

# SMC Based Bilateral Control

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**Abstract— Design of a motion control system should take into account (a) unconstrained motion performed without interaction with environment or other system, and (b) constrained motion with system in contact with environment or another system or has certain functional interaction with another system. Control in both cases can be formulated in terms of maintaining desired system configuration what makes essentially the same structure for common tasks: trajectory tracking, interaction force control, compliance control etc. It will be shown that the same design approach can be used for systems that maintain some functional relation – like bilateral or multilateral systems, relation among mobile robots or control of haptic systems..**

## I. INTRODUCTION

Modern motion control systems are acting as “agents” between skilled human operator and environment (surgery, microparts handling, teleoperation, etc.). In such situations design of control should encompass wide range of very demanding tasks. At the lower level one should consider tasks of controlling individual systems - like single DOF systems, motor control, robotic manipulator or mobile robot. On the system level control of bilateral or multilateral interaction between systems of the same or different nature, the remote control in master-slave systems, haptics etc. should be considered. In general design of motion control system should take into account (i) unconstrained motion - performed without interaction with environment or other system - like trajectory tracking, (ii) motion in which system should maintain its trajectory despite of the interaction with other systems - disturbance rejection tasks, (iii) constrained motion where system should modify its behavior due to interaction with environment or another system or should maintain specified interconnection - virtual or real - with other system and (iv) in remote operation control system should be able to reflect the sensation of unknown environment to the human operator.

Decentralized control seems a promising framework for application in motion control. There are many applications to robot control systems, with concepts such as multi-agent system [1, 2], cell structure [3], decomposition block control [4]. Under certain conditions overall control input can be designed by linear superposition [5], or a framework of controller design based on functionality [6].

In this paper we will present a framework in motion control systems with or without contact with environment. The possibility to enforce certain functional relations between coordinates of one or more motion systems represent a basis of

the proposed algorithm. We will demonstrate that all basic control problems - trajectory tracking, force control, hybrid position/force control scheme and the impedance control - can be treated in the same way while avoiding the structural change of the controller and guarantying stable behavior of the system. It will be shown that this framework can be naturally extended to the control of mechanical systems in interaction, like bilateral or multilateral control.

The paper is organized as follows. In section 2 mathematical background, formulation of problem and the control design are discussed. In section 3 the application to bilateral system is discussed along with experimental verification of the proposed algorithm.

## II. MATHEMATICAL FORMULATIONS

Mathematical description of fully actuated mechanical system may be presented as

$$\begin{aligned} \mathbf{M}(\mathbf{q})\ddot{\mathbf{q}} + \mathbf{L}(\mathbf{q}, \dot{\mathbf{q}})\dot{\mathbf{q}} + \mathbf{H}(\mathbf{q}, \dot{\mathbf{q}}) &= \mathbf{F} - \mathbf{F}_{ext} \\ \mathbf{L}(\mathbf{q}, \dot{\mathbf{q}})\dot{\mathbf{q}} + \mathbf{H}(\mathbf{q}, \dot{\mathbf{q}}) &= \mathbf{N}(\mathbf{q}, \dot{\mathbf{q}}) \end{aligned} \quad (1)$$

where  $\mathbf{q} \in \mathfrak{R}^n$  and  $\dot{\mathbf{q}} \in \mathfrak{R}^n$  stand for the vectors of generalized positions and velocities respectively,  $\mathbf{M}(\mathbf{q}) \in \mathfrak{R}^{n \times n}$   $M^- \leq \|\mathbf{M}(\mathbf{q})\| \leq M^+$  is the generalized positive definite inertia matrix,  $\mathbf{N}(\mathbf{q}, \dot{\mathbf{q}}) \in \mathfrak{R}^{n \times 1}$   $\|\mathbf{N}(\mathbf{q}, \dot{\mathbf{q}})\| \leq N^+$  represent the vector of coupling forces,  $\mathbf{F} \in \mathfrak{R}^{n \times 1}$ ,  $\|\mathbf{F}\| \leq F^+$  stands for the vector of generalized input forces,  $\mathbf{F}_{ext} \in \mathfrak{R}^{n \times 1}$ ,  $\|\mathbf{F}_{ext}\| \leq F_{0ext}$  stands for the vector of external forces.  $M^-$ ,  $M^+$ ,  $N^+$  and  $F^+$   $F_{0ext}$  are known scalars.

External force in general can be represented as

$$\mathbf{F}_{ext} = \begin{cases} \mathbf{F}_{ext}(\mathbf{q}, \mathbf{q}_e) & \text{if there is interaction with environment} \\ 0 & \text{if there is no interaction with environment} \end{cases} \quad (2)$$

In many cases interaction of the systems is modeled as spring-damper so than the interaction force is represented as a linear combination in the form  $\mathbf{F}_{ext} = \mathbf{K}_s(\mathbf{q} - \mathbf{q}_e) + \mathbf{K}_D(\dot{\mathbf{q}} - \dot{\mathbf{q}}_e)$  with  $\mathbf{q}_e \in \mathfrak{R}^n$  standing for the environment position vector.

