ASSESSMENT OF THE PERFORMANCE OF THE FAIR DYNAMIC PRICING POLICY IN A COMPETITIVE MARKET

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ABSTRACT

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Keywords: Queueing Models, Make-to-Stock Queues, Inventory/Production Policies, Lead-time Quotation, Dynamic Pricing

In this thesis, we consider two companies, each modeled as a production/inventory system, that are in a competition while serving the same market. Each company produces a single type of product, which is a substitute of the one produced by its competitor. Demand for a product depends on its price and – when there is no stock – the quoted lead-time as well as what the other company charges and quotes for the substitute product. By modeling each system as an $M_n/M/1$ type make-to-stock queue, we restrict one company to follow a fair dynamic pricing policy while the competitor is unrestricted in this regard. A fair pricing policy stipulates that customers be charged lower prices if they are quoted longer lead times. Yet, both companies are expected to deliver a high proportion of their backlogged products during the lead times they quote. We adapt one policy for both companies, originally considered for a monopoly in the literature, which offers a different price and lead-time bundle depending on the position a customer joins the backlog queue. We develop a simpler alternative policy, more for the company applying fair pricing principles, according to which only two prices are charged: a high price when there is stock and a low price with a single lead-time for all backlogged customers. This problem setting lets us explore if the company sticking to fair pricing principles can survive in the presence of its competitor and if the simple policy can be further preferred.

ÖZET

REKABETÇİ BİR PAZARDA ADİL DEVİNGEN FİYATLANDIRMA POLİTİKASININ PERFORMANSININ DEĞERLENDİRİLMESİ

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Tez Danışmanı: Prof. Dr. Ahmet Barış Balcıoğlu

Anahtar Kelimeler: kuyruk modelleri, stok için üretim kuyrukları, envanter/üretim politikaları, teslim süresi belirleme, dinamik fiyatlandırma

Bu tezde aynı pazarda hizmet verirken rekabet eden, her biri birer üretim/envanter sistemi olarak modellenmiş iki firmayı ele aldık. İki firma da rakibinin ürününün yerine geçebilecek tek bir ürün çeşidi üretmektedir. Ürünlere talep, ürünün fiyatına ve – stok olmadığı durumlarda – öngörülen teslim süresine ve bunların yanı sıra rakip firmanın eşdeğer ürün için sunduğu fiyat ve teslim zamanına bağlıdır. İki sistemi de $M_n/M/1$ tipi stok için üretim kuyruğu olarak modelleyerek, bir firmayı adil ve dinamik bir fiyatlandırma politikası izleyecek şekilde sınırlandırılırken rakibi bu konuda kısıtlamıyoruz. Adil fiyatlandırma politikasına göre, ürünü teslim almak için daha uzun süre bekleyeceği duyurulan müşterilerden daha düşük bir ücret tahsil edilmelidir. Ancak her iki firmanın da, stokları yokken sipariş veren müşterilerine yüksek bir oranla, belirttikleri süre içinde ürünü vermesi gerekmektedir. İki firma için de ilgili yazında aslında tekeller için düşünülmüş bir politikayı uyarlıyoruz ki buna göre stok yokken kaçıncı sırada beklemeye başladığına bakarak müşteriye fiyat ve teslim zamanı sunulur. Daha çok adil fiyatlandırma prensiplerini uygulayan firma için olmak üzere daha basit bir politika da geliştiriyoruz. Bu politikaya göre sadece iki fiyat sunulur: stok olduğu durumda duyurulan yüksek fiyat ve stok olmayan durumlarda, tek bir teslim süresi duyurulan müşteriler için daha düşük tek bir fiyat. Bu çerçeve, adil fiyatlandırma politikasına bağlı kalan bir firmanın rakibi karşısında pazarda kalmaya devam edip edemeyeceğini ve basit politikanın her şeye rağmen tercih edilebilecek bir seçenek olup olmayacağını araştırmamıza olanak sağlamaktadır.

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to my family

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LIST OF ABBREVIATIONS

FIS	Full Information Setting
LIS	Limited Information Setting
MDP	Markov Decision Process
RP	Refined Policy
SP	Simple Policy

LIST OF SYMBOLS

$\approx$	Approximately equal
$\int$	Integral
$\sum$	Summation
λ	State-dependent rate of demand
μ	Mean production/service rate

1. INTRODUCTION

The performances of policies or company decisions impacting the customer demand and that are, in return, affected by the customer response have to be tested in the presence of a competitor. It is known that the price of a product and how long it would take the company to deliver it would influence a customer's decision on choosing that offer. It is also clear that the utility of the customers or the demand rate expressed as functions decreasing in price and lead-time imposes constraints even on a monopoly. Yet, the presence of an alternative channel for customers, that is, a competitor for the company, can change the dynamics further. For instance, as Griffith & Rust (1997) state, when companies focus on relative performance, they end up making less profit than the maximized profits that could be achieved if companies cooperate, and cooperation may result in charging the same price. In this thesis, our main interest is to investigate if a company that adopts fair pricing principles would survive against a competitor selling a substitutable product without following such restrictions in its pricing strategy.

The three principles of fairness we consider for one of the companies were first proposed for a monopoly by Balcıoğlu & Varol (2022) that serves price and delay sensitive customers. According to them, *i*) customers facing the same experience need be quoted the same price, *ii*) customers quoted longer lead times need be quoted lower prices, and *iii*) the company should deliver a disclosed and high proportion of the items within the quoted lead-time. Thus, customers arriving when there is stock are charged the same price, which needs be higher than the prices for backlogged customers.

Their numerical examples show that being fair is not a costly constraint and the profit loss due to it is minimal. However, can we say the same thing in the presence of a competitor who does not adapt the first two of these principles and can charge prices more freely? We assume that the violation of the third principle would kill the reputation of a company in the long run whereas customers making their purchasing decisions knowing the prices would not be detrimentally penalizing a company for not following fair pricing policies. At the end, a customer can choose not to order from that company. With this in mind, the questions we would like to answer are as follows: *i*) What sort of dynamic pricing and lead-time quotation policies can these companies employ? *ii*) Can the fair company survive in this environment? *iii*) Can simpler policies be profitable in addition to their practicality? *iv*) Would the picture change radically if none of the companies follows fair pricing policies?

To this end, we model both companies as make-to-stock queues, each one being an $M_n/M/1$ queue. This gives rise to a continuous-time Markov chain to capture the underlying dynamics from which the relevant probability distributions in steady-state can be computed. The production/service times are assumed to be independent and identically distributed (i.i.d.) exponential random variables and customers arrive one at a time according to a Poisson process with state-dependent arrival rates. These state-dependent rates are a function of the prices and, in case of a stock-out, the quoted lead times by both companies. The production of each company is controlled by a base-stock policy. According to this, the production stops when the inventory level reaches the base-stock level and restarts as soon as the inventory level decreases by 1. We propose two policies that can be followed by the companies. The Refined policy (RP), presented in Section 3.1.1, can offer a different set of price and lead-time depending on whether there is stock and otherwise, depending on the position in which a customer joins the backlog queue. This policy forces companies to quote different lead times for customers finding different number of backlogs in front of them when they start waiting for their order. If the company sticks to fair dynamic pricing as well, it has to also charge different prices decreasing in lead-time. The competitor, not bound with this principle, can choose its prices freely with the knowledge that what both companies offer as price and lead-time changes the demand rate. The second policy, namely the Simple policy (SP), presented in Section 3.1.2, exploits the observation from the literature (e.g., Balcıoğlu & Varol (2022), Kim & Randhawa (2018)) that offering two prices would bring most of the benefit one would get via a more detailed dynamic pricing scheme. This policy is more suitable for the company with fair pricing goals: a high price when there is stock and a low price with a single lead-time for all backlogged customers. Obviously, the competitor can also offer two prices and can even charge a higher price to backlogged customers. However, since the RP is expected to be more profitable, we assume that the competitor would normally opt for it instead of the SP. For both the RP and SP, the companies would determine an optimal limit on the maximum number of backlogged customers. This is in line with what other work such as Hammami, Frein & Albana (2021), Balcıoğlu & Varol (2022) suggest: not accepting all demand can increase profits. Since quoted lead times can not be attained all the time we allow a tardiness cost proportional to the length of late delivery to be paid in line with Savaşaneril, Griffin & Keskinocak (2010) and Kahvecioğlu & Balcıoğlu (2016). We do not consider production cost separately but holding stock is assumed to have its own cost.

