Sliding-Mode Neuro-Controller for Uncertain Systems

Yildiray Yildiz, Asif Sabanovic and Khalid Abidi

***Abstract*—In this paper a method allowing to merge good features of the sliding mode control (SMC) and Neural network (NN) design is presented. Design is performed by applying NN to minimize cost function selected to depend on the distance from sliding mode manifold thus providing that NN controller enforces sliding mode motion in a closed loop system. It has been proven that selected cost function has global minima and that selection of the NN weights guaranty that the global minima is reached and then the sliding mode conditions are satisfied, thus closed loop motion is robust against parameter changes and the disturbances. For the controller design the system states and the nominal value of the control input matrix are used. The design for both MIMO and SISO systems is discussed. Due to the structure of the (M)ADALINE network used in the control calculation the proposed algorithm can be also interpreted as a sliding mode based control parameters adaptation scheme The stability proofs are given and the controller performance is verified by experimental results.**

***Index Terms*—Neural networks, Sliding mode control.**

I. Introduction

Due its robustness to parameter uncertainties and external disturbances sliding mode is a well-established control method for application in nonlinear systems [1], [2], [3]. Merging a well-established control structure like sliding mode control with neural network (NN) based algorithms appeared to be a good idea and many researchers published various control structures based on this idea. A comprehensive historical investigation and a literature survey can be found in [4]. In application of SMC methods to NN based control a few main ideas seem to be prevailing. The first one attempts to apply NN as an observer in estimation of equivalent control [5] and in some cases disturbances [6]. Such an application of NN leads to effective linearization of the system and thus allows simpler design of the main controller. The weights of NN are determined based on the evaluation of evaluation of the

distance from the sliding mode manifold. Good results in application to nonlinear systems linear with respect to control are reported. In [6], Jezernic, Rodic, Safaric and Curk applied the idea on a 3.D.O.F PUMA type DD - robot system. They used continuous sliding mode theory to establish a robust control scheme. To avoid the chattering effect, they estimated the equivalent control and used this estimation in the sliding mode control algorithm. The estimation of the equivalent control was done using an online neural network estimator. In [7], Rodic, Jezernic, Sabanovic and Safaric used a sliding-mode based learning algorithm for robust accurate tracking of a single axis DD robotic manipulator driven with an induction motor. In another work, [8], Fang, Y., Chow; T.W.S. and Li, X.D. proposed a control system on the basis of a discrete Lyapunov function. Part of the equivalent control is estimated by a recurrent neural network (RNN) and a real-time iterative learning algorithm is developed and used to train the RNN. They also proved the stability of the system by showing that the learning error converges to zero.

The adoption of a nonlinear dynamic adjustment strategy in an ADALINE based controller applied to a three dof robot is discussed in [10]. The idea is to impose a sliding mode control while an adaptation is imposed on the controller parameters in such a way that desired motion is achieved. PD controller with a bias term is applied so three parameters are to be adjusted.The rate of change of the parameters is discontinuous (sign function) which are approximated by the boundary layer continuous high gain. The sliding mode manifold is selected to be a difference between desired (unknown) and applied torque thus another functional mapping is needed in order to use available sliding mode function expressed as a linear combination of the position and velocity error. Similar idea is explored in [12] and [13] for a class of nonlinear systems.

In this work, the proposed controller is based on the minimization of a cost function that is obtained by satisfying Lyapunov stability criteria. The cost function is selected in such a way that its time derivative is explicit function of control thus allowing calculation of the control input that will guaranty a stable solution. This cost function is the same cost function used in [5] but different from their approach, the aim is not calculating the equivalent control but computing the whole control signal using the minimization process. Also, the neural network used is a one layer neural network that holds the linearity of parameters. Due to the structure of the (M)ADALINE network used in the control calculation the proposed algorithm can be also interpreted as a sliding mode

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based control parameters adaptation scheme. The difference with other approaches is in the selection of the NN error function which guarantee the existence and stability of the global minima and the stability of the sliding mode manifold when error function minima is reached. In addition selection of the control as a linear combination of the motion error and a bias term is straight forward with the guaranty of the stability of overall closed loop motion due to the fact that the sliding mode existence is proven.

To verify the performance of the control scheme, two different experimental setups are used. The first experimental setup is a single axis linear drive that is driven by a dc motor. This setup is used for the implementation of the controller designed for SISO systems. The other setup consists of two piezo stack actuators and used for the controller designed for MIMO systems. The proposed control schemes performed well in both of the experiments.

II. PROBLEM STATEMENT

In this paper we will consider dynamical systems consisting of m interconnected sub-systems described by $y_i^{n_i-1} = h_i + b_i u_i + g_i$ where $y_i^{l_i}$ stands for $l-th$ time derivative of y_i. By selecting the state vector $x = \left[y_1, \dot{y}_1, ..., y_1^{n_i-1}, ..., y_m, \dot{y}_m, ..., y_m^{n_i-1}\right]^T$ these interconnected sub-systems can be represented as a class of nonlinear systems linear with respect of control as depicted in (1)

$$\dot{x} = f(x) + B(x)u + d \qquad (1)$$

where $x^T \in \Re^n$ is the state vector $n = \sum_{i=1}^{m} n_i$, $u \in \Re^m$ is the control vector, $f(x) \in \Re^n$ is an unknown, continuous and bounded nonlinear function, $B(x) \in \Re^{nxm}$ is a known input matrix whose elements are continuous and bounded and $rank(B(x))\|_{\forall x} = m$, with $d \in \Re^n$ being an unknown, bounded external disturbance. Both $f(x) \in \Re^n$ and $d \in \Re^n$ satisfy the matching conditions and all their components are bounded $\|f_i(x)\|_{\forall x} \le M$ and $\|d_i(t)\|_{\forall t} \le N$. Fully actuated mechanical systems belong to the class of systems described by (1). Such systems can be interpreted as m interconnected sub-systems $\ddot{q}_i = h_i(q_i, \dot{q}_i) + b_i(q_i, t)u_i + g_i(q_i, q_j)$, $h_i(q_i, \dot{q}_i)$ in general represents Coulomb friction term, $g_i(q_i, q_j)$ represents the interaction term and is regarded as a disturbance.