### A. Control Problem Formulation

Vector of generalized positions and generalized velocities defines configuration  $\xi(\mathbf{q}, \dot{\mathbf{q}})$  of a mechanical system. The control tasks for the system (1) are usually formulated as selection of the generalized input such that: (i) system executes desired motion specified as trajectory tracking, (ii) system exerts a desired force while in the contact with environment and (iii) system reacts as a desired impedance on the external force input or in contact with environment. In literature these problem are generally treated separately [7, 8] and motion that requires transition from one to another task is treated in the framework of hybrid control [9]. Without loss of generality, in this paper, it will be assumed that system configuration can be expressed as  $\xi(\mathbf{q}, \dot{\mathbf{q}}) = \mathbf{C}\mathbf{q} + \mathbf{Q}\dot{\mathbf{q}}$ .

In the sliding mode framework control system requirements is equivalent to the enforcing sliding mode on the manifold  $S_q$  defined by

$$S_q = \left\{ (\mathbf{q}, \dot{\mathbf{q}}) : \mathbf{C}\mathbf{q} + \mathbf{Q}\dot{\mathbf{q}} - \xi^{ref}(t) = \boldsymbol{\sigma}(\mathbf{q}, \dot{\mathbf{q}}, \xi^{ref}) = \mathbf{0} \right\}, \quad (3)$$

$$\boldsymbol{\sigma}, \xi, \xi^{ref} \in \mathfrak{R}^{nx1}; \mathbf{C}, \mathbf{Q} \in \mathfrak{R}^{nxn}; \mathbf{C}, \mathbf{Q} > 0$$

where  $\xi^{ref}(t) \in \mathfrak{R}^{nx1}$  stands for the reference configuration of the system and is assumed to be smooth bounded function with a continuous first order time derivative, matrices  $\mathbf{C}, \mathbf{Q} \in \mathfrak{R}^{nxn}$  have full rank  $rank(\mathbf{C}) = rank(\mathbf{Q}) = n$ .

### B. Selection of control input

The simplest and the most direct method to derive control is to enforce Lyapunov stability conditions for solution  $\boldsymbol{\sigma}(\xi, \xi^{ref}) = \mathbf{0}_{nx1}$  on the trajectories of system (1), (2). Lyapunov function candidate may be selected as  $v = \frac{1}{2} \boldsymbol{\sigma}^T \boldsymbol{\sigma} > 0$  with first time derivative  $\dot{v} = \boldsymbol{\sigma}^T \dot{\boldsymbol{\sigma}}$ . To ensure stability the derivative of Lyapunov function is required to be negative definite so one can require that  $\dot{v} = \boldsymbol{\sigma}^T \dot{\boldsymbol{\sigma}} = -\boldsymbol{\sigma}^T \boldsymbol{\Psi}(\boldsymbol{\sigma}) < 0$ . For  $-\boldsymbol{\sigma}^T \boldsymbol{\Psi}(\boldsymbol{\sigma}) = -\rho v^\delta < 0$  with  $\rho > 0$  and  $\frac{1}{2} \leq \delta < 1$  stability conditions are satisfied and finite time convergence to sliding mode manifold is obtained. From  $\dot{v} = \boldsymbol{\sigma}^T \dot{\boldsymbol{\sigma}} = -\boldsymbol{\sigma}^T \boldsymbol{\Psi}(\boldsymbol{\sigma})$  one can derive  $\boldsymbol{\sigma}^T (\dot{\boldsymbol{\sigma}} + \boldsymbol{\Psi}(\boldsymbol{\sigma})) = 0$  and consequently control should be selected to satisfy  $\dot{\boldsymbol{\sigma}} + \boldsymbol{\Psi}(\boldsymbol{\sigma})|_{\boldsymbol{\sigma} \neq 0} = \mathbf{0}$ . By differentiating  $\boldsymbol{\sigma}$  and substituting (1) under the assumption that  $\mathbf{C}, \mathbf{Q} \in \mathfrak{R}^{nxn}$  are constant and  $(\mathbf{Q}\mathbf{M}^{-1})^{-1}$  exists, from  $(\dot{\boldsymbol{\sigma}} + \boldsymbol{\Psi}(\boldsymbol{\sigma}))|_{\boldsymbol{\sigma} \neq 0} = \mathbf{Q}\mathbf{M}^{-1}(\mathbf{F} - \mathbf{F}_{eq}) + \boldsymbol{\Psi}(\boldsymbol{\sigma}) = \mathbf{0}$  one can find equivalent control as  $\mathbf{F}_{eq} = (\mathbf{F}_{ext} + \mathbf{N}) - (\mathbf{Q}\mathbf{M}^{-1})^{-1}(\mathbf{C}\dot{\mathbf{q}} - \dot{\xi}^{ref})$  and then control input  $\mathbf{F} = \mathbf{F}_{eq} - (\mathbf{Q}\mathbf{M}^{-1})^{-1}\boldsymbol{\Psi}(\boldsymbol{\sigma})$ . The structure of control input depends on the selection of  $\boldsymbol{\Psi}(\boldsymbol{\sigma})$ , which should be determined in such a way so to ensure stability conditions for solution  $\boldsymbol{\sigma} = \mathbf{0}$  are guaranteed and that  $\boldsymbol{\sigma} \rightarrow \mathbf{0}$ .

