ESSAYS ON MICROECONOMICS

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ESSAYS ON MICROECONOMICS

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ABSTRACT

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In vertically related markets with downstream competition, an upstream firm sells inputs to competing downstream firms, and prices are determined through bilateral bargaining. Horn and Wolinsky (1988) propose a simultaneous bargaining model for input price determination. However, whether or not no information disclosure (hence, simultaneous bargaining) is in the best interest of the upstream firm needs to be analyzed. We demonstrate that when the downstream duopoly competes à la Cournot, the upstream firm achieves more profit with full information disclosure by adopting sequential bilateral bargaining. We argue that the upstream firm is entitled to choose the rules of conduct concerning negotiations as it is the single input supplier and has the ability to employ non-disclosure clauses in its agreements. Therefore, it is plausible to expect the upstream firm to disclose information by employing sequential bargaining rather than simultaneous bargaining. We also analyze the outcome when there is a price regulation in the market calling for the adoption of a single price for the input.

ÖZET

MİKROEKONOMİ ÜZERİNE MAKALELER

HÜSEYİN ÇELİK

EKONOMİ YÜKSEK LİSANS TEZİ, TEMMUZ 2024

Tez Danışmanı: Doç. Dr. Mehmet Barlo

Anahtar Kelimeler: Dikey Piyasalar, İşbirlikçi Pazarlık, Ardışık Pazarlık

Dikey ilişkili piyasalarda, üst endüstri firmaları alt endüstri firmalarına ara mal satarlar. Alt endüstrideki firmalar üretim yapmak için bu ara malı kullanır. Bu tezdeki modelde, üst endüstride tekel durumundaki firma, alt endüstride Cournot cinsi üretim miktarının belirlenmesi rekabetine giren firmalara girdi sağlamaktadır. Bu girdi fiyatları, firmalar arasında yürütülen ikili pazarlıklar sonucu belirlenir. Horn and Wolinsky (1988) bu piyasalar için eşzamanlı yürütülen bir pazarlık modeli öneriyor. Bu durumda, tekel pazarlıklar hakkında olan bilgiyi diğer firma ile paylaşmamış olmaktadır. Bunun yanında, eşzamanlı pazarlık yürütmek, üst endüstri firması için iyi bir strateji olmayabilir. Bu çalışmada, ardışık pazarlık sürecinin üst endüstri firması için daha yüksek kar getirdiğini gösteriyoruz. Bulgularımız, tekelin bilgi paylaşımını tercih ettiğini de ortaya koymaktadır. Devamında, fiyat kontrollerinin ve pazarlık sürecine getirilen kısıtlamaların firmaların karlılıklarına etkilerini inceliyoruz.

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1. INTRODUCTION

We analyze a vertical market where downstream firms have to acquire input from an upstream firm to produce a final homogeneous good, and the competition between the downstream firms is á la Cournot (resulting in an oligopolistic quantity competition). As usual, the price of the final good is determined with a linear inverse demand function. The price of the input, namely the input cost, is determined by bilateral negotiations conducted with the input supplier, while the price of the final homogeneous good is determined as a result of quantity competition in the downstream market. As a result, firms‘ profits depend not only on their bargaining outcome but also on the other firm's bargaining outcome.

Horn and Wolinsky (1988) proposed a solution concept for a simultaneous bargaining procedure in vertical markets. They also proposed a sequential bargaining procedure without determining the disagreement points. Obviously, the outcome of the sequential bargaining depends on the selection of the disagreement points.

In many settings of interest, buyers are assumed to be nonatomic and hence, are modeled as price takers. However, when a handful of downstream firms buy inputs from a single input supplier, bilateral negotiations are more plausible. The input price depends on the quantity of input demanded in a bargaining scenario which in turn depends on the input price. As we work under perfect information, when a contract (specifying the price of input) between the supplier and a downstream firm is signed, the corresponding quantity of input is fixed as the input price is determined. Thus, without a loss of generality, one can assume that the contract between the supplier and any one of the downstream firms specifies the input price as well as the input quantity.

Consequently, the bargaining between the supplier and one of the downstream firms (to determine the match specific input prices and quantities) depends on the total production of the homogeneous good; hence, the bargaining between the very same supplier and the other downstream firms.

If the bargaining outcome of the other downstream firms is not known by downstream firms during an input price/quantity negotiation with the upstream firm (alternatively, the supplier), it is as if the upstream firm bargains with the downstream firms simultaneously. This follows the linear marginal cost assumption we have in our model: The cost of producing one unit of input is fixed and equals zero for simplicity. This enables us to dismiss the emergence of interdependent bargaining considerations by the supplier as it views each bargaining separately (because it cares only about the total amount of input goods sold). In this case, the Nash equilibrium of the bargaining outcomes is used as a solution concept: The outcome of the bargaining between any one of the given downstream firms and the supplier is a best response to the bargaining outcomes that emerge between the other downstream firms and the supplier.

However, this is not the sole method to conduct input price negotiations. The input supplier may sequentially conduct price negotiations by sharing the outcome of previous negotiations with downstream firms engaging in competition. At each history, upstream and downstream firms observe the outcome of previous bargaining problems and co-determine their input price under complete information. As a result, the bargaining set at each history depends on the previous bargaining results. Therefore the subgame perfect equilibrium with dynamic best responses is the appropriate solution concept. In sequential bargaining, the outcome of the bargaining between any one of the given downstream firms and the supplier is a dynamic best response to the bargaining outcomes that have been realized and are expected to emerge (under populations of rationality) between the other downstream firms and the supplier. Indeed, the order of bargaining determines the profit realizations of each firm since the sequence determines the information available to the downstream firms, which leads to a network structure in bargaining. Whether or not the upstream firm, 'the informed/entitled party,' chooses to disclose information when the downstream firms are engaged in a competition among themselves is an interesting question in bargaining involving network structures. This model is just one example in that setting. We study a model where the upstream firm either discloses the information or does not under a fixed sequence (network) of bargaining. However, the network structure, as well as disclosure strategies, can be determined randomly with a probability distribution.

Depending on the disagreement points, either the first mover or the second mover has an advantage in sequential bargaining. However, selecting disagreement points should not be random but economically intuitive. By using a random dictatorship procedure, we compute the disagreement points and show that there is a second-mover advantage in a Cournot competition with two downstream firms. We also show that the upstream firm obtains a higher profit when the bargaining is conducted sequentially compared to a simultaneous bargaining model.

The information available to the firms affects the cooperative bargaining outcomes. While an input price is co-determined between a downstream firm and an upstream firm, they know their profit structures and those of other downstream firms. However, the downstream firm may or may not know the (realized) input costs of the other firms and the quantity of input provided to the other firms. As the sole input provider in the vertical market, the upstream firm can disclose information on the other bargaining outcomes if it chooses to do so. Indeed, by imposing no disclosure clauses to its contracts with the downstream firms, the supplier has the ability to prevent downstream firms from sharing the realized input prices and quantities. Consequently, the supplier is the only party that knows the input price charged and the amount of input supplied to every downstream firm. Thus, the supplier can strategically use the available information by disclosing or hiding it when bargaining with a downstream firm. This ability to use or hide this information brings additional profit opportunities to the supplier.

We show that the upstream firm can obtain a higher profit with full disclosure (sequential bargaining) compared to no disclosure (simultaneous bargaining) when there are two downstream firms. The downstream firms are adversely affected. The additional profit obtained by the disclosure should be regarded as the value-added of the supplier's information disclosure ability.

However, when there are three downstream firms, and Firm 2 and 3's simultaneous bargaining with the upstream firm follows the bargaining (and information disclosure) of the supplier with Firm 1, the upstream firm does not benefit from disclosing the outcome of bargaining with Firm 1. In this setting, we show that simultaneous bargaining with all the firms is what the supplier prefers. Consequently, in this network structure, there is no value-added of the supplier's information disclosure ability.

We add two additional specifications to the model: price regulations and bargaining regulations. If an input supplier cannot charge different prices to downstream markets due to equal treatment, we observe a first-mover advantage contrary to the baseline model. Another regulation imposed in the market may be the order of negotiation. When the upstream firm has to separate a downstream firm and negotiate with this firm before the other firms, the first negotiator has an advantage over the others, and the firms bargaining simultaneously are negatively affected by the regulation.

In Chapter 2, we discuss the related literature on vertical markets and bargaining. Chapter 3 covers cooperative bargaining theory and the selection of disagreement points. We present the model in Chapter 4 and solve a sequential bargaining model with two downstream firms, followed by a simultaneous bargaining model. We show that the upstream firm can obtain a higher profit in sequential than simultaneous bargaining. In Chapter 5, we analyze incentives for merges and test the countervailing hypothesis. Next, we discuss price regulations in a sequential bargaining game in Chapter 6. Another regulation regarding the bargaining procedure, a mixture of simultaneous and sequential bargaining, is discussed in Chapter 7. Finally, we generalize the number of firms in a simultaneous bargaining model in Chapter 8.

2. RELATED LITERATURE

In their seminal work, Horn and Wolinsky (1988) study a duopoly where inputs are provided by suppliers through bilateral bargaining to analyze incentives for merging. Following their contribution, many other studies used bilateral bargaining to analyze various markets such as health care. Gaynor and Town (2011) identify a bargaining problem between hospitals and health plans to determine the amount of money paid for each patient that the hospital treats. Gowrisankaran et al. (2015) study the effect of hospital mergers on patients with a bargaining model between hospitals and managed care organizations. This model has been applied to many other industries, including television markets (Crawford and Yurukoglu 2012). They compare à la carte pricing to the bundling of channels and find that input costs increase under à la carte pricing.

Disagreement points, as well as the profits of the firms, affect the outcome of the axiomatic Nash bargaining solutions. Abreu and Manea (2024) study exclusion strategies in sequential bargaining settings to increase the seller's profit. In many settings, buyers have individualistic consumption, and their utility depends only on their bargaining outcome. When there are enough goods to serve every buyer, there is no competition among the buyers, unlike vertical markets where downstream firms compete in quantities. Abreu and Manea (2024) use exclusion to ignite competition among buyers; negotiations occur sequentially, starting from the weakest type to dynamically improve the seller's outside option. They identify the optimal bargaining procedure in sequential bargaining with exclusion strategies.

In vertical markets, there is already competition among buyers due to the oligopoly structure. Therefore, without excluding firms, the seller (upstream firm) can profit more by improving the outside option, disagreement point, after a negotiation when bargaining is conducted sequentially.

The solution concept used in simultaneous bargaining procedures is called Nash in Nash Bargaining. According to this solution concept, $(c_i^*)_{i \in N}$ is a solution to the simultaneous bargaining problem if c_i^* is the outcome of (F_i, S) given $c_j^*, j \neq i$. This approach has been criticized (Collard-Wexler, Gowrisankaran, and Lee 2019) because it uses a cooperative solution concept (Nash bargaining) with a non-cooperative solution concept (Nash equilibrium). They propose an alternating offer bargaining model where the AO prices converge to Nash bargaining prices when the time interval between offers goes to 0, similar to Binmore, Rubinstein, and Wolinsky (1986).

Due to countervailing hypothesis (Galbraith 1954), a concentration in downstream competition may benefit consumers with stronger firms negotiating their input cost better. Due to lower input costs, firms can produce final goods at a lower price, which benefits consumers. However, Iozzi and Valletti (Iozzi and Valletti 2014) show that under quantity competition, a downstream concentration does not decrease the consumer price no matter what the market parameters are. They analyze a simultaneous bargaining problem with two specifications: observable and unobservable breakdowns of negotiations under quantity and price competitions.

The information available to the players in a cooperative or non-cooperative game affects how players engage in a strategic environment and the outcome of the game. Even though the information structure is sometimes assumed to be predetermined, it can be endogenously determined within the game. An informed party can fully or partially disclose information to the other players strategically. Therefore, a sender can determine an optimal information disclosure strategy to maximize utility; see: (Kolotilin 2018), (Rayo and Segal 2010).

3. COOPERATIVE BARGAINING

Nash (1950) proposed a cooperative solution concept for surplus sharing among two rational agents. Under complete information, they co-determine a variable of interest, determining their payoffs when they have a conflict of interest. After a while, non-cooperative bargaining models have been presented, and their relation to the cooperative bargaining model has been analyzed (Binmore, Rubinstein, and Wolinsky 1986). A general Nash bargaining model between two players consists of a set of feasible payoffs ($S \subseteq \mathbb{R}^2$) and a pair of disagreement points (d_1, d_2) that players obtain in case they fail to agree. (S, d) defining a bargaining problem satisfies the following properties.

