AN ESSAY IN MACROECONOMICS

by
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ABSTRACT

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Thesis Supervisor: Asst. Prof. Remzi Kaygusuz

Keywords: search model with multi-worker firm, unemployment insurance, propagation of unemployment insurance

This paper investigates the impact of unemployment insurance on the unemployment rate, wages, welfare, firms' growth, and profitability. To that end, I extend Acemoglu and Hawkins (2014)'s random search model by incorporating unemployment insurance. In order to ascertain the impact of UI, I conduct a quantitative analysis by calibrating my model to the U.S economy. The quantitative analysis states the following results: As the proportionate tax rate on earnings rises, the flow values of unemployment and employment rise, peak, then fall, respectively. This behavior is reflected in social welfare: it reaches the peak at a 1.7 percent tax rate and then declines. The higher tax rate leads to higher wages which in turn causes a decrease in the number of vacancies posted by firms. Accordingly, This increases the unemployment rate and decreases the labor market tightness. Further, the increase in wage levels results in declining in firms' growth rate and profit rate. Lastly, I also examine the impulse response of our key variables to a 1.7 percent proportional unemployment insurance tax. My analysis points out that the response of our key variables (wages, the flow value of unemployment and employment, labor market tightness, and welfare) shows their persistence.

ÖZET

MAKROEKONOMİ ÜZERINE BİR DENEME

MEHMET AKIN ŞİMŞEK

EKONOMİ YÜKSEK LİSANS TEZİ, TEMMUZ 2022

Tez Danışmanı: Dr. Öğr. Üyesi Remzi Kaygusuz

Anahtar Kelimeler: çok işçili firma ile iş arama modeli, işsizlik sigortası, işsizlik sigortasının yayılması

Bu makale, işsizlik sigortasının işsizlik oranı, ücretler, refah, firmaların büyümesi ve karlılık üzerindeki etkisini incelemektedir. Bu amaçla, Acemoglu and Hawkins (2014)'un rastgele arama modelinden yararlanıp, modele işşizlik sigortasını ekleyerek modeli geliştiriyorum. İşşizlik sigoratasının etkisini belirlemek için modelimi ABD ekonomisine göre kalibre ederek nicel bir analiz yapıyorum. Yaptığım nicel analiz ortaya koyduğu sonuçlar şu şekildedir: Kazançlar üzerindeki orantılı vergi oranı arttıkça, işsizlik ve istihdamın akış değerleri sırasıyla yükselir, zirveye ulaşır, ve düşer. Bu davranış sosyal refaha da yansır: sosyal refah yüzde 1,7 vergi oranıyla zirveye ulaşır ve sonra düşer. Ek olarak, vergi oranı artırmak daha yüksek ücretlere yol açar ve bu da firmalar tarafından ilan edilen açık pozisyonların sayısında bir azalmaya neden olur. Buna göre, bu durum işsizlik oranını artırmakta ve işgücü piyasasının sıkılığını azaltmaktadır. Ayrıca, ücret seviyelerindeki artış, firmaların büyüme oranlarında ve kâr oranlarında düşüşe neden olmaktadır. Son olarak, kilit değişkenlerimizin yüzde 1,7 oransal işsizlik sigortası vergisine tepkisini de inceliyorum. Analizim, temel değişkenlerimizin (ücretler, işsizlik ve istihdamın akış değeri, işgücü piyasasının sıkılığı ve refah) tepkisinin kalıcılığını gösterdiğine işaret ediyor.

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To my grandfather

TABLE OF CONTENTS

LIST OF TABLES

LIST OF FIGURES

1. INTRODUCTION

The unemployment insurance (UI) mechanism is one of the critical pillars of social insurance policies implemented in modern welfare states. UI mechanism allows workers to have higher utility and preserve their welfare above a certain level when they are unemployed. However, since it improves the value of unemployed workers, introducing UI may lead to an increase in wages, which is determined by bargaining between firms and workers. The rising wage levels may deter firms from the same number of job posting as before, which drives up the unemployment rate Amaral and Ice (2014). Since the number of job postings by firms declines, UI puts downward pressure on the labor market tightness, which defines the probability of job-finding rates for unemployed workers Landais, Michaillat, and Saez (2018). In other words, the introduction of UI may lower the job-finding probability for unemployed workers. This results in a decrease in the expected capital gains from the employment relationship. Accordingly, the flow value of unemployment exhibits a decline following the increase caused by the payment of unemployment insurance. Similarly, the flow value of employed workers exhibits an increase due to the rise in the wage level after the unemployment insurance implementation. However, the flow value of employment may get lower as the flow value of unemployment shrinks. After all, it is clear that the introduction of unemployment insurance influences the flow values of unemployment and employment through its effect on wages and the labor market tightness. Hence, the provision of UI impacts labor market outcomes and determines social welfare. Along with its labor market and welfare-related implications, UI provision might have firm-related consequences. Since UI implementation has an impact on wages and hiring costs through labor market tightness, its implementation may lead to a change in firms' profitability and growth levels.

In this paper, I investigate the impact of unemployment insurance on the unemployment rate, wages, welfare, firms' growth, and profitability. In addition, I assess the persistence of the labor market tightness and the unemployment rate responses to unemployment insurance implementation. To that end, I extend Acemoglu and

Hawkins (2014)'s random search model by incorporating unemployment insurance into the model. The model involves firms that aim to hire multiple workers simultaneously in a frictional labor market. It includes identical unemployed workers to be matched with firms with different productivity levels. The model involves several crucial components. The production function displays the property of a declining return to labor. Wages are determined through Nash bargaining between firms and workers Stole and Zwiebel (1996). Hiring an employee is a costly process, and the cost function of contacting workers exhibits convexity. The properties bring about some results related to firm size and wage dispersion. The costly hiring mechanism creates dispersion in firm size distribution because the new firms enter the economy and reach their optimal employment level in multiple steps of hiring, that is, firms are at different steps reaching their optimal employment level. Furthermore, the property of diminishing return to labor causes the marginal product of labor to differ by firms, since the economy involves firms with different employment levels at a point in time. Along with Nash's bargaining assumption, as firms have different marginal products of labor, wage dispersion emerges. In other words, even if all firms have a common target of employment, relatively new entering firms have higher marginal labor productivity than the older firms, so they hire more workers and grow faster. By Nash bargaining, they pay more because of the higher marginal productivity of labor that they have. This generates dispersion in wage levels.

The theoretical findings of the model with homogeneous firms are consistent with several empirical findings of the U.S labor market. The more profitable firms, the higher wages they pay (Reenen (1996)), and the faster firm growth, the higher wages paid to workers (Belzil (2000)). But, some implications of the model of homogeneous firms do not overlap with some features of the U.S economy. Empirical findings on the U.S economy imply the positive association between firm size and wages (Davis et al. (1991)), and firm size distribution complying with Zipf's law (Axtell (2001)) but the model with identical firms does not generate such results. Hence, I integrate firms' productivity heterogeneity into the model; it yields results that are consistent with all these aspects of the data. Introducing UI does not generate implications that are not inconsistent with the empirical evidence. Thus, the model with UI enables us to analyze the impact of UI on firms.

The unemployment insurance mechanism works as follows: the government levies a proportional tax rate on workers' wage and distributes the collected amount equal to the unemployed workers. We define social welfare as the weighted sum of average lifetime utility of unemployed and employed workers (Fredriksson and Holmlund (2001)) so as to examine the impact of unemployment insurance on social welfare. The insurance mechanism has an influence on welfare as follows: the insurance payment drives up the value of the outside option of unemployed workers and thereby increases wages through Nash's bargaining. This causes the value of employment to increase due to the wage and the flow value of unemployment are its components. The raising wage level makes production more costly, so firms might lay off some of their workers and reduce their recruitment effort. Since the number of vacancies posted by firms goes down and the unemployment rate climbs up (?), this reduces the labor market tightness, which lowers the job-finding probability for unemployed workers. The value of unemployment might exhibit a decline because the lower labor market tightness and the taxation on wages lead to a decrease in the expected value of employment. The potential reduction in the unemployment value might put downward pressure on the wage level. Similarly, the flow value of employment experience a decline. Therefore, the overall effect of UI on wages and welfare is analytically ambiguous. Additionally, its effect on firms is not analytically clear as well because its impact effect on wages is undetermined. In order to determine the impact of UI, I conduct a quantitative analysis by calibrating my model for the U.S economy with the parameter values which are consistent with the features of the U.S economy and studies in the literature.

My quantitative analysis suggests the following results: As the proportionate tax rate on earnings rises, the flow values of unemployment and employment rise, peak, and then begin to fall, respectively. The decline occurs because the rising proportional tax rate put downward pressure on labor market tightness. For lower proportionate tax rates, the insurance payment effect dominates the labor market tightness effect, causing the flow values to increase, but as the rate climbs up, the latter one becomes more dominant, precipitating a decline in the flow values. This behavior is transmitted to the social welfare: The social welfare achieves the highest value at the 1.7 percent tax rate and then begins to decline. Additionally, the higher the tax rate prompts the higher wage levels because the insurance payment increases the flow value of unemployment, which reflects in wages through Nash Bargaining. Since the increase in wage level causes firms to post a less vacancy and so unemployment rate displays an increase, which means, a higher proportional tax rate, a higher unemployment rate.

The firm-related consequences of UI are as follows: the increasing wages results in declining flow profit and profit per worker. The firms' growth rate also declines because the increase in wages brings about a reduction in the number of vacancies posted by firms. These results come from the model with identical firms and they hold under productivity heterogeneity as well.

I also investigate the persistence and impulse response of endogenous variables to 1.7

percent proportional unemployment insurance taxation on employed workers and its insurance payment to unemployed workers. With the introduction of unemployment insurance, labor market tightness decreases dramatically, but the reduction does not extend and eventually comes to a halt as firms increase their number of vacancy-posting. However, since the entry-level is not high enough to make up for the amount of the firm's exit, the number of active firms decreases by roughly 5 percent initially, grows somewhat with the entry of new firms during the transition, and decreases by 4 percent in the long term. Over the transition, wages and the flow values of unemployment and employment display a slight increase, drastic decrease, and extreme rise, respectively. In the long run, rV^u, V, and wages increase respectively by 7 percent, 8 percent, and 4 percent. Similarly, social welfare exhibits similar behavior as rV^u and V during the transition and rises roughly 6 percent in the long run. The adjustments in wages, the flow value of unemployment and employment, labor market tightness, and welfare over the transition indicate their persistence.

Section 2 reviews the previous literature. Section 3 describes the environment and the model. In Section 4, we determine key equations and establish the equilibrium with unemployment insurance. In Section 5, we conduct the quantitative application, provide the welfare-optimizing and the implications of UI on firms, and present the impulse response of the economy to the introduction of unemployment insurance. Finally, Section 6 concludes.