### C. Equations of motion

Assume that matrices  $\mathbf{C}, \mathbf{Q} \in \mathfrak{R}^{nxn}$  are constant and that inverse  $(\mathbf{Q}\mathbf{M}^{-1})^{-1}$  exists, and can be expressed as  $(\mathbf{Q}\mathbf{M}^{-1})^{-1} = \mathbf{M}\mathbf{Q}^{-1}$ . The application of the equivalent control for system (1) with the sliding mode enforced on the manifold (3)

$$\mathbf{F}_{eq} = (\mathbf{F}_{ext} + \mathbf{N}) - (\mathbf{Q}\mathbf{M}^{-1})^{-1}(\mathbf{C}\dot{\mathbf{q}} - \dot{\xi}^{ref}) \quad (4)$$

leads to the equations of motion in the following form

$$\mathbf{M}\ddot{\mathbf{q}} = (\mathbf{Q}\mathbf{M}^{-1})^{-1}(\dot{\xi}^{ref}(t) - \mathbf{C}\dot{\mathbf{q}}) = \mathbf{M}\ddot{\mathbf{q}}^{des} \Rightarrow \ddot{\mathbf{q}} = \ddot{\mathbf{q}}^{des}$$

$$\ddot{\mathbf{q}}^{des} = \mathbf{Q}^{-1}(\dot{\xi}^{ref}(t) - \mathbf{C}\dot{\mathbf{q}}) \quad (5)$$

The sliding mode motion (5) is equivalent to the acceleration control [8] with desired acceleration  $\ddot{\mathbf{q}}^{des} = \mathbf{Q}^{-1}(\dot{\xi}^{ref}(t) - \mathbf{C}\dot{\mathbf{q}})$  and the closed loop system behaves as a "nominal plant" defined by design parameters  $\mathbf{C}$  and  $\mathbf{Q}$ . Equations (4) show that in the ideal case motion of the system will not be modified when it comes in contact with environment. If closed loop motion (5) should be modified due to the contact with environment than the reference configuration must depend on the interaction force

### D. Discrete-time implementation of control

The discrete-time implementation of control  $\mathbf{F} = \mathbf{F}_{eq} - (\mathbf{Q}\mathbf{M}^{-1})^{-1}\boldsymbol{\Psi}(\boldsymbol{\sigma})$  requires evaluation of the equivalent control at the end of every sampling interval. The equivalent control is smooth function so one can resort of using its value at  $t = (k-1)T$  instead of the exact value at  $t = kT$ . By evaluating  $\dot{\boldsymbol{\sigma}} = \mathbf{Q}\mathbf{M}^{-1}(\mathbf{F} - \mathbf{F}_{eq})$  at  $t = (k-1)T$  it is easy to derive  $\mathbf{F}_{eq}(k-1) = (\mathbf{F}(k-1) - \mathbf{M}\mathbf{Q}^{-1}\dot{\boldsymbol{\sigma}}(k-1))$ . Approximation  $\dot{\boldsymbol{\sigma}}(k-1) = (\boldsymbol{\sigma}(k) - \boldsymbol{\sigma}(k-1))/T$  leads to  $\mathbf{F}_{eq}(k-1) = (\mathbf{F}(k-1) - \mathbf{M}\mathbf{Q}^{-1}T^{-1}(\boldsymbol{\sigma}(k) - \boldsymbol{\sigma}(k-1)))$  with an approximation error of the  $o(T^2)$  order. The approximated control input can be expressed as

$$\mathbf{F}(k) \cong \mathbf{F}_{eq}(k-1) - \mathbf{M}\mathbf{Q}^{-1}\boldsymbol{\Psi}(\boldsymbol{\sigma}_k)$$

$$\mathbf{F}(k) = (\mathbf{F}(k-1) - \mathbf{M}\mathbf{Q}^{-1}T^{-1}(\boldsymbol{\sigma}(k) - \boldsymbol{\sigma}(k-1))) - \mathbf{M}\mathbf{Q}^{-1}\boldsymbol{\Psi}(\boldsymbol{\sigma}(k)) \quad (6)$$

By inserting (6) into (1) one can evaluate system dynamics at  $t = kT$  as

$$\mathbf{M}\ddot{\mathbf{q}}(k) = \mathbf{F}_{eq}(k-1) - \mathbf{M}\mathbf{Q}^{-1}\boldsymbol{\Psi}(\boldsymbol{\sigma}(k)) - (\mathbf{F}_{ext}(k) + \mathbf{N}(\mathbf{q}(k), \dot{\mathbf{q}}(k))) \quad (7)$$

The error introduced by this approximation of control can be estimated from the following relation

$$\dot{\boldsymbol{\sigma}}(k) + \boldsymbol{\Psi}(\boldsymbol{\sigma}(k)) = -\mathbf{Q}\mathbf{M}^{-1}(\mathbf{F}_{eq}(k) - \mathbf{F}_{eq}(k-1)) \quad (8)$$

The thickness of the boundary layer of the sliding mode manifold can be determined by evaluating

$\sigma(kT + \tau) - \sigma(kT) = -\int_{kT}^{kT+\tau} \Psi(\sigma(t)) dt + o(T^2)$ . From here one can

see why the relay control in discrete-time implementation will result in motion with chattering within a boundary layer having thickness of the  $o(T)$  order.

