Investor's Increased Shareholding due to Entrepreneur–Manager Collusion

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Abstract

This study presents an investor/entrepreneur model in which the entrepreneur has opportunities to manipulate the workings of the project via hidden arrangements. We provide the optimal contracts in the presence and absence of such hidden arrangements. The contracts specify the shareholding arrangement between investor and entrepreneur. Moreover, we render an exact condition necessary for the credit market to form.

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1 Introduction

Analysis of investor–entrepreneur relations with the theory of contracts has provided important insight in recent years. In such models whenever it can be assumed that the entrepreneur has more control over the implementation of the project than the investor does, the following observation can be justified: when agency problems increase, entrepreneurs have more options to manipulate the operation of the project to their advantage.

This study presents an investor–entrepreneur model with collusion between the entrepreneur and the agent operating the project. The entrepreneur has the ability to influence the workings of the project via hidden arrangements.

Our model builds upon a two principal, one agent version of the one in Holmstrom and Milgrom (1991). The first and wealth-constrained principal, the entrepreneur, is risk-neutral and possesses an asset/project, but lacks the required startup capital and needs to employ a risk-averse agent (manager) to operate it. The critical feature of our model is that the project renders two dimensional verifiable and non-divertable returns, which can be interpreted respectively as money and power. Naturally, the technology is such that money and power are substitutes.

The entrepreneur can obtain the startup capital from an investor (the second, risk-neutral and non-wealth-constrained principal), who must be paid off from the returns of the project. In order to do that, the entrepreneur makes a take-it-or-leave-it offer to the investor, and this offer consists of a contract, a feasible monetary compensation scheme and shares of the project.[1] If possible an easy method of compensating the investor is to pay back the startup capital (possibly with interest) from the monetary returns of the project. However, when the monetary returns do not suffice, then the entrepreneur also has to give some portion of the project to the investor.[2] This arrangement, then, gives birth to non-trivial strategic interactions between the investor and the entrepreneur.

Indeed, we assume that the entrepreneur and the investor do not view the two dimensional returns the same, that is, their priorities over money and power differ. The fact that investors and

[1]Our model contains only one investor. Yet, due to the entrepreneur making a take-it-or-leave-it offer, our model can be interpreted as one in which there are many competing investors. This is because, in both of these formulations the results will not change due to the investor(s) not obtaining any additional surplus.

[2]The act of giving some portion of the project to the investor in exchange for the startup capital can be seen as the entrepreneur selling some of his shares. The price at which this transaction occurs can be derived from our results characterizing the optimal contracts between the entrepreneur and the investor, namely Propositions 1 and 3.

entrepreneurs might have different objectives is a well known phenomenon and needs mentioning at this point. For example, Shleifer and Vishny (1997) discusses "some major conglomerates, whose founders built vast empires without returning much to investors".

In our model, the monetary returns are transferable, but the second return (power) is not. The only way for the entrepreneur to transfer some of the second return is by giving the investor some shares of the project. Moreover, we assume that the entrepreneur assigns a higher value to the second return than the investor does. In particular, both of the principals' payoff functions aggregate the expected returns in a linear fashion, where the difference between the two is due to entrepreneur's coefficient for power being strictly higher than that of the investor.

There are two technical assumptions for the derivation of our results. Assumption 1 ensures that it is strictly beneficial for the investor to own the whole project while the entrepreneur cannot manipulate the agent. It should be pointed out that under this assumption the set of feasible contracts (between the entrepreneur and the investor) which makes both principals willing to participate is non-empty. Assumption 2 guarantees the participation constraint of the second principal.

As mentioned above, when the monetary returns from the project do not suffice to pay back the startup capital, the investor must be given some shares of the project. Consequently, he has a say in the arrangement/allocation of the resources on the two dimensional returns for this project. The process of deciding which arrangement to choose is modeled with a utilitarian bargaining problem between investor and entrepreneur, where their bargaining weights are given by the fraction of the project they own.[3]

Therefore, when the entrepreneur can commit to honor the outcome of the bargaining process between him and the investor,[4] the entrepreneur offers the optimal (incentive compatible and individually rational) contract to the agent that ensures the implementation of the allocation determined by the bargaining process. However, when such a commitment is impossible, the entrepreneur has an opportunity to have the agent implement another arrangement via a secret side contract between the agent and the entrepreneur. That is, in the no-commitment case the entrepreneur and the agent may collude, and this leads to an agency problem which is in the same spirit as those in the renegotiation proofness of Maskin and Moore (1999), and the collusion proofness of Laffont and

[3]We refer the reader to Thomson (1981) for more on utilitarian bargaining problems.

[4]Alternatively, the investor perfectly observes all interaction between entrepreneur and agent.

Martimort (2000).

We show that the optimal contract between the investor and the entrepreneur is not immune to collusion. Furthermore, characterizations of the optimal contracts in both commitment and no-commitment cases are provided. Based on those characterizations, we investigate the effect of collusion on investor's share of the project. We show the existence of cases where this share increase and decrease. Moreover, in both commitment and collusion the associated optimal contracts make the entrepreneur obtain strictly positive payoffs, while the investor is not given any additional surplus.

When the entrepreneur may collude with the agent, he has the opportunity to offer a hidden side contract to the agent. Hence, it must be that the investor is not paying any of the resulting additional costs, because otherwise he would become aware of this arrangement. Thus, the entrepreneur's benefit of collusion with the agent consists of the collection of additional returns from determining the allocation of resources on his own. Meanwhile, the entrepreneur's cost of collusion is due to him being restricted to pay all the additional costs on his own. Then, we prove that in the no-commitment case the investor (considering the entrepreneur's offer) knows the following: the entrepreneur will make sure that the project will be implemented with a weight (on market share) strictly lower than the one obtained from the bargaining between the two. That is, with collusion the entrepreneur is able to divert the payments that were supposed to be made to the agent, by making him work at an arrangement different than the one agreed by the investor.

Zingales (1994) and Barca (1995) provide some partial empirical support for our conclusions. Indeed they conclude that managers in Italy (whom are to be interpreted as the entrepreneurs in our setting) have significant opportunities to divert profits to themselves and not share them with shareholders uninvolved in the companies' operations.

Our model can be applied to shareholding by commercial banks, a topic of recent interest. In our model, the investor can be interpreted as a bank, providing funds, and it is not difficult to imagine that the bank/investor has little expertise on the particular field of the project. It should be noted that while in some countries, such as the USA, shareholding is prohibited, while in others such as Japan, Norway, and Canada banks are allowed to own equities of firms up to a certain legal limit. Santos (1999) reports that

> This limit is 50 percent in Norway; 25 percent in Portugal; 10 percent in Canada and

> Finland; 5 percent in Belgium, Japan, the Netherlands, and Sweden; and zero percent in the United States, because U.S. commercial banks are not allowed to invest in equity. Germany and Switzerland are examples of countries where banks' investments in equity are not limited by that form of regulation.

Moreover, Flath (1993) examines the situation in Japan reports that "largest debtholders ...[among Japanese banks] hold more stock if the firms ... [are more] prone to the agency problems of debt ...". James (1995) specifies conditions where banks are willing to own equity. It should be mentioned that Santos (1999) argues that "equity regulation is never Pareto-improving and does not increase the bank's stability".

We specify a condition, Assumption 2, which must be satisfied in order that the credit market form. In cases where this specification is not fulfilled, the investor does not have any incentives to provide the necessary funding regardless of the amount of shares offered to him. Hence, we provide a necessary condition for the participation constraint of the investor.

For the rest of the section, we wish to discuss some aspects of our model in more detail. First of all, it is imperative to stress that in our model collusion occurs between the entrepreneur and the agent. Hence, unlike the situation in Itoh (1991), Laffont and Martimort (2000), Laffont and Martimort (1997) and Barlo (2006), in this study collusion is not an ingredient of the strategic interaction among agents. Rather, it shares the same spirit as the renegotiation proofness of Maskin and Moore (1999), because the entrepreneur is restricted to offer contracts which are immune to his intervention in the later stages of the game.

The second point we wish to emphasize is about the structure of our model. We borrow the basic model of Holmstrom and Milgrom (1991) in which attention is restricted to CARA utilities (for the agent), normally distributed returns and linear contracts. [5] Our modifications consist of using two principals (instead of only one), and solving the interaction between the two principals with utilitarian bargaining in the commitment case, and incorporating collusion between one of the principals and the agent into this setting. Moreover, we need to mention that dispensing with the

[5] The reader may need to be reminded that the pioneering model in this field is given in Holmstrom and Milgrom (1987). This research was followed by Schattler and Sung (1993) and Hellwig and Schmidt (2002) who provided important extensions. Those studies feature repeated agency settings in which the lack of income effects (due to exponential utility functions) are employed to show the optimality of linear contracts. Lafontaine (1992) and Slade (1996), on the other hand, provide empirical evidence for the use of linear contracts.

agent in this model is a possibility, yet, we believe keeping the agent as a part of the analysis is more appealing in terms of applications.

The third and final aspect that we wish to discuss concerns principal's benefit functions. As mentioned above, we assume that both principals' returns are not transferable. On the other hand, the monetary returns from the project can be transferred without any frictions, yet, the only way to transfer utility using the second return involves transfer of shares. Moreover, we assume that each principals aggregate the expected two dimensional returns linearly, and the only difference between the two arises due to the multiplier of power. We argue that this form essentially captures the inherent distinction between an entrepreneur and an investor, and also allows us to come up with a clear presentation. Therefore, using these observations an alternative interpretation for the two types of returns in our model can be given as follows: let the first return be the immediate monetary ones, and the second be the "*market share*" of the project. Assuming that the level of personal authority that the entrepreneur derives from the project is not transferable and increases with market share, suffices for our purposes. It should be pointed out that this last assumption is consistent with our interpretation of the identities of the principals. Indeed, we think of the entrepreneur to be someone who is associated in the area of the project and has an "idea" but not the cash, and the investor to be a financial intermediary whose first priority is monetary, which is potentially followed by his investments' market shares.