- S is closed, convex, bounded
- $d \in S$ and there exists $x \in S$ such that $x > d$

A bargaining rule maps each bargaining problem to a pair of payoffs. We will use a generalized Nash rule, which maximizes the multiplication of the surplus of players assuming $x > d$

$$N(S,d) = arg\max_{x \in S}(x_1 - d_1)^{\frac{1}{2}}(x_2 - d_2)^{\frac{1}{2}}$$

Bargaining power determines the importance of surplus for each player. We assume that players have equal bargaining power. Considering a problem between a downstream firm (Firm 1) and an input supplier, we can define the set of feasible payoffs

$$S = \Big\{(\Pi_1, \Pi_s) \in \mathbb{R}^2_+ | \quad 0 \leq \Pi_1 \leq (1 - q_1 - q_2 - c_1)q_1 \quad \text{and} \quad 0 \leq \Pi_s \leq c_1 q_1 + c_2 q_2\Big\}$$

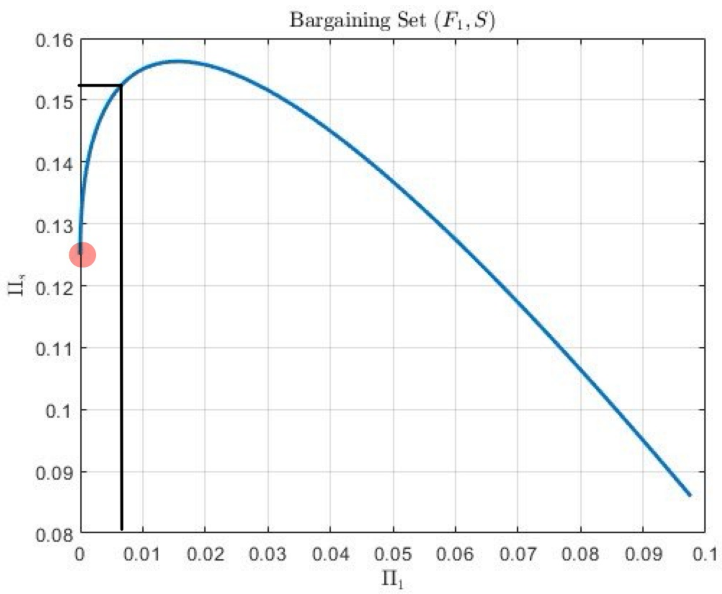
When players fail to agree on c_1, they obtain $d = (d_1, d_s) > 0$

Bargaining set S is d-comprehensive if $\forall x \in S, \forall y \in \mathbb{R}^2$ satisfying $d \leq y \leq x$, we have $y \in S$. This assumption means that players can freely discard their utilities up to the disagreement point. Comprehensiveness is a useful property since it implies a non-levelness axiom. Bargaining set S is non-level if every weakly Pareto optimal payoff vector is Pareto optimal. All of the studies presenting convergence of sub-game perfect equilibrium of non-cooperative bargaining games to Nash bargaining results assume that the bargaining set satisfies non-levelness (Herings and Predtetchinski 2011).

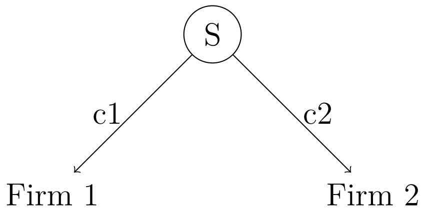
Barlo and Ilkılıc (2023) analyze a set of Nash Bargaining solutions failing to have comprehensive and non-level bargaining sets. While determining input prices, both parties can have a common interest in the decrease in input prices, which makes the bargaining solution redundant. The same problem persists in vertical markets when the disagreement points are determined as a breakdown of the negotiation, and firms obtain 0 profit in disagreement. While determining disagreement as a breakdown provides calculation convenience, the non-levelness condition may fail and can be restored with a dictatorship procedure (De Clippel 2007).

In vertical markets literature, if the disagreement point of the downstream firm is assumed to be 0, assuming that the downstream firm will not be able to produce in case of breakdown, the bargaining set does not satisfy d-comprehensiveness. Therefore applying a dictatorship procedure is plausible. Figure 3.1 displays the failure of d-comprehensiveness in a bargaining problem in vertical markets.

Figure 3.1 Failure of D-Comprehensiveness



4. MODEL



There are N downstream firms and a single upstream supplier (S) who sells perfectly divisible inputs to downstream firms. A single input is required for each output. Bilateral negotiations between downstream and upstream firms determine linear input costs. While the downstream firms incur their input costs, the upstream firm does not have any cost of production for the inputs or capacity constraints. Upstream and downstream firms do not have fixed costs. Downstream firms compete in quantities in the final goods market. In the previous studies (Horn and Wolinsky 1988), downstream quantity competition starts after the input prices are determined and announced. Foreseeing the outcome resulting from the quantity competition, firms conduct bargaining simultaneously or sequentially at the first stage. Then, in the second stage, production occurs with simultaneous quantity competition, regardless of whether bargaining is simultaneous or sequential. In our model, firms specify the quantity of input/output in the contract along with input cost. Therefore, downstream firms compete in a simultaneous quantity competition when the bargaining is simultaneous, whereas, with sequential bargaining, downstream competition becomes sequential. We assume that the input(output) amount is fixed with a contract right after the price is determined and announced to the public. Therefore, a downstream firm will observe the quantity ordered(produced) by other firms before deciding on its quantity. The profits of the firms are given below,

$$\Pi_1 = (1 - q_1 - q_2 - c_1)q_1$$

$$\Pi_2 = (1 - q_1 - q_2 - c_2)q_2$$

$$\Pi_s = c_1 q_1 + c_2 q_2$$

4.1 Sequential Bargaining

Suppose there are two downstream firms, Firm 1 (F_1) and Firm 2 (F_2), and an input supplier. At the game's first stage, Firm 1 bargains with the input supplier for c_1. When the bargaining is completed, Firm 1 decides on the number of quantity q_1. c_1 and q_1 are fixed with a contract. In the second stage, observing $(\bar{c}_1, \bar{q}_1)$, Firm 2 bargains with the input supplier.

The sequential bilateral bargaining with two downstream firms involves the determination of the bargaining set as well as the disagreement points based on the history of the bargaining. That is, the bargaining set and disagreement points involving the bargaining between the upstream firm and the second downstream firm depend on the agreement (resulting from the bargaining) between the upstream firm and the first downstream firm. This implies that dynamic rationality is at play in the first round of the bargaining, as the first downstream firm as well as the upstream firm in the first round, form time-consistent expectations about what will happen in the next period.

4.1.1 Bargaining with Firm 2

Given $(\bar{c}_1, \bar{q}_1)$, we can define the bargaining problem between Firm 2 and supplier, (F_2, S). Each $(\bar{c}_1, \bar{q}_1)$ results in a different bargaining set for Firm 2 and the supplier.

$$S_2 = \Big(\Pi_2(c_2), \Pi_s(c_2)\Big) \quad \text{such that} \quad c_2 \in [0,1]$$

$$S_2(\bar{c}_1, \bar{q}_1) = \Big\{(\Pi_2, \Pi_s) \in \mathbb{R}^2_+ | \quad \Pi_2 \leq (1 - \bar{q}_1 - q_2 - c_2)q_2 \quad \text{and} \quad \pi_s \leq \bar{c}_1 \bar{q}_1 + c_2 q_2\Big\}$$

After determining c_2, Firm 2 maximizes its profit by choosing optimal q_2^* that maximizes their profit.

$$\max_{q_2} \Pi_2 = (1 - \bar{q}_1 - q_2 - c_2)q_2$$

$$\textbf{F.O.C:} \quad \frac{\partial \Pi_2}{\partial q_2} = 1 - \bar{q}_1 - 2q_2 - c_2 = 0 \quad \Longrightarrow \quad q_2^* = \frac{1 - \bar{q}_1 - c_2}{2}$$

$$\textbf{S.O.C:} \quad \frac{\partial^2 \Pi_2}{\partial q_2^2} = -2 < 0$$

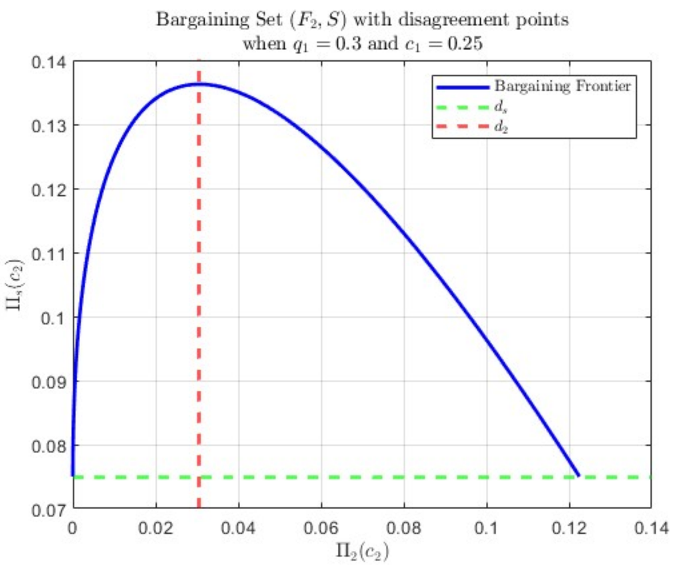
The determination of c_2 via bargaining will result in the following profit realizations:

$$\Pi_2(c_2) = (1 - \bar{q}_1 - q_2^*(c_2) - c_2)q_2 = \left(\frac{1 - \bar{q}_1 - c_2}{2}\right)^2$$

$$\Pi_s(c_2) = \bar{c}_1\bar{q}_1 + c_2\left(\frac{1 - \bar{q}_1 - c_2}{2}\right)$$

We can portray the bargaining set for a given pair of $(\bar{c}_1, \bar{q}_1) = (\frac{1}{4}, \frac{3}{10})$. Values of $c_2 \in (0, 1 - \bar{q}_1)$, (Π_2, Π_s) generates the bargaining set.

Figure 4.1 Bargaining Set (F_2, S)



Bargaining set (Π_2, Π_s) with disagreement points (d_2, d_s) defines a bargaining problem.

d_2 : The supplier maximizes its profit

$$\max_{c_2} \bar{c}_1\bar{q}_1 + c_2\left(\frac{1 - \bar{q}_1 - c_2}{2}\right) \Longrightarrow \hat{c}_2 = \frac{1 - \bar{q}_1}{2} \Longrightarrow d_2 = \Pi_2(\hat{c}_2) = \left(\frac{1 - \bar{q}_1}{4}\right)^2$$

d_s : Firm 2 would set its input price $c_2 = 0 \Longrightarrow d_s = \Pi_s(\bar{c}_1, \bar{q}_1, c_2 = 0) = \bar{c}_1\bar{q}_1$

Nash rule maps the bargaining problem (F_2, S) to a unique payoff profile $(\Pi_2(c_2^*), \Pi_s(c_2^*))$. c_2^* solves the following Nash program where firms have equal bargaining powers.

$$\max_{c_2 \in (0, 1-\bar{q}_1)} \Omega_2 = \Big(\Pi_2 - d_2\Big)^{\frac{1}{2}} \Big(\Pi_s - d_s\Big)^{\frac{1}{2}}$$

$$\max_{c_2 \in (0, 1-\bar{q}_1)} \Omega_2 = \Big(\Big(\frac{1-\bar{q}_1-c_2}{2}\Big)^2 - \Big(\frac{1-\bar{q}_1}{4}\Big)^2\Big)^{\frac{1}{2}} \Big(c_2\Big(\frac{1-\bar{q}_1-c_2}{2}\Big)\Big)^{\frac{1}{2}}$$

The bargaining problem is defined for c_2 values where Firm 2 and the upstream firm have a positive surplus.

$$\Big(\frac{1-\bar{q}_1-c_2}{2}\Big)^2 - \Big(\frac{1-\bar{q}_1}{4}\Big)^2 > 0 \implies \frac{1-\bar{q}_1}{2} > c_2$$

F.O.C:

$$\frac{\partial \Omega_2}{\partial c_2} = \frac{\sqrt{c_2\left(\frac{1-c_2-\bar{q}_1}{2}\right)}\left(\frac{c_2+\bar{q}_1-1}{2}\right)}{2\sqrt{\left(\frac{1-c_2-\bar{q}_1}{2}\right)^2 - \left(\frac{\bar{q}_1}{4}-\frac{1}{4}\right)^2}} + \frac{\sqrt{\left(\frac{1-c_2-\bar{q}_1}{2}\right)^2 - \left(\frac{\bar{q}_1}{4}-\frac{1}{4}\right)^2}\left(\frac{1}{2}-c_2-\frac{\bar{q}_1}{2}\right)}{2\sqrt{c_2\left(\frac{1-c_2-\bar{q}_1}{2}\right)}} = 0$$

There is only one solution in the bargaining domain

$$c_2^*(q_1) = \frac{(3-\sqrt{5})(1-q_1)}{4}$$

S.O.C: We need to verify that the first-order condition maximizes the problem. Strict convexity in the domain of the bargaining problem is needed.