The aim is to determine the control input $u = [u_1, ..., u_m]^T$ such that the outputs of the system $y_1(t), ..., y_m(t)$ track the desired trajectories $y_{d_1}(t), ..., y_{d_m}(t)$ while control error satisfies selected dynamical constraints.

III. Controller Design

The controller will be designed in the SMC framework by firstly selecting a suitable sliding manifold that will ensure desired systems dynamics and then selecting control such that Lyapunov stability conditions are satisfied.. Selecting the Lyapunov function candidate in terms of the sliding function is a natural way of guaranteeing the sliding mode existence on the selected manifold and thus having desired closed loop dynamics. Finally, the necessary control input should be selected that will fulfill the requirements of the Lyapunov stability criteria.

A. Sliding manifold

For system (1) the natural selection of the sliding manifold is in the following form

$$\sigma = Ge_t = 0, \qquad (2)$$

where, tracking error vector is defined as, $e_t = \left[e_1, ..., e_1^{(n_1-1)}, ..., e_m, ...e_m^{(n_m-1)}\right]^T \in \Re^n$, $e_i = y_{d_i} - y_i$. $\sigma = [\sigma_i, ..., \sigma_m]^T \in \Re^m$ $G \in \Re^{mxn}$. Matrix G is selected such that each component of vector $\sigma(e)$ is selected to be function of one output control error and its derivatives $\sigma_i(e_i) = 0$ having form $\sigma_i = \sum_{i=1}^{n_i-1} a_i e_i; a_i > 0, a_{i1} = 1$ with multiple real root being equal to $-C$.

B. Computing the Necessary Control Input

A Lyapunov Function candidate can be selected as

$$V = \frac{1}{2}\sigma^T \sigma \qquad (3)$$

where, $V \in \Re$. This function can also be stated as $V = (1/2)\|\sigma\|_2^2$, where $\|\bullet\|_2$ indicates Euclidian norm with $V(0) = 0$. The time derivative of the candidate Lyapunov function $\dot{V}$ should be negative definite. In order to use this condition in selection of the control, we may require that the $\dot{V}$ satisfies some preselected form. Equating the time derivative of this function to a negative definite function like in (4),

$$\dot{V} = -\sigma^T D\sigma - \mu \frac{\sigma}{\sigma^T \sigma}, \qquad (4)$$

where, D is a positive definite symmetric matrix, and $\mu > 0$ thus Lyapunov conditions are satisfied. By substituting (3) into (4), the following requirement is found.

$$\sigma^T\left(\dot{\sigma}+D\sigma+\mu\frac{\sigma}{\sigma^T\sigma}\right)=0 \tag{5}$$

Therefore, for $\sigma \neq 0$, the control law can be calculated by satisfying the following equation.

$$\left(\dot{\sigma}+D\sigma+\mu\frac{\sigma}{\sigma^T\sigma}\right)=0 \tag{6}$$

and the sliding mode conditions are satisfied. The discontinuous term can be selected as small in order to avoid chattering. It had been proven [14, 15] that in the discrete time implementation the sliding mode is guarantied with continuous control action. We are targeting the computer controller systems for which controller will be implemented in discrete-time so in our application the discontinuous term will be omitted and we will be determining the control action that satisfies conditions $(\dot{\sigma}+D\sigma)=0$ but all further analysis can be easily adopted for application of expression (6) if the term $(D\sigma)$ is replaced with $\left(D\sigma+\mu\,\sigma/\sigma^T\sigma\right)$.

For system (1) with sliding mode manifold (2) the control tha satisfies $(\dot{\sigma}+D\sigma)=0$ can be determined as

$$u=-(GB)^{-1}\left(G\left(f+d-\dot{x}_{d_i}\right)-D\sigma\right)=u_{eq}+(GB)^{-1}D\sigma \tag{7}$$

where, $x_d=\left[y_{d_1},...,y_{d_1}^{(n_1-1)},...,y_{d_m},...,y_{d_m}^{(n_m-1)}\right]$ and u_{eq} is so-called equivalent control obtained as a solution of the equation $\dot{\sigma}=0$. By substituting (7) into (1) the equations of motion of system (1) in manifold (2) are obtained as $\sigma=Ge_t=0$ and the approach to this solution is governed by equation (6). This is a result of the specific structure of the plant (1) in which states are selected as the derivatives of the measurable outputs and each sub-block is represented in the canonical form.

To implement this control input, information about the plant dynamics and external disturbances are needed, which is hard to achieve. Hence, this solution needs the information on the equivalent control thus may be applied for the plants when u_{eq} is known or can be estimated with sufficient accuracy. The approach in [4,5,6] is based on the application of neural network (NN) in the estimation of the equivalent control.

In this paper we will take different approach. Instead of estimating equivalent control and then applying (7) we will apply a least square minimization using neural networks to fulfill $(\dot{\sigma}+D\sigma)=0$.

C. *Structure and Working Principles of NN*

The structure of the NN used in this paper is presented in Fig. 1, where e_{t_i} is the i^{th} row of e_t . w_{ij} refers to the weight of the signal that comes from the j^{th} node and goes to the i^{th} node, whereas w_{i0} refers to the bias term of the i^{th} node. Control inputs, which are the outputs of the NN, can be defined as $u=[u_1,...,u_m]$, where,

$$u_i=\sum_{j=1}^{n}e_{t_i}w_{ij}+1w_{i0},\quad i=1,...m \tag{8}$$

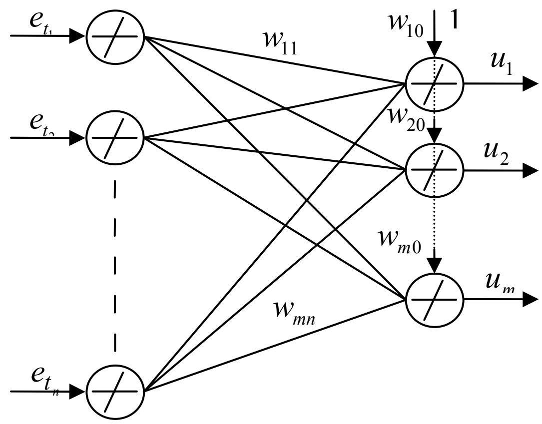


Figure 1. Structure of the Neural Network

As seen from (8), in the selected network the activation functions are linear and the network is static. In (8) the weights can be treated also as variable coefficients that should be adjusted in order to determine necessary control. For second order systems the structure (8) could be viewed as a PD controller with gain adaptation and for higher order systems it can be viewed as a state feedback controller with adaptation of the gain matrix. As follows from the structure of the network is such that if the inputs are zero (that means the control error vector is zero – thus the control objective is reached) the output is equal to the bias vector – thus the bias weights should compensate the system's disturbance