In literature behavior of a motion control system is mostly analyzed in three separate frameworks: (i) the trajectory tracking, (ii) the force control and (iii) impedance control. Due to the fact that in fully actuated systems interaction forces and system configuration cannot be set independently hybrid schemes had been developed to cope with position-force control tasks and the transitions from one to another [9].

In SMC framework the reference configuration for the trajectory tracking should be selected as  $\xi_q^{ref}(t) = -(\mathbf{Q}\dot{\mathbf{q}}^{ref} + \mathbf{C}\mathbf{q}^{ref})$  and consequently the sliding mode manifold becomes  $S_q = \{\mathbf{q}, \dot{\mathbf{q}} : \mathbf{Q}\dot{\mathbf{q}} + \mathbf{C}\mathbf{q} - (\mathbf{Q}\dot{\mathbf{q}}^{ref} + \mathbf{C}\mathbf{q}^{ref}) = \mathbf{0}\}$  thus the control (5) can be directly applied to obtain  $\mathbf{F} = \mathbf{F}_{eq} - \mathbf{M}\mathbf{Q}^{-1}\Psi(\sigma_q)$ . For example, if one selects  $\Psi(\sigma_q) = -\mathbf{D}\sigma_q; \mathbf{D} > 0$  the equations of motion can be determined in the following form  $\mathbf{Q}(\ddot{\mathbf{q}} - \ddot{\mathbf{q}}^{ref}) + (\mathbf{C} + \mathbf{D}\mathbf{Q})(\dot{\mathbf{q}} - \dot{\mathbf{q}}^{ref}) + \mathbf{C}\mathbf{D}(\mathbf{q} - \mathbf{q}^{ref}) = \mathbf{0}$ . This result is the same as the one obtained by application of the disturbance observer and PD controller as discussed in [8].

In the force control system with the reference  $\mathbf{F}^{ref}(t)$  the sliding mode manifold can be defined as  $S_F = \{\mathbf{q}, \dot{\mathbf{q}} : \mathbf{F}(\mathbf{q}, \dot{\mathbf{q}}) - \mathbf{F}^{ref}(t) = \sigma_F = \mathbf{0}\}$ . If measured force is modeled as  $\mathbf{F}(\mathbf{q}, \dot{\mathbf{q}}) = \mathbf{K}_p\Delta\mathbf{q} + \mathbf{K}_D\Delta\dot{\mathbf{q}}$ ,  $\Delta\mathbf{q} = \mathbf{q} - \mathbf{q}_e$ ,  $\mathbf{K}_p, \mathbf{K}_D$  are diagonal matrices of appropriate dimensions, then by defining the reference configuration as  $\xi_F^{ref}(t) = (\mathbf{F}^{ref}(t) + \mathbf{K}_p\mathbf{q}_e + \mathbf{K}_D\dot{\mathbf{q}}_e)$  the sliding mode manifold becomes  $S_F = \{\mathbf{q}, \dot{\mathbf{q}} : \mathbf{K}_p\mathbf{q} + \mathbf{K}_D\dot{\mathbf{q}} - \xi_F^{ref}(t) = \sigma_F = \mathbf{0}\}$ . This manifold has the same form as the one derived for the trajectory tracking, thus the structure of the control input is the same as for trajectory tracking with appropriate changes of the variables.

#### E. Modification of System Configuration

Since trajectory tracking is basic task in mechanical systems it will be natural to assume that function  $\xi^{ref}(\mathbf{q}^{ref}, \dot{\mathbf{q}}^{ref})$  depends on the desired trajectory and that the trajectory should be modified if system is in contact with environment in order to maintain desired interaction. For such a behavior of the system (1) the desired manifold (3) should be changed to include the environmental interaction control. In addition, while in contact with the environment motion system is required to modify its trajectory in order to control interaction between system and environment. Assume that the overall external force  $\mathbf{F}_{ext} = \mathbf{F}_d + \mathbf{g}_{ij}$  consists of the disturbance  $\mathbf{F}_d$  that should be rejected by the system controller and the interaction force

between system and environment  $\mathbf{g}_{ij}(\mathbf{q}, \mathbf{q}_e)$  that should be maintained. One possible structure that includes both requirements may be selected as in (13)

$$\begin{aligned} S_{qg} &= \{\mathbf{q}, \dot{\mathbf{q}} : \xi(\mathbf{q}, \dot{\mathbf{q}}) - \xi^{ref} - \vartheta(\Delta\mathbf{g}_{ij}) + \Gamma\mathbf{g}_{ij} = \sigma = \mathbf{0}\} \\ \xi^{ref}(\mathbf{q}^{ref}, \dot{\mathbf{q}}^{ref}) &= \mathbf{C}\mathbf{q}^{ref} + \mathbf{Q}\dot{\mathbf{q}}^{ref}; \xi(\mathbf{q}, \dot{\mathbf{q}}) = \mathbf{C}\mathbf{q} + \mathbf{Q}\dot{\mathbf{q}} \\ \mathbf{g}_{ij} &= \begin{cases} \mathbf{g}_{ij}(\mathbf{q}, \dot{\mathbf{q}}, \mathbf{q}_e, \dot{\mathbf{q}}_e) & \text{when in contact} \\ 0 & \text{without contact} \end{cases} \end{aligned} \quad (9)$$

