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CONTROLING INTERACTIONS IN MOTION CONTROL SYSTEMS

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Abstract: Design of motion control systems should take into account (a) unconstrained motion performed without interaction with environment or other systems, (b) constrained motion performed by certain functional interaction with environment or other system. Control in both cases can be formulated in terms of maintaining desired system configuration what makes essentially the same structure for common tasks: trajectory tracking, interaction force control, compliance control etc. It will be shown that the same design approach can be used for systems that maintain some functional relations like parallel robots. *Copyright © 2007 IFAC*

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1. INTRODUCTION

Modern motion control systems are acting as “agents” between skilled human operator and environment (surgery, microparts handling, teleoperation, etc.). In such situations design of control should encompass wide range of very demanding tasks. At the lower level one should consider tasks of controlling individual systems - like single DOF systems, motor control, robotic manipulator or mobile robot. On the system level control of multilateral interaction between systems of the same or different nature, the remote control in master-slave systems, haptics, parallel mechanisms etc. should be considered. In general design of motion control system should take into account (i) unconstrained motion - performed without interaction with environment or other system - like trajectory tracking, (ii) motion in which system should maintain its trajectory despite of the interaction with other systems - disturbance rejection tasks, (iii) constrained motion where system should modify its behaviour due to interaction with environment or another system or should maintain specified interconnection - virtual or real - with other system and (iv) in remote operation control system should be able to reflect the sensation of unknown environment to the human operator.

Decentralized control seems a promising framework for application in motion control. It posses many good features such as flexibility, fault tolerance, expandability, and fast response. There are many applications to robot control systems, with concepts such as multi-agent system (Sabattucci and Chella, 2003), cell structure (Ueyama, *et al*., 1992), and fault tolerant systems (Fujimoto, and Sekguchi, 2003). Decomposition block control (Hernandez, *et al*., 2001). Arimoto and Nguyen (Arimoto and Nguyen, 2001) showed that under certain conditions overall control input can be designed by linear superposition, Tatani and Nakamura proposed a method based on the singular value decomposition (Okada, *et al*., 2002). Tsuji, Nishi and Ohnishi proposed a framework of controller design based on functionality (Tsuji and Ohnishi, 2005), Onal and

Sabanovic implemented a bilateral control using sliding mode control applying functionality (Onal, and Sabanovic, 2005).

In this paper we will present a framework in motion control systems design based on the idea that by enforcing certain functional relations between coordinates one can determine the functional behaviour of the system. The approach will be demonstrated on the control of the Stewart Platform like parallel mechanism in which the position and orientation of platform is defined by the length of the supporting linear actuators. By enforcing certain relations among these actuators (for example if all are forced to maintain the same length the motion of the platform will be than in z axis only) the constrained motion of the system can be performed. By representing the task as a combination of these constrained motion in some cases the oveall controller design becomes simpler and decoupling of the nonlinear dynamics can be achieved. In essence, the method is using Sliding Mode Control (SMC) design procedure.

The body of paper begins in section II with mathematical formulations of control and motion of systems, extension to general systems in interactions and parallel manipulator example takes place in section III and in section IV, simulation results are presented to compare the performance of observers on robot and function coordinates. Finally, Section V, concludes the paper.

2. MATHEMATICAL FORMULATIONS

For fully actuated mechanical system (number of actuators equal to the number of the primary masses) mathematical model may be found in the following form

$$\begin{aligned} &\mathbf{M}(\mathbf{q})\ddot{\mathbf{q}} + \mathbf{L}(\mathbf{q},\dot{\mathbf{q}})\dot{\mathbf{q}} + \mathbf{H}(\mathbf{q},\dot{\mathbf{q}}) = \mathbf{F} - \mathbf{F}_{ext} \\ &\mathbf{L}(\mathbf{q},\dot{\mathbf{q}})\dot{\mathbf{q}} + \mathbf{H}(\mathbf{q},\dot{\mathbf{q}}) = \mathbf{N}(\mathbf{q},\dot{\mathbf{q}}) \end{aligned} \qquad (1)$$

where $\mathbf{q} \in \Re^n$ stands for vector of generalized positions, $\dot{\mathbf{q}} \in \Re^n$ stands for vector of generalized velocities, $\mathbf{M}(\mathbf{q}) \in \Re^{nxn}$ $M^- \le \|\mathbf{M}(\mathbf{q})\| \le M^+$ is generalized positive definite inertia matrix with bounded parameters, $\mathbf{N}(\mathbf{q},\dot{\mathbf{q}}) \in \Re^{nx1}$ $\|\mathbf{N}(\mathbf{q},\dot{\mathbf{q}})\| \le N^+$ represent vector of coupling forces including gravity $\mathbf{H}(\mathbf{q},\dot{\mathbf{q}})$ and friction $\mathbf{L}(\mathbf{q},\dot{\mathbf{q}})\dot{\mathbf{q}}$, $\mathbf{F} \in \Re^{nx1}$, $\|\mathbf{F}\| \le F^+$ stands for vector of generalized input forces, $\mathbf{F}_{ext} \in \Re^{nx1}$, $\|\mathbf{F}_{ext}\| \le F_{0ext}$ stands for vector of external forces. M^-, M^+, N^+ and F^+ F_{0ext} are known scalars.