2. RELATED LITERATURE

This study contributes to the literature that studies dynamic Stole and Zwiebel-type bargaining in an environment characterized by labor market frictions and decreasing returns to labor. In Caballero (1994), firms and workers share marginal surplus stemming from employment relationship through Nash bargaining. The bargaining approach in the steady-state is similar to Stole and Zwiebel's style, which makes their steady-state analysis comparable to my study. My study differs from their study in particular aspects. I show the existence of a steady-state equilibrium with unemployment insurance. In our model, firms have different productivity levels and aim to reach their long-run employment size through multiple hiring in the presence of unemployment insurance and I examine how UI affects firms and welfare, while Caballero (1994) investigates how idiosyncratic productivity shocks have an influence on labor reallocation. Furthermore, in Cahuc, Marque, and Wasmer (2008) and Elsby and Michaels (2013), the bargaining between firms and workers takes place in the presence of search frictions. Cahuc, Marque, and Wasmer (2008) supposes that firms always perform their production at their desired long-run employment level with the purpose of tractability. In my study, because of time-consuming hiring, firms carry out their operation while reaching their desired long-run employment level. Elsby and Michaels (2013) assumes that the vacancy posting cost function displays a constant return to scale, which results in firms reaching their target employment immediately by posting a huge number of vacancies. However, this study also abstracts from firms being further from the long-run employment level. These assumptions eliminate the variation in firms' size and productivity of labor caused by the costly hiring process, which is central to our analysis to observe the unemployment insurance effect on wages, firm profitability, and growth at different employment levels.

This study is also related to the literature that focuses on unemployment insurance in the search equilibrium. Fredriksson and Holmlund (2001), Lehmann and Linden

(2007) and Mitman and Rabinovich (2015) study optimal unemployment insurance in search equilibrium. Mitman and Rabinovich (2015) focuses on optimal unemployment insurance as a response to productivity increases and falls. Fredriksson and Holmlund (2001) investigates the optimal insurance payment structure: fixed unemployment insurance payment or increasing/decreasing payment structure over worker's unemployment spell. Lehmann and Linden (2007) establishes optimal insurance structure taking into account worker's search effort. In these studies, a firm is matched to a worker or vacancy position, which refers to one-to-one matching in search equilibrium. In my study, firms are able to hire multiple workers at the same time. A recent study in the literature Landais, Michaillat, and Saez (2018) analyzes optimal unemployment insurance and they consider both the labor-demand effect and rat-race effect simultaneously by introducing Bailey-Chetty model of UI into a matching model. However, they define social welfare as a combination of generosity of UI and labor market tightness, their welfare analysis does not take into account lifetime values of employment and unemployment as my study. Moreover, in the literature, as far as I know, there is no previous study that focuses on unemployment insurance's effect on firms' size, profitability, and growth in search equilibrium. This study will lead in that regard and contribute to the literature on unemployment insurance in the search equilibrium with multi-worker firms.

Lastly, this paper also relates to literature investigating the propagation mechanism through which a temporary shock has a permanent effect on the economy. Shimer (2005), Fujita and Ramey (2007) and Hagedorn and Manovskii (2011) consider the economy's response to transitory productivity shock through the propagation mechanism. Since the introduction of unemployment insurance and its extensions are an additional shock to the economy like productivity shock, they generate propagation through the economy. Hagedorn et al. (2019) analyzes the propagation resulting from introducing and extending unemployment insurance in response to productivity decline as a counter-cyclical policy. This study focuses on the propagation mechanism generated by the introduction of unemployment insurance and its extension when there is no productivity shock.

3. THE MODEL

The model is a generalized version of the standard Mortensen-Pissarides model with multi-worker firms, enhanced by integrating the unemployment insurance mechanism. The economy involves a unit measure of risk-neutral workers and a larger continuum of firms. Time is continuous and the time horizon is infinite. All agents in the economy discount their future value at the rate $r \geq 0$. With the payment of entry cost of k, inactive firms become active and get a permanent productivity level from a distribution. At the start, they have no workers and they plan to reach their long-run employment level by step-by-step multiple hiring.

Firms' production depends on only labor as an input which is combined with firms' productivity z, $y(n, z)$. I assume that the production function satisfies the following properties: strictly increasing, strictly concave, and continuously differentiable in n, strictly increasing in z, and lastly $y(0, z) = 0$ for any z.

Matching between firms and workers takes place in a frictional labor market, which corresponds to that firms need to maintain open vacancies in order to employ workers. The vacancy-posting cost function, $c(v)$ is strictly increasing, strictly convex, continuously differentiable and meets the Inada conditions below.

$$\lim_{v \to 0^+} c^{'}(v) = 0 \text{ and } \lim_{v \to \infty} c^{'}(v) = +\infty \tag{1}$$

The aggregate matching function depends on the number of unemployed workers u and total numbers of vacancies $\bar{v}$, and measures amounts of firm-worker matching. The aggregate matching function display a constant return to scale to $(u, \bar{v})$ jointly and decreasing return to scale u and $\bar{v}$ separately. The vacancy to unemployment ratio $\bar{v}/u$ refers to the labor market tightness represented by θ. An unemployed worker's job-finding rate is represented by $\theta q(\theta) = M/u$. A firm that posts v vacancy meets a worker at a rate $v.q(\theta)$.

Wages are determined according to the approach in Kirman, Aumann, and Shapley (1976) and Stole and Zwiebel (1996): a firm and worker bargain over the surplus raised by firm-worker matching. The equation below refers to how the wages are set between firms and workers.

$$\Phi J_n(n,t;z) = (1-\Phi)[V(n,t;z) - V^u(t)] \tag{2}$$

J corresponds to the value of a firm with n worker and productivity z at date t, V refers to the value of an agent working at such a firm and V^u denotes the value of unemployment for a worker, lastly, the ϕ is worker's bargaining power.

Because the workers are identical in terms of productivity, all workers at such a firm will get the same amount of wage. The worker-firm matching can break down in two ways: the destruction of firms at an exogenous Poisson rate δ and a worker's separation at exogenous rate s. When a firm's destruction takes place, all workers employed by the destroyed firm return to the unemployed worker pool and the destroyed firm has no scrap value. When separation shock hits a firm, the firm continues its production with the remaining workers. These shocks occur independently of firms' size, productivity, and employment relationships.

The unemployment insurance mechanism works as follows: the government imposes a proportional tax t on wages and uses its tax revenue to finance unemployment insurance, p. I assume the government follows a balanced budget policy, which means they distribute all tax revenue to unemployed workers. All unemployed workers receive the same amount of insurance payment. In addition to unemployment income $b > 0$, unemployed workers receive p as an insurance payment. I assume if firms grow above a certain employment level, the marginal product of labor reaches below $b+p$ for any productivity level, which is central to our equilibrium analysis.

$$\lim_{n\to\infty} y'(n,z) < b+p \tag{3}$$

Explaining firm dynamics in the model might be useful to better understand our model. Newly entering firms have fewer workers, so their marginal productivity of labor and marginal contribution of hiring new workers is relatively higher. Thus, it is optimal for the firms to post a high number of vacancies, and such firms experiences high growth. However, since the vacancy-posting cost function is convex, firms' growth rate is limited. The marginal contribution of hiring an employee display a decrease in time and firms post a lower number of open vacancy which results in a decrease in firms' growth rate. If firms survive the destruction shocks, firms reach their long-run desired unemployment level and hire additional workers to maintain their target level of employment, which is disrupted by the separation shocks. In addition to the mechanism lying behind a firms' path to their target level, the firms' target level of employment is determined by the following factors: productivity level of firms, the intensity of labor market frictions, worker's separation rate, market tightness, and the value of unemployment.

Because of heterogeneity in firms' productivity and the time-consuming hiring process, firms' size dispersion occurs. The firms with higher productivity levels set higher long-run target employment, whereas firms with lower productivity set lower long-run employment. If free entry is allowed, since some firms are destroyed at a Poisson rate δ, new firms enter the economy as a substitute for destroyed firms. This results in the economy involving firms of different ages: the younger the firm, the lower the number of employees that have conditional on the productivity level. These two reason causes firm size dispersion and firm size dispersion still occurs even if one of the reasons is non-existent. For example, when the destruction rate is equal to zero, firm size dispersion stems from productivity heterogeneity. In addition, if all firms have the same productivity level, the firm size dispersion still surfaces since the economy encompasses firms that are in a different stage of their hiring process toward their target level.

4. THE EQUILIBRIUM

4.1 Value Functions and Definitions

In this part, I will construct the equilibrium and determine the following endogenous variables: the labor market tightness, wages, the values of employment and unemployment, and the unemployment rate.

I assume that the firm's value function is strictly concave and twice continuously differentiable with respect to n. The HJB equation for firm's value is in the following form:

$$
\begin{aligned}
(r+\delta)J(n,t;z) - J_t(n,t;z) = {} & y(n;z) - nw(n,t;z) - snJ_n(n,t;z) \\
& + \max_{v \geq 0}\{-c(v) + q(\theta(t))vJ_n(n,t;z)\}
\end{aligned} \tag{4}
$$

On the left-hand side of the equation, the firm's effective discount rate $r+\delta$ refers to the combination of the firm's value of future and the firm's destruction rate. The right side involves current production and the wage cost of the production. Because of worker's separation s, sn amount of workers separate from the firm, and Jn denotes per worker firm's value loss. Since firms aim to reach the long-run target level, they post vacancy v to maximize net capital gains, which is the total flow gains qvJ_n minus the vacancy posting cost c.

I specify the optimal vacancy posting strategy v as follows:

$$v(n,t,z) = \arg\max_{v \geq 0}[-c(v) + q(\theta(t))vJ_n(n,t;z)] \tag{5}$$

First-order condition determines optimal vacancy strategy implies:

$$c^{'}(v) = qJ_n(n,t,z) \tag{6}$$

The HJB equation for the value of worker at a firm with n employee and z productivity is

$$\begin{aligned} rV(n,t;z) - V_t(n,t;z) = {} & w(n,t;z)(1-\tau(t)) + (s+\delta)\left[V^u(t) - V(n,t;z)\right] \\ & + \left[q(\theta(t))v(n,t;z) - sn\right]V_n(n,t;z) \end{aligned} \tag{7}$$

The left side of the equation corresponds to the discounted value of workers at such a firm. On the right-hand side, w denotes wage payment, the second term represents the expected loss from returning to unemployment and the last term refers to the change in the value of employment caused by a potential increase in the number of workers.

