OPERATIONS REVERSAL: AN INVESTIGATION OF CAPACITATED AND UNCAPACITATED MODELS

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OPERATIONS REVERSAL: AN INVESTIGATION OF CAPACITATED AND UNCAPACITATED MODELS

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ABSTRACT

Increasing product and part variety is one of the most distinctive characteristics of industrial competition today. It is generally accepted that proliferation of products results in deterioration in manufacturing/logistics performance. Higher product variety leads to higher forecast errors, excessive inventory for some products, and shortages for others. In addition, high product overhead and administrative costs, higher manufacturing costs due to more specialized process materials, changeovers and quality assurance methods are encountered.

During a manufacturing process consisting of multiple stages products with high degree of variety are produced. Product may start as a common single engine and takes certain features or personalities at each stage. As more and more identities are added, total number of end products increases. Due to the nature of end products' demand, especially the variation from period to period the production volumes of intermediate stages in a production process vary.

Researchers have tried to eliminate the negative effects of high product variety. The strategies produced for reducing product variety and the level of complexity it possesses can be grouped into two categories, lead time reduction strategies and non-lead time reduction strategies. This thesis is motivated to analyze a non-lead time reduction strategy; operations reversal, which reengineers the manufacturing process by reversing two consecutive stages of the process. It is observed that operation reversal can lead to variance reduction and improves the efficiency of the process. Throughout this thesis, models that characterize the impact of operations reversal are developed and used to derive insights on when operations reversal would be beneficial.

ÖZET

Artan ürün ve parça çeşitliliği günümüz endüstriyel rekabet ortamının en belirgin özelliği olmaya başlamıştır. Genel olarak ürün çeşitliliğindeki artışın üretim ve lojistik alanındaki performansı düşürdüğü bilinmektedir. Yüksek ürün çeşitliliği ciddi öngörü hatalarına sebep olmakta, bunun sonucunda da envanter yönetiminde sorunlar yaşanmaktadır. Ek olarak, bu tür üretim özelleşmiş proses malzemeleri, teknikleri ve kalite güvence metodları gerektirdiğinden genel gider ve yönetim maliyetlerinde de artışa sebep olmaktadır.

Genellikle yüksek ürün çeşitliliği çok aşamalı üretim sistemlerini de beraberinde getirir. Ürün, son ürün haline gelirken üretim sisteminin her aşamasında yeni özellikler kazanır. Son ürünün talep yapısından, özellikle de peryotlar arasındaki değişkenliğinden dolayı ara üretim aşamalarındaki ürün hacimleri çeşitlilik sergilemektedir. Bu durum da yukarıda bahsi geçen proses maliyetlerini oluşturmaktadır.

Ürün çeşitliliğini ve beraberinde getirdiği zorlukları hafifletmek için geliştirilen yöntemleri iki kategoriye ayırabiliriz. Bunlar önsüre kısaltmaya yönelik olan ve olmayan stratejiler diye ikiye ayrılır. Bu çalışmanın konusu önsüre kısaltmaya yönelik olmayan *operasyon sırası değiştirme* yöntemidir. Bu yöntem üretim sürecindeki ardışık iki üretim aşamasının yerini değiştirerek sistemi yeniden yapılandırmakta, böylece varyans azalmakta ve sistemin verimliliği artmaktadır. Bu çalışmada üretim sistemleri modellenerek *operasyon sırası değiştirme* yönteminin etkilerine bakılmakta ve yöntemin hangi durumlarda yararlı olduğu araştırılmaktadır.

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1. INTRODUCTION

Driven by the market's pull for increasingly differentiated products and by the manufacturer's push to seek finely targeted niche segments, the variety of products offered in most industries has increased steadily over the last several decades. The pull comes from customers who seem to reward companies that can offer high variety while matching the price and quality of competitors with narrower product lines. Modern marketing methods accelerate this trend by identifying once obscure specifics of customer preferences. As more companies compete internationally, product markets become more crowded and product differentiation more important. The push comes from new firm capabilities as the increased sophistication and declining price of flexible, programmable automation bring the opportunity for greater product variety within the grasp of many companies. (Fisher, 1994)

Another approach defines product variety on two dimensions: the breadth of products that a firm offers at a given time and the rate at which the firm replaces existing products with the new products. Both dimensions of variety have steadily increased in many industries so that the managerial challenge is to provide the high degree of variety that seems necessary and also retain the scale of economies required for low cost.

The approaches companies have taken in coping with this challenge can be classified as process based or product based. Process based strategies are based on enabling production and distribution processes with sufficient flexibility to accomodate a high level of variety at a resonable cost. Product based strategies seek product designs that allow high variety in the marketplace while representing the production and distribution system with a relatively low level of component variety and assembly complexity.

Important consideration is also being given to mass customization strategies by companies today. Additional impetus is coming from mass merchandisers expecting deliveries in small quantities directly to stores with extremely high service levels, specialized packaging, and unique promotion combinations. These strategies result in sales growth or higher prices and presumed profit gained by meeting more specialized demands. However, such decisions can have adverse implications for manufacturing and distribution systems that are not always captured in cost, margin, and non-financial performance estimates for such strategies.

There is also an increasing evidence which reveals that achieving competitive advantage through increased product variety is heavily dependent on ensuring the proper alignment between the marketing and manufacturing strategies pursued by a company.

Increasing product and part variety is one of the most distinctive characteristics of industrial competititon today. Its increasing role as determinant of operations strategy and performance has been noted by a series of studies. A survey of 1400 business units suggested that broadening product lines have a positive impact in competitiveness and that the firms in many businesses have to increase their product variety to remain competitive. (Prasad, 1998)

Researchers from diverse fields such as economics, marketing, and operations management have studied product variety and define it as the breadth and the depth of product lines. The main focus is on the cost of variety within a company. Several empirical studies showed that greater product variety increases the cost by making the business more complex. The problem with this complexity is the uncertainty it brings. Uncertainties in demand realization, delays in deliveries, etc., lead to either excess inventories or stocks that are insufficient to meet customer demands.

In this thesis, the main purpose is to analyze non-lead time reduction strategies, especially operations reversal to improve operating efficiency. It is generally accepted that, higher product variety leads to higher forecast errors, excessive inventory for some products and shortages for others, higher overhead and administrative costs, and higher manufacturing costs due to more specialized process materials, changeovers, and quality assurance methods. In discussion of how to deal with the operational problems related to the complexity of product variety, the main starting point of the study is to reduce a 2-by-N manufacturing system's overall variance using operations reversal. Multiple performance measures including service levels and inventory carrying costs can be incorporated in the analysis of the system. However, for the sake of simplicity, system's variance is chosen as the main performance criterion for the base case. Then other performance criteria such as backorder levels, queue length, and server capacities are incorporated in the model.

The model under consideration is a manufacturing process that consists of two stages. At each stage, a particular feature of the product is defined through an installation or customization process. In one stage there are two features and in the other stage, there are multiple features. The decision is how to sequence the stages to improve system's efficiency, namely to reduce overall variance of the system.

It is assumed that, the process operates like a pull system and order-up-to inventory policy is used. In each time period, the production volumes for the second stage are such that they equal the demands for the end products of the previous period. Sum of variances of the production volumes at each stage will constitute the performance measure. The main focus is on the variability of the production volumes in the manufacturing process since it drives buffer inventory and high variability of production volumes are often associated with degradation of quality, process yields, machine down-times, and staff planning troubles.

In the next chapter a brief literature on product variety, its origins and development is presented. Then the strategies to reduce the impact of product variety on processes are investigated via various models, mostly emphasizing the terms *delayed differentiation, postponement*, *part commonality, process sequencing*, and particularly *operations reversal*.

In chapter 3, the model to which operations reversal is applied is discussed in detail. The discussion begins with the description of the two-stage uncapacitated model, which concentrates on the overall variability of the system. General system behavior for two choice three feature (2-by-3) and two choice four feature (2-by-4) cases are investigated in detail. Then a model is developed to include the more realistic capacitated case using simulation. The new model concentrates on more than one performance criterion: namely variance, queue lengths and backorder levels are also incorporated. The discussion continues with the comparison of results for the deterministic capacitated case and the uncapacitated case.

In chapter 4 the results of the study are briefly summarized and further research opportunities in operations reversal methodology are discussed.

2. LITERATURE REVIEW

In today's competitive global markets, marketing managers through strategies such as market segmentation and niche marketing often suggest an increase in product variety and range to satisfy specific groups of customers' needs or wants. Variety can be defined as the breadth of products that a firm offers at a given time and the rate at which the firm replaces existing products with the new products. Both dimensions of variety have steadily increased in many industries so that the managerial challenge now is how to provide the high degree of variety that seems necessary for competitive success while retaining the economies of scale required for low cost.

High degree of variety brings system complexity. Complexity starts at companies with high product varieties having problems in acquiring accurate demand forecasts for individual product groups. They also have to control proliferation of inventory and provide a high service level for customers. Strategies for reducing the level of complexity can be grouped into two categories: lead time reduction strategies and non-lead time reduction strategies. It is known that short-term forecasts are more accurate than long-term forecasts. Lead-time reduction is useful in the sense that it reduces forecast horizon. As the error between actual demand and forecast gets smaller the safety stock levels decrease. Production Line Structuring and Quick Response (QR) (A strategy for reducing inventories by cutting lead times in production and improving coordination between different stages of the supply chain) systems are two effective means of reducing total lead times. The main objective of this class of strategies is to reduce the complexity of the system by reducing the number of parts and processes and by mitigating the effect of uncertainties on total cost in the system. Part commonality, postponement, and process sequencing are some of non-lead time reduction strategies that delay product differentiation. Since Anderson (1950) introduced the concept of delayed product differentiation, it became an emerging means to address these challenges.

In many manufacturing processes, multiple end products may share some common components or processes at the initial stages of the process. At some point work-in-process specializes into different end products. This point is known as the point of differentiation. Doremalen and Fleuren (1991), Zinn and Bowersox (1988), and Zinn (1990) use this concept extensively in logistics and distribution processes.

There exists a fixed and variable cost associated with delayed product differentiation. Depending on the system settings, the unit processing cost and the unit inventory holding cost as well as the cost of redesign may change. Lee and Billington (1994) discuss the cost drivers of delayed product differentiation in detail. The benefit of delayed product differentiation in the form of inventory reduction or service improvement is similar to the pooling effect of multi-echelon inventory systems (Eppen and Shrage, 1981; Federgruen and Zipkin, 1984; and Schwarz, 1989). In multi-echelon inventory systems, a central warehouse procures products and in turn supplies multiple retailers. Lee (1996) describes a model that captures the inventory reduction of delayed product differentiation that is essentially an adaptation of the multi-echelon inventory system. The model, however, assumes that no buffer inventories are held until the end of production process. Hence, the only possible inventory savings can come from the reduction of finished goods inventory. Lee (1994) studies the benefits of postponement in make-to-order (MTO) and make-to-stock (MTS) systems that he defines. In the MTO system an intermediate stage of the product is made to stock and is customized into various end products on demand. In the production system it takes t time units to produce the generic intermediate product and $T-t$ time units to customize the intermediate product into end products. He shows that the value added to the intermediate product determines the cost effectiveness of postponement. The MTS system operates under centralized control and stocks only finished products. The total lead-time to manufacture each end product is T time units. Demand for the end product is normally distributed with mean μ and variance σ^2. The demand for each product is independent across time but may be correlated within a time unit. He shows that delaying product differentiation would always result in lower inventories of finished products. The savings are greater in cases where demands for the end products are negatively correlated. A real-world example of delayed product differentiation by means of deferring the localization process for the deskjet printer at Hewlett Packard, where inventories are kept in finished goods form, is reported in Lee et al. (1993).

Lee and Tang (1997) consider a model that captures the costs and benefits of redesign strategy. They consider a situation in which the company has determined its product portfolio and focus on the concept of redesigning the product or the production process so that the point of differentiation is delayed as late as possible. This redesign increases the flexibility of the process to cope with market uncertainties and lower the inventory for the same target level. They present the factors that are affected by product differentiation, such as design cost, processing cost, inventory cost at intermediate stages, lead times, etc. They formalize three basic approaches for delayed product differentiation that some companies use: standardization, modular design, and process resequencing. Standardization can be defined as replacing a family of products by a standard product, and modularity is placing functionality in modules which can easily be added to a product. Resequencing refers to modifying the order of product manufacturing steps so that those operations that result in the differentiation of specific items or products are postponed as much as possible.

Another way where reengineering the manufacturing process leads to better operational performance in terms of inventories, customer service level, and operating costs, is variance reduction. Since demands of the end products are highly variable from period to period, production volumes of the intermediate stages in the manufacturing process are also variable. Variability of the production volumes may add cost to the process. One way in which variability can be controlled by reengineering is reversing the sequence of two consecutive stages in a manufacturing process or supply chain. The Benetton case (Harvard Business School Note, 1990; Daprian, 1992) depicts an example of such an effort.

Benetton used to produce its sweaters by first dying yarns into different colors and then knitting the colored yarns into different end products. Mismatch of inventory of finished garments with different colors resulted in costly end of season markdowns.

Luciano Benetton reversed the dyeing and knitting operations in the supply chain with his reengineering effort. Bleached yarns are knitted into the different styles and sizes,

and then dyed into the different colors due to season's fashion preferences. This change significantly improved Benetton's operational performance.

Gupta and Krishnan (1996) also describe how resequencing the steps in the assembly of a fountain pen can lead to process improvements. In one sequence, the rib is assembled to the nib head, followed by the assembly of inner and outer bodies. In another sequence, the nib head is first assembled to the inner and outer bodies before adding the nib. Gupta and Krishnan (1996) show how these two sequences can have very different process flexibility and efficiency in the manufacturing of fountain pens.

Lee and Tang (1998) expand the study on postponement. Postponement results in smaller standard deviations of production volumes at intermediate stages of the manufacturing process primarily because of the risk pooling effect at the standardized stages. Lee and Tang (1998) discuss operations reversal as a mechanism to reduce variability of production volumes in the intermediate stages of a manufacturing process. They also explore the conditions under which reversal of two process stages is desirable.

In Lee and Tang (1998), a product is manufactured in two stages; at each stage a particular feature of the product is added. The feature at each stage is supposed to have only *two* variants. In the Benetton example, style and color may be the two features. In general, these features are denoted by A and B. The aggregate demand of all products in a period is a random variable N with mean μ and standard deviation σ. Each customer has a probability p (q) of choosing variant 1 of feature A (B) and probability $1-p$ $(1-q)$ of choosing variant two of feature A (B). In addition, a customer's choice probabilities of A are independent of his/her choice probabilities of B. The system variance is defined as the sum of the variances of in-process production volumes. Lee and Tang demonstrate that if aggregate demand over all end products is relatively stable $(\mu > \sigma^2)$ and the choice probabilities associated with feature A are more distinctive than the choice probabilities associated with feature B $(p(1-p) < q(1-q))$ then sequence A-B would result in a lower system variance than sequence *B-A* The reverse is true if $(\mu < \sigma^2)$. The authors state that the nature of total demand uncertainty is very critical in determining whether operations reversal is an effective means to reengineer the supply chain. They expand their results for multiple choice-two feature, two choice-multiple feature, and multiple choice-multiple feature systems. Also a model is developed in which the lead times are included. This model is developed for the case where second stage choice probabilities are independent.

The key limitation in Lee and Tang (1998) is the use of total variability as a performance measure. Kapuscinski and Tayur (1999) show that if the analysis of the model presented by Lee and Tang (1998) used standard deviation rather than variance some nonintuitive predictions of their analysis would have been eliminated.

Jain and Paul (2001) generalize the operations reversal process of Lee and Tang (1998) to explicitly incorporate two important characteristics of fashion goods markets: heterogeneity among customers and unpredictability of customer preferences. They present a new approach to modeling the operations reversal problem. Their model explicitly captures the cost implications of production volume vulnerability by permitting a direct comparison of safety stocks with and without operations reversal. In addition, instead of requiring distributional information on customers' choice probabilities, the model utilizes macro-level data on the distribution of fraction of aggregate demand for each variant.

Another approach to manage broader product lines through delayed differentiation is using vanilla boxes (Swaminathan and Tayur, 1998). Their work is motivated by the characteristics of an IBM product line. They argue that in an environment where demands are stochastic, it seems a good strategy to store inventory in the form of semi-finished products (vanilla boxes) that can serve more than one final product. Finding the optimal configurations and inventory levels of vanilla boxes is a challenging task. In their paper, they model the problem as a two-stage integer program with recourse. They propose an effective solution procedure by utilizing structural decomposition of the problem and (sub)gradient derivative methods. They provide insights on the effect of demand variance, correlation, and capacity limitations on the optimal configuration and inventory levels of vanilla boxes and the performance of a vanilla assembly process. In addition, they compare the performance of the vanilla assembly process to MTS and ATO (assemble-to-order) processes.