External force is a result of system's interaction with environment at position $\mathbf{q}_e$ and in general can be represented as

$$\mathbf{F}_{ext}(\mathbf{q},\mathbf{q}_e) = \begin{cases} \mathbf{F}_{ext}(\mathbf{q},\mathbf{q}_e) & \text{if there is interaction} \\ 0 & \text{if no interaction} \end{cases} \qquad (2)$$

In many cases interaction of the systems is modeled as spring-damper so than the interaction force is represented as a linear combination in the form $\mathbf{F}_{ext} = \mathbf{K}_S(\mathbf{q}-\mathbf{q}_e) + \mathbf{K}_D(\dot{\mathbf{q}}-\dot{\mathbf{q}}_e)$. The same reasoning can be applied for modeling virtual interaction between systems.

2.1 *Control Problem Formulation*

Vector of generalized positions and generalized velocities defines configuration $\xi(\mathbf{q},\dot{\mathbf{q}})$ of a mechanical system. The control tasks for the system (1) are usually formulated as selection of the generalized input such that: (i) system executes desired motion specified as position tracking, (ii) system exerts a defined force while in the contact with environment and (iii) system reacts as a desired impedance on the external force input or in contact with environment. The task (i) requires tracking of the reference trajectory with or without interaction with environment – thus requiring very high stiffness and good disturbance rejection. The tasks (ii) and (iii) are specified for a system being in interaction with environment and both require modification of the system state in order to achieve desired behavior while in the contact. In literature these problems are generally treated separately (Onal, and Sabanovic, 2005)and motion that requires transition from one to another task are treated in the framework of hybrid control (Raibert, and Craig, 1981). The most general formulation of the fully actuated mechanical systems can be formulated as a task to maintain desired configuration $\xi^{ref}(\mathbf{q}^{ref},\dot{\mathbf{q}}^{ref})$ of the system. Assume that the control system requirements are satisfied if real and desired configurations of mechanical system satisfy an algebraic constraint expressed as $\boldsymbol{\sigma}(\xi(\mathbf{q},\dot{\mathbf{q}}),\xi^{ref}(\mathbf{q}^{ref},\dot{\mathbf{q}}^{ref})) = \mathbf{0}_{nx1}$ and for $\boldsymbol{\sigma} = \mathbf{0}_{nx1} \Rightarrow (\xi = \xi^{ref})$. Now the control problem can be formulated as selection of control input so that solution $\boldsymbol{\sigma}(\xi,\xi^{ref}) = \mathbf{0}_{nx1}$ is stable on the trajectories of system (1). Note that this formulation is similar to SMC control with a difference that in SMC the reaching time to $\boldsymbol{\sigma}(\xi,\xi^{ref}) = \mathbf{0}_{nx1}$ is required to be finite. In this paper, without loss of generality, it will be assumed that system configuration can be expressed as a linear combination of generalized positions and velocities $\xi(\mathbf{q},\dot{\mathbf{q}}) = \mathbf{C}\mathbf{q} + \mathbf{Q}\dot{\mathbf{q}}$ and consequently $\xi^{ref} = \mathbf{C}\mathbf{q}^{ref} + \mathbf{Q}\dot{\mathbf{q}}^{ref}$. Now control problem can be formulated as a selection of the control so that the state of the system is forced to remain in manifold S_q:

$$\begin{aligned} &S_q = \{\mathbf{q},\dot{\mathbf{q}} : \boldsymbol{\sigma}(\xi(\mathbf{q},\dot{\mathbf{q}}),\xi^{ref}(\mathbf{q}^{ref},\dot{\mathbf{q}}^{ref})) = \xi(\mathbf{q},\dot{\mathbf{q}}) - \xi^{ref}(\mathbf{q}^{ref},\dot{\mathbf{q}}^{ref}) = \mathbf{0}\}, \\ &\boldsymbol{\sigma},\xi,\xi^{ref} \in \Re^{nx1}; \mathbf{C},\mathbf{Q} \in \Re^{nxn}; \mathbf{C},\mathbf{Q} > 0, \\ &\boldsymbol{\sigma} = [\sigma_1,\sigma_2,\ldots,\sigma_n]^T \end{aligned} \qquad (3)$$

Where $\xi^{ref}(\mathbf{q},\dot{\mathbf{q}}) \in \Re^{nx1}$ stands for reference configuration of the system and is assumed to be smooth bounded function with continuous first order time derivative, matrices $\mathbf{C},\mathbf{Q} \in \Re^{n\times n}$ have full rank

$rank(\mathbf{C}) = rank(\mathbf{Q}) = n$. By selecting $\mathbf{C}, \mathbf{Q} \in \Re^{nxn}$ as diagonal (3) can be represented by a set of *n* first order equations $\sigma_i = g_i\left(q_i^{ref} - q_i\right) + h_i\left(\dot{q}_i^{ref} - \dot{q}_i\right) = 0$, *i=1,2,..,n*.

