ESSAYS ON MICROECONOMICS

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ABSTRACT

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Keywords: Nash Implementation, Behavioral Implementation, Anonymity, Maskin Monotonicity, Efficiency

We consider Nash implementation under complete information with the additional feature that planners need obey fairness restrictions when designing mechanisms and shaping individuals' unilateral deviation opportunities. An extreme form of such a notion of full implementation is anonymous implementation, which demands the following: First, any socially optimal alternative at any one of the given states is attainable via a Nash equilibrium (NE) at that state, which provides the same opportunity set for all individuals. Second, any such NE at any one of the states must be socially optimal at that state. We identify the necessary and (almost) sufficient conditions for the anonymous implementation of social choice correspondences. Further, we extend this concept to partitioned-anonymous implementation to allow for relaxed notions of fairness. First, agents are divided into equivalence classes (groups based on fairness considerations), delivering a partition over the set of individuals. Then, anonymity is required within each partition. This allows for a more flexible design while ensuring fairness within groups. We provide necessary and (almost) sufficient conditions for partitioned-anonymous implementation as well. Notwithstanding, we show that there are collective goals that are anonymously implementable but fail to be Nash implementable. Therefore, anonymity provides society with additional decentralizable social choice rules that are otherwise not Nash implementable. Unfortunately, anonymity imposes a heavy burden when implementing efficiency: The Pareto social choice correspondence is not anonymously implementable in the full domain.

ÖZET

ESSAYS ON MICROECONOMICS

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Anahtar Kelimeler: Nash Uygulaması, Davranışsal Uygulama, Anonimlik, Maskin Monotonluğu, Verimlilik

Bu tezde, tam bilgi altında, sosyal planlayıcıların mekanizma dizayn ederken ve bireylerin cayabileceği fırsatları şekillendirirken eşitlik sınırlamalarını da göz önünde bulundurmaları gereken bir Nash uygulaması modeli üzerinde duruyoruz. Eşitlik nosyonunun en ekstrem versiyonu olan anonim uygulamaları şöyle tanımlıyoruz: İlk olarak, herhangi bir durumdaki herhangi bir sosyal optimal alternatif, yine aynı durumda nash dengesiyle elde edilebilmeli ve öyle ki bu nash dengesi tüm bireylerin aynı fırsatlara sahip olduğu bir çevreden gelmeli. İkinci olarak, herhangi bir durumdaki herhangi bir NASH dengesi (ilk koşuldaki fırsat eşitliği çevresinden gelmiş) sosyal optimal dengesi olmalı. Devamında, anonim uygulama nosyonunu genişleterek parçalı-anonim uygulama kavramını tanımlıyoruz. Böylece bireyler arası fırsat eşitliğini biraz daha gevşetmiş oluyoruz. İlk olarak, bireyleri eşitlik sınıflarına(adil bir gruplama yaparak) ayırıyoruz. Sonrasında anonimliği her bir alt grup için uyguluyoruz. Anonim uygulamalar için gereklilik ve yeterlilik koşullarını sunduğumuz gibi parçalı-anonimlik için de bu koşulları tanımlıyoruz. Ek olarak, anonim uygulamaya uymasına rağmen NASH uygulamasına uymayan koletif amaçlar olabileceğini bir örnekle gösteriyoruz. Dolayısıyla, anonimliğin topluma, bazı merkezi olmayan sosyal seçim kurallarını sağlayabiliyor öyle ki bu kurallar NASH uygulamasına uymuyor.

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To my family

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1. INTRODUCTION

Can an authority influence the choices of members of a society to sustain goals the authority deems desirable? Is it possible to influence individuals' actions simply by presenting the right set of alternatives and ensuring that their final choices align with the desired outcomes? To accomplish this goal, the authority (planner) needs to design a mechanism so that the resulting stable behavior (equilibrium) when individuals freely make their choices parallels the desired objectives.

In a society where individuals have varying preferences over a set of alternatives, the state of the world captures the particular preference profile of the society at a given time. The collective goal, defined by a predetermined social choice rule, specifies the socially optimal alternatives (from the perspective of the planner) for each state of the world and is common knowledge among individuals. Each member of society knows both the realized state and the desirable outcomes for that state. A social planner aims to implement this predetermined social choice rule by designing a mechanism sustaining the desired outcomes in equilibrium.

The intriguing aspect of implementation theory lies in the social planner's lack of direct knowledge about individuals' preferences, i.e., the realized state of the world. To overcome this, the planner designs a mechanism, or game form, that includes a message space for individuals and an outcome function that determines an outcome based on the message profile. Thus, individuals become players in the game set by the mechanism designer, choosing strategies that align with their interests. A social choice rule is implemented by a mechanism if the set of equilibrium outcomes coincides with the set of socially optimal alternatives at every state of the world. The planner must carefully design the mechanism to prevent individuals from misreporting the realized state to obtain unilateral gains.

Nash implementation is crucial in ensuring that the desired social outcomes are achieved even when individuals have private information and may act strategically. It guarantees that the mechanisms designed by the social planner lead to equilibrium outcomes that match the socially optimal alternatives. This alignment is vital in various fields, including economics, politics, and public policy, where strategic behavior can significantly impact the outcomes of collective decisions.

There are some significant contributions to the theory of Nash implementation. Following the seminal contribution Maskin (1999), Eric Maskin, was awarded the Nobel Prize in Economic Sciences in 2007 (along with Leonid Hurwicz and Roger B. Myerson). In that study, Maskin (1999) provides the necessary and sufficient conditions for a social choice rule to be Nash implementable. This work shows that monotonicity is a necessary condition for Nash implementability, meaning every Nash implementable social choice rule must be monotonic. Additionally, he demonstrated that the no-veto-power property is also required, ensuring that no single individual can block an alternative that is favored by everyone else.

Following Maskin's seminal work, Moore and Repullo (1990) introduced condition μ, which further refined the necessary and sufficient conditions for Nash implementability. They argued that condition μ, although weaker and more complex than monotonicity and the no-veto-power property, effectively closes the gap between these conditions. Their constructive proof extended Maskin's framework and highlighted the importance of these conditions in designing implementable social choice rules. Barlo and Dalkıran (2009) provided necessary and sufficient conditions for a social choice correspondence to be implementable via epsilon-Nash equilibrium when agents' preferences can be represented by cardinal utilities. Their results also displayed robustness of Maskin's seminal result on Nash implementation. Korpela (2012) and de Clippel (2014) further generalized these results by investigating Nash implementation without relying on rational preference assumptions, introducing the concept of consistency. We refer the interested reader to Barlo and Dalkıran (2022*a*) for more details on behavioral implementation under complete information.

The notion of consistency resides at the heart of de Clippel (2014)'s characterization. A consistent collection of sets of alternatives is essentially a family of choice sets indexed for each individual, each state, and each socially optimal alternative at that state such that the following hold: A socially optimal alternative at a state is chosen by every individual at that state from the corresponding choice set; if an alternative is socially optimal at the first state but not at the second, then there is an individual who does not choose this alternative at the second state from her choice set corresponding to this alternative and the first state. This consistency condition ensures that individuals' choices align with the socially optimal outcomes, thereby facilitating the implementation of collective goals.

Fairness is a pivotal concept in mechanism design, particularly when considering how mechanisms are structured to ensure equitable treatment among participants. Korpela (2018) delineates two primary ways to conceptualize fairness: procedural fairness and end-state fairness. Procedural fairness focuses on the fairness of the decision-making process itself, whereas end-state fairness is concerned with the fairness of the outcomes produced by the mechanism. Korpela emphasizes the importance of procedural fairness, particularly in scenarios where agents are more concerned with the fairness of the process rather than the fairness of the outcomes. For instance, in the case of a firm bankruptcy with two parties having legal claims, if the parties differ significantly in their contributions or investments, they might accept a discriminatory division of assets. However, if the parties are similar in all relevant aspects except for their preferences, it becomes challenging to justify any discriminatory allocation based on preferences alone. Korpela's work provides an almost complete characterization of scenarios where agents are solely focused on procedural fairness, under the assumption of complete information.

Procedural fairness in mechanism design ensures that the process by which decisions are made is equitable, independent of the final outcomes. This type of fairness is critical when individuals prioritize being treated fairly during the decision-making process itself, rather than being primarily concerned with the results. Korpela introduces the concept of permutation monotonicity, a refinement of the traditional monotonicity condition, as necessary for implementing mechanisms that are procedurally fair in Nash equilibrium. This condition ensures that even when individuals can strategically report their preferences, the fairness of the process is maintained. By focusing on procedural fairness, Korpela highlights how mechanisms can be designed to treat all participants equally throughout the decision-making process, thereby upholding the integrity and fairness of the entire procedure.