$$\frac{\partial^2\Omega_2}{\partial c_2^2} = \frac{\sqrt{c_2\left(\frac{1}{2}-\frac{c_2}{2}-\frac{q1}{2}\right)}}{4\sqrt{\left(\frac{c_2}{2}+\frac{q1}{2}-\frac{1}{2}\right)^2-\left(\frac{q1}{4}-\frac{1}{4}\right)^2}} - \frac{\sqrt{\left(\frac{c_2}{2}+\frac{q1}{2}-\frac{1}{2}\right)^2-\left(\frac{q1}{4}-\frac{1}{4}\right)^2}}{2\sqrt{c_2\left(\frac{1}{2}-\frac{c_2}{2}-\frac{q1}{2}\right)}}$$
$$-\frac{\sqrt{\left(\frac{c_2}{2}+\frac{q1}{2}-\frac{1}{2}\right)^2-\left(\frac{q1}{4}-\frac{1}{4}\right)^2}\left(c_2+\frac{q1}{2}-\frac{1}{2}\right)^2}{4\left(-c_2\left(\frac{c_2}{2}+\frac{q1}{2}-\frac{1}{2}\right)\right)^{3/2}} - \frac{\sqrt{c_2\left(\frac{1}{2}-\frac{c_2}{2}-\frac{q1}{2}\right)}\left(\frac{c_2}{2}+\frac{q1}{2}-\frac{1}{2}\right)^2}{4\left(\left(\frac{c_2}{2}+\frac{q1}{2}-\frac{1}{2}\right)^2-\left(\frac{q1}{4}-\frac{1}{4}\right)^2\right)^{3/2}}$$
$$-\frac{\left(\frac{1}{2}-\frac{2c_2}{2}-\frac{q1}{2}\right)\left(\frac{1}{2}-\frac{c_2}{2}-\frac{q1}{2}\right)}{2\sqrt{\left(\frac{c_2}{2}+\frac{q1}{2}-\frac{1}{2}\right)^2-\left(\frac{q1}{4}-\frac{1}{4}\right)^2}\sqrt{c_2\left(\frac{1}{2}-\frac{c_2}{2}-\frac{q1}{2}\right)}}$$

The second derivative is composed of 5 fractions. The first one has a positive coefficient, whereas all the others are negative when $\frac{1-q_1}{2} > c_2$. Note that if $\frac{1-q_1}{2} < c_2$, the last fraction also becomes positive. By definition, $1 - c_2 - q_1 > 0$ is always satisfied for firms to have positive profits. If the absolute value of a fraction with a negative coefficient is larger than the first fraction, then the second derivative is negative. Therefore, the second-order condition holds. We can show this by comparing the first and the fourth fractions.

Claim:

$$\frac{\sqrt{c_2\left(\frac{1}{2}-\frac{c_2}{2}-\frac{q1}{2}\right)}\left(\frac{c_2}{2}+\frac{q1}{2}-\frac{1}{2}\right)^2}{4\left(\left(\frac{c_2}{2}+\frac{q1}{2}-\frac{1}{2}\right)^2-\left(\frac{q1}{4}-\frac{1}{4}\right)^2\right)^{3/2}} > \frac{\sqrt{c_2\left(\frac{1}{2}-\frac{c_2}{2}-\frac{q1}{2}\right)}}{4\sqrt{\left(\frac{c_2}{2}+\frac{q1}{2}-\frac{1}{2}\right)^2-\left(\frac{q1}{4}-\frac{1}{4}\right)^2}}$$

In the denominators, we have firm 1's surplus to the powers of $\frac{3}{2}$ and $\frac{1}{2}$, which is required to be positive by definition. Therefore, we can multiply each side with $4\left(\left(\frac{c_2}{2}+\frac{q1}{2}-\frac{1}{2}\right)^2-\left(\frac{q1}{4}-\frac{1}{4}\right)^2\right)^{3/2}$. Moreover, we have $\sqrt{c_2(\frac{1}{2}-\frac{c_2}{2}-\frac{q_1}{2})}$ at both of the numerators. So we can also cancel them out.

$$\Big(\frac{1}{2}-\frac{q_1}{2}-\frac{c_2}{2}\Big)^2 > \Big(\frac{1}{2}-\frac{q_1}{2}-\frac{c_2}{2}\Big)^2 - \Big(\frac{1}{4}-\frac{q_1}{4}\Big)^2$$

The inequality is satisfied for all $q_1 \neq 1$

4.1.2 Firm 1's Problem

For any agreement Firm 1 forms, (c_1, q_1), the outcome of the bargaining (F_2, S) will be $c_2^* = \frac{(3-\sqrt{5})(1-q_1)}{4}$ and $q_2^* = \frac{1-q_1-c_2}{2} = \frac{(1+\sqrt{5})(1-q_1)}{8}$. Therefore, during the first negotiation, Firm 1 and the upstream firm know what the outcome will be at the next stage.

$$\Pi_1 = (1 - q_1 - q_2 - c_1)q_1$$

For any c_1, Firm 1 determines the quantity of good to be produced, q_1^* that maximizes their profit. **F.O.C:**

$$\max_{q_1} \Pi_1 = \Big(1 - q_1 - \frac{(1+\sqrt{5})(1-q_1)}{8} - c_1\Big)q_1 = \Big(\frac{(7-\sqrt{5})}{8}(1-q_1) - c1\Big)q_1$$

$$\frac{\partial \Pi_1}{\partial q_1} = \frac{(7-\sqrt{5})}{8} - \frac{(7-\sqrt{5})}{4}q_1 - c1 = 0 \implies q_1^* = \frac{1}{2} - \frac{4}{7-\sqrt{5}}c_1$$

$$q_1^*(c_1) = \begin{cases} \left(\frac{1}{2} - \frac{4}{7-\sqrt{5}}c_1\right) & \text{if } c_1 \in \left[0, \frac{7-\sqrt{5}}{8}\right] \\ 0 & \text{if } c_1 \in \left[\frac{7-\sqrt{5}}{8}, 1\right] \end{cases}$$

S.O.C:

$$\frac{\partial^2 \Pi_1}{\partial q_1^2} = -\frac{7-\sqrt{5}}{4} < 0$$

The second-order condition is also satisfied. Therefore, the first-order condition provides the optimal q_1 for each value of c_1. Now we can compute $q_2^*(c_1), c_2^*(c_1)$.

$$q_2^*(c_1) = \frac{1-q_1-c_2}{2} = \frac{(1+\sqrt{5})(1-q_1)}{8} = \frac{1+\sqrt{5}}{8}\left(\frac{1}{2} + \frac{4}{7-\sqrt{5}}c_1\right)$$

$$c_2^*(c_1) == \frac{(3-\sqrt{5})(1-q_1)}{4} = \frac{3-\sqrt{5}}{4}\left(\frac{1}{2} + \frac{4}{7-\sqrt{5}}c_1\right)$$

First, we compute Firm 1 and the suppliers' profits as functions of c_1

$$\Pi_1(c_1) = \left(1 - \left(\frac{1}{2} - \frac{4}{7-\sqrt{5}}c_1\right) - \left(\frac{1+\sqrt{5}}{8}\right)\left(\frac{1}{2} + \frac{4}{7-\sqrt{5}}c_1\right) - c_1\right)\left(\frac{1}{2} - \frac{4}{7-\sqrt{5}}c_1\right)$$

$$\Pi_1(c_1) = \frac{2}{7-\sqrt{5}}c_1^2 - \frac{c_1}{2} + \frac{7-\sqrt{5}}{32}$$

$$\Pi_s = c_1\left(\frac{1}{2}-\frac{4}{7-\sqrt{5}}c_1\right)+\frac{(1+\sqrt{5})(3-\sqrt{5})}{32}\left(\frac{1}{2}+\frac{4}{7-\sqrt{5}}c_1\right)^2$$

We can define the bargaining problem between Firm 1 and the supplier. Bargaining set with disagreement points (d_1, d_s) generates the following problem.

$$S_1 = \left\{\Big(\Pi_1(c_1), \Pi_s(c_1)\Big) \in \mathbb{R}^2_+ \quad \text{such that } c_1 \in [0,1]\right\}$$

d_s : Firm 1 decides on $\hat{c_1}$ as a dictator

$$\max_{c_1} \Pi_1(c_1) : \frac{\partial \Pi_1}{\partial c_1} = \frac{4}{7-\sqrt{5}}c_1 - \frac{1}{2} = 0 \implies \hat{c_1} = \frac{7-\sqrt{5}}{8}$$

$$d_s = \Pi_s(\hat{c_1}) = \frac{2\sqrt{5}-2}{128} + \frac{26-2\sqrt{5}}{64} + \frac{32\sqrt{5}-118}{77(3-\sqrt{5})} \cdot \frac{54-14\sqrt{5}}{64} \approx 0.07$$

d_1 : The upstream firm decides on $\hat{c_1^s}$ that maximizes its profit as a dictator.

$$\max_{c_1} \Pi_s = \frac{2\sqrt{5}-2}{128} + \frac{26-2\sqrt{5}}{56-8\sqrt{5}}c_1 + \left(\frac{128\sqrt{5}-456}{896-304\sqrt{5}}\right)c_1^2$$

$$\frac{\partial \Pi_s}{\partial c_1} = \frac{26-2\sqrt{5}}{56-8\sqrt{5}} + 2\cdot c_1 \cdot \frac{128\sqrt{5}-456}{896-304\sqrt{5}} = 0 \implies \hat{c_1^s} = \frac{3292-1212\sqrt{5}}{7664-2704\sqrt{5}} \approx 0.3597$$

$$d_1 = \Pi_1(\hat{c_1^s}) = \frac{2}{7-\sqrt{5}}\left(\frac{3292-1212\sqrt{5}}{7664-2704\sqrt{5}}\right)^2 - \frac{1}{2}\left(\frac{3292-1212\sqrt{5}}{7664-2704\sqrt{5}}\right) + \frac{7-\sqrt{5}}{32} \approx 0.0233$$

S_1 accompanied with (d_1, d_s) defines a bargaining problem between Firm 1 and the supplier. c_1 that maximizes the following product is the solution to the bargaining problem. A graph for the Nash program is provided below.

$$\Omega_1 = \left(\frac{2}{7-\sqrt{5}}c_1^2 - \frac{c_1}{2} - \frac{2}{7-\sqrt{5}}\left(\frac{3292-1212\sqrt{5}}{7664-2704\sqrt{5}}\right)^2 + \frac{1}{2}\left(\frac{3292-1212\sqrt{5}}{7664-2704\sqrt{5}}\right)\right)^{\frac{1}{2}}$$
$$\left(\frac{26-2\sqrt{5}}{56-8\sqrt{5}}c_1 + \left(\frac{128\sqrt{5}-456}{896-304\sqrt{5}}\right)c_1^2 - \frac{26-2\sqrt{5}}{64} - \frac{32\sqrt{5}-118}{77(3-\sqrt{5})} \cdot \frac{54-14\sqrt{5}}{64}\right)^{\frac{1}{2}}$$

Nash program is the multiplication of the surpluses the firms get. For the program to consist of real numbers, firms' surpluses should be positive. $\Pi_1 - d_1$ is positive on $(0, 0.35967)$, and $\Pi_s - d_s$ is positive on $(0.12343, 0.59)$. Therefore, the program is defined on the intersection of these two intervals, which is $(0.12343, 0.35967)$

F.O.C: $\frac{\partial \Omega_1}{\partial c_1} = 0 \implies c_1^* = 0.2112$

We can display the bargaining set with disagreement points and the corresponding Nash program

Figure 4.2 Bargaining Set (F_1, S) with Disagreement Points

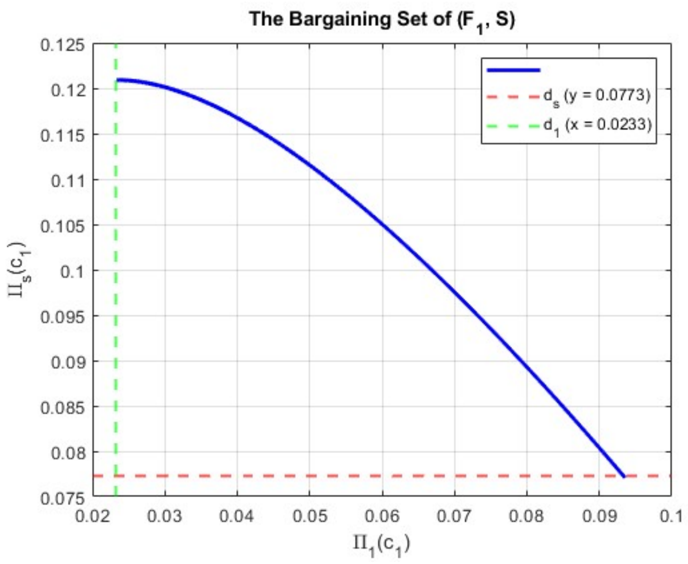
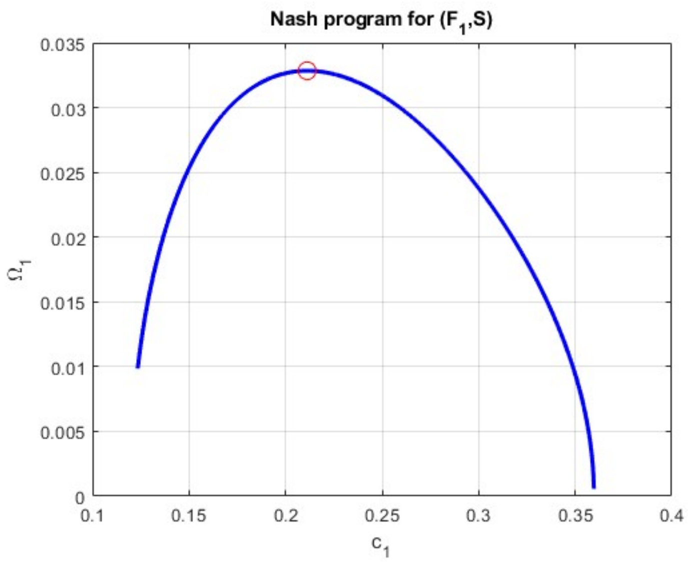