Section 2 develops the model with commitment: Proposition 1 characterizes its solution. In section 3 we extend the model to capture collusion between the entrepreneur and the agent, and in Proposition 2 we show that the commitment contract (between the entrepreneur and the investor) is not immune to collusion. Moreover, Proposition 3 characterizes the solution in the no-commitment case. Finally, Theorem 1 displays that the share of the investor is higher when there is collusion. Section 4 concludes.

2 The Model With Commitment

We will consider a linear, two–principal, single–agent, and two–task hidden–action model with state–contingent, observable and verifiable two–dimensional returns. Indeed it builds upon a two principal version of the one presented in Holmstrom and Milgrom (1991), and we will keep their notation.

Principal 1, the entrepreneur, owns an asset which requires a capital fixed cost of $K > 0$, and an agent. If operated, this asset delivers two–dimensional, state–contingent, observable and verifiable returns drawn from a normal distribution whose covariance matrix is assumed to be fixed. Through out this study, it is useful to assume that the first dimension of the returns is monetary, and the second related to individual power. Principal 1 does not possess the required capital investment of K, but has the option of obtaining it from principal 2, the investor; by paying him a fixed compensation R, and possibly making principal 2 be a partner with a share $(1 - \rho)$, where $\rho \in [0, 1]$ be the share of principal 1. Both of the principals are risk-neutral, and evaluate the two dimensional returns as follows: Given an expected monetary and power return $b = (b_1, b_2)$, the gross benefits (not including the costs of operating the project) to principal i is given by $\rho_i(b_1 + \lambda_i b_2)$, where $\rho_1 = \rho$, $\rho_2 = (1 - \rho)$, and $\lambda_i > 0$. Without loss of generality, we assume that $\lambda_1 > \lambda_2$. Moreover, given (ρ, R), the two principals will be involved in a utilitarian bargaining where each of them has a bargaining power given by the share of the project they possess.

After determining the nature of the point that they want to implement, they will seek to employ an agent, who has CARA utilities. Furthermore, the mean of the two–dimensional returns is determined by the employee's effort choice, which none of the principals can observe or verify. Hence, any contract to be offered cannot depend on agent's effort choice.

In summary, the timing of the game is as follows:

$t = 1:$ Principal 1 offers (ρ, R) to principal 2 for him to supply K, and principal 2 accepts or rejects. If principal 2 rejects the offer, the game ends and both principals get a payoff of 0; otherwise, it continues.

$t = 2:$ With bargaining weights given by their share of the project, the principals bargain over the feasible allocation of resources for the project. This determines a level of $\bar{\lambda} \in [\lambda_2, \lambda_1]$ that the principals have agreed upon.

$t = 3:$ Given $\bar{\lambda}$, the principals determine the optimal contract, and the entrepreneur offers it to the agent;

$t = 4:$ The agent chooses whether or not he should accept the offer, and exert the effort level the principals would like him to. Then, the observable and verifiable (by all) state is realized.

$t = 5:$ The entrepreneur makes the payments to the agent, and all of them are both observable and verifiable by the investor.

2.1 Agent's Problem

The agent determines a vector of efforts $t \in \Re^2_+$. The monetary and private cost of effort is given by $C : \Re^2_+ \to \Re_+$. We assume that $C(t_1, t_2) = \frac{k_1 t_1^2}{2} + \frac{k_2 t_2^2}{2}$, where k_1, k_2 are both strictly positive real numbers. We should note that C as defined above is a continuous and strictly convex function. Once t is determined, the returns are distributed with a two–dimensional normal distribution with mean

$$\mu(t) = \begin{pmatrix} \mu_1(t_1) \\ \mu_2(t_2) \end{pmatrix} = \begin{pmatrix} \gamma_1 t_1 \\ \gamma_2 t_2 \end{pmatrix}. \tag{1}$$

It should be noticed that $\mu : \Re^2_+ \to \Re^2$ is a continuous and concave function of t. The agent's effort choice creates a two–dimensional signal of information, $x \in \Re^2$, observable and verifiable by the two principals. x is given by $x = \mu(t) + \epsilon$, where ϵ is normally distributed with mean zero and covariance matrix

$$\Sigma = \begin{pmatrix} \sigma_1^2 & 0 \\ 0 & \sigma_2^2 \end{pmatrix}.$$

The agent has *constant absolute risk aversion* (CARA) utility functions, with a given CARA coefficient of $r \in \Re_{++}$. That is for $w \in \Re$, $u(w) = -\mathrm{e}^{-rw}$. Under a compensation scheme $w : \Re^2 \to \Re$, where $w(x)$ is often to be referred to as the wage at information signal x, the agent's expected utility is given by $u(CE) = \int_{-\infty}^{+\infty} - \exp\{-r(w(x) - C(t))\}\mathrm{d}x$, where CE denotes the certainty equivalent money payoff of the agent under the compensation scheme w. Moreover, the reserve certainty equivalent figure of the agent is normalized to 0.

We restrict attention to linear compensation rules of the form $w(x) = \alpha^T x + \beta$, where $\alpha \in \Re^2_+$, and $\beta \in \Re$. Making use of the CARA utilities and the normal distribution, it is easy to show that under our formulation the certainty equivalent of such a compensation scheme is

$$CE = (\alpha_1\gamma_1 t_1 + \alpha_2\gamma_2 t_2) - \left(\frac{k_1 t_1^2}{2} + \frac{k_2 t_2^2}{2}\right) - \frac{1}{2} r \left(\alpha_1^2\sigma_1^2 + \alpha_2^2\sigma_2^2\right) + \beta.$$

Consequently, by considering the first order conditions it is straightforward to see that given a linear compensation scheme, agent's optimal choice of effort is

$$t_\ell^* = \frac{\gamma_\ell \alpha_\ell}{k_\ell}, \tag{2}$$

$\ell = 1, 2$.

2.2 Optimal Offer To The Agent

The expected gross benefits from the project of principal i, $i = 1, 2$; is given by $B_i(t)$. As we mentioned above, we let $B_i(t) = \mu_1(t_1) + \lambda_i \, \mu_2(t_2)$, where $\lambda_i > 0$ for $i = 1, 2$.

At this stage it is useful to come back to the initial phase of the game. As mentioned above, first the principals will bargain to determine the weight $\bar{\lambda} \in [\lambda_2, \lambda_1]$ (recall that we have assumed without loss of generality that $\lambda_1 > \lambda_2$) to be used when the optimal contact is to be formulated. After agreeing on $\bar{\lambda}$, principal i's problem is

$$\max_{\alpha,\beta} \rho_i \left(\mu_1(t_1) + \bar{\lambda}\mu_2(t_2) - C(t) - \frac{1}{2} r \left(\alpha_1^2 \sigma_1^2 + \alpha_2^2 \sigma_2^2 \right) \right) \tag{3}$$

subject to (2), because while collecting ρ_i portion of the returns, principal i has to pay also ρ_i portion of the costs as well. Therefore, after agreeing on $\bar{\lambda}$, the incentives of the two principals are perfectly aligned; or formally, the solution to (3), is the same as the solution to the following (aggregated) maximization problem

$$\max_{\alpha,\beta} \left(\mu_1(t_1) + \bar{\lambda}\mu_2(t_2) - C(t) - \frac{1}{2} r \left(\alpha_1^2 \sigma_1^2 + \alpha_2^2 \sigma_2^2 \right) \right) \tag{4}$$

By Holmstrom and Milgrom (1991) we know that the optimal contract would not render any excess surplus to the agent. Thus, the optimal constant intercept, β^*, (which does not affect incentives due to lack of income effects thanks to CARA utility function) must be such that, at the optimal contract $CE = 0$. (Recall that the reserve certainty equivalent figure of the agent is normalized to 0.)

Working with first order conditions to solve the principals' problem, one can show that α_1^* and α_2^* are given as follows

$$\alpha_1^* = \frac{\frac{\gamma_1^2}{k_1}}{\frac{\gamma_1^2}{k_1} + r\sigma_1^2}, \tag{5}$$

and

$$\alpha_2^* = \frac{\frac{\gamma_2^2}{k_2}}{\frac{\gamma_2^2}{k_2} + r\sigma_2^2} \, \bar{\lambda} \, . \tag{6}$$

Now, substituting equations 5 and 6, into equation 2, and using 1, it can be obtained that when the principals agree on $\bar{\lambda}$, the project will deliver the following net benefit to principal i when

$\bar{\lambda} \in [\lambda_2, \lambda_1]$ is implemented:

$$\Pi_i(\bar{\lambda}) = \frac{1}{2}\Phi_1 + \bar{\lambda}\left(\lambda_i - \frac{1}{2}\bar{\lambda}\right)\Phi_2, \tag{7}$$

where

$$\Phi_\ell = \frac{\left(\frac{\gamma_\ell^2}{k_\ell}\right)^2}{\frac{\gamma_\ell^2}{k_\ell} + r\sigma_\ell^2}, \tag{8}$$

$\ell = 1, 2$.

