OPTIMAL SELECTION AND PLACEMENT OF MILITARY DEFENSE SYSTEMS

by

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ABSTRACT

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The problem is based on a variety of terrorist organizations acting over a region of three different governments, political entities. Due to different entities ruling on different sides of the border, there is no consensus on how to eliminate terrorists. In the region, threats are small groups, they may use guerrilla tactics, direct attacks, tribute collections. The threats are mobile, and they prefer ambushes, sabotage, hit and run. The hardness of cooperation creates an abundance of space for organizations in such regions. Terrorist organizations benefit from technological and physical emptiness in regions closer to the border. That is why military organizations consider surveillance systems to support their weapon systems. The case is modeled using data collected on the ground as a MaxiSum model. The objective is to maximize the benefit of using surveillance and weapon systems. The geographical properties of the region are embedded. Model is constructed in such a way that geographical and geopolitical changes can be easily incorporated. Military procurement decisions are strategic level decisions and are made once a year on a general basis. Performance of the model is evaluated based on the requirements of the special case.

ÖZET

ASKERİ SAVUNMA SİSTEMLERİNİN OPTİMUM SEÇİMİ VE YERLEŞTİRİLMESİ

İBRAHİM ENES YAVAŞ

ENDÜSTRİ MÜHENDİSLİĞİ YÜKSEK LİSANS TEZİ, ARALIK 2024

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Sorun, üç farklı hükümetin, siyasi varlığın bulunduğu bir bölgede faaliyet gösteren çeşitli terör örgütlerine dayanmaktadır. Sınırın farklı taraflarında farklı varlıklar hüküm sürdüğü için teröristlerin nasıl ortadan kaldırılacağı konusunda fikir birliği yoktur. Bölgede tehditler küçük gruplardır, gerilla taktiklerini, doğrudan saldırıları, haraç toplama yöntemlerini kullanabilirler. Tehditler hareketlidir ve pusu, sabotaj, vur-kaç yapmayı tercih ederler. İşbirliğinin zorluğu, bu tür bölgelerde örgütler için bol miktarda alan yaratır. Terörist örgütler, sınıra daha yakın bölgelerdeki teknolojik ve fiziksel boşluklardan faydalanır. Bu nedenle askeri örgütler, silah sistemlerini desteklemek için gözetleme sistemlerini değerlendirirler. Durum, sahada toplanan veriler kullanılarak MaxiSum modeli olarak matematiksel ifade edilmiştir. Amaç, gözetleme ve silah sistemlerinin kullanılmasından elde edilen faydayı en üst düzeye çıkarmaktır. Bölgenin coğrafi özellikleri modele gömülüdür. Model, yeni coğrafi ve jeopolitik özelliklerin kolayca dahil edilebileceği şekilde oluşturulmuştur. Askeri tedarik kararları stratejik düzeydeki kararlardır ve genel olarak yılda bir kez verilir. Modelin performansı, mekânsal durumun gereksinimlerine göre değerlendirilmiştir.

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...to peace at home

and

peace on earth...

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1 INTRODUCTION

The defense industry is growing and getting more critical. In Middle East and Africa, increasing terrorist activities near city centers and border security issues have increased demand for simplest defense industry products and caused an increase in prices of such products. Additionally, political instability in the Middle East, Africa, Black Sea and Southeast Asia has led many countries to stock weaponry, acquire new defense technologies and search for new suppliers. With this increase in demand and the global supply chain crisis that started after the COVID-19 pandemic, the prices of normally cheap raw materials, semi-conductors, connectors, sensors, cards and explosives have been inflated considerably. Simultaneously, world military expenditure has been increasing steadily since 2014 and it has seen the steepest increase in 2023 (SIPRI, 2024). The growth is expected to continue for the next 20 years. Billions of dollars' worth investment is being included into the strategic plans due to wars between Russia and Ukraine, Israel and Palestine. The agendas are being updated to include more military investments by governments, to satisfy internal and external security needs. A total of 2443 billion dollars expenditure realized in 2023, the number is the highest ever recorded. Additionally, the North Atlantic Treaty Organization (NATO,2024) announced that all members will be committing at least 2% of their gross domestic product starting from 2025. The latest developments support the arguments about the continuance of the increasing military expenditure trend. Considering these, procurement and investment decisions will be at the center of military planning for the next 20 years and these decisions will be among the most important decisions for many nations. Evaluating the current situation, the motivation of this thesis is to develop a model for the optimal selection and placement of available military defense systems considering geographic, economic and social properties of acquiring nation.

In particular, we consider a weighted coverage problem over a predetermined area, taken from real land, whose dimensions are 50 kilometers both in length and width. This area has a special situation, it is a preferred field of activity of most active terrorist groups due to its unique position. The area has borders in three different countries, one to the north, one to the southeast and one to the west. The borders provide easy escape routes for terrorists. We propose a model to solve the problem of finding an optimal selection and placement of defense technologies. The selection and placement decisions are made by the model considering threat types, budget, selected technology's effective range, geographic importance of regions to government, and expected attacks according to past attack patterns.

The problem is based on a variety of terrorist organizations acting over a region of three different governments. Due to different entities ruling different sides of the border, it is hard to have a consensus to eliminate terrorists in case of such an existence. In these kinds of regions, the most frequent threats are small groups using guerrilla tactics. The threats are mobile and prefer guerrilla warfare which may include ambushes, sabotage, hit and run. Against these kinds of threats, increasing military existence, weaponry and surveillance have been proven effective. Along with this, increased political cooperation guarantees finalization of the fight against terrorism. However, the political difference and hardness of cooperation exist in every region of the world. This lack of mutual understanding and cooperation creates an abundance of space for organizations in such regions. The terrorist organizations benefit from technological and physical emptiness in regions closer to border. Therefore, like governments, companies working in defense industry also need the solution to the defined problem, they have to come up with comprehensive solutions for such environments in order to make a difference for their customers.

The terrorists tend not to stop by harming only one side of the borders. They generate their means of living by harming surroundings, collecting tributes and disrupting government functions. To stop these kinds of organizations, as it is not realistic to make all bordering states cooperate, countries must focus on improving control over their land by decorating it with the right technology which will enable them to act proactively against attacks and overcome terrorism. With high tech equipment and well-trained personnel, the countries increase their chances to eliminate all terrorist elements.

However, the selection must be done so precisely that none of the selected equipment should be obsolete over the long term. Thus, there is the need for increased decision-making efficiency in military investments that will enable usage of high tech equipment to army personnel in a correctly distributed manner. In other words, countries dealing with terrorism need to depend on decision support mechanisms to make informed decisions about their military investments which will eventually help them control their borders proactively. To control the borders and to protect and enforce the military personnel conducting dangerous routine patrol duty, the investments consisting surveillance and weapons must ensure that every possible measure is taken and that is within the government's capacity to protect and navigate the personnel. This will be guaranteed by the usage of model designed and described in this work.

In the land used for the model construction, the borders, rural area and town centers are completely uncontrolled, unsupervised. Inability to comprehensively control the borders, reactive rather than proactive approaches, weak control mechanisms, lack of training, technology, and intelligence are causes of terrorism near borders (Gohel,1954). With emptiness created by lack of technology, the advances against terrorists cannot be finalized quickly, the terrorists reach borders easily as there is no information to navigate the army. Also, bureaucracy and rules of engagement stop forces at the borders where terrorists move freely. Army does not cross borders while tracking terrorists as they avoid any risk of retaliation from neighboring countries. Correct selection and placement of defense technologies is of utmost importance for the army to make informed decisions while responding to attacks or advancing over terrorist groups. In any case of defense against the terrorist organizations, the army can follow the terrorists up until the border of the other nation. This situation shows that reacting to attacks is not a feasible and sustainable way to prevent terrorism. Lack of synchronization and cooperation leads to delays in getting approvals to cross borders, crossing any border requires following long procedures. Thus, responding to attacks, in most cases, will not create any consequences as responding brigade should stop at the border. A cross-border operation to stop the terrorists is not possible, proactive measures must be taken, and terrorists must be detected even before entering the significant towns and institutions, and while inside the borders. The investment decisions to solve such problems are among strategic decisions. Strategic investments are needed for solving the terrorism issues near borders.

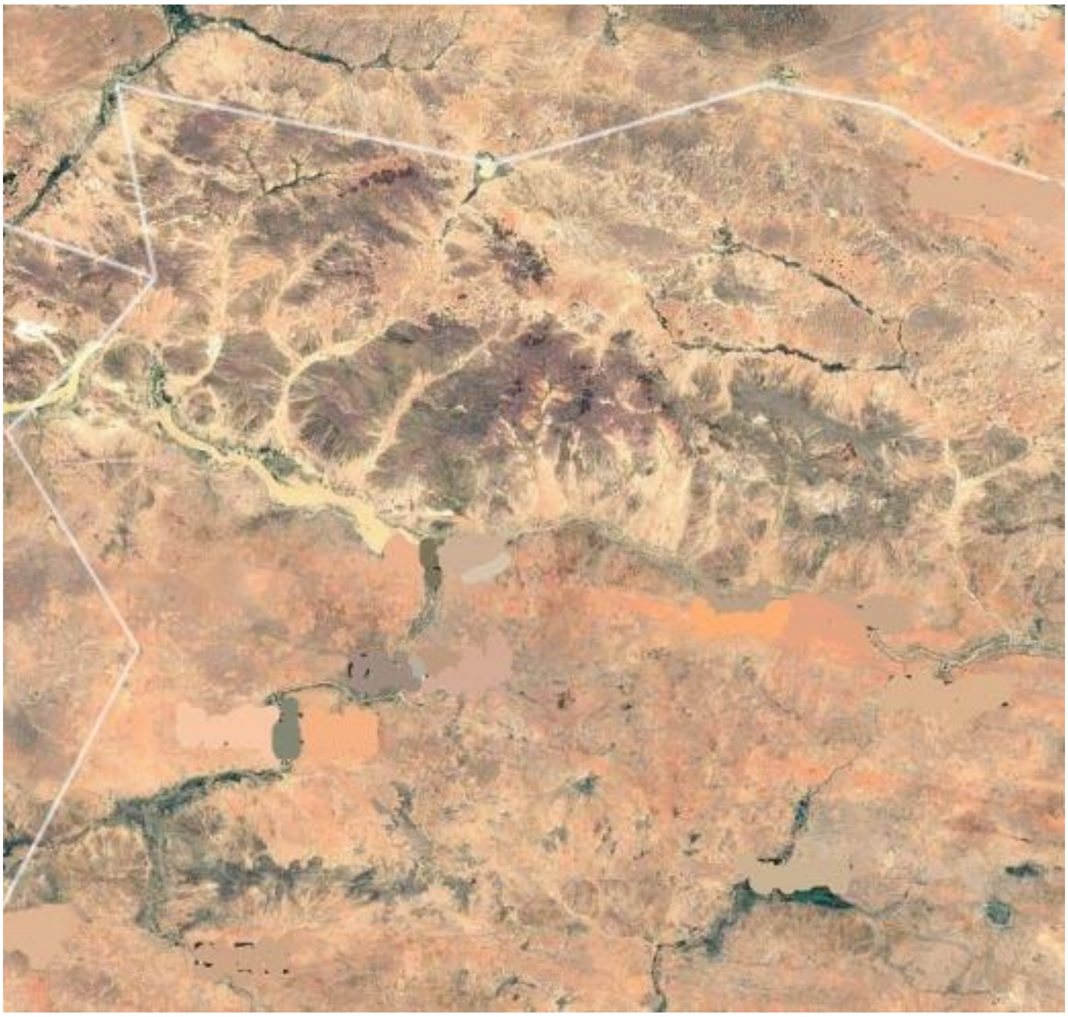

Figure 1.1 Border with neighboring countries, satellite image

Figure 1.1 shows an exemplary terrain next to the border area where terrorist activity is high. The terrain may be consisting of deserts, dirt roads, and green steps. The terrain types may change to include lakes, sea, and larger mountains. The change in terrain type affects the product portfolio that should be used to deal with terrorism. The region is a desert, rich in oil and gold, there is currently no military presence to secure the area from terrorists, the government is trying to set-up bases and surveillance systems to start securing the area before processing the resources. Military equipment and related surveillance locations will be determined. Borders with two bordering nations lay to the left and top of the map. There are small villages, consisting of around 100 people each, and few illegal dirt roads connecting the regional capital to bordering countries.

To secure borders and gold rich regions, the government targets procure and place surveillance and weapon systems. The procurement includes buying UAVs, missiles, machine guns, cameras, radars, radios, technology transfer agreements, long term integrated logistics support contracts, and advanced trainings. The systems and related services are expensive due to being military grade, meaning compliant to international regulations, encrypted, and costly to maintain. Considering utility against different types of threats, influence over time (shelf life), effective range, upgradeability and deterrence, the government aims to make an informed decision. In the foreseeable future, selecting and placing defense systems is standing out as the most important problem of the nation dealing with terrorism.

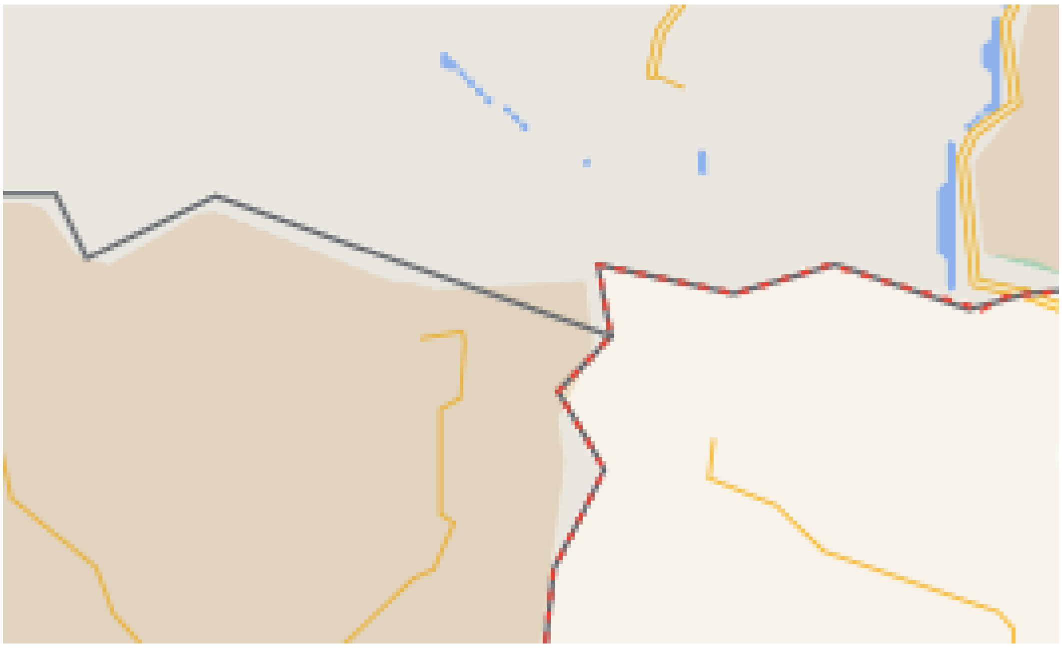

Figure 1.2 Border with neighboring countries

Figure 1.2 shows a bird-eye drawing of problematic borders represented with a satellite image in Figure 1.1. The flow of terrorists in or out of the borders is possible through dirt roads. Correct placement of any acquired system is a part of the problem for this government. The area's geographical conditions allow the use of radars and cameras without any loss of effective range. Thus, the installation of the acquired systems could be done considering the full coverage. During installation, benefit against certain attack types and costs are to be considered together. Different attack types require different technologies as counter measures. To solve such problems, the equipment selection must be done according to attack types and attack expectancies. Simulating a set of possible attacks and generating useful scenarios are good ways to visualize the situation. In the simulations, from terrorists' perspective, the attacks are created according to their benefits for terrorists. Using such support mechanisms to design a method for selection and placement of defense systems provide more mature and solid results. The problem discussed and detailed from different perspectives is a problem for both nations looking to finalize their defense industry investments and also for the companies trying to offer comprehensive solutions to their potential customers.