Figure 4.3 Nash Program for (F_1, S)



The Second-Order Condition

The Nash program is of the form

$$\Omega_1 = (x \cdot c_1 + y \cdot c_1^2 + z)^{\frac{1}{2}} \cdot (a \cdot c_1 + b \cdot c_1^2 + t)^{\frac{1}{2}}$$

- $x = \frac{-1}{2}$
- $y = \frac{2}{7-\sqrt{5}}$
- $z = -\frac{2}{7-\sqrt{5}}\left(\frac{3292-1212\sqrt{5}}{7664-2704\sqrt{5}}\right)^2 + \frac{1}{2}\left(\frac{3292-1212\sqrt{5}}{7664-2704\sqrt{5}}\right)$
- $a = \frac{26-2\sqrt{5}}{56-8\sqrt{5}}$
- $b = \frac{128\sqrt{5}-456}{896-304\sqrt{5}}$
- $t = -\frac{26-2\sqrt{5}}{64} - \frac{32\sqrt{5}-118}{77(3-\sqrt{5})} \cdot \frac{54-14\sqrt{5}}{64}$

$$\Omega_1 = \left(byc_1^4 + (bx+ay)c_1^3 + (ax+yt+bz)c_1^2 + (xt+az)c_1 + zt\right)^{\frac{1}{2}}$$

$$\frac{\partial \Omega_1}{\partial c_1} = \frac{1}{2}\left(byc_1^4 + (bx+ay)c_1^3 + (ax+yt+bz)c_1^2 + (xt+az)c_1 + zt\right)^{-\frac{1}{2}} \cdot \left(4byc_1^3 + 3(bx+ay)c_1^2 + 2(ax+yt+bz)c_1 + (xt+az)\right)$$

$$\frac{\partial^2 \Omega_1}{\partial c_1^2} = -\frac{1}{4}\left(byc_1^4 + (bx+ay)c_1^3 + (ax+yt+bz)c_1^2 + (xt+az)c_1 + zt\right)^{-\frac{3}{2}} \tag{4.1}$$

$$\cdot \left(4byc_1^3 + 3(bx+ay)c_1^2 + 2(ax+yt+bz)c_1 + (xt+az)\right)^2 \tag{4.2}$$

$$+ \frac{1}{2}\left(byc_1^4 + (bx+ay)c_1^3 + (ax+yt+bz)c_1^2 + (xt+az)c_1 + zt\right)^{-\frac{1}{2}} \tag{4.3}$$

$$\cdot \left(12byc_1^2 + 6(bx+ay)c_1 + 2(ax+yt+bz)\right) < 0 \tag{4.4}$$

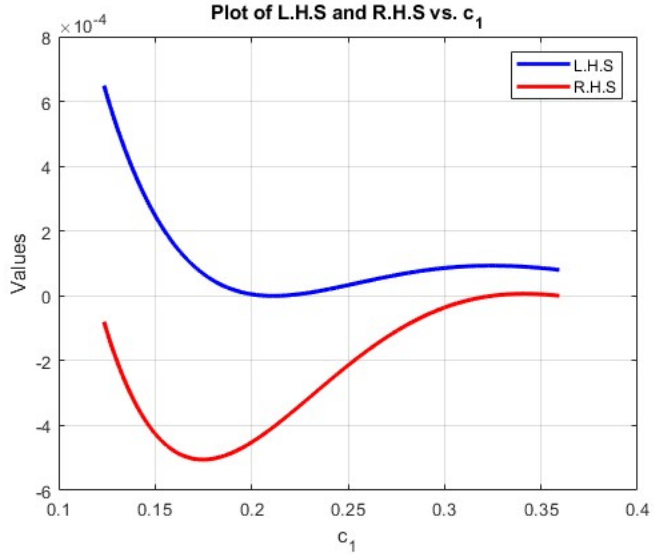
Taking the multiplication of (4.1) and (4.2) to the right-hand side of the equation:

$$\left(4byc_1^3 + 3(bx+ay)c_1^2 + 2(ax+yt+bz)c_1 + (xt+az)\right)^2 >$$

$$2\left(byc_1^4 + (bx+ay)c_1^3 + (ax+yt+bz)c_1^2 + (x+az)c_1 + zt\right)\left(12byc_1^2 + 6(bx+ay)c_1 + 2(ax+yt+bz)\right)$$

To show that the left-hand side is larger than the right-hand side, I will provide a plot

Figure 4.4 The Second-Order Condition



The outcome of the sequential bargaining game is summarized below,

- $c_1^*(\emptyset) = 0.2112$
- $q_1^*(c_1^*) = 0.3227$
- $c_2^*(c_1^*, q_1^*) = 0.1294$
- $q_2^*(c_1^*, q_1^*, c_2^*) = 0.2740$

We can compute the profits for Firm 1, Firm 2, and the input supplier (S).

- $\Pi_1 = (1 - q_1^* - q_2^* - c_1^*)q_1^* = 0.0620$
- $\Pi_2 = (1 - q_1^* - q_2^* - c_2^*)q_2^* = 0.0750$
- $\Pi_s = c_1^* q_1^* + c_2^* q_2^* = 0.1036$

As a result of quantity competition, the price of the final good is:

- $p = 1 - q_1 - q_2 = 0.4033$

4.2 Simultaneous Solution

Assuming that the negotiations take place simultaneously, we can adopt the Nash in Nash bargaining approach. Assuming (c_i, q_i) is given, we solve for (c_j, q_j). Simultaneous bargaining results in a symmetric outcome since Firm 1 and Firm 2 have identical cost structures.

$$c_2^*(q_1) = \frac{(3-\sqrt{5})(1-q_1)}{4} \qquad c_1^*(q_2) = \frac{(3-\sqrt{5})(1-q_2)}{4} \tag{4.5}$$

$$q_2^*(q_1) = \left(\frac{1+\sqrt{5}}{8}\right)(1-q_1) \qquad q_1^*(q_2) = \left(\frac{1+\sqrt{5}}{8}\right)(1-q_2) \tag{4.6}$$

$$q_1^* = q_2^* \implies q_1^* = q_2^* = \frac{1+\sqrt{5}}{9+\sqrt{5}} \tag{4.7}$$

$$c_1^* = c_2^* = \frac{3-\sqrt{5}}{4}\left(\frac{8}{9+\sqrt{5}}\right) = \frac{6-2\sqrt{5}}{9+\sqrt{5}} \tag{4.8}$$

$$\Pi_1^* = \Pi_2^* = \left(1 - \frac{1+\sqrt{5}}{9+\sqrt{5}} - \frac{1+\sqrt{5}}{9+\sqrt{5}} - \frac{6-2\sqrt{5}}{9+\sqrt{5}}\right)\frac{1+\sqrt{5}}{9+\sqrt{5}} = \left(\frac{1+\sqrt{5}}{9+\sqrt{5}}\right)^2 \approx 0.0829 \tag{4.9}$$

$$\Pi_s^* = c_1^* q_1^* + c_2^* q_2^* = 2\left(\frac{1+\sqrt{5}}{9+\sqrt{5}}\right)\left(\frac{6-2\sqrt{5}}{9+\sqrt{5}}\right) = 0.0783 \tag{4.10}$$

$$p = 1 - q_1 - q_2 = 0.4240 \tag{4.11}$$

When there are two firms operating in the downstream market, disclosing the outcome of (F_1, S) makes the bargaining problem sequential, while concealing information results in simultaneous bargaining. Sequential bargaining results in a higher profit for the input supplier compared to the simultaneous bargaining model. Therefore the supplier prefers to disclose the outcome of (F_1, S). Both of the downstream competitors obtain less profit under sequential bargaining compared to simultaneous bargaining. Therefore, they do not want the outcome of (F_1, S) to be disclosed. This leads to a conflict of interest in the information disclosure and the way bargaining is conducted between upstream and downstream firms. If there is no restriction on disclosure, the supplier would disclose the outcome of (F_1, S) as the informed party. As a result, we observe an asymmetric outcome where Firm 2 attains a higher profit than Firm 1 even though they have identical profit structures. The price of the final good paid by consumers depends on the amount of good produced in the downstream market. Sequential bargaining results in a higher number of production compared to the simultaneous bargaining solution. As a result, the price of the final good is lower with sequential bargaining, so consumers benefit from information disclosure.

Without a restriction on disclosure, the supplier would disclose the outcome of the first negotiation to bargain sequentially. The downstream firms may not prevent disclosure, but they can respond to the disclosure strategy by merging and operating as a monopoly. In the next section, We show that the downstream firms can achieve a higher profit with bargaining as a monopoly under equal profit shares compared to the sequential bargaining with two firms.

5. INCENTIVES FOR MERGES

The countervailing power hypothesis asserts that concentrated downstream markets may result in lower input prices with stronger downstream firms negotiating their input cost with the input suppliers. Due to lower input costs, downstream firms can produce more goods, which will reduce the price of the final good. As a result, consumers benefit from tight oligopolies. Iozzi and Valletti (2014) analyze this hypothesis with quantity and price competition in the downstream market. We will analyze the incentives for merges under quantity competition in the presence of two downstream firms. We will also use restored bargaining sets satisfying d-comprehensiveness and assume that the quantities are specified in the contract after the input price is determined, which leads to a sequential quantity competition in the downstream market under information disclosure. We show that the downstream firms benefit from a possible merger with equal profit shares.

In Chapter 4, we solve sequential and simultaneous bargaining models with two downstream firms. Now we will solve a bargaining model between a monopoly and an input supplier. In this model there is only one input cost implemented for the monopoly, hence only one bargaining problem. After the input cost c_m is determined by bilateral bargaining, the monopoly decides on the quantity of goods produced, q_m.

$$\Pi_M = (1 - q_m - c_m)q_m$$

$$\Pi_s = c_m q_m$$

The monopoly decides on the optimal level of production after the input cost is determined.

$$\max_{q_m} \Pi_M : \frac{\partial \Pi_M}{\partial q_m} = 0 \implies 1 - 2q_m - c_m = 0 \implies q_m^* = \frac{1 - c_m}{2}$$

we can set up a bargaining problem between the monopoly and the supplier where their profits depend on the variable of interest c_m by inserting q_m^*.

$$\Pi_M = (1 - q_m - c_m)q_m = \left(\frac{1-c_m}{2}\right)^2$$

$$\Pi_s = c_m\left(\frac{1-c_m}{2}\right)$$

Disagreement points are determined by a dictatorship procedure
d_m :

$$\max_{c_m} \Pi_s : \frac{\partial \Pi_s}{\partial c_m} = \frac{1-2c_m}{2} = 0 \implies \hat{c_m} = \frac{1}{2} \implies d_m = \Pi_M(\hat{c_m}) = \frac{1}{16}$$

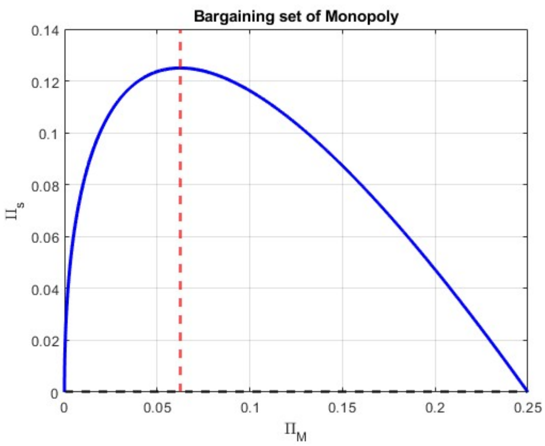
d_s : The monopolist would set $c_m = 0$ to maximize its profit.

$$d_s = \Pi_s(c_m = 0) = 0$$

Nash Program

$$\Omega_m = \left(\left(\frac{1-c_m}{2}\right)^2 - \frac{1}{16}\right)^{\frac{1}{2}} \left(c_m\left(\frac{1-c_m}{2}\right) - 0\right)^{\frac{1}{2}}$$