Lemma 1 *The following hold for $\Pi_i : [\lambda_2, \lambda_1] \to \Re$, $i = 1, 2$:*

1. *For all $\bar{\lambda} \in [\lambda_2, \lambda_1]$, $\Pi_1(\bar{\lambda}) - \Pi_2(\bar{\lambda}) = \bar{\lambda}(\lambda_1 - \lambda_2)\Phi_2 > 0$;*
2. *Π_i is strictly increasing for $\hat{\lambda} < \lambda_i$, and strictly decreasing for $\hat{\lambda} > \lambda_i$; and,*
3. *Π_i is strictly concave on $(0, 1)$, and $\partial\Pi_i(\bar{\lambda})/\partial\bar{\lambda}$ evaluated at $\bar{\lambda} = \lambda_i$ equals 0.*

Proof. While the first conclusion follows from employing equation (7), the others are due to the derivative of $\Pi_i(\bar{\lambda})$ being given by

$$\frac{\partial\Pi_i}{\partial\bar{\lambda}} = \Phi_2(\lambda_i - \lambda). \tag{9}$$

■

Thus, in order to guarantee the non-emptiness of the participation constraint of principal 1, the following technical assumption is needed:

Assumption 1 *The following holds:*

$$\frac{1}{2}(\lambda_2)^2\Phi_2 > K - \frac{1}{2}\Phi_1. \tag{10}$$

What Assumption 1 says is that in the case when principal 2 is the sole owner, it should be worthwhile to undertake this project. Notice that when this condition holds, then for any $\rho \in [0, 1]$, and for any $\lambda \in [\lambda_2, \lambda_1]$, the participation constraint of principal 1 will be non-empty. This is because

$$\begin{aligned}\rho\Pi_1(\lambda) + (1-\rho)\Pi_2(\lambda) - K &= \lambda\left((\rho\lambda_1 + (1-\rho)\lambda_2) - \frac{1}{2}\lambda\right)\Phi_2 - K + \frac{1}{2}\Phi_1 \\ &\geq \frac{1}{2}(\lambda_2)^2\Phi_2 - K + \frac{1}{2}\Phi_1 > 0.\end{aligned}$$

The inequality preceding the last is due to $\lambda \in [\lambda_2, \lambda_1]$. It should be pointed out that the participation constraint of the second principal is ensured by this very same condition. Because that it will be dealt later in greater detail, it suffices for now to mention that the participation constraint of principal 1 already takes care of that of the second principal due to the following: When player 1 has opportunities to make strictly positive profits, then he would make sure that principal 2 gets at least a payoff of K, ensuring his individual rationality.

2.3 Bargaining Over Implementable Contracts

Having determined the outcome and associated net returns, we may restrict attention to the bargaining between the two principals in the first phase of the game.

The two principals will bargain over the choice of $\bar{\lambda}$, and the set of admissible values must be in $[\lambda_2, \lambda_1]$. If the principals cannot agree in that bargaining, the project cannot go ahead, and thus, we assume each gets a return equal to their reserve value which is normalized to 0. Hence, the bargaining set is

$$S = \{(\pi_1, \pi_2) : \pi_i \in [0, \Pi_i(\lambda)], \text{ for some } \lambda \in [\lambda_2, \lambda_1]\}. \tag{11}$$

The Pareto optimal[6] frontier of S denoted by ∂S then is

$$\partial S = \{(\pi_1, \pi_2) : \pi_i = \Pi_i(\lambda), \text{ for some } \lambda \in [\lambda_2, \lambda_1]\}.$$

The following lemma establishes that $(S, \mathbf{0})$ is a well defined and "nice" bargaining problem. Moreover, such a bargaining set is given in figure 1 for the case when $\lambda_1 = 3/4$, $\lambda_2 = 1/4$, $\Phi_1 = 1$, and $\Phi_2 = 2$.

Lemma 2 *S is non–empty, compact and convex. Moreover, ∂S is strictly concave.*

Proof. Non–emptiness is trivial, because $\pi = (\Pi_1(\lambda_2), \Pi_2(\lambda_2))$ is both in S. Moreover, since all the variables are continuous, and $[\lambda_2, \lambda_1]$ is compact, compactness of S follows.

Since showing convexity of S is a standard exercise, it suffices to prove that ∂S is strictly concave. To that regard, let $\alpha \in (0, 1)$ and $\pi, \pi' \in \partial S$ with λ and λ' such that $\pi_i = \Pi_i(\lambda)$ and $\pi'_i = \Pi_i(\lambda')$, for all i. For a contradiction suppose that $\tilde{\pi} = \alpha\pi + (1 - \alpha)\pi'$ is in ∂S. Thus, there exists $\tilde{\lambda}$ such

[6]By Pareto optimality we mean the regular, non–strict, one.

that $\tilde{\pi}_i = \Pi_i(\tilde{\lambda})$. Hence, due to the strict concavity of Π_i, $i = 1, 2$, established in Lemma 1, we have

$$\Pi_i(\tilde{\lambda}) = \alpha\Pi_i(\lambda) + (1-\alpha)\Pi_i(\lambda') < \Pi_i(\alpha\,\lambda + (1-\alpha)\,\lambda'), \tag{12}$$

$i = 1, 2$. Finally, due to the same Lemma, we know that Π_1 is strictly increasing, therefore, inequality 12 implies $\tilde{\lambda} < \alpha\,\lambda + (1-\alpha)\,\lambda'$. The proof finishes, because the same inequality and Π_2 being strictly decreasing implies $\tilde{\lambda} > \alpha\,\lambda + (1-\alpha)\,\lambda'$, delivering the necessary contradiction. ■

These bargaining problems will be solved by the utilitarian solution concept. Please refer to Thomson (1981) for a detailed analysis of this bargaining solution. That is, for $(S, \mathbf{0})$, and for any given weights $\theta, (1-\theta) \in [0, 1]$, $\pi^\theta \in \tilde{S}$ is the *θ–utilitarian bargaining solution* of $(S, \mathbf{0})$ if and only if

$$(\pi_1^\theta, \pi_2^\theta) = \mathcal{N}(S, \mathbf{0}; \theta) \equiv \text{argmax}_{(\pi_1, \pi_2)\in S}\, \theta\pi_1 + (1-\theta)\pi_2. \tag{13}$$

Note that by Lemma 2, there exists a unique solution to $(S, \mathbf{0})$ for all $\theta \in [0, 1]$, thus $\mathcal{N}(S, \mathbf{0}; \theta)$ is a function. Moreover, it should be pointed out that we treat $\theta \in [0, 1]$ as exogenously given. For notational purposes, we let λ^θ be defined by $\Pi_i(\lambda^\theta) = \pi_i^\theta$, for $i = 1, 2$.

Lemma 3 *For every $\theta \in [0, 1]$, $\mathcal{N}(S, \mathbf{0}; \theta)$ is a function, and $\lambda^\theta \in [\lambda_2, \lambda_1]$ is strictly increasing in θ and is uniquely determined as follows:*

$$\lambda^\theta = \theta\,\lambda_1 + (1-\theta)\,\lambda_2\,. \tag{14}$$

Proof. The required conditions for the existence of the utilitarian bargaining solution f_θ have been shown to be satisfied. Namely, S is compact and convex, $\mathbf{0} \in S$, and by Assumption 1, there exists some $s \in S$ with $s_j > 0$, for $j = 1, 2$. Therefore, for any $\theta \in [0, 1]$ we have $\mathcal{N}(S, \mathbf{0}; \theta) \neq \emptyset$. Moreover, since ∂S is strictly concave, $\mathcal{N}(S, \mathbf{0}; \theta)$ is a function. By the Pareto efficiency axiom for the utilitarian bargaining solutions, $\mathcal{N}(S, \mathbf{0}; \theta) \in \partial S$, for all $\theta \in [0, 1]$. Recall that the definition of ∂S implies that there is some $\lambda^\theta \in [\lambda_2, \lambda_1]$ such that $\mathcal{N}(S, \mathbf{0}; \theta) = (\pi_1^\theta, \pi_2^\theta) = (\Pi_1(\lambda^\theta), \Pi_2(\lambda^\theta))$. This λ^θ is unique because Π_i are one-to-one (strictly monotone) functions of λ on $[\lambda_2, \lambda_1]$ by Lemma 1. Moreover, solving the following maximization problem with first order conditions

$$\max_{\lambda\in[\lambda_2,\lambda_1]} \theta\left(\frac{1}{2}\Phi_1 + \lambda\left(\lambda_1 - \frac{1}{2}\,\lambda\right)\Phi_2\right) + (1-\theta)\left(\frac{1}{2}\Phi_1 + \lambda\left(\lambda_2 - \frac{1}{2}\,\lambda\right)\Phi_2\right).$$

and noticing that the objective function is linear, and by Lemma 2 the boundary of the constraint set is strictly concave; and further noting that $\lambda_1 - \lambda_2 > 0$, delivers the conclusion. ■

Thus, given (ρ, R), that the principal 1 offered to principal 2 who accepted and supplied the capital investment of K, the net returns to principals are $\Pi_1(\rho, R) \equiv \rho\Pi_1(\lambda^\rho) - R$, and $\Pi_2(\rho, R) \equiv (1-\rho)\Pi_2(\lambda^\rho) + R$. It should be pointed out that by Lemma 1, $\Pi_1(\rho, R)$ is strictly increasing in ρ, and $\Pi_2(\rho, R)$ strictly decreasing.