The solutions to the discussed problem require large budgets, most of the products cost more than millions of dollars per unit. The magnitude of the problem makes it a strategic level problem. Decisions to solve this type of military problems are strategic level decisions. Strategic level military decisions are made bi-yearly when frequent and only reviewed yearly to supervise. The decisions are then converted to military projects with schedules that are distributed over the years. End products of these projects are planned to be sustainable for at least 20 years coming.

The global military investments consist more than 2% of gross domestic product generated according to SIPRI (2024). In the problem that is discussed this thesis, an example is given, government described in the example spends more than 40% of its gross domestic product as military expenditures. This shows how much importance is attributed to this problem. The problem is designed as a Mixed Integer Linear Programming (MILP) model. The model is then solved using Gurobi solver at default settings except for hard drive usage parameter which is set in a way that will allow usage of hard drive when soft memory limit is reached. In the default settings of this solver, different threads are used for concurrent optimization. The first concurrent thread is devoted to dual simplex, the second through fourth to a single parallel barrier solve, and the fifth to primal simplex. Additional threads are devoted to the parallel barrier solve. Considering the costs and effects of military decisions over the years, this thesis offers a comprehensive solution to the general problem of military procurement process and the proposed model is largely customizable according to the needs of nations or suggesting companies. The results showed that an integrated comprehensive weapon and surveillance system selection and placement solution can be achieved in acceptable time while considering important parameters involved in military decision making. This is a novel way of evaluating investment decisions in military, it is a critical decision support tool. With this work, the thesis introduces a new problem that is a combination of knapsack problem, facility location problem and maximal covering location problem (MCLP).

2 RELATED LITERATURE

The problem studied in this thesis combines structures from the knapsack problem, facility location problems and maximal covering location problems. The problem resembles maximal covering location problems due to its objective. The problem also shares similarities with the knapsack problem considering the constraints and their integrality, a selection is made from a set or more than one set then they are used while satisfying certain constraints such as budget. These similarities position the problem as a special case of both, resulting a large body of available literature to be covered. However, there are quite a few focusing on military applications of maximal covering location problems and the studies mostly focus on certain aspects of problems such as placing a single item or selecting systems offering similar benefits. All focus on special cases of the problem faced in this case. Thus, variety of methods have been developed but mostly to address comparatively smaller issues. Therefore, the related literature on location-allocation problems, maximal covering location problems, and military optimization problems are investigated thoroughly. Table 2.1 lists a set of papers is deemed related to maximal coverage location problems and a part of those includes military applications, that share similar conditions, expectations and constraints.

Table 2.1: List of papers considered related

Authors	Year	Solution technique
Lin	1965	Mathematical programming, k-lenght heuristic
Toregas, Swain, ReVelle, Bergman	1971	Mathematical programming applied to set covering problem
Church,Revelle	1974	Mathematical programming, greedy adding with substitution, branch and bound
Shih	1979	Branch and bound
Megiddo, Zemel, Hakimi	1983	Combination of algorithms

Schick	1992	Mathematical programming
Dell, Ewing, Tarantino	2008	Mathematical programming
Zarandi, Davari, Sisakht	2011	Genetic algorithm
Akkuş, Sarıçiçek	2015	ÉLECTRE
Yıldırım	2016	Mathematical programming, genetic algorithm
Yakıcı	2016	Mathematical programming
Razi, Karataş, Günal	2016	Mathematical programming, simulation
Çetinkaya, Haffar	2018	Mathematical programming
Xiong, Wang, Jiang	2019	Multiobjective genetic algorithm based on decomposition(MOEA/D)
Lai	2019	Simplified swarm optimization and AHP
Chauhan, Unnikrishnan, Figliozzi	2019	Novel greedy search heuristic with 3 stages
Wang, Chen, Liu, Yang	2019	Combination of algorithms
Karakaş, Erişkin	2021	Linearization, mathematical programming
Yakovlev, Wojciechowski, Podzeha, Illiashenko, Yakovleva	2023	Broyden-Fletcher-Goldfarb-Shanno algorithm, multistart algorithm
Chobar, Bigdeli, Chamami	2024	Meta heuristics, grey wolf optimizer(GWO)

The 0-1 knapsack problem (KP) is an NP-hard combinatorial optimization problem. The problem has many applications in resource allocation, project selection and investment decisions. Three main methods are preferred in solving 0-1 KPs, exact algorithms, approximate algorithms and heuristics algorithms. The advantage of using exact algorithms is to find the optimal solution. Applications of dynamic programming (Toth,1979) and branch-and-bound (Shih,1979) are widely employed. Despite the wide range of applications, exact algorithms tend to stop performing after reaching physical computational constraints as the solution space grows exponentially.

The literature on location theory includes papers and books overviewing efforts to optimize decision support models and laying out formulations along with their cases and assumptions . ReVelle, Eiselt and Daskin (2008) present a collection on discrete location problems. In the paper, recent studies are classified, and their contribution is discussed. Berman, Drezner, and Krass (2010) provides models and an overview of recent studies on Maximal Covering Location Problem, gradual coverage, cooperative coverage and variable radius. Daskin (2013) presents an introduction to models discussed in the field of covering, maximum covering location problem, along with center and median problems.

2.1 Facility Location Problem

Hekmatfar and Pishvaee (2009) introduces approaches in solving facility location problems for hub-spoke networks. The facility location theory and hub-spoke applications are presented with their mathematical representations. The paper starts with types of facility location problems, factors influencing decisions and continues with mathematical models used to optimize the decisions. The authors also discuss algorithms starting from classical methods to more advanced heuristic approaches. The hub covering location models share similar constraints with military location selection problems, but the models do not focus on selecting the system type or service type, the focus is mostly on solving the issue for a single case where most parameters are deterministic. The methods used in solving the problems are too general to be incorporated.

Zarinbal (2009) explains the usage of distance functions in location problems. To understand how far objects are from systems, distance functions are used. The analytic distance functions like Manhattan metrics are explained along with continuous distance models. Continuous distance models deal with geometrical representations, such as Euclidean distances. The distance functions are important in parameter calculation as the distance definition plays a vital role in determining the solution to facility location problems. Distance functions with different definitions create results that are incomparable. Each definition has its own domain, advantages and disadvantages to be considered during selection. At every time defining the distance functions, the semantic of the problem must be considered, along with the distance characteristic, and usage domain. The author also explains circle coverages.

Çetinkaya and Haffar (2018) present a new approach to military logistics planning by creating a risk-based location-allocation model. The risk is defined based on a risk arising from placing products to a region. Their objective is to minimize total transportation and setup cost along with the risk they defined. The paper eventual goal is to develop a location-allocation model specifically tailored for weapon logistics, i.e. carrying weaponry smoothly in time of urgent need. Focusing on minimizing risks associated with the storage and distribution of weapons, authors propose a model for the placement of weapon storage facilities considering vulnerability to threats, logistics, and sustainability. The proposed mathematical model integrates risk assessment into the decision-making process, aiming to minimize potential losses and enhance operational efficiency in weapon logistics. Incorporating risk factor into the mathematical model provides insights into better planning for military supply chains, emphasizing the importance of robust facility placement to mitigate threats and ensure readiness. Authors suggest that the paper contributes to the field of military logistics by introducing a novel method that addresses logistical needs and security concerns.

Chyh-Ming Lai (2019) present a hybrid approach combining simplified swarm optimization (SSO) with Analytic Hierarchy Process (AHP) to solve the capacitated military logistic depot location problem. The study investigates optimization of the placement of logistic depots considering capacity, logistic costs, and strategic military objectives. The objective maximizes the average utility of all requisitioned buildings. SSO starts by initializing a set of feasible solutions and according to predefined fitness function selects the current best and global best solution. Initialization is followed by generation of random numbers, according to a related stepwise function the numbers are checked and evaluated. Evaluation may result in keeping the solutions, changing the current best or changing the global best. Generation and evaluation iterations continue until predetermined termination criteria are met. SSO is employed to explore the solution space, while AHP facilitates the decision-making process by structuring criteria and evaluating alternatives based on their relative importance. By integrating the two methods, the researcher aims to provide a framework that balances operational efficiency and strategic goals for military logistics planning. The author suggests that the paper contributes to enhancing decision support systems in military logistics through a new integration of SSO and AHP to optimization techniques.

The paper by Chauhan, Unnikrishnan, and Figliozzi (2019) addresses a complex optimization problem that integrates traditional facility location strategies with the capabilities and constraints of drones. The objective of this problem is determining optimal location for certain facilities while achieving maximum coverage. Each facility has a limit of demand points it can serve and the service providing device has a limited range, i.e. coverage radius. Demand points, regions, are customers in the case and service providers, systems, are drones. The solution method is assumed to be used in service delivery setups where there is a complexity caused by range limitation. The paper contributes to field by addressing range constraints into facility location problems and offering heuristic approaches to solve them. The employed heuristic approaches are greedy approaches. Similar to earlier discussed greedy adding algorithms with substitution or Ignizio heuristics. Authors call the heuristic the three-stage heuristic(3SH). The 3SH solves the problem in three steps. First, algorithm solves a facility location problem and determines the facilities to be located and the demand points to be assigned to each facility. Second, knapsack problems are solved by assigning drones to facilities and demand points to drones. Third, substitutions applied to improve the solution.

The paper by Chobar, Bigdeli, and Chamami (2024) addresses previously solved military equipment warehouse location problem. Considering demand, authours designed a model to satisfy field requirements by using transfer hubs and they have found optimal locations by using both exact and heuristic methods. For their calculations, authors preferred using GAMS software. In order to achieve better computation times, after validating their model, authors offered heuristic methods. One of the methods is multi objective grey wolf optimizer (MOGWO), and authors preferred implementation using MATLAB software. The authors contribute to the literature with the introduction of their problem and the way authors used existing methods to solve their problem by customizing them is an example of how the existing methods can be applied to recent military decision-making problems.

2.2 Maximum Covering Location Problem

Fallah et al (2009) studies covering problems, customers receive service if distance between the customers and facilities is under certain range. In the problem, a system or service is selected with a certain range. The problem suggests that the customer can receive service by each facility that it is in the range of and there is no extra benefit of covering a location by more than one provider. This range value is called coverage distance or coverage radius.

Chuch and ReVelle (1974) modeled the maximization covering problem. The model is mathematically represented, and heuristic solution methods are discussed. They defined the problem as serving to people, one type of service, covering once is satisfactory and there is no extra benefit of covering more. The problem is similar to military services provided to regions, but only a single aspect of our problem is studied. Authors discuss the tradeoff related to not covering all the demand. The instance they used provided 90% coverage at half of the cost of a solution providing 100% coverage. In this kind of situation, it may be wiser to focus on other service areas where overall benefit may be increased much more. In the thesis, this overall picture is modeled, and the tradeoff is incorporated. Greedy Adding (GA) and Greedy Adding with Substitution (GAS) algorithms are discussed with their disadvantages. The algorithms can be applied to problems where the goal is to maximize coverage but they do not guarantee optimality. Chuch and ReVelle (1974) explains the similarity of replacing one facility at a time in the solution to achieve higher coverage to λ-optimum method for solving travelling salesman problem introduced by Lin (1965). Authors examine Ignizio heuristic, compared with GAS, the algorithm allocates systems to regions one at a time in a steepest ascent manner and employs a subroutine which replaces facilities in the solution which are no longer justified. Chuch and ReVelle (1974) also discussed Linear Programming approach, they ran their program on The Mathematical Programming System (MPS) on the IBM 360 Model 91 computer. Their objective was minimization of population uncovered. To solve the problem, a Fortran IV program was prepared to write the problem on a disc file. This disc file was prepared in such a manner that the MPS could access the problem file and perform the necessary linear programming algorithm. At the time, this was a great challenge to overcome. In the results of the runs, authors saw an 80-20 ratio of integral solutions to non-integral ones. Although they see little reason to work on the 20%, they suggested using Branch and Bound method to find integer solutions when the linear program terminates in fractional solutions. The linear programming solution provides a lower bound for the minimization problem. In the paper, the Branch and Bound method was applied to non-integer linear programming. By branching on a fractional variable, i.e., by setting it to either 1 or 0, two more linear programming problems are created. If the linear programming solution to each of these two problems has (0, 1), the solution that has the smaller value of the objective function, smaller uncovered population, is optimal. If the solution to the problem with the smaller value of the objective is fractional, additional branching and computations may be required.

According to their graphical representation, covering problems are divided into two as tree networks and general networks. Additionally, based on covering all or some demand points the problems are divided into two problems: total covering and partial covering problems. The total covering problem for emergency service locations is first modeled

modeled by Toregas, Swain, ReVelle and Bergman(1971). In the paper, authors first formulated the problem and solved the problem using a small dataset, along with comparison of the problem to the earlier initialized p-median problem. Up to the present time, many developments have occurred about total covering and partial covering problems in terms of solution techniques. Covering problems have many applications such as designing of switching circuits, data retrieving, assembly line balancing, airline staff scheduling, locating defend networks (at war), distributing products, warehouse location, locating of emergency service facility (Francis et al. 1992).

Megiddo, Zemel, and Hakimi (1983) address the Maximum Coverage Location Problem (MCLP). The study focuses on determining optimal locations for facilities to maximize the coverage of demand points or customers within a specified radius or service area. The work is done on the provider-requester network, facilities providing service to customers. The authors propose an algorithm to solve the defined problem, and their research shows the algorithm provides better computational complexity. Mathematical formulations and algorithms to tackle MCLP are compared considering required computational efforts. Authors consider factors such as facility capacities, demand weights, and distance decay functions while designing their algorithm. The research contributes to understanding how algorithms may improve computational complexity of exact optimization models. Overall, the paper provides insights into the theoretical foundations and computational approaches addressing the Maximum Coverage Location Problem.

Zarandi, Davari, and Sisakht address the Large-Scale Maximal Covering Location Problem (LSMCLP). This problem involves selecting a subset of locations for facilities to maximize the coverage of demand points within a specified distance or coverage radius. The objective is to maximize the number of demand points covered by a given number of facilities. The authors propose a genetic algorithm to efficiently solve LSMCLP instances of large scale. The genetic algorithm is a bio-inspired algorithm, after initialization the algorithm targets to have crossovers and mutations and searches the solution space in order to attain a global optimum. The algorithm has a selection phase for which autors preferred roulette wheel selection(RWS). RWS mimics a roulette wheel where the area of each section is proportional to the individual's fitness, and selection is made randomly, based on probabilities corresponding to each area. For reproduction, to achieve better solutions, authors define the set, $N(X)$, to be the set of solutions neighboring a solution, X. In each iteration, the next solution, Y, is generated from $N(X)$ by using a crossover, mutation or migration operator. While a crossover is carried out when two chromosomes are mated together to produce a new solution, a mutation is employed to make a diversification on a solution. Authors discuss the computational challenges associated with solving such large-scale problems and present computational results to demonstrate the effectiveness of their proposed method. The paper contributes to facility location theory by offering a practical approach to large scale MCLPs.