Another way to achieve delayed differentiation is to use part commonality / component sharing approach. Component sharing using the same version of a component across multiple products is increasingly viewed by companies as a way to offer high variety in the marketplace while retaining low variety in their operations. The commonality approach in automotive industry and including general structure of the industry and strategies to reduce product variety in mass production and especially in lean and craft production is examined by Fisher et al. (1994). Fischer et al.(1999), critically examine component sharing in automotive industry. They attempt to answer the following questions: What are the key drivers and trade-offs of component sharing decisions? How much variation exists in actual component sharing practice? And how can this variation be explained? To answer these questions they develop an analytical model of component sharing and show through empirical testing that this model explains much of the variation in sharing practice for automotive braking systems.

Aviv and Federgruen (2001) also explain and quantify the benefits of delayed differentiation. The paper characterizes the benefits in a more general setting. Parameters of the demand distribution fail to be known with accuracy or consecutive demands are correlated. In such a situation it is necessary to revise estimates of the parameters of demand distributions on the basis of observed demand data. They analyze these systems in a Bayesian framework, assuming that their initial information about the parameters of the demand distributions is characterized via prior distributions. They also characterize the structure of close to optimal ordering rules in these systems for a variety of types of order cost functions.

Another model by Mieghem and Dada (1999) examines postponement from a comparative point of view. The authors compare price versus production postponement and analyze possible postponement strategies in a two-stage decision model where firms make three decisions: capacity investment, production (inventory) quantity, and price.

They show how competition, uncertainty, and the timing of operational decisions influence the strategic investment decision of the firm. They also show that, in contrast

to production postponement, price postponement makes the investment and production decisions relatively insensitive to uncertainty. The result is that managers can make optimal capacity decisions by deterministic reasoning if they have some price flexibility. They point out consideration of a flexible ex-post pricing before production postponement reengineering is a high valued option.

A managerial approach to product variety by Da Silveira (1998) points out the importance of understanding how product variety may influence different levels of organizations, meaning to investigate the management of product variety from the assessment of market requirements within operations. In the research Da Silveira investigates five manufacturing companies, aiming to understand how they dealt with product variety requirements within different systems levels. Research findings lead to a framework for product variety management within strategy and operations.

Another approach to measure the success in managing product variety tries to measure the match of product variety with the supply chain structure (Randall and Ulrich, 2001). They examine the relation among product variety, supply chain structure, and firm performance and analyze product variety at the product attribute level, noting that the relative impact of variety on production and market median costs depends to a large extend on the attribute underlying variety. They argue that there is a coherent way to match product variety with supply chain structure. Empirical results suggest that firms which match supply chain structure to the type of product variety they offer outperform firms which fail to match such choices.

Berry and Cooper (1999) show that adding product variety can have adverse cost and margin implications when marketing and manufacturing strategies are misaligned. The critical strategic issues involve product pricing and manufacturing flexibility in the product mix. They report methods which provide a means of empirically diagnosing the degree of strategic mismatch using actual operating data.

Considering all these postponement strategies, results show that informational considerations have a paramount effect on the effectiveness of postponement strategies. Anand and Mendelson (1998) model a supply chain consisting of a production facility, a distribution center, and two different markets. Demand information is used to mitigate the effects of uncertainty in the output markets. They study the firm’s operational performance under alternative business processes, comparing early and delayed product differentiation. The comparison yields to the value of postponement. Results show that informational considerations enable researchers and decision makers to perform cost-benefit analysis and quantify the anticipated effects of implementing postponement strategies. They also provide qualitative guidance regarding factors that affect the value of postponement.

3. AN OPERATIONS REVERSAL MODEL

As mentioned in the previous chapter, Lee and Tang (1998) have discussed operations reversal as a mechanism to reduce variability of production volumes in the intermediate stages of a manufacturing process. This chapter is based on the model presented by Lee and Tang (1998) which explores the conditions under which reversal of two process stages is desirable. The model under consideration is a two-stage, uncapacitated process where each stage adds a specific feature of the end product.

The chapter first describes the two-stage, uncapacitated model where the variability of the end product options would not be affected by the operations reversal. The model concentrates on the variability of the intermediate stages, namely the first stage. Since each of the end products are being characterized by the choice defined on two features, i.e., A and B, the decision to be made is whether to sequence the supply chain such that A is installed first, followed by B, or vice versa. In Section 3.1, the overall system variance formulation for 2-by-2 case is derived. Then, in Section 3.2, the derivation is done for the 2-by-N case. The general behavior of 2-by-3 and 2-by-4 systems under this performance measure is investigated in Section 3.3. In Section 3.4, the base model is simulated to include more realistic capacitated case using Arena Simulation software. The new model has three performance criteria: variance, backorder level, and average queue length. The behavior of the new system is discussed in detail. Section 3.5 concentrates on comparison of the base model with the capacitated one. Hence, Arena is run for specific end product mix and results are compared with the formula derived in Section 3.1.

3.1. An Uncapacitated Two Stage Model: 2-by-2 System

As mentioned earlier, the manufacturing process consists of two stages. At each stage, a particular feature of the product is defined through an installation or customization process. In this study, the basic model with two features, each possessing two choices, is expanded with a more general case where one feature may have *N* choices, namely a 2-by-*N* system. The behavior of the 2-by-*N* system is compared with that of 2-by-2 system. To be able to analyze the results, one needs to know first the 2-by-2 system in detail. Thus, the model by Lee and Tang (1998) is first reviewed.

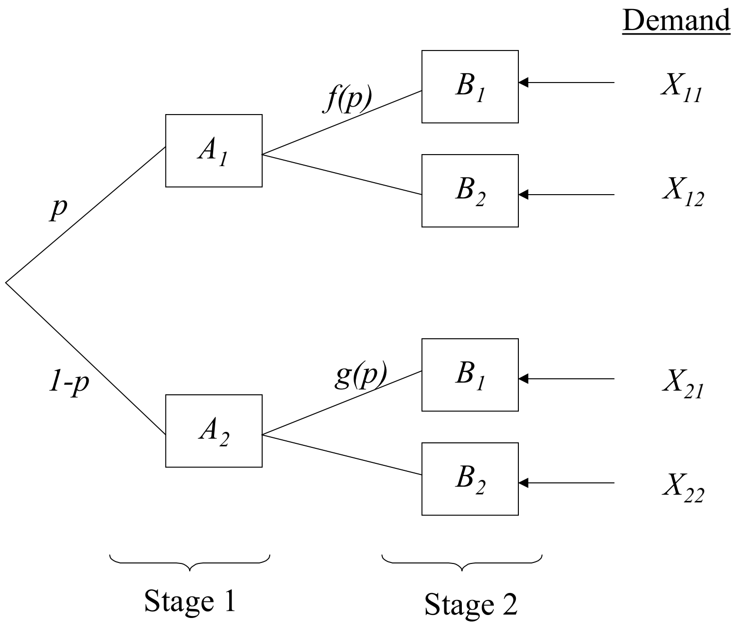


Figure 3.1 A two-stage uncapacitated model : 2-by-2 system

Figure 3.1 illustrates that in the first stage**,** there are two feature choices under consideration, A_1 and A_2. In the second stage, there are also two possible features choices, B_1 and B_2. Hence, there are four distinct end products, each being characterized by the choice defined on two features. The decision to be made is whether to sequence the supply chain such that *A* is installed first, followed by *B*, or vice versa.

For the above case, demands for the end product options $(X_{11}, X_{12}, X_{21}, X_{22})$ are random variables that are multinomially distributed with parameters $(N; \theta_{11}, \theta_{12}, \theta_{21}, \theta_{22})$.

N denotes the size of the total demand which follows a normal distribution with mean μ and standard deviation σ. It is also assumed that the total demands for different periods are independent.

$\boldsymbol{\theta_{ij}}$ denotes the probability that the customer will purchase the product that has choice *i* of feature *A* and choice *j* of feature *B*.
Then;

$$E(X_{ij} \mid N) = N\theta_{ij} \qquad Var(X_{ij} \mid N) = N\theta_{ij}(1-\theta_{ij})$$

$$Cov(X_{ij}, X_{mn} \mid N) = -N\theta_{ij}\theta_{mn} \qquad \text{for } \; ij \neq mn \tag{3.1}$$

The covariance formulation for the system shows that the demand is negatively correlated, which is a realistic assumption since these are substitute products.

The following notation will be used throughout the thesis:

p is the probability that a customer will purchase a product with choice 1 of feature *A* given that he/she will purchase a product.

f(p)**:** *Prob*(*B1*|*A1*) is the conditional probability that the customer buys the product with choice 1 of feature *B*, given that the customer has decided to purchase the product with choice 1 of feature *A*.

g(p)**:** *Prob*(*B1*|*A2*) is the conditional probability that the customer buys the product with choice 1 of feature *B*, given that the customer has decided to purchase the product with choice 2 of feature *A*.

C(p)**:** The difference between the overall variances of the two sequences.

$$C(p) = Var(A\text{-}B) - Var(B\text{-}A) \tag{3.2}$$

Variance of a process sequence *Var(A-B) or Var(B-A)* can be calculated using the following formula:

$$Var(X) = E(Var(X \mid N)) + Var(E(X \mid N)) \tag{3.3}$$

Using (3.1) and (3.2) *C(p)* value for the system can be calculated.

Given such a structure, the probabilities ϑ_{ij} are as follows:

$$\begin{aligned} \theta_{11} &= pf(p) & \theta_{12} &= p[1 - f(p)] \\ \theta_{21} &= (1-p)g(p) & \theta_{22} &= (1-p)[1 - g(p)] \end{aligned} \tag{3.4}$$

To calculate Var*(A-B)*, first $E(Var(X \mid N))$ is calculated. For the 2-by-2 system there are four distinct end products and the calculation of variance for the end products leaving A_1 of Stage 1 (upper branch) in Figure 3.1 is the same as end products leaving A_2 of Stage 1 (lower branch). Hence, the variance calculations are shown for upper branch only.

$$\begin{aligned} E(Var(X \mid N)) &= E(Var(X_{11} + X_{12} \mid N) + E(Var(X_{21} + X_{22} \mid N) \\ E(Var(X_{11} + X_{12} \mid N) &= E(Var(X_{11} \mid N) + Var(X_{12} \mid N) + 2Cov(X_{11} X_{12} \mid N) \\ &= E(N\theta_{11}(1-\theta_{11}) + (N\theta_{12}(1-\theta_{12}) - 2N\theta_{11}\theta_{12} \\ &= \mu(\theta_{11}(1-\theta_{11}) + \theta_{12}(1-\theta_{12}) - 2\theta_{11}\theta_{12}) \\ &= \mu(p(1-p) \end{aligned} \tag{3.5}$$

The total conditional variance for *A-B* is twice the conditional variance of the upper branch, and it is calculated as $2Np(1-p)$. The expectation of the conditional variance is $2\mu p(1-p)$.

To calculate $Var(E(X \mid N)$, upper and lower branch conditional expectations are calculated separately.

$$Var(E(X \mid N) = (E(X_{11} + X_{12} \mid N) + E(X_{21} + X_{22} \mid N))$$

$$\begin{aligned}(E(X_{11} + X_{12} \mid N)) &= Var(N\theta_{11} + N\theta_{12}) = Var(N(\theta_{11} + \theta_{12}) \\ &= \sigma^2(\theta_{11} + \theta_{12})^2 \\ &= \sigma^2 p^2 \end{aligned} \tag{3.6}$$

Hence, the upper branch variance of the conditional expectation is $\sigma^2 p^2$. The lower branch variance of the conditional expectation is calculated similarly and is found as $\sigma^2 + (1-p)^2$.

Thus, the total variance measure for *A-B* is :

$$2(\mu - \sigma^2)p(1-p) + \sigma^2 \tag{3.7}$$

For the sequence *B-A* total variance is:

$$2(\mu - \sigma^2)[pf(p) + (1-p)g(p)] \cdot [1 - [pf(p) + (1-p)g(p)]] + \sigma^2 \tag{3.8}$$

Subtracting (3.6) from (3.5) it is observed that the sequence *A-B* has a smaller variance than *B-A* if *C(p)<0*, where

$$C(p) = (\mu - \sigma^2)[p(1-p) - [pf(p) + (1-p)g(p)] \cdot [1 - [pf(p) + (1-p)g(p)]]] \tag{3.9}$$

From (3.7) it seen that due to $(\mu\text{-}\sigma^2)$ factor the sequencing of features is immaterial when $\mu=\sigma^2$ since *C(p)* equals zero.

The same model can be extended to include cases where the choice selection of one feature is independent of that of the other feature. In the independent case, *f(p)=g(p)=q*, where *q* is the probability of a customer selecting choice 1 of feature *B* and is a constant and independent of *p*. Here, $C(p)=(\mu-\sigma^2)[p(1-p)-\cdot q(1-q)]$. Hence, when $\mu>\sigma^2$ the sequence *A-B* has a lower total variance as long as the choice probabilities associated with feature *A* are more distinctive than feature *B*. This implies that one can lower the total variance of the system if the features with more distinctive choice probabilities are processed first.

3.2. An Uncapacitated Two Stage Model: 2-by-*N* System

In the 2-by-*N* system, second stage has *N* possible features. Therefore, there are *2N* distinct end products each being characterized by the choice defined on two features. Again the decision is whether to sequence the supply chain such that *A* is installed first, followed by *B*, or vice versa. What makes a sequence preferable over another is the total variability it possesses. As explained above, the variability measures of the two sequences with constant demand parameters are calculated and then the difference is calculated as *C(p)*.

The general variance formula (3.3) used for the 2-by-2 depicted in Figure 3.2 case can be formulated as follows:

$$\left(Var\left(\sum_{i=1}^{2}\sum_{j=1}^{N} X_{ij}\right)\right)=E\begin{bmatrix} Var\left(\sum_{i=1}^{N}\sum_{j=1}^{N} X_{if}\mid N\right)+2Cov\left(\sum_{i=1}^{N}\sum_{j=i+1}^{N} X_{1i}X_{1j}\right) \\ +2Cov\left(\sum_{i=1}^{N}\sum_{j=i+1}^{N} X_{2i}X_{2j}\right) \end{bmatrix}$$

$$+Var\left[\left(\sum_{i=1}^{N} E\left(X_{i1}+X_{i2}\mid N\right)\right)\right] \tag{3.10}$$

The formulation of the model for the 2-by-*N* system for sequence *A-B* is:

$$Var(X) = \mu\left(\sum_{i=1}^{2}\sum_{j=1}^{N}\theta_{ij}(1-\theta_{ij}) - 2\sum_{i=1}^{N}\sum_{j=1+1}^{N}\theta_{1i}\theta_{1j} - 2\sum_{i=1}^{N}\sum_{j=1+1}^{N}\theta_{2i}\theta_{2j}\right)$$

$$+\sigma^2\left(\left(\sum_{i=1}^{N}\theta_{1i}\right)^2 + \left(\sum_{i=1}^{N}\theta_{2i}\right)^2\right) \qquad (3.11)$$

Given that

$$\theta_{1j} = pa_j \qquad \theta_{2j} = (1-p)b_j \qquad (3.12)$$

a_j and b_j are the conditional end product probabilities of the second stage as shown in Figure 3.2.

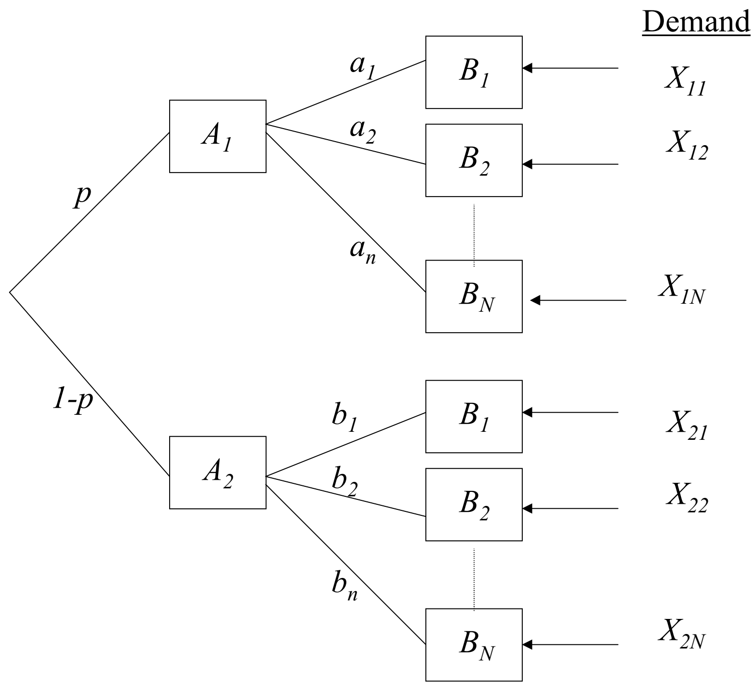


Figure 3.2 The model of 2-by-*N* uncapacitated system

The formulation of the 2-by-*N* system for sequence *B-A* is the same as formula (3.10), with new θ values. The derivation is carried out using conditional probability analysis and is given in Appendix A. The new values are:

$$\theta_{j1} = pa_j \qquad \theta_{j2} = (1-p)b_j \qquad (3.13)$$

The formulation of the model for the 2-by-*N* system for sequence *B-A* is:

$$Var(X)=\mu\sum_{j=1}^{N}\theta_{j1}\left(1-\theta_{j1}\right)+\theta_{j2}\left(1-\theta_{j2}\right)-2\theta_{j1}\theta_{j2}+\sigma^{2}\sum_{j=1}^{N}\left(\theta_{j1}+\theta_{j2}\right)^{2} \quad (3.14)$$

The sign of *C(p)* indicates whether the operations reversal is beneficial or not. If *C(p)*<0 then the sequence *B-A* has a larger variance so the reversal does not improve system efficiency. If *C(p)*>0 then reversing the consecutive operations (*A-B* to *B-A*) decreases variability.