2.4 Entrepreneur's Optimal Offer To the Investor

For (ρ, R) to be participatory for principal 2, who supplies the capital investment needed for the project, we need to have $\Pi_2(\rho, R) \geq K$. Thus, the program that the investor, principal 1, has to solve is $\max_{(\rho,R)} \Pi_1(\rho, R)$ subject to the participation constraints of the two principals, i.e. (1) $\Pi_1(\rho, R) \geq 0$, and (2) $\Pi_2(\rho, R) \geq K$. That is, the entrepreneur solves the following problem:

$$\max_{(\rho,R)} \quad \rho\Pi_1(\lambda^\rho) - R \tag{15}$$

subject to

$$\rho\Pi_1(\lambda^\rho) - R \geq 0$$
$$(1-\rho)\Pi_2(\lambda^\rho) + R \geq K.$$

Let $(\rho^\star, R^\star)$ solve this problem. Noticing that $\Pi_1(\rho, R)$ is strictly increasing in ρ, implies that the participation constraint of principal 2 will hold with equality at the solution $(\rho^\star, R^\star)$. Thus, ignoring the participation constraint of principal 1 for now, (15) is reduced to

$$\max_{(\rho,R)} \rho\Pi_1\left(\rho\,\lambda_1 + (1-\rho)\,\lambda_2\right) + (1-\rho)\Pi_2\left(\rho\,\lambda_1 + (1-\rho)\,\lambda_2\right) - K. \tag{16}$$

First notice that due to Lemma 1, the objective of this maximization is continuous in ρ. Moreover, principal 1 is solving a non-trivial utilitarian planner's problem where the weights assigned to the agents must be interpreted as their share of the project.

The following Proposition characterizes the solutions to (15):

Proposition 1 *Suppose that Assumption 1 holds. Then there exists a unique $(\rho^\star, R^\star)$ solving (15). Moreover, they are characterized as follows:*

1. *If $\frac{1}{2}\Phi_1 \geq K$, then $\rho^\star = 1$, and $R^\star = K$.*
2. *If $\frac{1}{2}\Phi_1 < K$, then $\rho^\star$ is the maximum real number in $[0, 1]$ solving*

$$(1-\rho^\star)\left(\rho^\star\,\lambda_1 + (1-\rho^\star)\,\lambda_2\right)\left(\lambda_2 - \frac{1}{2}\left(\rho^\star\,\lambda_1 + (1-\rho^\star)\,\lambda_2\right)\right)\Phi_2 \;=\; K - \frac{1}{2}\Phi_1, \tag{17}$$

and $R^\star = \rho^\star\left(\frac{1}{2}\Phi_1\right)$.

Proof. When Assumption 1 holds constraint set is non-empty and compact, thus, due to the continuity of the objective function of 16 there exists a solution.[7]

When $\frac{1}{2}\Phi_1 \geq K$ Because that $\Pi_1(\lambda_1) - \Pi_2(\lambda_2) = (1/2)\Phi_2(\lambda_1^2 - \lambda_2^2) > 0$ since $\lambda_1 > \lambda_2$, the optimal solution would be so that principal 1 would be the sole owner of the project, and on expected terms would pay off the capital investment borrowed from the second principal in full using only the monetary returns from the project. Notice that the solution in this case is unique.

Suppose $\frac{1}{2}\Phi_1 < K$. Let $A(\rho)$ be defined by

$$B(\rho) = (1-\rho^\star)\left(\rho^\star\,\lambda_1 + (1-\rho^\star)\,\lambda_2\right)\left(\lambda_2 - \frac{1}{2}\left(\rho^\star\,\lambda_1 + (1-\rho^\star)\,\lambda_2\right)\right)\Phi_2 - K + \frac{1}{2}\Phi_1.$$

It should be noticed that for any $\rho \in [0,1]$, the participation constraint of the second principal holds whenever $B(\rho) \geq 0$. Note that $B(1) = \frac{1}{2}\Phi_1 - K < 0$. Moreover, $B(0) = \frac{1}{2}\,\lambda_2^2\,\Phi_2 - K + \frac{1}{2}\Phi_1 > 0$ due to Assumption 1. Consequently, by the mean value theorem, there exists $\rho \in (0,1)$ such that $B(\rho) = 0$. But the key observation needed is that

$$\Pi_1\left(\rho\,\lambda_1 + (1-\rho)\,\lambda_2\right) - \Pi_2\left(\rho\,\lambda_1 + (1-\rho)\,\lambda_2\right) = \left(\rho\,\lambda_1 + (1-\rho)\,\lambda_2\right)(\lambda_1 - \lambda_2)\Phi_2 > 0.$$

Thus, in the optimal solution principal 2 should get as low as possible shares of the project, therefore, in the optimal contract he collects all the monetary returns from the project. Moreover, the remaining utility needed from the individual rationality constraint of principal 2 to be satisfied, is supplied to him by allocating as low as possible shares to him. [8] Note that this arrangement is unique. ■

In figure 2 the graph of $B(\rho)$ for given ρ is displayed for the following situations: $\lambda_1 = 0.75, \lambda_2 = 0.30$. The lowest curve happens when $\frac{K - \frac{1}{2}\Phi_1}{\Phi_2} = 0.045 = \frac{1}{2}\,\lambda_2^2$, i.e. when Assumption 1 holds with equality. The second lowest curve occurs for $\frac{K - \frac{1}{2}\Phi_1}{\Phi_2} = 0.03$, and finally the highest for $\frac{K - \frac{1}{2}\Phi_1}{\Phi_2} = 0.01$.

In order to display more details about the solution in the commitment case, consider the following example. Let $\lambda_1 = 0.75, \lambda_2 = 0.30$, and for the value of $\frac{K - \frac{1}{2}\Phi_1}{\Phi_2}$ we consider two levels 0.03 and 0.01.

[7] Notice that Assumption 1 holds trivially when $\frac{1}{2}\Phi_1 \geq K$.

[8] Alternatively, one can consider the objective function that Principal 1 paces:

$$H(\rho) = \left[\left(\rho\,\lambda_1 + (1-\rho)\,\lambda_2\right)\left(\rho\left(\lambda_1 - \frac{1}{2}\left(\rho\,\lambda_1 + (1-\rho)\right)\right) + (1-\rho)\left(\lambda_2 - \frac{1}{2}\left(\rho\,\lambda_1 + (1-\rho)\right)\right)\right)\right]\Phi_2 - K + \frac{1}{2}\Phi_1,$$

subject to $B(\rho) \geq 0$. Because that $\frac{\partial H}{\partial \rho} = \rho\,(\lambda_1 - \lambda_2)^2 + \lambda_2\,(\lambda_1 - \lambda_2) > 0$, we must choose the highest real number in [0,1], such that $A(\rho) = 0$.

In figure 3 $H(\rho)$ (the definition is in footnote 8) and $B(\rho)$ are given. Recall that principal 1 is maximizing $H(\rho)$ subject to $B(\rho) \geq 0$, and $H(0) = B(0)$. Thus, the solutions when $\frac{K - \frac{1}{2}\Phi_1}{\Phi_2}$ equals 0.03 and 0.01 are given in that figure and are labeled as $\rho_{(1)}$ and $\rho_{(2)}$, respectively.

3 Collusion Between The Entrepreneur And The Agent

Suppose that the entrepreneur has the opportunity to collude with the agent operating the project. Indeed, for simplicity we will assume that principal 1 has the option to convince an agent by using a hidden side contract to implement $\lambda \neq \lambda^{\rho^\star}$ even if he were not to own the whole project. This can be motivated as follows. After all, the entrepreneur is the party who came up with this project. Therefore it is conceivable that he has more access than principal 2 to the project, who we assumed is a financial investor not necessarily capable of understanding the nature of the project.

The timing of the game essentially is the same, with a difference happening towards the very end of the game:

$t = 1:$ Principal 1 offers (ρ, R) to principal 2 for him to supply K, and principal 2 accepts or rejects. If principal 2 rejects the offer, the game ends, otherwise, it continues.

$t = 2:$ With bargaining weights given by their share of the project, the principals bargain over the feasible allocation of resources for the project. This determines a level of $\bar{\lambda} \in [\lambda_2, \lambda_1]$ that the principals have agreed upon.

$t = 3:$ Given $\bar{\lambda}$, the principals determine the optimal contract, and the entrepreneur offers it to the agent;

$t = 4:$ Principal 1 can offer a hidden side contract to the agent, in which all resulting additional costs have to be covered by Principal 1.

$t = 5:$ The agent chooses whether or not he should accept one of these two offers, and exert the effort level desired. And, the observable and verifiable state is realized.

$t = 6:$ Finally, the entrepreneur makes the payments to the agent, and has the option of doing so in a way that the investor cannot observe or verify.

It should be pointed out that because the side contract needs to remain hidden, extra payments to the agent cannot be reflected to the investor. In order for the investor not to infer the true allocation of resources for the project, the entrepreneur needs to possess the ability of compensating the agent secretly. Thus, some payments to the agent (made by the entrepreneur) cannot be observable and/or verifiable by the investor.[9] Thus, having agreed on an allocation described by $\bar{\lambda}$ and on the fraction of shares given by ρ, the investor observes (and can verify) the state and pays only $(1-\rho)$ of the costs resulting at $\bar{\lambda}$. Otherwise, he would easily infer that the allocation is not given by $\bar{\lambda}$.

Consequently, the net payoff to the entrepreneur when he deviates to λ is his gross benefit from implementing the project at λ, minus all the cost of implementing the project at λ, plus the $(1-\rho)$ portion of the costs resulting from $\bar{\lambda}$. Thus, the investor continues to pay his share of the costs as if the project is being implemented at $\bar{\lambda}$, and additional costs are covered by the entrepreneur.