2.2 Selection of control input

Design of control inputs for system (1), (2) that will enforce the stability of $\boldsymbol{\sigma}\left(\xi, \xi^{ref}\right) = \mathbf{0}_{nx1}$ and that manifold (3) is reached asymptotically or in finite time. The simplest and the most direct method to derive control is to enforce Lyapunov stability conditions for solution $\boldsymbol{\sigma}\left(\xi, \xi^{ref}\right) = \mathbf{0}_{nx1}$ on the trajectories of system (1), (2). Lyapunov function candidate may be selected as $v = \frac{1}{2}\boldsymbol{\sigma}^T\boldsymbol{\sigma} > 0$ with first time derivative $\dot{v} = \boldsymbol{\sigma}^T\dot{\boldsymbol{\sigma}}$. To ensure stability the derivative of Lyapunov function is required to be negative definite so one can require that $\dot{v} = \boldsymbol{\sigma}^T\dot{\boldsymbol{\sigma}} = -\boldsymbol{\sigma}^T\boldsymbol{\Psi}(\boldsymbol{\sigma}) < 0$. For $-\boldsymbol{\sigma}^T\boldsymbol{\Psi}(\boldsymbol{\sigma}) = -\rho v^{\delta} < 0$ with $\rho > 0$ and $\frac{1}{2} \leq \delta < 1$ stability conditions are satisfied and finite time convergence to sliding mode manifold is obtained. From $\dot{v} = \boldsymbol{\sigma}^T\dot{\boldsymbol{\sigma}} = -\boldsymbol{\sigma}^T\boldsymbol{\Psi}(\boldsymbol{\sigma})$ one can derive $\boldsymbol{\sigma}^T\left(\dot{\boldsymbol{\sigma}} + \boldsymbol{\Psi}(\boldsymbol{\sigma})\right) = 0$ and consequently control should be selected to satisfy $\dot{\boldsymbol{\sigma}} + \boldsymbol{\Psi}(\boldsymbol{\sigma})\big|_{\sigma \neq 0} = 0$. By differentiating (3) and substituting (1) under the assumption that $\mathbf{C}, \mathbf{Q} \in \Re^{nxn}$ are constant and $\left(\mathbf{Q}\mathbf{M}^{-1}\right)^{-1}$ exists, from $\left(\dot{\boldsymbol{\sigma}} + \boldsymbol{\Psi}(\boldsymbol{\sigma})\right)\big|_{\sigma \neq 0} = \mathbf{Q}\mathbf{M}^{-1}\left(\mathbf{F} - \mathbf{F}_{eq}\right) + \boldsymbol{\Psi}(\boldsymbol{\sigma}) = 0$ one can find control input as in (4)

$$\begin{aligned} \mathbf{F} &= \mathbf{F}_{eq} - \left(\mathbf{Q}\mathbf{M}^{-1}\right)^{-1}\boldsymbol{\Psi}(\boldsymbol{\sigma}) = \mathbf{F}_{eq} - \left(\mathbf{Q}\mathbf{M}^{-1}\right)^{-1}\boldsymbol{\Psi}(\boldsymbol{\sigma}) \\ \mathbf{F}_{eq} &= \left(\mathbf{F}_{ext} + \mathbf{N}\right) - \left(\mathbf{Q}\mathbf{M}^{-1}\right)^{-1}\left(\mathbf{C}\dot{\mathbf{q}} - \dot{\boldsymbol{\xi}}^{ref}\right) \end{aligned} \tag{4}$$

The $\mathbf{F}_{eq}$ is the control input determined from algebraic equation $\dot{\boldsymbol{\sigma}} = 0$. This value of the control input will maintain solution $\boldsymbol{\sigma} = 0$ for zero initial conditions. Obviously the structure of control input depends on the selection of $\boldsymbol{\Psi}(\boldsymbol{\sigma})$, which should be determined in such a way so to ensure stability conditions for solution $\boldsymbol{\sigma} = 0$ are guarantied and that $\boldsymbol{\sigma} \to 0$.

Equations of motion for system (1) with control (4) enforcing stable solution (3) can be derived as

$$\begin{aligned} \mathbf{M}\ddot{\mathbf{q}} + \mathbf{N} &= \mathbf{F}_{eq} - \left(\mathbf{Q}\mathbf{M}^{-1}\right)^{-1}\boldsymbol{\Psi}(\boldsymbol{\sigma}) - \mathbf{F}_{ext} \\ &= \left(\mathbf{F}_{ext} + \mathbf{N}\right) - \left(\mathbf{Q}\mathbf{M}^{-1}\right)^{-1}\left(\mathbf{C}\dot{\mathbf{q}} - \dot{\boldsymbol{\xi}}^{ref}\right) - \left(\mathbf{Q}\mathbf{M}^{-1}\right)^{-1}\boldsymbol{\Psi}(\boldsymbol{\sigma}) - \mathbf{F}_{ext} \\ \mathbf{M}\ddot{\mathbf{q}} &= \left(\mathbf{Q}\mathbf{M}^{-1}\right)^{-1}\left[\left(\dot{\boldsymbol{\xi}}^{ref} - \mathbf{C}\dot{\mathbf{q}}\right) - \boldsymbol{\Psi}(\boldsymbol{\sigma})\right] = \mathbf{M}\ddot{\mathbf{q}}^{des} \end{aligned} \tag{5}$$

Since matrices $\mathbf{Q} \in \Re^{n \times n}$ and $\mathbf{M} \in \Re^{n \times n}$ are full rank matrices than $\left(\mathbf{Q}\mathbf{M}^{-1}\right)^{-1} = \mathbf{M}\mathbf{Q}^{-1}$ and (4) can be rewritten as

$$\ddot{\mathbf{q}}^{des} = \mathbf{Q}^{-1}\left[\left(\dot{\boldsymbol{\xi}}^{ref} - \mathbf{C}\dot{\mathbf{q}}\right) - \boldsymbol{\Psi}(\boldsymbol{\sigma})\right]; \quad \ddot{\mathbf{q}} = \ddot{\mathbf{q}}^{des} \tag{6}$$

