. These firms compete in a single market, and some assumptions constituting the marketing and production environment are described in the following sections.

3.1. Modeling Environment

3.1.1. Market Structure

Market structure is the description of the buyers and sellers in the market. There are two extreme points of the market structure:

i) Monopoly,
ii) Perfect competition.

In a monopolistic market, the firm sets the price levels. It can also decide in advertising and other marketing mix variables. Here, the firm only considers customers' willingness to pay the determined price under effects of other marketing mix variables' levels. Thus, basically it can be said that in monopoly, the firm takes customers' responses into account while determining the marketing mix decisions.

On the other hand, in a perfect competitive market, no players' actions affect the price level. The demand curve (price vs. quantity) is horizontal in a perfect competition. Here, market conditions are determined by participation of all players' decisions.

In this model, a duopolistic market is considered. Duopoly is a kind of market structure, where both customers' decisions and competition take place.

The market in which the firm serves is assumed to have the following properties:

a) There are two firms in the market, so the market is duopolistic. During the planning horizon no other competitors are allowed to join the market.
b) Potential market size (maximum demand) is constant and static during the planning horizon. Firms can reach this total demand value for the lowest price – maximum advertising case.
c) Production (or sales) capacities of firms are assumed to be equal to potential market in their maxima. This assumption allows increases or decreases in total actual sales in different periods, but increases are limited by a potential market volume. Total demand in a period can be altered by different marketing decisions.
d) Firms compete in the market with a single product.
e) All potential customers have to select the product of one of the firms or none. This means total actual market size (active demand) in a period can decrease compared to the previous period, since customers may not be willing to buy from any of the firms. This situation depends on the pricing and advertising policies of the firms in the period.
f) The product is assumed to be a fast moving consumer good (FMCG). It is assumed that customers may purchase a particular product repeatedly in many consecutive periods, so the market potential may not reach a saturation point during the planning horizon, with the demand becoming zero,.
g) It is assumed that seasonality does not exist in the market for the types of products considered.
h) The products of both firms are exact substitutes. There are no differences between products in terms of quality, usefulness, attractiveness, etc. However, reverse cross-price elasticity affects the willingness of the customers to shift between firms concerning price.

i) Periods are defined as quarters. Total planning horizon is assumed to consist of 12 periods, i.e. 3 years.

3.1.2. Pricing

Pricing decisions are one of more critical decisions in the marketing, since it is very effective on demand. In most cases, an increase in price leads to lower demands. As an example to exceptions, luxury items can be mentioned. However, since FMCG products are under consideration, it is expected that demand decrease whin prices increase and vice versa. In marketing literature, elasticities related with price and demand are defined. In this study, price elasticity and cross-price (as price difference) elasticity are used.

Price elasticity is defined as the change in demand due to a change in price. Mathematical representation of price elasticity is given as,

$$\alpha_1 = \frac{\Delta S}{\Delta P} * \frac{P}{S} \tag{3.1}$$

where P is price, S is sales (demand), and α_1 is the price elasticity.

Cross-price elasticity is described as the change in demand of a product due to a change in price of another product. The relative positions of the products in the market determine cross-price elasticity. Under the assumption of negative price elasticities for both of the products, if products are substitutes, then an increase in the price of one of the products would result in decreased demand of the product and in increased demand of the substitutive product, while other marketing conditions are static. However, if products were complementary, then an increase in the price of one of the products would generate lower demand for both of the products. Hence, if products are substitutes, then cross-product elasticity is expected to be positive. If products were complementary, then negative cross-product elasticity would be representative. Mathematical representation of price difference elasticity is given as,

$$\alpha_c = \frac{\Delta S_x}{\Delta P_y} * \frac{P_y}{S_x} \tag{3.2}$$

where P_y is price of product y, S_x is sales (demand) of product x, and α_c is cross-product elasticity.

In this thesis, pricing decisions are allowed under some assumptions. Price levels are allowed to be in a given interval in the model, where defined elasticities (price and price difference elasticities) affect both firms' decisions.

a) Price elasticity may change at different levels of price since the product may be purchased by different segments of the market. However, it is assumed that there is no segmentation in the market in terms of revenue or other demographic properties and price elasticity does not change over periods, since the total planning horizon is assumed to be sufficiently short.
b) Cross-price elasticity (price difference elasticity) of demand between firms does not change over periods, since the total planning horizon is assumed to be sufficiently short.
c) Customers base their purchasing decision on prices and advertising activities of both of the firms as well as the price difference between the firms. It is assumed that there are two types of customers in the market:
   i) Customers deciding by considering consistently only one of the firms.
   ii) Customers deciding according to the price difference between the prices of the products of the two firms.

   It is assumed that while the price difference decreases (in absolute terms) between products, number of customers considering price difference also decreases. These people decide on one of the firms and are affected only by this firm's price and advertising activities. However, when the price difference increases, more people focus on this difference and decide accordingly. On the other hand, when the prices are equal, then the customer segment (*ii*) becomes part of the customer segment *i*.

3.1.3. Advertising

Under these market structure conditions, firms decide on their individual pricing, advertising and production decisions. Advertising is one of the more important components of marketing mix variables as a result of its effect on demand. Advertising is a wide research topic in the marketing literature on its own. Advertising is used for many objectives in marketing, such as introducing a product or a service, creating and maintaining brand image, informing potential customers about other marketing conditions etc. In this study, advertising decisions are given under the following assumptions:

a) Advertising activities provide a certain increase (in absolute value) in demand. The interaction between a firm's advertising and pricing decisions as well as its competitor's decisions are reflected into the demand function in an additive fashion as described by Saunders (1987).
b) Advertising has a lagged effect on sales. Advertising spending in a period also affects the sales of consecutive *n* periods but its effect decays over the periods. The model of Rao and Miller (1975) concerning lagged effect of advertising is employed hereafter replacing exponential decay by linear decay. Since the product is assumed to be a FMCG, the lagged effect period is taken to be 3 consecutive periods as stated by Batra *et al.* (1996).
c) The competitor firm's advertising activities also affect the number of customers deciding based on the price difference. It is assumed that this advertising also creates product awareness in addition to brand awareness. So, when a firm's price is lower than its competitor's price, the competitor's advertising activities generate more demand for the firm, since this advertising creates product awareness. When the firm's price is higher than its competitor's price, the competitor's advertising affects negatively the firm's demand.
d) Advertising effectiveness is assumed to be increasing up to a certain level of spending and decaying when more advertising expenses are incurred after that level. This is defined as S-shaped curve of advertising spending versus sales. A model representing this situation is suggested by Little's (1970) ADBUDG model. This is defined with an S-shaped advertising expense vs. advertising

affected sales curve and a bell-shaped advertising expense vs. marginal advertising affected sales curve. Graphical representation of S-shaped advertising expense vs. advertising affected sales curve is given in Figure 3.1.

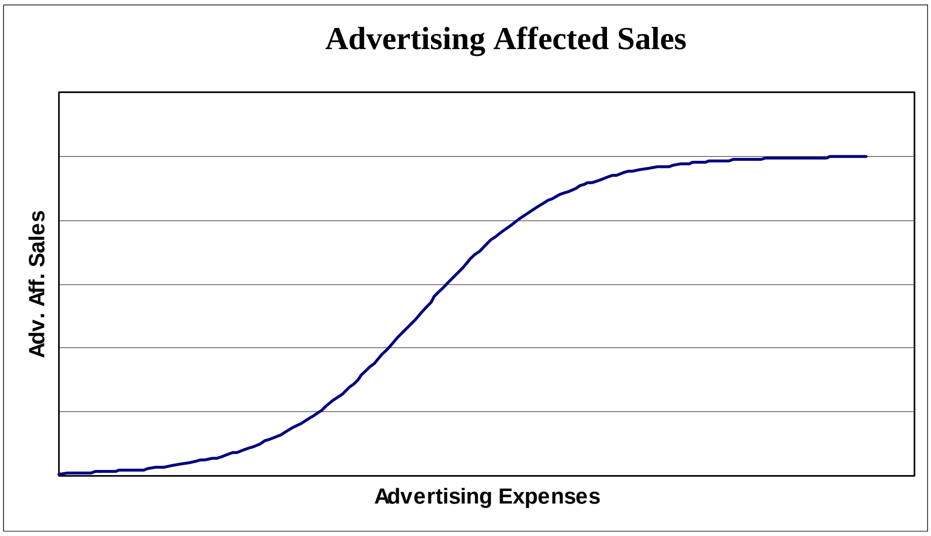


Figure 3.1 Advertising affected sales vs. advertising expenses

Figure 3.2 represents the bell-shaped advertising expense vs. marginal advertising affected sales curve.

The marginal increase in sales versus advertising expenses has also a maximum, which depends on price. While the price increases, the maximum marginal increase in sales versus advertising decreases. The level of advertising expenses needed to reach this maximum also increases when the price increases.

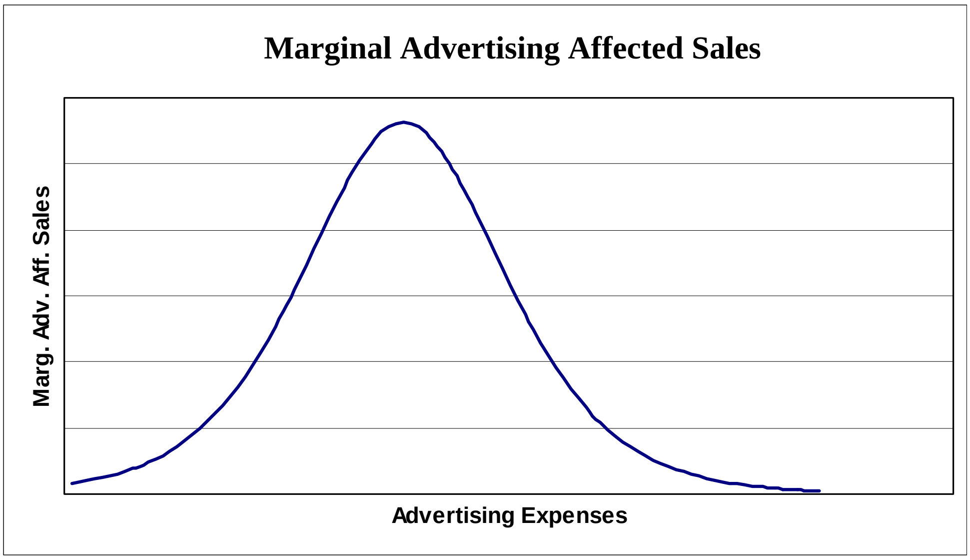


Figure 3.2 Advertising expenses vs. marginal advertising affected sales

The logistic model of the

$$Y = \frac{a}{1 + e^{-(b + cX)}} + d \tag{3.3}$$

is used to represent S-shaped advertising expense vs. advertising affected sales curve. Here, Y represents the advertising affected sales while X represents advertising expense. The parameter a stands for the saturation level, b for the location parameter, and c for the advertising impact factor, where d is a correction parameter. The saturation level, a, defines the maximum additional demand created by advertising activities when the maximum advertising expense is spent. The location parameter, b, shifts the advertising expense vs. marginal advertising affected sales curve. Thus, while b increases, the required advertising expense to reach the maximum advertising affected sales decreases and vice versa. Here, c, the advertising impact factor, affects both the maximum marginal increase of sales versus advertising expenses and the required advertising expenses to reach this maximum level. Therefore, parameter c is used to define the linkage between price and marginal increase of sales versus advertising. The formula

$$c = (\gamma_1 * P^2) + \gamma_2 \tag{3.4}$$

is used to represent this interaction, where P is the price and γ_1 and γ_2 are parameters. The advertising impact factor vs. price graph is given in Figure 3.3.

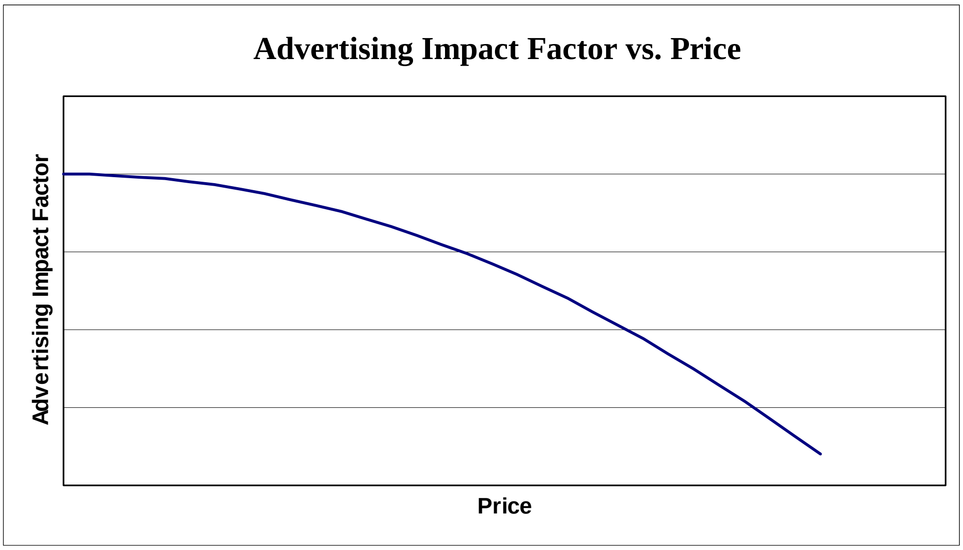


Figure 3.3 Advertising impact factor versus price curve

While *c* increases, the maximum marginal increase of sales increases and required advertising expense to reach this top level decreases. The correction parameter, *d,* allows setting the desired level of advertising affected sales by advertising expenses. For instance, *d* can be used to set the level of advertising affected sales to zero, when no advertising activity is done. With interaction of these parameters, the advertising affected sales and marginal advertising affected sales graphs versus price levels are given in Figure 3.4 and in Figure 3.5, respectively.

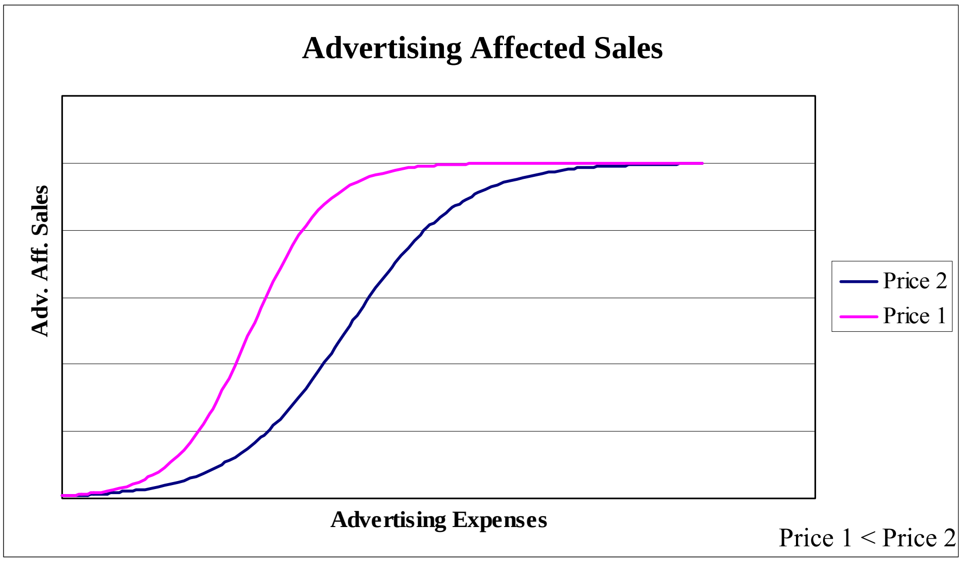


Figure 3.4 Change in advertising affected sales according to price

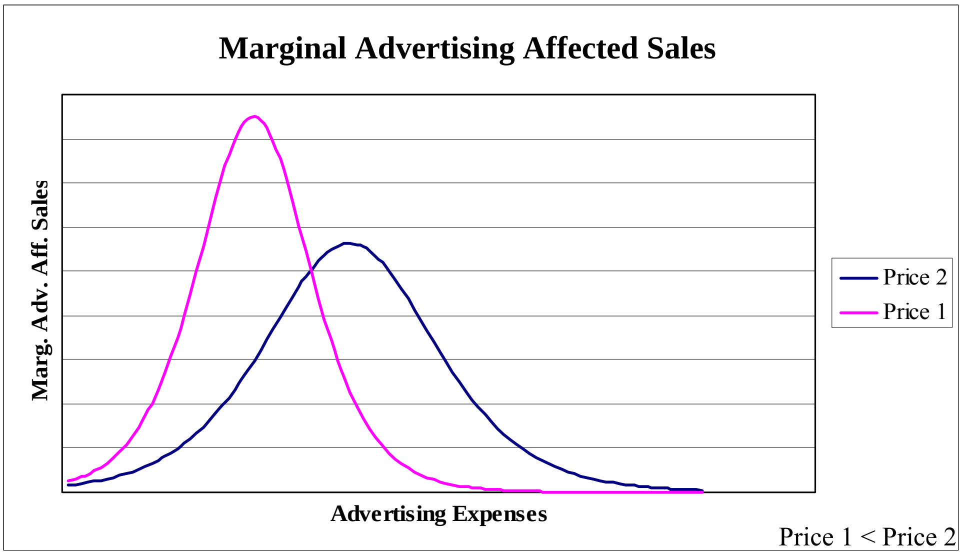


Figure 3.5 Change in marginal advertising affected sales according to price

3.1.4. Structure of Firms

Except from marketing environment affecting firm's decisions, their internal production conditions and managerial decisions have impacts on their strategies. Although it is assumed that both firms compete in the same market with perfect substitute products, they might have different internal conditions, such as manufacturing technologies, labor efficiency etc. The following are the assumptions about the structures of the firms:

a) Firms have similar manufacturing environments but their cost structures are slightly different. This might result from the firms employing different production technologies and / or different manufacturing systems.
b) It is assumed that firms set their total advertising budgets at about 5 – 10 % of their total revenue. Percentage of sales method is described by Lilien and Rangaswamy (1998).
c) It is assumed that economies of scale display the production costs of the firms. This is represented by a piecewise linear function. At certain levels of production volume, production cost per unit decreases, and then stays constant over a range of production values. Figure 3.6 represents this piecewise linear cost function.

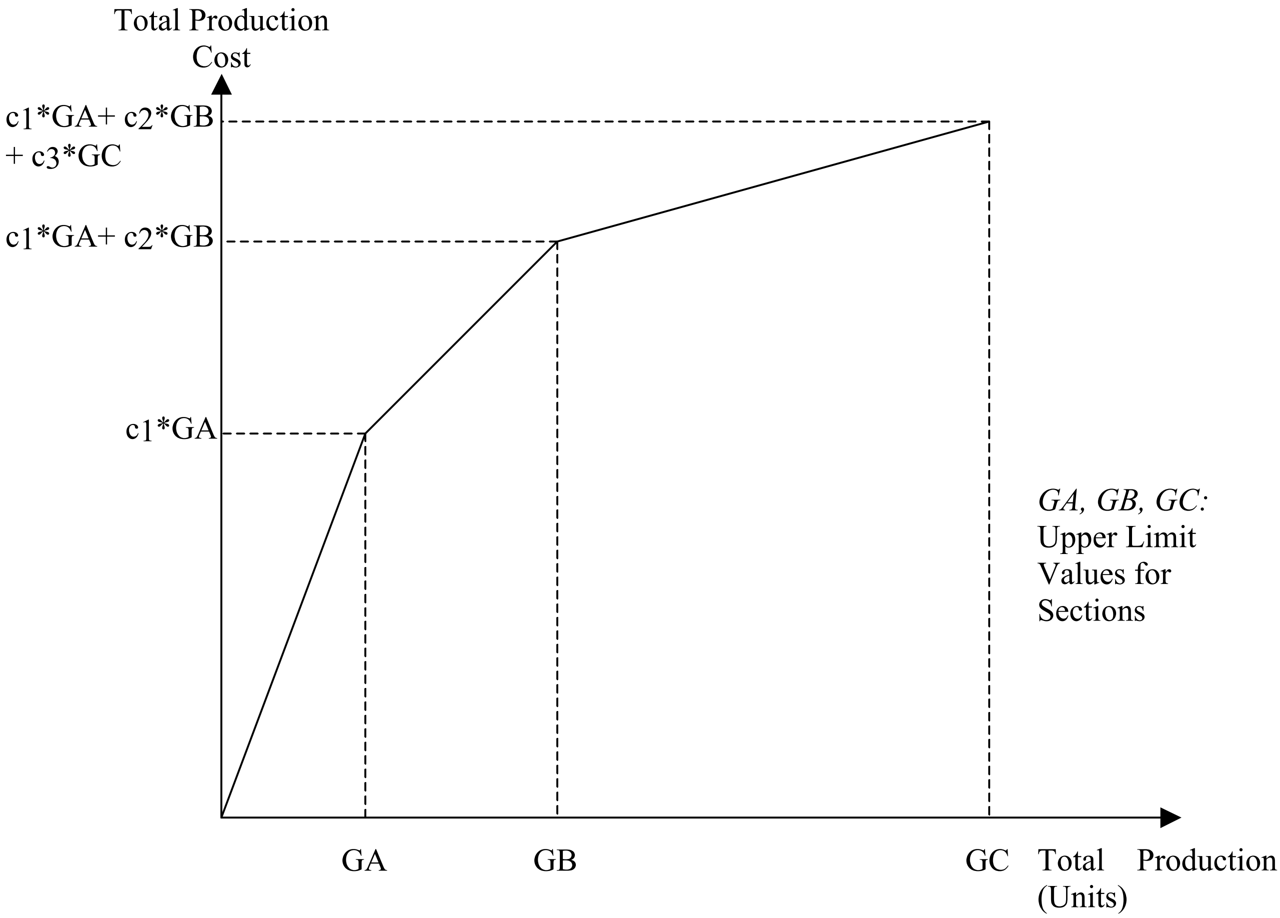


Figure 3.6 Piece-wise linear production cost function

Under environmental and internal conditions assumed, a mathematical model is developed. This model is described in the following section.

3.2. Proposed Model

The mathematical modeling of the problem is developed as a mixed integer non-linear programming model. First, the demand generation function is defined. The demand is assumed to depend on pricing and advertising decisions of the firm, as well as its competitor's pricing and advertising decisions, which are included as parameters. Second, a mathematical model, representing the firm's profit as the objective function and employing production and marketing constraints is introduced. In the following sections the demand function and the mathematical model are described in detail.

3.2.1. Demand Function

The demand generation function consists of three main parts, resulting in the demand value in sum. Each of these parts is related to marketing assumptions.

First, the demand generated by the firm’s own pricing decisions is included into the demand function. Here, price elasticity is employed having effect on demand according to price changes. A constant parameter is used to define the possible demand. With these parameters and price variable, this part of demand function is defined in equation (3.5):

$$\frac{\beta_1}{P_i^{\alpha_1}} \tag{3.5}$$

Here the parameter β_1 is demand parameter due to price. This parameter gives a constant demand value, and this demand value is subject to change according to pricing decisions. The price elasticity is employed as a power of price value. α_1 denotes the price elasticity and it is constant throughout the planning horizon. However, P_i takes different values at each period i.