It needs to be emphasized that investor's ability of observing (and verifying) the state is not sufficient to infer that the entrepreneur has deviated to some other allocation. This is because, the investor does not observe the real mean, but rather stochastic outcomes of the project. That is, when the entrepreneur deviates to an allocation λ not equal to $\bar{\lambda}$ (the level that they have agreed upon), the investor still thinks that his average return is given by $\bar{\lambda}$, and not by λ.

Recall that the gross benefit of the entrepreneur is given by $B_1(\lambda) = \mu_1(t_1) + \mu_2(t_2)\,\lambda_1$. Using the optimal efforts given in (2), the optimal contract parameters in (5), (6), and the definition of Φ_l in (8) we find

$$B_1(\lambda) = \Phi_1 + \Phi_2\,\lambda\,\lambda_1\,. \tag{18}$$

Cost incurred to principals when some λ is implemented is derived by employing the facts used in the derivation of (18), and the cost function present in (4). It is

$$\kappa(\lambda) = \frac{1}{2}(\Phi_1 + \Phi_2\,\lambda^2). \tag{19}$$

Thus,

$$\Pi_1\left(\lambda \mid \rho, \bar{\lambda}\right) = \rho B_1(\lambda) - \kappa(\lambda) + (1-\rho)\kappa(\bar{\lambda}).$$

Plugging in the definitions, and rearranging we find

$$\Pi_1\left(\lambda \mid \rho, \bar{\lambda}\right) = \rho\left(\frac{1}{2}\Phi_1 + \lambda\left(\lambda_1 - \frac{1}{2}\lambda\right)\Phi_2\right) - \frac{1}{2}\,(1-\rho)\,\Phi_2\left(\lambda^2 - \left(\bar{\lambda}\right)^2\right). \tag{20}$$

[9]On the other hand, the hidden contract between the entrepreneur and the agent is binding because each of them can verify its ingredients.

For a given $(\rho, \bar{\lambda})$ the partial derivative of $\Pi_1\left(\lambda \mid \rho, \bar{\lambda}\right)$ with respect to λ is given by

$$\frac{\partial \Pi_1\left(\lambda \mid \rho, \bar{\lambda}\right)}{\partial \lambda} = \Phi_2\left(\rho\, \lambda_1 - \lambda\right). \tag{21}$$

Hence, $\Pi_1\left(\lambda \mid \rho, \bar{\lambda}\right)$ is strictly concave, having a unique solution at $\rho\, \lambda_1$.

Next we display that in the case when $\frac{1}{2}\Phi_1 < K$ and Assumption 1 hold, the deviation of the entrepreneur from $\lambda^{\rho^\star}$ (as described in Proposition 1) to $\rho^\star \lambda_1$ is strictly profitable. Notice that when $\frac{1}{2}\Phi_1 \geq K$, $\rho^\star = 1$, thus, $\lambda^{\rho^\star} = \lambda_1$. Therefore, in that case there are no profitable deviations for the entrepreneur.

Proposition 2 *Suppose that $\frac{1}{2}\Phi_1 < K$ and Assumption 1 holds, and let $(\rho^\star, R^\star)$ be as given in Proposition 1. Then, under collusion principal 1 has a strictly profitable deviation.*

Proof. The optimal deviation of principal 1 at given levels of $\rho^\star$ and $\lambda^{\rho^\star}$ would be one that maximizes (20) subject to $\lambda > 0$. But we know that (21) implies $\rho^\star \lambda_1$ maximizes his objective. Further, due to $\frac{1}{2}\Phi_1 < K$ and Assumption 1, we know that $\rho^\star < 1$. Thus, $\rho^\star \lambda_1 < \rho^\star \lambda_1 + (1 - \rho^\star)\lambda_2 = \lambda^{\rho^\star}$. Let the payoffs from any deviation $\lambda \in [\rho^\star \lambda_1, \rho^\star \lambda_1 + (1-\rho^\star)\lambda_2]$ be given by $D(\lambda) \equiv \Pi_1\left(\lambda \mid \rho^\star, \lambda^{\rho^\star}\right) - \Pi_1\left(\lambda^{\rho^\star} \mid \rho^\star, \lambda^{\rho^\star}\right)$. Because that $\Pi_1\left(\lambda \mid \rho^\star, \lambda^{\rho^\star}\right)$ is strictly decreasing for all $\lambda \in (\rho^\star \lambda_1, \lambda^{\rho^\star}]$, and $D(\lambda^{\rho^\star}) = 0$, and $\Pi_1\left(\lambda^{\rho^\star} \mid \rho^\star, \lambda^{\rho^\star}\right)$ is constant in λ, we conclude that for all $\lambda \in [\rho^\star \lambda_1, \lambda^{\rho^\star})$, $D(\lambda) > 0$. ■

3.1 Optimal Arrangement With Collusion

In this section the important feature is that principal 2 knows that principal 1 and the agent can collude via a hidden contract between the two. Thus, when accepting principal 1's offer, $(\rho, R) \in [0,1] \times [0, K]$, principal 2 knows that the point that will be implemented, $\lambda(\rho)$ must solve the following problem:

$$\lambda(\rho) \in \text{argmax}_{\lambda \in [0,1]}\, \Pi_1\left(\lambda \mid \rho, \lambda(\rho)\right) \equiv \rho\left(\frac{1}{2}\Phi_1 + \lambda\left(\lambda_1 - \frac{1}{2}\lambda\right)\Phi_2\right) - \frac{1}{2}(1-\rho)\,\Phi_2\left(\lambda^2 - (\lambda(\rho))^2\right). \tag{22}$$

Because that it was already shown in the proof of Proposition 2 that $\Pi_1\left(\lambda \mid \rho, \lambda(\rho)\right)$ is continuous and strictly concave in λ, it can easily be proven that the unique solution to (22) for any given (ρ, R) is

$$\lambda(\rho) = \rho\, \lambda_1\,. \tag{23}$$

Consequently, the problem that principal 1 has to solve at the beginning of the game in order to identify the optimal offer that needs to be made to principal 2, is:

$$\max_{(\rho,R)} \rho\Pi_1(\rho\,\lambda_1) - R, \tag{24}$$

subject to

$$\begin{aligned} (1-\rho)\Pi_2(\rho\,\lambda_1) + R &\geq K, \\ \rho\Pi_1(\rho\,\lambda_1) - R &\geq 0. \end{aligned}$$

For what follows, first we will derive a condition that will ensure that the constraint set of this maximization problem is not empty. Let,

$$A(\rho) \equiv (1-\rho)\,(\rho\,\lambda_1)\left(\lambda_2 - \frac{1}{2}\,(\rho\,\lambda_1)\right)\Phi_2 - K + \frac{1}{2}\Phi_1.$$

Because that principal 1 always obtains strictly higher payoffs from the project than the principal 2, as was done in the proof of Proposition 1, all the monetary returns from the project, $\frac{1}{2}\Phi_1$, will be allocated to the second principal. Thus, it can be observed that the participation constraint of the second principal holds whenever $A(\rho) \geq 0$. Therefore, the question is whether or not there exists a $\rho \in [0,1]$ such that $A(\rho) \geq 0$.

Note that $A(\rho) < 0$ whenever $\rho < 0$. Moreover, for $\rho > 1$ high enough $A(\rho) > 0$. On the other hand, $A(1) = \frac{1}{2}\Phi_1 - K = A(0)$. Thus, the local maximum, $\tilde{\rho}$ must be in $(0,1)$. The condition we impose to guarantee that there exists a $\rho \in [0,1]$ such that $A(\rho) \geq 0$, requires $A(\tilde{\rho}) \geq 0$. That is why we consider $\frac{\partial A(\rho)}{\partial \rho} = 0$, and (as figure 4 displays) we have two roots, the lower one the local maximum, and the higher one the local minimum. In Assumption 2 we require that the local maximum providing ρ, the lower root of $\frac{\partial A(\rho)}{\partial \rho} = 0$ (that we labeled as $\tilde{\rho}$) is such that $A(\tilde{\rho}) \geq 0$. Because then, there exits $\rho \in (0,1)$ such that the participation constraint of the second player is nonempty.

Assumption 2 *Let $\lambda_1, \lambda_2, K, \Phi_1, \Phi_2$ be such that $A(\tilde{\rho}) \geq 0$ where*

$$\tilde{\rho} \equiv \frac{1}{3\,\lambda_1}\left(2\,\lambda_2 + \lambda_1 - \sqrt{\left(4\,\lambda_2^2 - 2\,\lambda_1\,\lambda_2 + \lambda_1^2\right)}\right) \in (0,1).$$

It is worthwhile to note that Assumption 1 fails to guarantee the participation constraint of the second principal. This can be observed in figure 4 which displays $A(\rho) < 0$ for $\rho \in [0,1]$ in the case when Assumption 1 holds: $\lambda_1 = 0.75, \lambda_2 = 0.30, \frac{K-\frac{1}{2}\Phi_1}{\Phi_2} = 0.03 < 0.045 = \frac{1}{2}\lambda_2^2$. But when $\frac{K-\frac{1}{2}\Phi_1}{\Phi_2} = 0.01$, then the same figure displays that participation constraint of the second principal holds for some ρ.

The importance of Assumption 2 is that when it does not hold, then the participation constraint of the second principal cannot hold for any $\rho \in [0,1]$. Thus, even though Assumption 1 holds, if Assumption 2 does not hold the market collapses, i.e. the project cannot be financed by the investor.

This situation happens for values $\lambda_1 = 0.75, \lambda_2 = 0.30, \frac{K-\frac{1}{2}\Phi_1}{\Phi_2} = 0.03 < 0.045 = \frac{1}{2}\lambda_2^2$. Note that then Assumption 1 holds, but (as figure 4 displays) Assumption 2 does not. Therefore, even though the solution under the commitment case is $\rho_{(1)}$ (as was shown in figure 3), the market collapses and the project is not financed by the second principal because his participation constraint cannot be satisfied at no $\rho \in [0,1]$.