In this paper we will demonstrate the selection of weights in (8) so that for system (1) so that requirement $(\dot{\sigma}+D\sigma)=0$ determined from the Lyapunov stability conditions are satisfied. In order to fulfill the above requirements we will apply the neural network that will minimize the error function (9)

$$E=\frac{1}{2}(\dot{\sigma}+D\sigma)^T(\dot{\sigma}+D\sigma) \tag{9}$$

By selecting the weights such that $E\to 0$ and that $E=0$ is a stable solution the condition $(\dot{\sigma}+D\sigma)=0$ will be satisfied and the stable sliding mode motion will be achieved in manifold (2). The selected error function depends on the control input and this allows take partial derivative of error function with respect to control. Due to the fact that, for selected structure of NN, the control is linearly dependent on weights the usual weight update is expected to give simple structure. In addition the selected error function does not depend on unknown variables so it can be evaluated on-line and it does not need off-line training.

1) Weight Updates: Weights are updated according to following rule.

$$\dot{w}_{ij} = -\eta \frac{\partial E}{\partial w_{ij}} \tag{10}$$

where, $\eta > 0$ is the learning constant. Using the chain rule, (10) can be written as

$$\dot{w}_{ij} = -\eta \frac{\partial E}{\partial u_i} \frac{\partial u_i}{\partial w_{ij}} \tag{11}$$

Substituting (9) into (11) taking the derivatives, the following equation is obtained.

$$\dot{w}_{ij} = -\eta (\dot{\sigma} + D\sigma)^T \frac{\partial \dot{\sigma}}{\partial u_i} e_{t_j} \tag{12}$$

Substituting (2) into (12),

$$\dot{w}_{ij} = -\eta (\dot{\sigma} + D\sigma)^T \frac{\partial (G\dot{x}_d - G\dot{x})}{du_i} e_{t_j} \tag{13}$$

is obtained. Rewriting (1),

$$\dot{x} = f(x) + [B_1(x) \vdots \ldots \vdots B_m(x)] \begin{bmatrix} u_1 \\ \vdots \\ u_m \end{bmatrix} + d \,, \tag{14}$$

and substituting (14) into (13) and taking the derivative gives the following result.

$$\dot{w}_{ij} = \eta (\dot{\sigma} + D\sigma)^T G B_i(x) e_{t_j} \tag{15}$$

where, $B_i(x)$ is the i^{th} column of the matrix $B(x)$. For the bias terms w_{i0}, the weight update can be computed using the same procedure as

$$\dot{w}_{i0} = \eta (\dot{\sigma} + D\sigma)^T G B_i(x) \,. \tag{16}$$

The weights update (15) is simple and depend on the fulfillment of the conditions $(\dot{\sigma} + D\sigma) = 0$. The weights will change as long as condition (6) is not satisfied. Both (15) and (16) depend on the plant gain matrix and the selected sliding mode manifold (2).

The reason for introduction of the bias term (16) can be easily seen from the analysis of the error function (9) rewritten as $E = \frac{1}{2}\left[G\left(\dot{x}_d - f(x) - B\sum_{i=1}^{n} w_i e^{(i-1)} - d\right) + DGe_t\right]^2$ from where is obvious that if all control errors are going to zero the error function will tend to finite value $E|_{e=0} = \frac{1}{2}[G(\dot{x}_d - f(x) - d)]^2$. By introducing the bias term the error function becomes $E = \frac{1}{2}\left[G\left(\dot{x}_d - f(x) - B\sum_{i=1}^{n} w_i e^{(i-1)} + B1w_{n+1} - d\right) + DGe_t\right]^2$ and terms that satisfy the matching conditions can be rejected so the error function will have minima in point *E=0*. After reaching *E=0* the bias term will asymptotically tends to the equivalent control $w_{n+1} \to -(GB)^{-1}(G(f + d - \dot{x}_d))$ thus allowing the compensation of the matching disturbances. Due to the assumption that components of vectors f, d and $\dot{x}_d$ are bounded the bias term will be also bounded, thus for bounded initial errors the control input will be also bounded.

The selection of the linear activation function is not essential to the solution. Single-valued continuous activation functions may be applied. For example if the model of the NN is described as $n_i = \sum_{j=1}^{n} e_{t_j} w_{ij} + 1w_{i0}$, $i = 1,\ldots m$ with activation function $u_i = g_i(n_i)$ then in (15) a multiplying term $g_i^{'} = \partial g_i(n_i)/\partial n_i$ will appear so we will have $\dot{w}_{ij} = \eta(\dot{\sigma} + D\sigma)^T g_i^{'} G B_i(x) e_{t_j}$. Proper selection of $g_i^{'}$ as single-valued positive definite function will preserve the validity of the proof given in the text.

This solution provides the rate of change of the NN weights as a function of the distance from the desired solution $(\dot{\sigma} + D\sigma) = 0$ and at the moment $(\dot{\sigma} + D\sigma) = 0$ is reached the weights are not further updated while the motion of the system reaches the sliding mode manifold according to $(\dot{\sigma} + D\sigma) = 0$. As a result the control determined by (7) at the moment the sliding mode manifold is reached is equal to the equivalent control and thus the sliding mode motion in manifold (2) is enforced. To verify these comments the convergence to the global minima and the stability must be proven.