The HJB equation for the value of unemployed workers is

$$rV^u(t) - V_t^u(t) = b + p(t) + \theta(t)q(\theta(t))E[V(n,t;z) - V^u(t)] \tag{8}$$

b denotes unemployment income, p is unemployment insurance payment, and the last term corresponds to expected value gains, which are generated by the employment relationship with a firm. Unemployed workers meet a firm at a Poisson rate θq Since firms differ in their employment level and productivity, the value of employment hinges on the firm's employment n and productivity z. Therefore, we need to calculate the expected value of employment considering firms' frequency and the number of vacancies posted by firms. To that end, I define firms' distribution by employment level n and productivity z. Suppose $x(t)$ is the total number of firms in the economy. Since firms' death does not depend on productivity level and firms discover their productivity level after their entry, we can obtain a cumulative distribution of firms' productivity $F(z)$ through the marginal distribution of firms' productivity. Let $G(n,t,z)$ be the total measure of firms that have productivity z and employees less than n [1]. We assume $G(n,t,z)$ satisfies the condition of being twice continuously differentiable in n and $g(n,t,z) = G_n(n,t,z)$. Thus, the total measure of firms takes the form below:

$$x(t) = \int_0^\infty \int_0^\infty g(n,t,z)\, dn\, dF(z) \tag{9}$$

[1] The firm size CDF of firms with the level of productivity z is represented as $G(n,t,z)/x(t)$. Writing the distribution of firm size in this manner allows us to construct PDE for change in firm size over time in equation (13).

Hence, The HJB equation for employed workers becomes:

$$rV^u(t) - V^u_t(t) = b + p(t) + \theta(t)q(\theta(t)) \frac{\int_0^\infty \int_0^\infty [V(v,t;z) - V^u(t)]v(v,t)g(v,t,z)\,dv\,dF(z)}{\int_0^\infty \int_0^\infty v(v,t)g(v,t,z)\,dv\,dF(z)} \tag{10}$$

Replacing V with its equivalent in the equation (2) establishes that:

$$rV^u(t) - V^u_t(t) = b + p(t) + \frac{\phi}{1-\phi}\theta(t)q(\theta(t)) \frac{\int_0^\infty \int_0^\infty J_v(v,t,z)v(v,t)g(v,t,z)\,dv\,dF(z)}{\int_0^\infty \int_0^\infty v(v,t)g(v,t,z)\,dv\,dF(z)} \tag{11}$$

Free entry condition implies :

$$\int_0^\infty J(0,t,z)\,dF(z) < k \tag{12}$$

Now, we need to determine how the firm distribution evolves. We assume that $g(n,t,z)$ is continuously differentiable with respect to z and t. The partial differentiation of $g(n,t,z)$ with respect to t, defines evolution of density distribution.

$$g_t(n,t;z) = -\frac{\partial}{\partial n}[(q(\theta(t))v(n,t;z) - sn)g(n,t;z)] - \delta g(n,t;z) + e(t)f(z)j(n) \tag{13}$$

e denotes entry rate at time t and j represents indicator function, whose value is 1 when $n = 0$, and 0 otherwise[2]

We also need to specify the labor market tightness, which suggests job-finding difficulty for the unemployed workers. The ratio of open vacancy to unemployment defines labor market tightness. Thus, firstly we need to characterize the unemployment rate and total vacancy that firms post.

[2] The derivation of the evolution equation as follows: Suppose that $n > 0$ and take $\epsilon > 0$. The amount of firms located in the interval of $(n-\epsilon/2, n+\epsilon/2)$ equals $\epsilon g(n,t,z)$. Considering the separation shock,s, and vacancies posted by the firms, the firms with employment level close to $n-\epsilon/2$ have the following growth rate: $q(\theta(t))v(n-\epsilon/2,t,z) - s(n-\epsilon/2)$. Thus in a short amount of time,$dt > 0$, the amount of firms with employment level less than $n-\epsilon/2$ that moves into the interval of $(n-\epsilon/2, n+\epsilon/2)$ through hiring is equal to $[q(\theta(t))v(n-\epsilon/2,t,z) - s(n-\epsilon/2)]\,g(n-\epsilon/2,t,z)dt$. Likewise, the amount of firms that leave the interval of $(n-\epsilon/2, n+\epsilon/2)$ through hiring is equal to $[q(\theta(t))v(n+\epsilon/2,t,z) - s(n+\epsilon/2)]\,g(n+\epsilon/2,t,z)dt$. Further, the due to destruction shock, δ, the amount of firms that leave the interval is equal to $\delta\epsilon g(n,t,z)$. Those allow us to establish that:

$$\begin{aligned}\frac{\partial[\epsilon g(n,t,z)]}{\partial t} &= [q(\theta(t))v(n-\epsilon/2,t,z) - s(n-\epsilon/2)]\,g(n-\epsilon/2,t,z) \\ &\quad - [q(\theta(t))v(n+\epsilon/2,t,z) - s(n+\epsilon/2)]\,g(n+\epsilon/2,t,z) - \delta\epsilon g(n,t,z).\end{aligned}$$

Multiplying that by $\frac{1}{\epsilon}$ and calculating its limit as ϵ approaches 0^+ gives the equation (13). Further, we can establish this equation for entry at $n = 0$, assuming that $f(z)$ is the density of firms of productivity z among the new entries.

Under the assumption that the total labor force is normalized to 1, we construct the measure of unemployed workers as follows:

$$u(t) = 1 - \int_0^\infty \int_0^\infty ng(n,t,z)\,dn\,dF(z) \tag{14}$$

The measure of total open vacancy which firms post is characterized below:

$$\bar{v}(t) = \int_0^\infty \int_0^\infty v(n,t,z)g(n,t,z)\,dn\,dF(z) \tag{15}$$

Thus, the labor market tightness, θ is $\bar{v}(t)/u(t)$.

Now, I need to specify the unemployment insurance taxation and insurance payment equations, total collected taxation amount takes the following form: amssymb

$$\mathcal{T}(t) = \int_0^\infty \int_0^\infty nw(n,t,z)\tau(t)g(n,t,z)\,dn\,dF(z) \tag{16}$$

Since the government pursues balanced-budget policy, it distributes the entire collected tax equally to the unemployed workers, the insurance payment is

$$p(t) = \frac{\mathcal{T}(t)}{u(t)} \tag{17}$$

We define the social welfare,χ, as the weighted sum of average life-time utilities of unemployed and employed workers:

$$\chi = (1-u)\frac{\int_0^\infty \int_0^\infty nrV(n,t,z)g(n,t,z)\,dn\,dF(z)}{\int_0^\infty \int_0^\infty ng(n,t,z)\,dn\,dF(z)} + urVu \tag{4.1}$$

4.2 Wage Determination

In this part, we characterize the wage equation by using a combination of HJB equations for firms and workers, and the Stole and Zwiebel (1996)'s bargaining equation. The following assumption on production function helps us to specify the wage in the equilibrium:

Assumption 1.

$$\lim_{n\to 0^+} n^{-\frac{(1-\tau(t)+\phi\tau(t))}{\phi}} \int_0^n v^{\frac{(1-\tau(t))(1-\phi)}{\phi}} y^{'}(v,z)\,dv = 0$$

Assumption 1 implies that as firms' worker number converges to zero, the marginal product of labor does not diverge to infinity. This assumption enables me to establish the wage equation. The condition holds if we choose a quadratic or Cobb-Douglas production function.

Lemma 1. *The unique solution for wages satisfying* $\lim sup_{n\to 0^+}|nw(n,z)| < +\infty$ *is*

$$w(n,t,z) = \frac{(1-\phi)}{(1-\tau(t)+\phi\tau(t))}[rV^u(t) - V_t^u(t)] + \frac{\phi}{(1-\tau(t)+\phi\tau(t))}\frac{\int_0^n v^{\frac{(1-\tau(t))(1-\phi)}{\phi}} y^{'}(v,z)\,dv}{\int_0^n v^{\frac{(1-\tau(t))(1-\phi)}{\phi}}\,dv} \tag{18}$$

Proof. Relegated to Appendix B. □

Incorporating unemployment insurance into the search model does not fundamentally change the wage equation commonly followed by the literature. The wage structure in my model is similar to those in Stole and Zwiebel (1996) and Elsby and Michaels (2013). Two main components establish the wage: the first one is the effect of the value of the outside option of the worker and the second component represents the worker's average contribution to the firm.

The wage function takes the following form if the production function, $y(n) = zn - \frac{\sigma n^2}{2}$, is quadratic, which we adopt through our analysis.

$$w(n,t,z) = \frac{(1-\phi)}{(1-\tau(t)+\phi\tau(t))}[rV^u(t) - V_t^u(t)] + \frac{z\phi}{(1-\tau(t)+\phi\tau(t))} - \frac{\sigma n\phi}{(1-\tau(t)+\phi\tau(t)+\phi)}$$

Lemma 2 shows wages and flow profits satisfy convenient boundary conditions. This enables us to establish the equilibrium of the model.

Lemma 2. *If* $rV^u(t) - V_t^u(t) > 0$*, then* $w(n,t,z) > 0$*,* $w_n(n,t,z) < 0$*, and* $w(n,t,z)$ *satisfies the followings*

$$\lim_{n\to 0^+} w(n,t,z) = \frac{(1-\phi)}{(1-\tau(t)+\phi\tau(t))}[rV^u(t) - V_t^u(t)] + \frac{\phi}{(1-\tau(t)+\phi\tau(t))}\lim_{n\to 0^+} y^{'}(n,z)$$

$$\lim_{n\to\infty} w(n,t,z) = \frac{(1-\phi)}{(1-\tau(t)+\phi\tau(t))}[rV^u(t) - V_t^u(t)] + \frac{\phi}{(1-\tau(t)+\phi\tau(t))}\lim_{n\to\infty} y^{'}(n,z)$$

$\pi(n,t,z)$ satisfies strict concavity and the following

$$\lim_{n\to 0^+} \pi(n,t,z) = 0$$

Provided that $rV^u(t) - V_t^u(t) > \lim_{n\to\infty} y^{'}(n,z)$, then $\lim_{n\to\infty} \pi(n,t,z) = -\infty$ and the profit maximizing n is finite. Further, provided that $rV^u(t) - V_t^u(t) < \lim_{n\to\infty} y^{'}(n,z)$, the profit maximizing n is strictly positive.

Proof. Relegated to Appendix B. □

If $\lim y < b+p$, the condition that, $\lim_{n\to\infty} y^{'}(n,z) < rV^u(t) - V_t^u(t)$, holds in the steady state. If the Inada condition related to production function, $\lim_{n\to 0^+} y'(n;z) = +\infty$, holds, Lemma 2 indicates $\lim_{n\to 0^+} w(n,t,z) = +\infty$ which corresponds to that optimal number of worker in a firm is strictly positive.

Considering the equations we have formulated up to this point, We characterize a dynamic equilibrium as follows:

Definition 1

A tuple $\langle\ \theta(t),\ V^u(t), G(n,t,z), g(n,t,z), x(t), J(n,t,z), V(n,t,z), v(n,t,z), w(n,t,z),$

$\mathcal{T}(t), p(t)\rangle$ establishes an equilibrium provided that for all time periods, the conditions below hold.