Motion (6) of the system (1), (2) under control (4) depends on selection of the manifold (3) (matrices **C** and **Q**) and the reference configuration $\boldsymbol{\xi}^{ref} \in \Re^{nx1}$. Closed loop system realizes an acceleration controller with desired acceleration defined by $\frac{d}{dt}\mathbf{Q}^{-1}\left[\left(\boldsymbol{\xi}^{ref} - \mathbf{C}\mathbf{q}\right) - \int \Psi(\sigma)dt\right]$. For $\boldsymbol{\Psi}(\boldsymbol{\sigma}) = \mathbf{D}\boldsymbol{\sigma}$ and $\boldsymbol{\xi}^{ref} = \mathbf{C}\mathbf{q}^{ref} + \mathbf{Q}\dot{\mathbf{q}}^{ref}$ motion (6) becomes

$$\begin{aligned} &\ddot{\mathbf{q}} = \left[\left(\mathbf{C}\dot{\mathbf{q}}^{ref} + \mathbf{Q}\ddot{\mathbf{q}}^{ref} - \mathbf{C}\dot{\mathbf{q}}\right) - \mathbf{D}\boldsymbol{\sigma}\right] \\ &\ddot{\mathbf{q}} = \ddot{\mathbf{q}}^{ref} - \mathbf{Q}^{-1}\left(\mathbf{C} + \mathbf{D}\mathbf{Q}\right)\left(\dot{\mathbf{q}}^{ref} - \dot{\mathbf{q}}\right) - \mathbf{Q}^{-1}\mathbf{D}\mathbf{C}\left(\mathbf{q}^{ref} - \mathbf{q}\right) \\ &\dot{\boldsymbol{\sigma}} + \mathbf{D}\boldsymbol{\sigma} = \mathbf{0} \end{aligned} \tag{7}$$

Motion (7) depend only on the selection of the design parameters (matrices C, Q and D) and if matrix $\mathbf{D} \in \Re^{n \times n}$ is selected diagonal and large enough the ε vicinity of the manifold (3) will be reached fast and then motion of the system will mostly determined by predominant pole defined by matrices C and Q. If control is selected in such a way that the manifold (3) is reached in finite time and sliding mode motion instead of n poles defined by D will have n poles in origin and the motion will be governed by $\mathbf{C}\Delta\mathbf{q} + \mathbf{Q}\Delta\dot{\mathbf{q}} = \mathbf{0}$. $\Delta\mathbf{q} = \mathbf{q}^{ref} - \mathbf{q}$ so that $\Delta\mathbf{q} = \mathbf{q} - \mathbf{q}^{ref} \to \mathbf{0}$ when $t \to \infty$. Equations (7) shows that in ideal case motion of the system will not be modified when it comes in contact with environment, thus this solution is suitable for solving position-tracking problem of mechanical systems.

Equivalent control $\mathbf{F}_{eq}$ is smooth bounded function and one can resort of using its value in $t = (k-1)T$ instead of the exact value at $t = kT$ and thus have some approximation it. The value of the $\mathbf{F}_{eq}$ at the end of interval $t = (k-1)T$ can be determined from the projection of the system motion in manifold (3) $\dot{\boldsymbol{\sigma}}\big|_{t=(1-k)T} = \mathbf{Q}\mathbf{M}^{-1}\left(\mathbf{F} - \mathbf{F}_{eq}\right)\big|_{t=(1-k)T}$ as $\mathbf{F}_{eq}(k-1) = \left(\mathbf{F}(k-1) - \mathbf{M}\mathbf{Q}^{-1}\dot{\boldsymbol{\sigma}}(k-1)\right)$ and with $\dot{\boldsymbol{\sigma}}(k-1) = \left(\boldsymbol{\sigma}(k) - \boldsymbol{\sigma}(k-1)\right)/T + o\left(T^2\right)$ one have $\mathbf{F}_{eq}(k-1) = \left(\mathbf{F}(k-1) - \mathbf{M}\mathbf{Q}^{-1}T^{-1}\left(\boldsymbol{\sigma}(k) - \boldsymbol{\sigma}(k-1)\right)\right)$ approximation error of $o(T^2)$ order. Now approximated control input can be expressed as

$$\begin{aligned} &\mathbf{F}(k) = \mathbf{F}_{eq}(k-1) - \mathbf{M}\mathbf{Q}^{-1}\boldsymbol{\Psi}(\boldsymbol{\sigma}_k) \\ &\mathbf{F}(k) = sat\left(\left(\mathbf{F}(k-1) - \mathbf{M}\mathbf{Q}^{-1}T^{-1}\left(\boldsymbol{\sigma}(k) - \boldsymbol{\sigma}(k-1)\right)\right) - \mathbf{M}\mathbf{Q}^{-1}\boldsymbol{\Psi}(\boldsymbol{\sigma}_k)\right) \end{aligned} \tag{8}$$

here $sat(\bullet)$ stands for saturation function. In implementation of algorithm (8) information on control, $\boldsymbol{\sigma}$ at $t = (k-1)T$ and at $t=kT$ and inertia matrix are needed. In order to verify the validity of approximation the evaluation of the $\boldsymbol{\sigma}$ within sampling interval $kT \leq t \leq (k+1)T$ should be considered. By inserting control (8) into (1) and assuming that its value is within unity gain of the saturation function, one has system dynamics at $t=kT$ as

$$\mathbf{M}\ddot{\mathbf{q}}(k) = \mathbf{F}_{eq}(k-1) - \mathbf{M}\mathbf{Q}^{-1}\boldsymbol{\Psi}(\boldsymbol{\sigma}(k)) - \left(\mathbf{F}_{ext}(k) + \mathbf{N}(\mathbf{q}(k), \dot{\mathbf{q}}(k))\right) \tag{9}$$

By adding and subtracting $\mathbf{F}_{eq}(k)$ to the right hand side of (8) after some algebra one can obtain the following relation

$$\mathbf{M}\ddot{\mathbf{q}}(k) = -\mathbf{M}\left(\mathbf{Q}^{-1}\boldsymbol{\Psi}(\boldsymbol{\sigma}(k)) + \mathbf{G}\dot{\mathbf{q}}(k) - \dot{\mathbf{f}}(k)\right) - \left(\mathbf{F}_{eq}(k) - \mathbf{F}_{eq}(k-1)\right) \tag{10}$$