In this thesis, we also focus on the fairness of the mechanism, emphasizing the importance of allowing and constraining agents to have the same opportunity sets. This approach ensures that the decision-making process is equitable and aligns with the principles of procedural fairness. Planners often face binding constraints in many economic scenarios. We explore Nash implementation in complete information environments where planners are externally restricted when forming individuals' opportunity sets. Specifically, we consider the scenario where planners must design mechanisms while adhering to fairness considerations. An extreme example of such fairness is the concept of anonymity, where the planner is required to offer each individual the same set of opportunities when designing mechanisms.

Thus, we introduce the concept of *anonymous implementation*: A social choice correspondence is anonymously implementable if (i) any socially optimal alternative at any given state can be achieved via a Nash equilibrium (NE) at that state, with the same opportunity set for all individuals, and (*ii*) any such NE at any state must be socially optimal at that state.

To illustrate the practicality of anonymous implementation, imagine a multidisciplinary team of doctors treating a patient.[1] In this scenario, requiring that equilibrium play in the mechanism leads to each expert facing the same set of treatment options is appealing: Each team member concurs on the treatment method and the permissible options. Conversely, maintaining NE with experts encountering different treatment options might cause objections and issues within the team. This example also shows why Nash implementation through a symmetric game form may be overly restrictive: The team members are doctors specializing in different areas, and demanding that they face identical opportunities for each decision may be highly limiting.

However, the requirement that every individual should have the same set of opportunities can be too restrictive in certain economic environments. In many real-world applications, it is impractical to ensure that every individual has the same set of opportunities. For example, in educational settings, students with different learning styles and abilities may require tailored resources and opportunities to succeed. Similarly, in public policy, citizens in different regions might need varied services and infrastructure investments based on local needs and conditions. In healthcare, patients with diverse medical conditions and histories often require personalized treatment plans rather than a one-size-fits-all approach. In corporate settings, employees across different departments or roles might need distinct sets of tools and resources to perform their tasks efficiently, reflecting the varying demands of their jobs.

To accommodate such intermediate fairness considerations, we introduce the concept of *partitioned-anonymous implementation*. By partitioning the agents, we can tailor the opportunity sets to be more relevant and manageable. Ensuring fairness across an entire society is challenging, but it is more achievable within smaller groups. Partitioning allows us to focus on equity within these smaller, more homogeneous groups, making the implementation more realistic and fair. Additionally, as the number of agents increases, maintaining a single, uniform opportunity set for everyone becomes increasingly complex and unwieldy. Partitioning helps in breaking down the problem into smaller, more manageable parts, enhancing the scalability of the implementation. Different groups of agents may have varying needs and preferences. Partitioning allows for specialization, where each group can have an op-

[1] We thank Atila Abdülkadiroğlu for suggesting this example.

portunity set that best suits their specific requirements, leading to more efficient and effective outcomes. The most extreme form of partitioned-anonymous implementation is to consider each individual as a separate partition. In such cases, we achieve the traditional implementation results of Maskin (1999) and de Clippel (2014). This demonstrates that partitioned anonymity encompasses all scenarios, from standard complete information implementation to fully anonymous implementation.

We present a necessary and (almost) sufficient condition for partitioned-anonymous implementation of social choice correspondences, which we term partitioned-anonymous consistency. This condition aligns with de Clippel (2014)'s consistency but with the restriction that choice sets do not depend on individual identities but rather on their partitioned groups. We prove that if a social goal is partitioned-anonymously implementable, then there is a collection of choice sets that are partitioned-anonymous consistent with the goal at hand (*necessity*). If a social goal has a partitioned-anonymous consistent profile of choice sets and satisfies the no-veto-power property with a given partition, then it is partitioned-anonymously implementable when there are at least three individuals (*sufficiency*).

We show that anonymous implementation does not necessarily limit the set of Nash implementable social goals: In Chapter 3, we describe an environment and a social choice correspondence that is anonymously implementable but is not Nash implementable. Indeed, anonymity broadens society's opportunities by enabling the decentralization of social choice rules that are otherwise not implementable in NE.

Conversely, we demonstrate that anonymity imposes significant constraints when considering efficiency: We identify a domain description which, if permitted, implies that the Pareto social choice correspondence is not anonymously implementable. Since the full domain of preferences includes this particular instance, we observe that the Pareto social choice correspondence is not anonymously implementable on the full domain.

Our results encompass both rational and behavioral environments. A closely related paper is Korpela (2018). On the other hand, Gavan and Penta (2023) proposes a new framework for implementation theory by requiring that any individual and group deviations (up to a fixed size) from the equilibrium must lead to acceptable outcomes, and hence, parallels the fault tolerant implementation of Eliaz (2002). Anonymous implementation aligns with the essence of Gavan and Penta (2023)'s approach in that we require unilateral deviations from the equilibrium to lead to the same set of alternatives for every individual. In a related paper, Barlo and Dalkıran (2022*b*) considers the implementation problem where planners have to ensure that the mechanism results in desirable outcomes even when they have partial information about individuals' state-contingent preferences.[2]

The organization of the paper is as follows: Chapter 2 provides the preliminaries, and Chapter 3 the example discussed above. In Chapter 4, we deal with the necessity and sufficiency of anonymous implementation and partitioned-anonymous implementation, while Chapter 5 provides our results concerning efficiency. Finally, Chapter 6 presents our concluding remarks.

[2]The implementation notion of Barlo and Dalkıran (2022*b*) rests on an ex-post approach under incomplete information; we refer to Barlo and Dalkıran (2023*a*,*b*) for more on implementation under incomplete information.

2. PRELIMINARIES

Let $N = \{1,...,n\}$ denote a *society* with at least two individuals, X a set of *alternatives*, 2^X the set of all subsets of X, and $\mathcal{X}$ the set of all non-empty subsets of X.

We denote by Ω the set of all *possible states* of the world, capturing all the payoff-relevant characteristics of the environment. In *behavioral environments*, the choice correspondence of individual $i \in N$ at state $\omega \in \Omega$ maps 2^X to itself so that for all $S \in 2^X$, $C_i^\omega(S)$ is a (possibly empty) subset of S. In *rational environments*, every individual's choice correspondence at every state satisfies the weak axiom of revealed preferences (WARP) and are represented by *preferences* of individual $i \in N$ at state $\omega \in \Omega$ captured by a complete and transitive binary relation, a ranking, $R_i^\omega \subseteq X \times X$, while P_i^ω represents its strict counterpart.[3] In rational environments, for all $i \in N$, all $\omega \in \Omega$, and all $S \in \mathcal{X}$, $C_i^\omega(S) := \{x \in S \mid xR_i^\omega y \text{ for all } y \in S\}$, and $L_i^\omega(x) := \{y \in X \mid xR_i^\omega y\}$ denotes the *lower contour set of individual i at state ω of alternative x*.

We refer to any $\tilde{\Omega} \subset \Omega$ as a *domain*. A *social choice correspondence* (SCC) defined on a domain $\tilde{\Omega}$ is $f : \tilde{\Omega} \to \mathcal{X}$, a non-empty valued correspondence mapping $\tilde{\Omega}$ into X. Given $\omega \in \tilde{\Omega}$, $f(\omega)$, the set of *f-optimal* alternatives at ω, consists of alternatives that the planner desires to sustain at ω. SCC f on $\tilde{\Omega}$ is ***unanimous*** if for any $\omega \in \tilde{\Omega}$, $x \in \cap_{i \in N} C_i^\omega(X)$ implies $x \in f(\omega)$. Moreover, SCC f on $\tilde{\Omega}$ satisfies ***no-veto-power property*** if for any $\omega \in \tilde{\Omega}$ and for any $x \in X$ there are $i, j \in N$ with $i \neq j$ such that $x \notin C_i^\omega(X)$ and $x \notin C_j^\omega(X)$.

The environment is of complete information in the sense that the true state of the world is common knowledge among the individuals but unknown to the planner as in Maskin (1999).

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[3] It is well-known that a choice correspondence satisfies WARP if and only if it satisfies the independence of irrelevant alternatives (IIA) and Sen's β. A choice correspondence C defined on $\mathcal{X}$ satisfies the IIA if for all $S, T \in \mathcal{X}$ with $S \subset T$, $x \in C(T) \cap S$ implies $x \in C(S)$, and Sen's β if for all $S, T \in \mathcal{X}$ with $S \subset T$, $x, y \in C(S)$ implies $x \in C(T)$ if and only if $y \in C(T)$. Further, a binary relation $R \subseteq X \times X$ is *complete* if for all $x, y \in X$ either xRy or yRx or both; *transitive* if for all $x, y, z \in X$ with xRy and yRz implies xRz.