However, when Assumption 2 holds, the solution can be found as follows. Consider the values $\lambda_1 = 0.75, \lambda_2 = 0.30, \frac{K-\frac{1}{2}\Phi_1}{\Phi_2} = 0.01 < 0.045 = \frac{1}{2}\lambda_2^2$. It should be noticed that both Assumptions hold at these values. Now principal 1 is maximizing

$$G(\rho) = \rho\,(\rho\,\lambda_1)\left(\lambda_1 - \frac{1}{2}\,(\rho\,\lambda_1)\right) + (1-\rho)\,(\rho\,\lambda_1)\left(\lambda_2 - \frac{1}{2}\,(\rho\,\lambda_1)\right) - \frac{K-\frac{1}{2}\Phi_1}{\Phi_2},$$

subject to $A(\rho) \geq 0$. Both G and A are depicted in figure 5. Note that A is equal to 0 in two spots. But because that

$$\frac{\partial G}{\partial \rho} = \rho\,(\lambda_1 - \lambda_2)^2 + \lambda_2\,(\lambda_1 - \rho\,\lambda_2) > 0,$$

the solution $\rho_{(3)}$ is the higher of the two roots of $A(\rho) = 0$.

The following Proposition will characterize the solutions to (24):

Proposition 3 *Suppose Assumption 1 holds. Then:*

1. *If* $\frac{1}{2}\Phi_1 \geq K$, *then the solution* (ρ^∇, R^∇) *is given by* $\rho^\nabla = 1$, *and* $R^\nabla = K$.

2. *If* $\frac{1}{2}\Phi_1 < K$ *and Assumption 2 does not hold, then the market collapses, and the project is not financed by the second player.*

3. *If* $\frac{1}{2}\Phi_1 < K$ *and Assumption 2 holds, then the solution* (ρ^∇, R^∇) *is such that* ρ^∇ *is the maximum real number in* $[0,1]$ *which solves*

$$(1-\rho^\nabla)\,(\rho^\nabla\,\lambda_1)\left(\lambda_2 - \frac{1}{2}\,(\rho^\nabla\,\lambda_1)\right)\Phi_2 \quad = \quad K - \frac{1}{2}\Phi_1, \tag{25}$$

and $R^\nabla = \rho^\nabla\left(\frac{1}{2}\Phi_1\right)$.

Proof. Note that for $(\rho^\triangledown, R^\triangledown)$, the individual rationality constraint of principal 1 is satisfied, because $(\rho^\triangledown \lambda_1)^2 (1 - 1/2\rho^\triangledown)\Phi_2 > 0$. Moreover, due to the rest of the proof being very similar to that of Proposition 1, it is omitted. ■

3.2 Collusion versus Commitment

We have some tools to visualize and compare the shareholding structure in collusion and commitment cases. Let us consider the only non-trivial case where $\frac{1}{2}\Phi_1 < K$, and Assumption 1, and Assumption 2 hold. Then, we have shown that $\rho^\star$ is the maximum real number in $[0, 1]$ solving

$$(1-\rho^\star)(\rho^\star \lambda_1 + (1-\rho^\star)\lambda_2)\left(\lambda_2 - \frac{1}{2}(\rho^\star \lambda_1 + (1-\rho^\star)\lambda_2)\right)\Phi_2 \quad = \quad K - \frac{1}{2}\Phi_1,$$

and that $\rho^\triangledown$ is the maximum real number in $[0, 1]$ which solves

$$(1-\rho^\triangledown)(\rho^\triangledown \lambda_1)\left(\lambda_2 - \frac{1}{2}(\rho^\triangledown \lambda_1)\right)\Phi_2 \quad = \quad K - \frac{1}{2}\Phi_1.$$

We can rewrite the former equation as $*(\rho^\star) = C$, and the latter as $\triangledown(\rho^\triangledown) = C$, where $*(\rho)$, and $\triangledown(\rho)$ are third degree polynomial functions in ρ, and $C = \frac{K - \frac{1}{2}\Phi_1}{\Phi_2}$. $\rho^\star$ is the greater of the two real numbers in [0,1] making $*(\rho)$ equal to C, and $\rho^\triangledown$ is the greater of the two real numbers in [0,1] making $\triangledown(\rho)$ equal to C.

To visualize the relationship between $\rho^\triangledown$, and $\rho^\star$ we will examine the relationship between the roots of $\triangledown(\rho)$, and $*(\rho)$. The real numbers solving $*(\rho) = 0$ are $-\frac{\lambda_2}{\lambda_1 - \lambda_2}$, $\frac{\lambda_2}{\lambda_1 - \lambda_2}$, and 1; whereas the real numbers solving $\triangledown(\rho) = 0$ are 0, $\frac{2\lambda_2}{\lambda_1}$, and 1.

We will consider three cases. The first case is when $\lambda_2 > \frac{\lambda_1}{2}$. It is straightforward to show that, in that case, the relationship between the roots is given by $-\frac{\lambda_2}{\lambda_1 - \lambda_2} < 0 < 1 < \frac{2\lambda_2}{\lambda_1} < \frac{\lambda_2}{\lambda_1 - \lambda_2}$, or

$$r_1^* < (r_1^\triangledown = 0) < (r_2^* = r_2^\triangledown = 1) < r_3^\triangledown < r_3^*,$$

where r^* are the roots of $*(\rho)$, etc. The second case is when $\lambda_2 < \frac{\lambda_1}{2}$. In that case, the relationship becomes $-\frac{\lambda_2}{\lambda_1 - \lambda_2} < 0 < \frac{\lambda_2}{\lambda_1 - \lambda_2} < \frac{2\lambda_2}{\lambda_1} < 1$, or

$$r_1^* < (r_1^\triangledown = 0) < r_2^* < r_2^\triangledown < (r_3^* = r_3^\triangledown = 1).$$

The last case is when $\lambda_2 = \frac{\lambda_1}{2}$. The relationship is

$$(-1 = r_1^*) < (r_1^\triangledown = 0) < (r_2^* = r_2^\triangledown = r_3^* = r_3^\triangledown = 1).$$

These relationships provide the general picture of the problem at hand. However, it fails to force an exact relation between ρ^{∇}, and $\rho^{\star}$. That is, by changing the specifics of the problem, we can have $\rho^{\nabla} < \rho^{\star}$, and $\rho^{\nabla} > \rho^{\star}$ while we are keeping the relationship between the roots.

Now, we will consider an example. Let $\lambda_2 = 2$, and $\lambda_1 = 3$. Hence, $*(\rho) = (1-\rho)(\rho+2)(1-\frac{\rho}{2})$, and $\nabla(\rho) = (1-\rho)(3\rho)(2-\frac{3}{2}\rho)$. The roots satisfy $(-2 = r_1^*) < (r_1^{\nabla} = 0) < (r_2^* = r_2^{\nabla} = 1) < (r_3^{\nabla} = \frac{4}{3}) < (r_3^* = 2)$. This alignment allows, with differen C's, different orderings for ρ^{∇}, and $\rho^{\star}$. For $C = 0.25$, $0.848 = \rho^{\star} < \rho^{\nabla} = 0.863$; whereas for $C = 0.95$, $0.494 = \rho^{\star} > \rho^{\nabla} = 0.488$.

4 Concluding Remarks

We wish to point out the fact that our research tries to explain the existing issue of shareholding by investors and shows the consequences of the structure of the issue. However, the fact that investors do not have effective control rights remains a problem. Typically, investors do not possess mechanisms to control how their funds are used. Thus, we have a mechanism design problem. Therefore, a future avenue for research is to attempt to devise mechanisms to force corporations pursue the rights of their investors. Once there are such mechanisms, external funding of corporations becomes easier.

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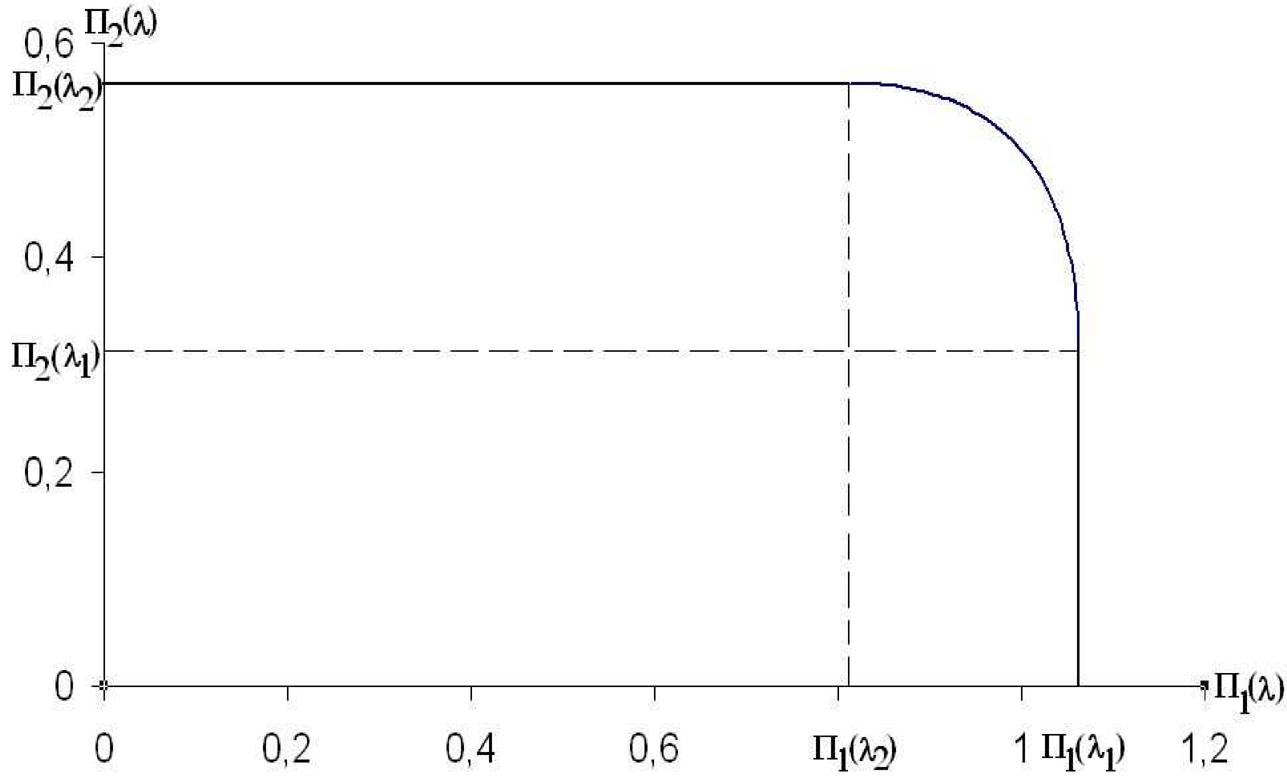