Taking into account $\dot{\boldsymbol{\sigma}}(k) = \mathbf{Q}\ddot{\mathbf{q}}(k) + \mathbf{C}\dot{\mathbf{q}}(k) - \dot{\boldsymbol{\xi}}^{ref}(k)$, (10) can be rearranged to

$$\dot{\boldsymbol{\sigma}}(k) + \boldsymbol{\Psi}(\boldsymbol{\sigma}(k)) = -\mathbf{QM}^{-1}\left(\mathbf{F}_{eq}(k) - \mathbf{F}_{eq}(k-1)\right) \tag{11}$$

The inter-sampling change of $\boldsymbol{\sigma}(t)$ can be evaluated

$$\boldsymbol{\sigma}(kT+\tau) - \boldsymbol{\sigma}(kT) = -\int_{kT}^{kT+\tau}\boldsymbol{\Psi}(\boldsymbol{\sigma}(t))dt + \int_{kT}^{kT+\tau}\mathbf{QM}^{-1}\left(\mathbf{F}_{eq}(k) - \mathbf{F}_{eq}(k-1)\right)dt \tag{12}$$

Since equivalent control is smooth function it is easy to show, by inspection of integral $\Delta = -\int_0^T \mathbf{QM}^{-1}\int_{(k-1)T-\lambda}^{kT-\lambda}\left(\dot{\mathbf{F}}_{eq}(\lambda)\right)d\lambda dt = o(T^2)$, that the error in Δ introduced by the control input approximation is of the $\Delta = o(T^2)$ order and that Δ remains within the $o(T^2)$ boundary layer during inter-sampling interval $kT \le t \le (k+1)T$. The thickness of the boundary layer of sliding mode manifold is defined by $\boldsymbol{\sigma}(kT+\tau) - \boldsymbol{\sigma}(kT) = -\int_{kT}^{kT+\tau}\boldsymbol{\Psi}(\boldsymbol{\sigma}(t))dt + o(T^2)$ and in obvious way depends on the selection of $\boldsymbol{\Psi}(\boldsymbol{\sigma}(t))$. By changing the reference configuration of the system $\xi^{ref}\left(\mathbf{q}^{ref}, \dot{\mathbf{q}}^{ref}\right)$, system's motion can be modified. Definition of control goal and behavior of the system is clearly resting on the selection of the reference configuration and its dependence on desired specification. In the following sections, we will concentrate on the selection of the reference configuration for problems of controlling systems required to satisfy certain functional relations (real or virtual). Assume that the overall external force consists of the disturbance $\mathbf{F}_d$ that should be rejected by the system controller and the interaction force between system and environment $\mathbf{g}_{ij}(\mathbf{q}, \mathbf{q}_e)$ that should be maintained so that $\mathbf{F}_{ext} = \mathbf{F}_d + \mathbf{g}_{ij}$. As a control task assume the requirement of trajectory tracking and the modification of the system configuration in such a way that the desired interaction between system and environment is maintained. Since trajectory tracking is basic task in mechanical systems it will be natural to assume that function $\xi^{ref}\left(\mathbf{q}^{ref}, \dot{\mathbf{q}}^{ref}\right)$ depends on the desired trajectory and that the trajectory should be modified is system is in contact with environment in order to maintain desired interaction. For such a behavior of the system (1) the desired manifold (3) should be changed to include the environmental interaction control. In addition, while in contact with the environment motion system is required to modify its trajectory in order to control interaction between system and environment. One possible structure that includes both requirements may be selected as in (13)

$$\begin{aligned} &S_{qg} = \left\{\mathbf{q}, \dot{\mathbf{q}} : \xi(\mathbf{q}, \dot{\mathbf{q}}) - \xi^{ref} - \vartheta\left(\Delta\mathbf{g}_{ij}\right) + \boldsymbol{\Gamma}\mathbf{g}_{ij} = \boldsymbol{\sigma} = \mathbf{0}\right\} \\ &\xi^{ref}\left(\mathbf{q}^{ref}, \dot{\mathbf{q}}^{ref}\right) = \mathbf{C}\mathbf{q}^{ref} + \mathbf{Q}\dot{\mathbf{q}}^{ref}; \xi(\mathbf{q}, \dot{\mathbf{q}}) = \mathbf{C}\mathbf{q} + \mathbf{Q}\dot{\mathbf{q}} \\ &\mathbf{g}_{ij} = \begin{cases} \mathbf{g}_{ij}\left(\mathbf{q}, \dot{\mathbf{q}}, \mathbf{q}_e, \dot{\mathbf{q}}_e\right) & \text{when in contact} \\ 0 & \text{without contact} \end{cases} \end{aligned} \tag{13}$$

The interaction control input $\vartheta(\Delta\mathbf{F}_e)$ should be determined the same way as the control (8) to maintain stability of system motion in manifold $S_F = \left\{(\mathbf{q}, \dot{\mathbf{q}}) : \mathrm{F}(\mathbf{q}, \dot{\mathbf{q}}) - F^{ref}(t) = \sigma_F = \mathbf{0}\right\}$. Selection of manifold as in (13) leads to the proposed control (8) $\mathbf{F}(k) = sat\left(\mathbf{F}(k-1) - \left(\mathbf{QM}^{-1}\right)^{-1}\left[T^{-1}\left(\boldsymbol{\sigma}(k) - \boldsymbol{\sigma}(k-1)\right) - \boldsymbol{\Psi}(\boldsymbol{\sigma}_k)\right]\right)$ and the only change is reflected in the calculation of the distance from manifold. $\vartheta_i\left(\Delta\mathbf{g}_{iji}\right)$ should have zero value if systems are not in interaction. For system (1), the sliding mode motion in the manifold (13) results in

$$\xi(\mathbf{q}, \dot{\mathbf{q}}) - \xi^{ref} = \vartheta\left(\Delta\mathbf{g}_{ij}\right) + \boldsymbol{\Gamma}\mathbf{g}_{ij} \tag{14}$$