A mechanism $\mu = (M, g)$ assigns each individual $i \in N$ a non-empty *message space* M_i and specifies an *outcome function* $g : M \to X$ where $M = \times_{j \in N} M_j$. Given a mechanism μ and $m_{-i} \in M_{-i} := \times_{j \neq i} M_j$, the *opportunity set* of individual i pertaining to others' message profile m_{-i} in mechanism μ is $O_i^{\mu}(m_{-i}) := g(M_i, m_{-i}) = \{g(m_i, m_{-i}) \mid m_i \in M_i\}$.

A message profile $m^* \in M$ is a ***Nash equilibrium of mechanism*** μ ***at state*** $\omega \in \Omega$ if $g(m^*) \in \cap_{i \in N} C_i^{\omega}(O_i^{\mu}(m_{-i}^*))$.[4] Given mechanism μ, the correspondence $NE^{\mu} : \Omega \twoheadrightarrow 2^X$ identifies ***Nash equilibrium outcomes of mechanism*** μ ***at state*** $\omega \in \Omega$ and is defined by $NE^{\mu}(\omega) := \{x \in X \mid \exists m^* \in M \text{ s.t. } g(m^*) \in \cap_{i \in N} C_i^{\omega}(O_i^{\mu}(m_{-i}^*)) \text{ and } g(m^*) = x\}$. A mechanism μ ***implements SCC*** f ***on domain*** $\tilde{\Omega}$***,*** $f : \tilde{\Omega} \to \mathcal{X}$***, in Nash equilibrium*** if $NE^{\mu}(\omega) = f(\omega)$ for all $\omega \in \tilde{\Omega}$.

Thanks to the necessity result for Nash implementability of an SCC by Maskin (1999), we know that if $f : \Theta \to \mathcal{X}$ is Nash implementable, then it is ***Maskin-monotonic***: $x \in f(\omega)$ and $L_i^{\omega}(x) \subset L_i^{\tilde{\omega}}(x)$ for all $i \in N$ implies $x \in f(\tilde{\omega})$. de Clippel (2014) generalizes Maskin's results on Nash implementation to behavioral domains. The resulting necessary condition behavioral implementation is equivalent to Maskin-monotonicity in the rational domain Barlo and Dalkıran (2022*a*) and calls for the existence of a profile of sets that are *consistent* with this SCC at hand: We say that a profile of sets $\mathbf{S} := (S_i(x, \theta))_{i \in N, \omega \in \tilde{\Omega}, x \in f(\theta)}$ is ***consistent*** with a given SCC $f : \tilde{\Omega} \to \mathcal{X}$ if

(*i*) if $x \in f(\omega)$, then $x \in \cap_{i \in N} C_i^{\omega}(S_i(x, \omega))$, and

(*ii*) if $x \in f(\omega) \setminus f(\tilde{\omega})$, then $x \notin \cap_{i \in N} C_i^{\tilde{\omega}}(S_i(x, \omega))$.

The current study aims to restrict the planner to anonymity when designing the mechanism and its choice sets. That is why we introduce the notion of anonymous implementation:

Definition 2.1. *A mechanism* μ ***anonymously implements SCC*** f ***on domain*** $\tilde{\Omega}$***,*** $f : \tilde{\Omega} \to \mathcal{X}$***,*** *if*

(*i*) *for all* $\omega \in \tilde{\Omega}$ *and all* $x \in f(\omega)$*, there is* $m^{(x,\omega)} \in M$ *such that* $g(m^{(x,\omega)}) = x \in \cap_{i \in N} C_i^{\omega}(O_i^{\mu}(m_{-i}^{(x,\omega)}))$*, and* $O_i^{\mu}(m_{-i}^{(x,\omega)}) = O_j^{\mu}(m_{-j}^{(x,\omega)})$ *for all* $i, j \in N$*; and*

(*ii*) *if* $m^* \in M$ *is such that* $g(m^*) \in \cap_{i \in N} C_i^{\tilde{\omega}}(O_i^{\mu}(m_{-i}^*))$ *and* $O_i^{\mu}(m_{-i}^*) = O_j^{\mu}(m_{-j}^*)$ *for all* $i, j \in N$*, then* $g(m^*) \in f(\tilde{\omega})$*.*

A practical shortcut to formalizing anonymous implementation involves the in-

[4]The notion of NE in behavioral domains, the behavioral Nash equilibrium, is introduced by Korpela (2012).

troduction of the following refinement of NE:[5] A message profile $m^* \in M$ is an ***anonymous Nash equilibrium of mechanism*** μ ***at state*** $\omega \in \Omega$ if $g(m^*) \in \cap_{i\in N} C_i^{\omega}(O_i^{\mu}(m_{-i}^*))$ and $O_i^{\mu}(m_{-i}^*) = O_j^{\mu}(m_{-j}^*)$ for all $i, j \in N$. So, a mechanism μ anonymously implements SCC f on domain $\tilde{\Omega}$ if and only if $ANE^{\mu}(\omega) = f(\omega)$ for all $\omega \in \tilde{\Omega}$, where $ANE^{\mu} : \Omega \twoheadrightarrow 2^X$, the set of ANE outcomes of mechanism μ at state $\omega \in \Omega$, is given by $ANE^{\mu}(\omega) := \{x \in X \mid \exists m^* \in M \text{ s.t. } m^* \in M \text{ is an } ANE \text{ of } \mu \text{ at } \omega\}$. We wish to emphasize that ANE implementation of an SCC does not necessitate a symmetric mechanism.

In many interesting economic environments, planners have to obey exogenous given fairness restrictions when implementing desirable social goals. Consequently, given equivalence classes on individuals (based on sex, or level of education, and etc.) we restrict the planner to "treat" individuals in the same equivalence class in the same way.

To that regard, the exogenously determined equivalence classes on individuals are captured by a given partition, namely $\{N_k\}_{k=1}^K$ with the defining property that for all $k, l \in \{1, \ldots, K\}$ with $k \neq l$ we have $N_k \cap N_l = \emptyset$ and $\cup_{k=1}^K N_k = N$.

Definition 2.2. *Given a partition* $\{N_k\}_{k=1}^K$*, A mechanism* μ ***partitioned-anonymously implements SCC*** f ***on domain*** $\tilde{\Omega}$***,*** $f : \tilde{\Omega} \to \mathcal{X}$***,*** *if*

(*i*) *for all* $\omega \in \tilde{\Omega}$*, all* $x \in f(\omega)$*, there is* $m^{(x,\omega)} \in M$ *such that* $g(m^{(x,\omega)}) = x$ *and*

  (*a*) $x \in \cap_{i\in N}\ C_i^{\omega}(O_i^{\mu}(m_{-i}^{(x,\omega)}))$*, and*

  (*b*) $O_i^{\mu}(m_{-i}^{(x,\omega)}) = O_j^{\mu}(m_{-j}^{(x,\omega)})$ *if* $i, j \in N_k$ *with* $k \in \{1, \ldots, K\}$*; and*

(*ii*) *if* $m^* \in M$ *is such that* $g(m^*) \in \cap_{i\in N} C_i^{\tilde{\omega}}(O_i^{\mu}(m_{-i}^*))$ *and* $O_i^{\mu}(m_{-i}^*) = O_j^{\mu}(m_{-j}^*)$ *for all* $i, j \in N_k$ *and all* $k = 1, \ldots, K$*, then* $g(m^*) \in f(\tilde{\omega})$*.*

This extension allows for a more flexible design while ensuring fairness within groups. By partitioning agents, we can maintain equitable treatment within partitions while allowing for differentiated opportunities across different partitions.

When formalizing partitioned-anonymous implementation, one could employ a practical shortcut similar to that we presented for anonymous implementation, which involves the introduction of the following refinement of NE: Given a partition $\{N_k\}_{k=1}^K$, a message profile $m^* \in M$ is a ***partitioned-anonymous Nash equilibrium (PANE) of mechanism*** μ ***at state*** $\omega \in \Omega$ if $g(m^*) \in \cap_{i\in N} C_i^{\omega}(O_i^{\mu}(m_{-i}^*))$ and $O_i^{\mu}(m_{-i}^*) = O_j^{\mu}(m_{-j}^*)$ for all $i, j \in N_k$ and all $k \in \{1, \ldots, K\}$. So, a mechanism μ partitioned-anonymously implements SCC f on domain $\tilde{\Omega}$ for the given partition

[5] We thank Kemal Yıldız for suggesting this approach.