Second, the advertising affected demand is modeled. In this part, all the assumptions given before are combined. The additional part related with advertising is given in equation (3.6):

$$\sum_{k=1}^{N} w_k * \left(\frac{a}{1+e^{-\left[b+\left(\left[(\gamma_1 * P_i^2)+\gamma_2\right] * A_{i-k+1}\right)\right]}} + d \right) \tag{3.6}$$

In this function, advertising affected demand is created according to pricing and advertising decisions. The function is based on mainly logistic function defined before. The parameters of a, b, and d represent their direct meanings defined in previous sections, where a is the upper limit coefficient for advertising attracted demand, b is the shifting parameter for marginal advertising curve, and d is correction parameter for advertising attracted sales curve. The advertising impact factor, c, is employed as $(\gamma_1 * P^2) + \gamma_2$ and it is related with advertising expenses. The logistic functions are summed up over k, which is the index of lagged effect periods. The number of periods, on which the lagged advertising effect impacts is N. w_k denotes the lagged effect weight corresponding to previous k^{th} period. An important point here is that the index of advertising is defined to be *(i-k+1)* to include each period's advertising to weighted calculation. However, only *i* is used for price index, since it is assumed that even though advertising is done before the current period, its demand generation efficiency depends on active period's price.

The third and the last part of demand function is related with competitor effects. Since the market is assumed to be duopolistic, this part is developed to reflect the impact of the competitor's decisions on the demand for the product of the firm. As it is defined previously, the demand of a firm is affected by price difference between products of firms. This price difference is employed by relative position of price difference with respect to actual price level. Advertising of competitor is also assumed to have impact on the demand for the product of the firm. This impact is formed within the equation by adding more potential customers to segment that is concerning the price difference. Under the stated assumptions, the third part of the demand function is formed and reported as equation (3.7):

$$[\beta_3 + [\beta_2 * (\sum_{k=1}^{N} w_k * [\frac{a}{1+e^{-\left[b+\left(\left[(\gamma_1 * P'^{2}_{i})+\gamma_2\right]*A'_{i-k+1}\right)\right]}} + d])] * \text{sgn}(\frac{P'_i - P_i}{P_i}) * [abs(\frac{P'_i - P_i}{P_i})]^{\alpha_2} \quad (3.7)$$

In the third part of the demand function, division of the price difference to the actual price of the firm's product represents relative price difference impact. This ratio is used in order to express the relative importance of the difference between product prices of firms in comparison to the actual price of the product of the firm. Here, the importance of the price difference can be evaluated by the total number of potential customers taking into consideration this price difference in their buying decision. As the actual price of the product of the firm decreases, the importance of the price difference increases. In such a low-priced product case, the unit price difference draws attention of more customers than it does in a high-priced product case. This situation is a result of increased demand when the price levels are low, since more customers would have willingness to buy the product. In contrast, if the price levels are high, the importance of the price difference diminishes because of the decreasing number of customers deciding to buy the product. Here, α_2 denotes reverse cross-price elasticity. Reverse cross-price elasticity is defined as $(1 / \alpha_c)$. Since reverse cross-price elasticity is employed as a power of the ratio *(price difference / price)*, absolute value is taken in order to avoid the cases where this ratio might take on negative values. A constant proportion of customers affected by competitor's advertising activities are also added to the potential customers concerning price difference. These customers are denoted by β_3 and the constant proportion of competitor advertising affected demand is denoted by β_2. This potential customer base is summed up and then recalculated by relative price conditions.

The total demand function is defined as the addition of these three parts and shaped as equation (3.8):

$$D_i = \frac{\beta_1}{P_i^{\alpha_1}} + \left(\sum_{k=1}^{N} w_k * \left[\frac{a}{\left(1+e^{-\left[b+\left(\left[(\gamma_1 * P_i^2)+\gamma_2\right] * A_{i-k+1}\right)\right]}\right)} + d \right] \right) +$$
$$\left[\left(\beta_3 + \left[\beta_2 * \left(\sum_{k=1}^{N} w_k * \left[\frac{a}{\left(1+e^{-\left[b+\left(\left[(\gamma_1 * P'^{2}_i)+\gamma_2\right] * A'_{i-k+1}\right)\right]}\right)} + d \right] \right) \right] \right) * \operatorname{sgn}\left(\frac{P'_i - P_i}{P_i}\right) * \left[abs\left(\frac{P'_i - P_i}{P_i}\right)\right]^{\alpha_2} \right]$$

(3.8)

where,

D_i : Demand of the product of firm in period *i*,

P_i : Price of the product of the firm in period *i*,

A_{i-k+1} : Advertising expenses of firm in period *(i-k+1),*

P'_i : Price of the product of the competitor in period *i*,

A'_{i-k+1} : Advertising expenses of competitor in period *(i-k+1),*

α_1: Price elasticity of demand,

α_2: Reverse cross-price elasticity of demand,

β_1: Demand parameter due to price,

β_2: Ratio coefficient for competitor advertising effect,

β_3: Cross-moving demand parameter,

γ_1, γ_2: Advertising impact factor parameters,

a: Upper limit coefficient for advertising affected demand,

b: Shifting parameter for marginal advertising curve,

d: Correction parameter for advertising affected sales curve,

w_k: Advertising related sales weight parameter for the k^{th} previous period.

3.2.2. Mathematical Model

In this section, the mathematical modeling of the problem is introduced. The model is an NLMIP model, which involves binary and continuous variables and includes non-linearity in the demand function and the objective function.

The proposed mathematical model of the problem is as follows:

$$\max z = \sum_i P_i.D_i - A_i - h.I_i - g.B_i - \sum_j c_j.X_{ij} \tag{3.9}$$

$$D_i = (3.8) \qquad \text{for } \forall i \tag{3.10}$$

$$D_i = B_i - I_i + I_{i-1} - B_{i-1} + \sum_j X_{ij} \qquad \text{for } \forall i \tag{3.11}$$

$$A_i \le AdvLimit \tag{3.12}$$

$$\sum_i A_i \le AdvBudget \tag{3.13}$$

$$\begin{aligned} X_{i1} &\le GA && \text{for } \forall i \\ X_{i2} &\le GB - GA && \text{for } \forall i \\ X_{i3} &\le GC - GB && \text{for } \forall i \end{aligned} \tag{3.14}$$

$$\begin{aligned} GA.IN_i &\le X_{i1} && \text{for } \forall i \\ (GB - GA).IN_i &\ge X_{i2} && \text{for } \forall i \\ (GB - GA).JN_i &\le X_{i2} && \text{for } \forall i \\ (GC - GB).JN_i &\ge X_{i3} && \text{for } \forall i \end{aligned} \tag{3.15}$$

$$m \le P_i \le m' \qquad \text{for } \forall i \tag{3.16}$$

$$\begin{aligned} A_i, X_{i1}, X_{i2}, X_{i3}, B_i, I_i &\ge 0 && \text{for } \forall i \\ IN_i, JN_i &\in \{0,1\} && \text{for } \forall i \end{aligned} \tag{3.17}$$

where,

X_{ij} : Number of units produced in the i^{th} period in the j^{th} section of production cost curve

B_i : Backlog level at the end of period i

I_i : Inventory level at the end of period i

h: Inventory holding cost per unit for one period

g: Backlog cost per unit for one period

c_j: Cost of production in the j^{th} section of production curve per unit

m, m' : Lower and upper limits for price

GA, GB, GC: Upper limit values for sections in the production cost curve

AdvLimit: Maximum allowed advertising expense in a period
AdvBudget: Total advertising budget for all planning horizon
IN_i, JN_i : Binary variables

In equation (3.9), the objective function of the model is defined. Profit maximization is considered as goal of the firms, so the objective function consists of revenue and cost parts. Revenue of a firm is generated by its sales, so total revenue during planning horizon is represented as

$$\sum_i P_i D_i \tag{3.18}$$

where P_i is the price valid in period i, and D_i is the demand generated in period i.

Cost part of the objective function is composed of advertising, production, inventory, and backlog costs. These four cost components are included in objective function as follows:

$$A_i + hI_i + gB_i + \sum_j c_j X_{ij} \tag{3.19}$$

Since the demand generation function is very complex, it is included into the mathematical model as a constraint for the sake of simplicity and represented in equation (3.10). The constraint supplied in equation (3.11) ensures that production is balanced with inventory, backlog, and demand. Demand generation plays an important role in production balancing. Whole planning horizon's production decisions are given on the base of demand. Inventory and backlog quantities are also related with demand and production decisions. Inventory can be defined as excess quantity of goods produced at the end of a period. Backlog is the demand that is not met at the end of a period and is postponed to next period to be met. This constraint ensures that the total demand either will be met in a period, if excess production is made this will be counted as inventory, or will be backlogged to be met in the following periods. It also ensures that backlogged demand and stocked goods in previous periods are carried to this period.

Equation (3.12) ensures that advertising expense in a period does not exceed periodic limitation. The next constraint, (3.13), limits total advertising expenses during the planning horizon and prevents exceeding expenses. These two constraints are double constraints and they control the distribution of advertising expenses over the planning horizon.

Equation sets (3.14) and (3.15) are piece-wise linear production cost constraints. First set, (3.14) enforces each segment's production volume is between segment intervals end points. The constraint sets (3.15) ensure that the first, second, and third sections of the production cost functions will be filled consequently and none of the sections will be used before the previous one is filled totally.

Next constraint (3.16) defines the upper and lower limits for pricing decisions. Since the price is assumed to be in an interval, this equation is used.

Last constraint set, (3.17), ensures that all variables are non-negative and two of them are binary.

4. SOLUTION PROCEDURE AND EXPERIMENT DESIGN

4.1. Solution Procedure

In this study, the proposed model is solved iteratively in order to investigate its ability to represent the real world conditions and to analyze how it reflects different marketing conditions. The solution procedure is described in the following sections in detail. However, before it is solved, experimental market environment is designed. Parameters both taking place in the model and in the demand function are determined. All values assigned to limits and parameters are evaluated and designed to create a representative market defined in market environment. Although these values are not taken from an empirical study, they are selected from a large set of possible values, which are best representing assumed market conditions. First, upper and lower limits of price level are determined. After that, primary parameters, including *demand parameter due to price, ratio coefficient for competitor advertising effect, cross-moving demand parameter, advertising impact factor parameters, upper limit coefficient for advertising attracted demand, shifting parameter for marginal advertising curve, correction parameter for advertising attracted sales curve,* and *advertising related sales weight parameter* are designated. For the basic market environment *price elasticity* is assumed to be 0.9 and *reverse cross-price elasticity* is assumed to be 1. By the determination of these parameters, total advertising budget and periodic maximum allowed advertising expenses are evaluated. By using percentage of sales method described, total advertising budget is set to 5 – 10% of revenue. Since marginal additional demand created by advertising approaches to zero while advertising expenses increase, periodic advertising level approaches to a stable level, which does not provide any meaningful increase in revenue for increasing advertising expense. This saturation level is taken as maximum allowed periodic advertising expense limit. Since this limit exceeds when summed up over the planning horizon in the maximum expense case, double constraint condition on advertising expenses is satisfied. After creating representative market environment for proposed model, the solution procedure is applied. Within the solution procedure, GAMS and GA is used as a part of procedure. In section 4.1.1, the theory and concepts of GA is introduced. In the following sections, the implementation of GA for the purposes of this study is described.

4.1.1. Genetic Algorithms

GA is a solution procedure based on natural genetic procedures. It follows the evolutionary steps of Darwinian Principle of Evolution. It includes survival of the fittest and concepts of biological genetic population events. It can handle solutions of NP-hard and large-scale combinatorial optimization problems in an acceptable amount of time. GA search differs from other search methods with its search concept on populations in some ways (Goldberg, 1989):

- GAs work with a coding of the parameter set, not the parameters themselves.
- GAs search from a population of points, not a single point.
- GAs use payoff (objective function) information, not the derivatives or other auxiliary knowledge.
- GAs use probabilistic transition rules, not deterministic rules.

Since GAs use a coding for parameters, it enables the search to continue without constraints and it excludes the possible barriers that can interrupt the search, such as non-existing derivatives. In many search techniques, search follows a path from one point to a next one by some rules. However, GAs search from multi-point to multi-point. This property reduces the risk of being trapped in a local optimum point when compared to other search methods. GAs do not need to calculus-based calculation techniques to continue searching. It only requires a payoff function (objective function) to search for a better solution. Finally, GAs use random choice as a tool to guide a search toward regions of the search space with likely improvement (Goldberg, 1989). These differences constitute the robustness and advantages of GAs over other search techniques.

The key elements of GAs are based on the natural genetics terminology. The basic concepts in GAs are as follows:

i) Chromosome: A predefined structure representing a solution of the problem.
ii) Population: Population is the set of feasible solutions.
iii) Generation: Generation is a step in GA consisting of a population.
iv) Gene: Smallest part of a chromosome representing a part of a solution.
v) Fitness Value: Relative closeness to the stated goal.

The chromosomes represent individuals in a population and are made of genes. Each chromosome corresponds to a solution for the problem and the search is based on these chromosomes. Encoding of chromosomes is an essential issue at this point. Chromosomes can be encoded in two ways: (i) direct representation; (ii) indirect representation. In direct representation, chromosomes include direct representations of solutions. Indirect representation is employed by using meaningful representatives for solutions in chromosomes. *Random key* is a kind of indirect representation, which uses random numbers to encode a solution. This representation eliminates the offspring feasibility problem by avoiding the issue (Bean, 1992). Genes are smallest parts of chromosomes and they denote a parameter or a variable. Genotype is defined as the total genetic code of an individual like it is in natural genetics. Phenotype is the decoded meaning of a genotype. In GAs, phenotypes represent a solution. However, to clarify a genotype, decoding has to be done. Decoding is the transformation of information stored in genotype of a chromosome into a solution alternative. Populations are solutions sets and composed of chromosomes. New generations of individuals are created from old generation's individuals by some GA operations. There are three operators employed in GAs (Goldberg, 1989):

i) Reproduction
ii) Crossover
iii) Mutation

In reproduction, the individuals having relatively better fitness values are cloned into new generation. This is based on the survival of the best theory of natural genetics. Crossover is based on the rationale of mating two individuals to produce off-springs. In crossover, two parents are selected from old population, recombined to form two new individuals, and one of the results is discarded at random (Back, 1996). The selection of parents from old generation can be done randomly or using roulette wheel technique. In roulette wheel technique, individuals are assigned selection probabilities according to their fitness function values. Thus, better fitness function value means higher probability to be chosen for crossover. In crossover, the selected gene segments of parent chromosomes are combined. The segments can be defined by using cutting points. In *n*-point crossover, *n* points are selected in a chromosome and parent chromosomes are divided into segments from these points. Then, each segment of new individuals are taken from these segments. In another type of crossover technique, uniform crossover, each gene of an offspring is cloned from randomly one of its parents. In figure 4.1 and figure 4.2, the crossover methods are represented.

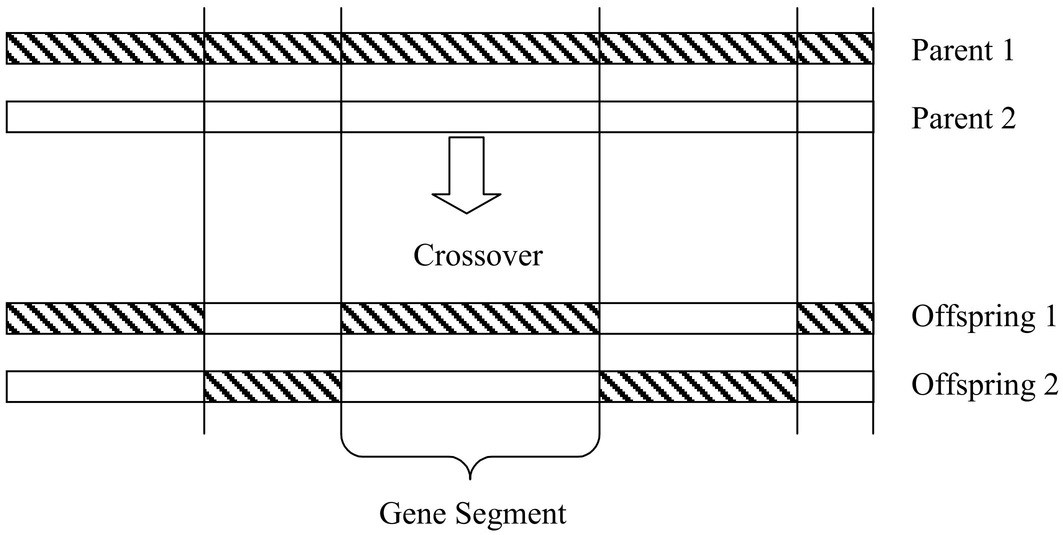


Figure 4.1 *n*-point crossover

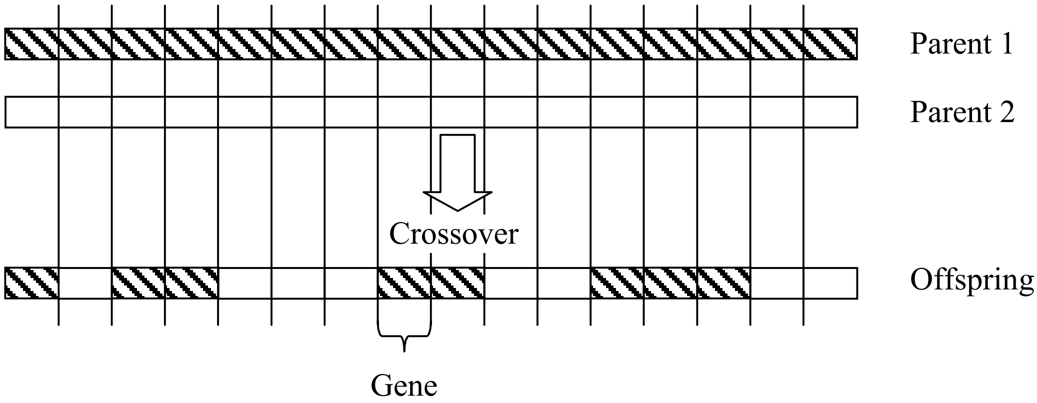


Figure 4.2 Uniform crossover

Another type of GA operator is mutation, which is based on altering some genes in a chromosome during new generation production. Mutation protects the search from losing some potentially useful material (Goldberg, 1989). Mutation creates some minor changes in chromosomes. This operation can be basically in three methods:

i) Swap mutation
ii) Bit mutation
iii) Inverse mutation

In swap mutation, two genes are interchanged in a chromosome. Bit mutation is altering a gene and assigning a different value to that gene. For instance, if chromosome representation is binary, bit mutation changes value of a gene from 1 to 0 or vice versa. Inverse mutation is performed on a selected gene segment in the chromosome. If binary representation is used, inverse mutation can be applied as bit mutation but on a gene segment rather than on a single gene.

In GAs, reproduction and crossover are mainly used to search the solution space. Mutation is a secondary operation, and it is used as an insurance policy against premature loss of important notions (Goldberg, 1989).

4.1.2. The Design of Genetic Algorithm-Based Solution Procedure

As defined before, GA is used to solve individual models of firms. Here, the details of the GA method used are described.

Chromosome Structure

The chromosome, as defined before, is used to represent a feasible solution of the problem in GAs. So, chromosome structure is important not only with its role but also with its usefulness in GA operations. In this study, chromosomes are defined as follows:

1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20	21	22	23	24	25	26	27	28	29	30	31	32	33	34	35	36

Price Decisions Advertising Decisions Production Decisions

Figure 4.3 The chromosome structure

As shown in Figure 4.3, each chromosome has 36 genes. The first 12 genes of the chromosome represent the price decisions; the second 12 genes represent the advertising decisions, while the last 12 genes represent the production decisions. Each group is represented by 12 genes, because the planning horizon is assumed to be 12 periods. Finally, each gene in each gene group represents the decision value of corresponding period for the corresponding decision variable type. All the genes of the chromosome are random numbers between 0 and 1, so *random key* is used to represent the solution. This allows chromosome to represent a feasible solution even after the GA operations are applied. However, these 3 groups of random numbers represent values in 3 different intervals. Since first group represents price levels, its transformation to actual numbers differs from other groups. The second and third groups represent also different decisions, so all these three have different actual value intervals even though all genes are random numbers between 0 and 1. The transformation of the gene groups into actual decision values is performed as introduced below:

For the first group (price) decisions:

(Genes between 1 and 12)

$Price\ level\ of\ period\ i = [(P_{max} - P_{min}) * gene\ (i)] + P_{min}$

For the second group (advertising) decisions:

(Genes between 13 and 24)

*Advertising level of period i = [(Periodical advertising limit / Advertising increment) * gene (i + 12)] * Advertising increment*

Here, advertising increment is defined as a constant value and it is used as unit increment level in advertising expenses. For example, an advertising expense of *50000* is represented by *(5*10000)*, where *10000* is advertising increment value and the multiplier *5* is obtained by evaluating the following expression: *[(Periodical advertising limit / Advertising increment) * gene (i + 12)]*

For the third group (production) decisions:

(Genes between 25 and 36)

*Production level of period i = (Periodical production capacity / 2) * gene (i + 24)*

Even though all these transformations of random numbers result in meaningful values within desired intervals, two types of infeasibilities can occur. One of them may occur if total advertising expense over 12 periods exceeds total advertising budget. If such a case occurs, the feasible values are obtained by the following method. First, the exceeding amount of advertising expenses is calculated. This value is denoted by ξ. After that, ξ is divided by advertising increment and result is represented by θ. If θ is divisible by 12 then [$(\theta / 12)$ * *(advertising increment)*] is subtracted from each period's advertising expenses. This calculation reduces the advertising expenses of each period by an equal amount such that the total advertising expense equals the total advertising budget. If $(\theta / 12)$ is not an integer, then all advertising expenses are reduced by multiplying by the factor *(total advertising budget / total advertising expense)*. In this case, while the general advertising expense structure is preserved, the increment structure is changed.

Another type of possible infeasibility may be caused by different total demand and total production values. Since no inventory or backlog is allowed at the end of the planning horizon, total demand must be equal to total production. If transformation of production values does not meet this condition, then one of two situations might have occurred. As the first case, total production may exceed total demand. In this case, total production is normalized to the total demand by subtracting average periodical difference [*(the difference between total production from total demand) / 12*] from each period's production. If production level of one of the period equals to a negative value after this calculation, then this chromosome is discarded and another chromosome is generated instead. In the second case, total production can be less than total demand. In this case, total production is again normalized to the total demand with the same procedure described above. The only difference here is that average periodical difference is added to each period's production. If the production level of one of the periods exceeds total periodical production capacity, this chromosome is discarded and another chromosome is generated instead.

First Generation

In the first step of GA, *n* chromosomes are generated, where *n* is the population size. Thirty six random numbers between 0 and 1 are assigned to each gene of the chromosomes and feasibility operations (repairs) are applied, if necessary. If this run is not the first step of the cycle for any of the firms, the best chromosome generated in the previous step of the cycle for the current firm is included in the population and *n-1* more new chromosomes are generated.

Genetic Operations and Following Generations

After the first generation, *m-1* generations are created, where *m* is the desired number of generations. In this part of the process, three operations are used:

i) Elitist chromosomes (Reproduction),
ii) Mutation,
iii) Crossover.

At the beginning of each generation after the first one, desired number of chromosomes are directly reproduced in the current generation. These chromosomes are selected from previous generations according to their fitness values. These chromosomes are named as *Elitist Chromosomes*. After this application, (*n – number of elitist chromosomes*) chromosomes are generated with one of the procedures, mutation or crossover. In generation of these chromosomes, a random number is generated for each chromosome. The decision of applying mutation, crossover, or reproducing without any change is given. If the decision is to apply mutation to generate one new chromosome, then a random chromosome is selected from previous generation's population, again according to their fitness values. The selection possibility of each chromosome is determined according to its fitness value; so firstly, fitness value of each chromosome is normalized to satisfy summation of them to be equal to 1. After selection of chromosome to mutate, desired number of genes are selected randomly. These genes' values are replaced by new random numbers. Then, feasibility operations (in terms of total advertising budget, and demand – production equality) are applied. Figure 4.4 represents the mutation operation. In this figure, genes 9, 17, and 30 of selected chromosome are decided to be mutated, and new random numbers are assigned to these genes during reproduction process. All other genes are copied with same values.