Figure 5.1 The Bargaining Set of Monopolist



c_m that maximizes the Nash program is the Nash bargaining solution.

$$\Omega_m = \left(c_m\left(\frac{1-c_m}{2}\right)^3 - \frac{1}{16}\left(\frac{c_m - c_m^2}{2}\right)\right)^{\frac{1}{2}}$$

First Order Condition

$$\frac{\partial \Omega_m}{\partial c_m} = \frac{1}{2}\left(c_m\left(\frac{1-c_m}{2}\right)^3 - \frac{1}{16}\left(\frac{c_m - c_m^2}{2}\right)\right)^{-\frac{1}{2}}\left(\left(\frac{1-c_m}{2}\right)^3 - \frac{3}{2}c_m\left(\frac{1-c_m}{2}\right)^2 - \frac{1-2c_m}{32}\right)$$

$$\frac{\partial \Omega_m}{\partial c_m} = 0 \implies c_m^* = \frac{3-\sqrt{5}}{4}$$

Second Order Condition

$$\frac{\partial^2 \Omega_m}{\partial c_m^2} = \frac{-\left(3\left(\frac{c_m}{2}-\frac{1}{2}\right)^2 + \frac{3c_m\left(\frac{c_m}{2}-\frac{1}{2}\right)}{2} - \frac{1}{16}\right)}{2\left(\frac{c_m^2}{32} - c_m\left(\frac{c_m}{2}-\frac{1}{2}\right)^3 - \frac{c_m}{32}\right)^{\frac{1}{2}}} - \frac{\left(\frac{3c_m\left(\frac{c_m}{2}-\frac{1}{2}\right)^2}{2} - \frac{c_m}{16} + \left(\frac{c_m}{2}-\frac{1}{2}\right)^3 + \frac{1}{32}\right)^2}{4\left(\frac{c_m^2}{32} - c_m\left(\frac{c_m}{2}-\frac{1}{2}\right)^3 - \frac{c_m}{32}\right)^{\frac{3}{2}}}$$

The second derivative is negative on the bargaining domain, $c_m \in [0, \frac{1}{2}]$. Therefore, the second-order condition holds.
As a result of bargaining, firms obtain the following profits

- $\Pi_M = \left(\frac{1-c_m}{2}\right)^2 = 0.1636$
- $\Pi_s = c_m\left(\frac{1-c_m}{2}\right) = 0.0773$
- $p = 1 - q_m = 0.5955$

When two downstream firms merge and operate as a monopolist, they make 0.818 each under equal profit shares. This is higher than what they would earn from sequential bargaining with information disclosure. We know that the upstream firm discloses the information from the first bargaining that makes downstream firms worse off. Downstream firms may respond to this disclosing strategy by merging. By operating as a monopolist and sharing the total profit equally, they can achieve higher profits than sequential bargaining with 2 firms. However, merging does not increase the amount of production, so the price of the final good does not decrease but increases. Therefore, this model does not support the countervailing power hypothesis.

6. PRICE REGULATIONS

In this section, I assume that the input supplier cannot charge different prices to different firms competing in the downstream market due to price regulations. Simultaneous bargaining already results in a symmetric outcome where input prices are equal. However, in the sequential bargaining procedure, we observe different prices as well as a second-mover advantage even though downstream firms have identical structures.

Similar to the previous sections, there are two downstream competitors and an upstream input supplier. Before determining input costs, a regulator announces that the upstream firm has to charge the same price to all downstream firms. While bargaining sequentially, (F_1, S) will determine c_1 and hence $c_2 = c_1 = \bar{c}$ due to the regulation. After agreeing on c_1, Firm 1 will choose q_1 level that maximizes its profit. Due to the price regulation, Firm 2 and the supplier will not be able to co-determine c_2 at the next stage. Observing c_1, q_1, Firm 2 will choose q_2 that maximizes its profit.

$$S_1(\emptyset) = \Big\{ \Big(\Pi_1, \Pi_s\Big) \in \mathbb{R}^2_+ \quad | \quad \Pi_1 \leq (1 - q_1 - q_2 - \bar{c})q_1, \quad \Pi_s \leq \bar{c}q_1 + \bar{c}q_2, \quad \bar{c} \in [0,1] \Big\}$$

$S_1(\emptyset)$ accompanied with disagreement points (d_1, d_s) define a bargaining problem between Firm 1 and the input supplier.

After observing $(\bar{c}_1, \bar{q}_1)$, Firm 2 decides on the output level q_2.

$$\Pi_2(q_2) = (1 - \bar{q}_1 - q_2 - \bar{c})q_2$$

$$\frac{\partial \Pi_2}{\partial q_2} = (1 - \bar{q}_1 - 2q_2 - \bar{c}) = 0 \implies q_2^* = \frac{1 - \bar{q}_1 - \bar{c}}{2}$$

$$q_2^*(\bar{c},\bar{q}_1) = \begin{cases} \frac{1-\bar{q}_1-\bar{c}}{2} & \text{if } 1-\bar{q}_1-\bar{c} \geq 0 \\ 0 & \text{otherwise} \end{cases}$$

After observing $(\bar{c})$ and foreseeing q_2^*, Firm 1 decides on the output level q_1.

$$\Pi_1 = (1-q_1-q_2^*-c)q_1 = \Big(1-q_1-\frac{1-q_1-\bar{c}}{2}-\bar{c}\Big)q_1 = \Big(\frac{1-q_1-\bar{c}}{2}\Big)q_1$$

$$\frac{\partial \Pi_1}{\partial q_1} = (1-2q_1-\bar{c})\frac{1}{2} = 0 \implies q_1^* = \frac{1-\bar{c}}{2}$$

Considering $q_1^* = \frac{1-\bar{c}}{2}$ and $q_2^* = \frac{1-\bar{c}}{4}$, Firm 1 and the upstream supplier can foresee the profits they achieve for each c_1 and engage in a bargaining model.

$$\Pi_1(\bar{c}) = (1-q_1-q_2-\bar{c})q_1 = \Big(1-\frac{1-\bar{c}}{2}-\frac{1-\bar{c}}{4}-\bar{c}\Big)\frac{1-\bar{c}}{2} = \frac{(1-\bar{c})^2}{8}$$

$$\Pi_s = \bar{c}q_1+\bar{c}q_2 = \bar{c}\Big(\frac{1-\bar{c}}{2}\Big)+\bar{c}\Big(\frac{1-\bar{c}}{4}\Big) = \frac{3}{4}\Big(\bar{c}-\bar{c}^2\Big)$$

- d_1 : The supplier decides on $\bar{c} = c^s$ as a dictator.

$$\max_c \Pi_s = \frac{3}{4}\Big(\bar{c}-\bar{c}^2\Big) \tag{6.1}$$

$$\frac{\partial \Pi_s}{\partial c} = \frac{3}{8}(1-2c) = 0 \implies c^s = \frac{1}{2} \implies d_1 = \Pi_1(c^s) = \frac{1}{32}$$

$$\text{S.O.C: } \frac{\partial^2 \Pi_s}{\partial c^2} = -\frac{6}{8} < 0$$

- d_s : Firm 1 decides on $\bar{c} = c^1$ as a dictator. Firm 1 attains its highest profit at $\bar{c} = 0 = c^1$. Therefore, $d_s^1 = \Pi_s(c^1) = 0$

Nash Program:

$$\Omega_1(\bar{c}) = \Big(\frac{(1-\bar{c})^2}{8}-\frac{1}{32}\Big)^{\frac{1}{2}}\Big(\frac{3}{4}\Big(\bar{c}-\bar{c}^2\Big)\Big)^{\frac{1}{2}}$$

Program is defined on $\bar{c} \in (0,\frac{1}{2})$, because Firm 1 gets a negative surplus if $\bar{c} > \frac{1}{2}$

Figure 6.1 Bargaining Set in a Regulated Market

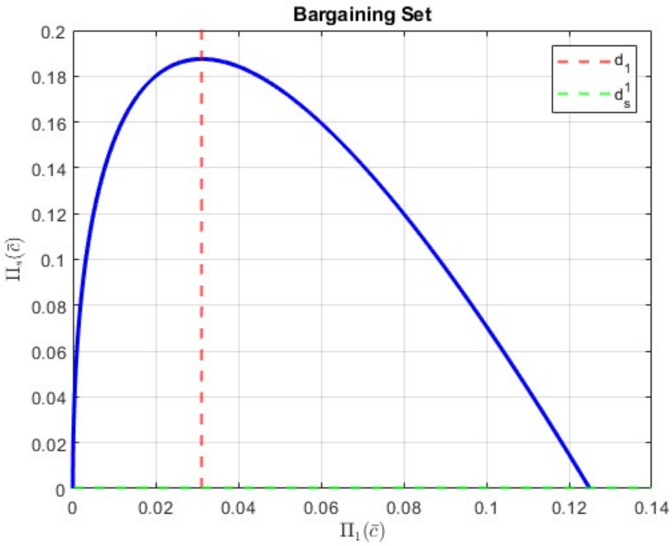
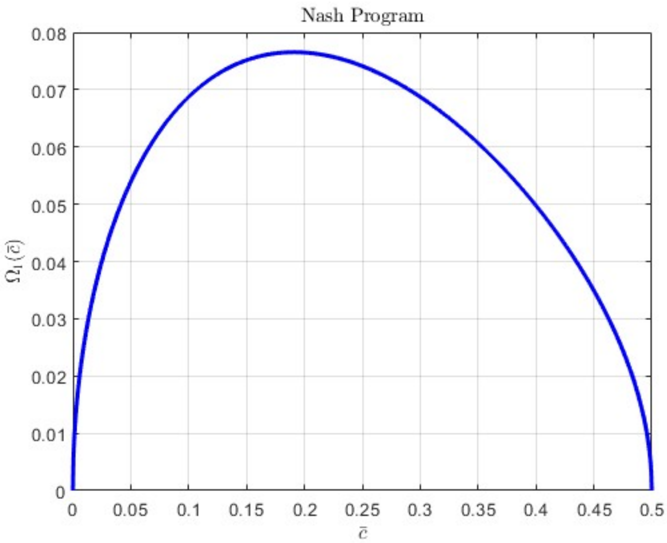


Figure 6.2 Nash Program



F.O.C:

$$\Omega_1 = \left(\left(\frac{(1-\bar{c})^2}{8} - \frac{1}{32}\right) \cdot \frac{3}{4}(\bar{c}-\bar{c}^2)\right)^{\frac{1}{2}} = \left(\frac{3}{32}\bar{c}(1-\bar{c})^3 - \frac{3}{128}(\bar{c}-\bar{c}^2)\right)^{\frac{1}{2}}$$

$$\frac{\partial\Omega_1}{\partial\bar{c}} = \frac{1}{2}\cdot\left(\frac{3}{32}\bar{c}(1-\bar{c})^3 - \frac{3}{128}(\bar{c}-\bar{c}^2)\right)^{-\frac{1}{2}}\left(\frac{3}{32}(\bar{c}(1-\bar{c})^3 - \bar{c}\cdot 3(1-\bar{c})^2) - \frac{3}{128}(1-2\bar{c})\right) = 0$$

$$\bar{c}^* = 0.1910$$

$$\frac{\partial^2\Omega_1}{\partial\bar{c}^2} = -\frac{1}{4}\cdot\left(\frac{3}{32}\bar{c}(1-\bar{c})^3 - \frac{3}{128}(\bar{c}-\bar{c}^2)\right)^{-\frac{3}{2}}\left(\frac{3}{32}((1-\bar{c})^3-\bar{c}\cdot 3(1-\bar{c})^2) - \frac{3}{128}(1-2\bar{c})\right)^2$$
$$+\frac{1}{2}\cdot\left(\frac{3}{32}\bar{c}(1-\bar{c})^3 - \frac{3}{128}(\bar{c}-\bar{c}^2)\right)^{-\frac{1}{2}}\left(\frac{3}{32}\left(-6(1-\bar{c})^2+6\bar{c}(1-\bar{c})\right)+\frac{3}{64}\right) < 0$$

Multiplying each side of the inequality with $\left(\frac{3}{32}((1-\bar{c})^3-\bar{c}\cdot 3(1-\bar{c})^2) - \frac{3}{128}(1-2\bar{c})\right)^{-\frac{3}{2}}$, we get:

$$-\frac{1}{4}\left(\frac{3}{32}((1-\bar{c})^3-\bar{c}\cdot 3(1-\bar{c})^2) - \frac{3}{128}(1-2\bar{c})\right)^2 +$$
$$\frac{1}{2}\left(\frac{3}{32}\bar{c}(1-\bar{c})^3 - \frac{3}{128}(\bar{c}-\bar{c}^2)\right)\left(\frac{3}{32}(-6(1-\bar{c})^2+6\bar{c}(1-\bar{c}))+\frac{3}{64}\right) < 0$$

We can write the left-hand side as a polynomial of $\bar{c}$.

$$\frac{9\bar{c}^6}{512} - \frac{81\bar{c}^5}{1024} + \frac{135\bar{c}^4}{1024} - \frac{405\bar{c}^3}{4096} + \frac{243\bar{c}^2}{8192} - \frac{81}{65536} < 0$$

The polynomial at the left-hand side is negative in $\bar{c} \in (0, \frac{1}{2})$. Therefore, the second-order condition holds, and the first-order condition providing $c_1^* = 0.1910$ maximizes the Nash program. The resulting quantities and profits are:

- $\bar{c}^*(\emptyset) = 0.1910$
- $q_1^*(\bar{c}^*) = \frac{1-\bar{c}}{2} = 0.4045$
- $q_2^*(\bar{c}^*, q_1^*) = \frac{1-\bar{c}}{4} = 0.2022$
- $\Pi_1 = (1-q_1-q_2-\bar{c})q_1 = 0.0818$
- $\Pi_2 = (1-q_1-q_2-\bar{c})q_2 = 0.0409$
- $\Pi_s = \bar{c}q_1 + \bar{c}q_2 = 0.1159$
- $p = 1-q_1-q_2 = 0.3933$

Regulators may implement a price regulation to eradicate the second mover advantage in the baseline model. However, this policy will result in differentiated optimum input decisions which does not solve the problem. With a price regulation, Firm 1 and the input supplier are better off, while the profit of Firm 2 decreases by 45.47%. In chapter 4, I show that there is a second mover advantage in sequential bargaining, however with price regulations there is a first mover advantage. The supplier also benefits from the price regulation, while Firm 2 is negatively affected. The reason is that Firm 2 does not have a say in price determination. The Nash bargaining rule maximizes the weighted surpluses of Firm 1 and the supplier. Therefore, while Firm 1 and the supplier achieve a higher profit with a regulation, Firm 2 is negatively affected. Moreover, we observe that the price of final good is lower due to higher level of production as a result of a price regulation. Therefore, consumers are positively affected by the regulation.