2.3 General Military Operations Research

In the thesis, along with a selection such as KP, there is another objective which is to decide where to install selected equipment, such problems are considered to have multiple objectives and widely discussed in the literature. Multi-objective optimization problems have been classified by a set of decision variables. Weapon selection and planning problems (WSPPs) include decision variables of weapon-type selection and weapon amount determination. The solution space is large and discontinuous, with non-convex Pareto front, the difficulty of problem increase. The paper of Xiong et al. (2019) solves the problem by using multi-objective evolutionary algorithm based on decomposition (MOEA/D). The proposed algorithm aims to overcome possible drawbacks of original MOEA/D with weighted sum approach for complex combinatorial problems.

McKean (1964) discusses defense planning, economizing defense choices, and cost-benefit analysis. The book explores different dimensions of military spending at management level; discusses the costs of defense, and its effect on nation's economy. The author addresses modeling military problems economically, applying cost-benefit analysis to create significant insights in defense, states the analysis helps to guide defense policy and to guide defense investments. Overall, the book suggests that for changing sizes of budgets, while correct decisions help to reach prosperity, incorrect ones carry to annihilation. The book also covers implications of decisions to guide decision makers and institutions to carefully determine the resource allocation, and to make investments based on analysis.

Karakaş, Yakıcı and Razi(2018) provide a focused overview of a wide variety of military location problems. The paper discusses mathematical models and optimization techniques to determine optimal locations for military facilities. The paper also covers classical and modern methods in optimization. They categorize location problems into five groups according to their types as logistics planning, infrastructure security and protection, sensor deployment, UAV location planning, search and rescue operations planning. The problems are also categorized according to solution methodologies such as Tabu Search, Simulated Annealing, Genetic Algorithm, Ant Colony Optimization, Greedy Heuristics, Gready Randomized Adaptive Search Procedure(GRASP), other heuristic methods, decomposition, and mathematical programming. The importance of terrain, threat proximity and type, reachability and logistics is emphasized.

Xiong, Wang, and Jiang (2019) address weapon selection problems and planning problems using Multi-Objective Evolutionary Algorithm based on Decomposition (MOEA/D) with distance-based divided neighborhoods. The paper focus on optimizing selection and deployment of weapons for each time point in military operations, considering a set of objectives like effectiveness, cost, and logistical feasibility. MOEA/D with distance-based divided neighborhoods is employed and helps to efficiently explore the solution space by dividing it into smaller neighborhoods based on distance metrics, enhancing the algorithm's convergence and diversity of solutions. The research aims to provide decision-makers with a set of Pareto-optimal solutions that balance conflicting objectives in weapon selection and planning, thereby the paper may be considered strategic decision support mechanism. Overall, authors claim that the paper contributes to advancing optimization techniques in military logistics and planning through innovative application of evolutionary algorithms.

Melese, Richter, and Solomon (2019) address the theoretical and practical uses of Military Cost Benefit Analysis (MCBA), the paper offers a comprehensive overview of MCBA applications. The paper presents utilization of MCBA for evaluating the costs and benefits of investments of military organization, including equipment supply, infrastructure, and operational investments. Using such decision support mechanisms' criticality considering both measurable and immeasurable factors including strategic value, risk, and long-term effects can be understood by examining the book. Real life cases in multi-objective scenarios, weapon procurement and field examples are explained to create a better understanding of MCBA usage in real-world military decision-making.

The book is useful for people needing introductory information on military decision support mechanisms. Policymakers, analysts, and military planners aiming to use reachable tools for taking cost effective decisions may benefit from the book.

Hitch and McKean (1960) published a book after mass weaponization of the cold war. The global race of acquiring lethal weaponry and defense mechanisms share a similarity with today's global geopolitical status. To understand how events unfolded back then and how governments should react to today's dynamics in planning military investment the book offers an in depth analysis. The book also discusses cost-benefit analysis in defense policy. The authors argue that defense policy should consider direct costs such as acquisition and operation of systems and results caused by the usage of such systems. Authors explain the deterrent effect of having weaponry and high technology against enemies.

Karataş and Erişkin (2021) present capacitated gradual and cooperative minimal covering location problem with distance constraints (cGC-MCLPD). It allows variable coverage radius and capacity of facilities to assess the effect of facility size on the network performance. The authors first formulate an Integer Non-Linear Program (INLP), develop two linear approximations that can be made arbitrarily close to the original nonlinear model, formulate an exact linearization of the INLP. Authors believe they introduced three novel properties to existing literature. First, paper extends the existing literature by allowing the effect of partial demand coverage such that the amount of coverage is represented with a non-increasing decay function. Second, their suggested method incorporates a cooperative coverage concept in which collective contributions of individual facilities are aggregated. Third, their systems or facilities do not have a fixed radius, they incorporated changing radii. Overall, authors suggest that the introduced solution method provides a balance between solution quality and computational speed.

This thesis contributes to literature by introducing a novel problem which considers new aspects that have not been considered and combines two main problems of the military investments, selection and placement. In the literature, there are no similar works. The works in the literature covered selection of defense systems considering a project timeline, depot location problems under different scenarios, deployment location problems, UAV hub location problems, border monitoring hub location problems, army base location problems, radar and other sensory systems placement problems.

3 PROBLEM DESCRIPTION

The problem that is the subject of this thesis is a weighted coverage problem in which the objective is to maximize the benefit of covering regions within a predetermined area. The benefit depends on a set of factors. The first factor is the effect of a selected technology against an attack type. Mainly, there are two groups of technologies which are surveillance and weapon systems. Every technology and attack type pair provides a different benefit to the region. The second factor is the coverage radius of products. Every product has a different coverage radius, which is affected by rain, light, and terrain conditions. This eventually effects the benefit that can be achieved by selecting different systems, the higher the distance sensitivity the lower the benefit. The third is the compliance of different technologies with each other when radiating a region together. Surveillance and weapon pairs provide higher benefits together against different attack types.

A model is designed to realize the objective of maximizing the benefit of acquiring and placing systems in an area. To maximize the benefit, an optimal selection and placement of defense technologies is required. These decisions are made by the model considering budget, selected technology's effective range, the distance sensitivity, the attack types, geographic importance of regions to the government and expected attacks according to past attack patterns.

The problem is solved in two phases. In the first phase, parameters are calculated, and the attack expectations are generated. Parameter calculations are explained later in this chapter. Attack expectations are generated by a model designed from terrorists' perspective. The objective of this model is to maximize terrorists' success of executing attacks. The model can be examined, it is shared in Appendix B. In the second phase, the equipment and the locations are selected while maximizing the total benefit of installing the equipment to selected locations considering the parameters. The overall benefit term is a scalable indicator of the effectiveness of installed systems that are working together as counter measures against terrorism. For every product, the benefit term remarks its effect against certain types of attacks on a certain coverage radius. We assume that the budget is given, and is in monetary units. Enemy actions are predicted in the first phase by generating attacks from terrorists' perspective and the results of this generation are incorporated into the model via defined parameters. In the following subsections, important parameters and preliminary calculation steps are explained thoroughly. After that, the mathematical model is presented with the notations.

3.1 Attack Types

There are several attack types considered during formulation of this problem. The list of attack types is presented in Table 3.1. This list can be modified according to requirements in different scenarios. The problem aims to maximize benefit against these types of attacks. Every technology selected has different effect and capabilities against attack types at different ranges, eventually creating the benefits.

Table 3.1: Attack types

Type Number	Definition
1	Tribute Collection
2	Military Attack
3	Transportation / Convoy Attack
4	Village Raid

Tribute collection attacks are done to villages where the enemy expects no resistance, this kind of village is considered to have an allegiance to the organization. The benefit of such attacks for terrorists is considerably low.

Military attacks target military institutions, it is important to protect an existing base from the attacks. Military attacks generally involve suicide bombers and raiders both results in casualties. This type of attacks is of comparatively higher importance.

Convoy attacks should be eliminated to protect the roads that support vital food, healthcare and other livelihoods to region. Roads of secondary importance when compared to town centers.

Village raids are attacks that end up with civilian casualties and happen where government existence is comparatively higher, thus resulting in military loss as well. Town centers are where people gather to feel more official presence and protection.

Attack types and effects are related. Measures against different types of attacks have different effects and a measure also may be ineffective on certain types of attacks. For instance, a radar only setup is without any defense against any attack and missiles or guns are blind without sensory systems.

3.2 Technology Types

A combination of available technologies can be used for any location. Using weaponry with surveillance systems enhances the effectiveness of both technologies. Surveillance systems provide early warning and help to direct weapon systems prior to enemy's entrance to the range. While using systems along with each other, the amount of increase in benefit may change based on the configuration of used technology and level of integration between systems.

Table 3.2 presents a list of surveillance and weapon technologies considered in this study. The selection of a variety of technologies is made according to national technological capabilities, this selection change based on availability of every single technological item. Systems can be used in detection and elimination of enemy units; they can be changed according to developing product portfolio.

Table 3.2: Selected technologies and their effective range

Definition	Range (km)	Type
Pan Tilt Zoom Night Vision Camera 2	2	Surveillance
Pan Tilt Zoom Night Vision Camera 4	4	Surveillance
Ground Surveillance Radar 4	4	Surveillance
Ground Surveillance Radar 10	10	Surveillance
Connected Movement Detection System	1	Surveillance
Automated Machine Gun 2	2	Weapon
Automated Machine Gun 4	4	Weapon
Automated Machine Gun 10	10	Weapon
Rifle	2	Weapon

Technologies have varying coverage, their effect against different types of attacks differ, the matrix base effect versus attack types can be examined in Table.App.A.4. Higher benefits may be achieved by selecting matching technologies together versus each attack type.

3.3 Determining The Benefits

The benefit calculation process depends on the selected equipment. Different equipment requires different methods in determining benefits. Two main methods of measuring benefits for surveillance systems are comparing results of different sensory systems and saturation threshold. In radar type of sensory systems, if the target is airborne, the results of Automatic Dependent Surveillance–Broadcast(ADSB) , Identification friend or foe (IFF) results and generated radar information is compared with each other to see the benefit of placement of the systems. For day-TV or IIR cameras, resolution and saturation of supporting chipset to alert is the main determinant in benefit. Converting the benefit of different systems into comparable numerical values is not covered in the existing published literature as the systems are considered incomparable due to being fundamentally different in theory. In the thesis, we attributed values to the benefit terms of different systems against different attack types by incorporating the effects of those systems. Rather than attributing numerical values to the benefit terms, attributing fuzzy values such as “good” or “mediocre” may be another way that can be used while calculating the terms.

Parameters used in determining benefits:

β_{ijt}/β'_{ijt}	benefit of weapon/surveillance at region *i* covering region *j*
$\beta''_{itt'}$	benefit of covering region *i* with surveillance *t* and weapon *t'*
$\beta_{ijtk}/\beta'_{ijtk}$	benefit of weapon/surveillance at region *i* covering region *j* against attack type *k*
$\beta''_{itt'k}$	benefit of covering region *i* with surveillance *t* and weapon *t'* against attack type *k*
P_i	importance of protecting region *i*
d_{ij}	the Euclidean distance between regions *i* and *j*
$\varepsilon_{tk}/\varepsilon'_{tk}$	base effect of having surveillance/weapon *t* against attack type *k*
$\varepsilon_{ijtk}/\varepsilon'_{ijtk}$	effect of surv./weapon *t* at reg. *i* covering reg. *j* vs. attack type *k*
π_{ijt}/π'_{ijt}	coverage health for surv./weapon t located at reg. *i* covering reg. *j*
π_t/π'_t	distance sensitivity for surveillance/weapon *t*
r_t/r'_t	range of surveillance/weapon *t*

The equipment may lose effectiveness as the distance from the radiation center increases. This phenomenon is called coverage health. Although a region is within the effective range of a product, due to decreased coverage health over the radius, the benefit decays. Coverage health is derived from distance sensitivity (π_t) for system *t*. Coverage health over region *j* of technology *t* located in *i* (π_{ijt}) is calculated as follows:

$$\pi_{ijt} = \begin{cases} (1 - \pi_t \dfrac{d_{ij}}{r_t}) & if\ d_{ij} \leq r_t \\ 0 & otherwise \end{cases} \qquad \forall i \in I_c, \forall t \in T, \forall j \in I \tag{3.1}$$

Where set *I* represent the set of all regions, set I_c represent set of candidate regions for locating technologies, set *T* represent the set of technologies, d_{ij} is the Euclidean distance between regions i and j, r_t is the range radius of system t.

Benefit of covering a region depends on the distance of technology to region and the attack types. We assumed every region has a set of expected attack types and then we were able to incorporate this effect into preliminary calculations. The effect of technology selection decreases as range increases if and only if the technology is distance sensitive. Effect of selected technology t placed in region i protecting region j from attack type k (ε_{ijtk}) is calculated as follows:

$$\varepsilon_{ijtk} = \varepsilon_{tk}\,\pi_{ijt} \qquad \forall i \in I_c, t \in T, \forall j \in I, \forall k \in K \qquad (3.2)$$

Where K is the set of attack types, K changes depending on the scenario, ε_{tk} is base effect of having technology t against attack type k. Once an area is covered with surveillance and weapon systems, then, in majority of the cases, there is a low expectancy of terrorist activity in the region. Following installation of systems, after the counter organization experiences first encounter with newly placed systems, they tend to learn the effect of new systems. To avoid getting caught or getting killed, the terrorists avoid regions decorated with these highly technological new systems. As shown in the above list radars have a higher range than weapons, they are also better than PTZ cameras in terms of distance sensitivity as they are not affected by weather or light conditions. Incorporating this distance sensitivity of information is done via calculating Euclidian distance between every region that is in the feasible domain.

Past events are used while calculating expectations/importance over the region that is considered for technological investment. The importance of regions is then used to calculate the benefit of covering that region. The importance is represented by P_j where j represents the region. The importance of a region increases when there is an attack pattern over the region, when region houses government institutions, when region is at a crossroads, when region is a village or town. Benefit of having tech t located at region i covering region j against attack type k (β_{ijtk}) is calculated as follows:

$$\beta_{ijtk} = P_j\,\varepsilon_{ijtk} \qquad \forall i \in I_c,\ \forall t \in T,\ \forall j \in I,\ \forall k \in K \qquad (3.3)$$

In regions covered by both surveillance and weapons, there may be an increased benefit due to the support these two systems may be providing each other. The delta occurring due to having better working conditions or inputs from other system is calculated using the base effect of having that technology. Therefore, the delta benefit of having both tech t and t' covering region j against attack type k ($\beta_{jtt'k}$) is calculated as follows:

$$\beta_{jtt'k} = P_j\ (\varepsilon_{tk}\ coef_t + \varepsilon_{t'k}\ coef_{t'}) \qquad \forall j \in I, \forall t \in S, \forall t' \in W, \forall k \in K \quad (3.4)$$

S represents set of surveillance systems, W represents set of weapon systems, $coef_t$ represents coefficient of increase in basic benefit for t while working alongside t', similarly $coef_{t'}$ represents coefficient of increase in basic benefit for t' while working alongside t. Coefficients depend on systems ability to improve each other while working together. For the simplicity of trials, $coef_t$ is set to 0.1 and $coef_{t'}$ is set to 0.2. A common example of this mutualist relationship of supporting systems is as the following, radar systems increase lethalness of weaponry as they raise a flag at regions where activity observed while weapon systems increase the benefit of radars by eliminating targets and preventing saturation of such surveillance systems.