We design our numerical study in Chapter 4 for two identical companies. In real markets, companies may have different service capacities, lead-time achievement rates, and produce different quality products, etc. However, since the objective of this study is to examine whether a company could make a difference solely with pricing, both companies would have the same level of service and production capacities. Moreover, although we develop our models in Chapter 3 for the full information setting (FIS), the numerical examples consider the limited information setting (LIS). In the former, a company would be able to see the production status of its competitor and this would give it a chance to maybe increase its price when the other one is out of stock. We think that this is less likely to be the practice and besides optimizing policies in the FIS would be computationally very expensive. In the LIS, the companies may not be able to see their competitor's production line and thus, have to rely only on the number of production orders they have when a new customer arrives. This has lead us to develop a demand model that would capture the impact of competition such that when one company charges very high prices or quotes very long lead times, the demand rate for the other company should be its demand if it were the monopoly. This model can still represent customer loyalty to a company in the sense that if the two offers are not radically different, some customers can still opt for the slightly higher price and/or the longer lead-time. Our models are developed free of a specific choice of demand function and other forms can be also tested.

According to our numerical results, as the price or delay sensitivity of customers increases, the optimal profit in both the monopoly and duopoly settings decreases. On the other hand, a larger potential market size increases the optimal profit in both settings. One important observation of our experiments is that the most profitable strategy for the less restricted company is to use the same fair dynamic pricing strategy followed by the other company. If these companies enter into an optimization race against each other, their optimal profits are never more than the optimal profit of the same fair policy that they can apply. Thus, if a company applies a fair pricing policy (RP or SP) the other company will have to apply the same policy in order to maximize its profit. As a result, the company whose survivability in a competition we initially question turns out to be actually the rule maker in the market.

Another important result is that the RP provides a slightly higher profit than the SP. The SP policy may not offer an improvement in computational terms (because it computes the common optimal lead-time for all backlogged customers in each iteration of an exhaustive search for finding the optimal prices), yet, it is a policy that is clearer/more understandable to the customers in practice. Therefore, it is possible to use the SP rather than the RP policy in some practical cases.

Finally, we allow both firms to implement pricing strategies without being concerned for fairness. The aim of this analysis is to see if being fair would cause the companies to lose profit. We see that being fair in dynamic pricing is not significantly costlier in a duopoly and the profit lost due to being fair in dynamic pricing is almost negligible. This result indicates that a company does not need to worry too much about adopting fairness principles for pricing even when in competition. This result broadens similar observations made by Balcıoğlu & Varol (2022) and Dede & Balcıoğlu (2023) for a monopoly.

The rest of the thesis is organized as follows. In Chapter 2, we make a brief summary of papers related to customers sensitive to price and delay. In Chapter 3, we present the problem analyzed. We discuss the proposed policies in Section 3.1 and present our numerical examples in Chapter 4. Chapter 5 is for the concluding remarks and possible future research questions.

2. LITERATURE REVIEW

Our study is related to the literature that focuses on dynamic pricing and lead-time quotation. These are established tools to regulate the stochastic demand so that higher profit and/or high levels of service can be attained with limited capacity. While early studies such as Naor (1969) and Mendelson (1985) focus on optimizing a static price for demand control, more recent studies, such as Kim & Randhawa (2018) or Balcıoğlu & Varol (2022), show that dynamic pricing is significantly better than static pricing at reducing the effect of uncertainty.

There are various studies that consider customers sensitive to price and delay. Some assume a single firm/ a monopoly, some consider competing companies. Some studies consider optimizing static prices for price and delay sensitive customers. We start with papers focusing on single companies optimizing a static price. Qian (2014) assumes a linear demand model that depends on other product related features in addition to the static price and lead-time. The model is further used to choose the supplier to match the market characteristics. The study concludes that the less costly supplier must be be preferred if the cost difference exceeds a threshold, while the better performing supplier is the right decision if the performance difference is above another value. Those values are determined by current performance characteristics and market characteristics. Kim, Kim & Lee (2023) model a single firm as a make-to-stock queue for a single product aimed at a single class of customers. The company, as in our study, uses the base-stock policy to control production. Customers purchase the product if there is inventory. They can also see the number of backlogged customers when there is no stock. The customer compares the value it gives to the product with the waiting cost and the price, and decides whether to join or balk. The company optimizes a single static price and the base-stock level. The customer finds her/his equilibrium strategy for joining the backlog given price and the base-stock level and the number of backlogged customers waiting in front of her/him. Cai & Li (2023) assume that the production cost is a function of the lead-time and the monopoly optimizes its static price accordingly. They find that if service level is above a threshold, the company aims at serving all customers. This would increase the lead-time and, in return, reduce the production cost. If the service rate is not high, some customers for whom the utility is less than the sum of the price and the waiting cost balk. While the majority of studies we summarize assume continuous but random flow of customers over time like us, which make queueing models the appropriate tool, we refer the reader to Wu, Kazaz, Webster & Yang (2012) for a newsvendor problem where a company has to consider determining the optimal price and lead-time that would affect the uncertain demand over a finite time horizon. Kuang & Ng (2018) study what a company can consider for pricing a product sold over two periods. It can either announce all its prices at the beginning of the first period or it can declare the price for a period only at the start of that period. Their study shows that announcing all prices at the beginning dominates dynamic pricing and the value of the former policy depends on the uncertainty and anticipated regret of the customers.

Among studies modeling competing companies that optimize static prices and lead times, we review the following studies. Armony & Haviv (2003) study two identical companies that post static prices for two types of customers. The customers differ in their assessment of the waiting cost and use the mean system time for cost computations. Thus, customers either choose one of the companies, modeled as an $M/M/1$ make-to-order queue or balk. In this setting, they show how a customer makes her/his decision. They then study how the companies can set their prices to maximize their profits. Shen & Zhang (2009) build a joint pricing and delivery lead-time decision model in a two-echelon system comprised of a manufacturer and two competing retailers. They study both the centralized and decentralized settings: In a decentralized supply chain, the retailers and the manufacturer decide independently but in the centralized supply chain all the decisions are made by the manufacturer. They show that the optimal lead-time in the centralized supply chain is larger than that in the decentralized one and the retail price in the centralized supply chain is lower than that in the decentralized supply chain. Additionally, each retailer's profit decreases with an increase in its price and the planned lead-time, but it increases with its rival's price. Huang, Chen & Ho (2013) consider two suppliers producing substitutable commodities, who have to make their pricing and lead-time decisions considering their capacity and cost concerns and the offer made by the competitor. One of the companies offers a lower price and a longer lead-time, while the other one offers a higher price and a shorter lead-time. There is also an e-retailer selling the commodity of these two suppliers. In a game theoretic setting, the suppliers are the leaders and the e-retailer is the follower, who can determine the retail prices for customer classes with different sensitivities based on the suppliers' prices. The players of the game do not consider dynamic pricing or lead-time quotation, but optimize static prices and lead times and the analysis is built on the assumption of linear-demand functions. Jayaswal & Jewkes (2016) consider two different classes of customers served: One class is price, and the other class is delay sensitive. Each company offers one price and lead-time for each class. The study assumes linear demand models that would increase if the price and the lead-time offered for that class by the competitor increase. The demand for a company from one class also increases if the price and the lead-time offered to the other class by the same company increase. Assuming that the companies operate in the FIS, a company optimizes its policy in response to the other company's policy. This is iteratively done (each round for one of the companies) until the equilibrium for both companies are obtained. The authors show that if companies fix their lead times and only compete with the prices they charge, this would force the companies to lower their prices. The companies cannot widen the gap between the prices they charge to different classes either. Pekgün, Griffin & Keskinocak (2017) explore whether the marketing department for the static price and the production department for the single lead-time should make their decisions jointly or independently to see which one gives better results in competition for a single class of customers. They also use a linear demand model that tries to capture the impact of what the competitor offers. They show that when price competition is intense, these departments should work in a centralized way. When lead-time competition is intense, a decentralized strategy giving the marketing department the leadership can increase profits more. Cai & Li (2023) also analyze the duopoly version of the monopoly problem we have summarized earlier. Knowing how the customers would behave and anticipating the competitor's policy, the company optimizes its price. In the case of homogeneous companies, we see that they charge the same prices. If they are heterogeneous, even when one of them captures the entire demand, its optimal price depends on the unit production cost of its competitor. Sun, Wu & Zhu (2022) allow more than two competitors and assume a linear demand function with a lead-time model that approximates the sojourn time in an $M/GI/1$ queue. They show that if price competition is intense, i.e., customers shift from one company to another more easily for the same price difference, the companies have to decrease their prices. If one observes that lead-time competition is intense, companies can increase the price but they have to shorten their lead times. The authors state that the lead-time competition has been traditionally weaker than the price competition. However, if companies compete more in that dimension, all companies will benefit from that.