	Simultaneous Model	**Sequential Model**	**With Price Regulation**
Π_1	0.0829	0.0620	0.0818
Π_2	0.0829	0.0750	0.0409
Π_s	0.0783	0.1036	0.1159
p	0.4240	0.4033	0.3933

7. SEMI-SEQUENTIAL BARGAINING WITH 3 FIRMS

Suppose that there are three firms in the downstream market and there is a single upstream input supplier. Due to regulations, the upstream firm has to conduct bargaining with Firm 2 and Firm 3 simultaneously. Bargaining with Firm 1 occurs before the simultaneous bargaining.

Firm 1 Enters Suppose that Firm 1 negotiates its input price, c_1 first. After the input price is determined by bilateral bargaining, q_1 will be specified with a contract. Firm 1 decides on q_1 that maximizes its profit after observing c_1. For any $(\bar{c}_1, \bar{q}_1)$ pair, simultaneous bargaining between downstream firms (Firm 2 and Firm 3) and the input supplier (S) is conducted.

$$\Pi_2 = (1 - \bar{q}_1 - q_2 - q_3 - c_2)q_2$$

$$\Pi_3 = (1 - \bar{q}_1 - q_2 - q_3 - c_3)q_3$$

7.1 Firm 3(2)'s Problem

Firm 3's best response to $(\bar{c}_1, \bar{q}_1)$ and $(\bar{c}_2, \bar{q}_2)$;

$$\frac{\partial \Pi_3}{\partial q_3} = 1 - \bar{q}_1 - \bar{q}_2 - 2q_3 - c_3 = 0 \implies q_3^* = \frac{1 - \bar{q}_1 - \bar{q}_2 - c_3}{2}$$

$$\Pi_3(c_3) = \left(\frac{1 - \bar{q}_1 - \bar{q}_2 - c_3}{2}\right)^2$$

$$\Pi_s(c_3) = \bar{c}_1\bar{q}_1 + \bar{c}_2\bar{q}_2 + c_3\left(\frac{1 - \bar{q}_1 - \bar{q}_2 - c_3}{2}\right)$$

$$S_3 = \Big\{(\Pi_3, \Pi_s) \in \mathbb{R}^2_+ | c_3 \in (0,1)\Big\}$$

S_3 with disagreement points (d_3, d_s) defines a bargaining problem between Firm 3 and the upstream firm.

d_3: The supplier acts as a dictator and determines c_3^s

$$\frac{\partial \Pi_s}{\partial c_3} = \Big(\frac{1-\bar{q}_1-\bar{q}_2-2c_3}{2}\Big) = 0 \implies c_3^s = \frac{1-\bar{q}_1-\bar{q}_2}{2}$$

$$d_3 = \Pi_3(c_3^s) = \Big(1-\bar{q}_1-\bar{q}_2 - \frac{1-\bar{q}_1-\bar{q}_2}{4} - \frac{1-\bar{q}_1-\bar{q}_2}{2}\Big)\cdot\Big(\frac{1-\bar{q}_1-\bar{q}_2}{4}\Big) = \Big(\frac{1-\bar{q}_1-\bar{q}_2}{4}\Big)^2$$

d_s: Firm 1 acts as a dictator and determines $c_3 = 0$

$$d_s = \Pi_s(0) = \bar{c}_1\bar{q}_1 + \bar{c}_2\bar{q}_2$$

Nash Program

$$(7.1) \quad \Omega_3(c_3) = \Bigg(\Big(\frac{1-q_1-q_2-c_3}{2}\Big)^2 - \Big(\frac{1-q_1-q_2}{4}\Big)^2\Bigg)^{\frac{1}{2}} \cdot \Big(c_3\frac{(1-q_1-q_2-c_3)}{2}\Big)^{\frac{1}{2}}$$

The program is defined on $c_3 \in \Big(0, \frac{1-q_1-q_2}{2}\Big)$ so that Firm 1 gets a positive surplus from the bargaining problem.

The First-Order Condition:

$$\frac{\partial \Omega_3}{\partial c_3} = -\frac{\Big(c_3\cdot\frac{(1-q_1-q_2-c_3)}{2}\Big)^{\frac{1}{2}}\cdot\Big(\frac{(1-q_1-q_2-c_3)}{2}\Big)}{2\Big(\Big(\frac{(1-q_1-q_2-c_3)}{2}\Big)^2 - \Big(\frac{(1-q_1-q_2)}{4}\Big)^2\Big)^{\frac{1}{2}}}$$

$$+\frac{2\Big(\Big(\frac{(1-q_1-q_2-c_3)}{2}\Big)^2 - \Big(\frac{(1-q_1-q_2)}{4}\Big)^2\Big)^{\frac{1}{2}}\cdot\Big(\frac{(1-q_1-q_2-2c_3)}{2}\Big)}{2\Big(c_3\cdot\frac{(1-q_1-q_2-c_3)}{2}\Big)^{\frac{1}{2}}} = 0$$

There are three roots of the equation, but only one of them is feasible for the bargaining problem.

$$c_3^* = \frac{(3-\sqrt{5})(1-q_1-q_2)}{4}$$

We need to check the second-order condition to verify that the first-order condition solves the maximization problem.

$$\frac{\partial^2 \Omega_3}{\partial c_3^2} = \frac{\left(c_3 \frac{(1-q_1-q_2-c_3)}{2}\right)^{\frac{1}{2}} \frac{1}{2}}{2\left(\left(\frac{1-q_1-q_2-c_3}{2}\right)^2 - \left(\frac{1-q_1-q_2}{4}\right)^2\right)^{\frac{1}{2}}} \tag{7.2}$$

$$- \frac{\left(c_3 \frac{(1-q_1-q_2-c_3)}{2}\right)^{-\frac{1}{2}} \cdot \left(\frac{1-q_1-q_2-2c_3}{2}\right)\left(\frac{1-q_1-q_2-c_3}{2}\right)}{4\left(\left(\frac{1-q_1-q_2-c_3}{2}\right)^2 - \left(\frac{1-q_1-q_2}{4}\right)^2\right)^{\frac{1}{2}}} \tag{7.3}$$

$$- \frac{\left(\frac{1-q_1-q_2-c_3}{2}\right)^2 \cdot \left(c_3 \frac{(1-q_1-q_2-c_3)}{2}\right)^{\frac{1}{2}}}{4\left(\left(\frac{1-q_1-q_2-c_3}{2}\right)^2 - \left(\frac{1-q_1-q_2}{4}\right)^2\right)^{\frac{3}{2}}} \tag{7.4}$$

$$- \frac{\left(\left(\frac{1-q_1-q_2-c_3}{2}\right)^2 - \left(\frac{1-q_1-q_2}{4}\right)^2\right)^{-\frac{3}{2}} \cdot \left(\frac{1-q_1-q_2-c_3}{2}\right) \cdot \left(\frac{1-q_1-q_2-2c_3}{2}\right)}{4\left(\frac{c_3(1-q_1-q_2-c_3)}{2}\right)^{\frac{1}{2}}} \tag{7.5}$$

$$- \frac{\left(\left(\frac{1-q_1-q_2-c_3}{2}\right)^2 - \left(\frac{1-q_1-q_2}{4}\right)^2\right)^{\frac{1}{2}}}{2\left(c_3\left(\frac{1-q_1-q_2-c_3}{2}\right)\right)^{\frac{1}{2}}} \tag{7.6}$$

$$- \frac{\left(\left(\frac{1-q_1-q_2-c_3}{2}\right)^2 - \left(\frac{1-q_1-q_2}{4}\right)^2\right)^{\frac{1}{2}} \cdot \left(\frac{1-q_1-q_2-2c_3}{2}\right)^2}{4\left(c_3\left(\frac{1-q_1-q_2-c_3}{2}\right)\right)^{\frac{3}{2}}} \tag{7.7}$$

All the fractions constituting the second derivative have negative coefficients except for the first fraction. If any of the other fractions is larger than the first fraction in absolute terms, then the second-order condition holds. I show that (7.4) is larger than (7.2) in absolute terms.

$$\frac{\left(\frac{1-q_1-q_2-c_3}{2}\right)^2 \cdot \left(c_3 \frac{(1-q_1-q_2-c_3)}{2}\right)^{\frac{1}{2}}}{4\left(\left(\frac{1-q_1-q_2-c_3}{2}\right)^2 - \left(\frac{1-q_1-q_2}{4}\right)^2\right)^{\frac{3}{2}}} > \frac{\left(c_3 \frac{(1-q_1-q_2-c_3)}{2}\right)^{\frac{1}{2}}}{4\left(\left(\frac{1-q_1-q_2-c_3}{2}\right)^2 - \left(\frac{1-q_1-q_2}{4}\right)^2\right)^{\frac{1}{2}}}$$

By multiplying each side with $4\left(\left(\frac{1-q_1-q_2-c_3}{2}\right)^2 - \left(\frac{1-q_1-q_2}{4}\right)^2\right)^{\frac{3}{2}}$ and dividing each side with $\left(c_3\frac{(1-q_1-q_2-c_3)}{2}\right)^{\frac{1}{2}}$ we get the following inequality that holds for all c_3 values in the range.

$$\left(\frac{1-q_1-q_2-c_3}{2}\right)^2 > \left(\frac{1-q_1-q_2-c_3}{2}\right)^2 - \left(\frac{1-q_1-q_2}{4}\right)^2$$

Outcome of Simultaneous Bargaining

Given $(\bar{c}_1, \bar{q}_1)$ and $(\bar{c}_2, \bar{q}_2)$, bargaining between firm 3 and the supplier results in the following input cost and quantity

$$c_3^* = \frac{(3-\sqrt{5})(1-\bar{q}_1-\bar{q}_2)}{4} \qquad q_3^*(\bar{q}_1, \bar{q}_2) = \frac{(1+\sqrt{5})}{8} \cdot (1-\bar{q}_1-\bar{q}_2)$$

Due to symmetry, given $(\bar{c}_1, \bar{q}_1)$ and $(\bar{c}_3, \bar{q}_3)$, bargaining between firm 2 and the supplier results in the following input cost and quantity

$$c_2^* = \frac{(3-\sqrt{5})(1-\bar{q}_1-\bar{q}_3)}{4} \qquad q_2^*(\bar{q}_1, \bar{q}_3) = \frac{(1+\sqrt{5})}{8} \cdot (1-\bar{q}_1-\bar{q}_3)$$

By intersecting quantity best responses q_2^* and q_3^*,

$$q_3 = \left(\frac{1+\sqrt{5}}{8}\right)(1-\bar{q}_1) - \left(\frac{1+\sqrt{5}}{8}\right)^2 (1-\bar{q}_1) + \left(\frac{1+\sqrt{5}}{8}\right)^2 q_3$$

$$q_3^*(\bar{q}_1) = \frac{\frac{1+\sqrt{5}}{8}(1-\frac{1+\sqrt{5}}{8})}{(1+\frac{1+\sqrt{5}}{8})(1-\frac{1+\sqrt{5}}{8})}(1-\bar{q}_1) = \frac{1+\sqrt{5}}{9+\sqrt{5}} \cdot (1-\bar{q}_1)$$

Due to symmetry $q_2^* = q_3^*$.