Overall benefit of covering a region ($\beta_{ijtk}, \beta'_{ijtk}, \beta''_{ijtt'k}$) depends on importance of covered/observed region (P_j) and effect of having the selected tech to cover the related region against the types of attack ($\varepsilon_{\mathrm{ijtk}}, \varepsilon'_{ijtk}$). In the thesis, it is assumed that a region can be covered more than once and there is a limit to the benefit that can be attained by covering a region. Thus, even though a region is covered five times, the last four may be providing no additional benefit to this region.

The benefit that can be achieved by covering a region with a technology is obtained by summing the benefit of covering a region over attack types.

$$\beta_{ijt} = \sum_{k \in K} \beta_{ijtk} \qquad \forall i \in I_c,\ \forall t \in T,\ \forall j \in I \qquad (3.5)$$

The delta in benefit that can be achieved by covering a region with surveillance and weapon technologies together is obtained by summing the delta over attack types.

$$\beta_{jtt'} = \sum_{k \in K} \beta_{jtt'k} \qquad \forall j \in I, \forall t \in S, \forall t' \in W, \forall k \in K \quad (3.6)$$

Lastly, the benefit function is a subadditive set function for every technology type, does not yield same return for additional systems.

3.4 Problem Notation and Formulation

The problem is two-fold, first determining a set of defense products to be selected from a portfolio including surveillance and weapon systems, second placing them to optimal locations according to geographical properties and use cases. According to a customer's area, the area's properties, the terrorists in the area, the villages, the importance of regions change. The problem considers a predetermined area. This area is taken from exemplary land, its dimensions are 50 kilometers both in length and width. This area has a special situation, it is the most preferred field of activity of most active terrorist groups due to its unique position. The area has borders in two different countries, one to the north and one to the west. The borders provide easy escape routes to terrorists. In this thesis, a model is designed and solved to find an optimal selection of defense technologies according to the threat types, budget and regional geographic properties. Selection and placement are made by the model considering selected technology's effective range, geographic or geopolitical importance of regions to government and expected attacks according to past attack patterns. The notation is summarized as follows.

Sets:

I	set of all regions
I_c	set of candidate regions for locating technologies
T	set of technologies
S	set of surveillance technologies, a subset of technologies
W	set of weapon technologies, a subset of technologies

Parameters:

c_t	installation cost of technology *t*
β_{ijt}/β'_{ijt}	benefit of weapon/surveillance at region *i* covering region *j*
$\beta''_{itt'}$	benefit of covering region *i* with surveillance *t* and weapon *t'*
β_i^{max}	maximum benefit that can be achieved by covering any region *i*
B	budget

$\alpha_{ijt}/\alpha'_{ijt}$	1 if region *j* can be covered by region *i* with surveillance/weapon *t*; 0 otherwise
β_{lim}/β'_{lim}	maximum benefit achievable by covering a region with technology type surveillance/weapon
P_j	importance of covered/observed region

Decision Variables:

y_{ijt}/y'_{ijt}	1 if region *j* benefits from surveillance/weapon *t* located at *i*, 0 otherwise
x_{it}/x'_{it}	1 if location *i* uses surveillance/weapon *t*, 0 otherwise
q_{jt}/q'_{jt}	1 if region *j* is covered by surveillance/weapon *t*, , 0 otherwise
$z_{itt'}$	1 if region *i* is covered by surveillance t and weapon *t'*, 0 otherwise

The objective is to maximize the benefits of selecting and placing defense systems to this area, where risk of terrorism is high, the model below solves the problem considering coverage radius of products in the portfolio, suitability of the field for installation of the technology, existence of double coverage, maximum benefit that can be achieved by covering a region by using weapons/surveillance, the existing knowledge and budget.

$$\text{maximize} \sum_{i \in I_c} \sum_{j \in I} \sum_{t \in S} \beta_{ijt} y_{ijt} + \sum_{i \in I_c} \sum_{j \in I} \sum_{t \in W} \beta'_{ijt} y'_{ijt} + \sum_{t \in S} \sum_{t' \in W} \sum_{j \in I} \beta''_{jtt'} z_{jtt'} \tag{3.7}$$

subject to:

$$\sum_{i \in I_c} \sum_{t \in S} c_t x_{it} + \sum_{i \in I_c} \sum_{t \in W} c_t x'_{it} \leq B \tag{3.8}$$

$$\alpha_{ijt} x_{it} \geq y_{ijt} \qquad \forall i \in I_c, \forall j \in I, \forall t \in S \tag{3.9}$$

$$\alpha'_{ijt} x'_{it} \geq y'_{ijt} \qquad \forall i \in I_c, \forall j \in I, \forall t \in W \tag{3.10}$$

$$\sum_{\forall i \in I_c} y_{ijt} \geq q_{jt} \qquad \forall j \in I, \forall t \in S \tag{3.11}$$

$$\sum_{\forall i \in I_c} y'_{ijt} \geq q'_{jt} \qquad \forall j \in I, \forall t \in W \qquad (3.12)$$

$$2z_{jtt'} \leq q_{jt} + q'_{jt'} \qquad \forall j \in I, \forall t \in S, \forall t' \in W \qquad (3.13)$$

$$\sum_{\forall i \in I_c} \sum_{\forall t \in S} \beta_{ijt} y_{ijt} \leq \beta_{lim} P_j \qquad \forall j \in I \qquad (3.14)$$

$$\sum_{\forall i \in I_c} \sum_{\forall t \in W} \beta_{ijt} y'_{ijt} \leq \beta'_{lim} P_j \qquad \forall j \in I \qquad (3.15)$$

$$\sum_{i \in I_c} \sum_{t \in S} \beta_{ijt} y_{ijt} + \sum_{i \in I_c} \sum_{t \in W} \beta'_{ijt} y'_{ijt} + \sum_{t \in S} \sum_{t' \in W} \beta''_{jtt'} z_{jtt'} \leq \beta_j^{max} P_j \qquad \forall j \in I \qquad (3.16)$$

$$y_{ijt}, y'_{ijt'}, z_{jtt'}, x_{it}, x'_{it'}, q_{jt}, q'_{jt'} \in \{0,1\} \qquad \forall \mathrm{i} \in I_c, \forall j \in I, \forall t \in S, \forall t' \in W \qquad (3.17)$$

The objective function (3.7) maximizes the total benefit gained at all regions by using surveillance, weapons or a combination of the systems. The direction of the objective function forces all of the constraints to their upper bounds. The first constraint (3.8) ensures the cost of placed technologies does not exceed the budget. The second (3.9) and third (3.10) constraints force x_{it} and x'_{it} to one as long as the target region is in the range of technology *t* when the technology is placed at region *i*, the constraints help to check whether a region is covered by surveillance/weapons located at a specific region by using the coverage radius of every system. The fourth (3.11) and fifth (3.12) constraints check whether a location is covered by a surveillance, or a weapon located anywhere. The sixth constraint (3.13) forces forth (3.11) and fifth (3.12) constraints to their upper bounds and the objective function forces the sixth constraint (3.13) to its upper bound, the constraint checks the region-technology pairs to see whether a location is covered by a specific surveillance-weapon match, each surveillance-weapon match provides a differing delta benefit. The seventh (3.14) and eight (3.15) constraints ensure the benefit earned by covering a location with surveillance and weapon systems does not exceed the maximum achievable benefit for each technology type, we assumed that for every square meter there is a maximum benefit achievable, when the resolution changes and the size of region in square meters change accordingly, the achievable benefit also changes. The objective function forces the seventh (3.14) and eight (3.15) constraints to their upper bounds. The ninth (3.16) constraint ensures the total benefit of covering a region using surveillance and weapon systems does not exceed the maximum achievable benefit of covering a region, we assumed there is a maximum achievable benefit for every square meter covered in a region and after reaching this limit there will not be additional benefits for covering the region again. The tenth constraint (3.17) describes domains for the variables.

3.4.1 Toy Example

An example case is selecting surveillance from a set of two types systems and placing them into a map divided into a 4x4 grid considering importance of grids with costs of systems 4000, 6000 and budget is 20000.

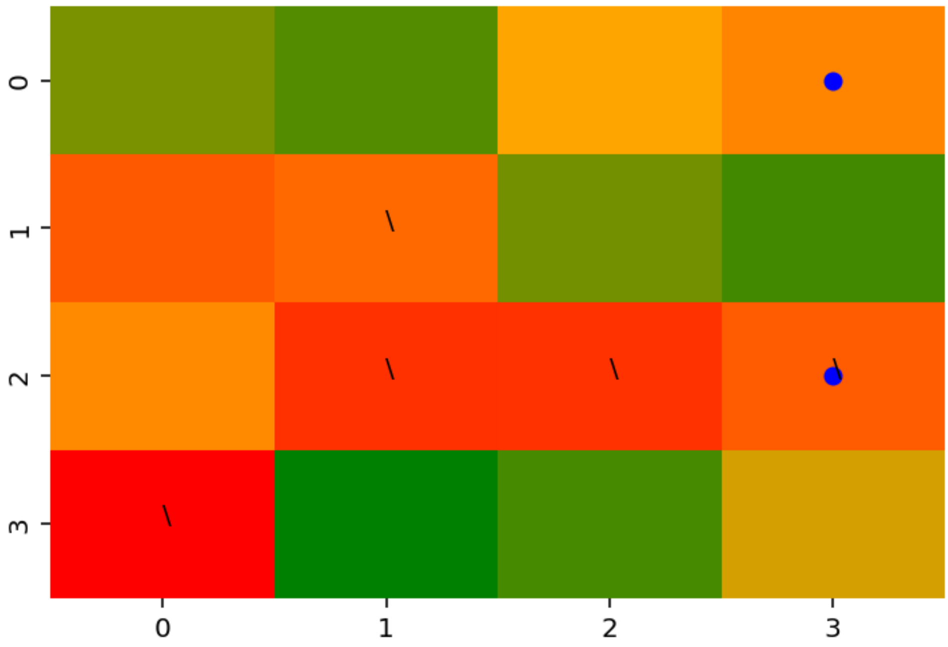

Figure 3.1: Placement of selected systems

Figure 3.1 displays a layout over a coordinate system. The figure shows a region in green if its importance is considerably low and shows the region in red if its importance is high, and the dots represent expected attacks. The line leaning towards left represents the region is covered with surveillance and if there were weapons, the weapons would be represented by lines leaning towards right. This map represents a decision of placement and selection. Along with the map, the variables generated by the model represent coordinates and type of the selected surveillance system. In this case, data used is given in Appendix A. The model selected using cheaper surveillance systems although the expensive option achieves higher benefits per placement. Five of cheaper systems together achieve higher benefits in total then three of expensive systems and a remaining budget of 10%.

4 EXPERIMENTAL STUDY

The country we have studied and examined thoroughly is facing issues at its western borders to its neighbors. Their government wants to implement an integrated solution to its borders. The government reached out to a set of defense industry companies, one of which is the company which wants to offer them an optimal selection and placement of their product portfolio customized for the field and threat properties of the country. The objective of the company is to create a comprehensive solution for this customer with a given budget using the existing product portfolio and the products' benefit coefficients versus expected targets. The country determined the area they wanted to install the solutions. The area is 50km in length and in depth. The country wants to acquire defense systems that are economically feasible and sustainable considering logistics, i.e. integrated logistic support and replacement parts. The company gathered geographic data about the land area upon request and mapped their scenario. Data about threat sets gathered via site visits, the threats use mostly motorcycles and very rarely pick-up trucks. Against this type of threats, the defense industry norm is using automated rifles, anti-tank missiles, small caliber missiles, and rifles. Small caliber missiles have warheads that have particle effects, thermobaric effects, high explosion effects, and armor piercing effects. An importance map of this area is shared by the government, there may be suitability issues for certain equipment, a suitability matrix is created using field properties.

The goal is to first solve this scenario and record it. To observe the solvability of the problem, we first started with a comparatively small model, which had a smaller number of regions because the area is divided into smaller integers in width and depth. To see how our computing pace is affected by the increase in regions, we kept the technologies the same, the area the same and we have increased the problem size. The number of square regions representing the area is the resolution. While solving the problem, we solved it for the same area at every iteration. To increase the resolution of the problem, we divided the area into more squares. Dividing the area into smaller squares helped us to have a more realistic coverage as the radius of effective range for every system became respectively larger and the difference between real and calculated coverage got much more negligible.

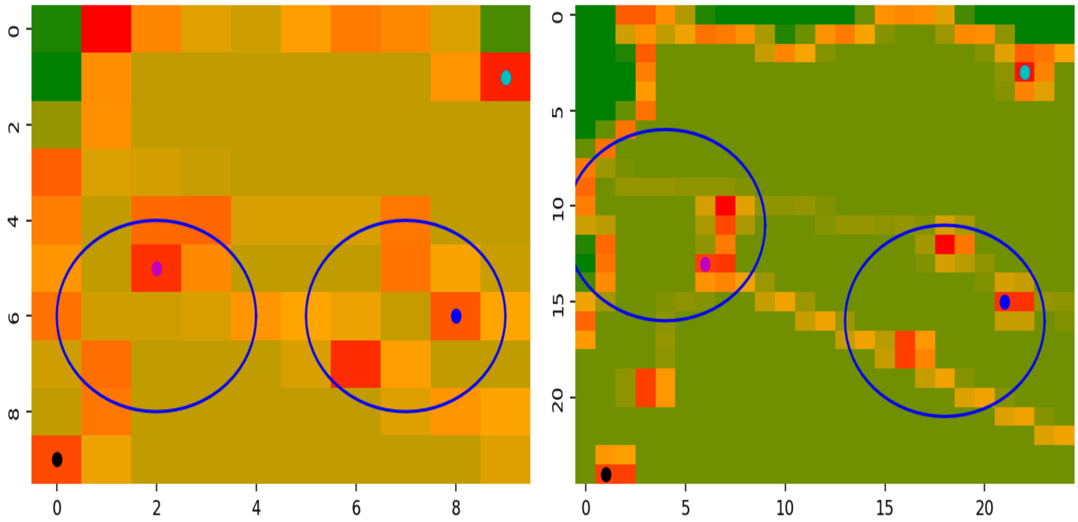

Figure 4.1: Comparison of details at different resolutions (10x10 vs. 25x25)

In Figure 4.1, maps of different resolutions are compared. As the number of regions and resolution decreases, the perception of importance changes along with the importance matrix. As the matrix scaled into lesser resolution, some critical details get lost. First, we started to solve the problem using the area divided into a hundred regions, the map on the left on Figure 4.1. Then, we continued solving the problem for a larger set of regions until it became insolvable. We changed the number of regions the area divided into while keeping every other parameter same. The parameters such as the importance map and the probability matrix of importance are scaled every time the resolution changes. As the resolution has increased, the total benefit that can be achieved by the area is divided into number of regions for both surveillance systems and weapon systems.