D. *Proof of Convergence*

One of the biggest problems in back propagation weight update algorithm is that system may not reach global minimum and may stay in some local minima. Investigating the shape of the error function (9), it can be shown that local minima do not exist for the selected formulation of the minimization problem.

1) The Shape of the Error Surface: If a function's second derivative w.r.t a function variable does not change sign, then the function does not have a change in the curvature sign through that variable, which means that the function does not have a local minimum through that variable. Taking the second derivative of the error function (9), w.r.t the weight w_{ij} gives the following result.

$$\frac{\partial^2 E}{\partial w_{ij}^2} = -\eta \left(\frac{\partial (\dot{\sigma} + D\sigma)^T}{\partial w_{ij}} \right) G B_i(x) e_{t_j} \tag{17}$$

Using the chain rule in derivation,

$$\frac{\partial^2 E}{\partial w_{ij}^2} = -\eta \left(\frac{\partial (\dot{\sigma} + D\sigma)^T}{\partial u_i} \frac{\partial u_i}{\partial w_{ij}} \right) GB_i(x) e_{t_j} \ , \tag{18}$$

and substituting (2) and (8) into (18), the following equation is obtained.

$$\frac{\partial^2 E}{\partial w_{ij}^2} = -\eta \left(\frac{\partial (G\dot{x}_d - G\dot{x})^T}{\partial u_i} \right) GB_i(x) e_{t_j}^2 \tag{19}$$

Substituting (1) into (19) gives the following equation.

$$\frac{\partial^2 E}{\partial w_{ij}^2} = \eta \left(\frac{\partial \left(f(x)^T G^T + u^T B(x)^T G^T + d^T G^T \right)}{\partial u_i} \right) GB_i(x) e_{t_j}^2 \tag{20}$$

Taking the derivative,

$$\frac{\partial^2 E}{\partial w_{ij}^2} = \eta B_i(x)^T G^T GB_i(x) e_{t_i}^2 = \eta \| GB_i(x) \|_2^2 e_{t_j}^2 \tag{21}$$

is obtained. Using the same procedure, second derivative of the error function (9) with respect to the bias weights are computed as

$$\frac{\partial^2 E}{\partial w_{i0}^2} = \eta B_i(x)^T G^T GB_i(x) = \eta \| GB_i(x) \|_2^2 .$$

The (21) and (22), show that the sign of the curvature of the error surface (9) is always positive; hence, there are no local minima, what indicates that, with a proper selection of the learning constant, the proposed network is capable of minimizing the function (9) up to its global minimum, which is nothing but zero. Thus, the tracking error vector has to converge to zero. Also, since η is a constant, G is a constant matrix and $B_i(x)$ is bounded, weight update algorithms (15) and (16) show that weights converge to a finite value. A finite value for the weights in steady state results in a bounded control input (8). As a result, all the signals in the control system are bounded.

E. Proof of Stability

Let the Lyapunov function candidate be the same function that is used for the cost function

$$V = \frac{1}{2} (\dot{\sigma} + D\sigma)^T (\dot{\sigma} + D\sigma). \tag{22}$$

It is easily seen that $V > 0$ for $\dot{\sigma} + D\sigma \neq 0$ and $V = 0$ for $\dot{\sigma} + D\sigma = 0$. Taking the time derivative of V, one obtains the following equation.

$$\dot{V} = -\sum_{i=1}^{m} \sum_{j=0}^{n} \frac{\partial V}{\partial w_{ij}} \frac{dw_{ij}}{dt} \tag{23}$$

Substituting (10) into (23) and using the identity $E = V$, gives the following expression.

$$\dot{V} = -\eta \sum_{i=1}^{m} \sum_{j=0}^{n} \left(\frac{\partial V}{\partial w_{ij}} \right)^2 \tag{24}$$

In the expression (24), η is a positive scalar, thus $\dot{V} \leq 0$. However, since it is proven that the error surface – hence, the Lyapunov function – does not have any local minima, the expression $\partial V / \partial w_{ij}$ becomes zero only at the global minimum, which is zero. So, $\dot{V} < 0$ for $\dot{\sigma} + D\sigma \neq 0$ and $\dot{V} = 0$ for $\dot{\sigma} + D\sigma = 0$. This proves that Lyapunov function converges to zero and the requirement $\dot{\sigma} + D\sigma = 0$ is satisfied, resulting in a stable system.

IV. A Special Case: SISO Systems

Since the MIMO system described in the previous section consists of a cluster of SISO systems, converting the above results to SISO systems is straightforward. In this section, first, the modified problem formulation for the SISO case is given and then related results are presented directly, without derivations.

A. Problem Statement

Consider the class of nonlinear systems described by the following differential equation.

$$\dot{x} = f(x) + B(x)u + d \tag{25}$$

where $x = \left[y, \ldots, y^{(n-1)} \right]^T \in \Re^n$ is the state vector, $y \in \Re$ is the system output, $u \in \Re$ is the control vector, $f(x) \in \Re^n$ is an unknown, continuous and bounded nonlinear function, $B(x) \in \Re^1$ is a known input gain coefficient, and $d \in \Re$ is an unknown, bounded external disturbance. Also, $y^{(n)} = d^n y / dt^n$. It is assumed that the system is controllable. The aim is to compute the control action u such that the output of the system y tracks the desired trajectory y_d, while desired state vector is defined as $x_d = \left[y_d, \ldots y_d^{(n-1)} \right]^T$. The tracking error vector is defined as $e_t = \left[e, \ldots, e^{(n-1)} \right]^T \in \Re^n$, where, $e = y_d - y$.