The area of the interest for the 2-by-*N* case is whether *C(p)* term includes $\left(\mu-\sigma^{2}\right)$ as a factor or not. The importance of the derivation is its assistance to guess the behavior of a 2-by-*N* model since it also helps to distinguish the behavior of 2-by-2 model for end product demand and standard deviations.

The derivation of *C(p)* for the 2-by-*N* case is as follows:

Since *C(p)* is *Var(A-B) -Var(B-A)* it can be calculated by subtracting (3.14) from (3.11);

$$\begin{aligned}C(p)=\mu&\left(\sum_{i=1}^{2}\sum_{j=1}^{N}\theta_{ij}\left(1-\theta_{ij}\right)-2\sum_{i=1}^{N}\sum_{j=1+1}^{N}\theta_{1i}\theta_{1j}-2\sum_{i=1}^{N}\sum_{j=1+1}^{N}\theta_{2i}\theta_{2j}\right)\\&+\mu\sum_{j=1}^{N}\theta_{j1}\left(1-\theta_{j1}\right)+\theta_{j2}\left(1-\theta_{j2}\right)-2\theta_{j1}\theta_{j2}+\sigma^{2}\sum_{j=1}^{N}\left(\theta_{j1}+\theta_{j2}\right)^{2}\\&+\sigma^{2}\left(\left(\sum_{i=1}^{N}\theta_{1i}\right)^{2}+\left(\sum_{i=1}^{N}\theta_{2i}\right)^{2}\right) \quad (3.15)\end{aligned}$$

Gathering all terms including μ in one bracket and all terms including σ^2 in another bracket gives:

$$C(p)=\mu\left[\begin{aligned}&\sum_{i=1}^{N}\theta_{1i}-\theta_{1i}^{2}-2\sum_{i\neq1}^{N}\sum_{k=i+1}^{N}\theta_{1i}\theta_{1k}+\sum_{i=1}^{N}\theta_{2i}-\theta_{2i}^{2}\\&-2\sum_{i\neq1}^{N}\sum_{k=i+1}^{N}\theta_{2i}\theta_{2k}-\sum_{i=1}^{N}\theta_{i1}-\theta_{i1}^{2}-\sum_{i=1}^{N}\theta_{i2}-\theta_{i2}^{2}+2\sum_{i=1}^{N}\theta_{i1}\theta_{i2}\end{aligned}\right]$$

$$+\sigma^2\left[\left(\sum_{i=1}^{N}\theta_{1i}\right)^2+\left(\sum_{i=1}^{N}\theta_{2i}\right)^2-\sum_{i=1}^{N}(\theta_{i1}+\theta_{i2})^2\right] \quad (3.16)$$

It is known from (3.10) and (3.11) that $\theta_{nj} = \theta_{jn}$. Thus the first, third, fifth and sixth terms in the first bracket cancel out. The resulting equation is as follows:

$$C(p)=\mu\left[-2\sum_{i\neq 1}^{N}\sum_{k=i+1}^{N}\theta_{1i}\theta_{1k}-2\sum_{i\neq 1}^{N}\sum_{k=i+1}^{N}\theta_{2i}\ \theta_{2k}+2\sum_{i=1}^{N}\theta_{i1}\theta_{i2}\right.$$
$$\left.+\sigma^2\left[\left(\sum_{i=1}^{N}\theta_{1i}\right)^2+\left(\sum_{i=1}^{N}\theta_{2i}\right)^2-\left(\sum_{i=1}^{N}(\theta_{i1}+\theta_{i2})\right)^2\right]\right]$$

(3.17)

Using the substitution,

$$(a_1+a_2+.....+a_n)^2 = (\,a_1^2+a_2^2+...+a_n^2+2a_1a_2+2a_1a_3+....+2a_{n-1}a_n)$$

The term multiplied with σ^2 becomes,

$$\left[\begin{array}{l}\sum_{i=1}^{N}\theta_{1i}{}^2+2\sum_{i\neq 1}^{N}\sum_{k=i+1}^{N}\theta_{1i}\theta_{1k}+\sum_{i=1}^{N}\theta_{2i}{}^2+2\sum_{i\neq 1}^{N}\sum_{k=i+1}^{N}\theta_{2i}\theta_{2k}\\ \quad -\sum_{i=1}^{N}\theta_{1i}{}^2-\sum_{i=1}^{N}\theta_{2i}{}^2-2\sum_{i=1}^{N}\theta_{i1}\theta_{i2}\end{array}\right] \quad (3.18)$$

The first, third, fifth and sixth terms cancel out and the resulting *C(p)* equation is as follows:

$$C(p)=\left(\mu-\sigma^2\right)\left[-2\left(\sum_{i\neq 1}^{N}\sum_{k=i+1}^{N}\theta_{1i}\theta_{1i}+\sum_{i=1}^{N}\sum_{k=i+1}^{N}\theta_{2i}\theta_{2k}-\sum_{i=1}^{N}\theta_{i1}\theta_{i2}\right)\right] \quad (3.19)$$

Formula (3.19) shows that *C(p)* term includes $\left(\mu-\sigma^2\right)$ as a factor. Thus the benefit of operations reversal can be related to the magnitude of μ and σ^2 values. In the following section the overall variance of the system is calculated for different $\left(\mu-\sigma^2\right)$ values and first and second stage branching probabilities. To be able to investigate the behavior of *C(p)* in detail the second stage with *N* different branching probabilities is analyzed. For 2-by-3 and 2-by-4 systems all probability combinations within a step size of 0.1 are investigated and the resulting average *C(p)*s are depicted in Figures 3.3 through 3.7.

3.3. Analysis of the 2-by-*N* Uncapacitated System

In this section the outputs of the 2-by-*N* model is simulated using C++. The objective of simulating the 2-by-*N* system is to investigate the behavior of *C(p)* with respect to various parameter values. The main branch probabilities vary from 0.1 to 0.5 with 0.1 increment and the second branch probabilities are assigned as shown in the flowing program routine covering the whole solution space with 0.01 increment.

The pseudo-code in Appendix B is a sample subroutine showing the probability assignments of the first and second branches of the 2-by-4 model for a given demand and variance combination of end product. The outmost loop assigns the *p* value, second loop assigns first of the upper second branch's four probabilities which is denoted as a_1 in Figure 3.2. When a_1 gets the value 0.01 in the first run a_2=0.01 a_3=0.01 and a_4=0.97 and the second branch probabilities, b_n are also 0.01, 0.01, 0.01, 0.97 respectively. In the second run all probabilities are same except b_3= 0.02 and b_4=0.96. 62720 runs are made to cover all probability pairs for each of the (μ, σ^2) pairs of (100,1), (100,5), (100,20), (100,50) and (1,100), (5,100), (20,100), (50,100). Positive and negative *C(p)* averages are calculated and overall averages are depicted in Figures 3.3 through 3.6.

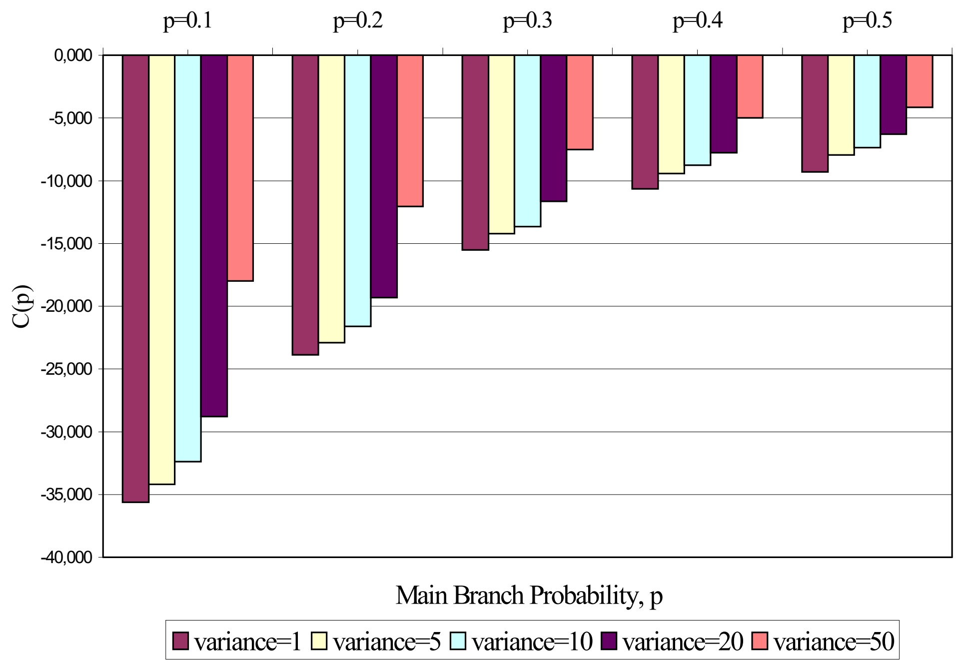


Figure 3.3 Average *C(p)* values for a 2-by-3 system at constant demand mean

In Figure 3.3 it can be noticed that as σ increases the absolute value of *C(p)* decreases regardless of the value of the main branch probability. All combinations point out that variance for sequence *A-B* is smaller than *B-A*. Hence, no operations reversal is beneficial. As seen in Table 3.1 that as *p* approaches to 0.5, the difference between two sequences decreases. This is also an intuitive result since when *p*=0.5 different features become almost interchangeable.

Table 3.1 Average *C(p)* values for a 2-by-3 system at constant demand mean

σ^2	*p*=0.1	*p*=0.2	*p*=0.3	*p*=0.4	*p*=0.5
1	-35,61	-23,87	-15,50	-10,63	-9,29
5	-34,17	-22,90	-14,21	-9,42	-7,93
10	-32,37	-21,60	-13,65	-8,76	-7,37
20	-28,77	-19,29	-11,63	-7,75	-6,30
50	-17,98	-12,05	-7,49	-5,00	-4,14

For the case of Figure 3.4, with the exception the case where the value of the demand mean is equal to the demand variance, all combinations have positive *C(p)*s, pointing out that at this fixed σ^2 value, operations reversal decreases variance considerably. Again, variance differences between two sequences decrease as p approaches to 0.5.

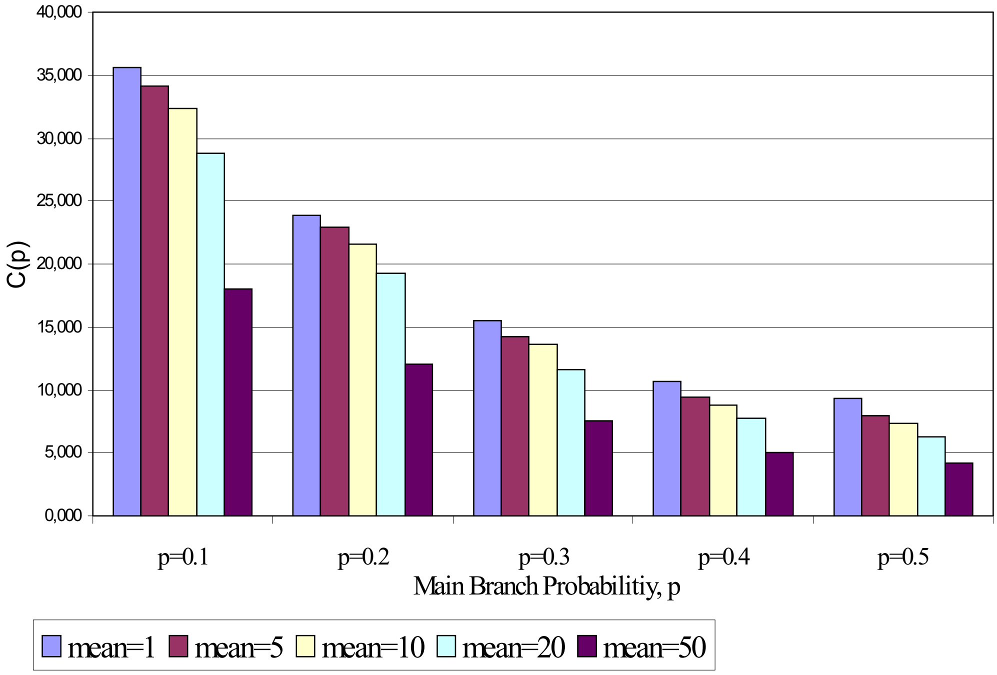


Figure 3.4 Average *C(p)* values for a 2-by-3 system at constant demand variance

For the case of 2-by-3 system at constant demand variance, the behavior of the model can be seen from Table 3.2.

Table 3.2. Average *C(p)* values for a 2-by-3 system at constant demand variance

μ	***p*=0.1**	***p*=0.2**	***p*=0.3**	***p*=0.4**	***p*=0.5**
1	35,61	23,87	15,50	10,63	9,29
5	34,17	22,90	14,21	9,42	7,93
10	32,37	21,60	13,65	8,76	7,37
20	28,77	19,29	11,63	7,75	6,30
50	17,98	12,05	7,49	5,00	4,14

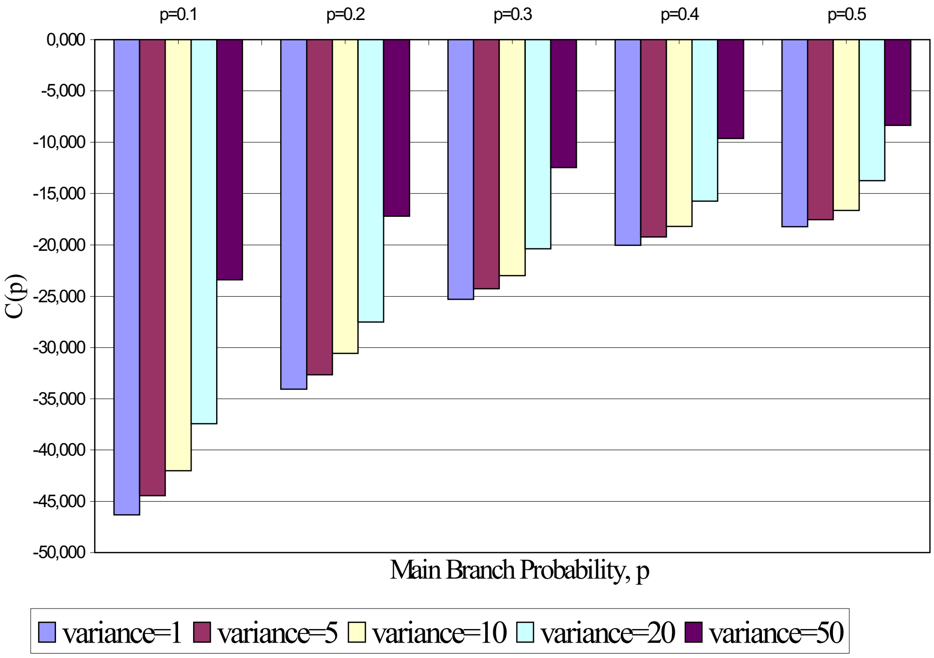


Figure 3.5 Average C(p) values for a 2-by-4 system at constant demand mean

Figure 3.5 shows average *C(p)* values for the 2-by-4 system at constant demand mean. The sign of *C(p)* for all *p* combinations is the same as that of 2-by-3 case. Hence, Figure 3.5 looks very similar to Figure 3.3. The main difference is the increase in the absolute value of *C(p)* which can be seen in Table 3.3. As *N* gets bigger, inappropriate choice of sequence of features has a deeper impact on the overall system variance. This is also an intuitive result since greater *N* means more end products and consequently the total variance is greater.