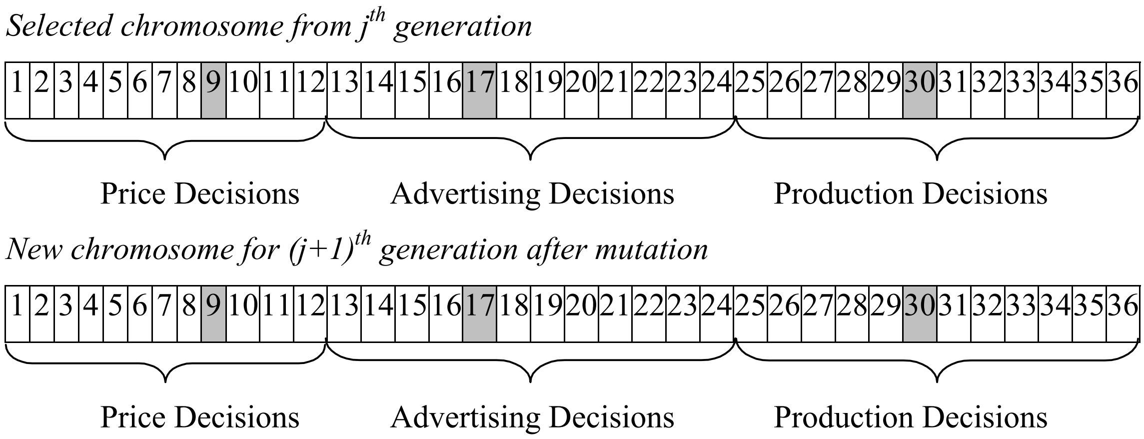


Figure 4.4 Application of mutation

If crossover operation is decided to be used to generate a new chromosome, then two random chromosomes are selected from previous population according to their fitness values. Both of these two chromosomes are divided into desired number of segments from random cut points. Here, the number of segments and cut points are same for both chromosomes. After that, first segment of first chromosome is taken. Then, second segment of the second chromosome is included. Next, the third segment of the first chromosome is added and this procedure is processed until all 36 genes are generated. Figure 4.5 represents the crossover operation. In this figure, each of the selected chromosomes are divided in to eight segments from seven cut points which are

10, 16, 21, 24, 29, 32, and 35. The new chromosome is formed by adding first, third, fifth and seventh segments of first selected chromosome before second, fourth, sixth and eighth segments of second selected chromosome, respectively.

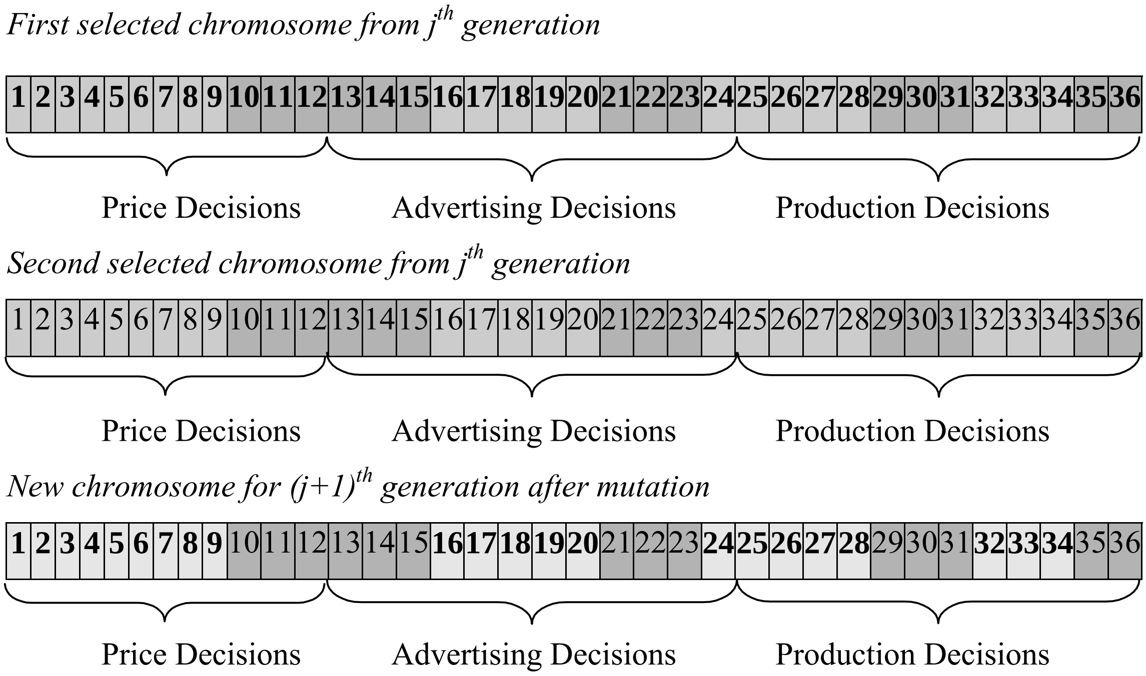


Figure 4.5 Application of crossover

After desired number of chromosomes are generated (new population is determined), the procedure repeats itself until desired number of generations are created.

When all generations are created, the best fitness-valued-chromosome of the last generation is taken and its price and advertising values are incorporated into the other firm's model. At the end of the second cycle and the following cycles, an equilibrium check is done and decision of continuing with the next cycle is taken. Figure 4.6 shows the flow of GA described.

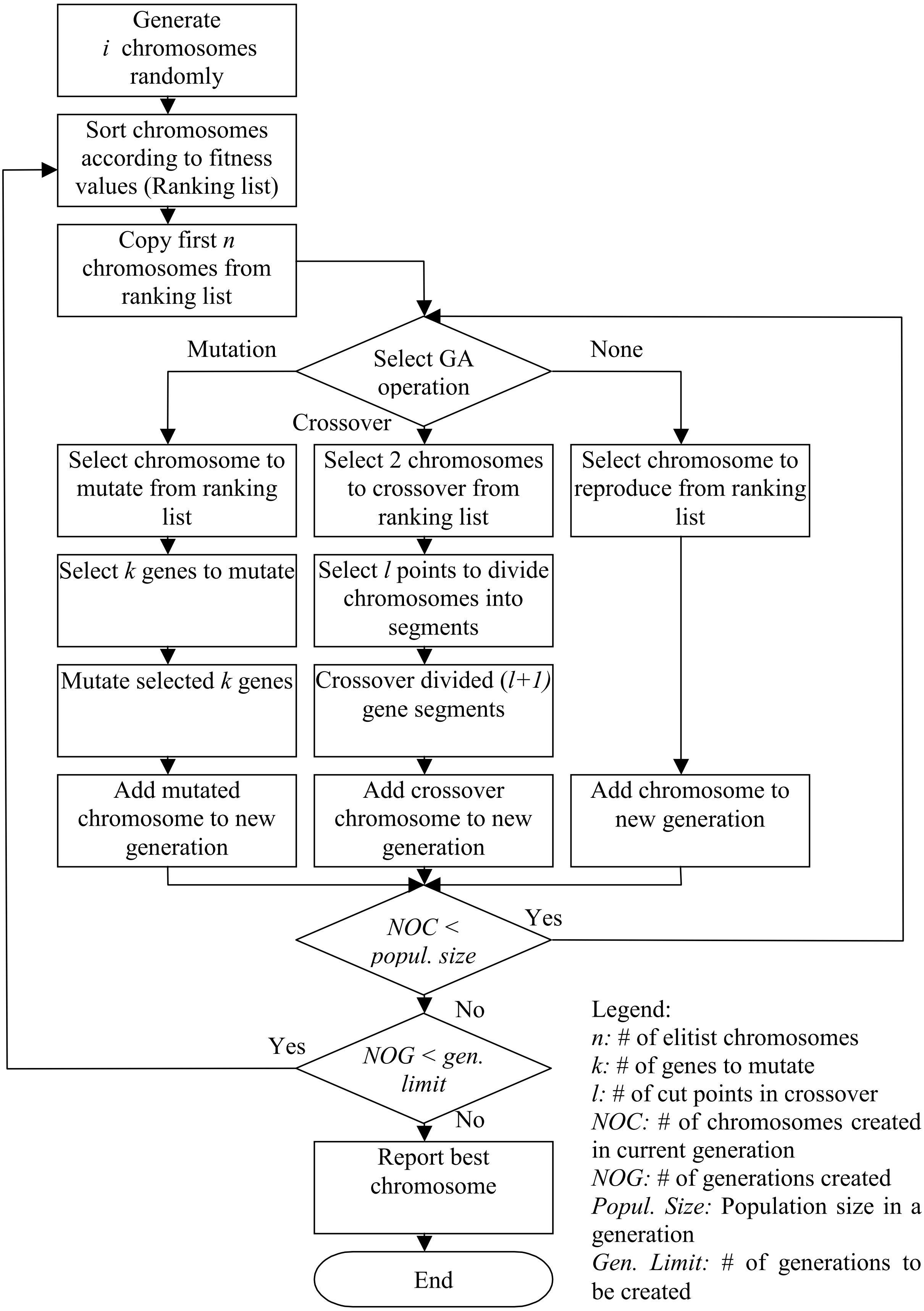


Figure 4.6 Flow of GA

4.2. Experimental Design

In this thesis, it is aimed to show how firms behave in terms of price, advertising, and production decisions under stated assumptions. The behaviour means whether firms reach equilibrium points in terms of their decision variables and if yes, at what levels these points are. The levels of equilibrium points for price, advertising, and production may also be affected by different conditions. In order to clarify how different conditions affect the equilibrium, many experiments with different parameters are designed. The following parameters are examined for their effects on the equilibrium points.

i) *Initial forecasts about competitor's price and advertising decisions*
ii) *Price elasticity*
iii) *Reverse cross-price elasticity*
iv) *Advertising lagged effect weights*
v) *Ratio coefficient for competitor advertising effect*
vi) *Cross-moving demand parameter*

As described earlier, price elasticity represents the change in demand due to change in price. So, what will be examined by changing price elasticity is how the firms set their prices to reach maximum profit and for which values of demand they achieve this profit. Reverse cross-price elasticity affects the responses of customers to the relative prices of both firms. The main goal of investigating reverse cross-price elasticity effects is to see how price and advertising decisions of firms changes and how relative prices are set. Advertising lagged effect weights determine how the additional demand generated by previous period's advertising action contributes to the current period. Increased weights are expected to influence more customers to choose the firm making advertising even after certain number of periods. The experiments about advertising weights are designed to see how high or low weight levels affect current period's advertising and thus the pricing decisions. It is also expected to expose the effects of different number of periods creating lagged effects. Ratio coefficient for competitor advertising effect is used to define the ratio of customers responding to competitor's advertising activities in addition to relative price levels. Therefore, value of ratio coefficient for competitor advertising effect is expected affect equilibrium price and advertising levels. Cross-moving demand parameter is used to define the number of potential customers who are concerning relative price levels. So, how different levels of this parameter affect equilibrium points is another issue to investigate. Finally, different initial forecasts about competitor are investigated in experiments. The main objective here is to see how both firms make their decisions under different forecasts. Since the solution cycle is initiated with forecasted values of the competitor's pricing and advertising expense decisions, effects of different forecasts are analyzed. The investigation aims to extract possible differences between equilibrium solution decisions and also between paths followed to reach these points. These initial conditions are thought to have two properties: *i)* different levels of price and advertising decisions, *ii)* different variances in the distribution of forecast levels. Decisions of firms under high, middle, and low levels of price and advertising forecast levels are investigated in experiments. The equilibrium point and decision paths followed to reach equilibrium can also be affected by the variance in the forecasts. Initial conditions that vary too much and that are smooth are examined. The Table 4.1 reports on the parameters to be examined and values assigned. The parameters assumed to be static during the planning horizon are given in the Table 4.2.

Table 4.1 Examined parameters

Parameter	**Value range**
Price elasticity	<1, =1, >1
Reverse cross-price elasticity	<1, =1, >1
Advertising lagged effect weights	1/0.5/0.3/0.1, 1/0.9/0.75/0.55, 1/0.1/0.01/0.001
Ratio coefficient for competitor advertising effect	0, 0.1, 0.5
Cross-moving demand parameter	0, 1000, 2000, 5000
Initial forecasts about competitor's price and advertising decisions	Combinations of low & high levels and variances

Table 4.2 Static parameters

Parameter	**Value**	**Parameter**	**Value for Firm**	**Value for Competitor**
β_1	17000000	c_1	150	175
γ_1	$-2.031\ 10^{-10}$	c_2	100	125
γ_2	$8.812\ 10^{-5}$	c_3	50	75
a	500	GA	1500	2000
b	-5	GB	3000	3500
d	-3.346425462	GC	9000	9000
m	200	h	30	40
m'	600	g	20	30
AdvBudget	2000000	*AdvLimit*	240000	240000

5. RESULTS AND CONCLUSIONS

5.1. Experimental Results

The proposed model is solved by GA approach and by GAMS®. As mentioned earlier, the model is solved with different parameter sets. Initial forecasts are also taken as different starting parameters, and sub-parameter experiments are executed for each initial forecast.

5.1.1. Initial Forecasts

As described above, sixteen initial forecasts containing competitor's pricing and advertising expense information are given as input for the first step of the cycle. Five experiments for the described parameters are made for each of these 16 initial cases. However, results of these different forecast experiments do not differ significantly. Since the model is non-linear and GA does not guarantee the global optimum solution, in some cases the experiments result in different decisions, which are not consistent with the conclusions stated above. All results of 16 experiments are analyzed. It is observed that most of them reach the same results in terms of general underlying decision structures although not in terms of numerical values. Finally, the results of the five experiments associated with the most consistent set are reported. The associated initial forecasts are reported in Table 5.1.

Table 5.1 The associated initial forecasts

Period	Price	Advertising Expense
1	460	80000
2	420	95000
3	520	70000
4	500	85000
5	540	75000
6	525	65000
7	485	75000
8	490	90000
9	510	85000
10	520	80000
11	550	90000
12	500	70000

5.1.2. Price Elasticity

The effects of price elasticity changes are investigated in these experiments even the product type is not changed. These experiments are generated in order to analyze how the model responds to price elasticity changes. As it defined before, price elasticity values are taken as smaller than one, equal to one, and greater than one. As price elasticity value increases in absolute terms (decreases in real values), the demand decreases, as it is expected even if same price levels are chosen. This also affects actual profits. Even though these results may be predicted mathematically by analyzing the proposed demand function; levels of advertising expenses, price level differences, possible changes in production policies, which cannot be easily predicted, are investigated with experiments. The results of experiments with different price elasticities are given in Table 5.2 (a) and Table 5.2 (b).

Table 5.2 (a). Experimental results for different price elasticity (GAMS results)

		Price Elasticity		
		0.9	1.0	1.1
Price	Firm	418.5	286.6	329.6
	Competitor	460.8	366.7	331.6
	Average	439.6	326.6	330.6
	Difference	42.2	80.1	1.9
Advertising	Firm	110,252	88,785	28,146
	Competitor	125,993	42,104	18,250
	Average	118,123	65,445	23,198
	Difference	15,741	46,681	9,896
Demand	Firm	8,249	7,240	3,122
	Competitor	7,228	4,500	3,000
	Average	7,739	5,870	3,061
	Difference	1,021	2,740	122
Profit	Firm	32,461,705	16,750,609	8,505,595
	Competitor	28,651,413	12,772,370	7,071,289
	Average	30,556,559	14,761,490	7,788,443
	Difference	3,810,292	3,978,239	1,434,305
Reverse Cross-Price Elasticity: 1.0 Advertising Weights: 1 / 0.5 / 0.3 / 0.1, Ratio Coefficient for Competitor Advertising Effect: 0.1 Cross-Moving Demand Parameter: 2000				

Table 5.2 (b). Experimental results for different price elasticity (GA results)

		Price Elasticity		
		0.9	1.0	1.1
Price	Firm	431.3	309.7	255.4
	Competitor	451.9	379.2	331.3
	Average	441.6	344.4	293.4
	Difference	20.7	69.5	75.9
Advertising	Firm	154,167	122,500	95,000
	Competitor	166,667	135,000	105,000
	Average	160,417	128,750	100,000
	Difference	12,500	12,500	10,000
Demand	Firm	8,171	6,960	5,282
	Competitor	7,675	4,951	3,178
	Average	7,923	5,955	4,230
	Difference	496	2,009	2,104
Profit	Firm	32,622,746	16,872,035	8,928,703
	Competitor	29,191,998	12,941,325	5,664,409
	Average	30,907,372	14,906,680	7,296,556
	Difference	3,430,749	3,930,710	3,264,294
Reverse Cross-Price Elasticity: 1.0 Advertising Weights: 1 / 0.5 / 0.3 / 0.1, Ratio Coefficient for Competitor Advertising Effect: 0.1 Cross-Moving Demand Parameter: 2000				

As it can be seen from experimental results, while price elasticity increases price levels decrease. Since demand parameter due to price stays constant for each problem, increasing price elasticity highly affects demand, relatively. Both firms reduce price levels in order to generate demand, but it is also seen that they do not keep previous demands even they make price reduction. An interesting result is that main firm reduces its price level as price elasticity goes from 0.9 to 1, but then it sets higher price as price elasticity goes from 1 to 1.1. This result is generated by GAMS solution. However, in GA results, firm continuously reduces its price. In general, it is seen that average price levels are decreasing.

Advertising expenses are also decreasing like price levels, while price elasticity increases. Since advertising efficiency increases while price levels are decreasing, both firms prefer to cut their advertising expenses, since they set lower prices. S-shaped advertising affected sales function generates very small changes in demand after a saturation point with respect to price. Since advertising expenses needed to reach this saturation level decrease while price decreases, both firms make less advertising and generate demand. It is seen that firms stop advertising, where cost of additional advertising is not covered by revenue gained by additional demand.

Lower price elasticity values mean that customer's main consideration is not price. In such a case, it is seen that firms prefer making advertising rather than reducing prices. However, since higher price elasticity values represent higher interest in prices, firms prefer price reductions and lower advertising instead of setting higher prices supported by high advertising. Graphical representations of these results are given in Figure 5.1 (a) and Figure 5.1 (b).

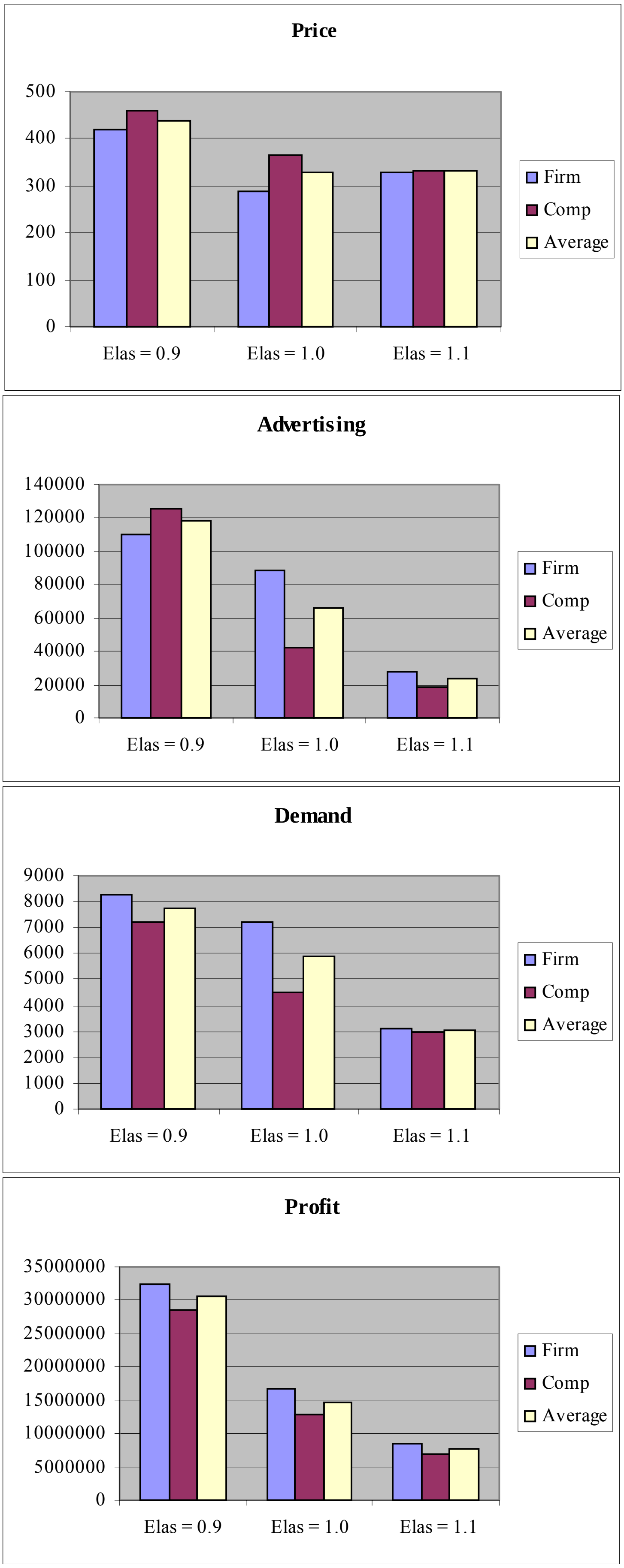


Figure 5.1 (a) Experimental results for different price elasticity (GAMS results)

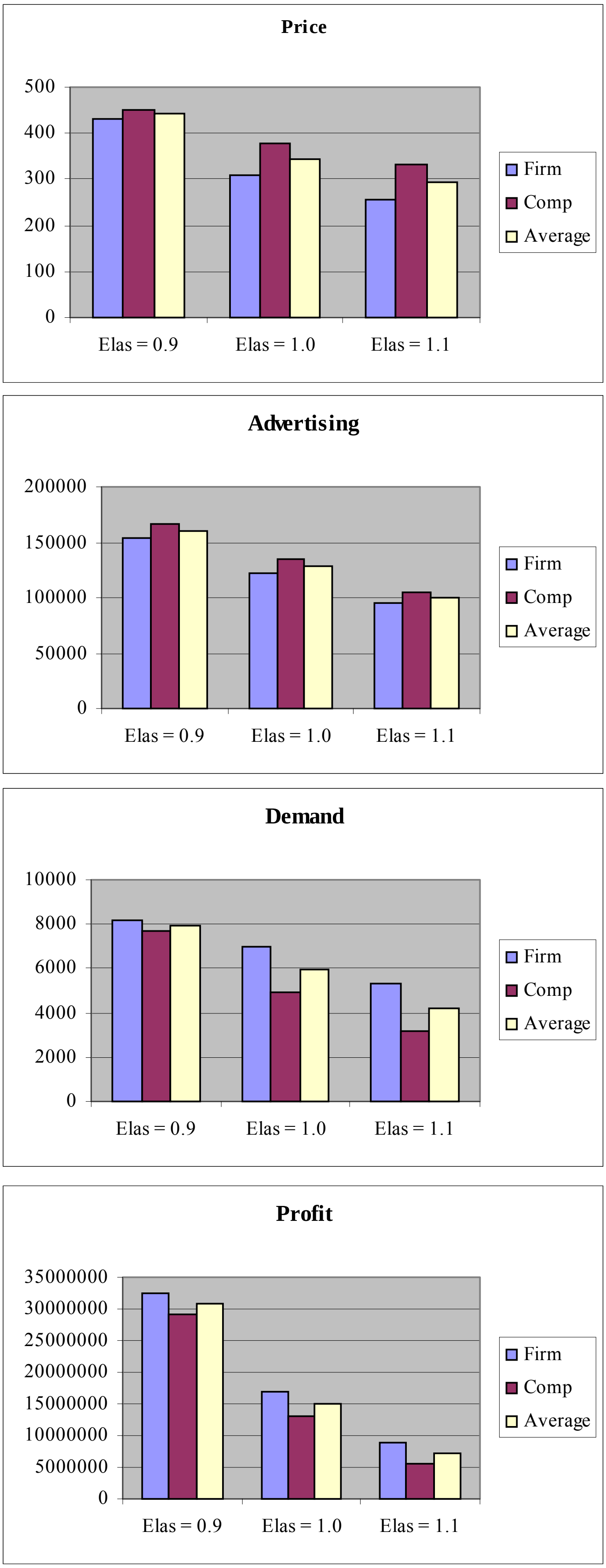


Figure 5.1 (b) Experimental results for different price elasticity (GA results)

5.1.3. Reverse Cross-Price Elasticity

The reverse cross-price elasticity is used to represent effects of changes in price of a product on demand of another product as described before. In these experiments, three values for reverse cross-price elasticity are taken. One of these values is smaller than one, the other is equal to one, and the last one is greater than one. Since firms are considered as competitors, their products are assumed to be substitutes. Thus, reverse cross-price elasticity values are taken positive. However, since ratio of price difference of products to actual price of each product is taken as basis of reverse cross-price elasticity, this parameter acts like cross-price elasticity. Higher cross-price elasticity is expected to generate more changes in demand of substitutive product when price of a product alters. Since the defined ratio is smaller than 1, when prices are not twice as high as each other, the reverse cross-price elasticity is used to represent actual effects of cross-price elasticity. The results of cross-price elasticity experiments are given in Table 5.3 (a) and Table 5.3 (b).