3. EXTENSION TO THE GENERAL SYSTEMS IN INTERACTIONS

In the situation depicted above control modifies the behaviour of only one of the systems in interaction while motion of other system is treated as a disturbance. For motion control systems of particular interest systems configurations maintain desired functional relation (for example bilateral control or cooperating robots etc.). In such systems control should be selected to maintain a functional relation by acting on all of the subsystems. Similar situation is examined in so-called "*function control*" framework (Tsuji, and Ohnishi, 2005; Tsuji, 2006; Tsuji, *et al*., 2006), where the notion of system role *"description on the requirement from a user to a robot"* and its representation by "*elementary functions*" defined as *"a minimum component of a system role*" is discussed and the following design procedure is suggested: (i) the system role is determined by a designer, (ii) the designer divides the system role into functions, (iii) a priority order of functions is determined, (iv) the transformation to a "*function subspace*" is derived, and (v) function-based controllers are designed individually and back transformation is applied in order to determine actual control inputs.

Assume a set of n single DOF motion systems each represented by $S_i : m_i(q_i)\ddot{q}_i + n_i(q_i, \dot{q}_i, t) = f_i - f_{iext}$ $i = 1, 2, \ldots, n$ or $\mathbf{S} : \mathbf{M}(\mathbf{q})\ddot{\mathbf{q}} + \mathbf{N}(\mathbf{q}, \dot{\mathbf{q}}, t) = \mathbf{BF} - \mathbf{d}_\Sigma$, $\mathbf{q} \in \Re^{n\times 1}, rank\mathbf{B} = rank\mathbf{M} = n$, vectors $\mathbf{N}, \mathbf{d}_\Sigma$ satisfy matching conditions. Assume also that required role $\boldsymbol{\Phi} \in \Re^{n\times 1}$ of the system $\mathbf{S}$ may be represented as a set of smooth linearly independent functions $\zeta_1(\mathbf{q}), \zeta_2(\mathbf{q}), \ldots, \zeta_n(\mathbf{q})$ and role vector can be defined as $\boldsymbol{\Phi}^T = \left[\zeta_1(\mathbf{q}) \ \ldots \ \zeta_n(\mathbf{q})\right]$. Consider problem of designing control for system $\mathbf{S} : \mathbf{M}(\mathbf{q})\ddot{\mathbf{q}} + \mathbf{N}(\mathbf{q}, \dot{\mathbf{q}}, t) = \mathbf{BF} - \mathbf{d}_\Sigma$ such that role vector $\boldsymbol{\Phi} \in \Re^{n\times 1}$ tracks its smooth reference $\boldsymbol{\Phi}^{ref} \in \Re^{n\times 1}$. Let sliding mode manifold $\boldsymbol{\sigma}_\Phi \in \Re^n$ be defined as

$$S_\Phi = \left\{(\mathbf{q}, \dot{\mathbf{q}}) : \xi_\Phi\left(\boldsymbol{\Phi}, \dot{\boldsymbol{\Phi}}\right) - \xi_\Phi^{ref}\left(\boldsymbol{\Phi}^{ref}, \dot{\boldsymbol{\Phi}}^{ref}\right) = \boldsymbol{\sigma}_\Phi = 0\right\} \tag{15}$$

By calculating $\dot{\boldsymbol{\Phi}} = \left[\frac{\partial\Phi}{\partial\mathbf{q}}\right]\dot{\mathbf{q}} = \mathbf{J}_\Phi\dot{\mathbf{q}}$ with $\mathbf{J}_\Phi = \left[\frac{\partial\Phi}{\partial\mathbf{q}}\right]$, one can determine $\ddot{\boldsymbol{\Phi}} = \hat{\mathbf{B}}\mathbf{F} + \hat{\mathbf{d}}_\Sigma$ where $\hat{\mathbf{B}} = \mathbf{J}_\Phi\mathbf{M}^{-1}\mathbf{B}$ and $\hat{\mathbf{d}}_\Sigma = \mathbf{J}_\Phi\mathbf{M}^{-1}\left(-\mathbf{N}(\mathbf{q}, \dot{\mathbf{q}}, t) - \mathbf{d}_\Sigma\right) + \dot{\mathbf{J}}_\Phi\dot{\mathbf{q}}$. By introducing $\left[\frac{\partial\xi_\Phi}{\partial\dot{\Phi}}\right] = \mathbf{Q}_\Phi$ and $\left[\frac{\partial\xi_\Phi}{\partial\Phi}\right] = \mathbf{C}_\Phi$ projection of the system motion on manifold S_Φ, can be expressed as $\frac{d\sigma_\Phi}{dt} = \mathbf{Q}_\Phi \hat{\mathbf{B}}\mathbf{F} + \left(\hat{\mathbf{d}}_\Sigma + \mathbf{C}_\Phi \dot{\mathbf{\Phi}} - \dot{\xi}_\Phi^{ref}\right)$. With $\hat{\mathbf{d}}_\Phi = \hat{\mathbf{d}}_\Sigma + \mathbf{C}_\Phi \dot{\mathbf{\Phi}} - \dot{\xi}_\Phi^{ref}$ and $\mathbf{F}_\Phi = \mathbf{Q}_\Phi \hat{\mathbf{B}}\mathbf{F}$, it can be simplified as $\dot{\sigma}_\Phi = \mathbf{F}_\Phi + \hat{\mathbf{d}}_\Phi$ what represents a virtual plant described by n first order systems of the form $\dot{\sigma}_{\Phi i} = F_{\Phi i} + \hat{d}_{\Phi i} \quad i = 1,\dots,n$ for which design of control $F_{\Phi i}$ is straightforward and the algorithm (7) or its modifications may be applied directly. If $\left(\mathbf{Q}_\Phi \hat{\mathbf{B}}\right)^{-1} = \left(\mathbf{Q}_\Phi \mathbf{J}_\Phi \mathbf{M}^{-1}\mathbf{B}\right)^{-1}$ exists then inverse transformation $\mathbf{F} = \left(\mathbf{Q}_\Phi \hat{\mathbf{B}}\right)^{-1}\mathbf{F}_\Phi$ gives control in the original state space. Since $\mathbf{M} \in \Re^{n \times n}$ and $\mathbf{B} \in \Re^{n \times n}$ are square full rank matrices then one can determine conditions that matrices $\mathbf{J}_\Phi$ and $\mathbf{Q}_\Phi$ should satisfy in order that $\left(\mathbf{Q}_\Phi \mathbf{J}_\Phi \mathbf{M}^{-1}\mathbf{B}\right)^{-1}$ exists. Since $\mathbf{J}_\Phi, \mathbf{Q}_\Phi, \mathbf{M}, \mathbf{B} \in \Re^{n \times n}$, sufficient conditions for having unique solution for control $\mathbf{F}$ is $rank\left(\mathbf{Q}_\Phi \mathbf{J}_\Phi\right) = n$.