$\{N_k\}_{k=1}^K$ if and only if $PANE^\mu(\omega) = f(\omega)$ for all $\omega \in \tilde{\Omega}$, where $PANE^\mu : \Omega \twoheadrightarrow 2^X$, the set of PANE outcomes of mechanism μ at state $\omega \in \Omega$, is given by $PANE^\mu(\omega) := \{x \in X \mid \exists m^* \in M \text{ s.t. } m^* \in M \text{ is an } PANE \text{ of } \mu \text{ at } \omega \text{ for partition } \{N_k\}_{k=1}^K\}$.

3. AN EXAMPLE

In what follows, we present an example in the rational domain involving an SCC that is anonymously implementable but is not implementable in NE. Here there is only partition so we restrict all the agents to have the same opportunity sets to obey the anoynmous implementability. We have two agents, Ann and Bob, and three alternatives, a, b, c. The domain $\tilde{\Omega}$ equals $\{\omega^{(1)}, \omega^{(2)}, \omega^{(3)}\}$, and individuals' state-contingent rankings are as in Table 3.1. The planner aims to implement SCC

Table 3.1 Individuals' state-contingent rankings

$\omega^{(1)}$		$\omega^{(2)}$		$\omega^{(3)}$	
$R_A^{\omega^{(1)}}$	$R_B^{\omega^{(1)}}$	$R_A^{\omega^{(2)}}$	$R_B^{\omega^{(2)}}$	$R_A^{\omega^{(3)}}$	$R_B^{\omega^{(3)}}$
b	a	a,b	c	c	c
a	b	c	a,b	a	a
c	c			b	b

$f : \tilde{\Omega} \to \mathcal{X}$ given by $f(\omega^{(1)}) = \{a\}$, $f(\omega^{(2)}) = \{b\}$, and $f(\omega^{(3)}) = \{c\}$. Consider the mechanism in Table 3.2.

Table 3.2 The mechanism

		Bob		
		L	M	R
Ann	U	a	c	a
	M	c	c	a
	D	a	a	b

The message profile (U,L) (shown as circled) is an ANE of μ at state $\omega^{(1)}$ as $a \in C_A^{\omega^{(1)}}(O_A^{\mu}(L)) \cap C_B^{\omega^{(1)}}(O_B^{\mu}(U))$ and $O_A^{\mu}(L) = O_B^{\mu}(U) = \{a,c\}$. Moreover, $NE^{\mu}(\omega^{(1)}) = \{a\}$ and hence $ANE^{\mu}(\omega^{(1)}) = \{a\} = f(\omega^{(1)})$. On the other hand, $b \in C_A^{\omega^{(2)}}(O_A^{\mu}(R)) \cap C_B^{\omega^{(2)}}(O_B^{\mu}(D))$ and $O_A^{\mu}(R) = O_B^{\mu}(D) = \{a,b\}$ enables us to conclude that (D,R) (depicted with a square around it) $b \in ANE^{\mu}(\omega^{(2)})$. Meanwhile, the other NE are given by (D,L) and (D,M). As $O_A^{\mu}(L) = O_A^{\mu}(M) = \{a,c\}$ and $O_B^{\mu}(D) = \{a,b\}$, we conclude that $ANE^{\mu}(\omega^{(2)}) = \{b\} = f(\omega^{(2)})$. Similarly,

$c \in C_A^{\omega^{(3)}}(O_A^\mu(M)) \cap C_B^{\omega^{(3)}}(O_B^\mu(M))$ and $O_A^\mu(M) = O_B^\mu(M) = \{a,c\}$ implies that (M,M) (depicted with a diamond around it) $c \in ANE^\mu(\omega^{(3)})$; $NE^\mu(\omega^{(3)}) = \{c\}$ we see that $ANE^\mu(\omega^{(3)}) = \{c\} = f(\omega^{(3)})$.

Therefore, μ anonymously implements SCC f.

To illustrate how NE that are not ANE may constitute grounds for objection based on justified envy, let us consider the message profile (D,M), an NE at state $\omega^{(2)}$ resulting in alternative a, which is not desirable at that state according to the given SCC. Then, only Ann (but not Bob) has alternative c as an additional opportunity while c is Bob's top choice. That is why Bob envies Ann's equilibrium opportunities in NE (D,M) at state $\omega^{(2)}$ even though the mechanism itself is symmetric.[6]

Meanwhile, (D,M) being NE at $\omega^{(2)}$ also shows that μ does not implement f in NE since $NE^\mu(\omega^{(2)}) = \{a,b\} \neq \{b\} = f(\omega^{(2)})$.

One may wonder if there is another mechanism that implements SCC f in NE. In what follows, we establish that in this example, the answer is negative: f is not Nash implementable.

To achieve a contradiction, suppose that SCC $f : \tilde{\Omega} \to \mathcal{X}$ were implementable in NE. Then, thanks to de Clippel's necessity result, we know there is a profile of sets $\mathbf{S} = (S_i(x,\omega))_{i\in N, \omega\in\tilde{\Omega}, x\in f(\omega)}$ consistent with f. In particular, for any $i \in N$, $\omega \in \tilde{\Omega}$, and $x \in f(\omega)$, $S_i(x,\omega)$ is given by $O_i^\mu(m_{-i}^{(x,\omega)})$ where $m^{(x,\omega)} \in M$ is a NE sustaining x, i.e., $g(m^{(x,\omega)}) = x \in \cap_{i\in N} C_i^\omega(O_i^\mu(m_{-i}^{(x,\omega)}))$. So, $f(\omega^{(2)}) = \{b\}$ and (i) of consistency implies $S_B(b,\omega^{(2)})$ equals either $\{b\}$ or $\{a,b\}$. If $S_B(b,\omega^{(2)}) = \{b\}$, then the mechanism μ has a NE $m^{(b,\omega^{(2)})} \in M$ such that $O_B^\mu(m_A^{(b,\omega^{(2)})}) = \{b\}$ (i.e., b constitutes Bob's only choice) and hence for all messages $m_B \in M_B$ we have $g(m_A^{(b,\omega^{(2)})}, m_B) = b$. So, $b \in O_A^\mu(m_B)$ for all $m_B \in M_B$. As b is Ann's top-ranked alternative at $\omega^{(1)}$ and $O_B^\mu(m_A^{(b,\omega^{(2)})}) = \{b\}$, we observe that $(m_A^{(b,\omega^{(2)})}, m_B)$ is a NE of μ at $\omega^{(1)}$ since $b \in C_A^{\omega^{(1)}}(O_A^\mu(m_B)) \cap C_B^{\omega^{(1)}}(O_B^\mu(m_A^{(b,\omega^{(2)})}))$. But, $b \notin f(\omega^{(1)}) = \{a\}$. Thus, $S_B(b,\omega^{(2)}) = \{a,b\}$ as $S_B(b,\omega^{(2)})$ cannot equal $\{b\}$. So, $S_B(b,\omega^{(2)}) = O_B^\mu(m_A^{(b,\omega^{(2)})}) = \{a,b\}$ and hence there exists $\tilde{m}_B \in M_B$ such that $g(m_A^{(b,\omega^{(2)})}, \tilde{m}_B) = a$; ergo, $a \in O_A^\mu(\tilde{m}_B)$. Then, because $a \in C_B^{\omega^{(2)}}(S_B(b,\omega^{(2)})) = C_B^{\omega^{(2)}}(\{a,b\}) = \{a,b\}$ and a is Ann's top-ranked alternative at $\omega^{(2)}$, a emerges as a Nash equilibrium outcome (and message profile $(m_A^{(b,\omega^{(2)})}, \tilde{m}_B)$ as a NE) at $\omega^{(2)}$ because $a \in C_A^{\omega^{(2)}}(O_A^\mu(\tilde{m}_B)) \cap C_B^{\omega^{(1)}}(O_B^\mu(m_A^{(b,\omega^{(2)})}))$. But, $a \notin f(\omega^{(2)}) = \{b\}$. Hence, we cannot have $S_B(b,\omega^{(2)})) = \{a,b\}$ as well, which implies the desired contradiction.

[6]Similarly, Ann envies Bob's equilibrium opportunities in NE (D,L) at state $\omega^{(1)}$: This NE results in alternative a, and in equilibrium only Bob has b as an additional opportunity while it is Ann's top ranked alternative at $\omega^{(1)}$.