Next, we summarize studies allowing dynamic pricing in a monopoly setting. Dong, Kouvelis & Tian (2009) can be given as a more traditional example considering dynamic pricing over a finite selling season without lead-time quotation as part of the problem. They show the benefits of dynamic pricing when inventory gets scarce. Suh & Aydin (2011) show that for a company that offers substitutable products, the optimal price may increase while approaching the end of the finite selling horizon or if the total inventory grows. Chen & Chen (2018) also analyze dynamic pricing for substitutable products over a finite period. At the beginning of each subperiod, the company can choose from one of the available finitely many prices not exceeding the last posted price. They show that frequent price changes is not necessary. Ceryan, Sahin & Duenyas (2013) allow inventory replenishment at the start of finitely many time periods along with pricing decisions for substitutable products. They assume finite capacity for production, some reserved for a product and some that can be used flexibly. They show that base-stock policies are optimal. If the production capacity is not sufficient to raise the inventory to this level, it is optimal to increase the price. Studies allowing replenishment over an infinite horizon are more related to our study: Kim & Randhawa (2018) consider price and delay sensitive customers served by a monopoly modeled as a single-server queue with exponential service times. Here, customers compare their value for service generated from a distribution with the sum of the price and waiting costs. Since customers can observe the queue length, they use the mean waiting time for their delay cost computation. If her/his service valuation is higher, the customer joins the queue, otherwise, she/he is simply lost. Thus, the company does not quote lead-time as in our study but focuses on an optimal dynamic pricing strategy employing a Markov decision process (MDP) approach. We, on the other hand, assume that the demand rate is a function of prices and the lead times offered by both companies at the times of customer arrivals. They propose a simple policy which suggests using only two prices: a low price when the congestion of the system is low and a high price when the congestion is high. They show that this policy garners most of the benefits of dynamic pricing. We show that the same holds in a duopoly. Çelik & Maglaras (2008) study a monopoly modeled as a single-server make-to-order queue offering a variety of products. When a customer arrives, she/he sees the price and lead-time pair for each product and can choose the product she/he wants. Dynamic pricing helps the company to divert demand from one product to another and this enables the company to meet the lead times quoted. Their study also attests to profit increase when dynamic pricing is used instead of static pricing.

According to our categorization in this chapter, our work is about a duopoly allowing dynamic pricing. However, we could not spot research in this domain. Without lead-time quotation, Sato (2019) can be given as an example studying competition over a finite selling horizon. Ranking as superior and inferior in the quality of the substitutable products they offer the market, two companies compete over a finite selling horizon and the study has a focus on the company selling the lower quality product. This company has a wider possibility for choosing its prices to survive in the competition. Yet, the results indicate that the competitor's prices need be taken into account for better pricing decisions. Given the scarcity of work studying dynamic pricing for competitors producing substitutable products over an infinite horizon, our study seems to be making an early contribution.

The most relevant studies to our problem consider monopolies that can dynamically set its prices and quote lead times for a single or two classes of customers. Balcıoğlu & Varol (2022) propose four different policies where they analyze fair dynamic pricing policies and compare them with fair static pricing policies. We consider one of these four policies, called the simple dynamic pricing policy. In this policy, only two prices are offered for the customers. One for the customers who arrive when there is stock and another price with a single lead-time for all backlogged customers. Since the firm adopts fair pricing principles, the price offered to a customer who arrives when there is stock should be higher than the price offered to a backlogged customer. We redesign this policy by increasing the number of firms in the system and limiting the number of backlogged customers. Our policy, namely the SP, optimizes this common lead-time using different mathematical derivations. This increases the computational time of the SP but quoting the same lead-time may be more understandable and practical for customers. Dede & Balcıoğlu (2023) study a monopoly serving two different classes of customers. One class of customers is more sensitive to delays and does not mind paying higher prices whereas the other class of customers can tolerate longer lead times but want to pay lower prices. We adapt one of their FCFS policies by adding a competitor. What we call as the RP quotes different lead times for customers seeing different number of customers in front of them when they join the backlog queue. Unlike the SP, determining these lead times is not computational, however, when there are more prices to optimize for each state of the production line, the computation can be prohibitive.

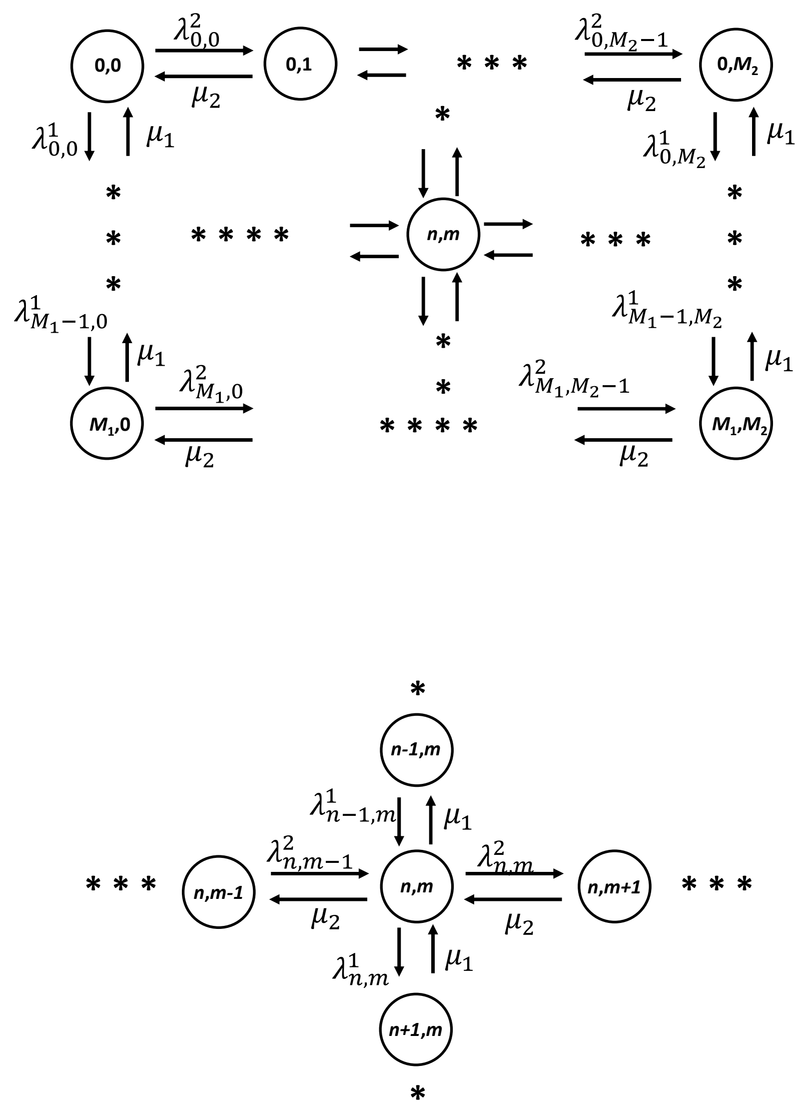
3. TWO $M_n/M/1$ QUEUES WITH PRICE AND LEAD-TIME QUOTATIONS

In this chapter, we consider two manufacturers, Companies 1 and 2, producing Products 1 and 2, respectively, which are substitutable products and sold in the same market. Customers appear one at a time according to an independent Poisson process for one unit of item. The demand rate for Product i, $i = 1,2$ depends on its price and, in case of shortage, the lead-time quoted for it together with the price of and, if necessary, the lead-time quoted for Product j, $j = 1,2$, $j \neq i$.