Reaching the limits of current processing power showed the limits of exact optimization methods, we evaluated the current result and processing pace is and discussed if it is satisfactory for business purposes considering following scenario.

4.1 Base Case Scenario

In the base case scenario, the problem of optimal selection and placement of defense system is solved using the model formulated. The problem is solved at different sizes. The size of the problem is changed by dividing the area into more regions, it is called the resolution throughout the text. All other parameters have been kept the same.

4.1.1 Results on a 10x10 grid

In this resolution, the selected equipment is placed to most beneficial regions between these 100 regions. Dividing 50 km to 50 km square area, there are 100 square regions with side length 5 km. The objective is maximizing their benefit according to coverage, importance of covered region, expected attacks, available technology. The selection is made with 9 types of technologies, 5 surveillance systems and 4 weapon systems. Each technology has different benefits, their benefit increase when combined with systems of other type. Used technologies are given in the Table App.A.2 in the Appendix A. Base effect of having tech t against attack type k (ε_{tk}) is given in the Table App.A.4 in Appendix A.

According to the scenario, when the area is divided into 100 regions, the attacks are mapped as in Table 4.1.

Table 4.1: 10x10 attack regions

Region	Attack Type	Figure Color
(1,9),(6,8)	Tribute Collection	Blue
(1,9)	Military Attack	Cyan
(9,0)	Convoy Attack	Black
(5,2)	Village Raid	Magenta

In the Figure 4.2, a representation of the attacks predicted to be realized by the terrorists in near future is mapped using the attack color pairs given in Table 4.1.

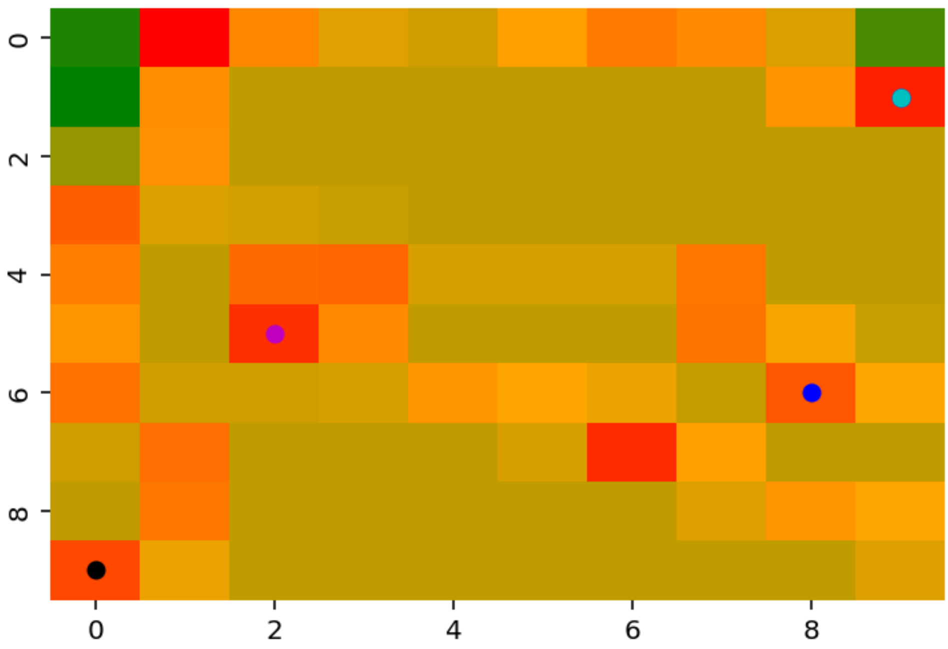

Figure 4.2: Map of predicted attacks 10x10

The problem solved using the importance of regions and predicted attacks. Covering foreseen attack regions gives more benefit to systems from both types. If a region is covered by both surveillance and weapons, the systems increase each other's performance at different levels. Initially, a budget of 150,000 is used to solve the problem. The maximum achievable benefit means the benefit gained when the whole area is covered against all type of attacks. Initially, the maximum achievable benefit is set to 176,000 for surveillance and 302,000 for weapons. Maximum achievable benefit for the area consisting of all regions is therefore initially set to 478,000. The benefit limits are divided according to the selected resolution and the benefit is chosen respectively to the size of the covered region. The solution of the problem when area is divided into 100 regions is as in Table 4.2.

Table 4.2: 10x10 selection, placement, coverage

Number	Product (S / W)	Unit Coverage(%)	Real Coverage(%)	Solution Coverage(%)
3	PTZ2 (S)	0.50	1.51	3.00
2	RCAW10 (W)	12.57	25.13	26.00

We have used python for calculations and called Gurobi solver in default settings for solving the model, the time of execution of the program is 16 seconds. According to this result defender achieves 22,629 in benefits. The surveillance systems and the weapon systems are located either to the same regions or to regions that are next to each other. The placement shows that selected technologies are selected and placed in a way to support each other. Weapon systems together provide 26.00% coverage as they coincide. Surveillance systems provide 3.00% coverage. Surveillance systems are only located in places where importance is comparatively higher.

The solution provides 25.13% coverage of all area with weapons, this is smaller than what the model assumed, but the difference is considerably small, and coverage is realistic.

The solution provides 1.51% coverage of all regions with surveillance, this is smaller than the assumed coverage of model which is 3%. The difference is high, the reason is the coverage radius of selected system. To avoid this issue and have more realistic coverage, the edge length must be smaller, and resolution must be higher.

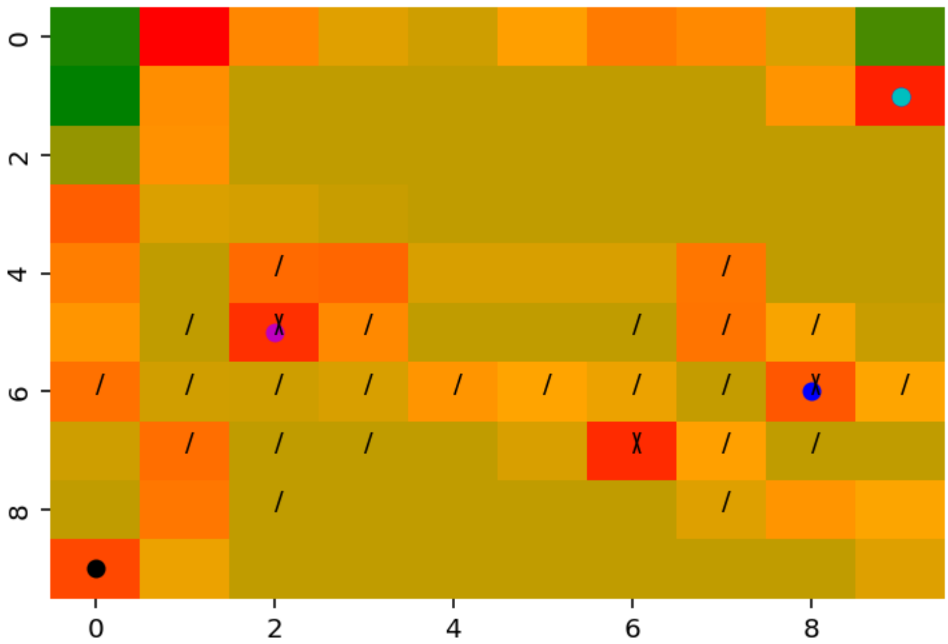

Figure 4.3: Coverage of systems 10x10

In Figure 4.3, the coverage can be seen. If there is a slash towards the right, that means the region is protected by weapons. If there is a slash towards the left, that means the region is observed by surveillance. If there is an X over the region on the map, that means the area is covered by both technology types. As discussed above, there is a difference in assumed coverage versus coverage provided by the solution. The difference became smaller by increasing the resolution of the solution. As the same area is divided into more regions, the difference between square regions and circular radius of ranges became somewhat negligible.

4.1.2 Results on a 25x25 grid

The procured equipment is located over the same area divided into 625 regions. When the region is divided into 625 regions, all parameters are kept same. The importance matrix is a derivative of the 100x100 map, thus there was no information lost while scaling the matrix. The attacks are also derived from 100x100 map and scaled as in Table 4.3.

Table 4.3: 25x25 attack regions

Region	Attack Type	Figure Color
(3,22),(15,21)	Tribute Collection	Blue
(3,22)	Military Attack	Cyan
(24,1)	Convoy Attack	Black
(13,6)	Village Raid	Magenta

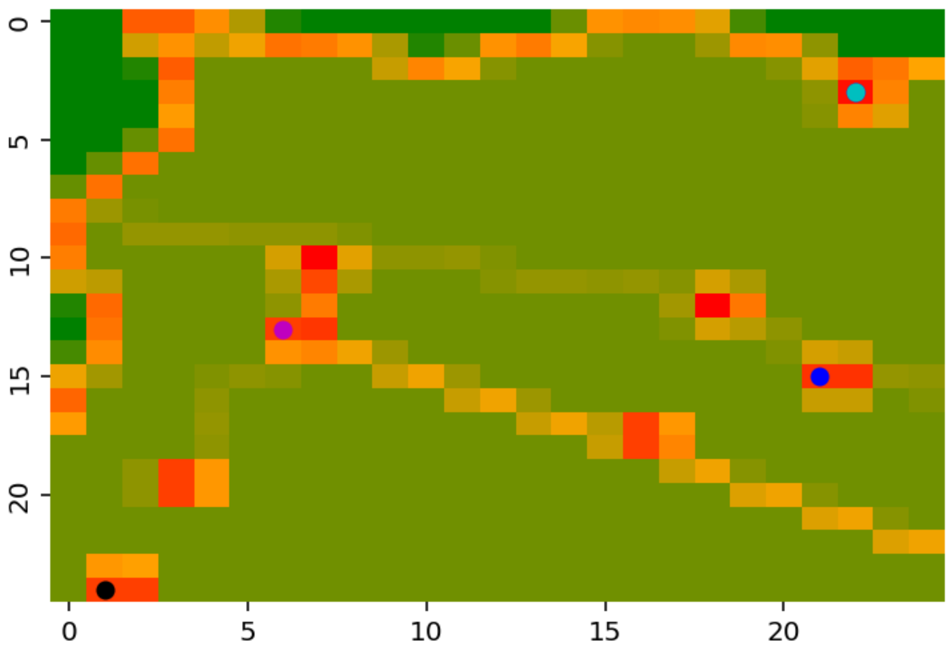

Figure 4.4: Map of predicted attacks 25x25

Figure 4.4 marks the predicted attacks according to attack-color pairs listed in Table 4.3. As the resolution increases, the representation of predicted attacks gets nearer to their real locations and the importance of nonimportant places is more clearly distinguished. The same scenario is solved at higher resolution, the solution is as in Table 4.4.

Table 4.4: 25x25 selection, placement, coverage

Number	Product(S/W)	Unit Coverage(%)	Real Coverage(%)	Solution Coverage(%)
2	PTZ2(S)	0.80	1.01	1.60
1	PTZ4(S)	2.08	2.01	2.08
2	RCAW10(W)	12.96	25.13	25.28

The execution time of the program is: 765 seconds, 48 times longer than the solution time of the problem when solved resolution 10x10. According to this solution, the defender achieves 23,179 in benefits. Benefit of the solution is similar, there is a very small increase in gained benefit of this selection and placement compared to the solution of resolution 10x10. Product-wise, the solution is like the solution of the resolution 10x10, only one thermal camera is upgraded to the one with a larger range. The increase in objective is a result of being able to see the importance of every region for what they are and placing the systems accordingly. When the resolution is downgraded to 10x10, there is a much higher loss of details. The increase in the number of regions provides a more realistic comparison of range radius for all products. Therefore, as seen in Table 4.4, the difference between real coverage of the solution and coverage assumed in the model decreases as the resolution increases.

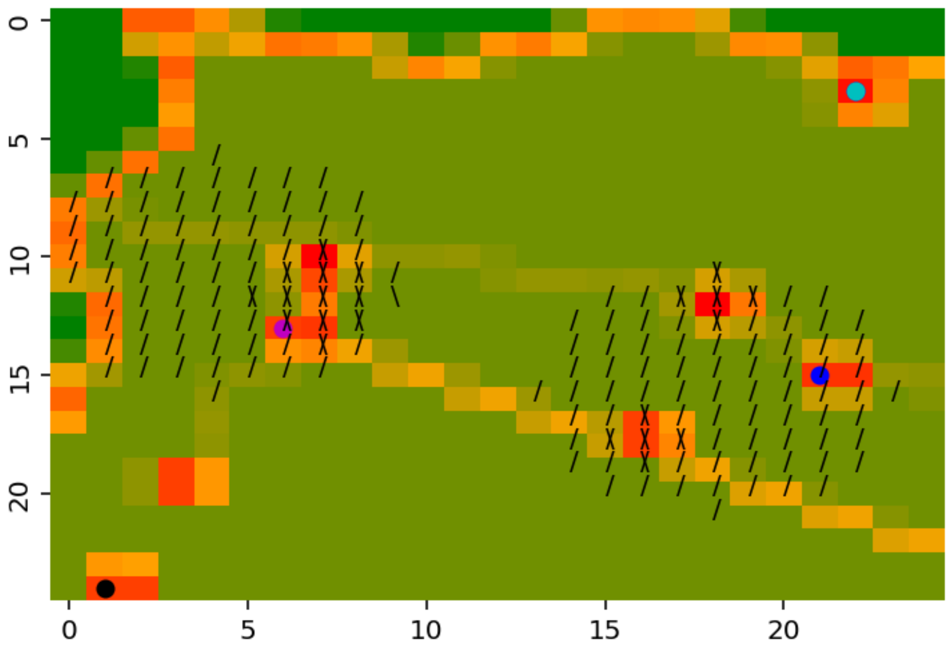

Figure 4.5: Coverage of systems 25x25

Weapon systems together provide 25.92% coverage. Surveillance systems provide 3.68% coverage. Surveillance systems are only located in places where importance is comparatively higher as seen in Figure 4.5.

4.1.3 Results on a 50x50 grid

On a 50 by 50 grid, the squares are much smaller, and the loss of detail is much smaller as well. The problem is solved with the same parameters except for the number of regions.

According to the scenario, when the area is divided into 2,500 regions, the attacks are predicted as in Table 4.5.