- (2),(4),(7) and (8) holds which provide equilibrium values of J, V and V^u.
- (5) holds which assures optimal vacancy posting v
- Free entry condition satisfies (12) and the equality $e > 0$ is satisfied.
- g(n,t,z) satisfies (13)
- (9) holds for calculation of total number of firms, x(t)
- $\theta(t) = \frac{\bar{v}(t)}{u(t)}$ holds where $u(t)$ and $\bar{v}(t)$ are obtained by equations (14) and (15)
- $\mathcal{T}(t)$ satisfies (16) and $p(t) = \frac{\mathcal{T}(t)}{u(t)}$ holds.
- w(n,t,z) satisfies (18)

4.3 Steady State Equilibrium under Identical Productivity Level

In this part, in order to investigate the effect of unemployment insurance on firms' growth and profit, and welfare, we characterize the steady-state equilibrium under the assumption that firms have the same level of productivity. In the following parts, we conduct the same analysis under firm heterogeneity.

Since the analysis is in the steady state, so the variables (θ, rV^u) and the distribution of firm size are independent of time.

The following partial different equation corresponds to how firms' size distribution evolves.

$$\frac{g^{'}(n)}{g(n)} = \frac{s-\delta-qv^{'}(n)}{qv(n)-sn} \tag{19}$$

The integration of the partial differential equation delivers:

$$g(n) = Dexp(\int_0^n \frac{s-\delta-qv^{'}(n)}{qv(n)-sn}dv) \tag{20}$$

where D is the integration constant, which allows g for integrating over the region to the total amount of firms in the economy x. In the steady-state, the amount of firm entry is equal to the number of destroyed firms, which gives $x = e/delta$

In addition, under the assumption of firms' homogeneity, free entry condition (12) in the steady-state implies:

$$J(0) \leq k \text{ and } \theta \geq 0 \tag{21}$$

The Homogeneous version of Definition 1 delivers the steady-state equilibrium definition under homogeneous productivity.

Definition 2

A tuple $\langle\ \theta(t),\ V^u(t), G(n,t,z), g(n,t,z), x(t), J(n,t,z), V(n,t,z), v(n,t,z), w(n,t,z),$

$\mathcal{T}(t), p(t)\rangle$ establishes an equilibrium provided that for all time periods, the conditions below hold.

- (2),(4),(7) and (8) holds which provide equilibrium values of J, V and V^u.
- (5) holds which assures the optimal vacancy posting v

- Free entry condition satisfies (21) and the equality $e > 0$ is satisfied.
- g(n,t,z) satisfies (20)
- (9) holds for the calculation of total number of firms, x(t)
- $\theta(t) = \frac{\bar{v}(t)}{u(t)}$ holds where $u(t)$ and $\bar{v}(t)$ are obtained by equations (14) and (15)
- $\mathcal{T}(t)$ satisfies (16) and $p(t) = \frac{\mathcal{T}(t)}{u(t)}$ holds.
- w(n,t,z) satisfies (18)

Equating the amount of entry $(1-u)(s+\delta)$ to unemployment pool and the amount of entry to employment pool $\theta q(\theta)u$ delivers the steady-state unemployment rate

$$u = \frac{s+\delta}{s+\delta+\theta q(\theta)}$$

Where $\theta q(\theta)$ is the job-finding probability for unemployed workers and $s+\delta$ refers to the total amount of employment relationship destruction.

In the steady-state, a firm creates vacancy positions to compensate for their worker loss because of separation shock, $v(n) = \frac{sn}{q}$. Such a firm seeks to preserve its employment level until a destruction shock delta hits, so the firm obtains a constant level of profit $\pi(n)$ and incurs the cost of vacancy posting. Hence, the firm value takes the following form:

$$(r+\delta)J(n^*) = \pi(n^*) - c(v(n^*)).$$

We also need to specify the optimal employment level at the steady-state. The following steps will provide optimal employment level:

Initially, the first order condition for $v(n)$ (6) at $n = n^*$ implies

$$J^{'}(n^*) = \frac{1}{q}c^{'}(v(n^*)) = \frac{1}{q}c^{'}(\frac{sn^*}{q}) \tag{22}$$

Secondly, the differentiation of the equation (4) at steady state with respect n and its equivalence in $v(n)'s$ first order condition and n^* definition $qv(n) - sn^*$ deliver the following

$$(r+\delta+s)J^{'}(n^*) = \pi^{'}(n^*)$$

With the incorporation of J'(n)'s equivalence in equation (21), we obtain that :

$$\frac{r+\delta+s}{q}c^{'}(\frac{sn^*}{q}) = \pi^{'}(n^*) \tag{23}$$

Lemma 2 implies profit-maximizing employment level is positive and the profit function is strictly concave, which corresponds to that $\pi(n)$ satisfies the condition of being strictly decreasing. By the assumption regarding vacancy posting cost function, $c'(v)$ exhibits being strictly increasing and satisfies Inada condition (1). Hence, we can find a unique n^* for the equation (21).

Alternatively, we can provide intuition for n^* specification through the wage equation. Differentiation of the profit equation and substituting it in the wage equation (16) delivers $w(n) = \frac{rV^u}{1-\tau} + \frac{\phi}{(1-\phi)(1-\tau)}\pi^{'}(n)$. Putting $\pi(n)'$ equivalence in 23, we get that :

$$w(n^*) = \frac{rV^u}{1-\tau} + \frac{\phi(r+\delta+s)}{(1-\phi)(1-\tau)}\frac{1}{q}c^{'}(v(n^*))$$

The equation suggests that firms hire workers to the point where the wage is equal to the sum of the post-unemployment insurance value of the outside option and the part which is in proportion to labor market friction. The second term appears in the wage equation since the hiring cost of a replacement employee leads to a match-specific quasi-rent which results in higher wage demand of workers in the bargaining process.

Consequently, the following proposition outlines the assessments in this part.

Proposition 1. *Provided that $q = q(\theta) > 0$ and $rV^u > 0$, then an allocation $(q(\theta), rV^u)$ in steady state exits if and only if the following statements satisfy:*

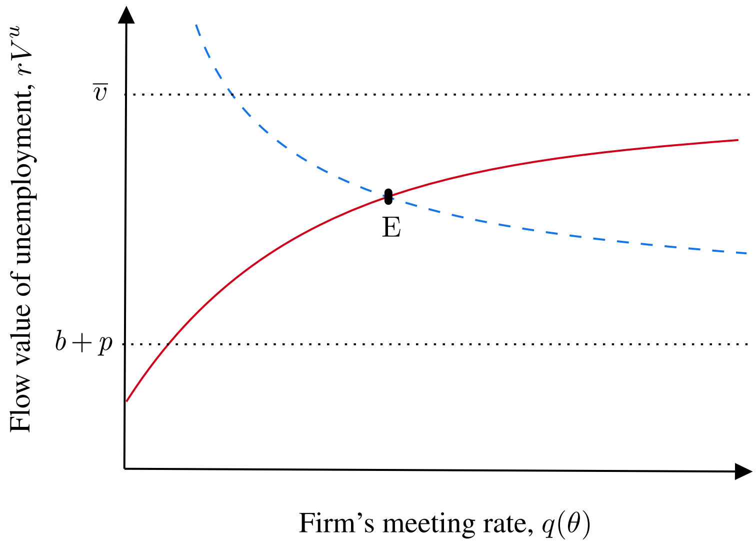
- *$J(.)$ provides a unique solution to HJB equation for firm's value (4) provided that the equation (22) holds and n^* satisfies (23).*
- *(11) holds for the unemployment value, rV^u*
- *$\mathcal{T}(t)$ satisfies (16) and $p(t) = \frac{\mathcal{T}(t)}{u(t)}$ holds.*
- *(6) gives optimal number of vacancies posted by firms*
- *(20) delivers $g(n)$*
- *The free-entry requirement (21) holds*

Theorem 1 enables us to establish the existence of the steady-state equilibrium.

Theorem 1. *There exists a steady-state equilibrium given that firms pursue threshold hiring strategy.*

We aim to demonstrate that there exits (q, rV^u) values which satisfy the hypothesis

Figure 1 Illustration of equilibrium existence.



The Downward-sloping curve represent the $(q(\theta), rV^u)$ relationship required by unemployed worker HJB equation (10) and the upward-sloping curve denotes the $(q(\theta), rV^u)$ relationship suggested by firms' zero-profit condition (21). The intersection of the curves gives us the equilibrium.

of Proposition 1. The proof requires that there exits an equilibrium which involves active firms if the following statement holds

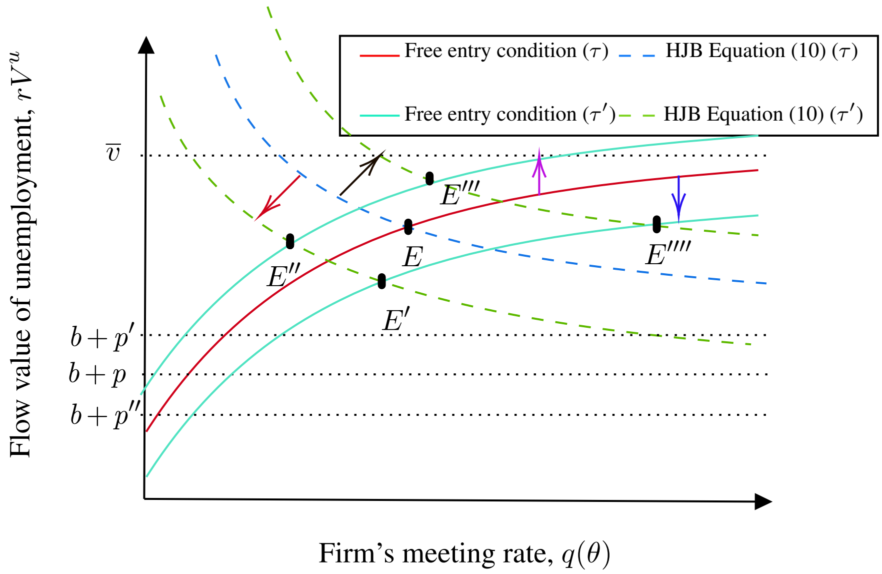
$$k < \frac{1}{r+\delta} \max_{n>0} \left\{ y(n) - n^{-(1-\phi)(1-\tau)/\phi} \int_0^n \nu^{(1-\phi)(1-\tau)/\phi} y'(\nu) d\nu - n(1-\phi)(b+p) \frac{1}{1-\tau+\theta\tau} \right\}$$

Proof. Relegated to Appendix B. □

We show the maximizing value of n exits in the proof of Lemma 2.

Figure 1 involves two curves: the upward and downward sloping curves. The former one indicates the relationship between q and rV^u implied by the free-entry condition (21). It suggests a positive relationship between q and rV^u because if rV^u exhibits an increase, it must be offset by an increase in q according to the free-entry condition (21). This occurs since higher rV^u leads to higher wage levels, thereby lowering firms' profit margins. In the case of a rise in rV^u, employee hiring of firms should be quick so that the value of $J(0) = k$ remains constant. The quicker hiring takes place in case of a decrease in hiring cost, which is obtained by rising q. This concludes the reason why the former one is upward-sloping. Further, the latter reflects the relationship between q and rV^u suggested by unemployed workers' HJB equation (10). It indicates the negative relationship between q and rV^u since an increase in rV^u should be offset by a decrease in q according to the HJB equation

Figure 2 The effect of an increase in proportional tax rate on equilibrium



The proportional tax rate increases from τ to τ' ($\tau' > \tau$).

for unemployed workers. This is because an increase rV^u results in an increase in wages. To ensure rV^u satisfies the equation (10), the employee hiring must be quicker. Thus, after being hired, the worker tends to display less effort to earn the high wages offered by small firms as firms grow quicker. This results in lower q [3]. In Theorem 1's proof, we observe that utilizing the argument of continuity requires these curves to intersect.