4. NECESSITY AND SUFFICIENCY

We proceed with the key condition for anonymous implementation. In what follows, we show that this condition is necessary and almost sufficient for anonymous implementation of SCCs. We note that the following condition applies both to the rational and the behavioral domains.

Definition 4.1. *Given an environment $\langle N, X, \Omega, (C_i^{\omega})_{i\in N}\rangle$ and SCC f on domain $\tilde{\Omega}$, $f:\tilde{\Omega}\to\mathcal{X}$, a profile of sets $\mathbf{S} := (S(x,\omega))_{\omega\in\tilde{\Omega},\ x\in f(\omega)}$ is* ***anonymous consistent*** *with f on domain $\tilde{\Omega}$ if*

(*i*) *for all $\omega\in\tilde{\Omega}$ and all $x\in f(\omega)$, $x\in\bigcap_{i\in N} C_i^{\omega}(S(x,\omega))$; and*

(*ii*) *$x\in f(\omega)\setminus f(\tilde{\omega})$ for any $\omega,\tilde{\omega}\in\tilde{\Omega}$ implies that $x\notin\bigcap_{i\in N} C_i^{\tilde{\omega}}(S(x,\omega))$.*

Next, we present our result, providing a full characterization of SCCs that are anonymously implementable (both in the rational and behavioral domains):

Theorem 4.1. *Given an environment $\langle N, X, \Omega, (C_i^{\theta})_{i\in N}\rangle$,*

(*i*) *if SCC $f:\tilde{\Omega}\to\mathcal{X}$ is anonymously implementable on domain $\tilde{\Omega}$, then there is a profile of sets anonymous consistent with f on domain $\tilde{\Omega}$.*

(*ii*) *if there is a profile of sets anonymous consistent with a unanimous SCC $f:\tilde{\Omega}\to\mathcal{X}$, then f is anonymously implementable on domain $\tilde{\Omega}$ whenever $n\geq 3$.*

Proof of (i) *of Theorem 4.1.* To prove (i) of Theorem 4.1, suppose that $f:\tilde{\Omega}\to\mathcal{X}$ is anonymously implementable in NE on domain $\tilde{\Omega}$. So, for all ω and all $x\in f(\omega)$, there is $m^{x,\omega}\in M$ such that $O_i^{\mu}(m_{-i}^{x,\omega}) = O_j^{\mu}(m_{-j}^{x,\omega})$ for all $i,j\in N$ and $g(m^{x,\omega}) = x\in\cap_{i\in N} C_i^{\omega}(O^{\mu}(m_{-i}^{x,\omega}))$. Define $\mathbf{S}$ as follows: for all ω and $x\in f(\omega)$, $S(x,\omega) := O_i^{\mu}(m_{-i}^{x,\omega})$ for any $i\in N$. Then $\mathbf{S}$ satisfies (i) of anonymous consistency as for all $\omega\in\tilde{\Omega}$, and $x\in f(\omega)$, $g(m^{x,\omega}) = x\in\cap_{i\in N} C_i^{\omega}(O^{\mu}(m_{-i}^{x,\omega}))$ and $O_i^{\mu}(m_{-i}^{x,\omega}) = O_j^{\mu}(m_{-j}^{x,\omega})$ for all $i,j\in N$. To show that $\mathbf{S}$ satisfies (ii) of anonymous consistency, suppose for some $\omega,\tilde{\omega}\in\tilde{\Omega}$, $x\in f(\omega)\setminus f(\tilde{\omega})$ and $x\in\cap_{i\in N} C_i^{\tilde{\omega}}(S(x,\omega))$. Then, $x\in\cap_{i\in N} C_i^{\tilde{\omega}}(O^{\mu}(m_{-i}^{x,\omega}))$. Since, $O_i^{\mu}(m_{-i}^{x,\omega}) = S(x,\omega) = O_j^{\mu}(m_{-j}^{x,\omega})$ for all $i,j\in N$, $m^{x,\omega}$ is an ANE at $\tilde{\omega}$ as

$x = g(m^{x,\omega})$. Thus, we obtain the desired contradiction as $x \in f(\tilde{\omega})$ (as μ implements f anonymously on $\tilde{\Omega}$). $\square$

Proof of (ii) of Theorem 4.1. Suppose SCC $f : \tilde{\Omega} \to \mathcal{X}$ is unanimous and the profile $\mathbf{S} = (S(x,\omega))_{\omega\in\Omega,\ x\in f(\omega)}$ is anonymous consistent with f on domain $\tilde{\Omega}$. Consider the canonical mechanism given as follows: $M_i = X \times \tilde{\Omega} \times X \times \mathbb{N}$ where $m_i = (x^i, \omega^i, y^i, k^i)$ with $x^i \in f(\omega^i)$, $y^i \in X$, $\omega^i \in \tilde{\Omega}$, and $k^i \in \mathbb{N}$ for all $i \in N$; the outcome function $g : M \to X$ defined by

Rule 1: If $m_i = (x,\omega,\cdot,\cdot)$ for all $i \in N$, then $g(m) = x$;

Rule 2: If $m_i = (x,\omega,\cdot,\cdot)$ for all $i \in N \setminus \{j\}$ for some $j \in N$ and $m_j \neq m_i$ with
$$m_j = (x',\omega',y',\cdot), \text{ then } g(m) = \begin{cases} x & \text{if } y' \notin S(x,\omega), \\ y' & \text{if } y' \in S(x,\omega). \end{cases}$$

Rule 3: In all other cases, $g(m) = y^{i^*}$ where $i^* = \max\{i \in N \mid k^i \geq k^j\ \forall j \in N\}$.

The result holds thanks to the following two claims.

Claim 4.1. *For all $\omega \in \tilde{\Omega}$ and $x \in f(\omega)$, $m^{(x,\omega)}$ defined by $m_i^{(x,\omega)} = (x,\omega,x,1)$ is an ANE of μ at ω s.t. $g(m^{(x,\omega)}) = x$.*

Proof. Let $\omega \in \tilde{\Omega}$, $x \in f(\omega)$, and $m^{(x,\omega)}$ be as in the statement of the claim. Then, Rule 1 holds under $m^{(x,\omega)}$. So, $g(m^{(x,\omega)}) = x$, and due to Rules 1 and 2, $O_i^{\mu}(m_{-i}^{(x,\omega)}) = S(x,\omega)$ for all $i \in N$. By (i) of anonymous consistency, $g(m^{(x,\omega)}) = x \in \cap_{i\in N} C_i^{\omega}(S(x,\omega))$. So, $m^{x,\omega}$ is an ANE of μ at ω. $\square$

Claim 4.2. *If m^* is an ANE of μ at $\omega \in \tilde{\Omega}$, then $g(m^*) \in f(\omega)$.*

Proof. Suppose m^* is an ANE of μ at $\omega \in \tilde{\Omega}$.

Suppose additionally that Rule 1 holds under m^*. So, let $m_i^* = (x',\omega',\cdot,\cdot)$ with $\omega' \in \tilde{\Omega}$ and $x' \in f(\omega')$ for all $i \in N$. By Rules 1 and 2, $O_i^{\mu}(m_{-i}^*) = S(x',\omega')$ for all $i \in N$ and $g(m^*) = x'$. If $x' \notin f(\omega)$, then $x' \notin \cap_{i\in N} C_i^{\omega}(S(x',\omega'))$ (by (ii) of anonymous consistency); this is equivalent to $x' \notin \cap_{i\in N} C_i^{\omega}(O_i^{\mu}(m_{-i}^*))$ thanks to Rule 1; i.e., m^* is not an ANE of μ at ω. This delivers the desired contradiction and establishes that $g(m^*) = x' \in f(\omega)$ when Rule 1 holds under m^*.

If Rule 2 holds under m^*, then (by Rules 1, 2, and 3) for all $i \in N \setminus \{j\}$ for some $j \in N$, $O_i^{\mu}(m_{-i}^*) = X$ and $O_j^{\mu}(m_{-j}^*) = S(x,\omega)$. Thus, $S(x,\omega) = X$ as m^* is an ANE. Then, as f is unanimous, $g(m^*) \in \cap_{i\in N} C_i^{\omega}(X)$ implies $g(m^*) \in f(\omega)$.