Table 4.5: 50x50 attack regions

Region	Attack Type	Figure Color
(7,45),(31,43)	Tribute Collection	Blue
(7,45)	Military Attack	Cyan
(48,3)	Convoy Attack	Black

(27,13)	Village Raid	Magenta

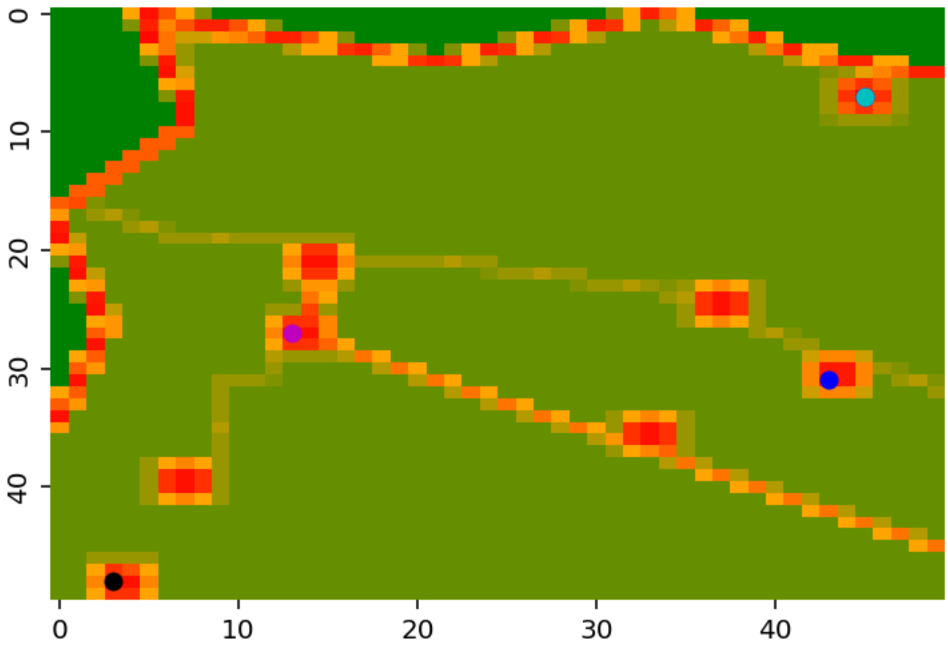

Figure 4.6: Where attacks are predicted, 50x50

The attacks are expected at locations marked on Figure 4.6 according to type-color pairs presented in Table 4.5. According to the importance of regions, with same budget and maximum achievable, the solution is as in Table 4.6.

Table 4.6: 50x50 selection, placement, coverage

Number	Product(S/W)	Unit Coverage(%)	Real Coverage(%)	Solution Coverage(%)
2	PTZ4(S)	2.01	4.02	3.92
2	RCAW10(W)	12.57	25.13	25.00

The execution time of the program is: 22,731 seconds, 30 times longer than the problem solved at resolution 25x25. According to this result defender achieves 23,062 in overall benefit, the objective value is less than the objective of resolution 25x25. The solution provides 3.92% coverage of all area with both systems and 21.08 % coverage of all area with only weapon systems.

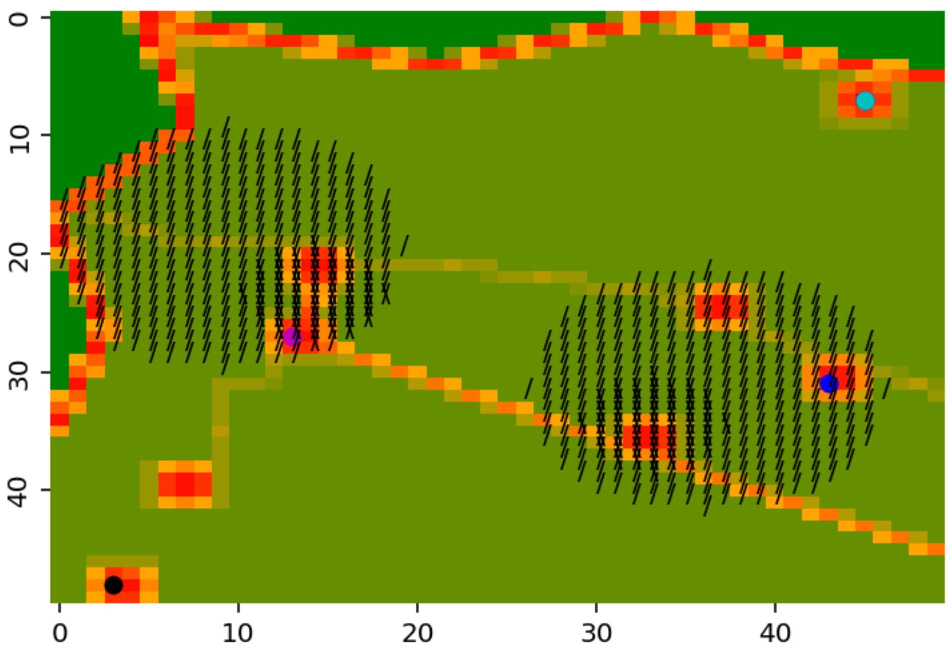

Figure 4.7: Coverage of systems 50x50

Figure 4.7 visualizes the coverage of solution of the model when solved at 50x50 resolution. Compared to previous solutions, selected systems and coverages are similar and more realistic. There is no major change in attained benefit.

4.1.4 Summary of Results

The problem size grows exponentially, 100x100 resolution requires more memory than available. The optimality gap (tolerance) has been increased and the number of threads has decreased. After that, the model completed the setup process but did not progress after pre-solve. According to results from experiments with different resolutions, the results are not significantly different from each other. As the model size grows, the coverage and benefit results get closer to the real values. However, selection of systems and their placement are similar. The resolution of 25x25 produces a meaningful solution at an acceptable time. The program takes around 22 minutes to solve the problem using Intel(R) Xeon(TM) E5-2643 CPU and DIMM 1600 MHz 128 Gb DDR3 RAM. Considering the importance of the strategic decision, such a strategic decision-making problem justifies larger CPU times, it is also important to note that the CPU may be improved considerably, we used the device that is available to us.

Table 4.7: Summary of results for different resolutions

Size	Benefit	Solution Time (s)	Weapon Coverage(%)	Surveillance Coverage(%)
10x10	22629	16	26.00	3.00
25x25	23179	765	25.28	3.68
50x50	23062	22731	25.00	3.92

Evaluation of the results in Table 4.7 show that the selected systems provide similar results. This similarity of the results is related to selected regional importance parameters. During the selection and placement, suitability of a region and its importance are considered.

For a better comparison of different resolutions, the results obtained by solving the problem at 10x10 and 25x25 grids are used on a 50x50grid. Figure 4.8 presents visualization of the results placed over a map on 50x50 grids.

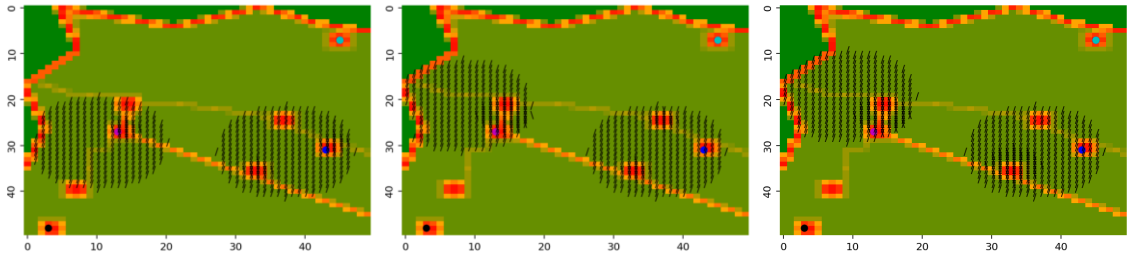

Figure 4.8 Result on 10x10 grid, result on 25x25 grid and result on 50x50 grid

According to Figure 4.8, the result obtained by solving the problem on 10x10 grid is somewhat different to the results obtained by solving the problem on 25x25 and 50x50 grids. The benefit achieved by 10x10 grid relocating at 50x50 grid is 21697 and resolving takes around additional 2133 seconds. The benefit achieved by 25x25 grid relocating at 50x50 grid is 22727 and resolving takes additional 3146 seconds. Considering the results, after switching to 50x50 grid for better comparison, the resolution of 25x25 still produces a meaningful solution at an acceptable time.

4.2 Sensitivity Analysis

The budget is the main constraint. Small changes in the budget can change the resulting benefit on a larger scale. To see its effect on the benefit, the value of the budget changed, and the results are compared with the initial problem in which the budget was 150,000. The problem is solved at 25x25 resolution, the selection of 25x25 resolution is done based on the analysis made using the results of previous runs. According to the analysis, while there is a loss of details in 25x25 resolution, the results do not deviate from the results obtained by solving the problem at 50x50 resolution. The problem is solved with 10% lower, 30% lower, 10% higher and 30% higher budgets.

Introducing lower bounds for achieved benefit changes the solution considerably. To understand how having lower bounds for gained benefit by placing equipment covering regions considered critical effect the result, new constraints are introduced. The results of the analysis are discussed at the end of this chapter.

4.2.1 10% Lower Budget

In the first case, the budget is set to 135,000 and other parameters are kept the same. The solution of the model is as in Table 4.8.

Table 4.8: 10% lower budget selection, placement, coverage

Number	Product(S/W)	Unit Coverage(%)	Real Coverage(%)	Solution Coverage(%)
2	RCAW10(W)	12.57	25.13	25.92

The solutions selected only 2 weapon systems with larger radius providing more coverage. The time of execution of the updated program is 933 seconds. The benefit gained by selecting the two systems is 20,561.

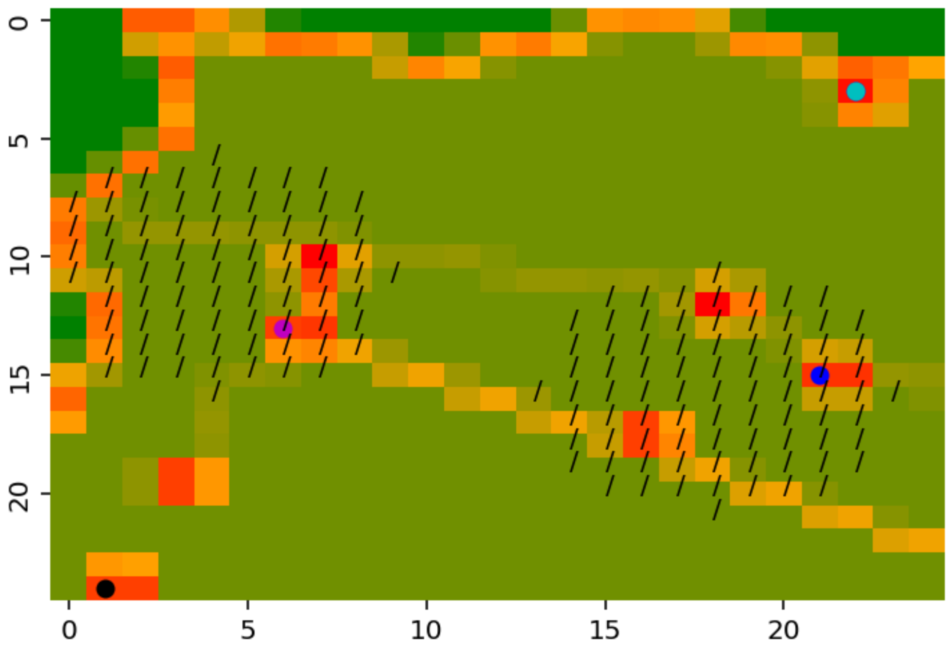

Figure 4.9: Coverage of systems at 10% lower budget

The solution provides 25.92% coverage of all regions with weapons as in Figure 4.9. There is no surveillance system selected to support the weapons.

4.2.2 30% Lower Budget

In this case, the budget is set to 105,000 and other parameters are kept the same. The solution of the model is as in Table 4.9.

Table 4.9: 30% lower budget selection, placement, coverage

Number	Product(S/W)	Unit Coverage(%)	Real Coverage(%)	Solution Coverage(%)
6	PTZ4(S)	2.01	2.01	11.52
1	RCAW10(W)	12.57	12.57	12.32

The solution selected only 6 cameras with night vision with a range of 4 km and 1 automated weapon with a range of 10 km. The time of execution of the updated program is 856 seconds. The benefit gained by selecting the two systems is 15,857.

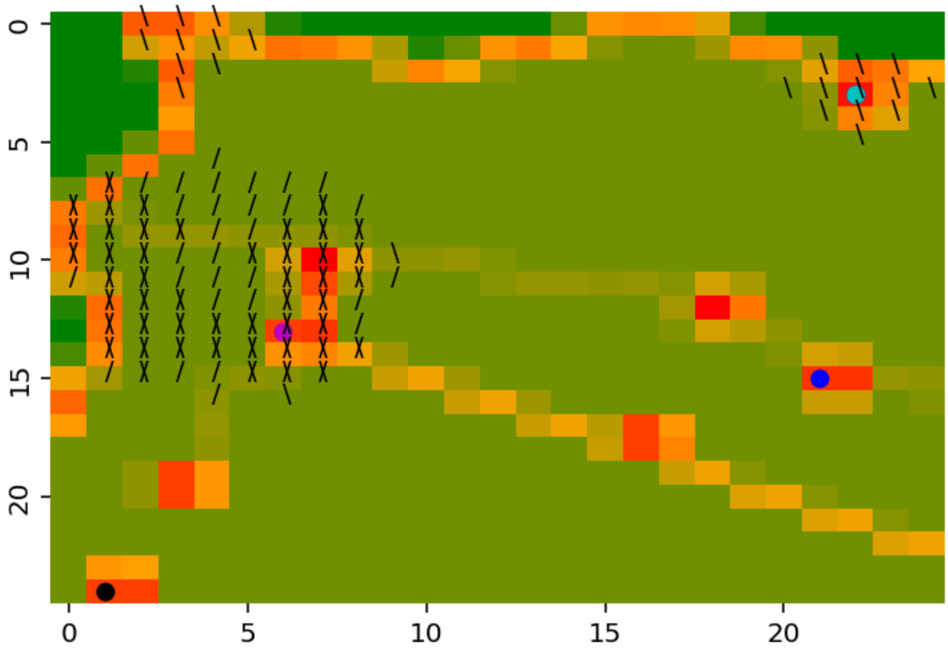

Figure 4.10: Coverage of systems at 30% lower budget

Figure 4.10 shows that the solution provides 12.32% coverage of all regions with weapons and 11.52% coverage of all regions with surveillance. The surveillance coverage is less than the coverage of the surveillance systems installed as some systems intersect.

4.2.3 10% Higher Budget

In the scenario, the budget is set to 165,000. All other parameters are kept the same. According to the execution of the program with new budget, solution is as in Table 4.10.

Table 4.10: 10% higher budget selection, placement, coverage

Number	Product (S/W)	Unit Coverage(%)	Real Coverage(%)	Solution Coverage(%)
5	PTZ4(S)	2.01	10.05	10.24
2	RCAW10(W)	12.57	25.13	25.92

When the budget is increased, the selected surveillance systems are increased. With increased surveillance, the benefit of the solution is 26,213. The time of execution of the new program is 642 seconds.