Now, we focus on the response of equilibrium to an increase in the proportional tax rate. The potential effect of raising the proportional tax rate on labor market tightness and the flow value of unemployment remains unclear. The upward-sloping curve shifts downward (upward) if the unemployment insurance payment, p, increases (decreases) following the increase in the tax rate because an increase (a decrease) in p causes a decrease (an increase) in the value of entry, $J(0)$, rV^u should lower (rise) for each level of q so that free entry condition (21) holds. Since the effect of the tax raises on unemployment insurance payments is ambiguous, the movement of the upward-sloping curve is ambiguous. Likewise, the adjustment of the downward-sloping curve remains uncertain. If the rise in the rate leads to an increase (a decrease) in wages, the downward-sloping curve shifts downward (upward). Since we are not able to determine the overall effect of the raise on wages analytically, the movement of the curve is unclear. As Figure 2 illustrates, equilibrium might end up with four different pairs of rV^u and q depending on the effect of an increase in the unemployment insurance tax on unemployment insurance payment and wages.

[3] The curve related to equation (10) might not be everywhere downward slopping, but it is downward-sloping in all calibrated examples.

Up to this part, we show that there exists a steady-state equilibrium that allows us to carry out our analysis, and equilibrium analysis of an increase in the unemployment insurance tax rate is not analytically convenient.

The potential effect of the introduction of unemployment insurance on key endogenous variables θ and rV^u is analytically ambiguous as well. The unemployment insurance payment increases the value of unemployment at first, which puts upward pressure on wages. Consequently, because of rising costs, some firms prefer exiting the economy or the remaining firms may update their long-run target level, which drives down labor market tightness. The reduction in the labor market tightness lowers the expected value of employment for unemployed workers, which may cause a decrease in the value of unemployment. The potential reduction in the value of unemployment might lead to an increase in the labor market tightness. Since we do not determine analytically which effect dominates, it is not possible for us to derive its effect on θ and rV^u. The impact of unemployment insurance on firms' growth and profit is not unclear because of the ambiguity of the effect on θ and rV^u. Hence, for the sake of identifying the impact, we conduct a simulation of the US economy in the following part.

5. QUANTITATIVE APPLICATION

In this part, my study exhibits how the introduction of unemployment insurance affects social welfare, labor market outcomes, and firms' growth and profit, and aims to determine the optimal unemployment insurance tax rate. Firstly, I focus on the mechanism through which it affects social welfare and its influence on social welfare. Since having productivity heterogeneity does not change the mechanism and its effect, for the sake of simplicity, I carry out welfare analysis through the model with homogeneous productivity. Secondly, I incorporate productivity heterogeneity into the model to illustrate its influence on firms with different productivity levels. To that end, I calibrate my model for the U.S economy with the parameter values, which are consistent with the features of the U.S economy and studies in the literature. I assume the production function and vacancy-posting cost function are in a quadratic form, as stated in Assumptions 2 and 3. Our parameter values match up with parameter values used in much of the literature. I benefit from Shimer (2005) in the determination of some parameter values. I follow the discrete-time approximation approach and take the time to be equal to a quarter. I adjust the discount rate to $r = 0.0123$ consistent with 4.7 percent annual discount rate ($\beta = 0.953$). I set the firm destruction rate to $\delta = 0.0167$ and the worker's separation rate, s to 0.0833 since the quarterly separation rate is equal to 10 percent and one-sixth of the separation stems from the firm's closure. I normalize the labor market tightness to 1 as in Shimer (2005), which requires the job-finding rate to be $q = 1.35$. I randomly choose the quadratic term of the production function to be $\sigma = 0.03$. I determine z and γ so that the average worker per firm is equivalent to 23.8 as in Davis, Faberman, and Haltiwanger (2006) and the flow value of unemployment is equal to 1. The values of z and γ allowing the model to reach these target levels are $z = 1.65$ and $\gamma = 0.06$. I assume that the matching function take the Cobb-Douglas form, $M(u, \bar{v}) = Zu^{\eta}\bar{v}^{1-\eta}$ with matching function constant $Z = 1.355$ and matching elasticity $\eta = 0.6$, based on estimates of Petrongolo and Pissarides (2001). The parameter values are consistent with a 6.87 percent unemployment rate. I set workers' bargaining power to $\phi = 0.4$ which is comparable with the values reported by Elsby and Michaels (2013), Millard and Mortensen (1997) and Botero et al. (2004). I choose unemployment income/value of leisure, $b = 0.4$ which is consistent with estimates given by Elsby and Michaels (2013) and Shimer (2005).

Parameter Values			
Parameters	Meaning	Value	Reason/Reference
η	Matching elasticity	0.6	Petrongolo and Pissarides (2001)
b	Unemployment income	0.4	Shimer (2005)
ϕ	Worker bargaining power	0.4	Elsby and Michaels (2013) and Botero et al. (2004)
z	Average productivity	1.65	Average employment per firm=23.8 and the flow value of unemployment=1
γ	Vacancy cost function parameter	0.06	Average employment per firm=23.8 and the flow value of unemployment=1
δ	Firm closure rate	0.013	Shimer (2005)
s	Employee's separation rate	0.087	Shimer (2005)
θ	Labor market tightness	1	Normalization by Shimer (2005)
r	Quarterly discount rate	0.012	Annual discount factor = 0.953

Since our aim is to make our model consistent with the U.S data and observe the effect of unemployment insurance on social welfare, firms' profit, and growth, concentrating on a particular parametric example is sufficient. For this reason, it is reasonable to assume that the production function and vacancy-posting cost function are quadratic[4].

Assumption 2. *All firms have the same production technology*

$$y(n; z) = zn - \tfrac{1}{2}\sigma n^2$$

where they have different productivity coefficient z but their coefficient $-\frac{1}{2}\sigma$ on the quadratic term is the same.

Assumption 3. *The cost function of vacancy-posting is in the following form:*

$$c(v) = \tfrac{1}{2}\gamma v^2$$

This enables us to derive closed-form solutions for a number of critical variables.

Lemma 3. *In a steady-state equilibrium, wages $w(n; z)$ and a firm's flow profit $\pi(n; z)$ take the following forms:*

$$w(n, z) = \frac{(1-\phi)(rV^u)}{(1-\tau+\phi\tau)} + \frac{\phi z}{(1-\tau+\phi\tau)} - \frac{\phi\sigma n}{(1-\tau+\phi\tau+\phi)}$$

[4]The findings stated in this part do not rely on the quadratic production function assumption. The same results can be obtained under the assumption of Cobb-Douglas production

$$\pi(n,z) = n\left[\frac{z(1-\tau)-rV^u}{(1-\tau+\phi\tau)/\ (1-\phi)}\right] - n^2\sigma\left[\frac{(1-\tau)(1-\phi)}{2(1-\tau(t)+\phi\tau+\phi)}\right]$$

The optimal employment level of a firm with productivity z is

$$n^*(z) = \frac{(z(1-\tau)-rV^u)}{(1-\tau(t)+\phi\tau)\left[\frac{\gamma s(r+\delta+s)}{q^2(1-\phi)} + \frac{(1-\tau)\sigma}{(1-\tau+\phi\tau+\phi)}\right]}$$

Vacancy-posting is in the following form:

$$v(n,z) = \frac{sn^*(z)}{q} + \lambda(n^*(z)-n)$$

where λ provides positive solution for the following equation

$$q\lambda^2 + (r+\delta+2s)\lambda - \frac{q(1-\phi)(1-\tau)}{\gamma(1-\tau+\phi\tau+\phi)}\sigma = 0$$

The percentage of firms of productivity z which employ more than n is

$$\bar{G}(n;z) \equiv 1 - \frac{G(n;z)}{x} = \begin{cases} \left[1-\frac{n}{n^*(z)}\right]^{\delta/(s+q\lambda)} & n < n^*(z) \\ 0 & n \geq n^*(z) \end{cases}$$

Proof. Relegated to Appendix B. □

Under these assumptions, we calibrate the model for the U.S economy with these parameter values and my model suggests the following implications.

5.1 The Results under Homogeneous Productivity

As Figure 3 illustrates, the flow values of unemployment and employment go up, reach their peak values and start to decline, respectively, as the proportional tax rate on wages increases. The adjustments boil down to the change in the labor market tightness. Since insurance payment leads to an increase in the flow value of unemployment, which results in higher wages. The increase in wage boosts the flow value of employment. However, the increasing wage level brings about a decrease in the labor market tightness because hiring an employee becomes more costly and thereby decreasing vacancy posting by firms. The decline in the labor market tightness reduces expected to gain from employment relationships for unemployed workers since job-finding probability goes down. This results in a decrease in the flow value of unemployment. Thus, I observe that there are two opposite forces determining the ultimate effect of unemployment insurance: the payment effect and the labor market tightness effect. Figure 3 exhibits the payment effect, which dominates the labor market effect up to a point and the flow values of employment and unemployment go up. After some point, these two flow values start to decline. In short, the flow values rise to some points and start to decline after the points as the proportional tax rate increases. This behavior through the proportional tax rate reflects the social welfare level. As Figure 5 shows, social welfare reaches the peak level at the 1.7 percent proportional tax rate and begins to fall above the rate. In addition, Figure 4 displays a higher proportional tax rate and a higher unemployment rate. This is because the increase in proportional tax rate results in fewer vacancy-posted by firms.

As shown in Figure 6, the implementation of the unemployment insurance policy increases the wage levels for all firms. The increase in wages results in declining flow profit and profit per worker. The firms' growth rate also declines because the increase in wages brings about a reduction in the number of vacancies posted by firms. The homogeneous model generates some results which are inconsistent with the US data in several aspects. The model suggests that smaller firms exhibit a higher growth rate than larger ones, which translates into wage payment[5]. But this implication is at odds with US data, which indicates larger firms pay higher than smaller ones. Secondly, since the firm dispersion stems from labor market friction under firm homogeneity, the firm distribution by employment at steady-state contradicts the US data. The model-generated firm size distribution indicates that firm density decreases until the optimal employment level and takes off around the optimal employment level. However, the U.S data demonstrates the density of firms exhibits a decrease at every point but has a right tail that complies with Zipf's laws. In addition, under firm homogeneity, we might not observe the effect of unemployment insurance on firms that have a different level of productivity. The magnitude of the effect of unemployment insurance on firms' growth and profit may differ because the magnitude of decrease/increase in their optimal employment level for firms is different under firm heterogeneity. For these reasons, we extend our model by incorporating firms' heterogeneity.