Table 5.3 (a) Experimental results for different reverse cross-price elasticity (GAMS results)

		Reverse Cross- Price Elasticity		
		0.9	1.0	1.1
Price	Firm	393.5	420.2	451.7
	Competitor	428.8	463.0	488.1
	Average	411.1	441.6	469.9
	Difference	35.3	42.8	36.4
Advertising	Firm	103,211	111,423	127,707
	Competitor	115,303	129,368	143,149
	Average	109,257	120,395	135,428
	Difference	12,092	17,945	15,441
Demand	Firm	8,737	8,245	7,728
	Competitor	7,719	7,238	6,990
	Average	8,228	7,741	7,359
	Difference	1,019	1,007	738
Profit	Firm	32,065,366	32,591,293	32,935,699
	Competitor	28,012,706	28,858,540	29,636,513
	Average	30,039,036	30,724,916	31,286,106
	Difference	4,052,660	3,732,754	3,299,186
Price Elasticity: 0.9, Advertising Weights: 1 / 0.5 / 0.3 / 0.1, Ratio Coefficient for Competitor Advertising Effect: 0.1, Cross-Moving Demand Parameter: 2000				

Table 5.3 (b). Experimental results for different reverse cross-price elasticity (GA results)

		Reverse Cross- Price Elasticity		
		0.9	1.0	1.1
Price	Firm	406.3	419.8	423.3
	Competitor	422.7	449.4	460.3
	Average	414.5	434.6	441.8
	Difference	16.4	29.7	36.9
Advertising	Firm	147,500	166,667	166,667
	Competitor	152,500	166,667	166,667
	Average	150,000	166,667	166,667
	Difference	5,000	0	0
Demand	Firm	8,603	8,451	8,410
	Competitor	8,122	7,677	7,513
	Average	8,363	8,064	7,961
	Difference	482	775	898
Profit	Firm	32,186,078	32,445,801	32,702,079
	Competitor	28,458,659	28,892,809	29,046,123
	Average	30,322,369	30,669,305	30,874,101
	Difference	3,727,419	3,552,992	3,655,956
Price Elasticity: 0.9 Advertising Weights: 1 / 0.5 / 0.3 / 0.1, Ratio Coefficient for Competitor Advertising Effect: 0.1, Cross-Moving Demand Parameter: 2000				

Since increasing reverse cross-price elasticity brings less importance to cross-prices, firms charge more different prices. In addition, general price levels are increasing with increasing reverse cross-price elasticity. Since increasing reverse cross-price elasticity generates less people concerning price differences, the main demand is generated by actual prices and advertising. In this case, firms increase their price levels to make more profit. As a conclusion, it can be said that when there are more customers basing their decisions on price differences, firms set lower prices to control this segment even they generate lower profit.

Firms also support their higher prices with high advertising, when importance of market segment concerning cross-prices decreases. When reverse cross-price elasticity reduces (this segment becomes more important), firms reduce prices as described before, and they make less advertising since less advertising generates high demand in such lower price levels.

Graphical representations of reverse cross-price elasticity changes are given in Figure 5.2 (a) and Figure 5.2 (b).

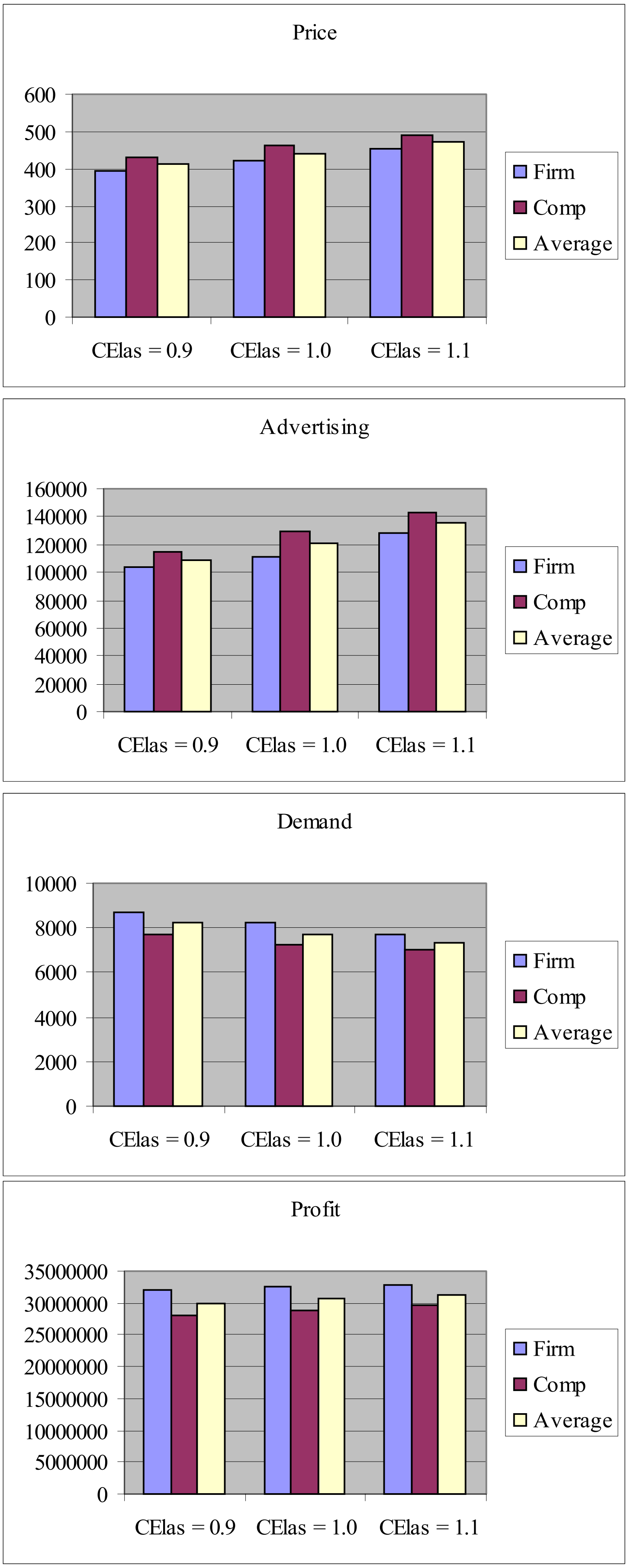


Figure 5.2 (a). Experimental results for different reverse cross-price elasticity (GAMS results)

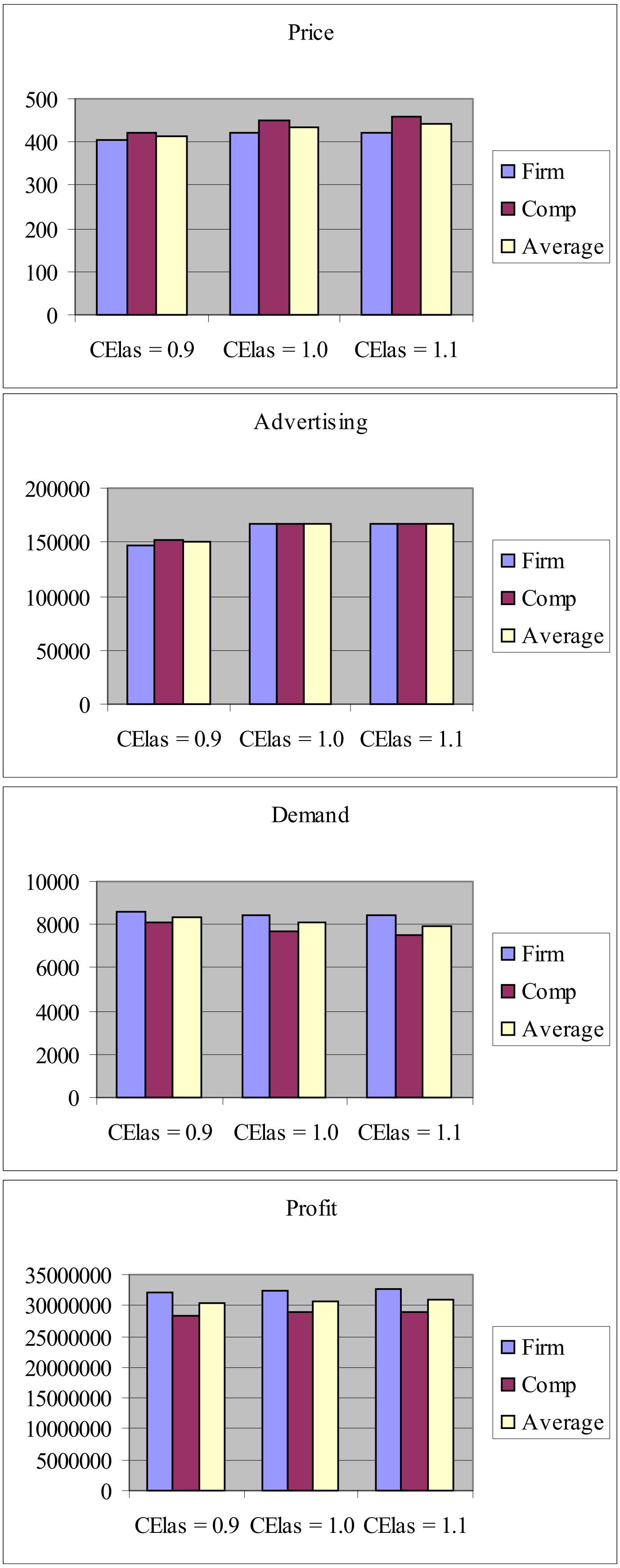


Figure 5.2 (b). Experimental results for different reverse cross-price elasticity (GA results)

5.1.4. Impact of Advertising Weights

The impacts of lagged effect of advertising expenses are analyzed by these experiments. Advertising activity made in a period is assumed to be having a decreasing effect during the next 3 periods. This decreasing effect is implemented by taking each period's advertising expenses with active period's price and recalculating advertising affected sales. These sales values are then multiplied by advertising weights. Three different advertising weights sets are analyzed with these experiments. Advertising weights are first reduced to minimum level representing faster decay of advertising effects. In the base case, middle level advertising weights are used. In the third case, higher advertising weights are used, meaning that it takes longer time customers to lose their interests in advertised product. The experimental results for different advertising weights are given in Table 5.4 (a) and Table 5.4 (b).

Table 5.4 (a). Experimental results for different advertising weights (GAMS results)

		Advertising Weights		
		1 / 0.1 / 0.01 / 0.001	1 / 0.5 / 0.3 / 0.1	1 / 0.9 / 0.75 0.55
Price	Firm	397.5	420.2	455.7
	Competitor	491.8	463.0	493.3
	Average	444.6	441.6	474.5
	Difference	94.3	42.8	37.6
Advertising	Firm	54,375	111,423	138,857
	Competitor	63,850	129,368	158,594
	Average	59,113	120,395	148,726
	Difference	9,475	17,945	19,738
Demand	Firm	8,454	8,245	8,134
	Competitor	6,275	7,238	7,369
	Average	7,364	7,741	7,752
	Difference	2,179	1,007	766
Profit	Firm	31,905,910	32,591,293	35,216,868
	Competitor	27,241,803	28,858,540	31,795,224
	Average	29,573,856	30,724,916	33,506,046
	Difference	4,664,107	3,732754	3,421,644
Price Elasticity: 0.9, Reverse Cross-Price Elasticity: 1.0, Ratio Coefficient for Competitor Advertising Effect: 0.1, Cross-Moving Demand Parameter: 2000				

Table 5.4 (b). Experimental results for different advertising weights (GA results)

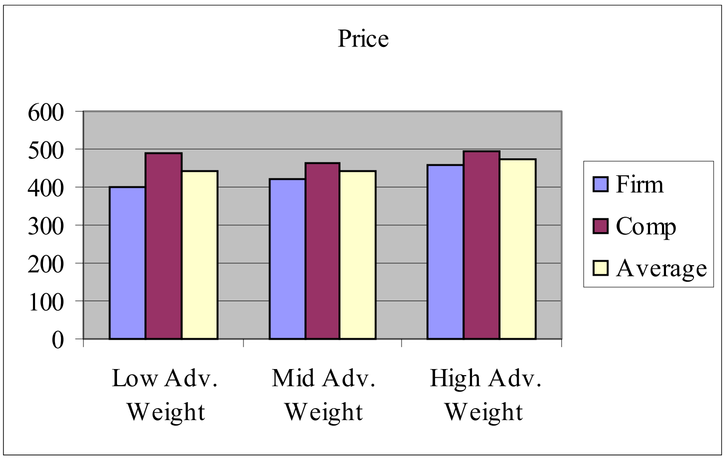
		Advertising Weights		
		1 / 0.1 / 0.01 / 0.001	1 / 0.5 / 0.3 / 0.1	1 / 0.9 / 0.75 0.55
Price	Firm	400.4	427.3	464.0
	Competitor	420.4	440.8	468.4
	Average	410.4	434.0	466.2
	Difference	20.0	13.4	4.4
Advertising	Firm	136,667	154,167	169,167
	Competitor	130,000	166,667	166,667
	Average	133,333	160,417	167,917
	Difference	6,667	12,500	2,500
Demand	Firm	8,379	8,206	8,108
	Competitor	7,819	7,883	7,976
	Average	8,099	8,045	8,042
	Difference	560	323	132
Profit	Firm	30,616,317	32,356,846	35,377,109
	Competitor	27,040,054	28,914,046	32,126,736
	Average	28,828,186	30,635,446	33,751,923
	Difference	3,576,263	3,442,800	3,250,373
Price Elasticity: 0.9, Reverse Cross-Price Elasticity: 1.0, Ratio Coefficient for Competitor Advertising Effect: 0.1, Cross-Moving Demand Parameter: 2000				

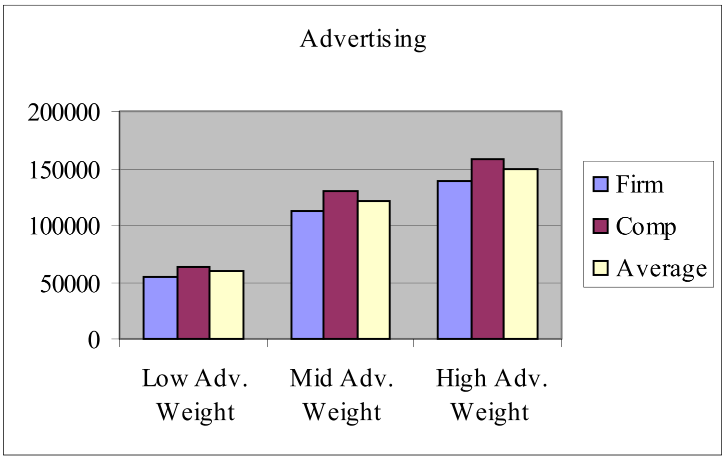
As the advertising weights increase, firms set higher prices, which are supported by higher advertising activities. Since advertising weights are higher, firms gain more customers with lower advertising expenses. In such a case, firms select higher prices to make more profit. Although advertising efficiency reduces because of increasing price levels, higher advertising weights compensate this reduction. In case of lower advertising weights, firms prefer to set lower prices in order to generate more demand and to make more profit. In this situation, while advertising activity in a period generates additional demand, the effect of this activity does not generate enough demand in consecutive periods to make profit. Since advertising benefits are lower, firms prefer lower prices to generate demand. They also are able to generate more demand for lower advertising costs, since advertising efficiency increases by reduced prices.

Another issue with different advertising weights is that by increasing advertising weights, firms select closer prices to each other. Since advertising expenses have longer effects on demand generation, firms avoid losing customers by setting higher prices than each other. As it is described in proposed demand function, each firm's demand is related with advertising activities of a firm's competitor as well as difference between product prices. Setting relatively higher price than its competitor, a firm would be losing higher demand in high advertising weights case, since both firms prefer to make high advertising. When advertising weights are lower, firms do not prefer to spend on advertising. Thus, higher gap between price levels does not lead to the loss of many customers.

Firms make more profits with higher advertising weights; even they set higher prices for their products. While more demand is generated by constant advertising expense relatively, profits made by unit advertising expense increases. This increase compensates the profit lost by increased price levels.

Graphical representations of impacts of advertising weights on prices, advertising expenses, demands, and profits are given in Figure 5.3 (a) and Figure 5.3(b).





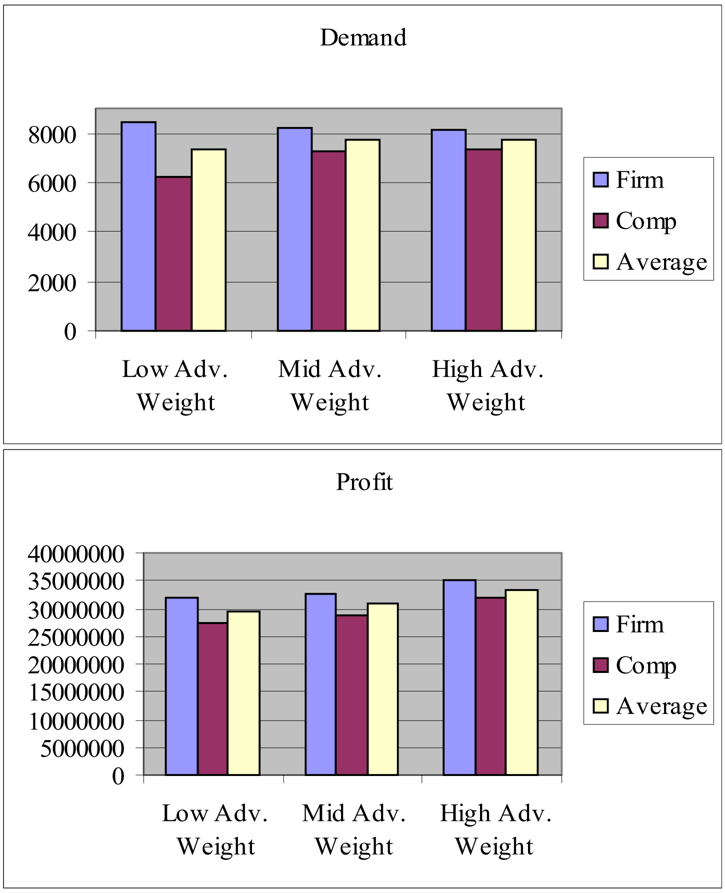
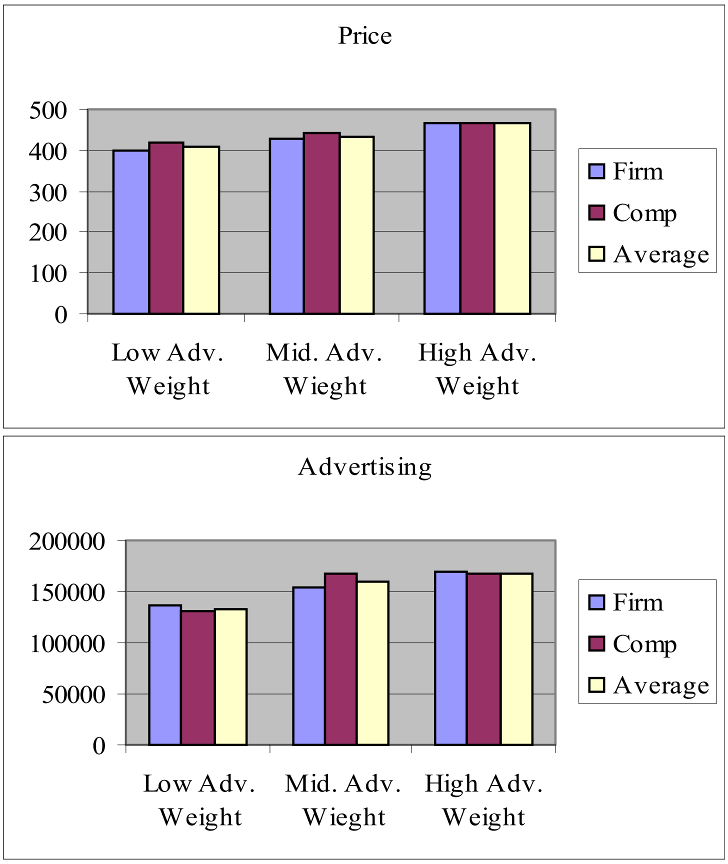


Figure 5.3 (a). Experimental results for different advertising weights (GAMS results)



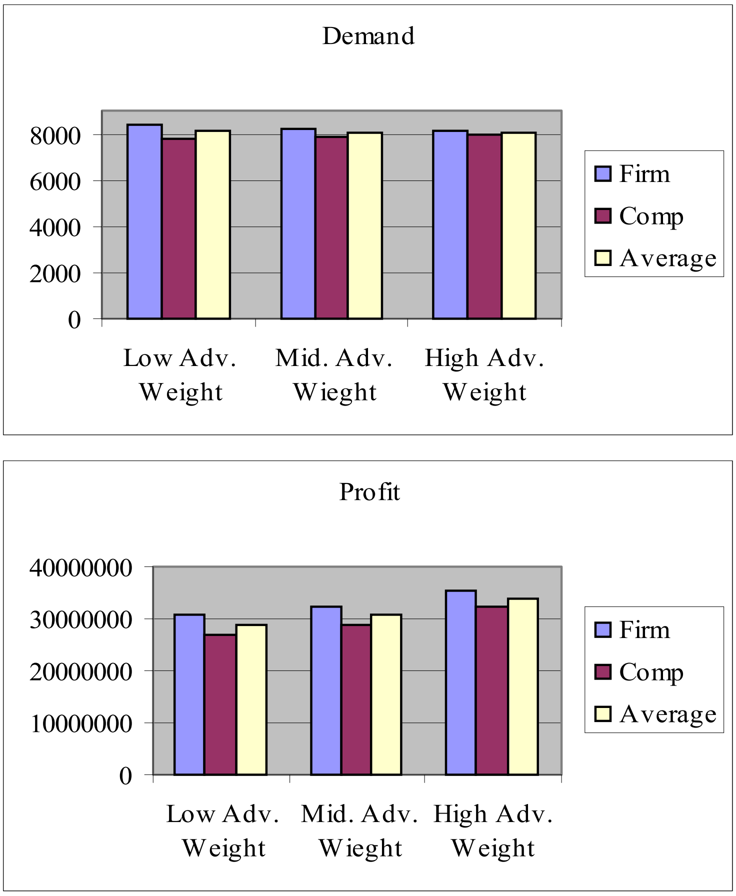


Figure 5.3 (b). Experimental results for different advertising weights (GA results)

5.1.5. Ratio Coefficient for Competitor Advertising Effect

One firms' advertising activities create product interest as well as brand awareness; so customers who are affected by advertising may decide to buy a product before they decide on which brand they would select. In the demand function, such customers are represented by a separate part and it is assumed that these customers decide on price levels of firms' products. If a firm sets higher prices for its product than its competitor does, then its competitor attracts potential customers and gets demand with lower prices. In such a case, firm's advertising activities also contribute to its competitors demand generation by creating product interest. In the demand function, the potential demand created by a firm's advertising activities is added to its competitor's demand by a ratio. This case occurs oppositely and firm gets more customers with its competitor's advertising activities if firm sets lower prices than its competitor does. The ratio coefficient for competitor advertising effect is used here to define what ratio of other firm's advertising affected demand would prefer to buy firm's product. Three cases for this parameter are analyzed. Setting this parameter equal to zero leads to the situation where no customers are affected by the advertising of the competitor firm. In other cases, increasing ratio values are used in order to investigate how firms behave in their decisions. In Table 5.5 (a) and Table 5.5 (b), the results of experiments with different ratio coefficient for competitor advertising effect values are given.