The interaction control input  $\vartheta(\Delta\mathbf{g}_{ij})$  should be determined the same way as the force control to maintain stability of system motion in manifold  $S_F = \{\mathbf{q}, \dot{\mathbf{q}} : \mathbf{g}_{ij}(\mathbf{q}, \dot{\mathbf{q}}) - \mathbf{g}_{ij}^{ref}(t) = \sigma_F = \mathbf{0}\}$ . The only change is reflected in the calculation of the distance from manifold. If systems are not in interaction  $\vartheta_i(\Delta\mathbf{g}_{ij})$  should have zero value. For system (1), the sliding mode motion in the manifold (9) results in

$$\xi(\mathbf{q}, \dot{\mathbf{q}}) - \xi^{ref} = \vartheta(\Delta\mathbf{g}_{ij}) + \Gamma\mathbf{g}_{ij} \quad (10)$$

Note that either, motion of the system or the environment can be modified in order to attain desired interaction and that interaction may represent a real or virtual force. Motion of systems in interaction is treated here the same way as force control. Actually, not only that the concept is the same but the structure of the controller remains the same. The only difference is in the selection of the interaction term and its measurement or estimation.

#### F. Extension to the General Systems in Interactions

In the situation depicted above control modifies the behavior of only one of the systems in interaction while motion of the other system is treated as disturbance. In motion control of particular interest could be a control under which systems maintain desired functional relation (for example bilateral control or cooperating robots etc.). In such cases control should maintain a functional relation by acting on all of the subsystems. Similar situation is examined in so-called "function control" framework [10,11,12,13], where the notion of system role "description on the requirement from a user to a robot" and its representation by "elementary functions" defined as "a minimum component of a system role" is discussed.

Assume a set of  $n$  single dof motion systems each represented by  $S_i : m_i(q_i)\ddot{q}_i + n_i(q_i, \dot{q}_i, t) = f_i - f_{iext}$   $i = 1, 2, \dots, n$  or in vector form  $\mathbf{S} : \mathbf{M}(\mathbf{q})\ddot{\mathbf{q}} + \mathbf{N}(\mathbf{q}, \dot{\mathbf{q}}, t) = \mathbf{B}\mathbf{F} - \mathbf{d}_\Sigma$ ,  $rank\mathbf{B} = rank\mathbf{M} = n$   $\mathbf{q} \in \mathfrak{R}^{n \times 1}$ , vectors  $\mathbf{N}, \mathbf{d}_\Sigma$  satisfy matching conditions. Assume also that required role  $\Phi \in \mathfrak{R}^{n \times 1}$  of the system  $\mathbf{S}$  may be represented as a set of smooth linearly independent functions  $\phi_1(\mathbf{q}), \phi_2(\mathbf{q}), \dots, \phi_n(\mathbf{q})$  and role vector can be defined as  $\Phi^T = [\phi_1(\mathbf{q}) \dots \phi_n(\mathbf{q})]$ . Consider problem of designing

control for system  $\mathbf{S} : \mathbf{M}(\mathbf{q})\ddot{\mathbf{q}} + \mathbf{N}(\mathbf{q}, \dot{\mathbf{q}}, t) = \mathbf{B}\mathbf{F} - \mathbf{d}_\Sigma$  such that role vector  $\Phi \in \mathfrak{R}^{n \times 1}$  tracks its smooth reference  $\Phi^{ref} \in \mathfrak{R}^{n \times 1}$ . The change of the function vector may be found as  $\dot{\Phi} = \left[ \frac{\partial \Phi}{\partial \mathbf{q}} \right] \dot{\mathbf{q}} = \mathbf{J}_\Phi \dot{\mathbf{q}}$  with  $\mathbf{J}_\Phi = \left[ \frac{\partial \Phi}{\partial \mathbf{q}} \right]$ , and its second derivative can be determine as

$$\ddot{\Phi} = \mathbf{F}_\Phi + \mathbf{d}_\Phi, \quad (11)$$

$$\mathbf{F}_\Phi = \mathbf{J}_\Phi \mathbf{M}^{-1} \mathbf{B} \mathbf{F}, \mathbf{d}_\Phi = \mathbf{J}_\Phi \mathbf{M}^{-1} (-\mathbf{N}(\mathbf{q}, \dot{\mathbf{q}}, t) - \mathbf{d}_\Sigma) + \dot{\mathbf{J}}_\Phi \dot{\mathbf{q}}$$

From (11) one can find control  $\mathbf{F}_\Phi^*$  such that vector  $\Phi$  tracks its reference  $\Phi^{ref}$ . Assuming  $(\mathbf{J}_\Phi \mathbf{M}^{-1} \mathbf{B})^{-1}$  exists original control can be found as  $\mathbf{F} = (\mathbf{J}_\Phi \mathbf{M}^{-1} \mathbf{B})^{-1} \mathbf{F}_\Phi^*$ .