For both companies, the underlying production system is modeled as a make-to-stock queue. Production for Company i is controlled by a base-stock policy: it stops when the continuously reviewed inventory level reaches the base-stock level S_i and starts as soon as the inventory level decreases to $S_i - 1$. When stock is depleted, customers are backlogged but their number cannot exceed a maximum of N_i. Therefore, when Company i has N_i backlogged customers, it does not make an offer to a new customer, which becomes a lost customer for that company at no additional cost. The production/service times are assumed to be independent and identically distributed (i.i.d) exponential random variables (r.v.s) with a mean of $1/\mu_i$. Thus, for each item sold by or ordered to Company i, a production order is created, making the arrival rate of production orders at the corresponding make-to-stock queue the same as the state-dependent demand rate for Product i. If n/m shows the number of production orders for Company 1/2 with $n = 0,\ldots,M_1$, $m = 0,\ldots,M_2$, $M_i = S_i + N_i$, $i = 1,2$, it gives the shortfall from S_1/S_2. In other words, when $n \leq S_1$, Company 1 carries $S_1 - n$ units in stock and when $n > S_1$, it has $n - S_1$ backlogged customers (and similar relation holds for Company 2 with m and S_2). Therefore, a two-dimensional continuous-time Markov chain with states (n,m), as given in Figure 3.1, gives a full description of the underlying model of the problem. Using standard techniques, we can compute $\pi_{n,m}$, the steady-state probability of being in state (n,m).

Due to this one-to-one correspondence between the rates of production orders and customer demand, we can assume that the prices charged and lead times quoted

Figure 3.1 The transition rate diagram of the underlying two $M_n/M/1$ queues



in state (n,m) yield the state-dependent rate of demand/production order, $\lambda^i_{n,m}$ for Company i. If the customer places an order to Company i but the produced item cannot be delivered during the quoted lead-time, $d^i_{n,m}$, a tardiness cost, l_i, is incurred/paid to the customer per unit time for her/his waiting time in excess of the quoted lead-time. Additionally, a holding cost of h_i is incurred by Company i per unit inventory per unit time.

In Section 3.1, where we present the policies that these companies can implement, we also provide the algorithms with which a company can determine the optimal parameters of the policy it employs if the policy of the other company is known. In

Chapter 4, we assume that each company, by employing these algorithms recursively in an iterative role-switching game, determines not only its but the competitors optimal policy, namely, S_i, N_i, and the prices and lead times to quote. A fundamental question that can arise is the following: Are the companies able to monitor the competitor's number of production orders as well? If this is possible, for example, a company holding two units of stock can post different prices when its competitor has stock or is out of stock. We refer to this situation as the "full information setting" (FIS) and stick to this setting as much as possible while developing the formulation.

Let $E[RV^i]$ denote the expected revenue per unit time; $E[C_H^i]$ and $E[C_D^i]$ the expected inventory holding and delay penalty cost rates, respectively, for Company i. With price and lead-time vectors $\mathbf{R}_i = [R^i_{0,0}, \ldots, R^i_{M_1,M_2}]$,$\mathbf{d}_i = [d^i_{0,0}, \ldots, d^i_{M_1,M_2}]$, respectively, with $d^i_{0,n_j} = \ldots = d^i_{S_i-1,n_j} = 0$ for $n_j = 0, \ldots, M_j$, corresponding to states in which Company i carries stock, the expected profit per unit time in the FIS for Company i is

$$\begin{aligned} Profit(S_i, N_i, \mathbf{R}_i, \mathbf{d}_i) &= E[RV^i] - E[C_H^i] - E[C_D^i], \\ &= \sum_{n_i=0}^{M_i-1} \sum_{n_j=0}^{M_j} \lambda^i_{n_i,n_j} \pi_{n_i,n_j} R^i_{n_i,n_j} \\ &\quad -h_i \sum_{n_i=0}^{S_i-1} (S_i - n_i) p_i(n_i) - l_i \sum_{n_i=S_i}^{M_i-1} \sum_{n_j=0}^{M_j} \lambda^i_{n_i,n_j} \pi_{n_i,n_j} L^i_{n_i,n_j}(d^i_{n_i,n_j}), \end{aligned} \tag{3.1}$$

and is subject to

$$P(T^i_{n,m} \le d^i_{n,m}) \approx \alpha_i, \;\; i = 1,2, \;\; n = 0, \ldots, M_1, \;\; m = 0, \ldots, M_2. \tag{3.2}$$

In Eq. (3.1), the steady-state probability of having a given number of orders (hereafter, we refer to production orders simply as orders) for each company is

$$p_1(n) = \sum_{m=0}^{M_2} \pi_{n,m}, \;\; n = 0, \ldots, M_1, \;\; p_2(m) = \sum_{n=0}^{M_1} \pi_{n,m}, \;\; m = 0, \ldots, M_2, \tag{3.3}$$

and $L^i_{n,m}(d^i_{n,m})$ is the expected waiting time in excess of $d^i_{n,m}$ of a customer that accepts the quoted lead-time $d^i_{n,m}$. In Eq. (3.2), $T^i_{n,m}$ is the r.v. showing the elapsed time from the moment a customer places this order when arriving in state (n,m) until she/he receives the finished item, and α_i is the proportion of deliveries that should be done within the quoted lead times. Then, with $g^i_{n,m}(\cdot)$, denoting the probability density function of $T^i_{n,m}$, we have

$$L^i_{n,m}(d^i_{n,m}) = \int_{d^i_{n,m}}^{\infty} (x - d^i_{n,m}) g^i_{n,m}(x) dx. \tag{3.4}$$

In practice, companies may not be able to see how many orders there are in the production line of their competitor and have to take only their own order queue length into account to determine the price and lead-time when a new customer appears. In other words, $R^i_{n_i,n_j} = R^i_{n_i}$ and $d^i_{n_i,n_j} = d^i_{n_i}$ for $n_j = 0, \ldots, M_j$, $j \neq i = 1, 2$. This is what we refer to as the "limited information setting" (LIS) and which is the main focus of the thesis. The customer, on the other hand, would have two offers from competitors, based on which she/he can choose to buy the product from one of them or is simply lost without causing any cost to any of the companies.

In the next section, we discuss the two fair policies that such a company can consider for profit maximization.

3.1 Alternative Policies

In this section, we consider two polices that differ from each other when there is no stock. While one policy considers in which order a backlogged customer joins the backlog queue, the other one quotes the same lead-time to all customers. Since the profit rate function in Eq. (3.1) is not in closed-form, and nor differentiable, we can find the optimal parameters "approximately": we compute profit rates by setting some discrete values to the prices the two companies can charge for each state and identify the set of the prices for all states, namely a policy, that gives the highest profit rate. Thus, after each policy is presented, we provide an algorithm that does an exhaustive search to optimize the policy parameters.

3.1.1 The R Policy: The Refined Policy

This policy is an adaptation of the refined FCFS (RF) policy designed by Dede & Balcıoğlu (2023) for a monopoly serving two different classes of customers. In our adapted version, namely, the refined (R) policy, Company i uses the total number of backlogs (k) for Product i when a new customer appears, and quotes $d^i_{n_i}$ as the lead-time where $n_i = k + S_i$. Hence, $d^i_{n_i} = 0$ for $n_i = 0, \ldots, S_i - 1$ and $k = 0$ implies that $n_i = S_i$ with neither stock nor backlogs. In other words, the lead-time quoted is independent of the number of orders that the competitor has at that instant even in the FIS. This is because a customer joining the backlog queue for Product i with k backlogs in front waits for a random time T^i_{k+1} (as the $k+1$st backlogged customer) with $k+1$-stage Erlang distribution with each exponential stage having a rate of μ_i, i.e., Erlang($\mu_i, k+1$). For such an Erlang r.v., T^i_{k+1} (e.g., Gross & Harris (1998)), Eq. (3.2) becomes

$$(3.5) \qquad P(T^i_{k+1} \le d^i_{n_i}) \quad = \quad G_{RP}(d^i_{n_i}, \mu_i, k) = 1 - \sum_{m=0}^{k} \frac{(\mu_i d^i_{n_i})^m e^{-\mu_i d^i_{n_i}}}{m!} = \alpha_i.$$

Since $L^i_{n_i,n_j}(d^i_{n_i,n_j}) = L^i_{n_i}(d^i_{n_i})$, from Kahvecioğlu & Balcıoğlu (2016), Eq. (3.4) can be rewritten as

$$(3.6) \qquad L^i_{n_i}(d^i_{n_i}) \quad = \quad e^{-\mu_i d^i_{n_i}} \left(\frac{k+1}{\mu} \sum_{m=0}^{k+1} \frac{(\mu_i d^i_{n_i})^m}{m!} - d^i_{n_i} \sum_{m=0}^{k} \frac{(\mu_i d^i_{n_i})^m}{m!} \right),$$

where $k = n_i - S_i$.