B. *Structure of the NN and Weight Updates*

Again NN minimizes $\dot{\sigma} + D\sigma$, where $\sigma = Ge_t$ is the sliding function and $G^T \in \Re^n$. Sliding manifold is defined as $\sigma = (d/dt + C)^{n-1} e = 0$. The NN used is presented in Fig. 2

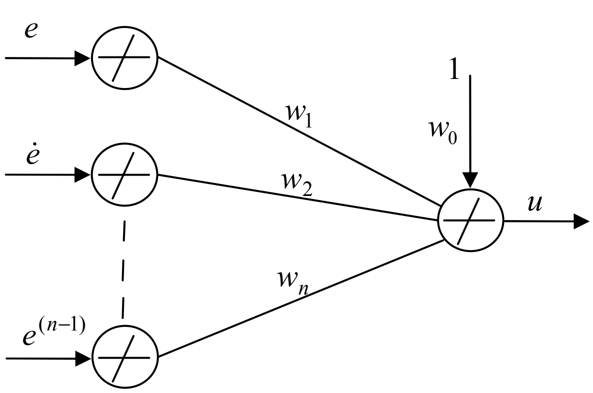


Figure 2. Structure of the NN for SISO Systems

This type of NN is called "adaptive linear element" (ADALINE). As seen from the figure, the output of the network (which is the control input u of the system (25)) is the weighted sum of the inputs, which are the individual elements of the tracking error vector, and the bias term. Using the same procedure as in the MIMO case, weight updates can be computed as

$$\dot{w}_i = \eta(\dot{\sigma} + D\sigma) GB(x) e_{t_i}, \quad i = 1,...,n \tag{26}$$

where e_{t_i} is the i^{th} row of the error vector e_t. For the bias term, weight update takes the following form.

$$\dot{w}_0 = \eta(\dot{\sigma} + D\sigma) GB(x) e \tag{27}$$

Convergence and the stability proofs are straightforward using the same procedure as in the MIMO case.

V. Experimental Results

A. *Experiments with a Single Axis, Linear Servo Drive – A SISO System*

The experiments to verify the theory for SISO systems are carried with a single axis, toothed belt, linear servo system, which is now in use at Sabanci University, Mechatronics Laboratory. The experimental setup scheme is presented in Fig. 3, where "M" and "E" refer to motor and the encoder respectively. This DGEL25-1500-ZR-KF linear drive is equipped by an electrical servo motor MTR-AC-70-3S-AA with a motor driver attached to a dSPACE DS1103 module hosted in the PC with dSPACE software Control Desk v.2.0 and the MATLAB 6.0.0.88.R12. The belt attached to the motor can carry different loads by a carriage. The load carried by the belt, the friction forces between the carriage and the rail and the friction on the motor bearings affect the motor as disturbance. A physical model of the overall system is presented in Fig. 4.

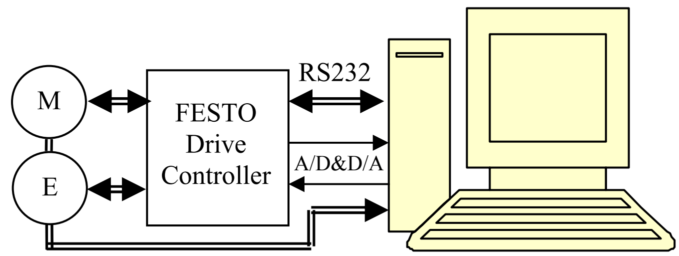


Figure 3. Simplified Structure of the Experimental Setup

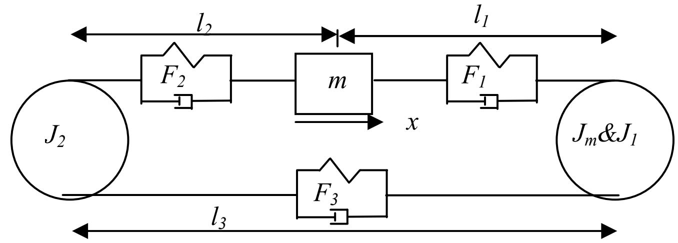


Figure 4. Physical Model of the Linear Servo Drive

In Fig.4, J_m, J_1 and J_2 refers to the inertias of the motor, the pulley that is driven by the motor and the idle pulley, while m refers to the mass of the load that is attached to the belt. The belt is modeled as a spring and damper system so that overall system is of the fourth order with scalar input – represented by the torque developed by the driving motor. Due to the belt force dependence on the belt stretch overall system can be presented as an dual mass system with flexible link. From the point of motor shaft control system can be taken as a second order system with motor current as an input, the motor shaft position as an output and the disturbance represented by the motor friction and the belt force reflected to the motor shaft [16]. The aim is to control the motor position without the information of the load or other disturbances. While designing the controller, the load m, the friction at the slider and at the motor bearings, and the belt force are assumed to be unknown. We assume the nominal value of the motor torque constant to be known. To control the position of the motor, the sliding manifold is chosen as $\sigma = \dot{e} + Ce$ where $e = \theta_r - \theta$ refers to the position error of the motor shaft. The control is implemented in a discrete-time form by implementing the calculation of weights as:

$$w_1(k+1) = w_1(k) + \eta \frac{K_t r}{J} (\dot{\sigma}(k) + D\sigma(k)) e(k) \tag{28}$$

$$w_2(k+1) = w_2(k) + \eta \frac{K_t r}{J} (\dot{\sigma}(k) + D\sigma(k)) \dot{e}(k) \tag{29}$$

$$w_3(k+1) = w_3(k) + \eta \frac{K_t r}{J} (\dot{\sigma}(k) + D\sigma(k)) \tag{30}$$

Controller parameters are selected as $C = 10 \quad D = 200$ $\eta = 0.00001$ and the sampling rate is 0.0001sec and the motor parameters K_t, J, r are assumed to be constant with their nominal values.

Fig. 5 – 11 show the response of the system to a smooth sigmoid position reference. The selection of such a reference is dictated by the limits on the acceleration timing-belt system may sustain on one hand and the usual profile of the velocity curve in the point-to-point industrial positioning systems. From Fig. 5, which shows the position tracking of the motor, it is very hard to distinguish the reference and actual motor positions. Fig. 6 shows the error in this tracking. As it is seen, the transient error makes a jump in the very beginning of the motion and then decreases fast during the tracking. In the end, steady state error reaches its theoretical limit, which is set by the position measurement device. In Fig. 7, the velocity tracking of the motor is presented. This figure also shows that after a deviation from the reference, the velocity catches its reference and tracks it. The initial deviation can be explained by the weights of the network starting from zero. After they reach certain values in a short time, the system behaves as desired. In Fig. 8, the control signal produced is shown. It is seen that the control signal is sufficiently smooth.