Figure 1: Bargaining set S, for $\lambda_1 = 3/4$, $\lambda_2 = 1/4$, $\Phi_1 = 1$, and $\Phi_2 = 2$

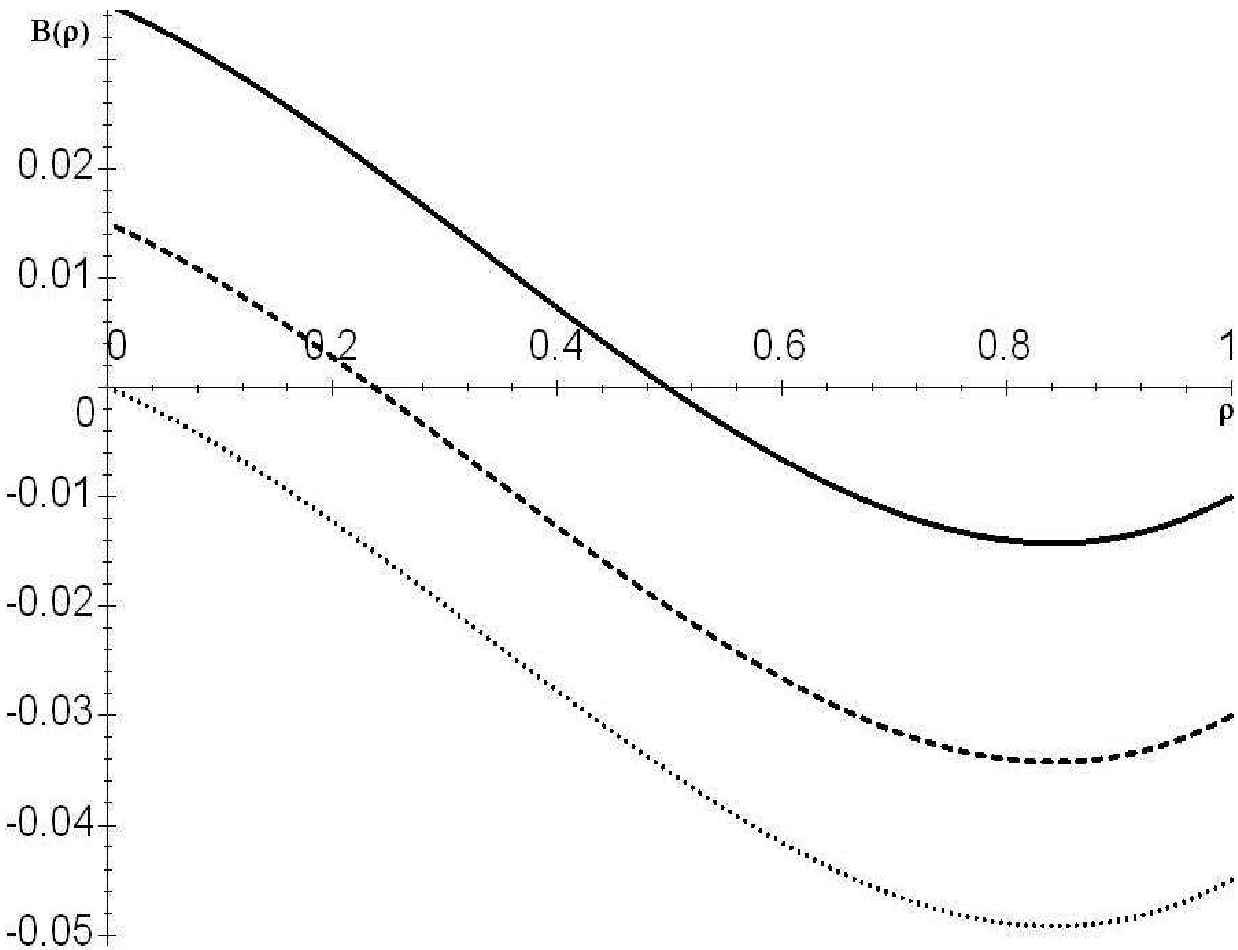

Figure 2: The graph of $B(\rho)$ for $\rho \in [0,1]$ and $\lambda_1 = 0.75, \lambda_2 = 0.30, \frac{K - \frac{1}{2}\Phi_1}{\Phi_2} = 0.01 < 0.045 = \frac{1}{2}\,\lambda_2^2$ is given in the solid (the highest) curve. The second highest one occurs when $\frac{K - \frac{1}{2}\Phi_1}{\Phi_2} = 0.03$. Finally, the lowest one of them happens when $\frac{K - \frac{1}{2}\Phi_1}{\Phi_2} = 0.045 = \frac{1}{2}\,\lambda_2^2$, i.e. when Assumption 1 holds with equality.

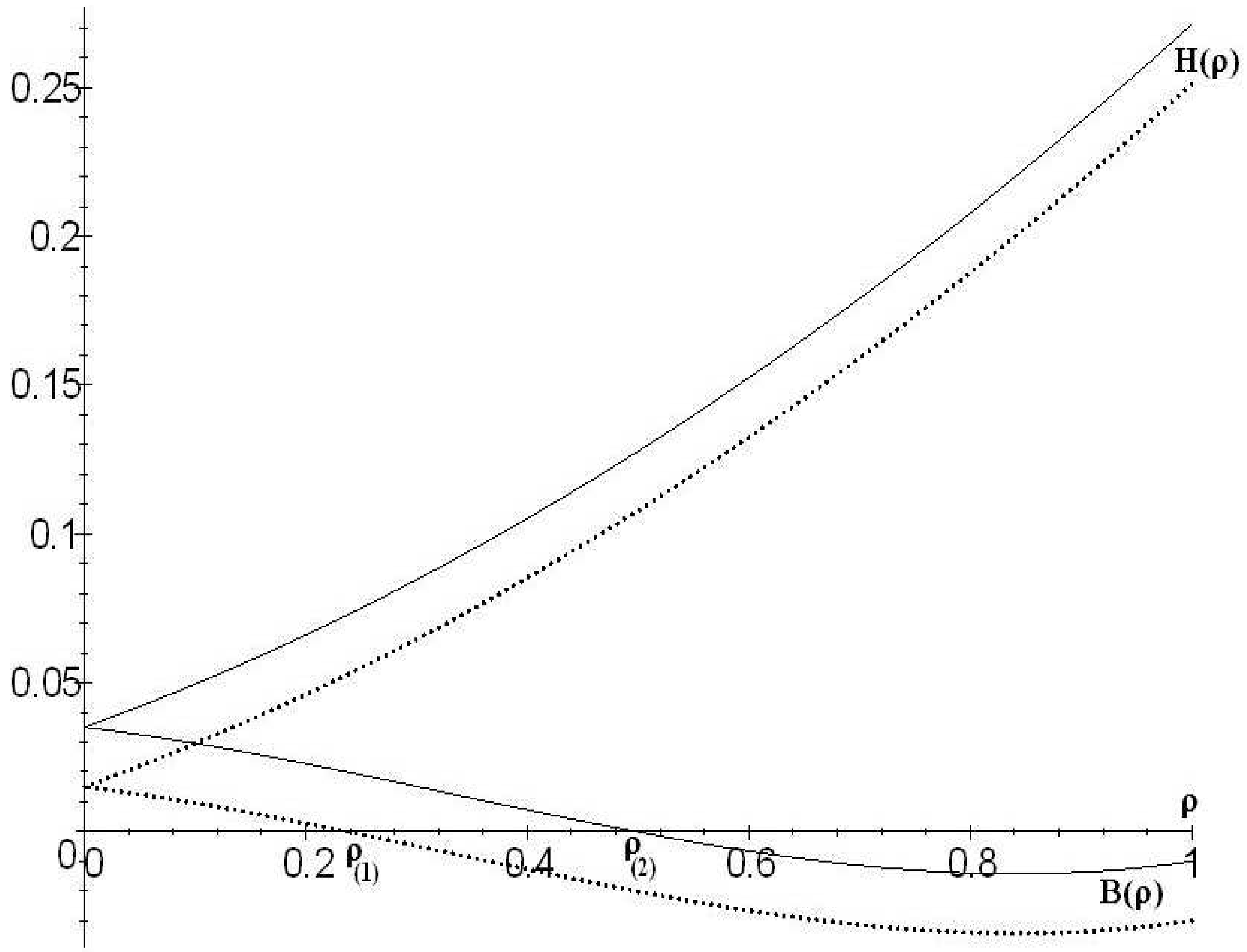

Figure 3: The graphs of $H(\rho)$ and $B(\rho)$ for $\rho \in [0,1]$ and $\lambda_1 = 0.75, \lambda_2 = 0.30, \frac{K-\frac{1}{2}\Phi_1}{\Phi_2} = 0.01$ (the solid curve) and $\lambda_1 = 0.75, \lambda_2 = 0.30, \frac{K-\frac{1}{2}\Phi_1}{\Phi_2} = 0.03$ (the curve with the dots). The solutions when $\frac{K-\frac{1}{2}\Phi_1}{\Phi_2}$ equals 0.03 and 0.01 are given in that figure and are labeled as $\rho_{(1)}$ and $\rho_{(2)}$, respectively.