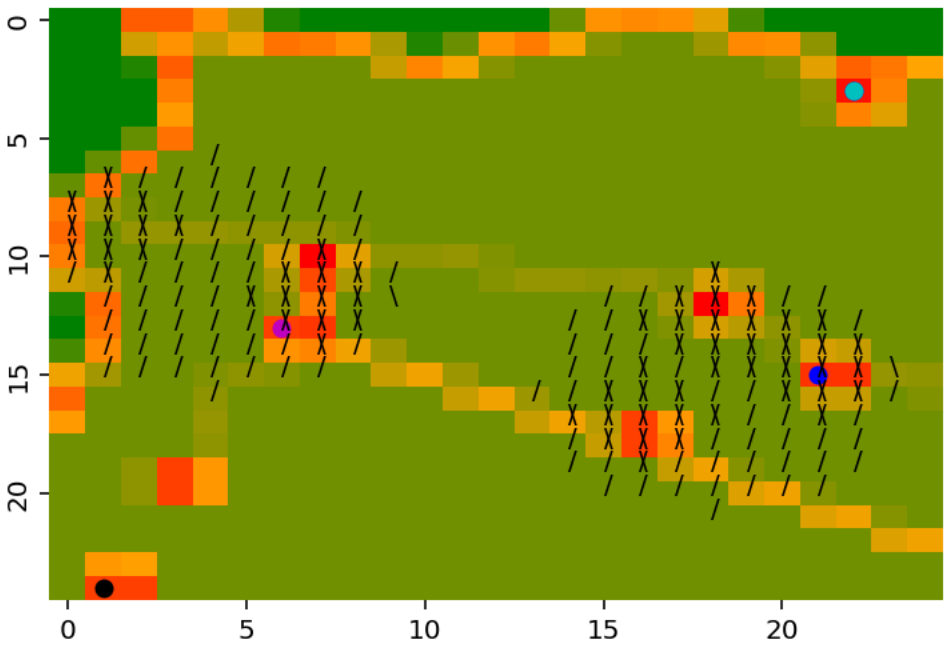

Figure 4.11: Coverage of systems at 10% higher budget

As visualized in Figure 4.11, the executed model with 10% higher budget provides 25.92% coverage with weapons and 10.24% with surveillance over all regions. 9.92% of all regions are covered by surveillance and weapons together as supporting systems. In regions covered with both types of systems, the realized benefit per region is higher.

4.2.4 30% Higher Budget

In the scenario, the budget is set to 195,000. All other parameters are kept the same. According to the execution of the program with the new budget, solution is as in Table 4.11.

Table 4.11: 30% higher budget selection, placement, coverage

Number	Product (S/W)	Unit Coverage(%)	Real Coverage(%)	Solution Coverage(%)
10	PTZ4(S)	2.01	20.11	19.84
2	RCAW10(W)	12.57	25.13	25.92

When the budget is increased, the selected surveillance systems are increased. With increased surveillance, the benefit of the solution is 30,146. The time of execution of the new program is 659 seconds.

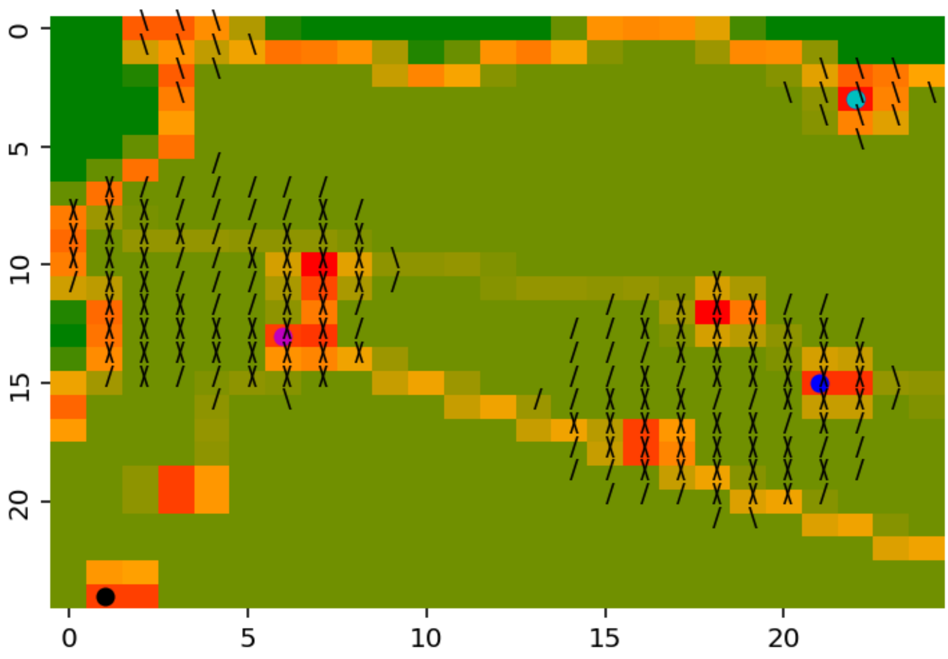

Figure 4.12: Coverage of systems at 30% higher budget

In Figure 4.12, the executed model with 10% higher budget provides 25.92% coverage with weapons and 19.84% with surveillance over all regions. 15.68% of all regions are covered by surveillance and weapons together as supporting systems.

4.2.5 Surveillance and Weapon Benefit Lower Bounds at Critical Regions

If a region is considered important for a customer, constraints to make the region covered may be added. In that case, to have coverage of both types are at a critical region, keeping every parameter the same, below parameters are incorporated to the model.

ζ_i	1 if region i is an critically important region, 0 otherwise
$\beta_{LSLim}/\beta'_{LWLim}$:	benefit lower bound for surveillance/weapon systems

The parameters are then used in the below constraints.

$$\sum_{\forall i \in I} \sum_{\forall t \in S} \sum_{\forall k \in K} \beta_{ijtk} y_{ijt} \geq \beta_{LSLim} \times \zeta_j \qquad \forall j \in I \qquad (4.1)$$

$$\sum_{\forall i \in I} \sum_{\forall t \in W} \sum_{\forall k \in K} \beta'_{ijtk} y'_{ijt} \geq \beta'_{LWLim} \times \zeta_j \qquad \forall j \in I \qquad (4.2)$$

Constraints are used to guarantee that if a region is of critical importance to the nation, then it is covered by both systems, otherwise the constraints are relaxed. Setup can be changed to guarantee either surveillance or weapon coverage. Keeping everything the same and setting the minimum benefit limit to 1, results are as in Table 4.12.

Table 4.12: Results for lower limits for both types

Number	Product (S / W)	Unit Coverage(%)	Real Coverage(%)	Solution Coverage(%)
5	PTZ2(S)	0.50	2.52	3.84
2	PTZ4(S)	2.01	4.02	4.16
1	RCAW10 (W)	12.57	12.57	12.96
5	RW(W)	0.50	2.52	3.84

According to to results of the model after constraints 4.1 and 4.2 added, the coverage of important regions remain the same. However, selected products and total benefit of these products are changed. The solution time is 719 seconds, the solution took about 30 seconds less compared to the model without the new constraints. The benefit of using this solution is 19890.

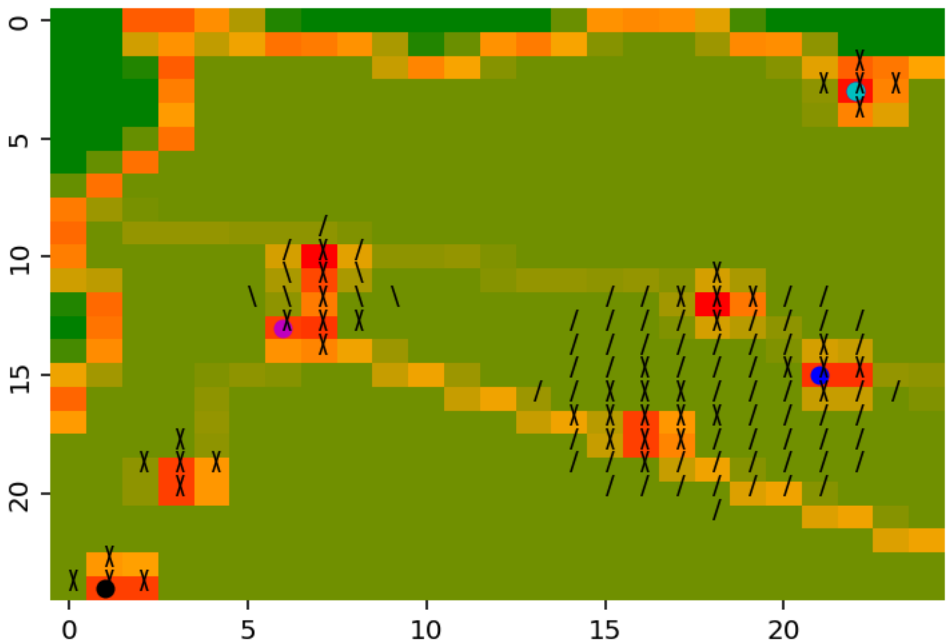

Figure 4.13: Coverage of systems with lower bound, both types

Installation of the selected equipment and its coverage is as in Figure 4.13. The solution provides 16.80% coverage with weapons and 8.00% coverage with surveillance of all regions. The regions covered using both technologies are 7.04% of all regions.

4.2.6 Lower Bound for Total Benefit for Regions at Critical Regions

If a region is considered critical, it must be covered, and it does not matter what type of technology the region is covered by. In that case, keeping every parameter the same, below parameters are incorporated into the model.

ζ_i: 1 if region *i* is a critically important region, 0 otherwise

β_{LLim}: Lower limit of benefit expected for covering the region

The parameters are then used in the below constraint.

$$\sum_{\forall i \in I} \sum_{\forall k \in K} \Big(\sum_{\forall t \in S} \beta_{ijtk} y_{ijt} + \sum_{\forall t \in W} \beta'_{ijtk} y'_{ijt} \Big) \geq \beta_{LLim} \times \zeta_j \qquad \forall j \in I \qquad (4.3)$$

Constraint 4.3 is used to guarantee that if a region is of critical importance to the nation, then it is by a system, if the regions is not considered critical then the constraints are relaxed.

Keeping other parameters the same and setting the lower benefit limit to 1 in order to ensure the critical region is covered, results are as in Table 4.13.

Table 4.13: Results for a total lower limit at critical regions

Number	Product (S / W)	Unit Coverage(%)	Real Coverage(%)	Solution Coverage(%)
2	PTZ2(S)	0.50	1.01	1.44
1	PTZ4(S)	2.01	2.01	2.08
2	RCAW10 (W)	12.57	25.14	25.28

According to the results of the model after adding constraint 20, the coverage of important regions remain the same. However, selected products and total benefit of these products are changed. The solution time is 797 seconds, the solution took about 30 seconds more compared to the model without the new constraints and about 10 seconds less then the model where coverage with both surveillance and weapons expected at critical regions. The benefit of using this solution is 21,911.

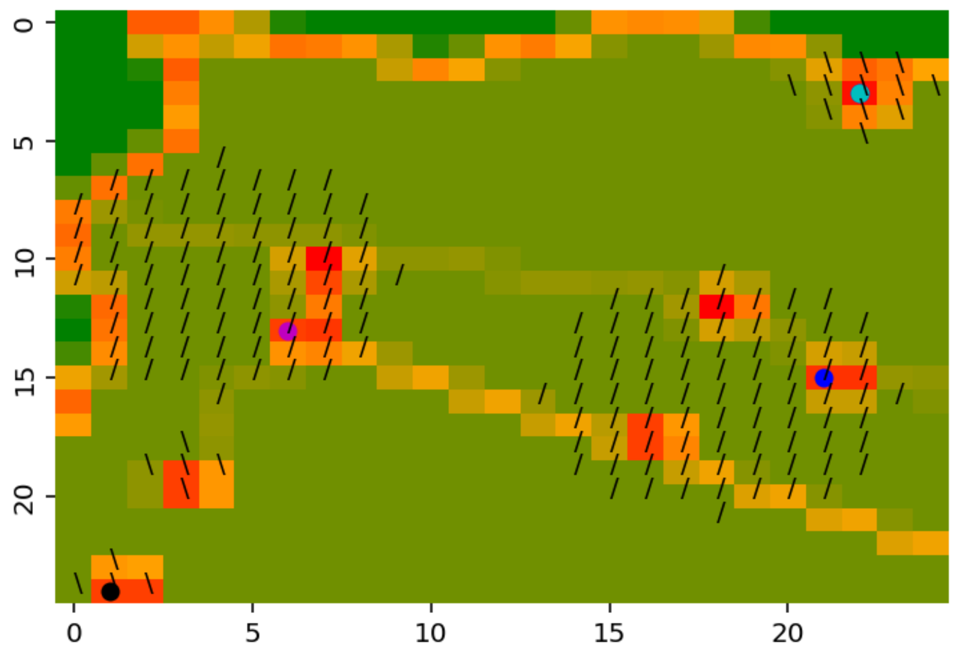

Figure 4.14: Coverage of systems for total lower bound

The solution provides 25.28% coverage with weapons and 3.52% coverage with surveillance of all regions. The installation plan is as seen in Figure 4.14.

4.2.7 Sensitivity Analysis Results

Evaluation of results at different resolutions showed that there will not be major increases in benefits gained overall after reaching resolution 25x25. According to this result sensitivity analysis conducted on changes in budget parameter and incorporating new lower bound parameters for regional benefits. The results of sensitivity analysis and the result of the base case scenario solved at 25x25 resolution are given in Table 4.14.

Table 4.14: Comparison table

Problem	Benefit	Solution Time(s)	Weapon Coverage(%)	Surveillance Coverage(%)
Original	23179	765	25.92	3.68

10% Low B.	20561	933	25.92	0.00
30% Low B.	15857	856	12.32	11.52
10% High B.	26213	642	25.92	10.24
30% High B.	30146	659	25.92	19.84
Separate L. Bounds	19890	719	16.80	8.00
L. Bound for Total	21911	797	25.28	3.52

According to the table, the solution time increases when new lower bounds for regions are introduced. The introduced lower bounds guarantee the coverage of respectively more important regions. However, with the introduction of lower bounds the overall benefit of selection and placement decreases.

The table shows that changing the budget about 10% in a positive direction yields much better results. However, increasing the budget more does not provide an increase in the benefit at the same pace after passing the 10% threshold. The reason is that after increasing the budget 10%, the model covers most of the important regions. After this budget level, expanding the coverage more will not achieve a benefit per region as high as before.

4.3 Minimizing Budget While Covering Important Regions

The complementary problem to the benefit maximization problem studies is the budget minimization problem designed to achieve similar benefit levels. Using the same case and dataset, the problem formulation changes as the following.

The budget parameter is discarded. Instead of the budget, the minimum benefit gained is introduced to the model which will ensure that at least that amount of benefit will be gained through the acquisition and placement of the systems that will be selected from the portfolio. The new parameter:

β_{min}: minimum benefit gained

The decision variables will be kept the same, and the objective function (3.5) will be modified as the following.

$$minimize \sum_{i \in I} \sum_{t \in W} c_{it} x'_{it} + \sum_{i \in I} \sum_{t \in S} c_{it} x_{it} \tag{4.4}$$

The budget constraint (3.6) will be removed. Instead of the budget constraint, the constraint that ensures minimum benefit gained will be introduced as follows.

$$\sum_{j \in I} \omega_j \geq \beta_{min} \tag{4.5}$$

The newly introduced constraint (4.5) ensures the sum of benefits over all regions satisfy the minimum benefit expected for this investment.

4.3.1 Result of Minimization Problem on A 25 By 25 Grid

The modified problem offers a budget for changing requirements. The new model can incorporate must-covered regions as well geographical changes while guaranteeing minimum cost. The results of the problem while everything is kept as the base case scenario is as in Table 4.15. The minimum benefit gained parameter is set to the number the base case scenario achieved.

Table 4.15: 25x25 for cost minimization

Number	Product(S/W)	Unit Coverage(%)	Real Coverage(%)	Solution Coverage(%)
2	PTZ2(S)	0.50	1.01	1.60
1	PTZ4(S)	2.01	2.01	2.08
2	RCAW10(W)	12.57	25.13	25.92

The selected products are the same with the base case scenario. Execution time of the program is 1,445 seconds, about 2x longer than the original problem. According to this solution, the defender achieves 23,179 benefits, which is the same.