7.2 Firm 1's Problem

While bargaining, Firm 1 and the input supplier know that Firm 2 and Firm 3 will co-determine their input costs with the upstream firm via simultaneous Nash bargaining. For each $(\bar{c}_1, \bar{q}_1)$ pair, q_2^*, q_3^* will be determined as previously shown.

$$\Pi_1(c1, q_1) = (1 - q_1 - q_2^* - q_3^* - c_1)q_1 = \left(1 - q_1 - 2 \cdot \frac{1+\sqrt{5}}{9+\sqrt{5}} \cdot (1-q_1) - c_1\right)q_1$$

$$\Pi_1(c1, q_1) = (1 - q_1 - q_2^* - q_3^* - c_1)q_1 = \left(\frac{7-\sqrt{5}}{9+\sqrt{5}}(1-q_1) - c_1\right)q_1$$

After the input price c_1 is determined, Firm 1 will determine the optimal quantity q_1 that maximizes its profit.

$$\frac{\partial \Pi_1}{\partial q_1} = \frac{7-\sqrt{5}}{9+\sqrt{5}} - 2q_1\frac{7-\sqrt{5}}{9+\sqrt{5}} - c1 = 0$$

$$q_1^* = \frac{1}{2} - c_1\frac{9+\sqrt{5}}{14-2\sqrt{5}}$$

$$q_2^*(q_1^*) = q_3^*(q_1^*) = \frac{1+\sqrt{5}}{9+\sqrt{5}} \cdot (1-q_1^*) = \frac{1+\sqrt{5}}{9+\sqrt{5}} \cdot \left(\frac{1}{2} + c_1\frac{9+\sqrt{5}}{14-2\sqrt{5}}\right)$$

$$c_2^*(q_1^*) = c_3^*(q_1^*) = \frac{(3-\sqrt{5})}{4}(1 - q_1^* - q_2^*(q_1^*)) = \frac{(3-\sqrt{5})}{4}\left(\frac{8}{9+\sqrt{5}}\right)\left(\frac{1}{2} + c_1\frac{9+\sqrt{5}}{14-2\sqrt{5}}\right)$$

Now that we have q_1^*, q_2^*, q_3^*, we can write down the profits of Firm 1 and the supplier in terms of c_1 and set up a bargaining problem.

$$\Pi_1(c_1) = \left(\frac{7-\sqrt{5}}{9+\sqrt{5}}\left(\frac{1}{2} + c_1\frac{9+\sqrt{5}}{14-2\sqrt{5}}\right) - c_1\right)\left(\frac{1}{2} - c_1\frac{9+\sqrt{5}}{14-2\sqrt{5}}\right)$$

$$\Pi_s(c_1) = c_1\left(\frac{1}{2} - c_1\frac{9+\sqrt{5}}{14-2\sqrt{5}}\right) + 2\left(\frac{6-2\sqrt{5}}{9+\sqrt{5}}\right)\left(\frac{1+\sqrt{5}}{9+\sqrt{5}}\right)\left(\frac{1}{2} + c_1\frac{9+\sqrt{5}}{14-2\sqrt{5}}\right)^2$$

d_s : Firm 1 acts as a dictator and determines $\hat{c_1} = 0$

$$d_s = \Pi_s(\hat{c_1}) = \frac{1}{2}\left(\frac{6-2\sqrt{5}}{9+\sqrt{5}}\right)\left(\frac{1+\sqrt{5}}{9+\sqrt{5}}\right) \approx 0.0196$$

d_1 : The upstream firm acts as a dictator and determines $\hat{c_1^s}$

$$\frac{\partial \Pi_s}{\partial c_1} = \frac{1}{2} - \frac{2c_1(\sqrt{5}+9)}{14-2\sqrt{5}} - \frac{2(12-4\sqrt{5})\left(\frac{c_1(\sqrt{5}+9)}{2\sqrt{5}-14} - \frac{1}{2}\right)(\sqrt{5}+1)}{(14-2\sqrt{5})(\sqrt{5}+9)} = 0$$

$$\hat{c_1^s} = \frac{320\sqrt{5}-40}{80+1056\sqrt{5}} \approx 0.2767 \implies d_1 = \Pi_1(\hat{c_1^s}) = 0.0128$$

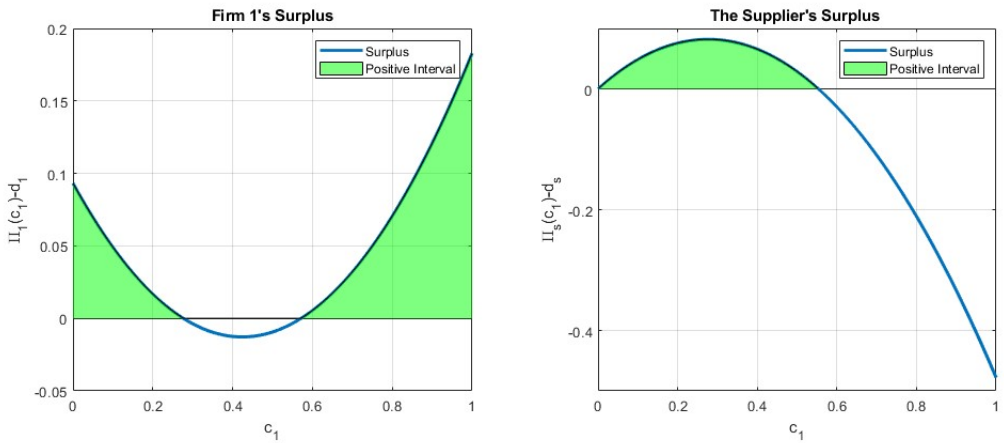
Profits and disagreement points define the following Nash program. The program is defined where Firm 1 and the supplier have a positive surplus. Below I present the intervals that provide positive surpluses and graphs.

$$\Omega_1 = \left(\left(\frac{7-\sqrt{5}}{9+\sqrt{5}} \left(\frac{1}{2} + c_1 \frac{9+\sqrt{5}}{14-2\sqrt{5}} \right) - c_1 \right) \left(\frac{1}{2} - c_1 \frac{9+\sqrt{5}}{14-2\sqrt{5}} \right) - 0.0128 \right)^{\frac{1}{2}}$$
$$\left(c_1 \left(\frac{1}{2} - c_1 \frac{9+\sqrt{5}}{14-2\sqrt{5}} \right) + 2 \left(\frac{6-2\sqrt{5}}{9+\sqrt{5}} \right) \left(\frac{1+\sqrt{5}}{9+\sqrt{5}} \right) \left(\frac{1}{2} + c_1 \frac{9+\sqrt{5}}{14-2\sqrt{5}} \right)^2 - 0.0196 \right)^{\frac{1}{2}}$$

$$\Pi_1(c_1) - d_1 > 0 \implies c_1 \in (0, 0.2766) \cup (0.5713, 1) \qquad \Pi_s(c_1) - d_1 > 0 \implies c_1 \in (0, 0.5534)$$

The program is defined on the intersection of these two intervals, which is $c_1 \in (0, 0.2766)$

Figure 7.1 Range of c_1 Giving Positive Surplus



Now we can plot the bargaining set and the Nash program in the bargaining domain.

Figure 7.2 Semi-Sequential Bargaining Set

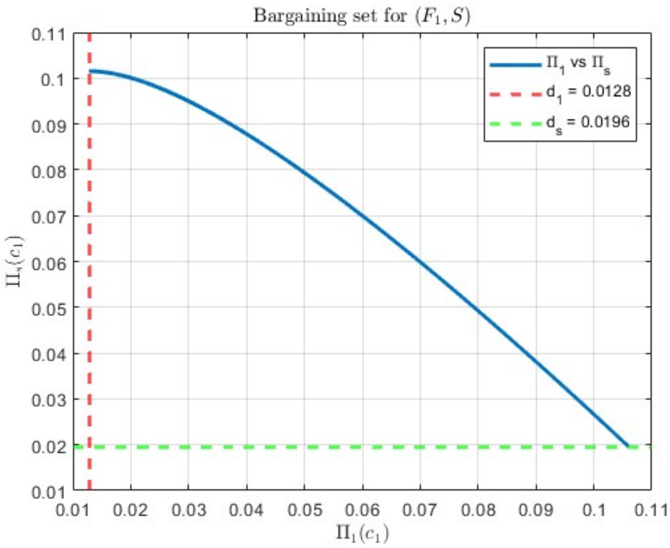
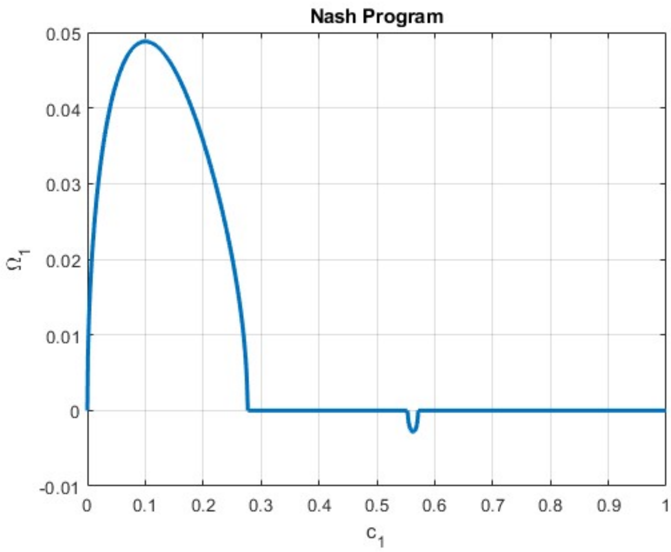


Figure 7.3 Semi-Sequential Nash Program



The First and The Second-Order Conditions Due to the complexity of the maximization problem, we can redefine the Nash program.

$$\Omega_1 = \left(\left(a\left(\frac{1}{2}+c_1b\right)-c_1\right)\left(\frac{1}{2}-c_1b\right)-0.0128\right)^{\frac{1}{2}}\left(c_1\left(\frac{1}{2}-c_1b\right)+2cd\left(\frac{1}{2}+c_1b\right)^2-0.0196\right)^{\frac{1}{2}}$$

- $a = \frac{7-\sqrt{5}}{9+\sqrt{5}}$
- $b = \frac{9+\sqrt{5}}{14-2\sqrt{5}}$

- $c = \frac{6-2\sqrt{5}}{9+\sqrt{5}}$
- $d = \frac{1+\sqrt{5}}{9+\sqrt{5}}$

The First-Order Condition:

$$\frac{\partial \Omega_1}{\partial c_1} = 0 \implies c_1^* = 0.1001$$

The Second-Order Condition:

$$\begin{aligned}\frac{\partial^2 \Omega_1}{\partial c_1^2} = &-\frac{(7.5733c_1^2) - (5.3068c_1) + 0.7919}{2\left(-(0.6311c_1^4) + (0.8845c_1^3) - (0.3960c_1^2) + (0.0552c_1) - 0.000001\right)^{\frac{1}{2}}} \\ &- \frac{\left((2.5244c_1^3) - (2.6534c_1^2) + (0.7919c_1) - 0.0552\right)^2}{4\left(-(0.6311c_1^4) + (0.8845c_1^3) - (0.3960c_1^2) + (0.0552c_1) - 0.000001\right)^{\frac{3}{2}}} < 0\end{aligned}$$

The denominator of the first fraction is positive due to the definition of the bargaining problem. Each side is multiplied by the denominator of the first fraction. For the second-order condition to hold,

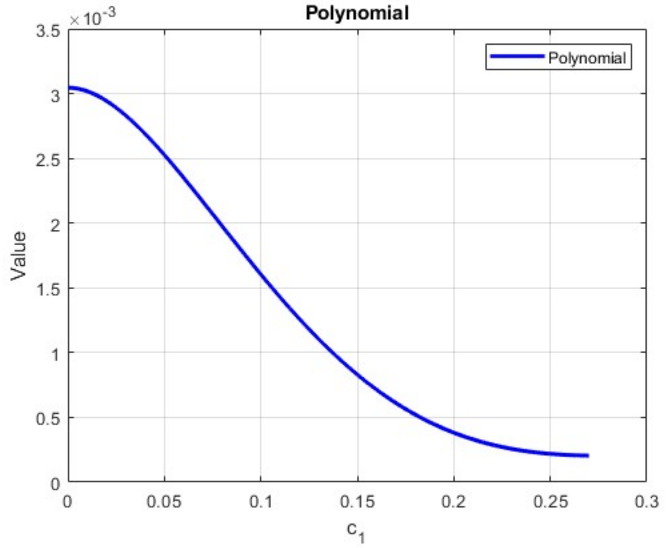
$$\frac{\left((2.5244c_1^3) - (2.6534c_1^2) + (0.7919c_1) - 0.0552\right)^2}{4\left(-0.631c_1^4 + 0.884c_1^3 - 0.396c_1^2 + 0.055c_1 - 0.000001\right)} > -\frac{7.573c_1^2 - 5.307c_1 + 0.792}{2}$$

We can convert this inequality to a polynomial to see if it is positive in the bargaining domain. I provide a graph to see that.