Table 3.3 Average *C(p)* values for a 2-by-4 system at constant demand mean

σ^2	***p*=0.1**	***p*=0.2**	***p*=0.3**	***p*=0.4**	***p*=0.5**
1	-46,31	-34,05	-25,29	-20,04	-18,23
5	-44,44	-32,67	-24,27	-19,23	-17,55
10	-42,01	-30,56	-22,99	-18,21	-16,62
20	-37,42	-27,51	-20,38	-15,73	-13,73
50	-23,39	-17,19	-12,46	-9,63	-8,36

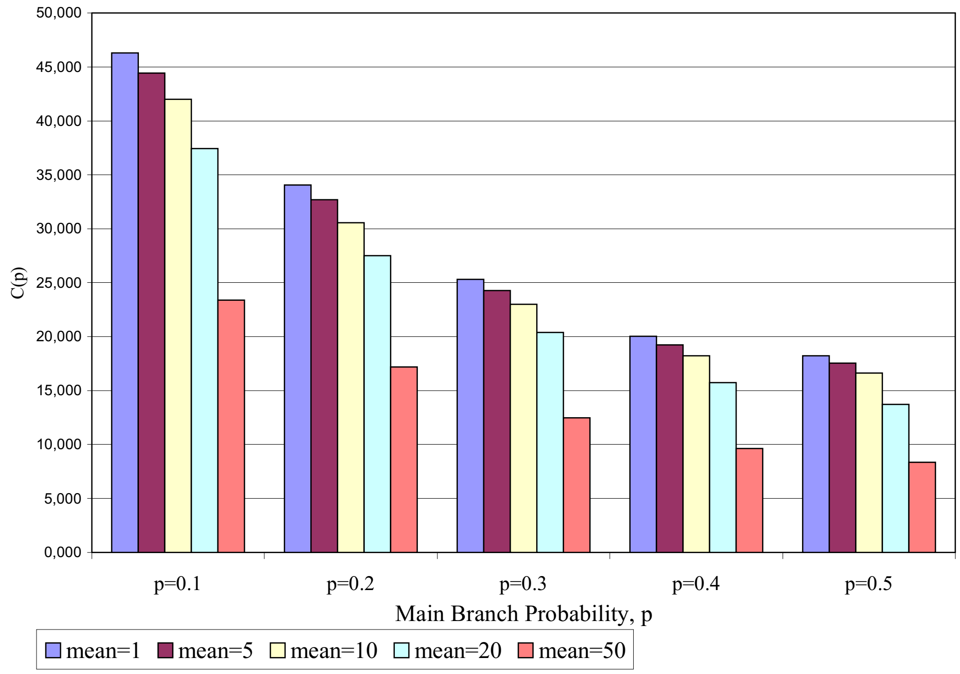


Figure 3.6 Average *C(p)* values for a 2-by-4 system at constant demand variance

Table 3.4 Average *C(p)* values for a 2-by-4 system at constant demand variance

μ	***p*=0.1**	***p*=0.2**	***p*=0.3**	***p*=0.4**	***p*=0.5**
1	46,31	34,05	25,29	20,04	18,23
5	44,44	32,67	24,27	19,23	17,55
10	42,01	30,56	22,99	18,21	16,62
20	37,42	27,51	20,38	15,73	13,73
50	23,39	17,19	12,46	9,63	8,36

Figure 3.6 follows exactly the same trends with Figure 3.4. The values of *C(p)* as can be seen from Table 3.4 are again larger then those of 2-by-3 model. From the graphs another expected result of formulation (3.17) is observed: *C(p)* value with $\mu=X$ and $\sigma^2=Y$ gives same absolute *C(p)* value with the pair $\mu=Y$ and $\sigma^2=X$.

The values calculated above are average results for all possible combinations of probability choices *A* and *B*. The general trends show that, when $\mu > \sigma^2$, the system has a lower variance with the sequence *A-B* and reversing the operations is not beneficial. However, when $\mu < \sigma^2$, it is beneficial to reverse the operations at every combination of probability, mean, and variance values.

When the individual *C(p)* values are further investigated, it is observed that at some probability combinations, especially when one end product has a very high probability to be produced than the other (such as *Prob*(*B1*|*A1*)=0.8, *Prob*(*B2*|*A1*)=0.1, and *Prob*(*B3*|*A1*)=0.1 for a 2-by-3 system), the *C(p)* value deviates from the general trends. The reversal is not only affected by the relative magnitudes of the means and variances but also by the homogeneity of the probabilities of the end products.

Another observation to mention is the change in the average value of *C(p)*s as the main branch probability, *p*, changes. For all $(\mu - \sigma^2)$ combinations the absolute value of *C(p)* decreases as the value of *p* changes from 0.1 to 0.5.

3.4. A Capacitated Model Analysis by Simulation for 2-by-2 System

In this section, the *2-by-2* system is modeled considering capacity limitations of the servers since individual features (A_1, A_2, B_1, B_2) are added to the end products during a period determined by the capacity and speed of the servers. Formula (3.11) is obtained under the assumption of infinite capacity. However, in real life applications, capacity is a vital problem having direct effects on queue lengths and work-in-process and buffer inventories. All these intermediate products carried in the inventories add up to the production cost, decreasing a firm's competitive advantage. Another important issue about the capacity limitation is the backorder level. High backorder levels are perceived as a firm's lack of liability, leading into customer dissatisfaction and loss of market share. The models are simulated using Arena, a software by systems Modeling Corp. The flow charts are displayed in Appendix C and Appendix D. These models consider all these important concepts and the obtained data are used to derive comparative results of manufacturing systems in terms of average queue length at each stage, backorder level and overall system variance.

The simulation model consists of a two-stage process and with two parallel servers with the same speed and capacity in each stage is modeled. Simulation is carried out covering 52-week production horizon. Demands for four different end products, $(A_1B_1, A_1B_2, A_2B_1, A_2B_2)$ are created via normal distribution. A week is assumed to have 40 working hours and demands are created for each week of the production horizon.

The probabilities of end products are sampled from a uniform distribution for each week. Average queue times, queue lengths, and server utilizations are measured. Backorders are monitored for each end product. The system is run for three specific demand sets with (μ,σ)[1] parameters of (100,5), (100,10), (100,20). Three specific server times (38, 43, and 48) are used to meet 80, 90, and 100% utilization levels.

To calculate the resulting *C(p)* values for each simulation run, two stages are reversed and the model is run with the initial seed for the given demand and server time values. Periodic variance data are calculated via collecting number of items processed by the servers of the first stage during each 480-minute period (a working day). Table 3.5 summarizes the parameters of the capacitated simulation model.

Table 3.5 Parameters of the capacitated model

# of Stages:	2
# of Servers:	4
Capacity of Servers:	1 item at a time
Speed of Servers:	38/ 43/ 48 minutes
Simulation Run:	1 production year/ 52 weeks
Demand Interval:	1 week
Demands:	$N(100,5)$, $N(100,10)$, $N(100,20)$
End Products:	$A_1B_1, A_1B_2, A_2B_1, A_2B_2$
Product Choice Probabilities:	U(0,1)

Throughout the thesis, the notation *(x,y)* is used to refer μ and σ parameters where $x=\mu$ and $y=\sigma$.

3.4.1. Investigation of the *C(p)* values

Since capacity issues are not incorporated while deriving the model in Section 3.1, it is of great interest to see if the result of the formulation of *C(p)* derived in the previous chapter is still valid for the system. Figures 3.7 through 3.9 are the graphs of the capacitated system's behavior under operations reversal at different mean and variations of demand.

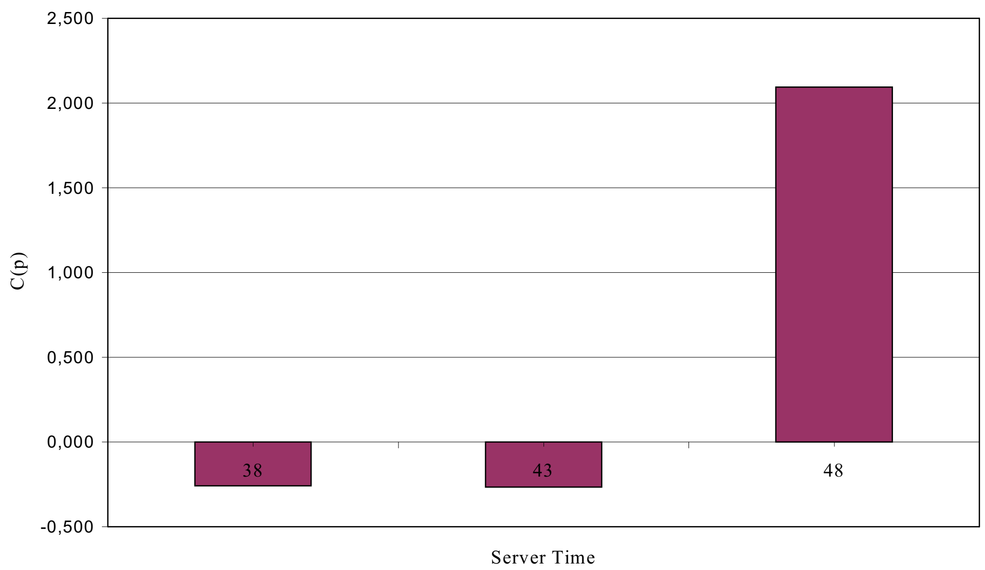


Figure 3.7 *C(p)* values for (100,5) case

Figure 3.7 shows how *C(p)* changes with changing server times for (100,5). *C(p)* value is negative and close to zero for server times 38 and 43, preferring sequence *A-B* and positive for server time 48 favoring sequence *B-A*. The reversal is beneficial only when server time is 48.

Figure 3.8 displays the outputs for (100,10) the system and exhibits the same tendency to operations reversal for all server times. In this case, reversal is not favored, *C(p)* value is very close to zero for server times 38 and 43, which is expected due to the $(\mu-\sigma^2)$ factor in the formulation of *C(p)*.

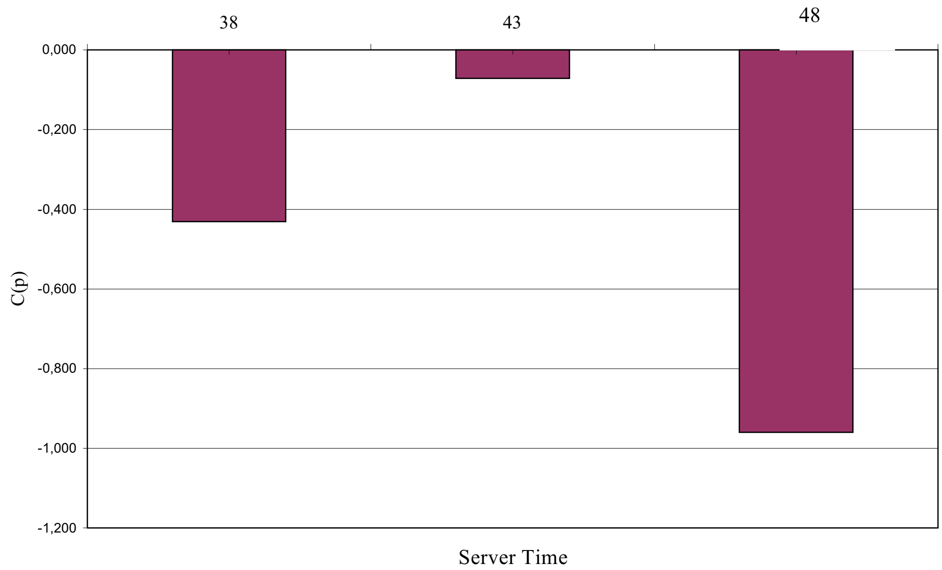


Figure 3.8 *C(p)* values for (100,10) case

Figure 3.9 indicates that as the value of σ^2 increases, the sequence *B-A* has a higher variance and operations reversal is less favored. For (100,20) case operations reversal is not favored for all server times. Another point of interest is the distinct increase in the absolute value of *C(p).* Since the variation of demand is larger than its mean the sign of *C(p)* is negative and operations reversal is not favored in accordance with formula (3.11). In addition, it is observed that server time, thus capacity limitation has a direct impact on the systems variation. Although it does not change the preferred sequence of the system, it affects the magnitude of overall variance of the system directly. Summary of the results of this section is given in Table 3.6.

Table 3.6 Comparison of operations preferred sequences for the capacitated model

σ	**Server Time**		
	38	**43**	**48**
5	A-B	A-B	B-A
10	A-B	A-B	A-B
20	A-B	A-B	A-B

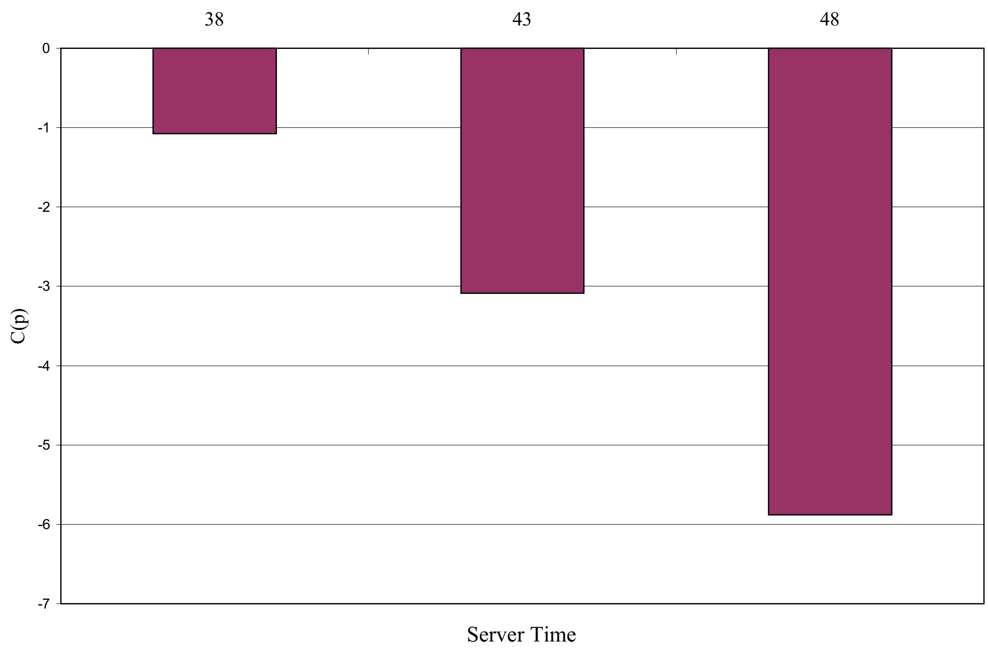


Figure 3.9 *C(p)* values for (100,20) case

3.4.2. Queue Lengths

Another measure to examine whether operations reversal is beneficial is the magnitude of the queue lengths of the servers. Although our model mainly focuses on the variance caused during the intermediate stages, it is of interest to investigate the number of items waiting in the queues due to the capacity of intermediate servers. In Figures 3.10 through 3.13 average queue lengths of the four servers in the two stages of the model are graphed for sequences *A-B* and *B-A* for various values of σ at constant server times.

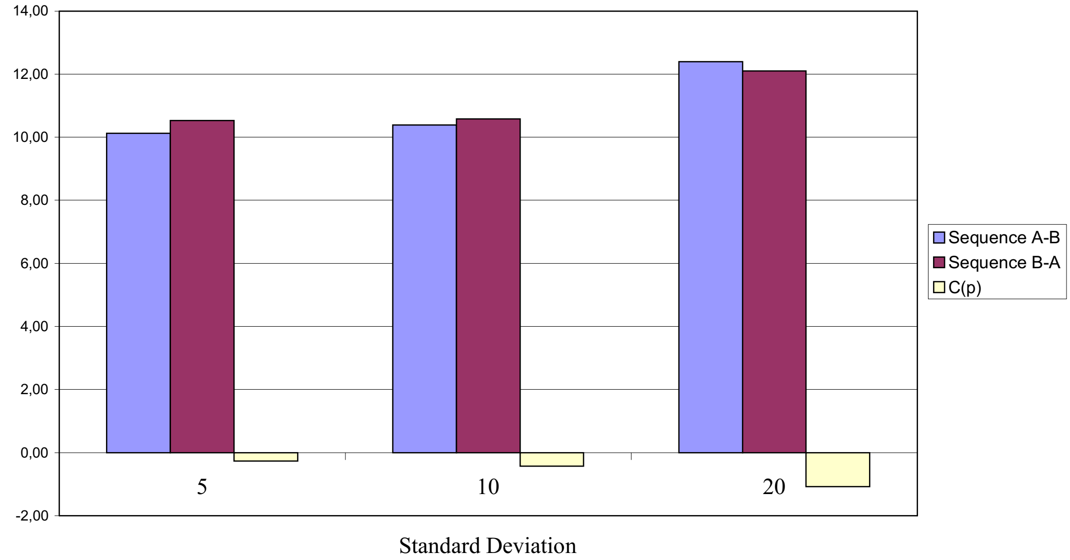


Figure 3.10 Queue lengths for *A-B* and *B-A* sequences when server time is 38

Figure 3.10 shows that when standard deviation of the demand is 5 or 10, average queue length for sequence *A-B* is shorter than sequence *B-A*. In addition, negative *C(p)* value also indicates that sequence *A-B* is better and operations reversal is favored by neither of the performance measures. However, for the case when standard deviation is 20, queue length for sequence *A-B* is longer than sequence *B-A*, though positive *C(p)* value favors sequence *A-B*.

In Figure 3.11, the server time is 43. For the set with a standard deviation of 5, the difference between queue lengths of the sequences *A-B* and *B-A* is positive favoring operations reversal, also positive *C(p)* value favors sequence *B-A*. For the set with standard deviation equal to 10, the length of the average server queues for sequence *A-B* is shorter than sequence *B-A,* and also negative *C(p)* value indicates that sequence *A-B* has a lower variance. Here, it should be noted that (Figures 3.10 and 3.11) the distinction between queue lengths and overall variance values for the two sequences is very negligible when $\sigma^2=\mu$. When $\sigma^2>\mu$, it is observed that two performance measures favor different sequences. Sequence *B-A* has a shorter queue but higher overall variance.