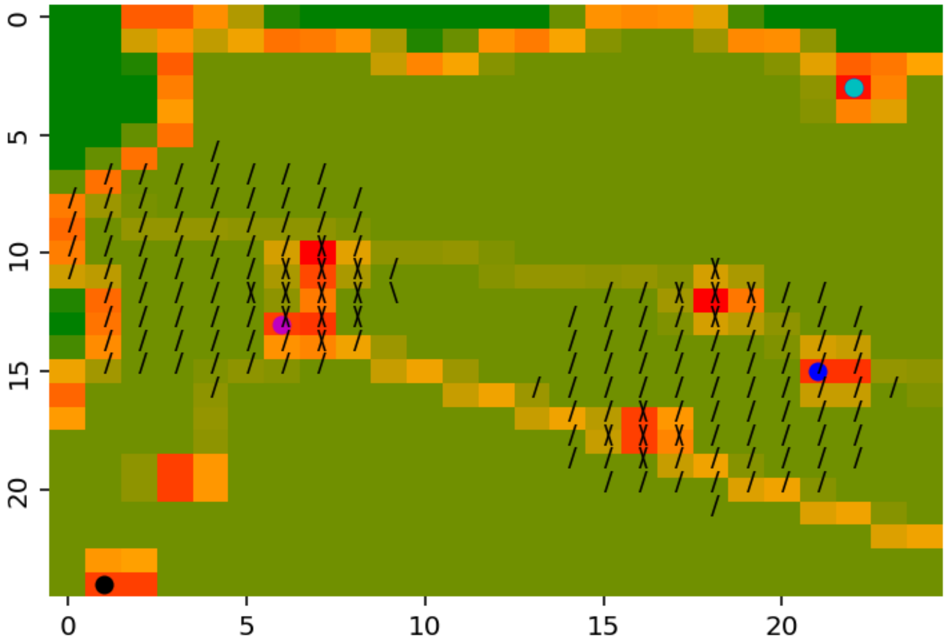

Figure 4.15: Coverage of systems for budget minimization problem

Figure 4.15 presents the coverage of systems selected. According to the figure, the coverage is the same as the original problem. The cost is also the same as the original problem as the selection is same.

5 DISCUSSION AND CONCLUSION

The defense industry has special market dynamics and transactions are mainly decided Government-to-Government (G2G). In the industry, the procuring government can describe its requirements at high clarity and the market players try to satisfy those requirement sets. Nature of defense transactions requires a larger analysis of the government that is executing the procurement of defense products. The analysis includes examining the intentions of the government, and an in-depth consideration of the history of relationships, potential conflicts of interest, risk of reverse engineering. Thus, the government of the defense company filters and eliminates most of the potential customers and the company has restricted customer portfolio. On the other hand, the customer government also has a restricted candidate supplier portfolio. Therefore, marketing teams working in defense industry have a minor chance and such decision support systems help the teams finalize such transactions. The decision support system described in this thesis helps to generate comprehensive solutions. Being able to provide such solutions makes a striking difference for a company working in the defense industry.

The products of defense industry cannot be examined using internet or other information sources because of the secrecy culture. The important properties of products such as integrability, technology, and guidance are not accessible. Thus, the defense exhibitions organized by governments to advertise their defense industry products to their allies are critical places to encourage potential customers to purchase offered solutions. Governments with intention of purchasing defense products have a limited time interval to examine the products being presented at exhibitions. The exhibitions are great places to direct potential customers by presenting integrated solutions to where a company has idle capacity or desire to sell.

The governments targeting complete defense mechanisms need integrated solutions consisting surveillance and weapon systems that can communicate with each other. On the other hand, selling a single product is not evaluated as success for companies, the target is either to sell a family of products with exchangeable parts that can be easily sustained by smaller technician teams or integrated solutions that can ameliorate the performance of all systems in the network. Simultaneously, defense industry services are tailor-made and include high-level customization. Considering after sales support, selling a single item or selling in smaller quantities may become a burden. The success would be to sell an integrated defense solution where margins are higher and supporting systems increase benefit of each other. That is why the model described in this thesis that facilitates quick comprehensive solutions can be a game changer.

Procuring defense products is discussed at the highest level at every country as they require large investments. The situation is considered a strategic investment problem due to the greatness of required budget and longitude of lifespan. The strategic investment decisions are the most examined decisions and taken with extreme caution. That is why leaders tend to look for investments that are backed profoundly, and solutions with proven effectiveness. The model offers a solution to the described problem that is exactly optimal and that creates an improved accountability for decision makers.

In the defense industry, different products supporting each other with their capabilities are manufactured by a set of companies working in this industry. The sales of integrated comprehensive solutions include many stakeholders. In these kinds of contracts, the major slice of the contract amount is charged by the leading contractor. Being able to offer an integrated comprehensive solution while preparing the offers creates conditions to lead such contracts. The model described in the thesis is therefore a great leverage for leading larger projects including many companies in the defense industry.

The optimal selection and placement of military defense systems problem is an NP Hard problem. The constraint set is large, and problem size exponentially grows as the resolution increases. Thus, considering heuristic approaches such as Greedy Adding with Substitution as an extension to solve the problem may offer near optimal solutions that can be reached quicker. Additionally, the preliminary problem to this problem is converting subjective benefit term to a more standardized calculation methodology, there is no agreed upon way to calculate value attribution to benefits achieved by using such systems and it would do a great contribution to the literature. Furthermore, the model may be simplified using Lagrange relaxations and introducing required penalties in order to solve it for larger instances and examining the performance of the model for the whole land of a nation, seeing results with larger instances would be a great contribution to military operations research literature. Lastly, we overlooked the saturation of systems as there is no existing literature on this either, and it is our belief that studying how many different location weapon or surveillance systems can cover inside their range would create valuable inputs for the model described in this thesis. In the case investigated, the decision was a strategic decision as mentioned. Strategic decisions are made rarely over the years. The investment decisions are the decisions that are questioned the most and are taken by considering all option. Considering the situation, the model provided a good and comprehensive solution in an acceptable time. The defense investments require large sums of spending, consequently, optimization of these investment decisions lead to majestic savings.

APPENDIX A: TABLES

In this appendix, we present the dataset used for solving the model described in this thesis. The dataset includes attack types, technology set, range, distance sensitivity, cost, base effect vs. attack types

Attack Types

There are four attack types, their definitions and abbreviations are listed in Table App.A.1. Their relation to the problem is discussed in the problem description of the thesis. The expected attack increases benefit of covering the region against that attack type by 5%.

Table App.A.1: Types of attack

Definition	Abbreviation
Tribute Collection	TC
Military Attack	MA
Transportation / Convoy Attack	CA
Village Raid	VR

Tribute Collection attacks represent attacks where no violence is expected unless there is resistance by civilians. Military Attack represents attacks on military institutions where casualties are expected. Transportation Attack represents the attacks that took place in dirt roads connecting villages to each other. Village Raid attacks represent mass killings and burglary near town centers.

Technology Set Used in Main Scenario

Technology set in Table App.A.2 represents the technologies used in the base case scenario.

Table App.A.2: Technology set used in main scenario

Definition	Range (km)
Pan Tilt Zoom Night Vision Camera	2
Pan Tilt Zoom Night Vision Camera	4
Ground Surveillance Radar	4
Ground Surveillance Radar	10
Connected Movement Detection System	1
Remote Controlled Automated Weapon	2
Remote Controlled Automated Weapon	4
Remote Controlled Automated Weapon	10
Regular Weapon	2

Technology Set

Technology set in Table App.A.3 presents the range, distance sensitivity and cost of different technologies.

Table App.A.3: Technologies

Abbr.	Definition	Range	π_t*	Cost
PTZC2	Pan Tilt Zoom Night Vision Camera 2	2	0.4	4000
PTZC4	Pan Tilt Zoom Night Vision Camera 4	4	0.49	6000
GSR4	Ground Surveillance Radar 4	4	0	25000
GSR10	Ground Surveillance Radar 10	10	0.06	75000
GSR60	Ground Surveillance Radar 60	60	0.10	125000
GSR100	Ground Surveillance Radar 100	100	0.13	200000
RCAW2	Remote Controlled Automated Weapon 2	2	0	25000
RCAW4	Remote Controlled Automated Weapon 4	4	0	37500
RCAW10	Remote Controlled Automated Weapon 10	10	0	50000
CMDS	Connected Movement Detection System	1	0.7	67500
RW	Regular Weapon	2	0	10000

*π_t: Distance Sensitivity

Technologies' Base Effect versus Attack Types

Technologies have different effects versus different attack types. In Table App.A.4, assumed effects of every technology versus every attack type used while solving the model presented in this thesis are listed.

Table App.A.4: Technologies' base effect against attack types

Technology	Vs. TC	Vs. MA	Vs. CA	Vs. VR
PTZC2	3	3	3	3
PTZC4	3	3	3	3
GSR4	4	4	4	4
GSR10	4	4	4	4
CMDS	1	2	2	2
RCAW2	1	9	9	9
RCAW4	1	9	9	9
RCAW10	1	9	9	9
RW	1	7	7	7

The values in Table App.A.4 can be updates according to changing attack types or environmental conditions.

Attack Type vs. Probability of Getting Killed, Cost, Return

In the data preparation phase, the problem described in Appendix B is solved. The problem sees the circumstances from the terrorists' point of view. The terrorists are called considered as the attacker in the problem. Below data is used for calculating where the attacker should execute the next attacks to gain maximum benefit. The assumption is that the attacker knows the importance of every region to the government and makes the plans accordingly. Probability of getting killed (PoGK) and returns for every type of attack is presented in Table App.A.5.

Table App.A.5: Attack type vs. probability of getting killed, costs, returns

Definition	PoGK	Returns	Costs
Tribute Collection	0.135	2.022x	1.157x
Military Attack	0.494	6.574x	2.094x
Transportation / Convoy Attack	0.302	5.007x	2.263x
Village Raid	0.681	21.929x	4.649x

APPENDIX B: ATTACK SCENARIO GENERATION

The benefit of installing equipment to a location is correlated with the possibility of receiving an attack, the expected attacks change the relative importance of the region. The efficiency of a certain equipment against a certain attack type is also a determining factor of benefit. This relationship between attack type versus equipment is documented by military organizations and manufacturers. The distribution of attacks to regions is calculated in the first phase. The motivation of terrorists is to increase their benefit earned by executing attacks, e.g. killing army members, stealing and recruiting.

The terrorists have knowledge about existing military defenses. Throughout their encounters with the army, the weaknesses of military defenses are tested by terrorists. Therefore, terrorists can make conclusions about the risks of getting killed for each type of attack targeting different regions. The terrorists therefore have their tools to optimize benefits considering their options. To simulate the terrorists' decisions to select and execute attacks, below model is prepared.

The attack types have different benefits for terrorists, and they also have costs. Visits to villages having allegiance to terrorist organizations to recruit new members are providing minor benefits for terrorists, and attacks to town centers yield higher benefits. The detailed list of probability of getting killed, returns, and costs are in Table App. A.5.

The governments know the magnitude of the threat, probability of having a threat at a location, and its consequences related to threat types. The information creates the importance maps for the governments. If there is a pattern of repeating attacks of a certain type to a certain location. Then this location is considered more important, as it is most probably a first line of defense versus terrorists. Therefore, this knowledge is incorporated into the main problem. However, terrorists are also aware of the situation, and they may consider attacking places that does not expect the attacks, that are more vulnerable to attacks and can still provide considerable benefits to terrorists. Rather than repeating the pattern, terrorists try to make the best decision about using their resources to plan new attack.

The model simulates the attacker's problem, copying its decision-making objectives. Then the results of the model provide inputs to the main problem. The attacks this model generates increase the importance of the regions attacks took place.

Some attack types historically caused more casualties, benefit of organizing this kind of attacks is higher. Map of importance shows the importance of regions to government, executing terrorist activity at the respectively more important regions is much more dangerous to the governments, these attacks provide more benefits to terrorists. Therefore, the importance matrix is an input to the terrorists' problem.

In the region, terrorists' main resources are trade, attacks and tributes terrorists receive when they visit ally villages. Tribute collection activities, recruitment activities are incorporated into the model and represented as attack types. Terrorists also:

- Attack convoys to collect fuel, gadgets, money and weaponry.
- Attack civilian and medical facilities, attack military bases to harm the government and collect healthcare products and weaponry.
- Raid villages to collect money and other resources that are scarce in the region.

The villages in the region operate small gold mines, in an old-fashioned manner, and they do not have the means to sell the products in big cities. Terrorists collect the raw gold from villages and pay comparatively smaller prices to villagers. As terrorists operate near borders, they change countries via dirt roads to sell as civilian traders. In the investigated case, villagers do not have the means to reach larger cities. Due to not having any other way of selling products and not understanding the real value of the raw gold, some villagers show allegiance to terrorists and even help them recruit. As villages gets closer to the border, terrorists act more and more freer. Near border, when there are disagreements in the villages, the terrorists organize mass shootings to set an example. This lack of government presence creates a distrust towards the government. When there is refusal to trade with the terrorists, terrorists remind the importance of allegiance to them by repeating these kinds of shootings.

The terrorists understand that once they are seen in PTZ (pan, tilt, zoom) thermal cameras, or detected in radars, they risk being hunted. They map threatening technologies and avoid them, setting surveillance and radio-controlled weapon systems cease the terrorist activities.

They aim to learn and test the army defenses so that they can expand, they quickly learn newly placed technologies and have the exact coverage of the existing systems owned by governments. They avoid the learned locations unless they want to destroy the installed system or institution. Destroying such government institutions and installations gives a higher reputation, eventually yielding higher benefits.

Terrorists' problem is to select where to attack next. In planning, they have information about villages, including their allegiance, and their defense, in this scenario they also know the importance of regions to the government. The terrorists can predict outcome of attacks and related costs, they know chances of getting killed, they can differentiate technology (radars, cameras, and automated weaponry). Considering their budget, terrorists conduct attacks.

Sets:

I: set of regions

K: attack types

Parameters:

B: budget

ρ_i: probability of getting killed at village *i*

r_i: cost of visiting village *i*

g_{ik}: basic gain of visiting village *i* for attack type *k*

δ_{ik}: benefit of visiting village *i* for attack type *k* considering the risks

Decision Variables:

θ_{ik}: 1 if village *i* is raided by terrorists in attack type *k*, 0 otherwise

$$maximize \sum_{i \in I} \sum_{k \in K} \delta_{ik}\, \theta_{ik} \qquad \text{(App.B.1)}$$

$subject\ to$:

$$\sum_{k \in K} \sum_{i \in I} (r_i\, \theta_{ik}) \leq B \qquad \text{(App.B.2)}$$

The benefit of an attack is calculated by either collected amount or recruits, in the problem a generalized benefit function is used which can be customized according to investigated case. Objective function of terrorists' problem (App.B.1) maximizes the benefit considering gain types and attack decisions. Their constraint (App.B.2) is their resources.

Benefit function (App.B.3) is given below, expected gain of visiting a village according to attack type and the probability of getting killed.

$$\delta_{qj} = g_{qj}(1 - \rho_j) \qquad \forall j \in I, \forall q \in Q \qquad \text{(App.B.3)}$$

The results of attackers' problem are used when calculating the benefits of selecting and placing technologies. The government considers the regions where attacks took place as high benefit yielding regions. According to the importance of the regions, the benefit of covering attacks increases.

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