Table 5.5 (a). Experimental results for different ratio coefficient for competitor advertising effect (GAMS results)

		Ratio Coefficient for Competitor Advertising Effect		
		0	0.1	0.5
Price	Firm	427.8	420.2	387.0
	Competitor	467.8	463.0	448.1
	Average	447.8	441.6	417.5
	Difference	40.0	42.8	61.1
Advertising	Firm	114,788	111,423	67,107
	Competitor	131,789	129,368	120,824
	Average	123,289	120,395	93,965
	Difference	17,000	17,945	53,717
Demand	Firm	8,118	8,245	8,724
	Competitor	7,194	7,238	7,295
	Average	7,656	7,741	8,009
	Difference	924	1,007	1,429
Profit	Firm	32,725,973	32,591,293	31,736,789
	Competitor	29,037,240	28,858,540	27,910,222
	Average	30,881,607	30,724,916	29,823,505
	Difference	3,688,733	3,732,754	3,826,567
Price Elasticity: 0.9, Reverse Cross-Price Elasticity: 1.0, Advertising Weights: 1 / 0.5 / 0.3 / 0.1, Cross-Moving Demand Parameter: 2000				

Table 5.5 (b). Experimental results for different ratio coefficient for competitor advertising effect (GA results)

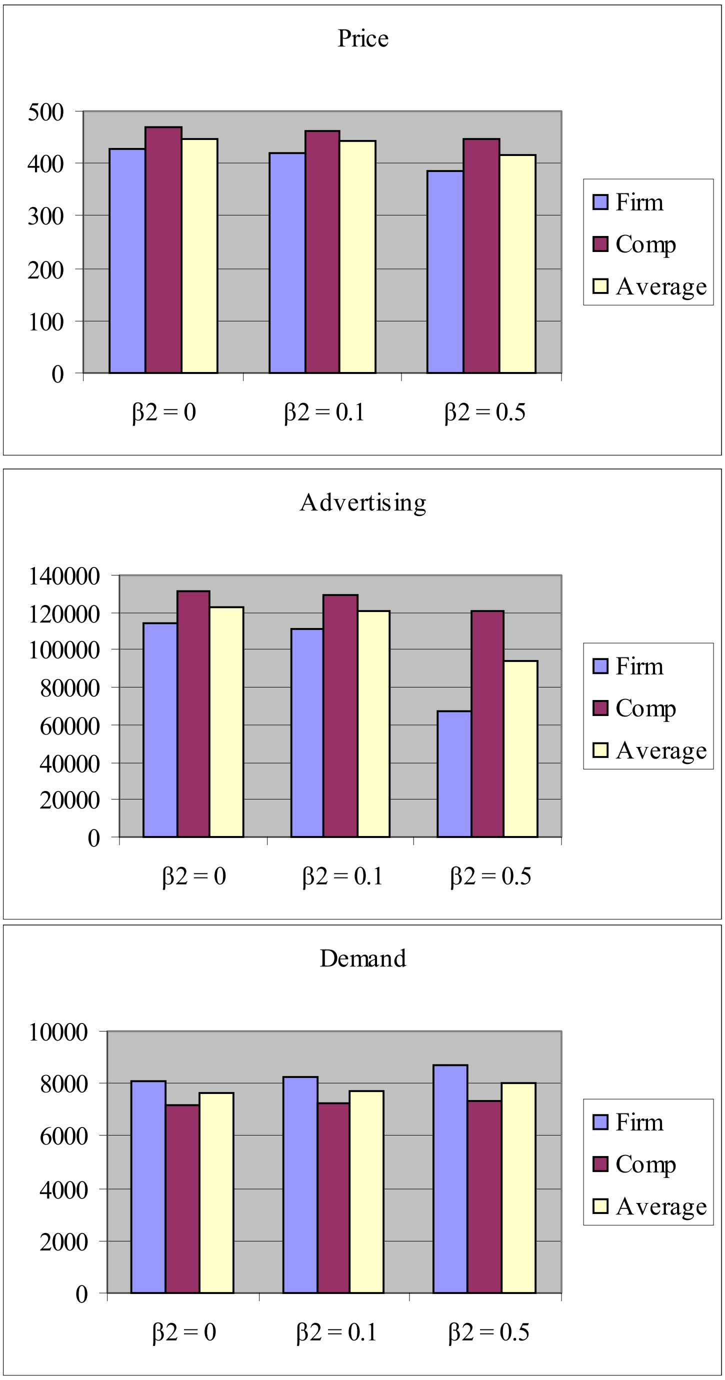
		Ratio Coefficient for Competitor Advertising Effect		
		0	0.1	0.5
Price	Firm	426.2	423.1	406.8
	Competitor	480.6	459.5	408.3
	Average	453.4	441.3	407.5
	Difference	54.4	36.4	1.5
Advertising	Firm	166,667	150,833	148,333
	Competitor	166,667	166,667	140,833
	Average	166,667	158,750	144,583
	Difference	0	15,833	7,500
Demand	Firm	8,425	8,391	8,528
	Competitor	7,080	7,475	8,451
	Average	7,752	7,933	8,490
	Difference	1,346	916	77
Profit	Firm	33,192,927	32,782,516	31,515,437
	Competitor	28,903,731	29,004,064	28,359,253
	Average	31,048,329	30,893,290	29,937,345
	Difference	4,289,196	3,778,452	3,156,184
Price Elasticity: 0.9, Reverse Cross-Price Elasticity: 1.0, Advertising Weights: 1 / 0.5 / 0.3 / 0.1, Cross-Moving Demand Parameter: 2000				

While ratio coefficient for competitor advertising effect increases, firms make price reductions for their products. When GA results are analyzed, it can be said that firms prefer closer prices. This can be explained by the logic behind the selected parameter. Since the value of parameter increases, more people are affected by other firm's advertising activities. In such a case, firms prefer to set closer prices in order to prevent its customers who are attracted by its own advertising activities to select other firm's product.

As the ratio coefficient for competitor advertising effect value increases, firms also spend less in advertising. Two rationales behind this action can be stated. First, since both firms reduce their product's price levels, they can attract more customers with lower advertising expenses. This kind of advertising expense reduction can be seen in almost each case of price reductions. Second, since this parameter value increases, firms decide in a more risky environment. In order to prevent losing customers that they attracted by their own advertising activities, firms avoid making more advertising. Instead, they prefer to decrease prices and to generate more demand by this price reduction.

Even though firms generate more demand, on the average their profits reduce as the ratio coefficient for competitor advertising effect parameter value increases. They reduce their product's price levels but in response to action, they cannot generate enough demand increase. Since it becomes more risky to make advertising, they set their prices closer to reduce risk of making advertising. In this case, the segment deciding based on price difference is distributed with less deviation.

Graphical representations of these experiments are given in Figure 5.4 (a) and Figure 5.4 (b).



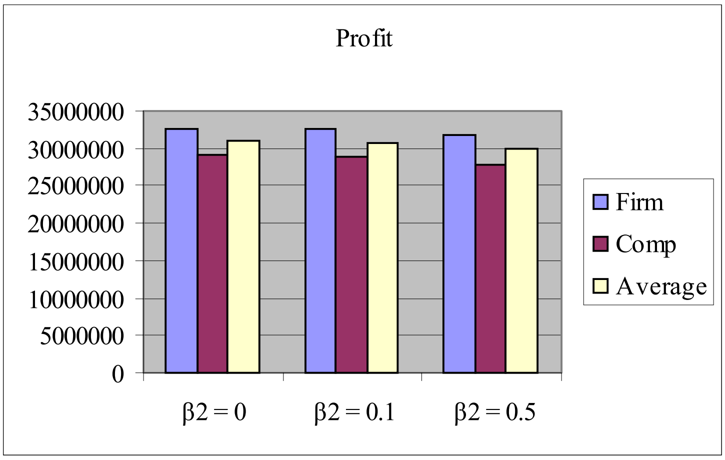
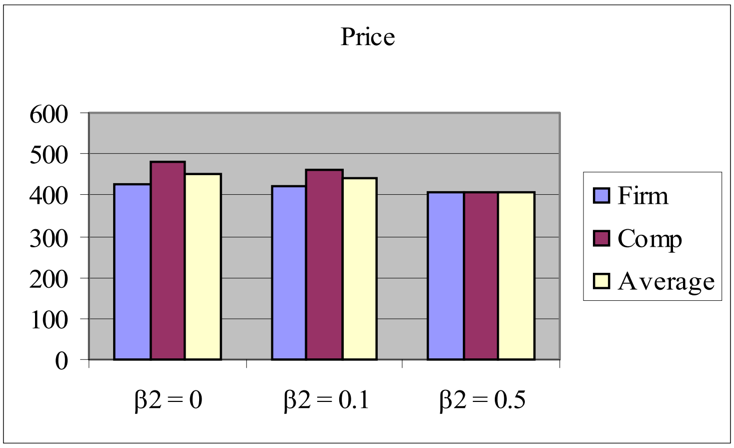
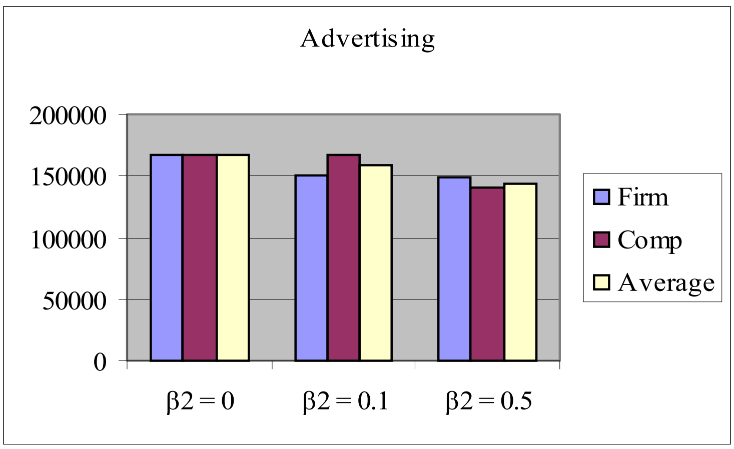


Figure 5.4 (a). Experimental results for different ratio coefficient for competitor advertising effect (GAMS results)





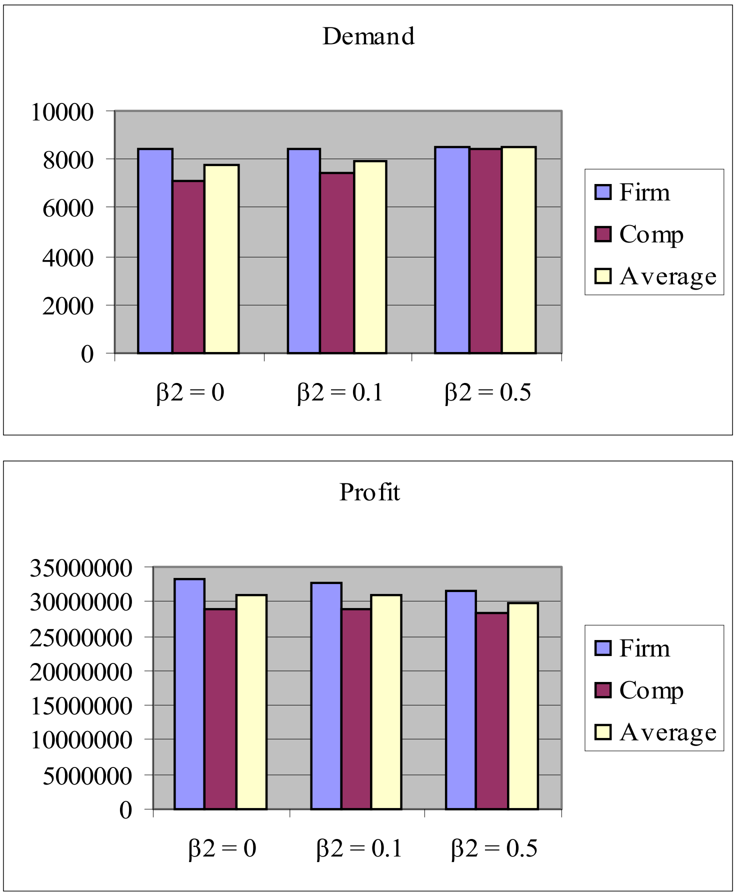


Figure 5.4 (b). Experimental results for different ratio coefficient for competitor advertising effect (GA results)

5.1.6. Cross-Moving Demand Parameter

In addition to advertising effect on the cross-moving segment in the market, a constant number of people are assumed to exist paying attention on price difference even if no advertising is made. Both firms' pricing decisions are assumed to be effective on the decisions of these customers. Four experiments are done in order to analyze how both firms' decisions are affected by different number of this type customers. In first experiment, it is assumed that this parameter is equal to zero, meaning that there are no such customers. In other experiments, this parameter value is increased. Table 5.6 (a) and Table 5.6 (b) show the results of these experiments.

Table 5.6 (a). Experimental results for different cross-moving demand parameter (GAMS results)

		Cross-Moving Demand Parameter			
		0	1000	2000	5000
Price	Firm	600.0	505.0	420.2	375.9
	Competitor	600.0	520.6	463.0	375.9
	Average	600.0	512.8	441.6	375.9
	Difference	0.0	15.6	42.8	0.0
Advertising	Firm	80,000	155,382	111,423	125,579
	Competitor	80,000	166,667	129,368	125,578
	Average	80,000	161,024	120,395	125,579
	Difference	0	11,285	17,945	1
Demand	Firm	1,075	6,945	8,245	9,000
	Competitor	1,075	6,701	7,238	9,000
	Average	1,075	6,823	7,741	9,000
	Difference	0	244	1,007	0
Profit	Firm	5,811,966	33,343,703	32,591,293	30,989,734
	Competitor	5,417,830	30,522,841	28,858,540	27,689,344
	Average	5,614,898	31,933,272	30,724,916	29,339,539
	Difference	394,136	2,820,861	3,732,754	3,300,390
Price Elasticity: 0.9, Reverse Cross-Price Elasticity: 1.0, Advertising Weights: 1 / 0.5 / 0.3 / 0.1, Ratio Coefficient for Competitor Advertising Effect: 0.1					

Table 5.6 (b). Experimental results for different cross-moving demand parameter (GA results)

		Cross-Moving Demand Parameter			
		0	1000	2000	5000
Price	Firm	510.4	483.6	422.9	379.3
	Competitor	530.3	478.1	463.0	379.4
	Average	520.3	480.8	443.0	379.3
	Difference	19.8	5.5	40.1	0.2
Advertising	Firm	162,500	168,194	153,333	127,500
	Competitor	162,500	166,667	166,667	130,000
	Average	162,500	167,431	160,000	128,750
	Difference	0	1,528	13,333	2,500
Demand	Firm	6,899	7,256	8,401	9,000
	Competitor	6,611	7,373	7,417	8,981
	Average	6,755	7,314	7,909	8,990
	Difference	288	117	984	19
Profit	Firm	33,008,874	32,853,800	32,912,648	30,680,129
	Competitor	30,413,672	30,049,372	29,004,410	27,389,049
	Average	31,711,273	31,451,586	30,958,529	29,034,589
	Difference	2,595,201	2,804,428	3,908,238	3,291,080
Price Elasticity: 0.9, Reverse Cross-Price Elasticity: 1.0, Advertising Weights: 1 / 0.5 / 0.3 / 0.1, Ratio Coefficient for Competitor Advertising Effect: 0.1					

While cross-moving demand parameter increases, firms prefer to set lower prices for their products. They also reduce their advertising expenses. As seen in previous parameter analyses, firms make less advertising when they make price reductions. This is a result of increasing advertising efficiency with price reductions. However, an interesting result which can be concluded by cross-moving demand parameter analysis is that the gap between price levels increases for second and third values of the parameter and then decreases for fourth value of the parameter. Since price levels are relatively higher for lower parameter value, the difference between two firms' product prices cause relatively less demand difference. This situation occurs because of two reasons. First, when price levels are relatively higher then different price levels have relatively less effect on directly price related demand. This is related with $\beta_1 / P^{\alpha 1}$ part of demand function, which defines demand generated by only pricing decisions of a firm. For higher values of prices, this value changes less than it does for lower values of prices. Second, in cases where price gap is higher, it can be seen that cross-moving demand is less. This affects the demand generated from price difference concerning segment, so lower parameter values relax firms to set more individually and to consider its competitor's prices less.

As the price levels reduce, the total demand generated by both firms increases. However, the profits of firms decrease, since demand increase does not provide enough additional revenue to compensate for this price reduction. While firms set approximately equal prices for higher cross-moving demand parameter values, they share this demand almost equally. As described in previous chapters, in such a case this segment acts according to price levels rather than price differences. Since price values are lower in these cases, total profit generated reduces.

Graphical representations of experimental results of cross-moving demand parameter are given in Figure 5.5 (a) and Figure 5.5 (b).

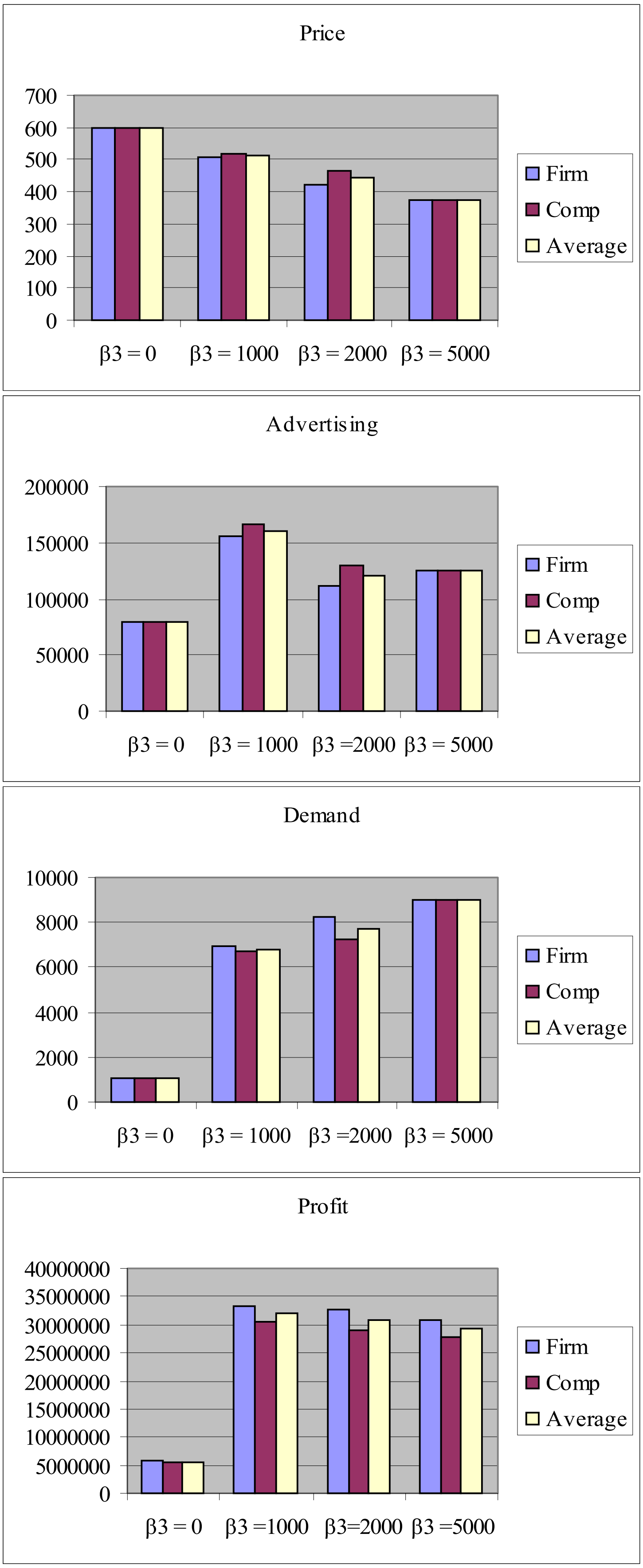


Figure 5.5 (a). Experimental results for different cross-moving demand parameter (GAMS results)

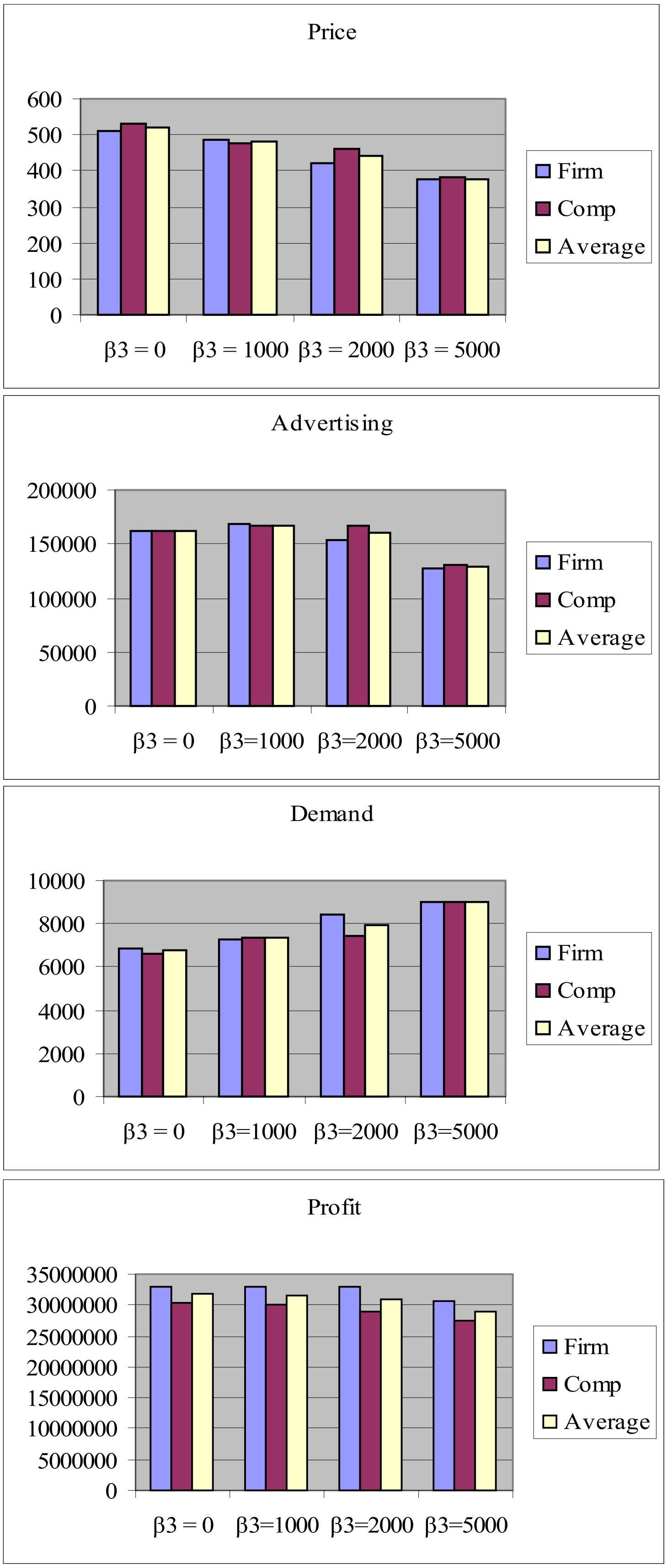


Figure 5.5 (b). Experimental results for different cross-moving demand parameter (GA results)

5.1.7. Impact of Cost Structure

The cost structures of firms are assumed to be slightly different in terms of both unit production costs and piece-wise linear cost curve segment volumes. In order to analyse the impact of the cost structure differentiation between the firms on the decisions, experiments for each of the selected 5 parameters are executed.