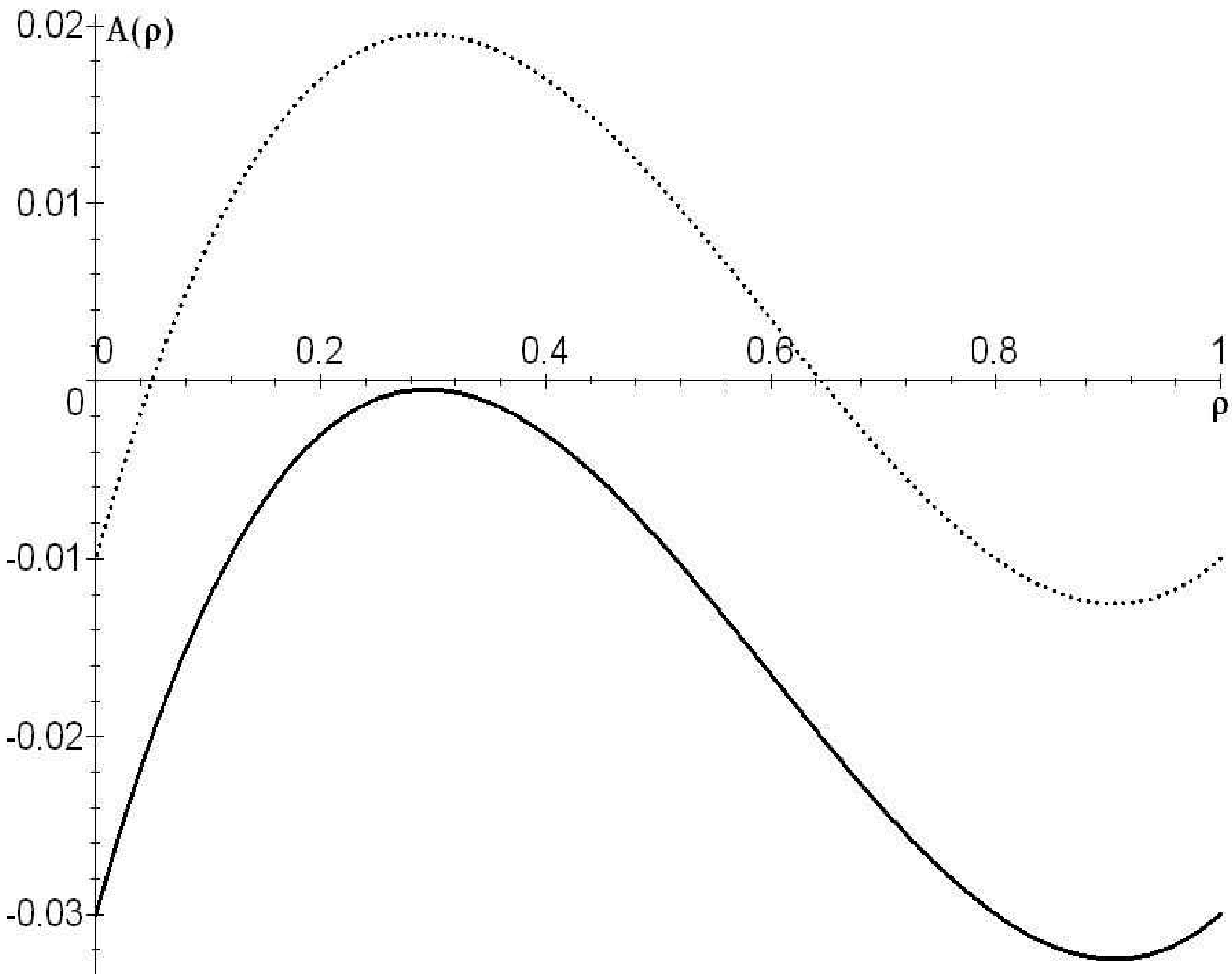

Figure 4: The graph of $A(\rho)$ for $\rho \in [0,1]$ and $\lambda_1 = 0.75, \lambda_2 = 0.30, \frac{K-\frac{1}{2}\Phi_1}{\Phi_2} = 0.03 < 0.045 = \frac{1}{2}\lambda_2^2$ is given in the solid curve. Whereas, the same situation when $\frac{K-\frac{1}{2}\Phi_1}{\Phi_2} = 0.01$ is depicted in the curve with the dots.

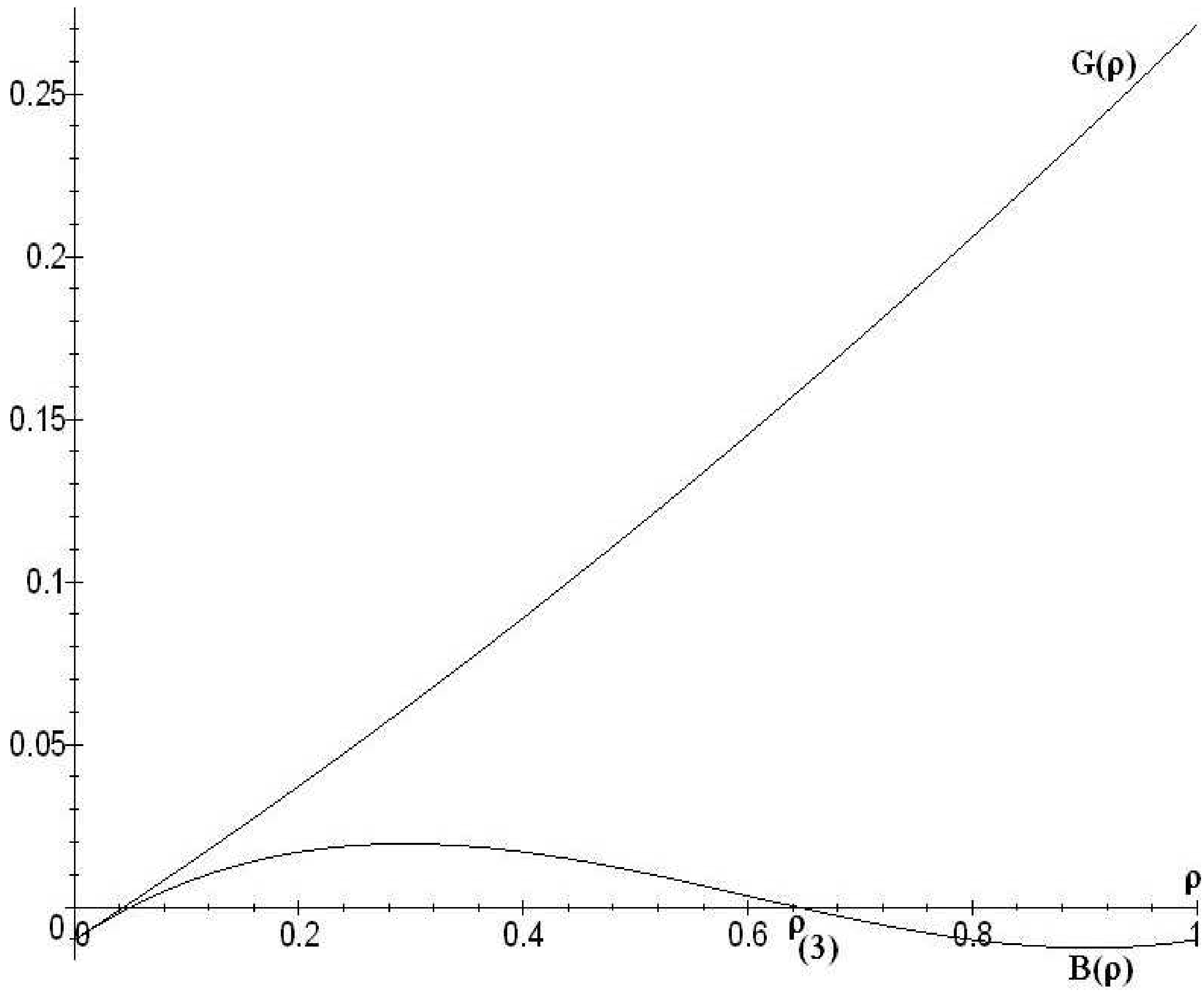

Figure 5: The solution for the no-commitment case for values $\lambda_1 = 0.75, \lambda_2 = 0.30, \frac{K-\frac{1}{2}\Phi_1}{\Phi_2} = 0.01 < 0.045 = \frac{1}{2}\lambda_2^2$. $G(\rho)$ and $A(\rho)$ are depicted, and the solution is $\rho_{(3)}$.