On the other hand, if Rule 3 holds under m^*, then for all $i \in N$, $O_i^{\mu}(m_{-i}^*) = X$. As m^* is an ANE, $g(m^*) \in \cap_{i\in N} C_i^{\omega}(X)$. This implies that $g(m^*) \in f(\omega)$ since f is unanimous. $\square$

Before proceeding further with the analysis of partitioned-anonymous implementation and efficiency, we wish to display the relation of anonymous consistency with Maskin-monotonicity in the following lemma:

Lemma 4.1. *Given a rational environment $\langle N, X, \Omega, (C_i^\omega)_{i\in N}\rangle$ and SCC $f : \tilde{\Omega} \to \mathcal{X}$, there exists a profile of sets anonymous consistent with f on domain $\tilde{\Omega}$ if and only if f satisfies the following (anonymous Maskin-monotonicity) condition on domain $\tilde{\Omega}$: For any $\omega, \tilde{\omega} \in \tilde{\Omega}$,*

$$x \in f(\omega) \text{ and } \cap_{i\in N} L_i^\omega(x) \subset \cap_{i\in N} L_i^{\tilde{\omega}}(x) \text{ implies } x \in f(\tilde{\omega}).$$

Proof of Lemma 4.1. Suppose that the environment $\langle N, X, \Omega, (C_i^\omega)_{i\in N}\rangle$ is rational and SCC f defined on domain $\tilde{\Omega}$ is given by $f : \tilde{\Omega} \to \mathcal{X}$.

For the necessity direction of the lemma, suppose that $\mathbf{S} := (S(x,\omega))_{\omega\in\tilde{\Omega},\ x\in f(\omega)}$ is anonymous consistent with f on domain $\tilde{\Omega}$ and adopt the hypothesis that $\omega, \tilde{\omega} \in \tilde{\Omega}$, $x \in f(\omega)$, and $\cap_{i\in N} L_i^\omega(x) \subset \cap_{i\in N} L_i^{\tilde{\omega}}(x)$. Hence, by (i) of anonymous consistency, we see that $S(x,\omega) \subset \cap_{i\in N} L_i^\omega(x)$. Ergo, $S(x,\omega) \subset \cap_{i\in N} L^{\tilde{\omega}}(x)$. If $x \notin f(\tilde{\omega})$, then by (ii) of anonymous consistency, there is $j \in N$ such that $x \notin C_i^{\tilde{\omega}}(S(x,\omega))$. So, there is $j \in N$ and $y^* \in S(x,\omega)$ such that $y^* P_j^{\tilde{\omega}} x$; i.e., $y^* \notin L_j^{\tilde{\omega}}(x)$. But, $y^* \in S(x,\omega)$ and $y^* \notin L_j^{\tilde{\omega}}(x)$ contradicts $S(x,\omega) \subset \cap_{i\in N} L_i^{\tilde{\omega}}(x)$.

To establish the sufficiency direction, define $\mathbf{S}$ so that for any $\omega \in \tilde{\Omega}$ and $x \in f(\omega)$, we have $S(x,\omega) := \cap_{i\in N} L_i^\omega(x)$. Then, $\mathbf{S}$ satisfies (i) of anonymous consistency trivially due to the definition of lower contour sets. To obtain (ii) of anonymous consistency, suppose that $x \in f(\omega) \setminus f(\tilde{\omega})$ for some $\omega, \tilde{\omega} \in \tilde{\Omega}$. So, $S(x,\omega) = \cap_{i\in N} L_i^\omega$ is not a subset of $\cap_{i\in N} L_i^{\tilde{\omega}}(x)$. Thus, there is $j \in N$ and $y^* \in S(x,\omega)$ with $y^* \notin L_j^{\tilde{\omega}} x$; i.e. $y^* P_j^{\tilde{\omega}} x$. Ergo, $x \notin C_j^{\tilde{\omega}}(S(x,\omega))$. $\square$

When it comes to partitioned-anonymous implementation, we consider the following condition:

Definition 4.2. *Given a partition $(N_1, N_2, \ldots, N_k) \in \mathcal{N}$ and an environment $\langle N, X, \Omega, (C_i^\omega)_{i\in N}\rangle$ and SCC f on domain $\tilde{\Omega}$, $f : \tilde{\Omega} \to \mathcal{X}$, a profile of sets $\mathbf{S} := (S_k(x,\omega))_{\omega\in\tilde{\Omega},\ x\in f(\omega)\ k\in\{1,\ldots,K\}}$ is **partitioned-anonymous consistent** with respect to the the partition $\{N_1, N_2 \ldots, N_K\}$ with f on domain $\tilde{\Omega}$ if*

- *(i) for all $\omega \in \tilde{\Omega}$, all $x \in f(\omega)$, and all $k = 1, \ldots, K$, we have $x \in \bigcap_{i\in N_k} C_i^\omega(S_k(x,\omega))$; and* (*ii*) *$x \in f(\omega) \setminus f(\tilde{\omega})$ for any $\omega, \tilde{\omega} \in \tilde{\Omega}$ implies that there is $j \in N_k$ with $x \notin C_j^{\tilde{\omega}}(S_k(x,\omega))$.*

As is documented in the following theorem, we need a no-veto power condition in our sufficiency result:

Definition 4.3. *Given environment $\langle N, X, \Omega, (C_i^{\omega})_{i\in N}\rangle$, we say that SCC f on domain $\tilde{\Omega}$, $f : \tilde{\Omega} \to \mathcal{X}$ satisfies the no-veto-power property if*

$$x \in C_i^{\omega}(X) \text{ for all } i \in N \setminus \{j\} \text{ for some } j \in N \text{ implies } x \in f(\omega).$$

We now can present our characterization result for partitioned-anonymous implementation:

Theorem 4.2. *Given an environment $\langle N, X, \Omega, (C_i^{\theta})_{i\in N}\rangle$ with an equivalence partition on individuals $\{N_k\}_{k=1}^K$,*

(*i*) *if SCC $f : \tilde{\Omega} \to \mathcal{X}$ is partitioned-anonymously implementable on domain $\tilde{\Omega}$ given partition $\{N_k\}_{k=1}^K$, then there is a profile of sets partitioned-anonymous consistent with f on domain $\tilde{\Omega}$ for partition $\{N_k\}_{k=1}^K$.*

(*ii*) *if there is a profile of sets partitioned-anonymous consistent with an SCC $f : \tilde{\Omega} \to \mathcal{X}$ given partition $\{N_k\}_{k=1}^K$ and satisfies the no-veto-power property, then f is anonymously implementable on domain $\tilde{\Omega}$ for partition $\{N_k\}_{k=1}^K$ whenever $n \geq 3$.*

Proof of (i) of Theorem 4.2. To prove (*i*) of Theorem 4.2, suppose that $f : \tilde{\Omega} \to \mathcal{X}$ is partitioned-anonymously implementable on domain $\tilde{\Omega}$ given partition $\{N_k\}_{k=1}^K$. So, for all ω, all $x \in f(\omega)$, there is $m^{x,\omega} \in M$ such that $g(m^{x,\omega}) = x \in \cap_{i\in N} C_i^{\omega}(O^{\mu}(m_{-i}^{x,\omega}))$ with the requirement that $O_i^{\mu}(m_{-i}^{x,\omega}) = O_j^{\mu}(m_{-j}^{x,\omega})$ if $i,j \in N_k$ for some $k \in \{1,\ldots,K\}$.

Define $\mathbf{S}$ as follows: for all ω, all $x \in f(\omega)$, and all $k \in \{1,\ldots,K\}$, $S_k(x,\omega) := O_i^{\mu}(m_{-i}^{x,\omega})$ for any $i \in N_k$.

Then $\mathbf{S}$ satisfies (*i*) of partitioned-anonymous consistency as for all $\omega \in \tilde{\Omega}$, and $x \in f(\omega)$, $g(m^{x,\omega}) = x \in \cap_{i\in N} C_i^{\omega}(O^{\mu}(m_{-i}^{x,\omega}))$ (as $\{N_k\}_{k=1}^K$ is a partition of N) and $O_i^{\mu}(m_{-i}^{x,\omega}) = S_k(x,\omega) = O_j^{\mu}(m_{-j}^{x,\omega})$ for all $i,j \in N_k$ and all $k \in \{1,\ldots,K\}$ by construction.