In terms of implementing the R policy, the only difference between the two companies is that for all m, $R^1_{0,m} = \ldots = R^1_{S_1-1,m} > R^1_{S_1,m} > \ldots > R^1_{M_1-1,m}$ should hold for Company 1. This is what we expect from a fair dynamic pricing scheme. Such a restriction is not imposed on Company 2 for $R^2_{n,0}, \ldots, R^2_{n,S_2}, \ldots, R^2_{n,M_2-1}$ for any n.

We employ the R (**R**efined: the capital letter in bold yields the acronym) Algorithm in Chapter 4 to optimize the base-stock level, the maximum number to backlog, and the prices to charge together with the lead times to quote. In this algorithm and the one to be presented in Section 3.1.2, the parameter values to consider are varied over appropriately chosen ranges in loops contained in other loops. For instance, assuming that the policy of Company j is known (or to be forced to be identical to that of Company i) the R Algorithm in the outermost loop starts from 0 and increments S_i by 1 until an $S_{i,\max}$ is reached. Within this loop, it also varies N_i in increments of 1 starting from 0 until an $N_{i,\max}$ is hit. The inner loops aim to capture every n, m so that the $R^i_{n,m}$ is varied from a minimum to a maximum value. Obviously, this type of search can become computationally expensive quite easily if the ranges of the parameters get wider. To shorten the description of the algorithms (the R Algorithm and the other one to be discussed), we present what they do for a given policy (a set of S_i, N_i, and $\mathbf{R}_i$) for Company i. At the end, out of all the policies generated, each algorithm identifies the policy that maximizes the profit.

The R Algorithm: This algorithm explains how the optimal R policy parameters, S^*_i, N^*_i, $\mathbf{d}^*_i$, $\mathbf{R}^*_i$, and the corresponding vector $\boldsymbol{\lambda}^*_i$ containing $\lambda^i_{n,m}$ for $n = 0, 1, \ldots, M_1$ and $m = 0, 1, \ldots, M_2$ are found.

Main Step For the $(S_i, N_i, \mathbf{R}_i)$ values considered, and the lead times determined beforehand (see Remark 1): If Company j is expected to follow the same policy, set $S_j = S_i, N_j = N_i, \mathbf{R}_j = \mathbf{R}_i$. Otherwise, Company j has its own policy $(S_j, N_j, \mathbf{R}_j)$. Using policies of both companies determine all $\lambda^i_{n,m}$ and $\lambda^j_{n,m}$ for all $n = 0, \ldots, M_1$ and $m = 0, \ldots, M_2$ according to a given function. Obtain the related steady-state probabilities as discussed in Chapter 3. Then, compute and store $Profit(S_i, N_i, \mathbf{R}_i, \mathbf{d}_i)$ using Eq. (3.1) to be compared in the Final Step.

Final Step Among all the instances with positive $Profit(S_i, N_i, \mathbf{R}_i, \mathbf{d}_i)$ values coming from the Main Step, the highest one gives the optimal instance and its parameters are the optimal S_i^*, N_i^*, $\mathbf{R}_i^*$, and $\mathbf{d}_i^*$.

Remark 1 *As many lead times as needed can be obtained beforehand using the following algorithm extracted from the RF Algorithm in Dede & Balcıoğlu (2023). From Eq.* (3.5)*, for $k = 0$, we have $d^i_{S_i} = -ln(1-\alpha_i)/\mu_i$. The rest can be recursively found. For $k > 0$, set LB=$d^i_{S_i+k-1}$ and UB=$d_{\max}$, respectively, as the lower and upper limits for the interval over which the following binary search is conducted to determine the $d^i_{S_i+k}$ value:*

Step a *Set $d^i_{S_i+k} = (LB + UB)/2$, and go to Step b.*

Step b *If $|G_{RP}(d^i_{S_i+k}, \mu_i, k) - \alpha_i| \leq \epsilon_\alpha$ for some tolerance ϵ_α, then $d^i_{S_i+k}$ is the lead-time to announce. Otherwise, go to Step c.*

Step c *If $G_{RP}(d^i_{S_i+k}, \mu_i, k) < \alpha_i$ (implying that a longer lead-time is needed), then set LB=$d^i_{S_i+k}$ and go to Step a. If $G_{RP}(d^i_{S_i+k}, \mu_i, k) > \alpha_i$ (implying that a shorter lead-time is needed), set UB=$d^i_{S_i+k}$ and go to Step a.*

3.1.2 The S Policy: The Simple Policy

This policy is inspired from the SDP policy by Balcıoğlu & Varol (2022) originally designed for a monopoly that follows the fair pricing principles. We first explain the policy for Company 1 and then we note how Company 2 can also use it in a less restrictive way. Under this policy, Company 1 charges a high price R_H when it has stock and a low price R_L when there is no stock. The same R_L necessitates Company 1 to quote the same lead-time $d^1_{\alpha_1}$ to all backlogged customers. However, differently from the SDP policy, we impose a limit on the maximum number of backlogs, which complicates determining this lead-time. Observe that for the k-th backlog, the probability of receiving its order within the lead-time is $G_{RP}(d^1_{\alpha_1}, \mu_1, k)$ as given in Eq. (3.5). Recalling that for $n \geq S_1$ we have $k = n - S_1$ and letting T^1 denote the random time for a backlogged customer to receive a Product 1, Eq. (3.2) becomes

$$
\begin{aligned}
P(T^1 \leq d^1_{\alpha_1}) &= G_{SP}(d^1_{\alpha_1}, \mu_1, S_1, M_1) \\
&= \frac{\sum_{n=S_1}^{M_1-1} \sum_{m=0}^{M_2} \lambda^1_{n,m} \pi_{n,m} G_{RP}(d^1_{\alpha_1}, \mu_1, n - S_1)}{\lambda^1_B} = \alpha_1, \qquad (3.7) \\
\lambda^1_B &= \sum_{n=S_1}^{M_1-1} \sum_{m=0}^{M_2} \lambda^1_{n,m} \pi_{n,m},
\end{aligned}
$$

where λ^1_B is the mean backlog rate for Product 1. One can still compute $L^1_n(1_{\alpha_1})$ from Eq. (3.6).

We think that Company 2 would prefer using the R policy which gives it a chance to post more than two prices. Even when fair pricing principles are not imposed, it is not straightforward to state that the optimal S policy cannot yield a higher profit than the optimal R policy. Although the latter can find the same two prices as optimal that the optimal S policy can find, the lead times quoted will not be the same (if the maximum number to backlog is more than 1) and this would change the arrival rates and hence everything. Company 2 can still implement the S policy without forcing $R_H > R_L$.

We employ the S (**S**imple: the capital letter in bold yields the acronym) Algorithm in Chapter 4 to optimize the base-stock level, the maximum number to backlog, and the prices to charge together with the lead times to quote.

The S Algorithm: This algorithm explains how the optimal S policy parameters, S^*_i, N^*_i, $\mathbf{d}^*_i$, R^*_H, R^*_L and the corresponding vector $\boldsymbol{\lambda}^*_i$ containing $\lambda^i_{n,m}$ for $n = 0, 1, \ldots, M_1$ and $m = 0, 1, \ldots, M_2$ are found.

Main Step For the $(S_i, N_i, \mathbf{R}_i = [R_H, R_L])$ values considered, if Company j is expected to follow the same policy, set $S_j = S_i, N_j = N_i, \mathbf{R}_j = \mathbf{R}_i$. Otherwise, Company j has its own policy $(S_j, N_j, \mathbf{R}_j)$. Set LB=0 and UB=$d_{\max}$, respectively, as the lower and upper limits for the interval over which the following binary search is conducted to determine the $d^i_{\alpha_i}$ value:

Step a Set $d^i_{\alpha_i} = (LB + UB)/2$. Now Company i has also a policy. Using policies of both companies determine all $\lambda^i_{n,m}$ and $\lambda^j_{n,m}$ for all $n = 0, \ldots, M_1$ and $m = 0, \ldots, M_2$ according to a given function. Obtain the related steady-state probabilities as discussed in Chapter 3. Go to Step b.

Step b If $|G_{SP}(d^i_{\alpha_i}, \mu_i, S_i, M_i) - \alpha_i| \leq \epsilon_\alpha$ for some tolerance ϵ_α, then $d^i_{S_i+k}$ is the lead-time to announce. Go to Step d. Otherwise, go to Step c.