[5]The model with homogeneous firms predicts that firm size is inversely related to firm growth, profit per worker, and the wage a firm pays. This case is inconsistent with the U.S data. In addition, the model indicates that if the value $(s+q\lambda)/\delta$ is bigger than 1, the firm size distribution has a bound and shows increasing density around the bound. The case $s+q\lambda > 1$ is the empirically holding condition, so firm size distribution does not match the empirical findings.

5.2 The Results of the Model with Heterogeneous Productivity

In this part, we introduce productivity heterogeneity to the model. In our model, firms obtain their productivity level z from a productivity distribution after they pay the cost of entry k. Their productivity level does not change over time.

The generalization of Definition 1 gives the steady-state definition under firms' heterogeneity. In addition, the existence of steady-state equilibrium in this environment can be shown by extending the argument in the proof of Theorem 1. Thus, we leave out the formal proof of existence for the sake of briefness.

Since the homogeneous model poorly performs in matching the features of the U.S economy in some aspects. We introduce productivity heterogeneity and observe that adding productivity heterogeneity improves the model's ability to match the features of the U.S economy.

If I assume that productivity distribution is Pareto[6], the firm size distribution is Pareto Axtell (2001). The following proposition is related to this statement.

Proposition 2. *The distribution of firm size has a Pareto tail provided that the firms' productivity is Pareto distributed.*

Proof. Relegated to Appendix B. □

The assumption on the productivity distribution allows the model to comply with Gibrat's law.

Proposition 3. *If the firms' productivity is Pareto distributed, hence Gibrat's law applies to firms with high employment levels.*

Proof. Relegated to Appendix B. □

Under firms' productivity heterogeneity, if we condition on firm's age, we observe the following result

Proposition 4. *Given that the age of firms is the same, the employment level of the firm is directly related to the growth rate of the firm, wages offered by the firm, and profitability of the firm.*

[6] Pareto distribution of random variable z is parameterized by a lower bound z_m and shape parameter k such that z has density function, $f(z) = kz_m z^{-(k+1)}$. We construct the Pareto distribution with k=1 and with minimum value z_m=1. I assume that $z - rV^u = z - 1$ is distributed Pareto so that Proposition 3 and Proposition 4 apply to all firms which means that Gibrat's law holds for all firms.

Proof. Relegated to Appendix B. □

To sum up, assuming that the productivity distribution is Pareto makes the model consistent with the stylized facts of the U.S economy.

We can better figure out the implications of the model of firms' heterogeneity if a simulation of the U.S economy is carried out. To that end, I take 7 different productivity levels (20^{th} percentile, 30^{th} percentile, 40^{th} percentile up to 80^{th} percentile) from the Pareto distribution and conduct the same analysis that we did in the homogeneous case. As can be seen from Figures 7a and 7b and Table 1, the model with heterogeneous firms suggests that wages and profit per worker exhibit a positive association with firm size, and growth rate shows a weak positive correlation with firm size. The associations are consistent with empirical findings which correspond that under frictional labor market and productivity heterogeneity, my model is able to match up the features of the US economy.

As shown in Figures 7a and 7b, introducing unemployment insurance does not lead to a drastic shift in findings of the former model. Under productivity heterogeneity, we observe that the implication of the model is consistent with that of the homogeneous case. The introduction of unemployment insurance increases the wages that firms pay whereas it drives down profit per worker and flows profit. In addition, firms' growth rate displays a decrease since labor market tightness declines and thereby reducing the probability of a successful meeting between firms and workers. Similarly, the firm's profit rate and per worker profit rate go down because of rising wages and decreasing labor market tightness. Further, as shown in Table 2, the implications of the model with unemployment insurance policy do not contradict stylized features of the U.S economy.

Table 1 Pre-insurance cross correlation between firm size, firm growth, wages, profits and profit per worker in a numerical example

	Firm Size	Firm Growth	Wages	Profit	Profit Per Worker
Firm Size	1	0.0189	0.5293	0.9491	0.9181
Firm Growth		1	0.8583	0.1222	0.4192
Wages			1	0.5908	0.8301
Profit				1	0.9128
Profit Per Worker					1

Note: We use the parameter values in the homogeneous model. We conduct the simulation on 90 thousand firms before and after the introduction of unemployment insurance.

Table 2 Post-insurance cross correlation between firm size, firm growth, wages, profits and profit per worker in a numerical example

	Firm Size	Firm Growth	Wages	Profit	Profit Per Worker
Firm Size	1	0.0553	0.6465	0.9463	0.9310
Firm Growth		1	0.7975	0.1374	0.4207
Wages			1	0.6769	0.8862
Profit				1	0.9122
Profit Per Worker					1

Note: We use the parameter values in the homogeneous model. We conduct the simulation on 90 thousand firms before and after the introduction of unemployment insurance.

5.3 The Propagation of Unemployment Insurance

Up to this part, we have analyzed the welfare and firm-related implications of unemployment insurance and seen that my model is not at odds with important features of U.S data. In this part, we aim to observe the reaction of labor market outcomes to the unemployment insurance introduction. The persistence mechanism functions by the channels of the outside option of unemployed worker rV^u and wage. If the government launches an unemployment insurance program, then the UI payment increases the value of the outside option rV^u. Through the Nash bargaining assumption, this puts upward pressure on wages, which increases the cost of production. Due to the raising cost of production, the number of vacancies posted by incumbent firms gets lower. Because of the destruction shock with the Poisson rate δ, some firms shut down their operations. However, newly entrant firms do not offset the loss in the active firms because only firms above a certain productivity level enter the economy due to the increasing operational cost. Thus, the number of active firms declines, which also reduces the number of vacancy-posting. On top of that, the unemployment rate goes up since employees of exiting firms become unemployed. The reduction in vacancy posting and raised unemployment rate translates into a decrease in labor market tightness which drives down job finding probability for the unemployed workers. As a result, the flow value of unemployment starts to go down and thereby reducing the wage paid to employees. Hence, this leads to a decrease in the average wage level in the economy. The persistence mechanism takes place at this stage due to the fact that wages are temporarily lower than the long-run level, which makes it profitable for firms aiming to reach their long-run target employment level. Consequently, the reduction in wages level allows firms to increase their vacancy posting, which puts upward pressure on labor market tightness. Following the increase in labor market tightness, the flow value of unemployment together with wages starts to go up. In order to observe the adjustments of endogenous variables in the transition, I conduct a simulation utilizing parameter values through which I make calibration of the model.

Figure 8 display endogenous variables' impulse response to 1.7 percent proportional unemployment insurance taxation on employed workers and its insurance payment to unemployed workers. The panel exhibits that on introducing unemployment insurance, the average wage level goes up through the Nash bargaining since the insurance payment causes an increase in the outside option of unemployed workers. The increase in wage leads to a reduction in the number of vacancies posted by incumbent firms. In addition, the raising wage results in entry-level which is not sufficient to compensate for the firm loss stemming from firm destruction shock δ. Furthermore, the economy will have fewer active firms than before since following the unemployment insurance introduction, the increasing cost of operations causes a positive mass of firms to exit the economy. This brings down the number of vacancy-posting by firms. Consequently, the labor market tightness goes down and the flow value of the outside option for unemployed workers starts to decline over time. These two effects together cause a decrease in wages. The decline is not permanent and over time the average wage starts to increase. This is because that after some decline in wages, the cost of operation shrinks, and firms increase their vacancy posting, which drives up the flow value of unemployment. The increase in wages leads to a gradual decrease in labor market tightness over the transition. With the introduction of unemployment insurance, the labor market tightness goes down drastically, but the decline can not maintain its pace and stops at some point as firms start to intensify their vacancy-posting. However, since the entry-level is not high enough or because of the firm's exit, the number of active firms falls by nearly five percent initially increases a bit during the transition, and the number of active firms declines by 4 percent in the long run. Over the transition, wages and the flow value of unemployment display a slight increase, drastic decrease, and extreme rise, respectively. In the long run, rV^u and wages increase respectively by 0.65 percent and 0.27 percent. The adjustment in wages, the flow value of unemployment, θ, and social welfare through the process indicates their persistence. After one quarter, the labor market tightness and the unemployment complete nearly more than 80 percent and 50 percent of their total adjustment, respectively.

I also conduct the propagation analysis by keeping the number of active firms the same before introducing shock. I observe that the reduction in labor market tightness is lower compared to the analysis with the loss of active firms and notice that through the transition, the endogenous variables (wage, rV^u, V, Welfare) do not display a drastic adjustment (dramatic decline and rise) as the previous case. The variables steadily decline to their long-run level following the initial increase. This analysis confirms that a reduction in the number of active firms leads to an additional decline in the labor market tightness and intensification of the decline following the initial increase.

6. CONCLUSION

This paper assesses the influence of UI on the unemployment rate, wages, welfare, firms' growth, and profitability and investigates the propagation of introducing UI in a search model with multi-worker firms.

My quantitative application yields the following conclusions: The flow values of unemployment and employment climb, peak, and then begin to decline as the proportional tax rate on earnings rises. The decrease in the flow values takes place when the proportionate tax rate rises, putting downward pressure on labor market tightness. For lower proportional tax rates, the insurance payment impact outweighs the labor market tightness effect, causing flow values to rise; but, as the tax rate rises, the latter becomes more dominant, leading flow values to fall. We observe this behavior in social welfare: social welfare achieves its peak value at the 1.7 percent tax rate and thereafter begins to drop. Furthermore, higher tax rates result in higher wage levels because insurance payments raise the flow value of unemployment, which is reflected in wages via Nash Bargaining. Because wage increases drive firms to post fewer vacancies and hence raise the unemployment rate, higher proportionate tax rate, higher unemployment rate. As for firm-related implications of UI, Increasing salaries lead to a decrease in flow profit and profit per worker. The firms' growth rate also slows when wages rise, causing a decrease in the number of vacancies posted by firms. Lastly, the impulse response analysis suggests that the behaviors of wage, the flow value of unemployment and employment, the labor market tightness, and welfare over the transition demonstrate their persistence.

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APPENDIX A

In this appendix, I report the proofs of Lemmas, Proposition and Theorems in my paper.