The experiments stated here are executed for the same costs as in the previous analysis for one firm. However, the cost structure of the competitor is changed and its unit costs are increased. In addition, production volumes representing the segment end points in the production cost curve are increased. The parameters representing the cost structure are given in Table 5.7.

Table 5.7 The different cost structures of the firms

Parameters	**Firm**	**Competitor (Slightly differentiated cost structure)**	**Competitor (Highly differentiated cost structure)**
GA	1500	2000	4000
GB	3000	3500	7000
GC	9000	9000	9000
c_1	150	175	225
c_2	100	125	200
c_3	50	75	125

The experiments represented here are executed using GA and the results are compared with GA outputs that are given in the previous chapters.

Table 5.8 The price elasticity results for highly differentiated cost structure

		Price Elasticity		
		0.9	1.0	1.1
Price	Firm	434.7	335.4	228.8
	Competitor	524.0	474.2	413.7
	Average	479.3	404.8	321.2
	Difference	89.3	138.8	184.9
Advertising	Firm	150,833	130,833	99,166
	Competitor	166,666	153,333	125,833
	Average	158,750	142,083	112,500
	Difference	15,833	22,500	26,666
Demand	Firm	8,403	6,890	6,854
	Competitor	6,291	3,708	2,066
	Average	7,347	5,299	4,460
	Difference	2,112	3,181	4,788
Profit	Firm	34,117,748	18,762,133	10,575,859
	Competitor	21,347,394	9,120,390	3,047,241
	Average	27,732,571	13,941,262	6,811,550
	Difference	12,770,354	9,641,742	7,528,617
Reverse Cross-Price Elasticity: 1.0 Advertising Weights: 1 / 0.5 / 0.3 / 0.1, Ratio Coefficient for Competitor Advertising Effect: 0.1 Cross-Moving Demand Parameter: 2000				

As the costs of the competitor firm increase, both of the firms charge higher prices (Table 5.8). Even though the firm increases its product's price, it is advantageous since also its competitor charges higher prices in order to make profit with increased costs. When compared with the previous results, it is seen that firms spend more on advertising. They use advertising to support their high pricing policy, although the advertising efficiency reduces with higher prices. The graphical representations of the price elasticity experiment results are given in Figure 5.6.

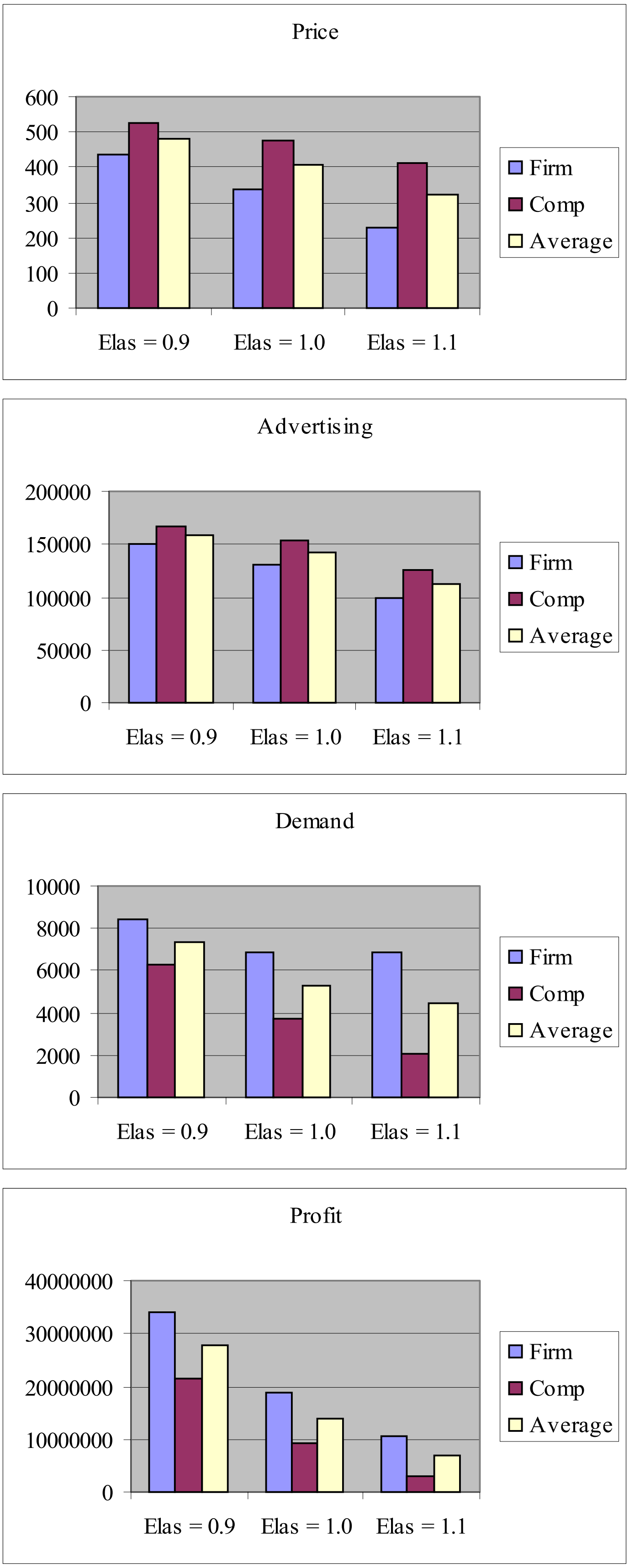


Figure 5.6 The price elasticity results for highly differentiated cost structure

Table 5.9 Reverse cross-price elasticity results for highly differentiated cost structure

		Reverse Cross- Price Elasticity		
		0.9	1.0	1.1
Price	Firm	425.8	430.6	453.4
	Competitor	495.2	536.3	532.3
	Average	460.5	483.5	492.8
	Difference	69.3	105.8	78.8
Advertising	Firm	153,333	159,166	170,833
	Competitor	166,736	158,333	166,944
	Average	160,034	158,750	168,888
	Difference	13,402	833	3,888
Demand	Firm	8,560	8,598	8,083
	Competitor	6,739	6,077	6,288
	Average	7,649	7,338	7,186
	Difference	1,820	2,521	1,794
Profit	Firm	33,816,967	34,435,335	33,959,387
	Competitor	20,735,369	21,364,517	21,854,216
	Average	27,276,168	27,899,926	27,906,801
	Difference	13,081,598	13,070,817	12,105,170
Price Elasticity: 0.9, Advertising Weights: 1 / 0.5 / 0.3 / 0.1, Ratio Coefficient for Competitor Advertising Effect: 0.1 Cross-Moving Demand Parameter: 2000				

As the reverse cross-price elasticity increases, firms prefer to charge higher prices and the price difference increases (Table 5.9). However, as the costs for the competitor increase, the competitor increases its product's prices and the firm benefits from this increase by increasing its own product's price. They use advertising to support their high pricing policy. It is observed that firm's profits increase with respect to previous experiments while the competitor's profits reduce.

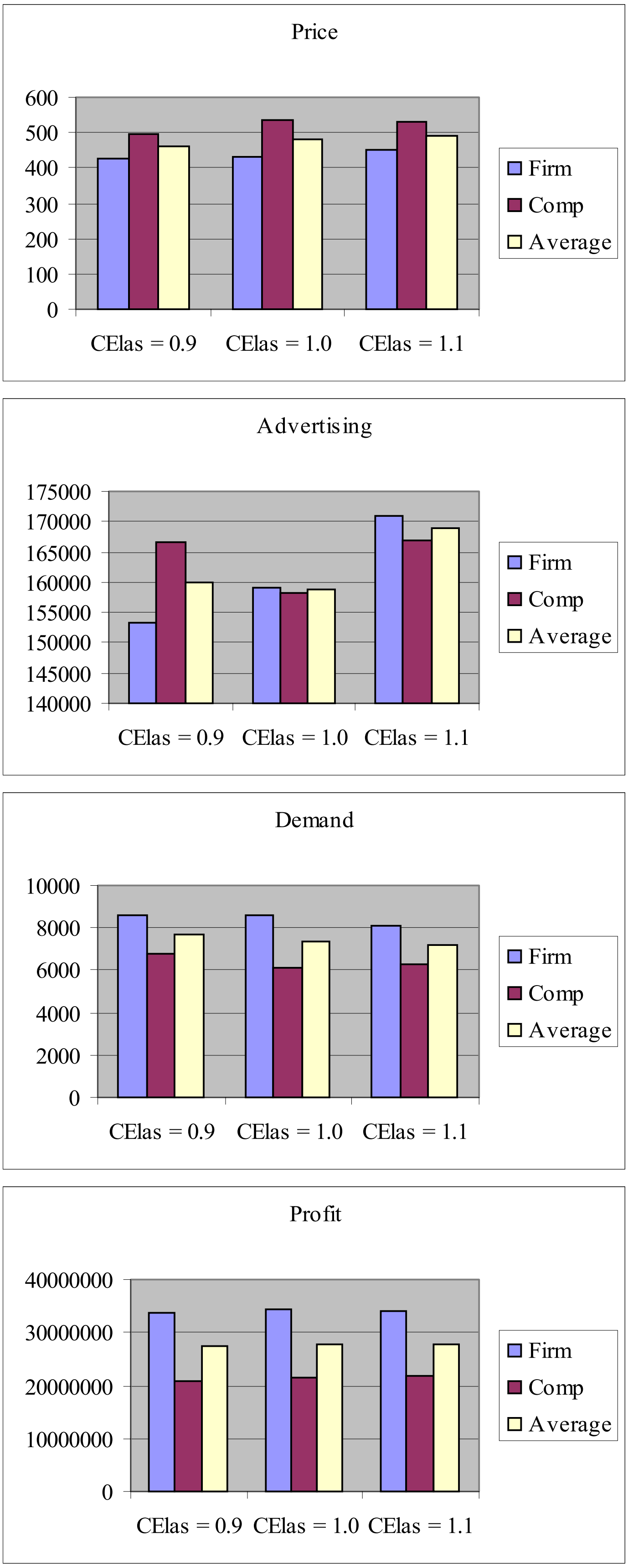


Figure 5.7 Reverse cross-price elasticity results for highly differentiated cost structure

Table 5.10 Advertising weight results for highly differentiated cost structure

		Advertising Weights		
		1 / 0.1 / 0.01 / 0.001	1 / 0.5 / 0.3 / 0.1	1 / 0.9 / 0.75 0.55
Price	Firm	418.3	430.4	452.7
	Competitor	568.3	529.3	507.1
	Average	493.3	479.9	479.9
	Difference	149.9	98.9	54.4
Advertising	Firm	140,833	164,166	189,166
	Competitor	11,666	160,833	167,500
	Average	76,250	162,500	178,333
	Difference	129,166	3,333	21,666
Demand	Firm	8,668	8,586	8,554
	Competitor	5,110	6,178	7,169
	Average	6,889	7,382	7,861
	Difference	3,557	2,408	1,385
Profit	Firm	33,631,353	34,289,886	36,246,921
	Competitor	20,990,341	21,230,078	23,489,619
	Average	27,310,847	27,759,982	29,868,270
	Difference	12,641,012	13,059,808	12,757,302
Price Elasticity: 0.9, Reverse Cross-Price Elasticity: 1.0, Ratio Coefficient for Competitor Advertising Effect: 0.1, Cross-Moving Demand Parameter: 2000				

As the advertising weights for lagged effect increase, firms maintain their price increasing strategy as in the previous experiments (Table 5.10). However, both firms charge higher prices when compared to the price levels in the previous experiments. This is due to the increased production cost for the competitor and the firm taking advantage of this situation. They also support these prices with advertising since advertising generates more demand as the lagged effect weights increase. It is observed that firm's profits increase while the competitor's profits decrease when compared with the previous experiments.

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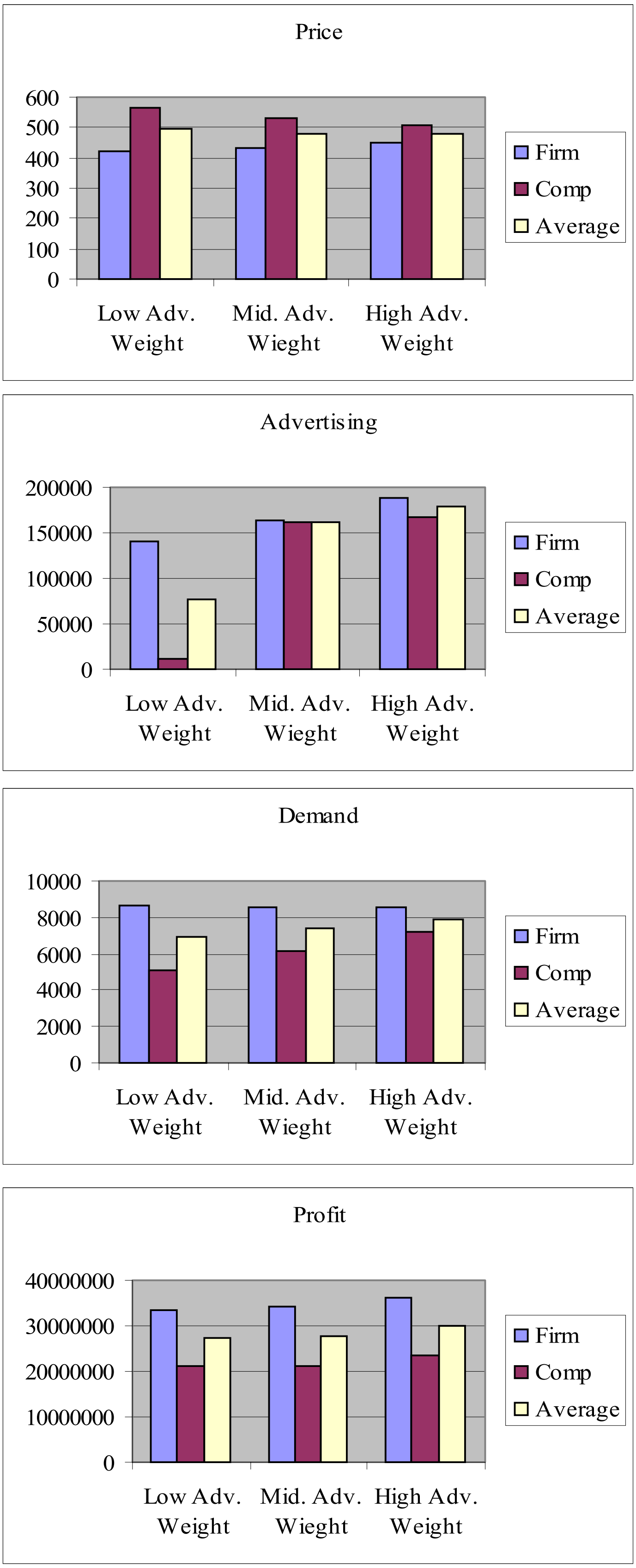

Figure 5.8 Advertising weight results for highly differentiated cost structure

Table 5.11 Ratio coefficient for competitor advertising effect results for highly differentiated cost structure

.		Ratio Coefficient for Competitor Advertising Effect		
		0	0.1	0.5
Price	Firm	419.0	437.4	424.3
	Competitor	522.2	513.9	492.3
	Average	470.6	475.6	458.3
	Difference	103.1	76.5	68.0
Advertising	Firm	157,500	157,500	167,500
	Competitor	167,152	167,430	166,666
	Average	162,326	162,465	167,083
	Difference	9,652	9,930	833
Demand	Firm	8,770	8,326	8,590
	Competitor	6,335	6,530	6,795
	Average	7,553	7,428	7,692
	Difference	2,435	1,795	1,795
Profit	Firm	34,079,793	33,893,826	33,520,943
	Competitor	21,232,394	21,493,960	20,728,382
	Average	27,656,093	27,693,893	27,124,662
	Difference	12,847,399	12,399,866	12,792,561
Price Elasticity: 0.9, Reverse Cross-Price Elasticity: 1.0 Advertising Weights: 1 / 0.5 / 0.3 / 0.1, Cross-Moving Demand Parameter: 2000				

As the ratio coefficient for competitor advertising effect increases, firms charge closer prices for their products, so the price difference reduces (Table 5.11). However, the firm does not strictly make price reductions as the parameter value increases although its competitor does. This is a result of higher prices that the competitor prefers because of its increased production cost. Although the firm does not reduce its product's price in absolute terms, it generates additional demand by charging lower prices compared to its competitor. Both firms prefer higher advertising. However, the firm increases its advertising expenses as the parameter value increases, since its price difference advantage reduces because of its competitor's price reductions. When its competitor prefers higher prices, the firm easily generates demand by the increased price difference without any additional advertising. When the competitor reduces its product's prices, the firm makes advertising to maintain the demand volume.

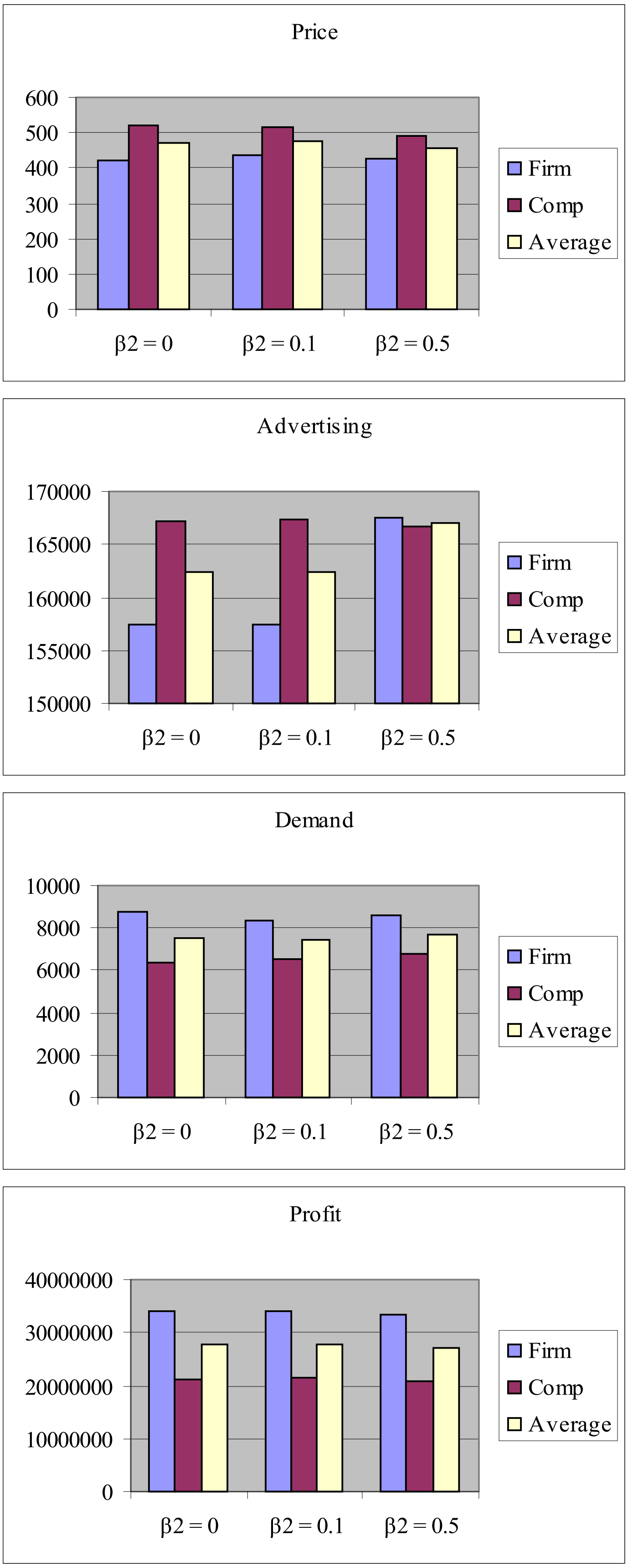

Figure 5.9 Ratio coefficient for competitor advertising effect results for highly differentiated cost structure

Table 5.12 Cross-moving demand results for highly differentiated cost structure

		Cross-Moving Demand Parameter			
		0	1000	2000	5000
Price	Firm	519.5	492.7	425.1	400.2
	Competitor	542.5	539.2	534.5	429.1
	Average	531.0	516.0	479.8	414.7
	Difference	23.0	46.5	109.3	28.9
Advertising	Firm	205,000	204,166	155,000	155,833
	Competitor	149,166	166,805	161,666	142,500
	Average	177,083	185,486	158,333	149,166
	Difference	55,833	37,361	6,666	13,333
Demand	Firm	6,853	7,337	8,723	8,955
	Competitor	6,354	6,375	6,084	7,720
	Average	6,603	6,856	7,404	8,337
	Difference	499	962	2,638	1,234
Profit	Firm	33,213,341	33,710,639	34,513,408	32,585,770
	Competitor	23,125,767	22,790,155	21,291,129	18,906,540
	Average	28,169,554	28,250,397	27,902,268	25,746,155
	Difference	10,087,575	10,920,484	13,222,279	13,679,230
Price Elasticity: 0.9, Reverse Cross-Price Elasticity: 1.0 Advertising Weights: 1 / 0.5 / 0.3 / 0.1, Ratio Coefficient for Competitor Advertising Effect: 0.1					

Firms maintain price reduction strategy, as the cross-moving demand value increases (Table 5.12). However, the prices are higher (in absolute terms) when compared to previous experiments because of the increase in the competitor's production cost. They also reduce their advertising expenses as the parameter value increases as they did in the previous experiments. The demand generated by price difference increases by an increase in the parameter value, so firms do not need to achieve high levels of advertising. Another reason for the advertising expense reduction is the reduced prices and increased adverting efficiency.