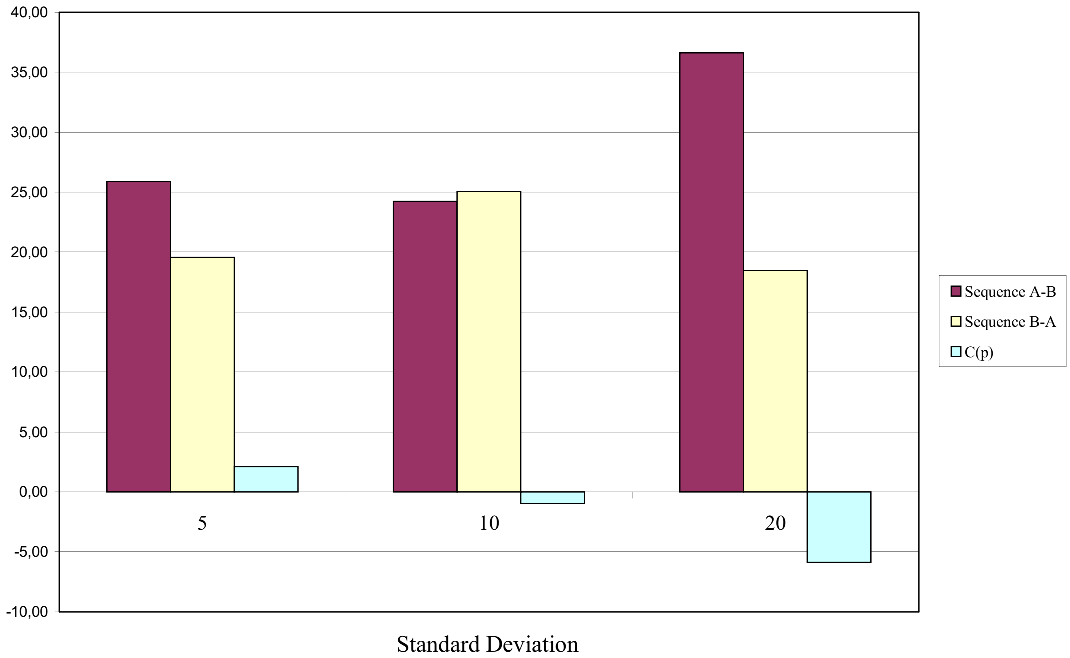


Figure 3.11 Queue length for *A*-*B* and *B*-*A* sequences when server time is 43

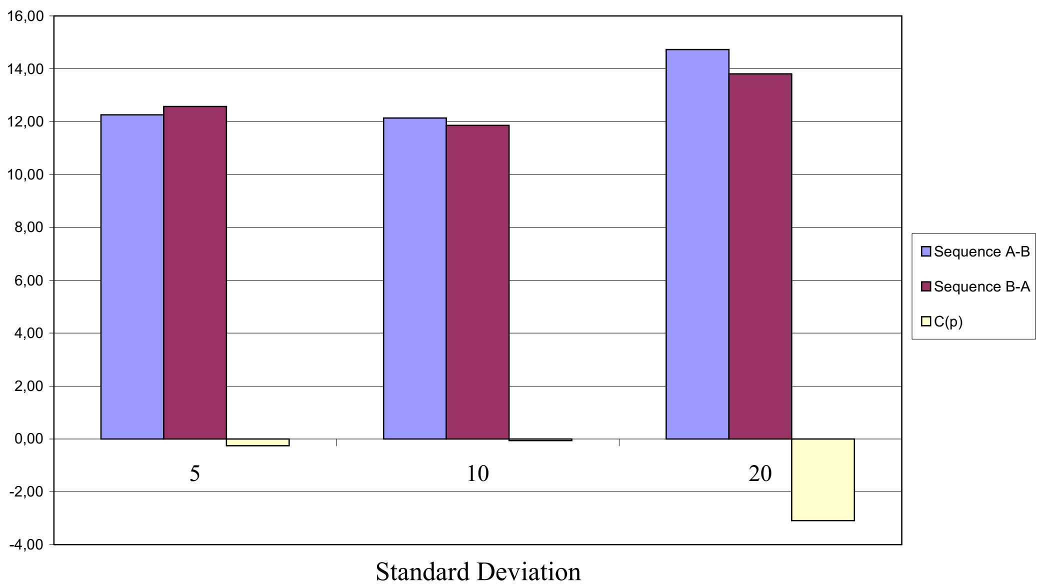


Figure 3.12 Queue length for *A*-*B* and *B*-*A* sequences when server time is 48

According to Figure 3.12, both measures indicate that sequence *B-A* is better when standard deviation is 5. When this value increases to 10, the queue length of sequence *B-A* is longer and also *C(p)* value favors *A-B*. For demand with standard deviation of 20, *C(p)* value favors sequence *A-B* although the queue length of *A-B* is twice as long as the queue length of *B-A*.

Table 3.7 shows the compliance of queue lengths and *C(p)* values for the capacitated model at three different server utilizations. Letter *Y* designates that two criteria agree and letter *N* designates that they disagree. Since for the pair with server time=48, σ=10 the calculated *C(p)* value is zero, that cell is left empty.

Table 3.7 Compliance of queue length and *C(p)* values for the capacitated model

σ	**Server Time**		
	38	**43**	**48**
5	Y	Y	Y
10	Y	Y	
20	N	N	N

When Figures 3.10, 3.11, and 3.12 are examined, it can easily be noticed that operations reversal has nearly no effect when $\mu=\sigma^2$. *C(p)* values are close to zero and queue lengths are nearly equal for two sequences. In addition, when $\mu\leq\sigma^2$, it is immaterial whether the queue length or *C(p)* value is taken as performance measure, since they both point at the same sequence of feature installations. However, when standard deviation is 20 the two performance measures disagree. For the three figures mentioned above, when $\sigma^2>\mu$, the performance criteria conflict at all server utilization levels. *C(p)* value favors the sequence with longer queue length.

To explore if the same situation holds for all server times between 43 and 48 (between 90% and 100% utilization) the model is again simulated in Arena. The results are depicted in Figure 3.13.

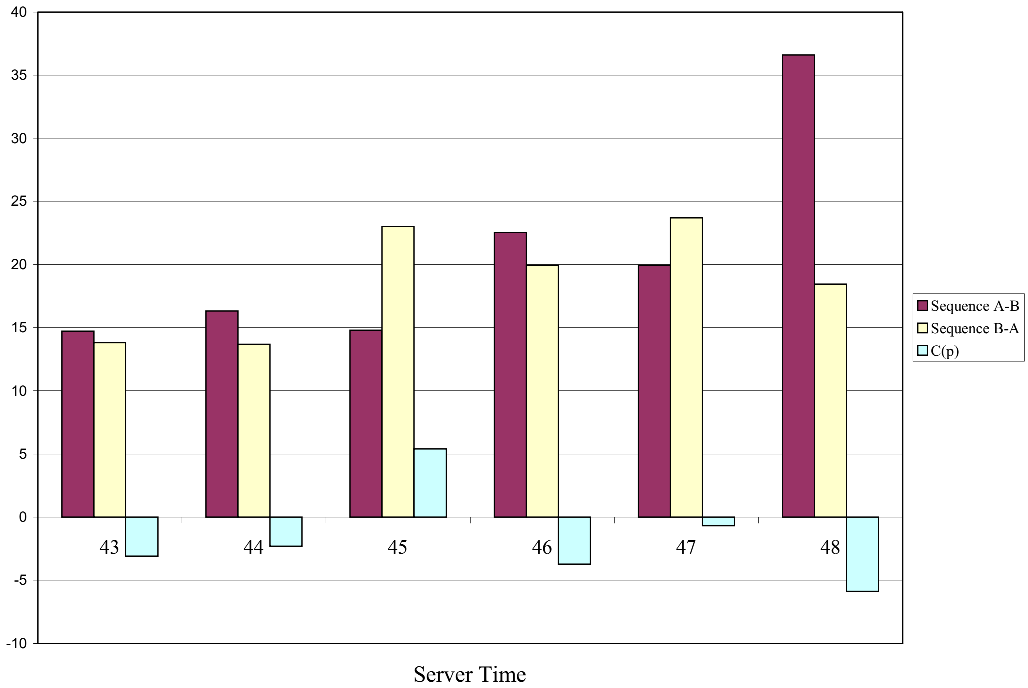


Figure 3.13 The effect of different server times when standard deviation is 20

Figure 3.13 shows that all *C(p)*s, except the one for server time 45, are negative, preferring sequence *A-B*. However, nearly in all combinations except when server time is 45, the queue length for *A-B* is longer than *B-A*. The result is in accordance with what is concluded for the above three server times.

3.4.3. Backorder

Another measure for the efficiency of operations reversal is the ability of the process to meet demand. For that purpose, the backorder level is recorded for each period (Figure 3.14) and the average backorder for each demand set is plotted at constant server times.

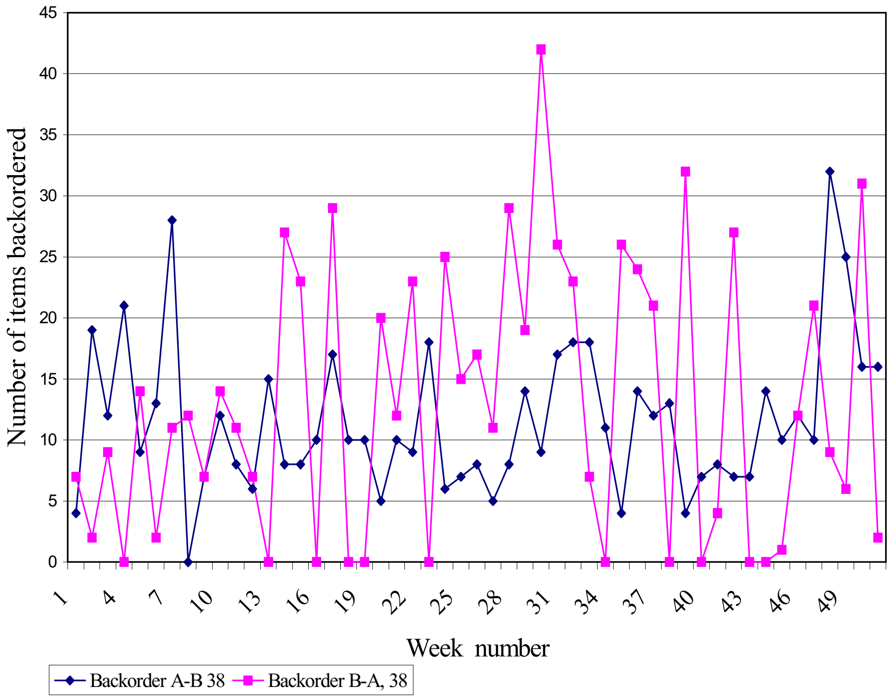


Figure 3.14 Backorder values when $\mu=\sigma^2$ and server time = 38

Figure 3.14 illustrates that the average number of items backordered is smaller for sequence *A-B*. The exact average backorder values for sequences are 11.59 and 12.94 items per demand, respectively. This result is in accordance with *C(p)* and queue length measures. All three measures favor sequence *A-B*.

Individual results for server time 38 are depicted in Figure 3.14. To have a broader view and generalize the behavior of the model under operations reversal Figure 3.15 is plotted for average backorder values at three different utilization levels. This figure summarizes the behavior of the model when performance measure is taken as the backorder level. For the demand of each week the backorder data of the two sequences (*A-B*, *B-A*) are recorded. As clearly seen from the figure, when server time is 38 no significant difference exists between backorder levels. Nevertheless, the backorder level for sequence *A-B* is higher and operations reversal seems useful. When higher utilization levels, namely server times 43 and 48, are investigated, it is observed that backorder level as a performance measure favors sequence *A-B*. The difference between backorder levels of the two sequences becomes more distinct as server time increases.

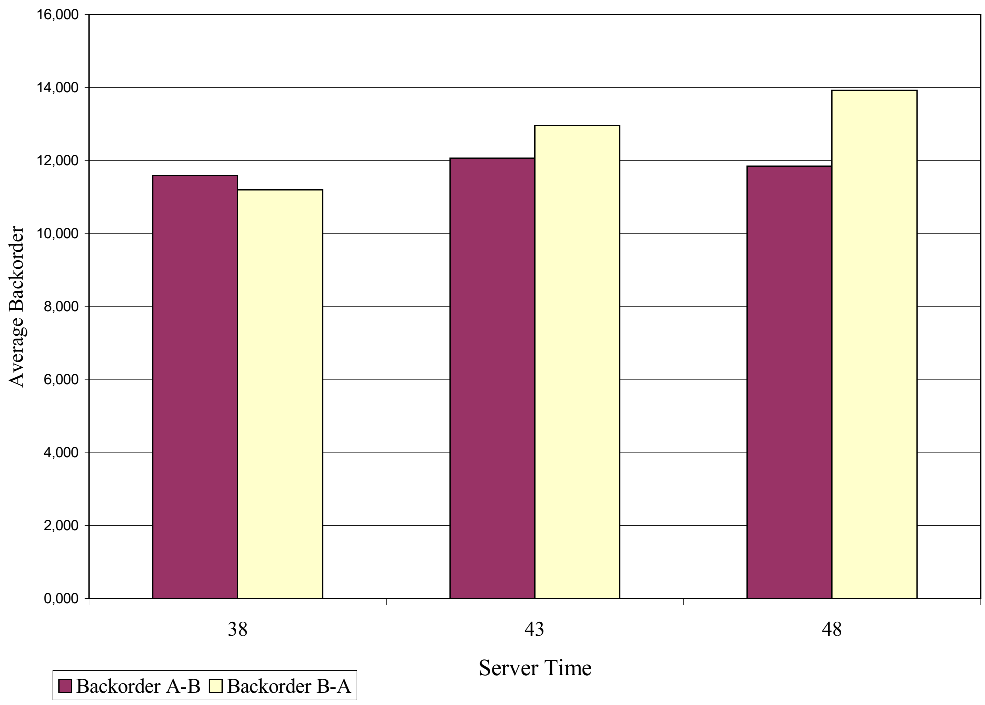


Figure 3.15 Average backorder values for different server utilizations

Table 3.8 Comparison of operations reversal for three performance criteria

Server time	Sigma	Queue Length	Backorder	Cp
38	5	A-B	B-A	A-B
	10	A-B	B-A	A-B
	20	B-A	B-A	A-B
43	5	A-B	A-B	A-B
	10	B-A	A-B	A-B
	20	B-A	A-B	A-B
48	5	B-A	A-B	B-A
	10	A-B	A-B	A-B
	20	B-A	A-B	A-B

Table 3.8 summarizes the outputs depicted for μ=100 in section 3.4. Backorder data is calculated for this demand level only since the aim is to take a snapshot of the match of different performance measures at changing server times and end product standard deviations. The sequences preferred by three performance measures differ in all cases except when server time=43, σ=5 and server time=48, σ=10. It is also observed that as server time (utilization) increases match between queue length and backorder sequence preferences decreases. In addition, *C(p)* and queue length pair prefer the same sequence more than *C(p)* and backorder pair.

3.5. A Deterministic Capacitated Two Stage Model by Arena: 2-by-2 System

The purpose of simulating the model of section 3.4 again with deterministic server times is to check if there is a relation in the *C(p)* values between the uncapacitated and capacitated model. In this section the only performance measure under consideration is the overall variance of the system. Since the difference between the overall variance of the two sequences generated by the operations reversal is reflected by the sign of *C(p),* a closer look at the *C(p)* outcomes of the uncapacitated and capacitated models are given. The primary objective in doing so is to investigate whether *C(p)* as a performance measure gives same results for both models.

Parameter setting is the same as the model presented in Section 3.4. The only difference lies in the creation of demand for four different end products, $(A_1B_1, A_1B_2, A_2B_1, A_2B_2)$. In this model, the probability of the first stage, *p,* for sequence *A-B* is taken as either 0.1, 0.3, or 0.5. The second stage probability pairs *f(p)* and *g(p)* are: 0.2-0.2; 0.2-0.4; 0.4-0.2; 0.4-0.4. The probabilities of the corresponding *B-A* sequence are calculated specifically for each *p*, *f(p)*, *g(p)* set using conditional probability calculations.

The system is run for three specific demand sets, namely (100,5), (100,10), and (100,20) corresponding to $(\mu > \sigma^2)$, $(\mu = \sigma^2)$, and $(\mu < \sigma^2)$, respectively, and for specific server times 38, 40, and 43 as in the model of Section 3.4. Periodic variance data are calculated via collecting number of items processed by the servers of the first stage during each 480-minute period (a working day).

Then the calculated overall variance data are compared with the formula of the uncapacitated model derived in Section 3.1. A total of 216 runs are made with server times mentioned. The results for server time 38 are depicted individually through Figures 3.16 to 3.24 to give a general view about the compliance of capacitated and uncapacitated models. The overall summary of the simulation runs with three specific server times and match/mismatch with uncapacitated model formulation is given in the table at the end of the section.