In Fig. 12 the transients for a small pulse changes in the motor position reference are depicted. The smooth transient without overshoot is achieved and as shown in Fig. 13 on a phase plot diagram the sliding mode manifold is reached and sliding mode motion is maintained in the system. This shows the capability of the proposed controller structure to cope with nonlinear disturbance which depends on the plant state variable (the dependence of the belt force on the motor position and velocity) while taking nominal value of the plant gain (the motor torque constant).

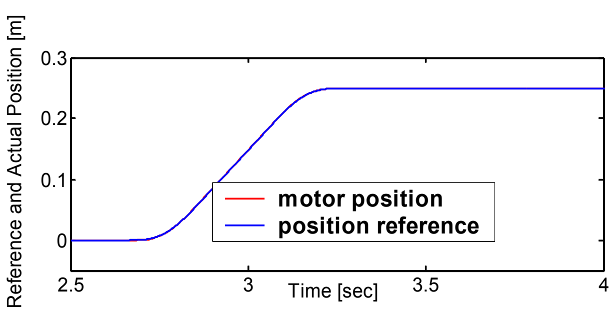


Figure 5. Position Tracking of the Motor

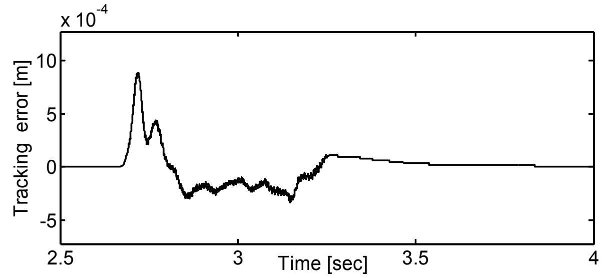


Figure 6. Position Tracking Error of the Motor

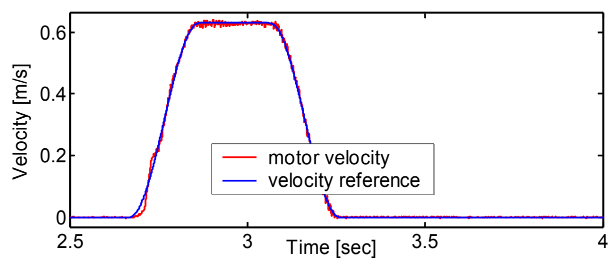


Figure 7. Velocity Tracking of the Motor

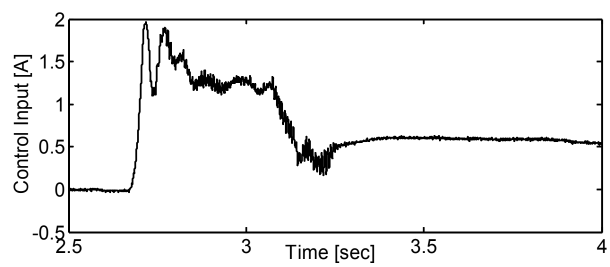


Figure 8. Control Input Applied to the Motor

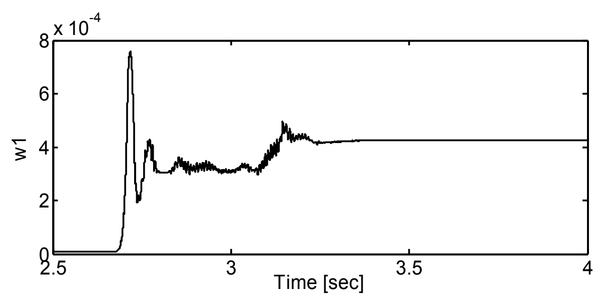


Figure 9. Time Evolution of w1

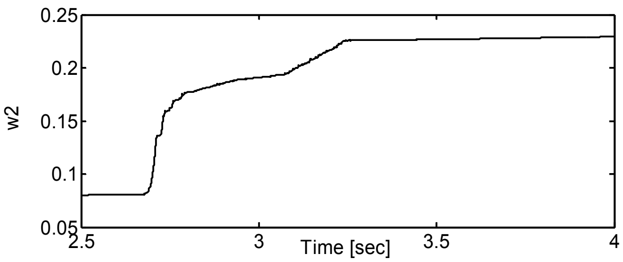


Figure 10. Time Evolution of w2

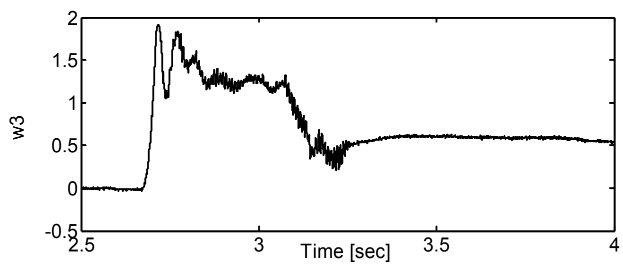


Figure 11. Time Evolution of w3

Fig. 9 – 11 indicate that after having a transient period, the weights are converging to a finite value, matching with the theoretical results.

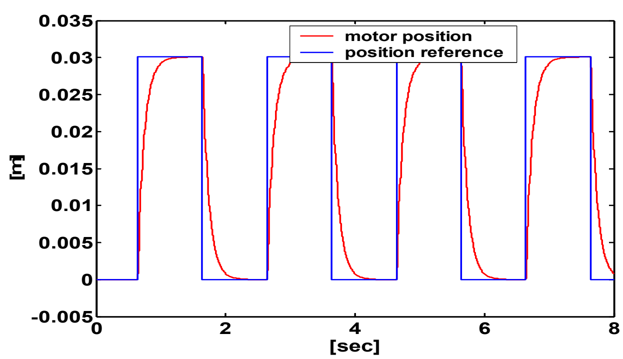


Figure 12. Transients for step position reference

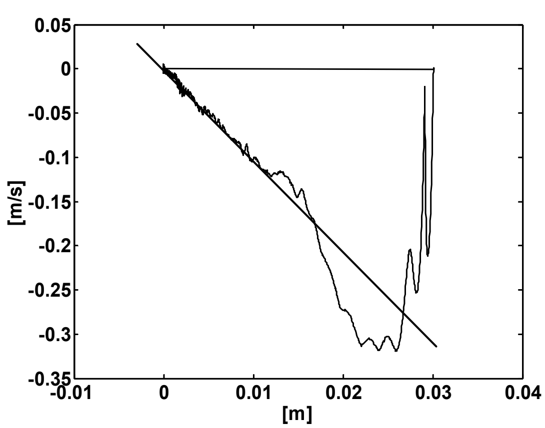


Figure 13. The phase plane for step position reference

The presented results confirm what has been proven in the previous sections. The sliding mode motion is achieved by simple weight update algorithm and very limited knowledge on the system's parameters.