Model of 3 DOF parallel manipulator is shown in Fig. 1. Each of the legs can be described by $m_i \ddot{x}_i + n_i\left(x_i, \dot{x}_i\right) = F_i - F_{disi} \quad i = 1,2,3$. Motion of the platform consists of the translational, which relates to the sum of the three legs positions and rotational motion with respect to some axis the simplest being defined by one leg length constant and the others varying in time so the rotation appears related to the difference in length of two legs. Based on this one can define the following functions to be controlled:

$\varepsilon = x_1 + x_2 + x_3$ translation along z axis (16)

$\varepsilon_{12} = x_1 - x_2$ rotation along AM_3 axis (17)

$\varepsilon_{13} = x_1 - x_3$ rotation along BM_2 axis (18)

$\varepsilon_{23} = x_2 - x_3$ rotation along CM_1 axis (19)

The projection of the parallel mechanism motion on the subspace defined by these functions may be easily obtained as in the following forms:

Translational movement of common mode

$$\ddot{\varepsilon} = \frac{1}{m_1}F_1 - \frac{F_{dis1}}{m_1} + \frac{1}{m_2}F_2 - \frac{F_{dis2}}{m_2} + \frac{1}{m_3}F_3 - \frac{F_{dis3}}{m_3} \quad (20)$$

$$u_i = \frac{F_i}{m_i}, i = 1,2,3, d_{123} = \sum_{i=1}^{3} \frac{F_{disi}}{m_i}$$

$$\ddot{\varepsilon} = u_1 + u_2 + u_3 - d_{123} \rightarrow \ddot{\varepsilon} = u_{123} - d_{123} \quad (21)$$

The dynamics on differential coordinates according to one of the rotating axis (AM_3, BM_2) are figured out as follows:

Rotation through the AM_3 axis

$$\ddot{\varepsilon}_{12} = \frac{1}{m_1}F_1 - \frac{F_{dis1}}{m_1} - \left(\frac{1}{m_2}F_2 - \frac{F_{dis2}}{m_2}\right) \quad (22)$$

$$\ddot{\varepsilon}_{12} = u_1 - u_2 - d_{12} \rightarrow \ddot{\varepsilon}_{12} = u_{12} - d_{12} \quad (23)$$

Rotation through the BM_2 axis

$$\ddot{\varepsilon}_{13} = \frac{1}{m_1}F_1 - \frac{F_{dis1}}{m_1} - \left(\frac{1}{m_2}F_3 - \frac{F_{dis3}}{m_3}\right) \quad (24)$$

$$\ddot{\varepsilon}_{13} = u_1 - u_3 - d_{13} \rightarrow \ddot{\varepsilon}_{13} = u_{13} - d_{13} \quad (25)$$

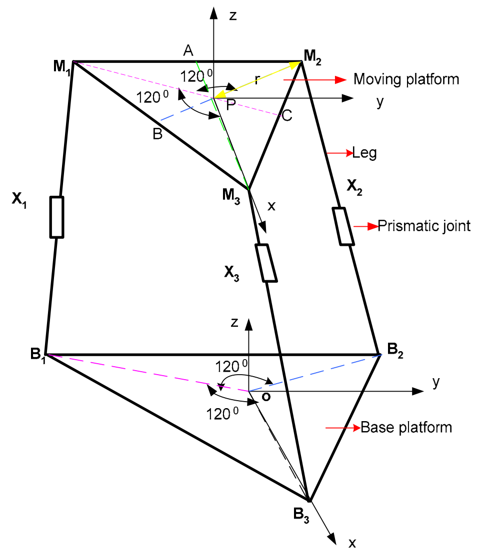

Fig. 1 General structure of bilateral control system

Following results presented in section 3 one should select such a set of functions so that transformation of control from functional space back to original space is unique. In our case we can select only three functions to be controlled at the same time. Assume we select ε, ε_{12}, ε_{13} for which transformation matrix from original to function space can be written as in (26) and selected functions (or "virtual plants") are defined as in (21), (23), (25).