Proof 1 (**Proof of Lemma 1**). *Rearranging The HJB Equation for the employed worker(7), gives the following relation:*

$$\begin{aligned} rV(n,t;z) - V_t(n,t;z) &= w(n,t,z)(1-\tau(t)) + (s+\delta)\left[V^u(t) - V(n,t;z)\right] \\ &\quad + [q(\theta(t))v(n,t;z) - sn]V_n(n,t,z) \end{aligned} \tag{a.1}$$

$$\begin{aligned} [r+\delta+s][V(n,t;z) - V^u(t)] - V_t(n,t;z) &= w(n,t;z)(1-\tau(t)) - rV^u(t) \\ &\quad + [q(\theta(t))v(n,t;z) - sn]V_n(n,t;z) \end{aligned} \tag{a.2}$$

The bargaining equation (2) and its derivatives with respect to n and t provide those:

$$J_n(n,t;z) = \frac{(1-\phi)}{\phi}[V(n,t;z) - V^u(t)] \tag{a.3}$$

$$J_{nn}(n,t;z) = \frac{(1-\phi)}{\phi}[V_n(n,t;z)] \tag{a.4}$$

$$J_{nt}(n,t;z) = \frac{(1-\phi)}{\phi}[V_t(n,t;z) - V_t^u(t)] \tag{a.5}$$

The derivative of HJB equation (4) for firms with respect to n establishes that:

(a.6)

$$\begin{aligned} (r+\delta)J_n(n,t;z) - J_{nt}(n,t,z) &= y_n(n,z) - w(n,t;z) - nw_n(n,t,z) - snJ_{nn}(n,t,z) - sJ_n(n,t,z) \\ &\quad + \max_{v\geq 0}\left\{-c^{'}(v)v_n + q(\theta(t))v_nJ_n(n,t;z) + q(\theta(t))vJ_{nn}(n,t,z)\right\} \end{aligned}$$

Firms take optimal vacancy strategy as given $v(n,z)$ in (5) which implies $c^{'}(v) = q(\theta(t))J_n(n,t;z)$ (6) and using the relations in (a.6) establishes the following:

(a.7)

$$[r+\delta+s]J_n(n,t,z)-J_{nt}(n,t,z)-[q(\theta(t))v-sn]J_{nn}(n,t,z)=y_n(n,z)-w(n,t;z)-nw_n(n,t,z)$$

Plugging (a.3),(a.4) and (a.5) into (a.7), generates that:

(a.8)

$$[r+\delta+s]\frac{(1-\phi)}{\phi}[V(n,t;z)-V^u(t)]-\frac{(1-\phi)}{\phi}[V_t(n,t;z)-V_t^u(t)]-[q(\theta(t))v-sn]\frac{(1-\phi)}{\phi}[V_n(n,t;z)]$$
$$=y_n(n,z)-w(n,t;z)-nw_n(n,t,z)$$

Using (a.2) to replace some terms involving the worker's value in (a.8) yields the following relationship:

$$(1-\phi)[(1-\tau(t))w(n,t,z)-(rV^u(t)-V_t^u)]=\phi[y_n(n,z)-w(n,t;z)-nw_n(n,t,z)] \quad \text{(a.9)}$$

After some algebra, I get the relationship below:

$$[1-\tau(t)+\phi\tau(t)]w(n,t,z)+\phi w_n(n,t,z)n=\phi y_n(n,t,z)+(1-\phi)(rV^u(t)-V_t^u) \quad \text{(a.10)}$$

I need to solve for the partial differential equation to construct the wage equation. I utilize the method of integrating factor.

$$\frac{[1-\tau(t)+\phi\tau(t)]}{\phi n}w(n,t,z)+w_n(n,t,z)=\frac{1}{n}y_n(n,t,z)+\frac{(1-\phi)}{\phi n}(rV^u(t)-V_t^u) \quad \text{(a.11)}$$

The integrating factor: $M=e^{\int\frac{[1-\tau(t)+\phi\tau(t)]}{\phi n}dn}$. *This corresponds to* $M=n^{\frac{[1-\tau(t)+\phi\tau(t)]}{\phi}}$. *Multiplying both sides of the equation (a.11) with M and integrating with respect to n present that:*

$$\begin{aligned} w(n,t,z) &= n^{-\frac{[1-\tau(t)+\phi\tau(t)]}{\phi}}\left(c+\int_0^n v^{\frac{(1-\tau(t))(1-\phi)}{\phi}}y_n(v,z)\,dn\right) \\ &+\frac{(1-\phi)}{1-\tau(t)+\phi\tau(t)}(rV^u(t)-V_t^u) \end{aligned} \quad \text{(a.12)}$$

The assumption 1 implies that $nw(n,t,z)$ is finite as n approaches to zero which corresponds to $c=0$. Writing $n^{-\frac{[1-\tau(t)+\phi\tau(t)]}{\phi}}=\frac{\phi}{1-\tau(t)+\phi\tau(t)}(\int_0^n v^{\frac{(1-\tau(t))(1-\phi)}{\phi}}dv)^{-1}$ in (a.12), gives the statement in the Lemma 1.

$$\begin{aligned} w(n,t,z) &= \frac{(1-\phi)}{(1-\tau(t)+\phi\tau(t))}[rV^u(t)-V_t^u(t)] \\ &+\frac{\phi}{(1-\tau(t)+\phi\tau(t))}\frac{\int_0^n v^{\frac{(1-\tau(t))(1-\phi)}{\phi}}y^{'}(v,z)\,dv}{\int_0^n v^{\frac{(1-\tau(t))(1-\phi)}{\phi}}dv} \end{aligned} \quad \text{(a.13)}$$

Proof 2 (Proof of Lemma 2).

$$\psi(n,z)=\frac{\int_0^n v^{\frac{(1-\tau(t))(1-\phi)}{\phi}}y^{'}(v,z)\,dv}{\int_0^n v^{\frac{(1-\tau(t))(1-\phi)}{\phi}}\,dv}$$

(b.1) $$\psi(n,z)=\left[n^{-\left(\frac{1-\tau(t)+\phi\tau(t)}{\phi}\right)}\right]\left(\frac{1-\tau(t)+\phi\tau(t)}{\phi}\right)\int_0^n v^{\frac{(1-\tau(t))(1-\phi)}{\phi}}y^{'}(v,z)\,dv$$

Since $\left(\frac{\phi}{1-\tau(t)+\phi\tau(t)}\right)\psi(n,z)=w(n,z)-\left(\frac{1-\phi}{1-\tau(t)+\phi\tau(t)}\right)[rV^u(t)-V_t^u(t)]$, *in order to construct the statements about the wages, it is enough to prove that* $\psi(n,z)>0$, $\psi^{'}(n,z)<0$, $\lim_{n\to\infty}\psi^{'}(n,z)=\lim_{n\to\infty},y^{'}(n,z)$ *and* $\lim_{n\to 0^+}\psi^{'}(n,z)=\lim_{n\to 0^+}y^{'}(n,z)$. *The first one holds obviously. The second property satisfies as follows:*

(b.2) $$\frac{\phi}{1-\tau(t)+\phi\tau(t)}\psi^{'}(n,z)=-\left[n^{-\left(\frac{1-\tau(t)+\phi+\phi\tau(t)}{\phi}\right)}\right]\left(\frac{1-\tau(t)+\phi\tau(t)}{\phi}\right)\times \int_0^n v^{\frac{(1-\tau(t))(1-\phi)}{\phi}}y^{'}(v,z)\,dv+n^{-1}y^{'}(n)$$

(b.3) $$\frac{\phi}{1-\tau(t)+\phi\tau(t)}\psi^{'}(n,z)=\left[n^{-\left(\frac{1-\tau(t)+\phi+\phi\tau(t)}{\phi}\right)}\right]\left(\frac{1-\tau(t)+\phi\tau(t)}{\phi}\right)\times \int_0^n v^{\frac{(1-\tau(t))(1-\phi)}{\phi}}\left[y^{'}(n,z)-y^{'}(v,z)\right]dv$$

Since $n\geq v$ *and* $y(.,z)$ *is strictly concave,* $\psi^{'}(n,z)$ *is strictly negative . The proof of third property is as follows:*

(b.4) $$y^{'}(n,z)<\psi(n,z)$$

(b.5) $$\psi(n,z)=n^{-1}y(n,z)-\left[n^{-\left(\frac{1-\tau(t)+\phi\tau(t)}{\phi}\right)}\right]\left(\frac{(1-\tau(t))(1-\phi)}{\phi}\right)\times \int_0^n v^{\frac{(1-\tau(t))(1-\phi)}{\phi}-1}y(v,z)\,dv$$

$$y^{'}(n,z)<\psi(n,z)<n^{-1}y(n,z)$$

the first inequality holds since $y^{'}(.,z)$ *is a decreasing function and* $\psi(n;z)$ *is a weighted average of values of* $y^{'}(v;z)$ *on the* $v\in[0,n]$, *while the second one holds due to the fact that* $y(.;z)$ *is strictly positive. Because* $\lim_{n\to\infty}n^{-1}y(n;z)=\lim_{n\to\infty}y^{'}(n,z)$, *we obtain* $\lim_{n\to\infty}\psi^{'}(n,z)=\lim_{n\to\infty},y^{'}(n,z)$ *by the sandwich theorem. Finally, the last propert holds because* $\psi(n,z)$ *is a weighted average of the values of* $y^{'}(v,z)$ *on the interval* $v\in(0,\ n)$.

To show the property of profit function, we utilize the characterization of wage equation, so profit function takes the following form:

$$\pi(n,t,z) = y(n,z) - \left(n^{-}\left[\frac{(1-\tau(t))(1-\phi)}{\phi}\right]\right)\int_0^n v^{\frac{(1-\tau(t))(1-\phi)}{\phi}} y'(v,z)\,dv - \frac{(1-\phi)n}{(1-\tau(t)+\phi\tau(t))}[rV^u(t) - V_t^u(t)] \tag{b.6}$$

The first order condition of the profit function suggests that:

$$\pi_n(n,t,z) = \frac{(1-\phi)}{(1-\tau(t)+\phi\tau(t))}\left[(1-\tau(t))\psi(n,z) - (rV^u(t) - V_t^u(t))\right] \tag{b.7}$$

since $\psi(.,t,z)$ is strictly decreasing, $\pi(.,t,z)$ is strictly concave. Thus we can find a unique point maximizing $\pi(.,t,z)$ on the interval $[0,\infty]$. Since $\lim_{n\to 0^+}\psi(n;t,z) = \lim_{n\to 0^+} y'(n,z)$, the maximizing point is strictly positive if $\lim_{n\to 0^+} y'(n,z) > rV^u(t) - V_t^u(t)$. To demonstrate that $\pi(n,t,z) \to 0$ as $n \to 0^+$, notice that $y(0,z) = 0$ by assumption, it is simple to prove $\pi(n,t,z) \to 0$ as $n \to 0^+$, through the wage equation (18) and Assumption 1, that $nw(n,z) \to 0$ as $n \to 0$. To prove that $\pi(n,t,z) \to -\infty$ as $n \to \infty$, notice that under the condition in the lemma's argument,

$$\frac{(1-\tau(t)+\phi\tau(t))}{(1-\phi)}\lim_{n\to\infty}\pi_n(n,t,z) = -[rV^u(t) - V_t^u(t)] + (1-\tau(t))\lim_{n\to\infty}\psi(n,z) \tag{b.8}$$

$$\frac{(1-\tau(t)+\phi\tau(t))}{(1-\phi)}\lim_{n\to\infty}\pi_n(n,t,z) = -[rV^u(t) - V_t^u(t)] + (1-\tau(t))\lim_{n\to\infty} y'(n,z) \tag{b.9}$$

thus n for sufficiently large n, the range of $\pi_n(n,t,z)$ is uniformly bounded away from zero.