In the SMC framework the sliding mode manifold  $\sigma_\Phi \in \mathfrak{R}^n$  should be represented in a similar way as in (3). Let this manifold be defined as

$$S_\Phi = \{(\mathbf{q}, \dot{\mathbf{q}}) : \xi_\Phi(\Phi, \dot{\Phi}) - \xi_\Phi^{ref}(\Phi^{ref}, \dot{\Phi}^{ref}) = \sigma_\Phi = 0\} \quad (12)$$

By introducing  $\left[ \frac{\partial \xi_\Phi}{\partial \Phi} \right] = \mathbf{Q}_\Phi$  and  $\left[ \frac{\partial \xi_\Phi}{\partial \dot{\Phi}} \right] = \mathbf{C}_\Phi$  projection of the system motion on manifold  $S_\Phi$ , can be expressed as

$$\frac{d\sigma_\Phi}{dt} = \mathbf{F}_{\sigma\Phi} + \mathbf{d}_{\sigma\Phi}. \quad (13)$$

where  $\mathbf{F}_{\sigma\Phi} = \mathbf{Q}_\Phi (\mathbf{J}_\Phi \mathbf{M}^{-1} \mathbf{B}) \mathbf{F}$  and  $\mathbf{d}_{\sigma\Phi} = \mathbf{d}_\Sigma + \mathbf{C}_\Phi \dot{\Phi} - \dot{\xi}_\Phi^{ref}$  and what represents a virtual plant described by n first order systems of the form  $\dot{\sigma}_{\Phi_i} = F_{\Phi_i} + \hat{d}_{\Phi_i}$   $i = 1, \dots, n$  for which design of control  $F_{\sigma\Phi_i}$  is straightforward. If  $(\mathbf{Q}_\Phi \mathbf{J}_\Phi \mathbf{M}^{-1} \mathbf{B})^{-1}$  exists then inverse transformation  $\mathbf{F} = (\mathbf{Q}_\Phi \mathbf{J}_\Phi \mathbf{M}^{-1} \mathbf{B})^{-1} \mathbf{F}_{\sigma\Phi}$  gives control in the original state space. Sufficient conditions for having unique solution for control  $\mathbf{F}$  is  $rank(\mathbf{Q}_\Phi \mathbf{J}_\Phi) = n$ .

### III. APPLICATION EXAMPLES

Bilateral control is a specific arrangement of the master-slave system in which slave is required to track master's position as dictated by operator and the force of interaction with environment on the slave side is to be transferred to the master as a force opposing its motion. Transparency is crucial to any bilateral controller as much as the stability of the overall system is [14, 15, 16, 17].

Assume two single dof mechanical systems defined by  $m_{ii}\ddot{x}_i + n_i(x_i, \dot{x}_i) = F_i - F_{exti}$   $i = m, s$  one of the acting as a master system and other one as a slave system as depicted in Fig. 1.

Functional relation between master and slave systems assuming position and force scaling requires that the error in position tracking  $\varepsilon_x = x_m - \alpha x_s$  and the error in force tracking  $\varepsilon_F = F_m + \beta F_s$  are nullified [10]. In [7] for single DOF identical master and slave systems, first by applying

disturbance feedback master and slave subsystems are reduced to double integrator plants  $\ddot{x}_i = F_i, i = m, s$ . Selection of controls for double integrator plants  $\ddot{\varepsilon}_x = \ddot{x}_m - \ddot{x}_s = F_x$  and  $\ddot{\varepsilon}_F = \ddot{x}_m + \ddot{x}_s = F_F$  is simple (for example PD controller with acceleration feed forward term guaranties stability and desired dynamics). Assume  $F_x^*$  and  $F_F^*$  are selected. The real control inputs are then obtained as  $F_m = \frac{1}{2}(F_x + F_F)$  and  $F_s = \frac{1}{2}(F_F - F_x)$ . The result can be extended to systems with scaling between master and slave side and to multilateral control.

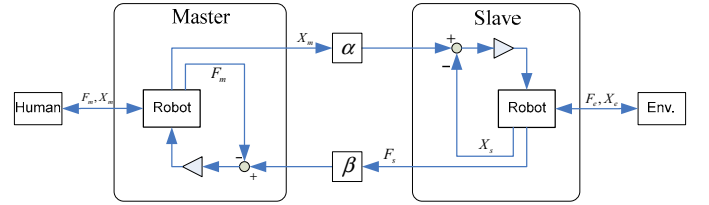


Fig. 1. General structure of bilateral control system

In bilateral control system consisting of functionally related master and slave subsystems SMC manifold should be selected as an intersection of the manifold defining the position tracking and manifold defining force tracking [18].

$$S_x = \{(x_m, x_s) : \xi_m(x_m, \dot{x}_m) - \xi_s(x_s, \dot{x}_s) = \sigma_x = 0\} \\ S_F = \{(x_m, x_s) : F_m(x_m, \dot{x}_m) + F_{sext}(x_s, \dot{x}_s) = \sigma_F = 0\} \quad (14)$$

Taking into account the human operator impedance  $F_m = Z_m \dot{x}_m = C_m \dot{x}_m + Q_m x_m$ , the master side force control can be formulated as maintaining SMC on manifold  $S_F = \{(x_m, x_s) : (C_m \dot{x}_m + Q_m x_m) + F_{sext} = \sigma_F = 0\}$  and the bilateral control is achieved on the intersection of the above manifolds:

$$S_B = \{(x_m, \dot{x}_m, x_s, \dot{x}_s) : S_x \cap S_F, \sigma_x = 0 \cap \sigma_F = 0\} \quad (15)$$

In the above formulation the coefficients  $Q_m$  and  $C_m$  can be selected in such a way that impedance perceived by the human operator is *shaped*. By using a bilateral controller and impedance shaping the task space impedance can be altered in a way that operator perceives manipulation task as in macro scale.