$$^{f}T_{l} = \begin{bmatrix} 1 & 1 & 1 \\ 1 & -1 & 0 \\ 1 & 0 & -1 \end{bmatrix} \quad (26)$$

Since all "virtual plants" are of the second order the controller should be designed in such a way that sliding mode is enforced on the intersection of the manifolds $S_{\varepsilon i}$ (i=1,2,3) :

$$S_{\varepsilon_i} = \left\{(\mathbf{q}, \dot{\mathbf{q}}) : \xi_{\varepsilon_i}\left(\varepsilon_i, \dot{\varepsilon}_i\right) - \xi_{\varepsilon_i}^{ref}\left(\varepsilon_i, \dot{\varepsilon}_i\right) = \boldsymbol{\sigma}_{\varepsilon_i} = 0\right\}$$

Controllers that enforce sliding mode on each of the surfaces are easy to determine as in (8). In order to look at different scenarios in designing the controllers we have developed two computer simulation models to check the dynamic formulation of three-legged parallel manipulator and compare the performance of disturbance observer on functional coordinate and on robot coordinates. The structure of the control system is depicted in Fig. 2. For simulation of the proposed system three-legged parallel manipulator and Faulhaber 2642 012 CR series motor parameters ($J=11*10^{-7}$ kgm^2, K_t=16.9Nm/A) are used, the parameters of sliding mode controller are $Ku=10^{-5}$, D=50, C=30, g=500 rad/s (cut off frequency of DOB), sampling time 0.1 ms.

4. Simulation Results

System responses are shown in robot space by Fig.2 & Fig.3 and functional space by Fig.4 & Fig.5 for $2x10^{-6}xsin(t)$ m reference with band-limited white noise (Amplitude: $2x10^{-6}$). As translational movement of common mode and rotational motions of difference mode of three legs positions are shown in the figures. The simulation results show that performance of functional controllers is satisfactory. When we have disturbance in our functional space, controller performs better in function coordinate with disturbance observer.

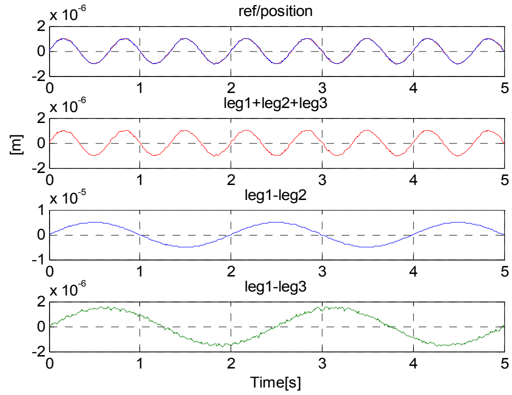

Fig. 2 Positions with disturbance

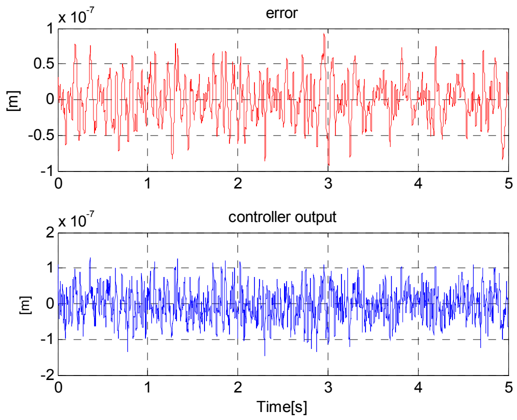

Fig. 3 Error and control output with disturbance

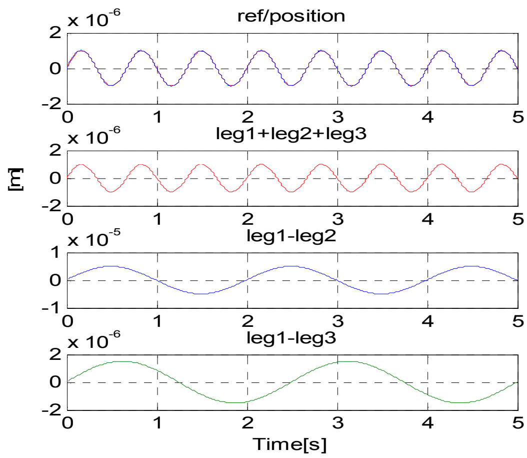

Fig. 4 Positions with disturbance

5. Conclusions

In this paper we presented a generalized approach to motion control system and a possibility project the system motion to a “functional space” in which a

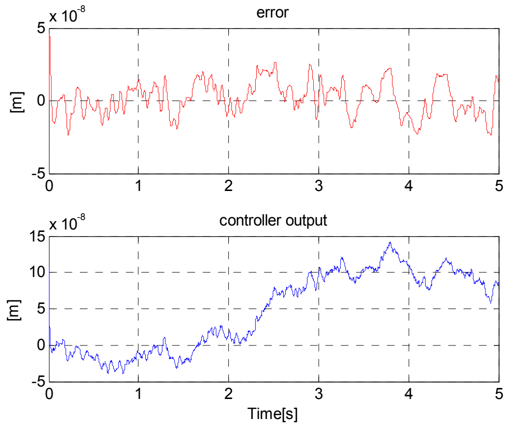

Fig. 5 Error and control output with disturbance

natural tasks of the system are presented (like move up, move down, role for this angle etc.). It has been shown that due to system structure design can be performed so to guaranty the tracking in the “function space”. The conditions for stability and integrity of such system design are found. As an example the manipulation of three-legged parallel manipulator is presented.

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