Step c If $G_{SP}(d^i_{\alpha_i}, \mu_i, S_i, M_i < \alpha_i$ (implying that a longer lead-time is needed), then set LB=$d^i_{S_i+k}$ and go to Step a. If $G_{SP}(d^i_{\alpha_i}, \mu_i, S_i, M_i > \alpha_i$ (implying that a shorter lead-time is needed), set UB=$d^i_{\alpha_i}$ and go to Step a.

Step d Compute and store $Profit(S_i, N_i, \mathbf{R}_i, \mathbf{d}_i)$ using Eq. (3.1) to be compared in the Final Step.

Final Step Among all the instances with positive $Profit(S_i, N_i, \mathbf{R}_i, \mathbf{d}_i)$ values coming from the Main Step, the highest one gives the optimal instance and its parameters are the optimal S^*_i, N^*_i, R^*_H, R^*_L, and $d^{i,*}_{\alpha_i}$.

4. NUMERICAL EXPERIMENT

In this chapter, we primarily investigate the situation where the two companies are comparable. They have the same production capacity. In other words, neither company can produce faster than the other one. Similarly, the inventory holding and tardiness costs are assumed to be the same for both. Additionally, we consider customers that do not (at least significantly) differentiate between Products 1 and 2. This leaves out examples from our study where a higher quality product sold at a higher price can still attract significant customer attention. Since Products 1 and 2 are easily substitutable for a customer, the fundamental tool with which a company can differentiate itself from the other company can be its pricing policy. With our numerical study we explore: *i*) if a company sticking to fair dynamic pricing policy can survive while competing with another company that does not restrict itself with those principles, *ii*) what happens when both companies refrain from these fair pricing principles, and *iii*) if a simple policy offering only two prices is robust in this competitive environment.

To make the production capacities the same, we assume that $\mu_1 = \mu_2 = 1$. We use the numerical example setting designed by Balcıoğlu & Varol (2022). We consider the linear demand model

$$\lambda(R,d) = \lambda_{\max} - aR - bd, \tag{4.1}$$

where $\lambda_{\max} > 0$ shows the potential market size and coefficients $a > 0$ and $b > 0$ capture the customer demand sensitivity to price and delay in delivery. Here a higher $\lambda_{\max}$ implies a bigger market and higher a/b indicates that customers are more sensitive to increase in price/delay in delivery.

Since we mainly focus on the LIS, we assume that the companies post price and lead-time offer based only on the number of orders they have. To reflect the impact of the presence of a competitor, we obtain the arrival rates as follows. Suppose that there are n/m orders for Company 1/2 and they offer $(R^1_n, d^1_n)/(R^2_m/d^2_m)$ to a newly arriving customer. Then, $\lambda(R^1_n, d^1_n) = X$ and $\lambda(R^2_m, d^2_m) = Y$ would give us the demand rate for Company 1 and 2, respectively, if the other company did not exist. We assume that these demand rates for a monopoly would indicate the desirability of the offer for Product i for a customer. To reflect the impact of the other company's presence onto the actual demand rate, we assume a linear model. This way, for a customer, even when X and Y are different, purchasing the product with smaller X or Y is possible.

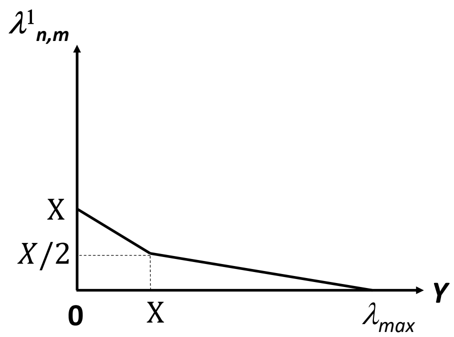
a. If $X \geq Y \geq 0$, then $\lambda^1_{n,m} = X - \frac{Y}{2}$ and $\lambda^2_{n,m} = \frac{Y}{2}\left(1 - \frac{X-Y}{\lambda_{\max}-Y}\right)$.

b. If $Y \geq X \geq 0$, then $\lambda^1_{n,m} = \frac{X}{2}\left(1 - \frac{Y-X}{\lambda_{\max}-X}\right)$ and $\lambda^2_{n,m} = Y - \frac{X}{2}$.

For this demand model, we also present Figure 4.1. This helps us see that when the difference, that is the desirability gap between the two offers, increases, demand gets less for the product with a poor price and lead-time offer.

In all the examples, the proportion of backlogged customers receiving their orders within the quoted lead-time is $\alpha_1 = \alpha_2 = 0.9$. The holding cost and penalty cost rates are set as $h_1 = h_2 = 4$ and $l_1 = l_2 = 4$, respectively. We have run the algorithms presented in Section 3.1 to search for the optimal price for a given state (e.g., in determining R^1_0 when stock for Company 1 is full in the LIS) by decreasing the price for that state from a maximum to a minimum value by $\Delta = 1$ in each round. We summarize the results of our numerical study in Table 4.1. In Appendix A, the reader can find the optimal policy parameters along with the optimal revenues and costs for the cases summarized here. In the first four columns of Table 4.1, we list the eight different demand functions considered. When the R and S algorithms in Sections 3.1.1 and 3.1.2, respectively, are run for a single company (setting $S_2 = 0$ and $N_2 = 0$) we find the optimal profit Company 1 would make as a monopoly under the R and S policies, respectively. The corresponding results are presented in columns 5 and 6 under the Monopoly heading. The R policy yields slightly higher profit than the S policy. In these eight sets, the highest profit difference is 2.07% for set 6 from 50.27 under the S policy to 50.57 under the R policy when Company 1 is a monopoly. On average, the R policy yields 1.23% higher profit than the S policy over the eight sets considered.

Figure 4.1 The demand rates in state (n,m) under imperfect substitution



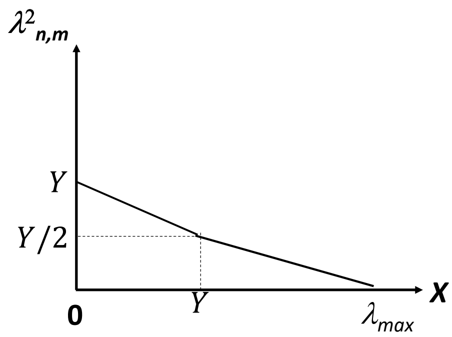


Table 4.1 The different demand functions considered and the summary of the maximized profit rates

				Monopoly		Duopoly RP			Duopoly SP		
						Same	Different		Same	Different	
Set	$\lambda_{\max}$	a	b	RP	SP	C_1/C_2	C_1	C_2	C_1/C_2	C_1	C_2
1	2	0.02	0.1	36.87	36.27	20.35	19.14	19.06	20.21	17.75	18.38
2		0.02	0.2	34.67	34.42	19.23	17.26	17.57	19.22	17.28	17.44
3		0.028	0.1	25.27	24.66	13.87	13.16	13.1	13.77	12.45	14.26
4		0.028	0.2	23.11	22.88	13.05	11.64	12.3	13.05	11.56	12.88
5	2.4	0.02	0.1	52.56	52.27	30.16	27.78	28.79	29.91	27.77	29.88
6		0.02	0.2	50.57	50.27	28.76	27.54	27.4	28.76	26.14	28.1
7		0.028	0.1	36.51	35.77	20.94	19.1	19.87	20.77	18.32	20.02
8		0.028	0.2	34.17	33.91	19.64	18.2	18.63	19.56	18.79	18.83

Column 7 under the heading Same gives the optimal profit for each company (C_i is short for Company i) if they were to use the same optimal policy. Thus, both companies employ fair dynamic pricing, keep the same base-stock level and the maximum number to backlog and the same prices for the same number of orders. Not surprisingly, not being a monopoly would decrease the profit sharply for Company 1. For instance, in Set 1, Company 1 would yield 20.35 in competition while 36.87 as a monopoly. However, in the duopoly setting the total production capacity is twice as much as that in the monopoly setting. This helps both companies to lower the customer loss and in return the total profit increases by 10% from 36.87 to $(2 \times 20.35 =)40.7$. The same policy followed (as given in Table A.1) would be $S = 1$, $N = 2$, and price vector $\mathbf{R} = [52, 51, 50]$. Let us refer to this policy as the policy at iteration 1 to help explain the results in columns 8 and 9.