By defining errors  $\varepsilon_x = x_m - \alpha x_s$  and  $\varepsilon_F = x_m + x_s$  the sliding mode manifold (22) can then be expressed as  $S_x = \{(x_m, x_s) : Q_m \dot{\varepsilon}_x + C_m \varepsilon_x = \sigma_x = 0\}$  for position control and as  $S_F = \{(x_m, x_s) : (Q_m \dot{\varepsilon}_F + C_m \varepsilon_F) - ((Q_m \dot{x}_s + C_m x_s) - \beta F_{sext}(x_s, \dot{x}_s)) = \sigma_F = 0\}$  for force control. Now projection of the system motion in the selected manifolds can be expressed as

$$\dot{\sigma}_x = f_x - d_x \\ \dot{\sigma}_F = f_F - d_F \quad (16)$$

where  $f_x = Q_x(f_m/m_m - \alpha f_s/m_s)$ ,  $f_F = Q_m(f_m/m_m + \beta f_s/m_s)$ ,  
 $d_x = (n_m/m_m - \alpha(n_s + g_{sext})/m_s) + C_x \dot{\epsilon}_x$ , and  
 $d_F = (n_m/m_m + (n_s + g_{sext})/m_s) + ((Q_m \ddot{x}_s + C_m \dot{x}_s) - \dot{g}_{sext})$ .  
 Stability of solution  $\sigma_x = 0$  and  $\sigma_F = 0$  will guaranty the fulfillment of the requirements for bilateral system. Assume the control inputs for (15) are master and slave systems can then be determined from  $f_x^*$  and  $f_F^*$  then actual control inputs for master and slave systems are

$$\begin{aligned} F_m &= \frac{m_m}{\alpha + \beta} \left( \frac{\beta}{Q_x} f_x^* + \frac{\alpha}{Q_m} f_F^* \right) \\ F_s &= \frac{m_s}{\alpha + \beta} \left( \frac{1}{Q_m} f_F^* - \frac{1}{Q_x} f_x^* \right) \end{aligned} \quad (17)$$

For verification of the proposed approach experimental system consisting of: x-y parallel manipulator with Faulhaber 2642 012 CR series a motors as a master side device and the Cartesian linear microstage with PI M-232.17 actuators in x and y directions as a slave device. The dSPACE® 1103 real-time controller with 100  $\mu$ sec measurement sampling rate and a 1 ms control output sampling rate is used. Structure of the overall system is depicted in Fig. 2. In the system the position is scaled by factor  $x_m = (20/3)x_s$ . The force is not scaled. The structure of the master side manipulator with its force diagram is depicted in Fig. 2. The actuation motors are placed in P<sub>1</sub> and P<sub>5</sub>.

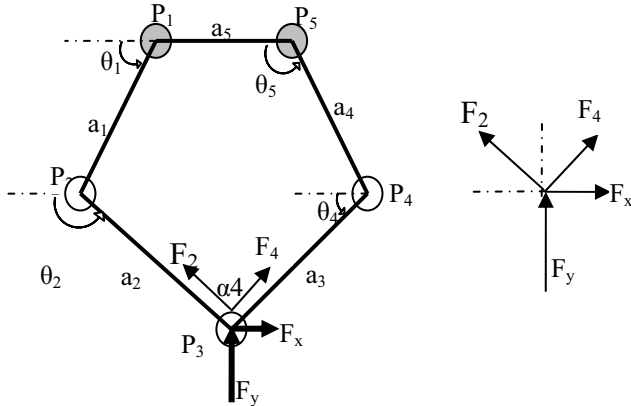


Fig. 2 The structure and the force diagram of the master side device.

The forces on the slave system are decoupled (due to the kinematical structure). On the master side the relation between forces  $F_x, F_y$  and torques is nonlinear and is defined as:

$$F_2 = \frac{F_x + F_y \cot \theta_4}{\sin \theta_2 \cot \theta_4 - \cos \theta_2}, \quad F_4 = \frac{F_y - F_2 \sin \theta_2}{\sin \theta_4} \quad (18)$$

$$T_1 = F_2 a_1 \sin(\theta_2 - \theta_1), \quad T_5 = -F_4 a_4 \sin(\theta_5 - \theta_4) \quad (19)$$

The controller on the master side (force control) and on the slave side (position tracking and force limit) are designed as presented in section 2 and all have the form

$$u(k) = (u(k-1) - \Gamma(\sigma(k) - \sigma(k-1))) - \Lambda d\sigma(k) \quad (20)$$

where  $\Gamma, \Lambda, d$  are positive constants. The sliding manifolds are selected as  $(Q_m \dot{\epsilon}_F + C_m \epsilon_F) - ((Q_m \dot{x}_s + C_m x_s) - \beta F_s) = \sigma_F$ ,  $\epsilon_F = x_m + x_s$  for master side force control and  $Q_x \dot{\epsilon}_x + C_x \epsilon_x = \sigma_x$ ,  $\epsilon_x = x_m - \alpha x_s$ , for position tracking.