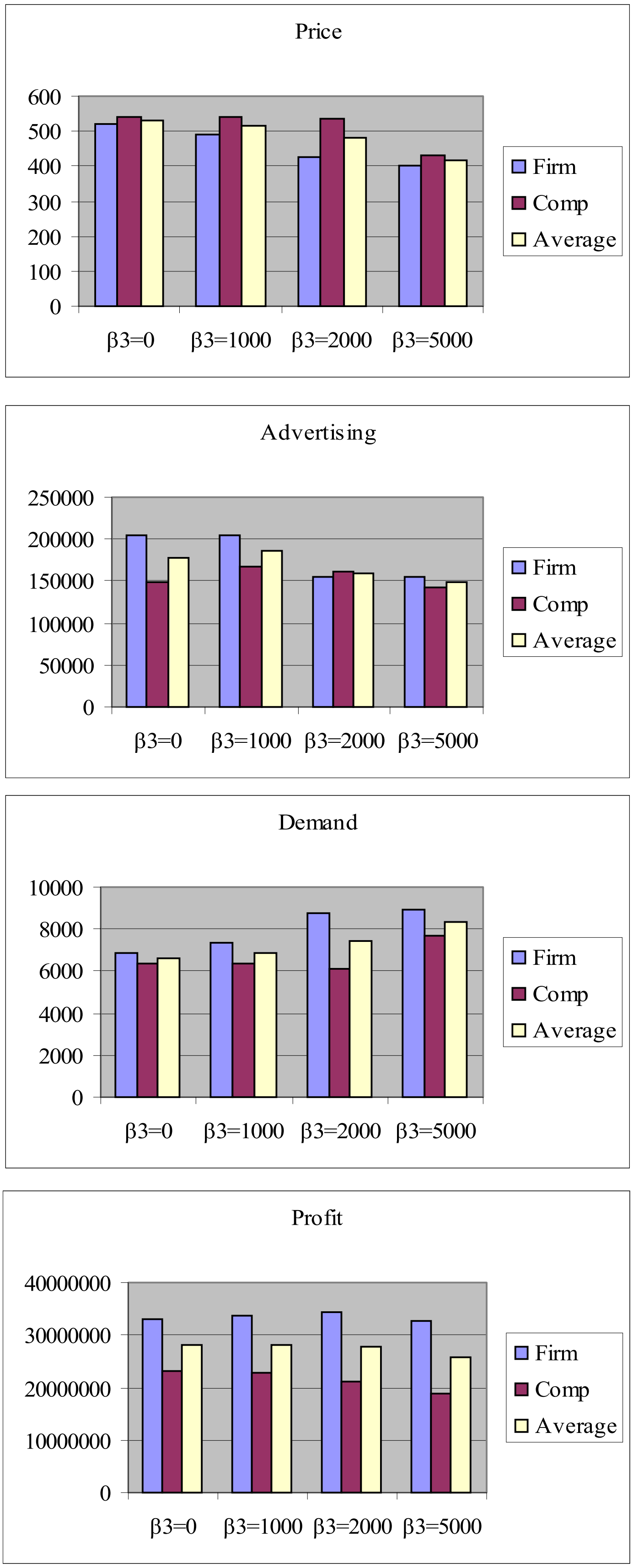

Figure 5.10 Cross-moving demand results for highly differentiated cost structure

5.1.8. Logit Function Parameter Analysis

In this section, the impacts of some parameters in the logit function are analysed. The two parameters investigated are *a*, the upper limit coefficient for advertising affected demand, and *b*, shifting parameter for marginal advertising curve. As the value of *a* increases, the total potential number of customer base, that can be generated by advertising activities, increases. So, the importance of advertising in terms of demand generation increases. As the parameter value for *b* increases, the marginal advertising affected sales curve shifts, resulting in less advertising expense needed to generate maximum marginal advertising affected sales. The results of logit function parameter analysis are given in Table 5.13 and Table 5.14.

Table 5.13 Experimental results for different upper limit coefficient for advertising affected demand

		a			
		100	500	1000	1500
Price	Firm	390.7	431.3	450.1	470.9
	Competitor	483.8	451.9	443.3	470.7
	Average	437.2	441.6	446.7	470.8
	Difference	93.0	20.7	6.8	0.1
Advertising	Firm	15,833	154,167	166,944	165,833
	Competitor	15,000	166,667	166,666	166,944
	Average	15,416	160,417	166,805	166,388
	Difference	833	12,500	278	1,111
Demand	Firm	8,475	8,171	8,554	8,994
	Competitor	6,268	7,675	8,777	8,903
	Average	7,371	7,923	8,665	8,949
	Difference	2,207	496	223	91
Profit	Firm	31,187,264	32,622,746	36,194,520	40,535,447
	Competitor	26,313,653	29,191,998	33,365,628	36,768,824
	Average	28,750,459	30,907,372	34,780,074	38,652,136
	Difference	4,873,612	3,430,749	2,828,892	3,766,622
Price Elasticity: 0.9, Reverse Cross-Price Elasticity: 1.0 Advertising Weights: 1 / 0.5 / 0.3 / 0.1, Ratio Coefficient for Competitor Advertising Effect: 0.1, Cross-Moving Demand Parameter: 2000					

It is observed that firms reduce their advertising expenses as the upper limit coefficient for advertising affected demand parameter value decreases (Table 5.13). Since this parameter represents the maximum demand that can be generated by advertising activities, profitability of advertising activities reduces. In this case, firm reduces its product's prices in order to generate demand. As the parameter value increases, firms spend more on advertising, since the additional demand generated by advertising increases. Firms increase their product's price levels. The decreases in the demand reasoned by price increases are covered by increased demand generated by advertising activities.

Table 5.14 Experimental result for different shifting parameter for marginal advertising curve

		b				
		-1	-3	-5	-7	-10
Price	Firm	491.1	437.5	431.3	387.5	389.2
	Competitor	521.5	448.5	451.9	443.0	407.6
	Average	506.3	443.0	441.6	415.2	398.4
	Difference	30.4	10.9	20.7	55.4	18.4
Advertising	Firm	107,500	132,500	154,167	163,333	166,666
	Competitor	115,833	118,333	166,667	166,666	137,500
	Average	111,666	125,416	160,417	165,000	152,083
	Difference	8,333	14,166	12,500	3,333	29,166
Demand	Firm	7,477	8,083	8,171	9,000	8,692
	Competitor	6,834	7,798	7,675	7,504	8,015
	Average	7,155	7,940	7,923	8,316	8,354
	Difference	643	284	496	1,623	677
Profit	Firm	35,019,252	32,968,858	32,622,746	31,761,477	30,339,870
	Competitor	31,477,707	29,780,567	29,191,998	27,510,641	26,655,771
	Average	33,248,480	31,374,712	30,907,372	29,636,059	28,497,821
	Difference	35,415,449	3,188,290	3,430,749	4,250,837	3,684,099
Price Elasticity: 0.9, Reverse Cross-Price Elasticity: 1.0 Advertising Weights: 1 / 0.5 / 0.3 / 0.1, Ratio Coefficient for Competitor Advertising Effect: 0.1, Cross-Moving Demand Parameter: 2000						

As the shifting parameter for marginal advertising curve value increases, firms prefer to charge higher prices for their products although they spend less on advertising since the advertising efficiency is increased (Table 5.14). However, firms reduce prices of their products, as this parameter value decreases. The advertising expenses needed to generate maximum marginal demand increases and firms prefer to make more advertising to generate this demand. Their price reductions are result of reduced advertising efficiency because of the decreases in the parameter value. The graphical representations of the experimental results of analysis of parameters *a* and *b* are given in Figure 5.11 and Figure 5.12.

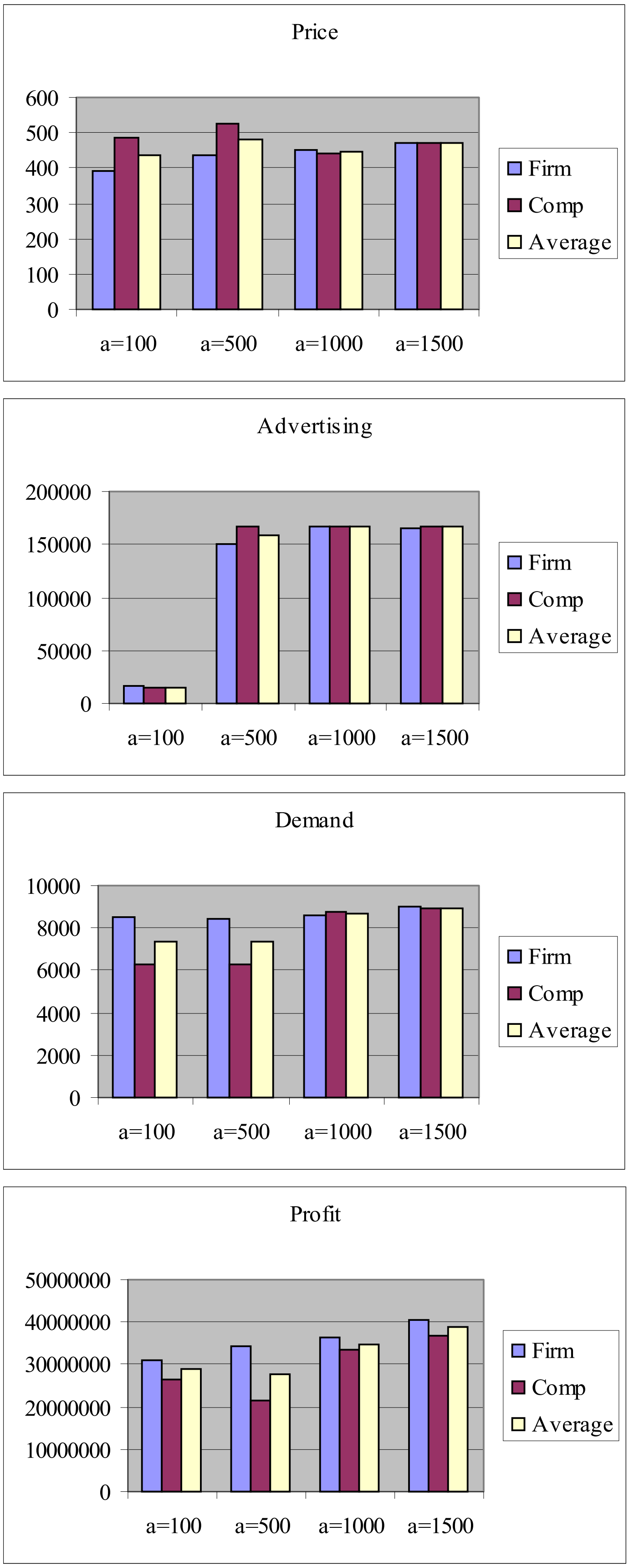

Figure 5.11 Experimental results for different upper limit coefficient for advertising affected demand

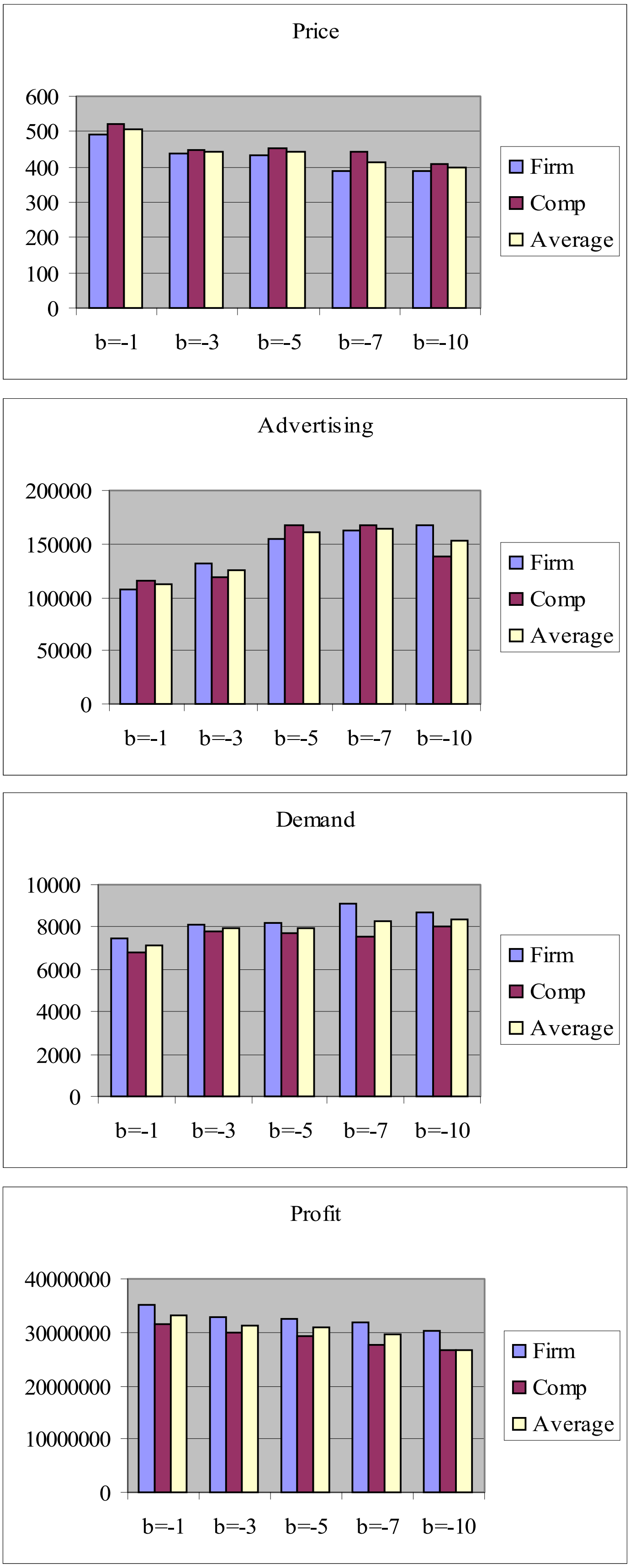

Figure 5.12 Experimental result for different shifting parameter for marginal advertising curve

5.2. Conclusions

A large number of experiments using different parameter sets are made. As a consequence of the non-linearity of the model there appear to be a large number of local optima in the solution space. The results of GAMS® and the solutions obtained by using a GA developed to treat this problem are compared in terms of robustness. The robustness is used to imply here a set of consistent solutions under different initial conditions and parameter values resulting in similar solutions in terms of general underlying decision structure comprised of pricing, production and advertising decisions.

It is observed that GA solutions are consistent in most cases. On the other hand, GAMS® produces a set of various solutions, which are not consistent in the above defined sense. There is a need to provide an initial feasible solution for GAMS®. The problem is solved for a large number of different initial feasible solutions. It is observed that in most cases GAMS does not produce acceptable solutions, since these solutions are inferior in terms of their objective value to the solutions produced by GA. Different solvers provided with GAMS® are used with different initial feasible solutions. However, in many experiments GAMS® resulted in solutions that are very close to the initial feasible solution provided, thus with only little improvement. In addition, it is observed that GAMS® solutions are highly dependent on the initial feasible solution. On the other hand, GA is more successful in reaching better solutions, although global optimality of its results is not guaranteed. The use of random keys makes it easier to search the solution space by crossover and mutation operations. One important point about GA is that the best solution earned at a step of the solution cycle is included into the first generation of the next step of the same firm. This policy prevents the possible loss of best solutions between the steps of the cycle.

The impacts of 5 analyzed parameters on the pricing and advertising decisions, demand volumes, and profits are represented in this Chapter. It is observed that price elasticity has the greatest effect on these values. This is a result of the demand function, since the price elasticity directly affects the customer type, which depends on the price of the product of the firm. However, price elasticity analysis is done in order to investigate the responses of the model according to these changes. In reality, changes in price elasticity value in such amounts can only be observed when different products are considered. Cross-moving demand parameter has the second greatest effect on results. The importance of price difference increases too much, as this parameter value increases. This parameter reflects the number of potential customers mainly concerning the price difference, and the importance of this kind of customers in total demand generation increases. Firms take these customers' priorities into consideration more as the parameter value increases. So, the impact of this parameter is relatively higher than the others, except the impact of price elasticity. Advertising weights' impact is not negligible on decisions of the firms, since these parameters have high effects on the benefits of advertising actions. Increase in advertising weights lets firms to charge higher prices, since it becomes easier to maintain the demand with the support of advertising activities. The impacts of ratio coefficient for competitor advertising effect and reverse cross-price elasticity are also important although they are less than the impacts of the other parameters. The importance and the effects of competitor's advertising and pricing are affected by these parameters. Firms prefer lower prices for their products as the reverse cross-price elasticity decreases, since the increases in prices causes loss of more demand in this case. As the impact of competitor's advertising activities increases, the other firm prefers to charge a price closer to its competitor's price to prevent the customers, whose product awareness has been increased through its own advertising, to buy its competitor's product.

It is observed that although the importance of different parameters changes, all parameters have noticeable effects on the pricing and advertising decisions of the firms and on the demand and profit values, accordingly.

Production scheduling outputs are given in Appendices A - E. It is observed that GAMS® results are better than GA outputs in terms of inventory and backlog decisions. GAMS® resulted in solutions consisting of less inventory and backlog. In most cases, GAMS® does not provide any inventory or backlog in production plans. On the other hand, GA results include inventory and backlog but these volumes do not result in too much increase in production costs.

The effect of cost structure is analysed and it is observed that changes in cost structures result in different prices and advertising expenses in absolute terms. However, general underlying decision structure does not change significantly. When the cost structure of the competitor is altered, both firms' pricing, advertising, and production decisions change in the same structure as in the previous experiments of the standard cost structure. However, it is observed that firm uses the benefits of increasing price levels of its competitor. Although the firm increases its prices, it takes the advantage of additional demand generated as a result of the price difference although it increases its prices.

Finally, the logit function parameters, a and b, are changed over a wide range in order to investigate their effects on the firms' decisions. The parameter a affected the pricing and advertising decisions since it is directly related to the advertising affected sales. As the value of a is increased, it is observed that firms prefer advertising as a response to increasing benefits of advertising in terms of additional demand with less advertising expenses. Firms also increase their prices when the benefits of advertising increase. The parameter b affected the advertising decisions, since it is related to maximum marginal advertising affected demand value and needed advertising expenses to generate this maximum demand. As the value of b is decreased, corresponding to reduced advertising efficiency, firms prefer to charge lower prices to increase the benefits of advertising activities. With these parameters' analysis, it is observed that the decisions of the firms highly depends on the values of these parameters, since both of these parameters affects the potential benefits of advertising activities in addition to price levels.

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7. APPENDICES

Appendix A: Production Plans for Price Elasticity Experiments

GA Runs

Price Elasticity = 0.9						
	Firm			Competitor		
Period	**Production**	**Inventory**	**Backlog**	**Production**	**Inventory**	**Backlog**
1	8408.11	809.49	0	6627.38	0	89.88
2	6594.73	0	618.58	7103.48	0	492.51
3	9000	0	289.87	8528.85	575.88	0
4	7467.05	0	289.21	6893.5	0	728.48
5	9000	379.32	0	8309.98	0	127.61
6	8081.19	137.96	0	8495.53	247.37	0
7	7645.87	0	2.5	7306.05	0	206.09
8	7701.49	34.86	0	8167.48	31.22	0
9	7420.37	0	521.79	7893.57	84.69	0
10	9000	0	80.77	9000	1146.62	0
11	8731.95	220.54	0	6590.26	0	155.28
12	9000	0	0	7180.28	0	0
Costs	7602538	47465	36054.13	10207226	83430.85	53995.62
Total Costs	7686057			10344653		

Price Elasticity = 1.0						
Period	**Production**	**Inventory**	**Backlog**	**Production**	**Inventory**	**Backlog**
1	6965.14	72.95	0	5278.9	353.75	0
2	6707.25	32.63	0	4305.91	188.88	0
3	6657.79	36.11	0	4633.36	9.2	0
4	8877.67	0	140.43	3325.21	0	1515.12
5	8592.56	0	165.94	5739.31	146.7	0
6	6284.43	153.1	0	5167.65	169.95	0
7	8420.58	14.57	0	4310.32	0	379.77
8	6369.2	0	110.67	5978.92	214.05	0
9	5934.9	0	71.81	5062.77	0	316.5
10	6530.71	0	1129.28	5022.6	0	92.11
11	5676.26	0	920.28	6314.08	1078.82	0
12	6503.69	0	0	4269.96	0	0
Costs	6876009	9280.86	50767.89	7746935	86453.53	69105.08
Total Costs	6936058			7902493		

Price Elasticity = 1.1						
Period	Production	Inventory	Backlog	Production	Inventory	Backlog
1	6059.93	0	273.57	2007.06	0	654.48
2	4691.06	0	210.89	4140.79	57.23	0
3	5881.67	112.4	0	4233.75	706.79	0
4	5084.44	0	433.44	858.99	0	1491.44
5	4219.21	0	964.25	5108.94	0	65.44
6	6440.91	0	1.59	4678.87	1294.84	0
7	4831.15	3.33	0	699.17	0	731.39
8	6082.02	0	208.23	5013.69	947.5	0
9	5377.03	0	233.93	1193.87	0	1147.4
10	4952.4	0	454.06	4623.23	282.35	0
11	4565.18	0	71.49	720.21	0	2291.73
12	5199.65	0	0	4862.21	0	0
Costs	5869233	3472.02	57029.01	5333134	131548.2	191456.1
Total Costs	5929734			5656138		

GAMS Runs

Price Elasticity = 0.9						
	Firm			Competitor		
Period	Production	Inventory	Backlog	Production	Inventory	Backlog
1	8186.82	0	0	7017.89	0	0
2	8242.99	0	0	7154.02	0	0
3	8253.53	0	0	7358.46	0	0
4	8315.93	0	0	7419.96	0	0
5	8112.44	0	0	7374	0	0
6	8239.36	0	0	7427.49	0	0
7	8201.61	0	0	6939.32	0	0
8	8121.63	0	0	6872.34	0	0
9	8149.97	0	0	6869.47	0	0
10	8264.29	0	0	7190.99	0	0
11	8369.15	0	0	7443.59	0	0
12	8541.98	0	0	7668.72	0	0
Costs	7649985	0	0	9805219	0	0
Total Costs	7649985			9805219		

Price Elasticity = 1.0						
Period	**Production**	**Inventory**	**Backlog**	**Production**	**Inventory**	**Backlog**
1	7940.66	0	0	0	0	4117.3
2	7394.33	0	0	9000	0	0
3	7194.18	0	0	0	0	4150.37
4	7058	0	0	9000	0	0
5	7055.24	0	0	0	0	4138.48
6	7019.39	0	0	9000	0	0
7	7076.13	0	0	0	0	4113.75
8	7096.46	0	0	9000	0	0
9	7656.75	0	0	0	0	4082.76
10	7156.31	0	0	9000	0	0
11	7217.24	0	0	0	0	4065.7
12	7026.93	0	0	9000	0	0
Costs	7044581	0	0	7350000	0	740050.8
Total Costs	7044581			8090051		

Price Elasticity = 1.1						
Period	**Production**	**Inventory**	**Backlog**	**Production**	**Inventory**	**Backlog**
1	0	0	3136.78	0	0	2883.05
2	9000	2524.71	0	9000	2767.3	0
3	467.36	0	0	0	0	0
4	0	0	3149.8	0	0	2922.33
5	9000	2378.02	0	9000	2695.29	0
6	569.06	0	0	0	0	0
7	0	0	3030.49	0	0	2869.87
8	9000	2656.31	0	9000	2723.91	0
9	193.33	0	0	0	0	0
10	0	0	3082.43	0	0	2869.17
11	9000	2638.13	0	9000	2766.93	0
12	240.38	0	0	0	0	0
Costs	4573507	305915.1	247990	6000000	438137.2	346332.6
Total Costs	5127412			6784470		