(7.8)

$$-3.1864\,c_1^6 + 6.6989\,c_1^5 - 5.3466\,c_1^4 + 1.9588\,c_1^3 - 0.2930\,c_1^2 + 0.00001\,c_1 + 0.0030 > 0$$

Figure 7.4 Values of Polynomial 7.8



The outcome:

- $c_1^*(\emptyset) = 0.1001$
- $q_1^*(c_1^*) = 0.3820$
- $c_2^*(c_1^*, q_1^*) = 0.0840$
- $q_2^*(c_1^*, q_1^*) = 0.1780$
- $c_3^*(c_1^*, q_1^*) = 0.0840$
- $q_3^*(c_1^*, q_1^*) = 0.1780$
- $\Pi_1 = 0.0618$
- $\Pi_2 = \Pi_3 = 0.0317$
- $\Pi_s = 0.0681$
- $p = 1 - q_1 - q_2 - q_3 = 0.2620$

7.2.1 Simultaneous Bargaining With Three Firms

In the previous section, I computed the best response of Firm 3 to $(\bar{c}_1, \bar{q}_1)$ and $(\bar{c}_1, \bar{q}_1)$.

$$q_3^* = \frac{1 - \bar{q}_1 - \bar{q}_2 - c_3}{2} \qquad c_3^* = \frac{3 - \sqrt{5}}{4}(1 - \bar{q}_1 - \bar{q}_2)$$

Due to symmetry, Firm 1 and Firm 3 also have similar best responses.

$$q_2^* = \frac{1-\bar{q}_1-\bar{q}_3-c_2}{2} \qquad c_2^* = \frac{3-\sqrt{5}}{4}(1-\bar{q}_1-\bar{q}_3)$$

$$q_1^* = \frac{1-\bar{q}_2-\bar{q}_3-c_1}{2} \qquad c_1^* = \frac{3-\sqrt{5}}{4}(1-\bar{q}_2-\bar{q}_3)$$

Solving the model simultaneously, we get the following quantities. Then we insert them into the bargaining outcomes of input costs.

$$q_1^* = \frac{1+c_2+c_3-3c_1}{4} \qquad c_1 = \frac{3-\sqrt{5}}{4}\left(1+\frac{c_2+c_3-c_1-1}{2}\right)$$

$$q_2^* = \frac{1+c_1+c_3-3c_2}{4} \qquad c_2 = \frac{3-\sqrt{5}}{4}\left(1+\frac{c_1+c_3-c_2-1}{2}\right)$$

$$q_3^* = \frac{1+c_1+c_2-3c_3}{4} \qquad c_3 = \frac{3-\sqrt{5}}{4}\left(1+\frac{c_1+c_2-c_3-1}{2}\right)$$

By solving input cost equations $c_1^* = c_2^* = c_3^* = \frac{3-\sqrt{5}}{5+\sqrt{5}} \approx 0.1056 \implies q_1^* = q_2^* = q_3^* = \frac{1+\sqrt{5}}{10+2\sqrt{5}}$

$$\text{(7.9)} \qquad \Pi_1 = \Pi_2 = \Pi_3 = \left(\frac{1+\sqrt{5}}{10+2\sqrt{5}}\right)^2 = \frac{1}{20} \qquad \Pi_s = \frac{6\sqrt{5}-6}{60+20\sqrt{5}} \approx 0.0708$$

$$\text{(7.10)} \qquad p = 1-q_1-q_2-q_3 = 0.3292$$

When there are two firms in the downstream market, sequential bargaining is more profitable to the input supplier. However, when there are three downstream firms, we see that information disclosure is not always profitable for the upstream firm. Compared to semi-sequential bargaining, where the bargaining outcome of Firm 1 is disclosed, simultaneous bargaining with 3 firms provides a higher profit to the input supplier. In this comparison, Firm 1 benefits from the information disclosure. If there is no restriction on downstream competitors to disclose their bargaining outcomes, Firm 1 discloses the information after the negotiation is completed. However, the upstream firm would prefer to disclose the bargaining outcome with Firm 1.

8. SIMULTANEOUS BARGAINING WITH N FIRMS

Assume that there are $N > 1$ firms in the downstream market. Firms engage in simultaneous bargaining for their input price with the input supplier. Firm i's best response to other bargaining outcomes $(c_j, q_j)_{j \neq i}$:

$$\Pi_i = (1 - \sum_{j=1}^{N} q_j - ci)q_i \tag{8.1}$$

$$\frac{\partial \Pi_i}{\partial q_i} = (1 - \sum_{j \neq i} q_j - 2q_i - c_i) = 0 \implies q_i^* = \frac{1 - \sum_{j \neq i} q_j - c_i}{2}$$

$$\Pi_i(c_1) = \left(1 - \sum_{j \neq i} q_j - \frac{1 - \sum_{j \neq i} q_j - c_i}{2} - c_i\right) \cdot \left(\frac{1 - \sum_{j \neq i} q_j - c_i}{2}\right) = \left(\frac{1 - \sum_{j \neq i} q_j - c_i}{2}\right)^2$$

$$\Pi_s = \sum_{j \neq i} c_j q_j + c_i \cdot \frac{1 - \sum_{j \neq i} q_j - c_i}{2}$$

d_i: The supplier acts as a dictator

$$\frac{\partial \Pi_s}{\partial c_i} = \frac{1 - \sum_{j \neq i} q_j - 2c_i}{2} = 0 \implies c_i^s = \frac{1 - \sum_{j \neq i} q_j}{2}$$

$$d_i = \Pi_i(c_i^s) = \left(1 - \sum_{j \neq i} q_j - \frac{1 - \sum_{j \neq i} q_j}{4} - \frac{1 - \sum_{j \neq i} q_j}{2}\right) \cdot \left(\frac{1 - \sum_{j \neq i} q_j}{4}\right) = \left(\frac{1 - \sum_{j \neq i} q_j}{4}\right)^2$$

d_s: Firm i acts as a dictator and determines $c_i = 0$

$$d_S = \Pi_s(c_i = 0) = \sum_{j \neq i} c_i q_j$$

Nash Program

$$\Omega_i = \left(\left(\frac{1 - \sum\limits_{j \neq i} q_j - c_i}{2} \right)^2 - \left(\frac{1 - \sum\limits_{j \neq i} q_j}{4} \right)^2 \right)^{\frac{1}{2}} \left(\sum_{j \neq i} c_j q_j + c_i \cdot \frac{1 - \sum\limits_{j \neq i} q_j - c_i}{2} - \sum_{j \neq i} c_j q_j \right)^{\frac{1}{2}}$$

The First-Order Condition

$$\frac{\partial \Omega_i}{\partial c_i} = \frac{-1}{2} \left(\left(\frac{1 - \sum\limits_{j \neq i} q_j - c_i}{2} \right)^2 - \left(\frac{1 - \sum\limits_{j \neq i} q_j}{4} \right)^2 \right)^{-\frac{1}{2}} \cdot \left(c_i . \frac{1 - \sum\limits_{j \neq i} q_j - c_i}{2} \right)^{\frac{1}{2}} \frac{1 - \sum\limits_{j \neq i} q_j - c_i}{2}$$

$$+ \frac{1}{2} \left(\left(\frac{1 - \sum\limits_{j \neq i} q_j - c_i}{2} \right)^2 - \left(\frac{1 - \sum\limits_{j \neq i} q_j}{4} \right)^2 \right)^{\frac{1}{2}} \cdot \left(c_i . \frac{1 - \sum\limits_{j \neq i} q_j - c_i}{2} \right)^{-\frac{1}{2}} \frac{1 - \sum\limits_{j \neq i} q_j - 2c_i}{2} = 0$$

$$\left(\frac{1 - \sum\limits_{j \neq i} q_j - c_i}{2} \right)^2 \cdot c_i = \left(\left(\frac{1 - \sum\limits_{j \neq i} q_j - c_i}{2} \right)^2 - \left(\frac{1 - \sum\limits_{j \neq i} q_j}{4} \right)^2 \right) \cdot \frac{1 - \sum\limits_{j \neq i} q_j - 2c_i}{2}$$

Let $a = 1 - \sum\limits_{j \neq i} q_j$

$$c_i \cdot \left(\frac{a - c_i}{2} \right)^2 = \left(\left(\frac{a - c_i}{2} \right)^2 - \frac{a^2}{16} \right) \cdot \frac{a - 2c_i}{2}$$

$$22a^2 c_i - 36 a c_i^2 + 16 c_i^3 - 3a^3 = 0$$

Solving the equation for c_i

$$c_i^* = \frac{3 - \sqrt{5}}{4} a = \frac{3 - \sqrt{5}}{4} \Big(1 - \sum_{j \neq i} q_j \Big) \qquad q_i^* = \frac{1 + \sqrt{5}}{8} \Big(1 - \sum_{j \neq i} q_j \Big) \tag{8.2}$$

Due to the symmetric structure of best responses;

$$q = \frac{1+\sqrt{5}}{8}\big(1-(N-1)q\big) \implies q^* = \frac{1+\sqrt{5}}{(1+\sqrt{5})N+7-\sqrt{5}}$$

$$c^* = \frac{3-\sqrt{5}}{4}\left(1-(N-1)\frac{1+\sqrt{5}}{(1+\sqrt{5})N+7-\sqrt{5}}\right) = \frac{6-2\sqrt{5}}{(1+\sqrt{5})N+7-\sqrt{5}}$$

Now we can compute the profits of each downstream firm and the input supplier

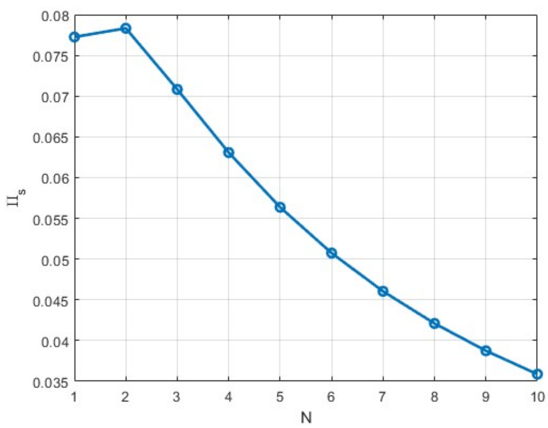
$$\Pi_i = \left(1-\frac{N(1+\sqrt{5})}{(1+\sqrt{5})N+7-\sqrt{5}} - \frac{6-2\sqrt{5}}{(1+\sqrt{5})N+7-\sqrt{5}}\right)\frac{1+\sqrt{5}}{(1+\sqrt{5})N+7-\sqrt{5}}$$

$$\Pi_i = \left(\frac{1+\sqrt{5}}{(1+\sqrt{5})N+7-\sqrt{5}}\right)^2 \quad \forall i \in \{1,2,...,N\}$$

$$\Pi_s = \frac{N(1+\sqrt{5})(6-2\sqrt{5})}{((1+\sqrt{5})N+7-\sqrt{5})^2}$$

The upstream firm attains the highest profit when there are only two downstream competitors. Therefore, the supplier only supplies the input to two downstream firms by excluding the others if possible. The downstream firms' profit is a decreasing function of the number of the firms. The exclusion strategy of the supplier benefits the surviving downstream firms, while the others are negatively affected due to exclusion.

Figure 8.1 The Profit of the Input Supplier



9. CONCLUSION

We show that the upstream firm can obtain a higher profit in sequential bargaining with two downstream firms competing in quantities. As the sole input supplier in the market, the upstream firm may choose how to conduct price negotiations. Therefore, sequential bargaining may be preferred. If downstream firms can not prevent information disclosure, they can respond to disclosure strategy by merging under equal profit shares. In this way, downstream firms achieve a higher profit than what they would earn in sequential bargaining with two firms.

Information disclosure results in an asymmetric outcome in the downstream market. Even though two downstream firms have identical cost structures, they obtain different profits in sequential bargaining. Due to equal treatment, an upstream firm may not charge different prices to different firms. When the upstream firm has to charge the same price to two identical downstream firms, the second-mover advantage is eliminated in an undesirable way. Due to the regulation, a first-mover advantage emerges. Moreover, the upstream firm makes a higher profit compared to the unregulated market. The downstream firm, which is not involved in price determination, is negatively affected.

Another restriction regulating the market may be how upstream firms conduct their negotiations, sequentially or simultaneously. With 3 downstream firms, two downstream firms may have to bargain simultaneously while the other firm is bargaining first. In this case, there is a first-mover advantage. Firm 1, negotiating their input cost first, makes a higher profit compared to Firm 2 and Firm 3. Disclosing the outcome of Firm 1 is not profitable for the supplier but for Firm 1 itself.

Lastly, we solve a simultaneous bargain model generalizing the number of firms to N. The profit of the upstream firm is maximized when the number of the firm is 2, which supports integration and narrowing in the downstream industry.

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