The overall experimental set-up and the control are realized as depicted in Fig. 3.

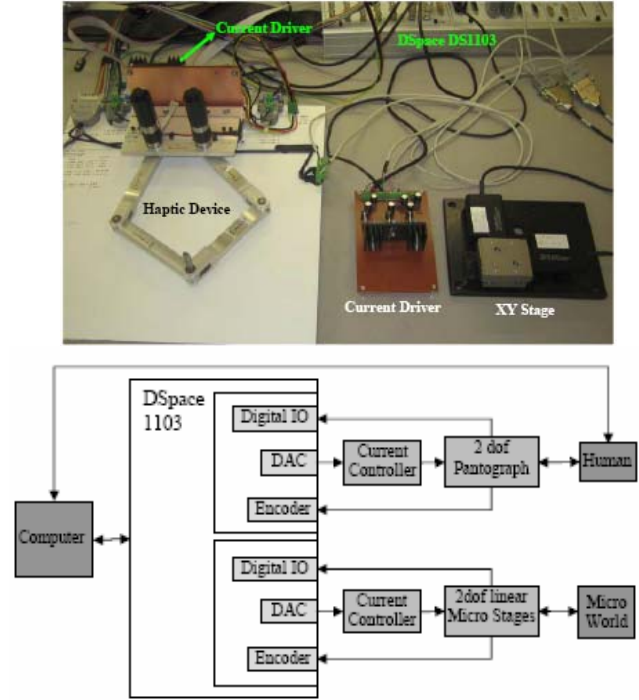


Fig. 3 Experimental system and the overall structure of the control system

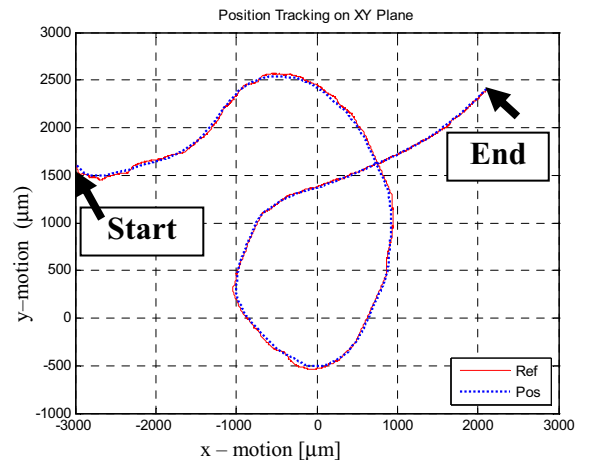


Fig. 4 Position tracking in the master and slave systems (a) and master and slave motion in x-y plane

The x-y motion of the master and slave systems is depicted in Fig. 4. The starting and ending points are marked in order to make comparison with time diagram of the same motion depicted in Fig. 4.b. The real slave position and the scaled master position are depicted in order to be able to compare

them more clearly. The diagram shows tracking capability of the system. The position vs. time for the motion shown in Fig. 4 is depicted in Fig. 5.a. The corresponding forces are depicted in Fig. 5.b.

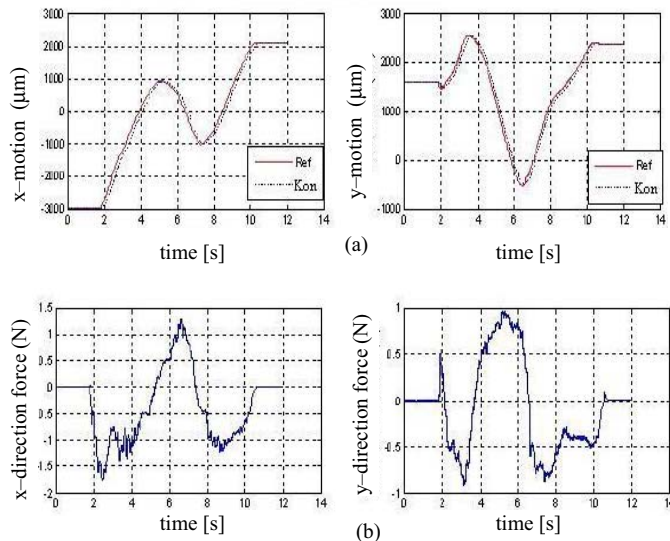


Fig. 5 Position vs. time (a) and forces vs. time for motion depicted in Fig. 4

#### IV. CONCLUSIONS

It has been shown that motion control tasks can be formulated as a requirement to enforce stability in selected manifold in state space of the system. The approach is applicable for systems with and without contact with environment that leads to unified formulation of the control tasks. In addition it has been shown that the same approach can be used in controlling systems in interaction and establishing desired functional relation between systems and allowing application of the same framework to bilateral and “function control” systems. The equivalency of the closed loop behavior under selected control with acceleration control is shown.

#### ACKNOWLEDGMENT

The presented work had been partially supported from TÜBİTAK project 104E024.

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