Appendix B: Production Plans for Reverse Cross-Price Elasticity Experiments

GA Runs

Reverse Cross-Price Elasticity = 0.9						
	Firm			Competitor		
Period	**Production**	**Inventory**	**Backlog**	**Production**	**Inventory**	**Backlog**
1	8637.94	320.88	0	6907.12	0	394.58
2	9000	25	0	8158.99	37.24	0
3	7428.99	0	949.03	8053.36	49.02	0
4	9000	75	0	9000	0	15.8
5	9000	0	119.73	6837.88	47.35	0
6	9000	208.75	0	8614.06	87.41	0
7	7802.04	0	438.44	7908.47	0	113.9
8	9000	99.54	0	9000	91.97	0
9	9000	325.65	0	8130.42	83.47	0
10	7641.33	0	478.63	9000	0	131.48
11	9000	0	209.08	9000	480.91	0
12	8730.89	0	0	6850.47	0	0
Costs	7862060	31644.62	43898.32	10609558	35094.87	19672.81
Total Costs	7937603			10664326		

Reverse Cross-Price Elasticity = 1.0						
Period	**Production**	**Inventory**	**Backlog**	**Production**	**Inventory**	**Backlog**
1	7917.75	244.01	0	6994.51	0	69.75
2	7043.37	0	612.4	9000	68.62	0
3	9000	0	40.83	6640.42	0	541.18
4	8827.96	168.72	0	9000	0	90.02
5	9000	0	996.83	7397	0	76.53
6	9000	112.5	0	7534.53	0	214.92
7	8715.19	147.12	0	8649.72	194.31	0
8	7005.04	0	905.03	6982.16	0	508.74
9	9000	279.85	0	8238.8	321.98	0
10	9000	0	195.85	7010.02	178.19	0
11	7903.33	47.89	0	8141.03	211.34	0
12	9000	0	0	6530.02	0	0
Costs	7770632	30002.43	55018.66	10208866	38977.63	45034.18
Total Costs	7855653			10292878		

Reverse Cross-Price Elasticity = 1.1						
Period	**Production**	**Inventory**	**Backlog**	**Production**	**Inventory**	**Backlog**
1	9000	103.93	0	6174.99	0	265.13
2	7235.07	0	1122.91	7152.63	0	32.64
3	7308.38	0	1073.39	7629.83	0	1000.36
4	9000	0	15.59	8378.96	0	388.43
5	8460.45	106.79	0	8137.49	292.24	0
6	8086.37	12.6	0	7685.71	0	209.54
7	9000	18.21	0	6818.05	0	155.65
8	9000	0	71.21	7380.65	0	309.98
9	7619.83	0	505.35	7198.04	0	1220
10	8877.74	0	2.69	6681.32	0	1462.99
11	9000	0	567.89	8454.1	0	299.8
12	8336.39	0	0	8459.82	0	0
Costs	7746212	7245.99	67180.63	10061368	11689.58	160335.5
Total Costs	7820639			10233393		

GAMS Runs

Reverse Cross-Price Elasticity = 0.9						
	Firm			**Competitor**		
Period	**Production**	**Inventory**	**Backlog**	**Production**	**Inventory**	**Backlog**
1	8546.4	0	0	8310.24	0	0
2	8659.56	0	0	7655.96	0	0
3	8754.69	0	0	7728.57	0	0
4	8604.38	0	0	7616.79	0	0
5	8573.71	0	0	7169.63	0	0
6	8604.02	0	0	7584.95	0	0
7	8680.25	0	0	8134.5	0	0
8	8765.48	0	0	8373.94	0	0
9	8925.62	0	0	7565.8	0	0
10	8851.42	0	0	7675.82	0	0
11	8995.31	0	0	7647.33	0	0
12	8885.4	0	0	7159.39	0	0
Costs	7942312	0	0	10246719	0	0
Total Costs	7942312			10246719		

Reverse Cross-Price Elasticity = 1.0						
Period	**Production**	**Inventory**	**Backlog**	**Production**	**Inventory**	**Backlog**
1	8186.82	0	0	7017.89	0	0
2	8242.99	0	0	7154.02	0	0
3	8253.53	0	0	7358.46	0	0
4	8315.93	0	0	7419.96	0	0
5	8112.44	0	0	7374	0	0
6	8239.36	0	0	7427.49	0	0
7	8201.61	0	0	6939.32	0	0
8	8121.63	0	0	6872.34	0	0
9	8149.97	0	0	6869.47	0	0
10	8264.29	0	0	7190.99	0	0
11	8369.15	0	0	7443.59	0	0
12	8541.98	0	0	7668.72	0	0
Costs	7649985	0	0	9805219	0	0
Total Costs	7649985			9805219		

Reverse Cross-Price Elasticity = 1.1						
Period	**Production**	**Inventory**	**Backlog**	**Production**	**Inventory**	**Backlog**
1	7382.87	0	0	6776	0	0
2	7466.91	0	0	7030.86	0	0
3	7547.11	0	0	7177.78	0	0
4	7351.92	0	0	6726.57	0	0
5	7338.59	0	0	6985.49	0	0
6	7693.36	0	0	7104.57	0	0
7	7863.31	0	0	7204.34	0	0
8	7689.61	0	0	7230.24	0	0
9	7980	0	0	7180.8	0	0
10	8018.69	0	0	6754.34	0	0
11	8006.99	0	0	7038.17	0	0
12	8399.96	0	0	6672.08	0	0
Costs	7336966	0	0	9591093	0	0
Total Costs	7336966			9591093		

Appendix C: Production Plans for Impact of Advertising Weights Experiments

GA Runs

Advertising Lagged Effect Weights 1 / 0.1 / 0.01 / 0.001						
	Firm			Competitor		
Period	Production	Inventory	Backlog	Production	Inventory	Backlog
1	8521.5	239.2	0	6838.84	0	260.53
2	7832.31	0	605.95	8357.27	280.42	0
3	8965.61	0	148.53	9000	0	15.09
4	7815.64	26.2	0	7549.55	199.32	0
5	9000	0	321.49	6858.31	0	498.28
6	8476.79	99.18	0	7508.12	102.83	0
7	8601.3	59.67	0	8481.72	629.73	0
8	8318.29	0	760.81	6610.56	111.91	0
9	8126.85	0	206.34	9000	57.11	0
10	8133.78	432.1	0	9000	456.34	0
11	9000	222.32	0	8023	77.19	0
12	7756.16	0	0	6603.17	0	0
Costs	7727411	32360.18	40862.19	10337291	76593.61	23217.01
Total Costs	7800633			10437102		

Advertising Lagged Effect Weights 1 / 0.5 / 0.3 / 0.1						
Period	Production	Inventory	Backlog	Production	Inventory	Backlog
1	7646.99	0	243.08	7021.47	75.75	0
2	7930.5	437.96	0	9000	597.19	0
3	9000	0	110.25	9000	1342.67	0
4	7911.71	0	128.34	7000.03	57.58	0
5	7645.44	0	58.56	8425.06	669.3	0
6	7646.62	0	476.34	8768.06	829.38	0
7	8813.95	242.14	0	6187.7	0	351.21
8	7458.85	0	843.49	9000	19.15	0
9	9000	0	28.89	7455.67	234.05	0
10	7495.53	0	800.3	7219.72	157.5	0
11	8921.04	0	548.87	6522.8	0	657.13
12	9000	0	0	9000	0	0
Costs	7623532	20403.27	64762.49	10395038	159302.5	30250.27
Total Costs	7708698			10584591		

Advertising Lagged Effect Weights 1 / 0.9 / 0.75 / 0.55						
Period	Production	Inventory	Backlog	Production	Inventory	Backlog
1	6859.06	0	199.67	8652.62	243.21	0
2	8984.87	475.26	0	7540.83	0	166.74
3	7111.57	0	456.39	7661.35	36.54	0
4	9000	0	239.77	7554.6	49.72	0
5	8987.51	510.15	0	8533.56	763.76	0
6	7396.84	0	547.44	7067.06	0	52.01
7	8004.41	0	63.69	8689.9	349.17	0
8	8980.87	168.41	0	8178.76	683.48	0
9	7683.13	0	602	8125.21	0	43.57
10	8733.81	430.74	0	8998.83	467.56	0
11	8421.03	475.46	0	6624.85	0	400.8
12	7137.69	0	0	8089.78	0	0
Costs	7565040	61800.31	42179.06	10478803	103737.6	19893.35
Total Costs	7669020			10602434		

GAMS Runs

Advertising Lagged Effect Weights 1 / 0.1 / 0.01 / 0.001						
	Firm			Competitor		
Period	Production	Inventory	Backlog	Production	Inventory	Backlog
1	8424.28	0	0	7273.96	0	0
2	8429.73	0	0	9000	2049.61	0
3	8761	0	0	0	0	3234.61
4	8526.02	0	0	9000	0	0
5	8395.25	0	0	6756.4	0	0
6	8371.32	0	0	9000	2122.06	0
7	8522.89	0	0	0	0	3185.62
8	8396.1	0	0	9000	0	0
9	8389.37	0	0	9000	2131.53	0
10	8521.64	0	0	0	0	3180.13
11	8395.06	0	0	9000	0	0
12	8312.67	0	0	7265.81	0	0
Costs	7772267	0	0	8947213	252128	288010.8
Total Costs	7772267			9487352		

Advertising Lagged Effect Weights 1 / 0.5 / 0.3 / 0.1						
Period	**Production**	**Inventory**	**Backlog**	**Production**	**Inventory**	**Backlog**
1	8186.82	0	0	7017.89	0	0
2	8242.99	0	0	7154.02	0	0
3	8253.53	0	0	7358.46	0	0
4	8315.93	0	0	7419.96	0	0
5	8112.44	0	0	7374	0	0
6	8239.36	0	0	7427.49	0	0
7	8201.61	0	0	6939.32	0	0
8	8121.63	0	0	6872.34	0	0
9	8149.97	0	0	6869.47	0	0
10	8264.29	0	0	7190.99	0	0
11	8369.15	0	0	7443.59	0	0
12	8541.98	0	0	7668.72	0	0
Costs	7649985	0	0	9805219	0	0
Total Costs	7649985			9805219		

Advertising Lagged Effect Weights 1 / 0.9 / 0.75 / 0.55						
Period	**Production**	**Inventory**	**Backlog**	**Production**	**Inventory**	**Backlog**
1	8472.9	0	0	6543.54	0	0
2	7891.83	0	0	7066.7	0	0
3	7950.35	0	0	7467.52	0	0
4	8169.16	0	0	7744.2	0	0
5	8320.72	0	0	7766.65	0	0
6	7891.26	0	0	7306.9	0	0
7	7928.78	0	0	7362.14	0	0
8	8000.26	0	0	7439.68	0	0
9	8112.98	0	0	7539.85	0	0
10	8365.87	0	0	7815.17	0	0
11	8435.4	0	0	7374.1	0	0
12	8074.12	0	0	6997.25	0	0
Costs	7580682	0	0	9931778	0	0
Total Costs	7580682			9931778		

Appendix D: Production Plans for Ratio Coefficient for Competitor Advertising Effect Parameter Experiments

GA Runs

Ratio Coefficient for Competitor Advertising Effect Parameter = 0						
	Firm			Competitor		
Period	**Production**	**Inventory**	**Backlog**	**Production**	**Inventory**	**Backlog**
1	7716.74	0	313.6	6980.67	0	50.85
2	8213.53	0	159.37	6689.51	416.71	0
3	8330.23	0	55.79	6070.24	0	269.34
4	9000	123.81	0	7977.07	526.23	0
5	8445.44	3.4	0	6564.13	224	0
6	9000	0	6.34	6441.22	0	565.25
7	8019.37	0	431.87	8316.84	449.96	0
8	8111.66	0	175.74	7459.05	0	107.69
9	9000	0	100.76	7551.89	0	36.95
10	7381.4	0	1394.8	7464.6	389.53	0
11	9000	0	284.5	7098.87	721.45	0
12	8882.91	0	0	6340.59	0	0
Costs	7755064	3816.13	58455.39	9671600	109115.5	30902.59
Total Costs	7817335			9811618		

Ratio Coefficient for Competitor Advertising Effect Parameter = 0.1						
Period	**Production**	**Inventory**	**Backlog**	**Production**	**Inventory**	**Backlog**
1	7647.15	0	78.81	7226.21	0	45.9
2	8832.62	158.52	0	7539.2	1.81	0
3	8409.07	0	327.4	7358.78	107.76	0
4	8686.48	71.77	0	7215	0	373.22
5	9000	56.48	0	7144.07	154.04	0
6	8472.61	0	115.69	7143.03	32.49	0
7	7969.3	185.48	0	6315.84	0	1441.22
8	7828.84	0	314.96	9000	0	70.31
9	8046.39	21.78	0	8145.59	106.71	0
10	8073.02	0	133.17	7179.01	0	665.85
11	8724.12	1041.01	0	8697.28	70.97	0
12	9000	0	0	6732.71	0	0
Costs	7734480	46051.08	19400.56	10027255	18951.32	77894.91
Total Costs	7799932			10124101		

Ratio Coefficient for Competitor Advertising Effect Parameter = 0.5						
Period	Production	Inventory	Backlog	Production	Inventory	Backlog
1	8518.34	89.6	0	7577.88	333.08	0
2	8564.59	150.75	0	9000	0	73.23
3	9000	486.55	0	9000	0	76.77
4	9000	0	1403.57	9000	0	208.14
5	9000	3.34	0	9000	0	859.39
6	8021.23	262.04	0	8614.46	163.16	0
7	9000	304.65	0	9000	0	43.91
8	8094.01	0	779.44	7733.55	21.46	0
9	9000	280.37	0	8907.33	370.31	0
10	7906.29	127.76	0	7477.21	0	424.33
11	7232.86	58.03	0	9000	45.53	0
12	9000	0	0	7104.51	0	0
Costs	7816865	52892.41	43660.28	10906120	37341.26	50573.25
Total Costs	7913418			10994035		

GAMS Runs

Ratio Coefficient for Competitor Advertising Effect Parameter = 0						
	Firm			Competitor		
Period	Production	Inventory	Backlog	Production	Inventory	Backlog
1	7976.89	0	0	6706.49	0	0
2	8092.82	0	0	7015.83	0	0
3	8173.36	0	0	7232.62	0	0
4	8206.18	0	0	7362.9	0	0
5	8221.46	0	0	7401.68	0	0
6	7994.13	0	0	7357.7	0	0
7	8121.58	0	0	7378.53	0	0
8	8037.64	0	0	7331.38	0	0
9	8090.3	0	0	7391.3	0	0
10	8173	0	0	6975.02	0	0
11	8278.71	0	0	7272.11	0	0
12	8049.33	0	0	6907.31	0	0
Costs	7570770	0	0	9774965	0	0
Total Costs	7570770			9774965		

Ratio Coefficient for Competitor Advertising Effect Parameter = 0.1						
Period	**Production**	**Inventory**	**Backlog**	**Production**	**Inventory**	**Backlog**
1	8186.82	0	0	7017.89	0	0
2	8242.99	0	0	7154.02	0	0
3	8253.53	0	0	7358.46	0	0
4	8315.93	0	0	7419.96	0	0
5	8112.44	0	0	7374	0	0
6	8239.36	0	0	7427.49	0	0
7	8201.61	0	0	6939.32	0	0
8	8121.63	0	0	6872.34	0	0
9	8149.97	0	0	6869.47	0	0
10	8264.29	0	0	7190.99	0	0
11	8369.15	0	0	7443.59	0	0
12	8541.98	0	0	7668.72	0	0
Costs	7649985	0	0	9805219	0	0
Total Costs	7649985			9805219		

Ratio Coefficient for Competitor Advertising Effect Parameter = 0.5						
Period	**Production**	**Inventory**	**Backlog**	**Production**	**Inventory**	**Backlog**
1	8502.6	0	0	7182.55	0	0
2	8286.04	0	0	7254.16	0	0
3	8203.38	0	0	7537.39	0	0
4	8567.37	0	0	7394.73	0	0
5	8755.95	0	0	7507.91	0	0
6	9000	0	0	7377.93	0	0
7	8969.43	0	0	7185.17	0	0
8	9000	0	0	6872.85	0	0
9	8571.29	0	0	7213.95	0	0
10	8826.8	0	0	7093.01	0	0
11	9000	0	0	7252.25	0	0
12	9000	0	0	7667.57	0	0
Costs	7934143	0	0	9865460	0	0
Total Costs	7934143			9865460		

Appendix E: Production Plans for Cross-Moving Demand Experiments

GA Runs

Cross-Moving Demand = 0						
	Firm			Competitor		
Period	**Production**	**Inventory**	**Backlog**	**Production**	**Inventory**	**Backlog**
1	5381.05	0	28.08	7204.41	796.49	0
2	6645.63	261.62	0	8746.31	2527.59	0
3	6958.59	0	130.82	4602	0	135.22
4	5547.91	0	1442.29	7073.13	4.77	0
5	8471.03	153.18	0	6649.99	0	502.56
6	7555.26	0	551.13	7247.28	56.51	0
7	6424.78	0	1537.38	7263.46	566.4	0
8	8118.89	0	424.22	5986.15	0	479.48
9	7715.29	472.85	0	6747.45	0	494.17
10	5394.17	0	1403.37	7410.88	590.16	0
11	7305.43	0	918.77	5775.92	847.94	0
12	7273.33	0	0	4629.09	0	0
Costs	6839567	26629.51	128721.1	9250206	215594.3	48342.68
Total Costs	6994918			9514143		

Cross-Moving Demand = 1000						
Period	**Production**	**Inventory**	**Backlog**	**Production**	**Inventory**	**Backlog**
1	8065.73	748.92	0	8196.51	617.57	0
2	6505.36	280.24	0	7293.26	1037.99	0
3	6770.63	0	156.69	6346.86	0	280.26
4	7732.76	522.32	0	8579.48	362.51	0
5	6238.57	0	651.91	6526.82	224.32	0
6	8169.23	93.78	0	7137.23	82.83	0
7	7334.39	0	65.13	6794.05	0	302.64
8	8043.13	237.55	0	7670.83	0	297.31
9	7572.97	1262.1	0	8631.66	20.69	0
10	6156.72	245.73	0	7473.47	585.71	0
11	7585.07	61.61	0	6093.16	0	843.44
12	6892.96	0	0	7729.4	0	0
Costs	7053377	103567.3	17474.59	9935455	117264.6	51709.56
Total Costs	7174419			10104430		

Cross-Moving Demand = 2000						
Period	**Production**	**Inventory**	**Backlog**	**Production**	**Inventory**	**Backlog**
1	8109.07	0	37.06	7088.47	10.81	0
2	7625.9	0	693.82	7992.46	0	136.89
3	9000	0	246.37	7179.94	0	239.63
4	9000	94.45	0	7905.4	0	39.91
5	7993.21	0	130.3	7028.86	0	50.34
6	8722.08	42.72	0	7590.81	45.91	0
7	9000	6.95	0	7962.6	207.25	0
8	7149.96	0	138.58	7436.27	0	359.05
9	8740.5	0	25.53	6812.87	0	699.33
10	9000	126.88	0	8760.67	108.85	0
11	8108.15	129.07	0	6416.98	0	598.23
12	8360.01	0	0	6830.67	0	0
Costs	7740444	12001.93	25433.36	9975451	14912.77	63701.49
Total Costs	7777879			10054065		

Cross-Moving Demand = 5000						
Period	**Production**	**Inventory**	**Backlog**	**Production**	**Inventory**	**Backlog**
1	9000	99.43	0	9000	0	440.54
2	9000	0	1987.71	9000	0	124.44
3	9000	0	239.93	9000	0	67.33
4	9000	0	40.97	9000	209.5	0
5	9000	95.59	0	9000	0	2021.63
6	9000	0	543.18	9000	0	70.94
7	8994.82	496.73	0	9000	0	20.74
8	9000	0	154.47	8797.67	651.94	0
9	9000	5.88	0	9000	913.66	0
10	9000	0	1555.12	9000	0	246.43
11	9000	311.96	0	9000	0	413.09
12	9000	0	0	8970.37	0	0
Costs	8099741	30288.18	90427.45	11382603	71004.14	102154.4
Total Costs	8220456			11555762		

GAMS Runs

Cross-Moving Demand = 0						
	Firm			Competitor		
Period	**Production**	**Inventory**	**Backlog**	**Production**	**Inventory**	**Backlog**
1	1092.61	0	0	1092.61	0	0
2	1081.32	0	0	1081.32	0	0
3	1074.54	0	0	1074.54	0	0
4	1072.28	0	0	1072.28	0	0
5	1072.28	0	0	1072.28	0	0
6	1072.28	0	0	1072.28	0	0
7	1072.28	0	0	1072.28	0	0
8	1072.28	0	0	1072.28	0	0
9	1072.28	0	0	1072.28	0	0
10	1072.28	0	0	1072.28	0	0
11	1072.28	0	0	1072.28	0	0
12	1072.28	0	0	1072.28	0	0
Costs	1934849	0	0	2257323	0	0
Total Costs	1934849			2257323		

Cross-Moving Demand = 1000						
Period	**Production**	**Inventory**	**Backlog**	**Production**	**Inventory**	**Backlog**
1	6684.81	0	0	6460.27	0	0
2	6386.07	0	0	6756.66	0	0
3	6888.77	0	0	6911.32	0	0
4	7055.87	0	0	6407.86	0	0
5	7162.75	0	0	6705.61	0	0
6	7213.61	0	0	6816.81	0	0
7	6690.98	0	0	6920.71	0	0
8	6962.86	0	0	6418.36	0	0
9	7076.68	0	0	6719.12	0	0
10	7193.92	0	0	6844.34	0	0
11	7277.55	0	0	6978.08	0	0
12	6745.63	0	0	6469.42	0	0
Costs	6866975	0	0	9330642	0	0
Total Costs	6866975			9330642		

Cross-Moving Demand = 2000						
Period	**Production**	**Inventory**	**Backlog**	**Production**	**Inventory**	**Backlog**
1	8186.82	0	0	7017.89	0	0
2	8242.99	0	0	7154.02	0	0
3	8253.53	0	0	7358.46	0	0
4	8315.93	0	0	7419.96	0	0
5	8112.44	0	0	7374	0	0
6	8239.36	0	0	7427.49	0	0
7	8201.61	0	0	6939.32	0	0
8	8121.63	0	0	6872.34	0	0
9	8149.97	0	0	6869.47	0	0
10	8264.29	0	0	7190.99	0	0
11	8369.15	0	0	7443.59	0	0
12	8541.98	0	0	7668.72	0	0
Costs	7649985	0	0	9805219	0	0
Total Costs	7649985			9805219		

Cross-Moving Demand = 5000						
Period	**Production**	**Inventory**	**Backlog**	**Production**	**Inventory**	**Backlog**
1	9000	0	0	9000	0	0
2	9000	0	0	9000	0	0
3	9000	0	0	9000	0	0
4	9000	0	0	9000	0	0
5	9000	0	0	9000	0	0
6	9000	0	0	9000	0	0
7	9000	0	0	9000	0	0
8	9000	0	0	9000	0	0
9	9000	0	0	9000	0	0
10	9000	0	0	9000	0	0
11	9000	0	0	9000	0	0
12	9000	0	0	9000	0	0
Costs	8100000	0	0	11400000	0	0
Total Costs	8100000			11400000		