3.5.1. Investigation of the *C(p)* Values

Figure 3.16 shows that for the given σ-*p* pair, the sign of *C(p)*[2] calculated by the formula of uncapacitated model and by simulation are the same. Only the magnitude of variance difference between two sequences is larger for the capacitated model. Both models favor *A-B* sequence regardless of the probability values of the second stage.

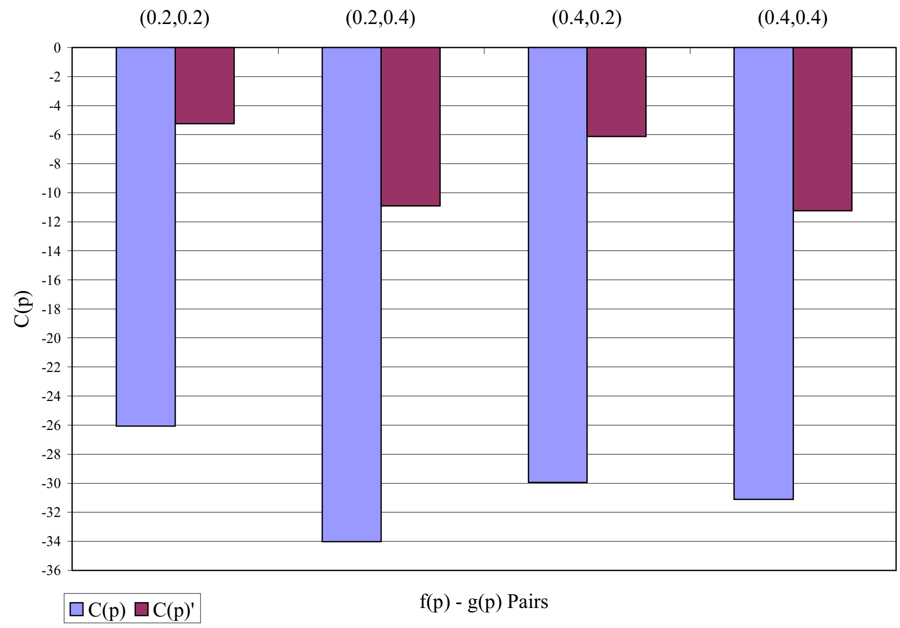
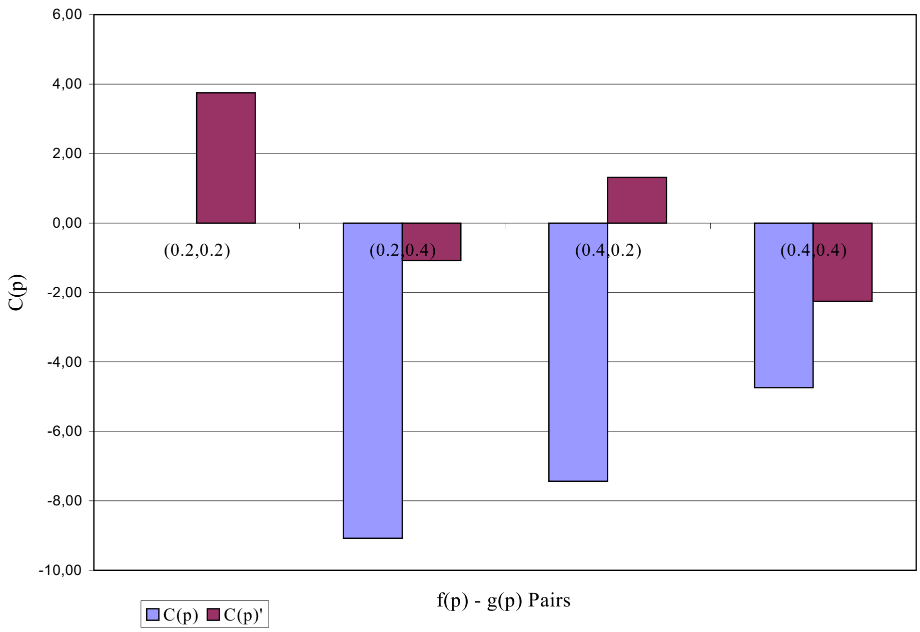


Figure 3.16 Comparison of *C(p)* and *C(p)'* values with σ=5 and *p*= 0.1



[2] Throughout the section the notation *C(p)'* is used for the uncapacitated model and *C(p)* is used for the capacitated model.

Figure 3.17 Comparison of *C(p)* and *C(p)'* values with σ=5 and *p*= 0.3

When probability of the main branch increases to 0.3, Figure 3.17 shows that the signs of *C(p)* and *C(p)'* do not match for second branch probabilities (0.4,0.2). Also due to C*(p)* the sequencing is immaterial for second branch probabilities (0.2,0.2) although *C(p)'* suggests sequence *B-A*. For the cases where capacitated and uncapacitated models suggest different sequences a decision can be made considering other performance measures such as queue length or backorder level.

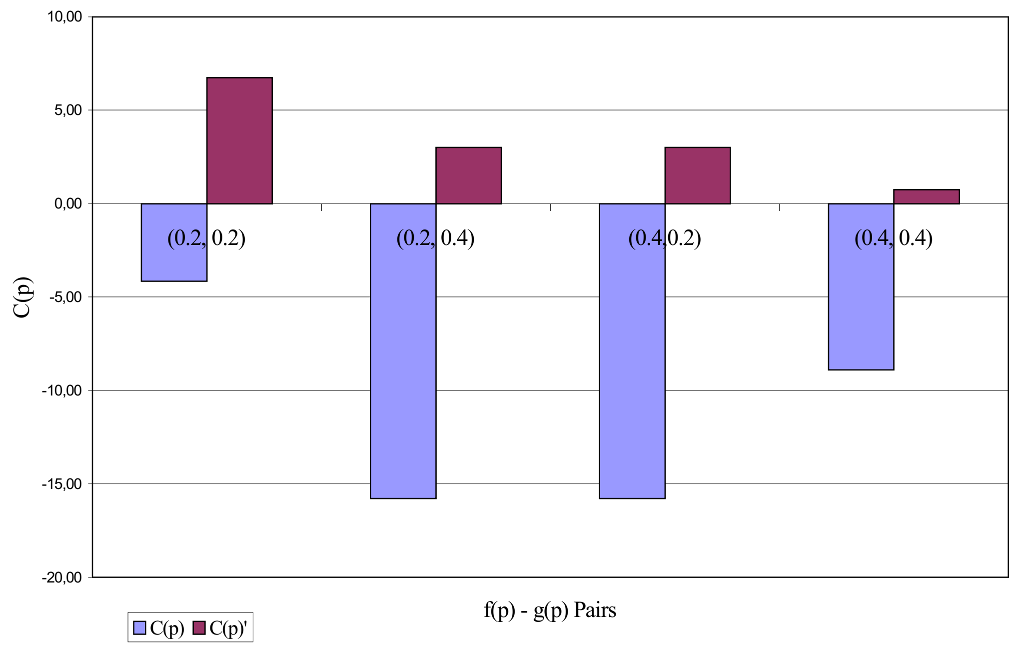


Figure 3.18 Comparison of *C(p)* and *C(p)'* values with σ=5 and *p*= 0.5

In Figure 3.18, it is observed that as the probability of the first stage increases to 0.5, in other words when, on the average, half of the demand has feature A_1 and the other half has feature A_2, the two models totally disagree with each other. The general trends for *C(p)* values is same in both figures except that the sequencing is important in Figure 3.18 for second branch probabilities (0.2,0.2). The observed *C(p)* values prefer sequence *A-B* for all second branch probability pairs. It is also observed that *C(p)* and *C(p)'* values are higher compared to the ones with *p*=0.3 in Figure 3.17.

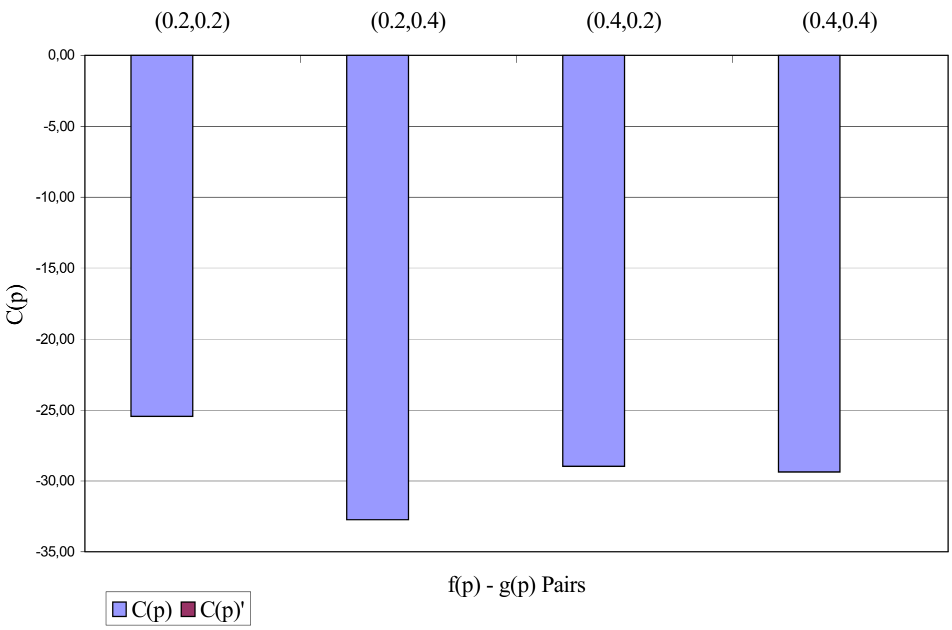


Figure 3.19 Comparison of *C(p)* and *C(p)'* values with σ=10 and p= 0.1

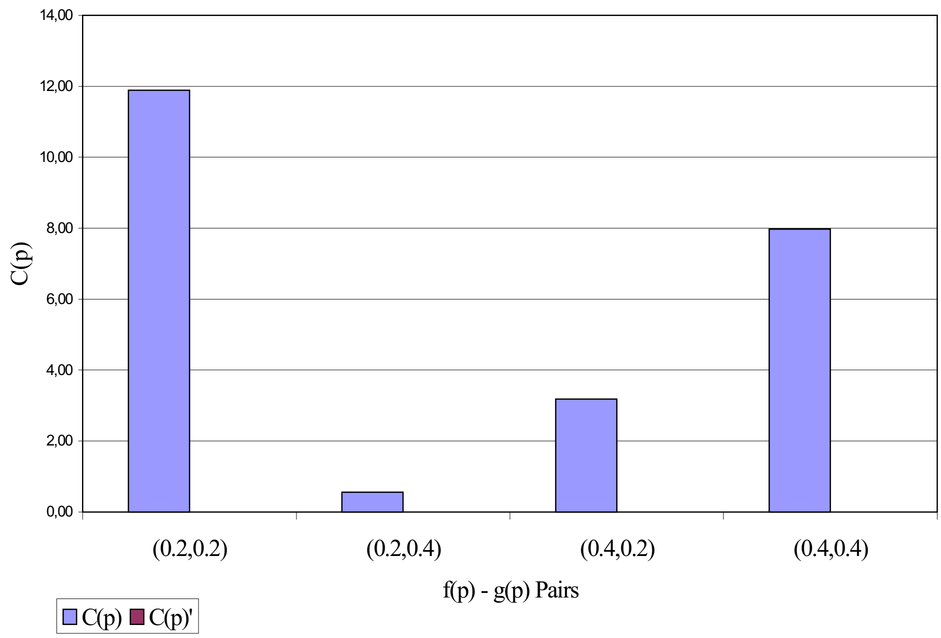


Figure 3.20 Comparison of *C(p)* and *C(p)'* values with σ=10 and p= 0.3

Figures 3.19 and 3.20 illustrate that as the main branch probability increases the operation reversal becomes effective. When p=0.1 all *C(p)'* values are negative, favoring sequence *A-B*. However when p increases to 0.3, keeping all other variables constant, all *C(p)* values favor sequence *B-A*. When $\mu=\sigma^2$, comparing *C(p)'* with *C(p)* is immaterial because *C(p)'* value is zero. Hence, Figures 3.19, 3.20, and 3.21 show only the *C(p)* values calculated.

When Figure 3.21 is compared to Figure 3.20, it is seen that *C(p)* values change sign. Therefore, probability 0.3 acts as a turning point of a concave function in a way. In this simulation the absolute values of *C(p)*s obtained are less than those shown in Figure 3.20, meaning that the difference in the variance between two sequences is decreasing. With these parameters at hand, it is still not beneficial to reverse the consecutive operations *A* and *B*.

The strongest result that can be observed from Figures 3.19, 3.20, and 3.21 is that in contrast with the uncapacitated model, capacitated model disproves the fact that sequencing of the features is immaterial when $\mu=\sigma^2$.

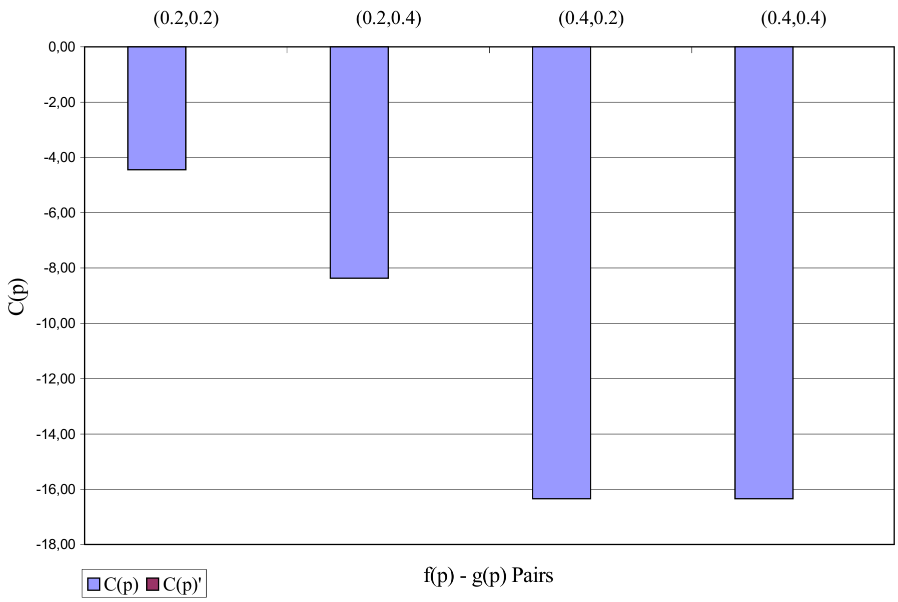


Figure 3.21 Comparison of *C(p)* and *C(p)'* values with σ=10 and p= 0.5

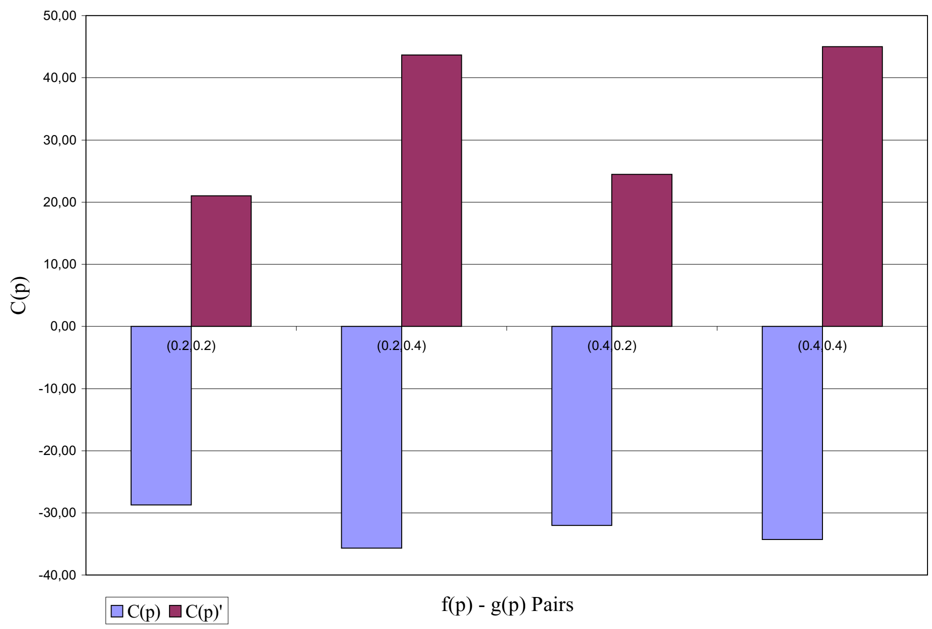


Figure 3.22 Comparison of *C(p)* and *C(p)'* values with σ=20 and *p*= 0.1

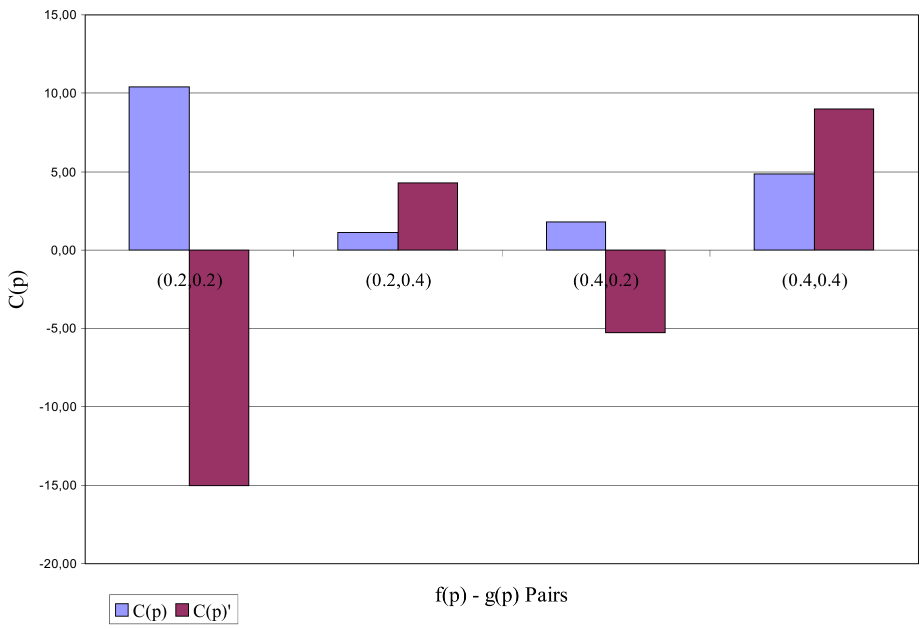


Figure 3.23 Comparison of *C(p)* and *C(p)'* values with σ=20 and *p*= 0.3

In Figure 3.22, the *C(p)* values for the two cases have nearly the same absolute values but opposite signs.