To show that $\mathbf{S}$ satisfies (*ii*) of partitioned-anonymous consistency, suppose for some $\omega, \tilde{\omega} \in \tilde{\Omega}$, $x \in f(\omega) \setminus f(\tilde{\omega})$ and $x \in \cap_{i\in N_k} C_i^{\tilde{\omega}}(S_k(x,\omega))$ for all $k \in \{1,\ldots,K\}$. Then, $x \in \cap_{i\in N} C_i^{\tilde{\omega}}(O^{\mu}(m_{-i}^{x,\omega}))$ as $\{N_k\}_{k=1}^K$ is a partition of N. Since, $O_i^{\mu}(m_{-i}^{x,\omega}) = S_k(x,\omega) = O_j^{\mu}(m_{-j}^{x,\omega})$ for all $i,j \in N_k$ and all $k \in \{1,\ldots,K\}$, $m^{x,\omega}$ is a PANE at $\tilde{\omega}$ as $x =$

$g(m^{x,\omega})$. Thus, we obtain the desired contradiction as $x \in f(\tilde{\omega})$ (as μ implements f partitioned-anonymously on $\tilde{\Omega}$). $\square$

Proof of (ii) of Theorem 4.2. Suppose SCC $f : \tilde{\Omega} \to \mathcal{X}$ satisfies the no-veto-power property and the profile $\mathbf{S} = (S_k(x,\omega))_{\omega\in\Omega,\ x\in f(\omega),\ k\in\{1,\dots,K\}}$ is partitioned-anonymous consistent with f on domain $\tilde{\Omega}$ for the given partition $\{N_k\}_{k=1}^K$ (which is fixed throughout the following proof).

Consider the canonical mechanism given as follows: $M_i = X \times \tilde{\Omega} \times X \times \mathbb{N}$ where $m_i = (x^i,\omega^i,y^i,\kappa^i)$ with $x^i \in f(\omega^i)$, $y^i \in X$, $\omega^i \in \tilde{\Omega}$, and $\kappa^i \in \mathbb{N}$ for all $i \in N$; the outcome function $g : M \to X$ defined by

Rule 1: If $m_i = (x,\omega,\cdot,\cdot)$ for all $i \in N$, then $g(m) = x$;

Rule 2: If $m_i = (x,\omega,\cdot,\cdot)$ for all $i \in N \setminus \{j\}$ for some $j \in N_k$ with $k \in \{1,\dots,K\}$ and $m_j \neq m_i$ with $m_j = (x',\omega',y',\cdot)$, then $g(m) = \begin{cases} x & \text{if } y' \notin S_k(x,\omega), \\ y' & \text{if } y' \in S_k(x,\omega). \end{cases}$

Rule 3: In all other cases, $g(m) = y^{i^*}$ where $i^* = \max\{i \in N \mid \kappa^i \geq \kappa^j\ \forall j \in N\}$.

The result holds thanks to the following two claims.

Claim 4.3. *For all $\omega \in \tilde{\Omega}$ and $x \in f(\omega)$, $m^{(x,\omega)}$ defined by $m_i^{(x,\omega)} = (x,\omega,x,1)$ is a PANE of μ at ω s.t. $g(m^{(x,\omega)}) = x$.*

Proof. Let $\omega \in \tilde{\Omega}$, $x \in f(\omega)$, and $m^{(x,\omega)}$ be as in the statement of the claim. Then, Rule 1 holds under $m^{(x,\omega)}$. So, $g(m^{(x,\omega)}) = x$, and due to Rules 1 and 2, $O_i^\mu(m_{-i}^{(x,\omega)}) = S_k(x,\omega)$ for all $i \in N_k$ with $k \in \{1,\dots,K\}$. Ergo, $i,j \in N_k$ for some $k \in \{1,\dots,K\}$ implies $O_i^\mu(m_{-i}^{(x,\omega)}) = S_k(x,\omega) = O_j^\mu(m_{-j}^{(x,\omega)})$. By (i) of partitioned-anonymous consistency, $g(m^{(x,\omega)}) = x \in \cap_{i\in N_k} C_i^\omega(S_k(x,\omega))$ for all $k = 1,\dots,K$ as $\{N_k\}_{k=1}^K$ is a partition of N. So, $m^{(x,\omega)}$ is a PANE of μ at ω. $\square$

Claim 4.4. *If m^* is an PANE of μ at $\omega \in \tilde{\Omega}$, then $g(m^*) \in f(\omega)$.*

Proof. Suppose m^* is an PANE of μ at $\omega \in \tilde{\Omega}$.

Suppose additionally that Rule 1 holds under m^*. So, let $m_i^* = (x',\omega',\cdot,\cdot)$ with $\omega' \in \tilde{\Omega}$ and $x' \in f(\omega')$ for all $i \in N$. By Rules 1 and 2, $O_i^\mu(m_{-i}^*) = S_k(x',\omega')$ for all $i \in N_k$, all $k \in \{1,\dots,K\}$; and, $g(m^*) = x'$. If $x' \notin f(\omega)$, then there is $k \in \{1,\dots,K\}$ such that $x' \notin \cap_{i\in N_k} C_i^\omega(S_k(x',\omega'))$ (by (ii) of partitioned-anonymous consistency). Ergo, $x' \notin \cap_{i\in N} C_i^\omega(O_i^\mu(m_{-i}^*))$; i.e., m^* is not a PANE of μ at ω. This delivers the desired contradiction and establishes that $g(m^*) = x' \in f(\omega)$ when Rule 1 holds under m^*.

If Rule 2 holds under m^*, then (by Rules 1, 2, and 3) for all $i \in N \setminus \{j\}$ for some $j \in N_k$, $O_i^{\mu}(m^*_{-i}) = X$ and $O_j^{\mu}(m^*_{-j}) = S_k(x,\omega)$. So, as f satisfies the no-veto-power property, $g(m^*) \in \cap_{i \in N\setminus\{j\}} C_i^{\omega}(X)$ implies $g(m^*) \in f(\omega)$.[7]

On the other hand, if Rule 3 holds under m^*, then for all $i \in N$, $O_i^{\mu}(m^*_{-i}) = X$. As m^* is an PANE, $g(m^*) \in \cap_{i \in N} C_i^{\omega}(X)$. This implies that $g(m^*) \in f(\omega)$ since f satisfies the no-veto-power-property. □

□

Lemma 4.2. *Given a partition $\{N_1,\ N_2,\ \ldots,\ N_k\} \in \mathcal{N}$ and a given a rational environment $\langle N, X, \Omega, (C_i^{\omega})_{i\in N}\rangle$ and SCC $f : \tilde{\Omega} \to \mathcal{X}$, there exists a profile of sets partitioned anonymous consistent with f on domain $\tilde{\Omega}$ if and only if f satisfies the following (partitioned anonymous Maskin-monotonicity) condition with respect to the the partition $\{N_1, N_2 \ldots, N_K\}$ with f on domain $\tilde{\Omega}$: For any $\omega, \tilde{\omega} \in \tilde{\Omega}$,*

$$x \in f(\omega) \textit{ and } \cap_{i\in N_k} L_i^{\omega}(x) \subset \cap_{i\in N_k} L_i^{\tilde{\omega}}(x) \textit{ implies } x \in f(\tilde{\omega}).$$

Proof of Lemma 4.2. Suppose that the environment $\langle N, X, \Omega, (C_i^{\omega})_{i\in N}\rangle$ is rational and SCC f defined on domain $\tilde{\Omega}$ is given by $f : \tilde{\Omega} \to \mathcal{X}$.

For the necessity direction of the lemma, suppose that $\mathbf{S} = (S_k(x,\omega))_{\omega\in\Omega,\ x\in f(\omega)}$ is partitioned anonymous consistent with f on domain $\tilde{\Omega}$ and adopt the hypothesis that $\omega, \tilde{\omega} \in \tilde{\Omega}$, $x \in f(\omega)$, and $\cap_{i\in N_k} L_i^{\omega}(x) \subset \cap_{i\in N_k} L_i^{\tilde{\omega}}(x)$. Hence, by (i) of partitioned anonymous consistency, we see that $S_k(x,\omega) \subset \cap_{i\in N_k} L_i^{\omega}(x)$. Ergo, $S_k(x,\omega) \subset \cap_{i\in N_k} L^{\tilde{\omega}}(x)$. If $x \notin f(\tilde{\omega})$, then by (ii) of partitioned anonymous consistency, there is $j \in N_k$ such that $x \notin C_i^{\tilde{\omega}}(S(x,\omega))$. So, there is $j \in N$ and $y^* \in S_k(x,\omega)$ such that $y^* P_j^{\tilde{\omega}} x$; i.e., $y^* \notin L_j^{\tilde{\omega}}(x)$. But, $y^* \in S(x,\omega)$ and $y^* \notin L_j^{\tilde{\omega}}(x)$ contradicts $S_k(x,\omega) \subset \cap_{i\in N} L_i^{\tilde{\omega}}(x)$.