B. Experiments with Piezoelectric Actuators – A MIMO System

In order to demonstrate applicability to the MIMO case the PZT dual actuator is controlled in such a way that it follows desired trajectory while enforcing desired grasping force. The experimental setup consists of a Piezomechanik's PSt150/5/60 stack actuators ($x_{max} = 60\,\mu m$, $F_{max} = 800\,N$, $v_{max} = 150$ Volt) connected to SVR150/3 low-voltage, low-power amplifiers. The actuators have built-in strain-gages for position measurement. The structure of the setup is presented in Fig. 14. In this setup, two piezo-drives (PD) are attached to each other via a load cell that is used for force measurement. The aim is to control the position of one actuator while controlling the force that is created due to the reaction of the load cell. Force control is achieved by moving the other actuator. Thus, there are two outputs of the system, position of one actuator and the force created in the load cell. Also there are two inputs: the voltage input to the actuator whose position is controlled and the voltage input to the other actuator by the help of which, the force is controlled. The overall system is described as two second order systems in interaction via load cell which is assumed without static. The conversion from the input voltage to the force is nonlinear having a hysteresis characteristics [17] what results in the plant gain being non single valued function of the input voltage and PZT stretch. Presence of the hysteresis nonlinearity in the system in addition to the unmodeled dynamics of the load cell makes design of the controller a challenging task. The sliding mode manifold is selected being intersection of the position tracking sliding mode function for PD-1 as $\sigma_x = \dot{e} + Ce$ where $e = x_{1r} - x_1$ and the sliding mode function for PD-2 as $\sigma_F = F_r - F$. The weights are updated the same way as in (28), (29) and (30) with respective changes of the switching functions. The nonlinear gain (due to hysteresis) is assumed to have constant value represented by the symmetry line of the hysteresis and its variation is treated as a matched disturbance in the system to be compensated by the bis term of the controller.

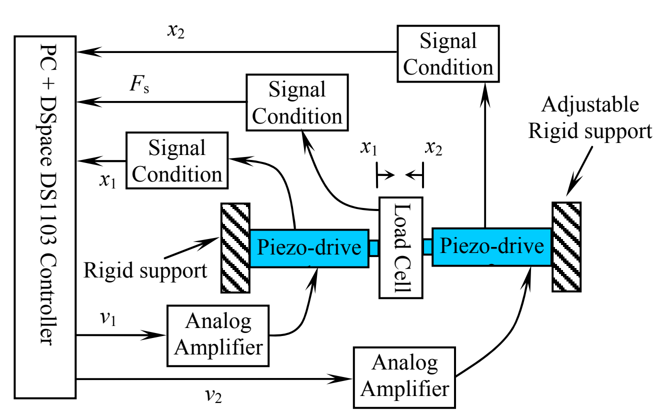


Figure 14. Simplified Experimental Setup

Fig. 15 – 21 depict the response of the system for a sigmoid reference for each of the actuators. For all the experiments, the controller parameters are $D = 400, \eta = 1.5$, and sampling time is selected as 0.0001 second. As shown the references are applied at the same time and PD-1 is able to track a sigmoid position reference while simultaneously PD-2 moves in such a way that the force created also tracks a sigmoid reference. Fig. 20 presents the trajectory when PD-2 follows in order to maintain the sigmoid force reference while PD-1 tracks its trajectory reference. The drift visible in Fig. 20 is due to the drift of the force transducer. Also, Fig. 17 and Fig. 20 show that both control inputs are bounded and well behaving.

In Fig. 22-23 the behavior of the same system for sinusoidal changes in position and the force is depicted. It shows the capability of the system to cope with harmonic change in both references.

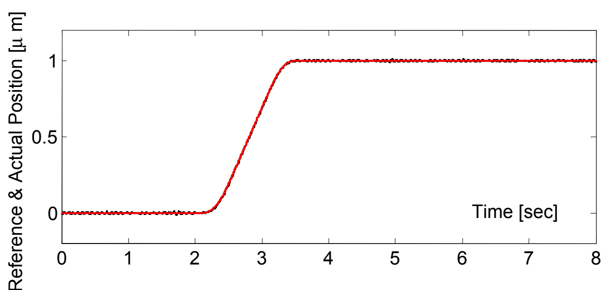


Figure 15. Position Tracking of PD-1

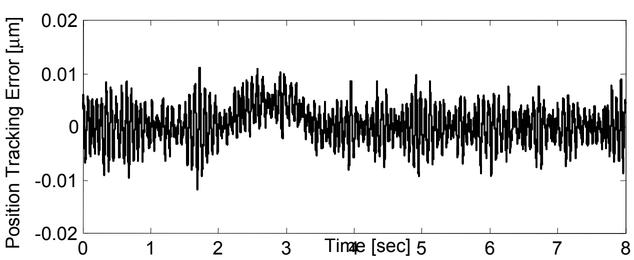


Figure 16. Tracking Error of PD-1

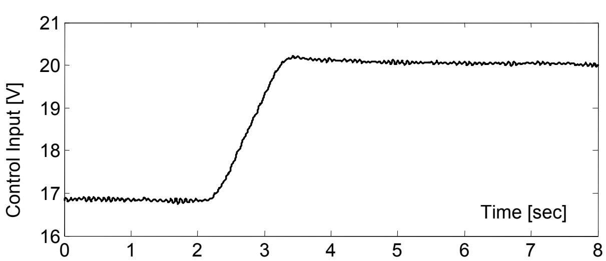
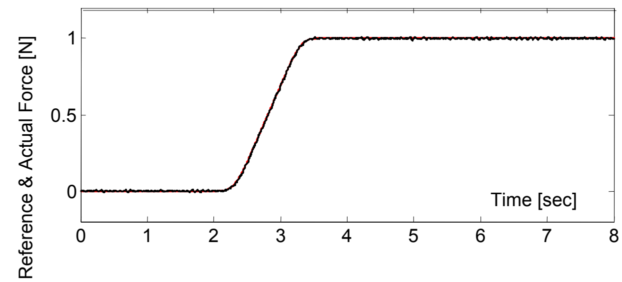