In Figure 3.23, both *C(p)* and *C(p)'* have the same sign when second branch probability pairs are (0.2,0.4) and (0.4,0.4) only. It is also seen that magnitudes of *C(p)'s* are far greater than *C(p)s*.

In Figure 3.24, it is observed that both models propose the same sequence, sequence *A-B*. *C(p)* values are not close and for all cases accept the one with second order probabilities (0.2,0.2), *C(p)* values are larger.

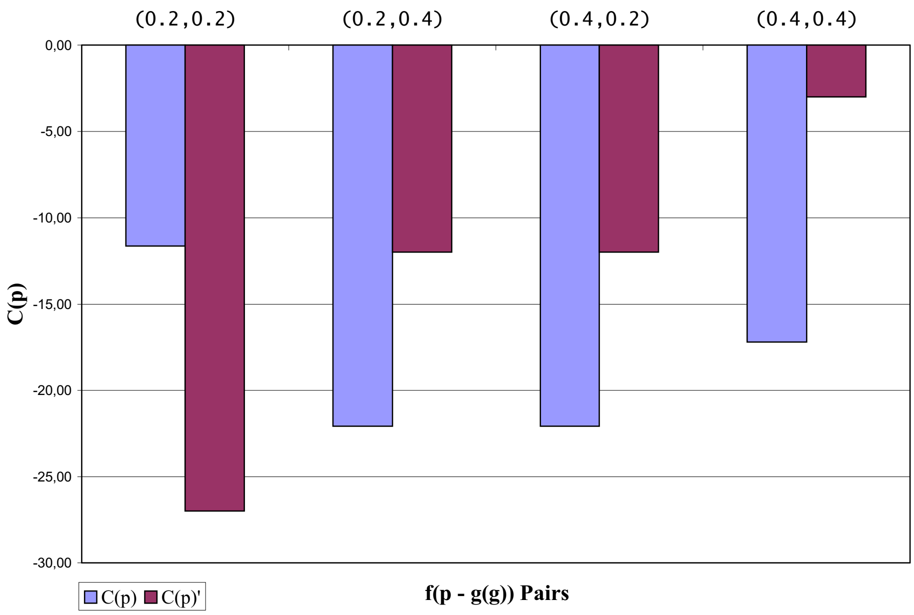


Figure 3.24 Comparison of *C(p)* and *C(p)'* values with σ= 20 and p= 0.5

To make a summary of the behavior of the capacitated model throughout this section three parameters of the model are selected. These parameters are standard deviation, main branch, and second branch probabilities. For the specified 80% utilization level the above graphs show how the formulation derived for the uncapacitated 2-by-2 system is applicable to the capacitated 2-by-2 system.

In order to reach to general results more data are needed. Therefore, the simulation carried for the above three parameters is enlarged to include %90 and 100%

utilization levels. Although these high percentages may not be frequently applicable in practice, it is known that the benefit of operations reversal comes from the absolute value of *C(p)* gets larger with increasing utilizations since the system is more sensitive to minor increase in variance. This is due to the fact that most emphasis is given to the distinction of end products demand distributions to decide on whether to reverse consecutive operations or not. In this step another grouping is made to strengthen the previous comparisons on how far can one push utilization up and still apply the formulation of uncapacitated system derived in section 3.1.

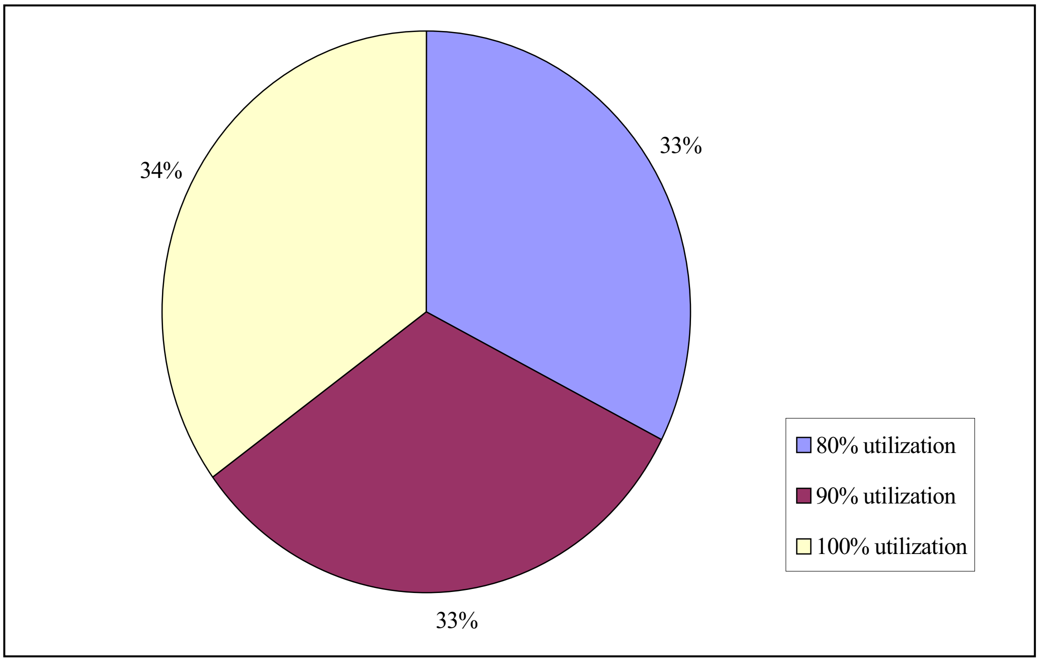


Figure 3.25 Percent correspondence of *C(p)* and *C(p)'* values with different utilizations

It can be observed from the Figure 3.25 that there is no significant distinction between different level of utilizations. The signs of *C(p)* and *C(p)*' match only 33% in the total 144 runs of 80% and 90% utilizations, and 34% in 100% mode. The obtained results do not give conclusive clues about how to use operations reversal tool in capacitated systems with moderate to high utilizations.

Another aspect to investigate is the percentage of compliance depending on the standard deviation of demand. To what extent is Formula (3.11) reliable for capacitated systems and how does reliability change with altering standard deviations? The answers for these questions are summarized in Table 3.9.

Table 3.9 Percent correspondence of *C(p)* and *C(p)'* values with different main branch probabilities and utilizations

σ	***p*=0.5**	***p*=0.3**	***p*=0.1**	**Total**
5	4/36	4/36	8/36	44.4%
10	0/36	0/36	0/36	0.0%
20	12/ 36	8/36	0/36	55.5%
Total	44.4%	33.3%	22.2%	

According to Table 3.9, the highest correspondence of *C(p)* signs for capacitated and uncapacitated models belongs to p=0.5, σ= 20 pair. In none of the runs when $\mu=\sigma^2$ the simulation gives a *C(p)* value of zero. The results are not even close to zero. Hence, the argument that the sequencing of operations is immaterial when $\mu=\sigma^2$ fails to hold for capacitated systems. The column with the highest matching value is the one with p=0.5, meaning that capacitated and uncapacitated models resemble most when the main branch probability of the system is less distinctive. The result is intuitive since the queue length, thus buffer inventories for the capacitated system is most homogeneous for the p=0.5 case. The row with the highest matching value is the one with σ=5, which points out that at all first branch probability levels the capacitated and uncapacitated systems behave most similarly when the variance of the end product is smallest. This result is in accordance with the theory of how overall system variance responds to standard deviation and how capacity becomes more important as product variance increases. However, it fails to explain why *C(p)* signs with highest match belong to capacitated and uncapacitated models with p=0.5, σ= 20 pair.

4. CONCLUSION

In this thesis, the effects of operations reversal on multi-stage production systems with high degree of product variety have been investigated. The results show that reengineering efforts require careful planning, and understanding the nature of demand variations, the choice probabilities of each product in the product mix, capacity limitations of the manufacturing process and other characteristics of the supply chain.

Uncapacitated and capacitated systems have been modeled and analyzed under demand uncertainty. The variability of production volumes in the intermediate stages of production was proposed as a performance measure for such production systems. In addition, the average queue lengths and backorder levels are considered in this study. The specific results of this study can be summarized as follows.

First, using total variability as a performance measure 2-by-2 and 2-by-*N* uncapacitated systems are modeled under demand uncertainty. It is observed that for both models, performance measure *C(p)* includes $(\mu-\sigma^2)$ as a factor. For 2-by-*N* case, operations reversal is not favored when $\mu>\sigma^2$, except for cases when one end product has a very high choice probability compared to the others. For end product demands where $\mu<\sigma^2$, at every combination of choice probabilities it is found to be beneficial to reverse the operations.

Then, since in real life applications, capacity is a vital problem having direct effects on queue lengths, work-in-process and buffer inventories, models are simulated using Arena considering all these important concepts and obtained data are used to derive comparative results of manufacturing systems in terms of average queue length at each stage, backorder level and overall system variance. The choice probabilities for the end product mix are created via uniform distribution. The observations are as follows: Operations reversal has nearly no effect when $\mu=\sigma^2$. *C(p)* values are close to zero and queue lengths are nearly equal for two sequences. In addition, when $\mu\leq\sigma^2$, it is immaterial whether the queue length or *C(p)* value is taken as performance measure, since they both point at the same sequence of feature installations. However, when $\mu>\sigma^2$ there exists disagreement between two performance measures, the performance criteria conflict at all server utilization levels. When backorder records are investigated it is seen that, at high utilization levels, operations reversal is not favored for $\mu=\sigma^2$. In addition, when three performance criteria are compared, it is observed that sequences preferred by three performance measures differ in all cases except low utilizations and demand variation. It is also observed that as utilization increases match between queue length and backorder sequence preferences decreases. In addition *C(p)* and queue length pair prefer the same sequence more than *C(p)* and backorder pair.

Lastly, a deterministic 2-by-2 capacitated model is created in order to find out whether for fixed choice probabilities, similar results are obtained for capacitated and uncapacitated cases. The results show that there is no compatibility between the two cases. Only one in three systems behaves similarly under operations reversal at all utilization levels. In addition an important argument that sequencing of operations is immaterial when $\mu=\sigma^2$ fails to exist for capacitated systems. It is observed that capacitated and uncapacitated models resemble most when the main branch probability of the system is less distinctive. Another generalization can be made when variance of the end product is smallest all first branch probability levels the capacitated and uncapacitated systems behave most similarly.

Further Study

Since it is observed that different performance measures conflict with each other, one may argue that somehow the relevant performance measures should be combined and turned into one global performance measure. In effect, one has to derive a total cost function and try to find the impact of operations reversal in the production system. One such model is proposed in Lee, Tang, (1997) which is discussed in Further Study section. Actually the model tries to find the optimal point of differentiation. As an extension of this study, this model can be applied in a real life case with relevant cost data.

Consider a manufacturing system with two end products, where each end product requires processes performed in N stages. The manufacturing system has a buffer that stores the work-in–process inventory after each operation. To focus on the result of operations reversal in a model with various parameters such as inventory, server times, and set-up cost, we assume that the system will have the first k operations common to both products and apply operations reversal for the $k+1^{st}$ and $k+2^{nd}$ stages.

The model is a discrete time model. The assumptions are; at the beginning of each period, unit size customer orders arrive for each end product. Intermediate product is used to satisfy all orders, and excess is backlogged. Inventory position is defined as inventory level plus work-in-process. At each review the inventory position is raised to S by placing a new order.

The parameters of the model are:

S_i : average investment cost per period if operation i becomes a common operation.

$n_i(k)$: lead time of operation i when operation k is the last common operation.

$p_i(k)$: the processing cost per unit associated with operation i

$h_i(k)$: inventory holding cost for holding one unit of inventory at buffer i for one period.

z : safety factor

Z(k) : total relevant cost per period for the case when operation k is the last common operation.

$$Z(k) = \sum_{i=1}^{k} S_i + \sum_{i=1}^{N} p_i(k)(\mu_1 + \mu_2) + \sum_{i=1}^{N} h_i(k)[n_i(k)(\mu_1 + \mu_2)]$$
$$+ \sum_{i=1}^{N} h_i(k)\left[n_i(k)(\mu_1 + \mu_2)/2 + z\sigma_{12}\sqrt{(n_i(k)+1)}\right]$$
$$+ \sum_{i=1}^{N} h_i(k)[(\mu_1 + \mu_2)/2] + z(\sigma_1 + \sigma_2)\sqrt{(n_i(k)+1)}$$

(3.18)

First term represents the total investment cost per period, the second term corresponds to total processing cost per period, the third term represents the total WIP inventory cost, and the fourth and fifth terms represent the total buffer inventory cost per period. In this case the performance measure is *Z(k+1)-Z(k+2).* The sign of the result shows whether operations reversal is beneficial or not.

It is expected to get trends in results for different settings such as server times, reversal cost and variance values. Using these results, algorithms for systems can be developed such that given parameters of the above model, the algorithm automatically decides to reverse operations or not.

Another future research direction could be analyzing the capacitated case analytically. To conduct such a research, one has to set up a framework to capture the most relevant features of the capacitated case and at the same time make simplifying assumptions to keep the model analytically tractable.

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APPENDIX A: Conditional Probability Derivations for Sequence *B-A*

According to Bayes' Theorem:

$$P(A \mid B) = \frac{P(B \mid A)P(A)}{P(B)} \tag{A.1}$$

In Formula (3.4) the ϑ_{ij} values for sequence *A-B* is given. Using these values and Formula (A.1) ϑ_{ij} values for sequence *B-A* are calculated as follows:

$$P(A_1 \mid B_1) = \frac{P(B_1 \mid A_1)P(A_1)}{P(B_1)} = \frac{f(p)p}{pf(p) + (1-p)g(p)} \tag{A.2}$$

$$P(A_2 \mid B_1) = \frac{P(B_1 \mid A_2)P(A_2)}{P(B_1)} = \frac{g(p)(1-p)}{pf(p) + (1-p)g(p)} \tag{A.3}$$

$$P(A_1 \mid B_2) = \frac{P(B_2 \mid A_1)P(A_1)}{P(B_2)} = \frac{(1-f(p))p}{p(1-f(p)) + (1-p)(1-g(p))} \tag{A.4}$$

$$P(A_2 \mid B_2) = \frac{P(B_2 \mid A_2)P(A_2)}{P(B_2)} = \frac{(1-g(p))(1-p)}{p(1-f(p)) + (1-p)(1-g(p))} \tag{A.5}$$

Then ϑ_{ij} values for sequence *B-A* are calculated as follows given in Formulation (3.11).