Assuming that Company 1 would use the policy at iteration 1, Company 2 can re-use the R algorithm to see if it can improve its profit more, giving us what we refer to as the policy at iteration 2. With $S_2 = 2$, $N_2 = 3$, and $\mathbf{R}_2 = [40, 44, 42, 42, 46]$, Company 2 increases its profit to 21.97 while Company 1 has a lower profit of 16.59. Then, Company 1, has to act in return. Using the R policy, it would find a new optimal policy against the Company 2 policy found at iteration 2. This gives $S_1 = 1$, $N_1 = 3$, and $\mathbf{R}_2 = [40, 39, 38, 37]$ at iteration 3. Now Company 1 has a profit of 18.47 while Company 2 has a profit of 18.77. Although, we see slight increases in later iterations, this competition does not bring the companies to a significantly different place from 19.14 and 19.06 as profits for Companies 1 and 2, respectively as listed in columns 8 and 9, respectively. Both companies can do this recursive search and would know that if one side re-optimizes its policy, the other one would also do the same and they would not arrive at a better place than what they would have with the same policy obtained at iteration 1, results of which are listed in column 7.

The same exercise can be repeated with the S Algorithm if Company 1 chooses to use the S policy. If Company 2 chooses to follow the same policy as the former, each would generate the expected profit rates listed in column 10. For instance, in set 1, using the S policy would decrease the profit by 0.7% (to 20.21 from 20.35 of the R policy). The average decrease for all the eight sets would be 0.44%. If Company 2 decides to employ the R policy while Company 1 sticks with the S policy, and they still try to optimize their policies in a similar recursive game, we see that their profits would decline as listed in columns 11 and 12.

Recalling that, sets 1-4 (5-8) cover the smaller (larger) market examples and in each market segment, sets 1 and 5 (4 and 8) are where customers are the least (the most) sensitive to both price and delay, based on these results, we make the following observations:

- Higher price (a) or delay sensitivity (b) decreases the optimal profit in each setting (monopoly or duopoly). The larger market size ($\lambda_{\max}$), as expected, increases it.

- If Company 2 does not have anything that differs Product 2 from Product 1, which would create customer loyalty, it does not increase its profits solely by an unrestricted dynamic pricing policy. The best approach would be to use the same fair dynamic pricing policy as Company 1. This is an important result because even when we ignore the potential customer respect and loyalty for the fair Company 1, the market does not give more chance to its competitor to follow a less restrictive policy. While exploring our first question presented at the beginning of this chapter regarding whether a company using the fair dynamic pricing policy can survive in a competition, in fact, we see that it is the rule maker in the market.

- Determining optimal prices for each state could also be difficult in real life. It is worthwhile to see that a two-price policy, namely, the S policy can be followed at the expense of little profit loss. Such a choice by either of the company would make the other follow the suit as well. This answers the third question posed at the start.

- To find out the answer for the second question, we repeated the duopoly study, this time not restricting Company 1 either. In other words, both companies were free to choose their prices as they liked. We still observe similar results as presented in Table 4.2. Be it the R policy or the S policy, both companies maximize their profits only when they follow the same policies, the profits of which are listed in columns 5 and 8. The profits increase slightly, with an average of 0.33% and 0.22% for the R and S policies, respectively, when both companies use the same optimal policy with profits in columns 5 and 8 in Table 4.2 instead of those with the corresponding profits in columns 7 and 10 in Table 4.1. This results agrees with what Balcıoğlu & Varol (2022) and Dede & Balcıoğlu (2023) observe for the monopoly company. In other words, the profit loss due to being fair in dynamic pricing is almost negligible.

Table 4.2 The summary of the maximized profit rates when neither company follows fair pricing

				Duopoly RP			Duopoly SP		
				Same	Different		Same	Different	
Set	$\lambda_{\max}$	a	b	C_1/C_2	C_1	C_2	C_1/C_2	C_1	C_2
1	2	0.02	0.1	20.43	19.25	19.22	20.27	18.17	19.38
2		0.02	0.2	19.24	17.36	17.4	19.23	17.19	17.4
3		0.028	0.1	13.94	13.2	13.18	13.82	12.99	13.32
4		0.028	0.2	13.07	11.64	12.3	13.06	11.59	12.64
5	2.4	0.02	0.1	30.27	28.19	28.23	29.98	28.41	28.60
6		0.02	0.2	28.85	26.63	27.24	28.84	26.34	27.56
7		0.028	0.1	21.09	19.72	19	20.85	19.81	19.2
8		0.028	0.2	19.67	18.37	18.47	19.58	18.17	18.95

5. CONCLUSION AND FUTURE WORK

In this thesis, we propose two dynamic pricing and lead-time quotation policies for two companies competing in the same market. Each company produces a single type of product, which is a substitute of the one produced by its competitor. The production system of each company is modeled as an $M_n/M/1$ queue and customers are both price and lead-time sensitive. In both policies, prices and lead times are determined based on the number of production orders in both companies, which yield state-dependent demand rates. Our policies are eventually tested in the limited information setting (LIS) according to which the companies may not be able to see the number of orders in their competitor's production line and consider only their own order queue length to this end. We consider one company to be fair in pricing implying that it charges the same price to customers that are quoted the same lead-time and lower prices to those who are quoted longer lead times. Such a restriction does not apply to the other one. Yet, both companies are expected to be reliable in delivering the goods within the lead times they quote.

Among the two policies proposed, the refined policy (RP) quotes different lead times to customers that arrive when the backlog queue has differ numbers of customers waiting. Thus, a backlogged customer that is anticipated to wait longer gets a longer quote. The simple policy (SP), on the other hand, quotes a single lead-time to all backlogged customers. Via our numerical examples, the main question we explore is whether a company using the fair dynamic pricing policy can survive in a competition. We propose a new linear demand function according to which customers do not (at least significantly) differentiate between the alternative products. The algorithms are run iteratively so that both companies could use it to foresee what the other company would do in response to its policy. This way, the base-stock levels, the maximum number of backlogs to permit and the prices for each number of production orders are optimized. Since we are interested in if a company can differentiate itself only with its pricing strategy, we consider comparable companies in competition.

According to the results of our numerical experiments, the fair company not only survives but is actually the rule maker in the market. Whatever policy the fair company chooses and optimizes is the only viable policy for the competitor as well if it wants to maximize its profits. Even when we ignore the potential customer respect and loyalty for the fair company, the market does not give more chances to its competitor to follow a less restrictive policy. Although the SP is designed more for the fair company, which forces it to offer only two prices, a higher price when there is stock and a lower price when there is no stock, we see that its profit loss with respect to the RP is minuscule. Thus, the SP policy can be used for its ease for practical purposes. Additionally, we extend our experiments by not restricting the fair company to apply fair pricing principles as well. The results indicate that being fair in dynamic pricing is not causing significant profit loss.

In our numerical study, we assume that the companies have the same production capacity. This means that neither company can produce faster than the other one. However, this may not be possible in the real markets. Companies may have different service capacities, lead time achievement rates, or products with different qualities, etc. Therefore, our model can be used to explore similar questions in those settings as well. Especially, companies who serve their own markets, can test if penetrating into the market of another company is worthwhile and possible.

We conduct our experiments in the LIS. Our study can be extended to cover the full information setting where each company considers the production line of its competitor in addition to its own order queue length when determining the price and lead-time bundle.

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APPENDIX A TABLES

In Tables A.1-A.8, we list the optimal control parameters S_i, N_i, the prices $\mathbf{R}_i$, d^i_α for the SP, with the expected revenue $E[RV^i]$, and the expected inventory holding/tardiness $E[C^i_H]/E[C^i_D]$ costs, followed by the profit that these policies yield are provided. When a company employs the RP, its lead time vector has S_i 0's and then as many as needed from the following values in sequence for each backlog: [2.30, 3.89, 5.33, 6.68, 7.98, 9.27, 10.53, 11.79, 13.01, 14.21]. For instance in Table A.1, for the RP in the monopoly with 3 as the base-stock level and 5 as the maximum to backlog, $\mathbf{d}_1 = [0, 0, 0, 2.30, 3.89, 5.33, 6.68, 7.98]$. Therefore, we do not list these vectors for this policy. For the SP, we just list the same lead-time announced. Otherwise, it also uses S_i 0's and then the same lead-time for all backlogs. For example, when the companies use the same policy with 1 as the base-stock level and 2 as the maximum to backlog, $\mathbf{d}_1 = [0, 2.83, 2.83]$.