Figure 17. Control Input for PD-1



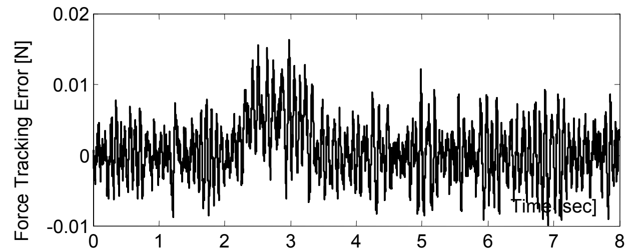


Figure 19. Tracking Error of PD-2

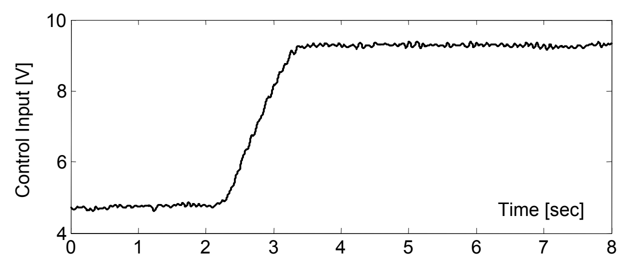


Figure 20. Control Input for PD-2

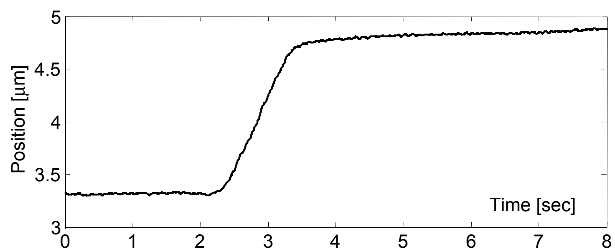
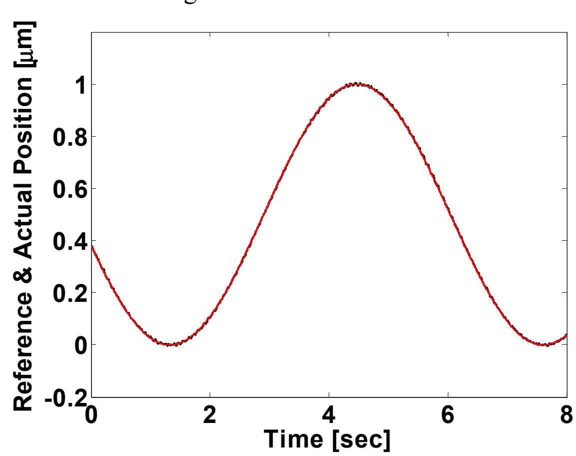
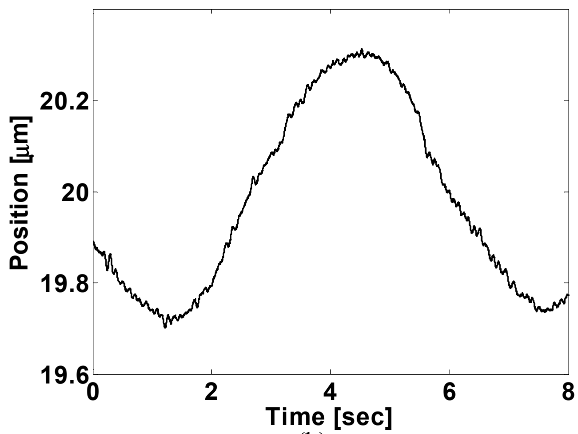


Figure 21. Position of PD-2



(a)



(b)

Figure 22. (a) Position Tracking of PD-1 and (b) position of PD-2 producing the force as depicted in Fig. 22

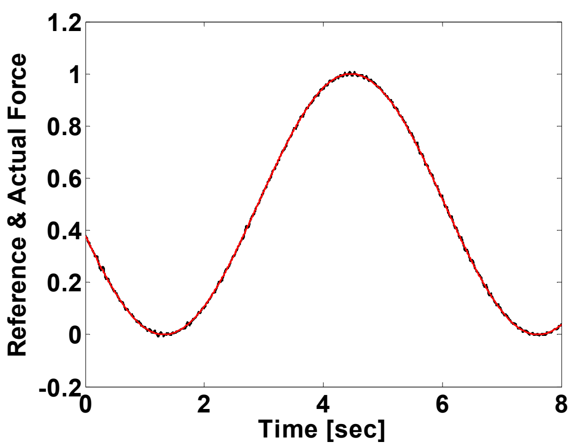


Figure 23. Force tracking

VI. CONCLUSION

In this work, a structure of adapting the weights of a neuro-sliding mode controller for uncertain systems is proposed..

Proposed controller ensures the overall stability of the closed loop dynamic of nonlinear system with bounded disturbance and guaranty asymptotic transient towards sliding mode manifold, thus guarantying a robust properties of the sliding mode control systems without any off-line training. The weight update is derived from the stability conditions and for its implementation the sliding mode function and the nominal value of the plant gain matrix are needed thus proposed algorithm seems simple enough for real applications. The rejection of disturbances that are satisfying matching condition is proven and demonstrated via experimental results. The applicability of the theoretical results is demonstrated for the nonlinear SISO and MIMO systems and results are shown to conform to predictions The proposed structure seems promising for application in controller parameters adaptation.

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