To establish the sufficiency direction, define $\mathbf{S}$ so that for any $\omega \in \tilde{\Omega}$ and $x \in f(\omega)$, we have $S_k(x,\omega) := \cap_{i\in N_k} L_i^{\omega}(x)$. Then, $\mathbf{S}$ satisfies (i) of partitioned anonymous consistency trivially due to the definition of lower contour sets. To obtain (ii) of partitioned anonymous consistency, suppose that $x \in f(\omega) \setminus f(\tilde{\omega})$ for some $\omega, \tilde{\omega} \in \tilde{\Omega}$. So, $S_k(x,\omega) = \cap_{i\in N_k} L_i^{\omega}$ is not a subset of $\cap_{i\in N_k} L_i^{\tilde{\omega}}(x)$. Thus, there is $j \in N$ and $y^* \in S(x,\omega)$ with $y^* \notin L_j^{\tilde{\omega}} x$; i.e. $y^* P_j^{\tilde{\omega}} x$. Ergo, $x \notin C_j^{\tilde{\omega}}(S_k(x,\omega))$. □

[7] Note that if $|N_k| > 1$, then there is $\tilde{i} \neq j$ such that $O_{\tilde{i}}^{\mu}(m^*_{-\tilde{i}}) = X$ and hence $S_k(x,\omega)$ equals X. So, when the partitions are so that for all $k \in \{1,\ldots,K\}$, we have $|N_k| > 1$, the sufficiency result follows from unanimity; and hence, we may dismiss the no-veto-power condition. On the other hand, we need to employ the no-veto-power condition for cases involving $|N_k| = 1$ for some $k \in \{1,\ldots,K\}$.

5. EFFICIENCY

In rational environments, the Pareto SCC on the full domain Ω, $PO : \Omega \to \mathcal{X}$, is defined by

$$PO(\omega) := \{x \in X \mid \nexists y^* \in X \text{ s.t. } y^* P_i^\omega x \ \forall i \in N\}$$

for any $\omega \in \Omega$. On the other hand, in behavioral environments, we consider the efficiency SCC introduced by de Clippel (2014), $E^{\text{eff}} : \Omega \to \mathcal{X}$, which is defines as follows

$$E^{\text{eff}}(\omega) := \{x \in X \mid \exists (S_i)_{i \in N} \in \mathcal{X}^N \text{ s.t. } x \in \cap_{i \in N} C_i^\omega(S_i) \text{ and } \cup_{i \in N} S_i = X\}$$

for any $\omega \in \Omega$. We know that when $\tilde{\Omega}$ is a subset of the rational domain, then these two notions coincide, and hence efficiency SCC is an extension of the Pareto SCC to behavioral domains de Clippel (2014). Moreover, as choices are nonempty-valued, so are these SCCs: We observe that for all ω (in rational or behavioral domains) $x \in C_1^\omega(X)$ implies $x \in E^{\text{eff}}(\omega)$ by setting $S_1 = X$ and $S_j = \{x\}$ for all $j \neq 1$.

Below, we report bad news about the anonymous implementation of these efficiency notions.

We observe that PO is not anonymously implementable in the full rational domain whenever choices are non-empty valued due to the following: Suppose PO were anonymously implementable on the full rational domain and consider two states $\omega, \tilde{\omega}$ such that $L_1^\omega(x) = X$, $L_2^\omega(x) = \{x\}$, and $\cup_{i \in N} L_i^{\tilde{\omega}}(x) \neq X$. Then, $x \in PO(\omega) \setminus PO(\tilde{\omega})$. Further, $L_2^\omega(x) = \{x\}$ implies $O_i^\mu(m_{-i}^{\omega,x}) = \{x\}$ for all $i \in N$ where $m^{\omega,x} \in M$ is an ANE sustaining x at ω. But then, $m^{\omega,x}$ is also an ANE at state $\tilde{\omega}$ as $x \in \cap_{i \in N} C_i^{\tilde{\omega}}(\{x\})$.

We show that the failure of the anonymous implementability of efficiency extends to the behavioral domain whenever there are two states ω and $\tilde{\omega}$ in the domain $\tilde{\Omega}$ on which efficiency SCC is defined and an alternative $x \in X$ with $x \in E^{\text{eff}}(\omega) \setminus E^{\text{eff}}(\tilde{\omega})$ such that for any $S \in \mathcal{X}$, x is chosen from a set S at ω by all individuals implies x

continues to be chosen from S at $\tilde{\omega}$ by all agents.

Proposition 5.1. *Given an environment $\langle N, X, \Omega, (C_i^\theta)_{i\in N}\rangle$, efficiency SCC $E^{\mathit{eff}}: \tilde{\Omega} \to \mathcal{X}$ is not anonymously implementable on domain $\tilde{\Omega}$ whenever there are $\omega, \tilde{\omega} \in \tilde{\Omega}$ and $x \in E^{\mathit{eff}}(\omega) \setminus E^{\mathit{eff}}(\tilde{\omega})$ such that for all $S \in \mathcal{X}$, $x \in \cap_{i\in N} C_i^{\omega}(S)$ implies $x \in \cap_{i\in N} C_i^{\tilde{\omega}}(S)$.*

Proof of Proposition 5.1. Let $\tilde{\Omega} \subset \Omega$ be a domain such that there are $\omega^{(1)}, \omega^{(2)} \in \tilde{\Omega}$ and $x^* \in E^{\text{eff}}(\omega^{(1)}) \setminus E^{\text{eff}}(\tilde{\omega}^{(2)})$ such that for any $S \in \mathcal{X}$, $x^* \in \cap_{i\in N} C_i^{\omega^{(1)}}(S)$ implies $x^* \in \cap_{i\in N} C_i^{\omega^{(2)}}(S)$. Then, from the above we know that $m^{x^*,\omega^{(1)}}$ is such that $g(m^{x^*,\omega^{(1)}}) = x^*$ and $O_i^\mu(m_i^{x^*,\omega^{(1)}}) = S^*$ for all $i \in N$; and $x^* \in \cap_{i\in N} C_i^{\omega^{(2)}}(S^*)$. But then, $m^{x^*,\omega^{(1)}}$ is also an ANE of μ at $\omega^{(2)}$ which implies (thanks to μ anonymouysly implementing E^{eff}) $x^* \in E^{\text{eff}}(\omega^{(2)})$, a contradiction. $\square$

Notwithstanding, anonymous implementation of the Pareto SCC on rational subdomains can be achieved as the following example demonstrates: Let us refer to two individuals as Ann and Bob, $X = \{a, b, c\}$, $\tilde{\Omega} = \{\omega^{(1)}, \omega^{(2)}\}$, where individuals' strict rankings are as in Table 5.1. Pareto SCC PO on $\tilde{\Omega}$ is given by $PO(\omega^{(1)}) = \{a, b\}$

Table 5.1 Anonymous implementation of Pareto SCC on a rational subdomain

$\omega^{(1)}$		$\omega^{(2)}$	
$R_A^{\omega^{(1)}}$	$R_B^{\omega^{(1)}}$	$R_A^{\omega^{(2)}}$	$R_2^{\omega^{(2)}}$
a	b	b	c
b	a	c	b
c	c	a	a

and $PO(\omega^{(2)}) = \{b, c\}$. One can verify that the mechanism in Table **??** anonymously implements the Pareto SCC on domain $\tilde{\Omega}$ (where we depict ANE at $\omega^{(1)}$ by circling the corresponding cells and those at $\omega^{(2)}$ by using squares):

Table 5.2 The mechanism implementing SCC PO on a rational subdomain

		Bob			
		L	M_1	M_2	R
Ann	U	a	c	c	a
	C_1	c	b	c	b
	C_2	c	c	c	a
	D	a	b	a	b

6. CONCLUDING REMARKS

Implementing SCCs anonymously requires the planner to adhere to anonymity during mechanism design. This entails ensuring that any socially optimal alternative at any given state is achievable through an ANE at that state, and that any ANE at any given state must be socially optimal at that state. We identify *anonymous consistency* as the necessary and (almost) sufficient condition for anonymous implementation. This condition mirrors de Clippel (2014)'s consistency, with the additional constraint that choice sets are independent of individuals' identities. We demonstrate that anonymous implementation does not necessarily restrict the set of Nash-implementable social goals: In our example in Chapter 3, we present an SCC that is anonymously implementable but not Nash implementable. Our observation confirms that anonymity can expand society's range of decentralizable SCCs beyond those attainable through Nash implementation. Notwithstanding, we show that anonymity imposes a heavy burden when dealing with efficiency: The Pareto SCC cannot be anonymously implemented on the full domain.

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