APPENDIX B: C++ Pseudo-Code:

The code written for 2-by-3 system in Section 3.3 is as follows:

```
for  z = 1,…,6  do
probability = 0.1* z
    Assign Nu = 50
    Assign Sigma = sqrt(100)

        for  j = 1,…,10 do
ProbabilityArray (0,0) = probability * 0.1 * (j+1)
for  i = 1,..,10-(j+1) do
ProbabilityArray (0,1) = probability * 0.1* (i+1)
for  w = 0,.., (10-j+i+2) do
     ProbabilityArray (0,2) = probability * 0.1 * (w+1)
                        ProbabilityArray (0,3) = probability *
                  (1(0.1*(j+1)+0.1*(w+1)))
```

Assign second lower branch probabilities:

```
        ProbabilityArray (1,0) = (1-probability) * 0.1 * (l+1)
                for m=0,…,10-(l+1) do
    ProbabilityArray (1,1) = (1-probability)*0.1* (m+1)
                for  v = 0,.., v <(10-l-m-2) do
                       ProbabilityArray (1,2) = (1-probability) * 0.1 * (v+1)
                            ProbabilityArray (1,3) = (1-probability)*(1-(
                  0.1* (l+1) + 0.1 * (m+1) + 0.1* (v+1)))
end for
          enf for
          end for
```

Second branch lower loop closes.

end for

end for

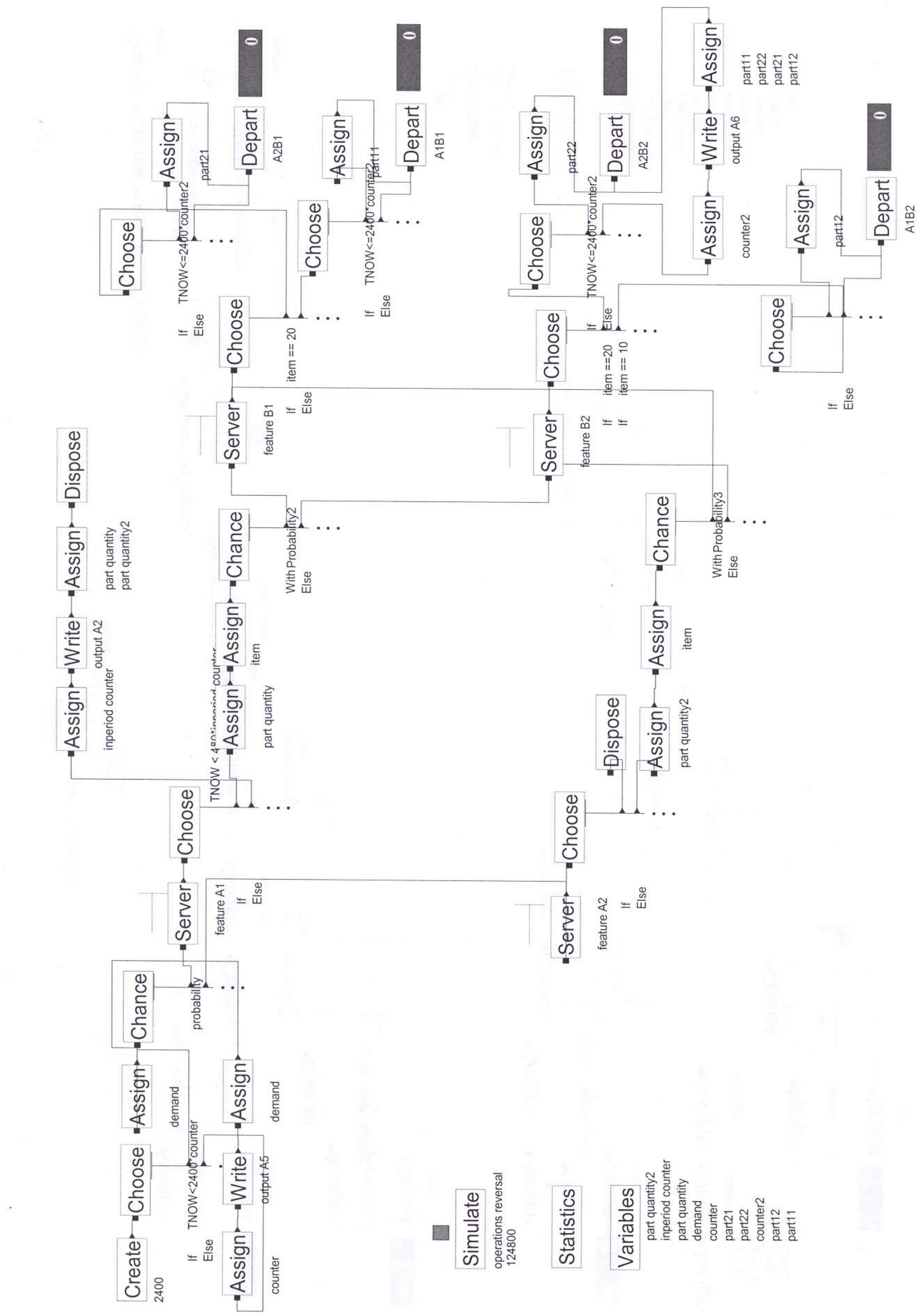
end for

Second branch upper loop closes.

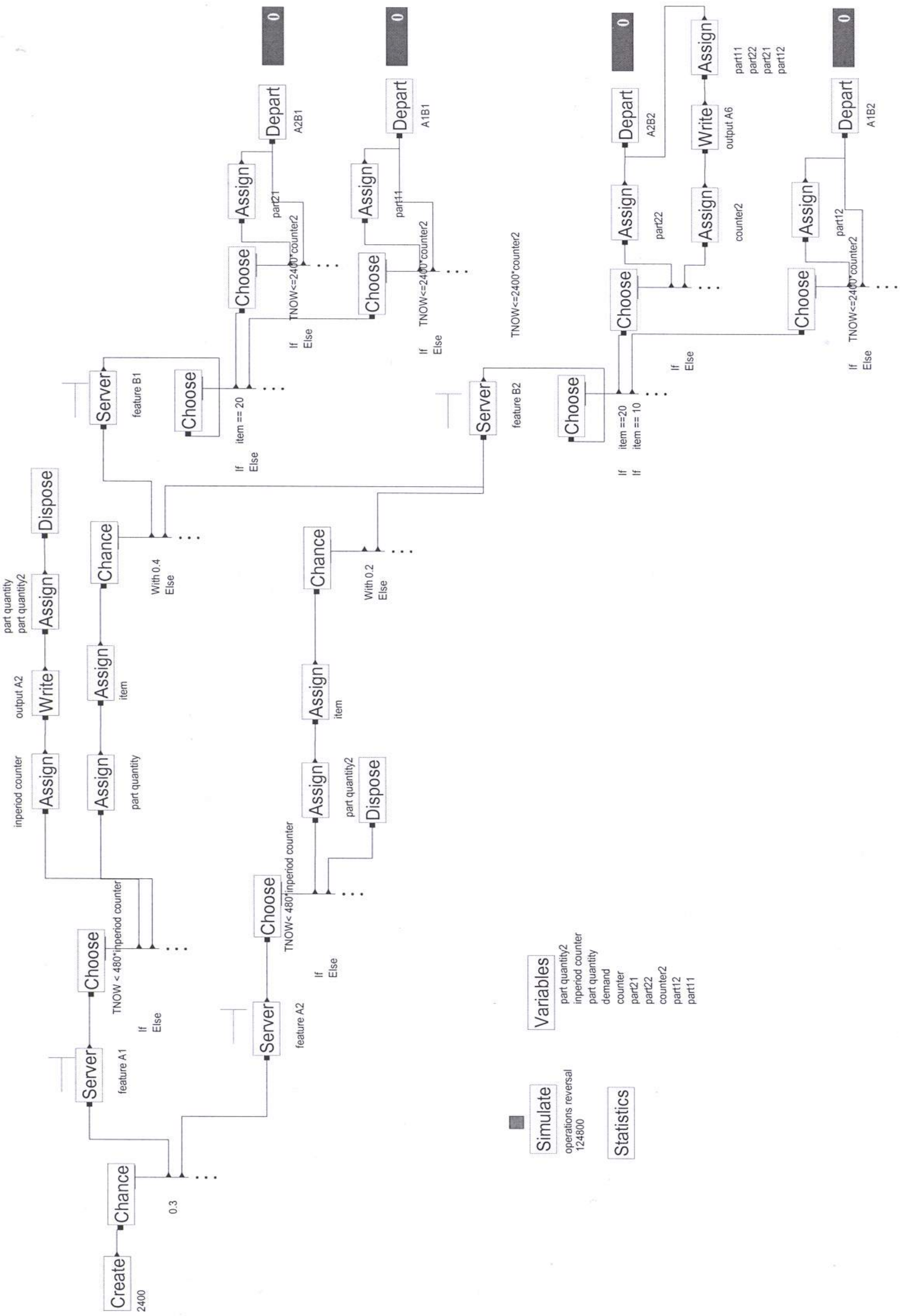
end for

p loop closes.

APPENDIX C: Arena Simulation Flow Chart For Section 3.4



APPENDIX D: Arena Simulation Flow Chart For Section 3.5



APPENDIX E: Sample Simulation Outputs

Outputs for capacitated model in Section 3.4:

A-B sequence for σ=10, server time 38 is given first then followed by *B-A* sequence for σ =10, server time 38.

```
ARENA Simulation Results
                              MFG1 - License: 1033

                        Summary for Replication 1 of 1

Project:operations reversal                      Run execution date :
4/29/2002
Analyst:MFG1                                     Model revision date:
4/29/2002

Replication ended at time      : 124800.0

                            TALLY VARIABLES

Identifier                Average   Half Width  Minimum    Maximum
______________________________________________________________________

A2B2_Ta                   1068.3     46.128     76.000     2736.0
feature A1_R_Q Queue T    952.74     44.980     .00000     2394.0
feature A2_R_Q Queue T    940.92     55.178     .00000     2622.0
feature B1_R_Q Queue T    59.411     (Corr)     .00000     342.00
A1B1_Ta                   1095.2     47.053     114.00     2432.0
A1B2_Ta                   1123.0     47.700     76.000     2394.0
feature B2_R_Q Queue T    64.551     12.223     .00000     380.00
A2B1_Ta                   1048.9     51.157     76.000     2660.0


                       DISCRETE-CHANGE VARIABLES

Identifier                Average   Half Width  Minimum    Maximum
______________________________________________________________________

feature B1_R Available    1.0000     (Insuf)    1.0000     1.0000
feature B2_R Available    1.0000     (Insuf)    1.0000     1.0000
# in feature A2_R_Q       19.248     1.8177     .00000     69.000
# in feature B2_R_Q       1.2460     .27483     .00000     10.000
feature B1_R Busy         .69514     .03732     .00000     1.0000
feature B2_R Busy         .73351     .02828     .00000     1.0000
feature A1_R Available    1.0000     (Insuf)    1.0000     1.0000
feature A2_R Available    1.0000     (Insuf)    1.0000     1.0000
# in feature A1_R_Q       19.970     1.6746     .00000     63.000
```

# in feature B1_R_Q	1.0868	.27956	.00000	9.0000
feature A1_R Busy	.79623	.03170	.00000	1.0000
feature A2_R Busy	.77705	.03504	.00000	1.0000

COUNTERS

Identifier	Count	Limit
A2B2_C	1184	Infinite
A1B1_C	1132	Infinite
A1B2_C	1225	Infinite
A2B1_C	1151	Infinite

Simulation run time: 0.08 minutes.
Simulation run complete.

ARENA Simulation Results
MFG1 - License: 1033

Summary for Replication 1 of 1

Project:operations reversal   Run execution date :
4/29/2002
Analyst:MFG1   Model revision date:
4/29/2002

Replication ended at time   : 124800.0

TALLY VARIABLES

Identifier	Average	Half Width	Minimum	Maximum
A2B2_Ta	1062.2	58.560	76.000	2660.0
feature A1_R_Q Queue T	962.87	(Corr)	.00000	2470.0
feature A2_R_Q Queue T	942.85	48.251	.00000	2546.0
feature B1_R_Q Queue T	71.464	(Corr)	.00000	456.00
A1B1_Ta	1123.5	45.497	114.00	2660.0
A1B2_Ta	1136.1	54.429	114.00	2508.0
feature B2_R_Q Queue T	68.991	11.571	.00000	418.00
A2B1_Ta	1072.0	58.459	76.000	2812.0

DISCRETE-CHANGE VARIABLES

Identifier	Average	Half Width	Minimum	Maximum
feature B1_R Available	1.0000	(Insuf)	1.0000	1.0000
feature B2_R Available	1.0000	(Insuf)	1.0000	1.0000
# in feature A2_R_Q	19.408	1.7391	.00000	67.000
# in feature B2_R_Q	1.2996	.25357	.00000	11.000
feature B1_R Busy	.72042	.03732	.00000	1.0000
feature B2_R Busy	.71585	.03464	.00000	1.0000
feature A1_R Available	1.0000	(Insuf)	1.0000	1.0000

feature A2_R Available	1.0000	(Insuf)	1.0000	1.0000
# in feature A1_R_Q	20.275	1.7180	.00000	65.000
# in feature B1_R_Q	1.3548	.38032	.00000	12.000
feature A1_R Busy	.79989	.03674	.00000	1.0000
feature A2_R Busy	.78192	.02863	.00000	1.0000

COUNTERS

Identifier	Count	Limit
A2B2_C	1170	Infinite
A1B1_C	1187	Infinite
A1B2_C	1181	Infinite
A2B1_C	1179	Infinite

Simulation run time: 0.07 minutes.
Simulation run complete.

The following two outputs are for the capacitated model with deterministic probability values. The parameters are σ=10, server time 38, *p*=0.1, *f(p)*=0.2,and *g(p)*=0.2. Again the outputs for sequences A-B and B-A are given.

ARENA Simulation Results
MFG1 - License: 1033

Summary for Replication 1 of 1

Project:operations reversal   Run execution date :
5/16/2002
Analyst:MFG1   Model revision date:
5/16/2002

Replication ended at time   : 124800.0

TALLY VARIABLES

Identifier	Average	Half Width	Minimum	Maximum
A2B2_Ta	18596.	(Corr)	76.000	38702.
feature A1_R_Q Queue T	189.49	18.384	.00000	570.00
feature A2_R_Q Queue T	19468.	(Corr)	.00000	38626.
feature B1_R_Q Queue T	3.1360	(Insuf)	.00000	38.000
A1B1_Ta	372.03	(Insuf)	152.00	646.00
A1B2_Ta	433.28	(Insuf)	114.00	814.00
feature B2_R_Q Queue T	80.561	11.871	.00000	290.00
A2B1_Ta	19817.	(Insuf)	456.00	38702.

DISCRETE-CHANGE VARIABLES

Identifier	Average	Half Width	Minimum	Maximum
feature B1_R Available	1.0000	(Insuf)	1.0000	1.0000
feature B2_R Available	1.0000	(Insuf)	1.0000	1.0000
# in feature A2_R_Q	728.37	(Corr)	.00000	1457.0
# in feature B2_R_Q	.41184	.12065	.00000	8.0000
feature B1_R Busy	.05146	(Insuf)	.00000	1.0000
feature B2_R Busy	.19426	.03061	.00000	1.0000
feature A1_R Available	1.0000	(Insuf)	1.0000	1.0000
feature A2_R Available	1.0000	(Insuf)	1.0000	1.0000
# in feature A1_R_Q	.79715	.15950	.00000	15.000
# in feature B1_R_Q	.00425	(Insuf)	.00000	1.0000
feature A1_R Busy	.15955	.01796	.00000	1.0000
feature A2_R Busy	1.0000	.00000	.00000	1.0000

COUNTERS

Identifier	Count	Limit
A2B2_C	427	Infinite
A1B1_C	58	Infinite
A1B2_C	211	Infinite
A2B1_C	111	Infinite

Simulation run time: 0.13 minutes.
Simulation run complete.

ARENA Simulation Results
MFG1 - License: 1033

Summary for Replication 1 of 1

Project:operations reversal Run execution date : 5/16/2002
Analyst:MFG1 Model revision date: 5/16/2002

Replication ended at time : 124800.0

TALLY VARIABLES

Identifier	Average	Half Width	Minimum	Maximum
A2B2_Ta	13112.	(Corr)	76.000	25498.
feature A1_R_Q Queue T	385.22	27.572	.00000	1102.0
feature A2_R_Q Queue T	13604.	(Corr)	.00000	26392.
feature B1_R_Q Queue T	1.6190	(Insuf)	.00000	38.000
A1B1_Ta	535.92	(Insuf)	152.00	1102.0
A1B2_Ta	780.81	52.234	114.00	1864.0
feature B2_R_Q Queue T	269.05	21.621	.00000	722.00
A2B1_Ta	13307.	(Insuf)	380.00	24922.

DISCRETE-CHANGE VARIABLES

Identifier	Average	Half Width	Minimum	Maximum
feature B1_R Available	1.0000	(Insuf)	1.0000	1.0000
feature B2_R Available	1.0000	(Insuf)	1.0000	1.0000
# in feature A2_R_Q	437.85	(Corr)	.00000	851.00
# in feature B2_R_Q	3.7576	.70243	.00000	19.000
feature B1_R Busy	.05755	.00955	.00000	1.0000
feature B2_R Busy	.53072	.05126	.00000	1.0000
feature A1_R Available	1.0000	(Insuf)	1.0000	1.0000
feature A2_R Available	1.0000	(Insuf)	1.0000	1.0000
# in feature A1_R_Q	3.2750	.46484	.00000	29.000
# in feature B1_R_Q	.00245	(Insuf)	.00000	1.0000
feature A1_R Busy	.32276	.03135	.00000	1.0000
feature A2_R Busy	1.0000	.00000	.00000	1.0000

COUNTERS

Identifier	Count	Limit
A2B2_C	1020	Infinite
A1B1_C	81	Infinite
A1B2_C	723	Infinite
A2B1_C	108	Infinite

Simulation run time: 0.12 minutes.
Simulation run complete.