Table A.1 Results for Data Set 1 when $\lambda(R,d) = 2 - 0.02R - 0.1d$

	Company	S_i^*	N_i^*	Prices	Lead-Time	$E[RV^i]$	$E[C^i_H]$	$E[C^i_D]$	Profit
Monopoly									
RP	1	3	5	[55,55,55,54,53,52,51,49]		42.08	5.12	0.1	36.87
SP	1	3	3	[55,55,55,54,54,54]	3.63	41.71	5.35	0.1	36.27
Duopoly RP									
Same	1/2	1	2	[52, 51, 50]		22.65	2.24	0.06	20.35
Different	1	1	3	[40, 39, 38, 37]		21.1	1.86	0.1	19.14
	2	1	3	[40, 39, 41, 45]		21.04	1.89	0.09	19.06
Duopoly SP									
Same	1	1	2	[52,51,51]	2.83	22.53	2.26	0.06	20.21
Different	1	2	1	[40,40,39]	2.3	22.42	4.64	0.04	17.75
	2	2	3	[41,44,42,42,46]		23.15	4.73	0.04	18.38

Table A.2 Results for Data Set 2 when $\lambda(R,d) = 2 - 0.02R - 0.2d$

	Company	S_i^*	N_i^*	Prices	Lead-Time	$E[RV^i]$	$E[C^i_H]$	$E[C^i_D]$	Profit
Monopoly									
RP	1	3	3	[56,56,56,54,48,41]		40.65	5.92	0.05	34.67
SP	1	3	2	[56,56,56,52,52]	2.89	40.46	5.99	0.05	34.42
Duopoly RP									
Same	1/2	1	2	[53, 52, 51]		21.62	2.36	0.03	19.23
Different	1	2	3	[41, 41, 40, 38, 34]		22.11	4.82	0.03	17.26
	2	2	3	[39, 41, 42, 39, 35]		22.33	4.73	0.03	17.57
Duopoly SP									
Same	1	1	1	[53,52]	2.3	21.62	2.36	0.03	19.22
Different	1	2	1	[39,39,38]	2.3	21.96	4.66	0.03	17.28
	2	2	3	[39,42,44,40,34]		22.27	4.81	0.03	17.44

Table A.3 Results for Data Set 3 when $\lambda(R,d) = 2 - 0.028R - 0.1d$

	Company	S_i^*	N_i^*	Prices	Lead-Time	$E[RV^i]$	$E[C^i_H]$	$E[C^i_D]$	Profit
Monopoly									
RP	1	2	5	[39,39,38,37,36,35,34]		28.41	3	0.13	25.27
SP	1	2	3	[40,40,39,39,39]	3.6	28.14	3.35	0.12	24.66
Duopoly RP									
Same	1/2	1	2	[37,36,35]		16.16	2.23	0.06	13.87
Different	1	1	3	[29,28,27,26]		15.12	1.87	0.1	13.16
	2	1	3	[29,28,29,32]		15.09	1.9	0.09	13.1
Duopoly SP									
Same	1	1	2	[36,35,35]	2.84	16.04	2.2	0.07	13.77
Different	1	1	1	[30,29]	2.3	14.55	2.04	0.06	12.45
	2	1	3	[30,29,29,32]		16.18	1.82	0.1	14.26

Table A.4 Results for Data Set 4 when $\lambda(R,d) = 2 - 0.028R - 0.2d$

	Company	S_i^*	N_i^*	Prices	Lead-Time	$E[RV^i]$	$E[C_H^i]$	$E[C_D^i]$	Profit
Monopoly									
RP	1	3	3	[39,39,39,38,33,29]		28.86	5.7	0.05	23.11
SP	1	3	2	[39,39,39,36,36]	2.92	28.7	5.77	0.05	22.88
Duopoly RP									
Same	1/2	1	2	[37,36,35]		15.41	2.32	0.04	13.05
Different	1	1	3	[30,28,26,23]		13.81	2.12	0.05	11.64
	2	2	3	[30,33,30,28,24]		17.11	4.79	0.03	12.3
Duopoly SP									
Same	1	1	1	[37,36]	[0,2.30]	15.41	2.32	0.03	13.05
Different	1	1	1	[31,29]	2.3	13.8	2.19	0.04	11.56
	2	2	3	[31,33,31,31,31]		17.64	4.73	0.03	12.88

Table A.5 Results for Data Set 5 when $\lambda(R,d) = 2.4 - 0.02R - 0.1d$

	Company	S_i^*	N_i^*	Prices	Lead-Time	$E[RV^i]$	$E[C_H^i]$	$E[C_D^i]$	Profit
Monopoly									
RP	1	3	3	[71.,71, 71, 70, 69, 68]		57.15	4.47	0.12	52.56
SP	1	4	3	[70,70,70,70,69,69,69]	3.78	58.71	6.33	0.11	52.27
Duopoly RP									
Same	1/2	1	3	[63,62,61,60]		32.18	1.94	0.09	30.16
Different	1	2	2	[50, 50, 49, 48]		31.79	3.94	0.07	27.78
	2	2	3	[47, 50, 52, 58, 60]		32.71	3.85	0.07	28.79
Duopoly SP									
Same	1	1	2	[63,62,62]	2.93	31.96	1.96	0.09	29.91
Different	1	2	1	[51,51,50]	2.3	31.92	4.1	0.05	27.77
	2	2	3	[51,55,55,55,62]		33.97	4.02	0.07	29.88

Table A.6 Results for Data Set 6 when $\lambda(R,d) = 2.4 - 0.02R - 0.2d$

	Company	S_i^*	N_i^*	Prices	Lead-Time	$E[RV^i]$	$E[C_H^i]$	$E[C_D^i]$	Profit
Monopoly									
RP	1	4	3	[70,70, 70, 70, 69, 65,59]		57.56	6.93	0.06	50.57
SP	1	4	2	[70,70,70,70,69,69]	2.93	57.44	7.11	0.05	50.27
Duopoly RP									
Same	1/2	2	1	[62,62,61]		33.61	4.82	0.03	28.76
Different	1	2	3	[50,50,49,48,47,0]		31.64	4.05	0.05	27.54
	2	2	3	[49, 53, 55, 53, 50, 0]		31.7	4.27	0.04	27.4
Duopoly SP									
Same	1	2	1	[62,62,61]	2.3	33.61	4.82	0.03	28.76
Different	1	2	1	[51,51,50]	2.3	30.54	4.37	0.04	26.14
	2	3	3	[48,51,58,56,55,51]		34.97	6.84	0.03	28.1

Table A.7 Results for Data Set 7 when $\lambda(R,d) = 2.4 - 0.028R - 0.1d$

	Company	S_i^*	N_i^*	Prices	Lead-Time	$E[RV^i]$	$E[C_H^i]$	$E[C_D^i]$	Profit
Monopoly									
RP	1	3	6	[50,50,50,49,48,47,46,45,44]		40.78	4.13	0.15	36.51
SP	1	3	3	[50,50,50,49,49,49]	3.78	40.4	4.49	0.14	35.77
Duopoly RP									
Same	1/2	1	3	[45,44,43,42]		22.96	1.93	0.09	20.94
Different	1	1	3	?[36, 35, 34, 33]		20.86	1.63	0.13	19.1
	2	2	2	[33, 36, 36, 43]		23.68	3.74	0.07	19.87
Duopoly SP									
Same	1	1	2	[45,44,44]	2.93	22.82	1.95	0.09	20.77
Different	1	2	1	[35,35,34]	2.3	22.36	3.98	0.06	18.32
	2	2	3	[35,38,38,39,44]		24	3.91	0.08	20.02

Table A.8 Results for Data Set 8 when $\lambda(R,d) = 2.4 - 0.028R - 0.8d$

	Company	S_i^*	N_i^*	Prices	Lead-Time	$E[RV^i]$	$E[C_H^i]$	$E[C_D^i]$	Profit
Monopoly									
RP	1	4	3	[49,49,49,49,48,46,41]		40.85	6.62	0.06	34.17
SP	1	4	2	[49,49,49,49,48,48]	2.96	40.77	6.8	0.06	33.91
Duopoly RP									
Same	1/2	1	2	[45, 44, 43]		21.75	2.05	0.06	19.64
Different	1	2	3	[37, 37, 36, 35, 34]		22.49	4.24	0.05	18.2
	2	2	3	[34, 37, 38, 37, 35]		22.77	4.1	0.04	18.63
Duopoly SP									
Same	1	1	1	[46,45]	2.3	21.71	2.1	0.05	19.56
Different	1	2	1	[37,37,36]	2.3	23	4.17	0.04	18.79
	2	2	3	[38,42,40,40,36]		23.36	4.49	0.04	18.83