Proof 3 (Proof of Theorem 1)**.** *Let $\chi,\omega : \mathbb{R}_+ \times \mathbb{R} \to \mathbb{R}$ be two functions. We identify $\chi(q, rV^u)$ as the term $J(0) - k$. $J(.)$ is the unique solution to HJB equation for the value of firm in the steady state where the wage is specified by (18), where the solutions n^* to (23) and $J(n^*)$ to (22) define the boundary condition for the initial value problem , $(n^*, J(n^*))$. Let ω be the following*

$$\omega(q, rV^u) = rV^u - \left[b + p + \frac{\phi}{(1-\phi)}\theta q \frac{\int_0^{n^*} v(n)J'(n)g(n)dn}{\int_0^{n^*} v(n)g(n)dn}\right]$$

where $g(\cdot)$ is defined by (20) and $\chi(q, rV^u)$ corresponds to the net surplus obtained by an entering firm that assumes the values of q and rV^u as given by the equilibrium, gives the wages $w(\cdot)$ specified by (18), and sets its optimal vacancy posting strategy according to (5); $rV^u - \omega(q, rV^u)$ denotes the value of unemployment in an economy involving the firms.

(q, rV^u) is part of an equilibrium allocation if and only if the following conditions hold

$$\chi(q, rV^u) = \omega(q, rV^u) = 0.$$

We will show that the condition holds. Let H be the set of tuples (q, rV^u) such that: $k = \chi(q, rV^u)$. We will prove that H is a connected subset of $\mathbb{R}^+ \times \mathbb{R}$. By the intermediate value theorem, we conclude that confining ω to the set H establishes a continuous function with both negative and positive values. The theorem of maximum implies that $\chi(\cdot)$ is a continuous function, it is sufficient to prove that net surplus is non-decreasing in its first term while it is non-increasing in its second, in order to demonstrate that the set of values satisfying $\chi(q, rV^u) = 0$ is a continuous 1-manifold. (both properties are strict provided that hiring is the optimal strategy for a firm with no workers.). To figure out the reason, notice that $\chi(q, rV^u)$ is the maximized term of the problem. a decrease in rV^u leads to an increase in $\chi(q, rV^u)$ for any q. This is because provided that the firm pursues the same strategy of hiring, then it would cause an increase in the firm value as $w(n)$ declines for each n. The increase in the value is strict if the number of the firm hiring is positive, which is assured by Lemma 2 provided that $y'(0) > rV^u$.

Next, if q rises to $q' > q$, the firm will update its vacancy-posting strategy $v(\cdot)$ with $qv(\cdot)/q'$ which increases the firm value. This would result in the same size dynamics as before but at a lower cost (The relationship is strict if the number of vacancies posted by the firm is positive).

Further, let $\bar{v}$ be the solution to the following equation

$$\text{(c.1)}\;\; k = \frac{1}{r+\delta} \max_{n>0} \left\{ y(n) - n^{-(1-\phi)(1-\tau)/\phi} \int_0^n \nu^{(1-\phi)(1-\tau)/\phi} y'(\nu) d\nu - n(1-\phi) \frac{\bar{v}}{1-\tau+\theta\tau} \right\}$$

The wage paid by a firm takes the following form:

$$w(n) = (1-\phi)\frac{\bar{v}}{1-\tau+\theta\tau} + n^{-(1-\phi)(1-\tau)/\phi} \int_0^n \nu^{(1-\phi)(1-\tau)/\phi} y'(\nu) d\nu$$

such a firm will reach a break-even point if and only if it is able to reach the optimal employment n^* which maximizes the right-hand side of (c.1) at the time of the entry and at no cost. If the following statement, $\bar{v} - b - p \leq 0$ holds, then there exists an equilibrium in which there is no firm. if $\bar{v} > b + p$, we prove that there are points $(q_1, v_1), (q_2, v_2) \in H$ such that $\omega(q_1, v_1)$ and $\omega(q_2, v_2)$ have opposite signs.

To determine a point at which $\omega(\cdot)$ becomes positive, notice the followings $\lim_{q\to\infty} \chi(q, \bar{v}) = 0$ and for $v > \bar{v}$, $\chi(q, v) < k$. If $q \to \infty$, then then any firm will instantly reach optimal employment level by hiring n^*; thus in the limit, for all

$n \in [0, n^*]$, we observe $J(n) = J(n^*)$. Consequently, the definition of $\omega(\cdot)$ implies that for q sufficiently large, that is, for $\theta q(\theta)$ sufficiently small, $\omega(q, \bar{v}) = \bar{v} - b - p > 0$.

To determine a point at which $\omega(\cdot)$ becomes negative, let $\hat{q} > 0$ such that $\chi(\hat{q}, b, p) = 0$. There exists such a $\hat{q}$ since $\bar{v} > b + p$.

$$\omega(\hat{q}, b, p) = -\frac{\phi}{(1-\phi)}\theta q \frac{\int_0^{n^*} v(n) J'(n) g(n) dn}{\int_0^{n^*} v(n) g(n) dn}$$

because $J'(n)$ is strictly positive for any $n < n^*$ and $\theta q > 0$, $\omega(\cdot)$ takes a negative value. Hence, if $\bar{v} - b - p > 0$, then H involves points at which ω have opposite signs. By the intermediate value theorem, we conclude the proof of the existence of an equilibrium with unemployment insurance.

Proof 4 (Proof of Lemma 3)**.** *Assuming the quadratic form of the production function, the equation for wages is given by (18), and substituting the wage equation into the profit function gives the equation of firm profit.*

Further, substituting for wages into the firm's HJB equation (4) under steady-state establishes the following

$$(r+\delta)J(n;z) = \pi(n;z) - snJ_n(n;z) + \frac{1}{2\gamma}q^2 J_n(n;z) \tag{d.1}$$

We can obtain a closed-form solution through the guess and verify method. Suppose that there is a quadratic solution for $J(\cdot)$ which takes the following form:

$$J(n;z) = A(z) + B(z)n - \frac{1}{2}Cn^2$$

When we solve for the unknown coefficients, we get the followings.

$$\begin{aligned} C &= \frac{\gamma}{2q^2}\left(-(r+\delta+2s) + \sqrt{(r+\delta+2s)^2 + \frac{4q^2(1-\phi)(1-\tau)}{\gamma(1-\tau+\phi\tau+\phi)}\sigma}\right) \\ B(z) &= \frac{2(1-\phi)\left(z(1-\tau) - rV^u\right)}{r+\delta+\sqrt{(r+\delta+2s)^2 + \frac{4q^2(1-\phi)(1-\tau)}{\gamma(1-\tau+\phi\tau+\phi)}\sigma}} \\ A(z) &= \frac{q^2 B(z)^2}{2\gamma(r+\delta)} \end{aligned} \tag{d.2}$$

The coefficient values present us a unique concave solution.

Next, notice that optimal vacancy posting (6) implies $c'(v(n;z)) = qJ_n(n;z)$. Assuming the vacancy posting function takes the quadratic form, substituting for $J_n(n;z)$

into (6) establishes that

$$v(n;z) = \frac{q}{\gamma}(B(z) - Cn) \tag{d.3}$$

The relationship, $qv(n^*(z)) = sn^*(z)$ *identifies the optimal employment level* $n^*(z)$*. Substituting the optimal employment level* $n^*(z)$ *into the vacancy equation (d.3) provides the statement regarding vacancies in Lemma 3.*

Next, we need to specify the firm size distribution, substituting the vacancy function into (19) and integrating delivers the density of firms whose productivity is z and employment level is n:

$$\frac{g(n)}{x} = \delta\left(\frac{\gamma}{q^2B(z)}\right)^{\delta/\left(s+q^2C/\gamma\right)}\left(\frac{q^2B(z)}{\gamma} - \left(s + \frac{q^2C}{\gamma}\right)n\right)^{-1+\delta/\left(s+q^2C/\gamma\right)} \tag{d.4}$$

and a second integration gives the statement regarding the distribution, $\bar{G}(n;z)$*.*

Proof 5 (Proof of Proposition 2)**.** *Utilizing the equations of* $n^*(z)$ *and Lemma 3, delivers the percentage of firms of pr productivity z whose size is greater than n:*

$$\bar{G}(n;z) = \begin{cases} \left(1 - \frac{c_1 n}{z(1-\tau) - rV^u}\right)^{c_2} & n < n^*(z) \\ 0 & n \geq n^*(z) \end{cases} \tag{e.1}$$

the constants j_1 *and* j_2 *are independent of z and n For any* $n > 0$*, the equation* $n^*(z) = n$ *for z , that is,* $z = \frac{rV^u + j_1 n}{1-\tau}$ *delivers the maximum z satisfying* $\bar{G}(n;z) = 0$*. Thus the following equation gives the percentage of firm whose employment level is greater than n:*

$$\bar{G}(n) = \int_{\frac{rV^u + j_1 n}{1-\tau}}^{\infty} \bar{G}(n;z) f(z) dz \tag{e.2}$$

Let $\hat{z}$ *be the change variable such that* $\hat{z} = z - \frac{rV^u}{1-\tau}$*, thus*

$$\bar{G}(n) = \int_{\frac{j_1 n}{1-\tau}}^{\infty} \left(1 - \frac{j_1 n}{\hat{z}}\right)^{j_2} \frac{k z_m^k}{\left(\hat{z} + \frac{rV^u}{1-\tau}\right)^{k+1}} d\hat{z} \tag{e.3}$$

Next, notice that provided that $\lambda \geq 1$ *is a constant, we get the following equation,*

$$\bar{G}(\lambda n) = \int_{\frac{j_1 n}{1-\tau}}^{\infty} \left(1 - \frac{j_1 \lambda n}{\hat{z}}\right)^{j_2} \frac{k z_m^k}{\left(\hat{z} + \frac{rV^u}{1-\tau}\right)^{k+1}} d\hat{z} \tag{e.4.1}$$

$$\bar{G}(\lambda n)=\int_{\frac{j_1 n}{1-\tau}}^{\infty}\left(1-\frac{j_1\lambda n}{\lambda\tilde{z}}\right)^{j_2}\frac{kz_m^k}{\left(\lambda\tilde{z}+\frac{rV^u}{1-\tau}\right)^{k+1}}\lambda d\tilde{z} \tag{e.4.2}$$

$$\bar{G}(\lambda n)=\lambda^{-k}\int_{\frac{j_1 n}{1-\tau}}^{\infty}\left(1-\frac{j_1 n}{\tilde{z}}\right)^{j_2}\frac{kz_m^k}{\left(\tilde{z}+\frac{rV^u}{1-\tau}\right)^{k+1}}\left(\frac{1+rV^u/\tilde{z}(1-\tau)}{1+rV^u/(\lambda\tilde{z}(1-\tau))}\right)^{k+1}d\tilde{z} \tag{e.4.3}$$

in (e.4.2), I use $\hat{z}$ in the place of $\lambda\tilde{z}$ Since in the steady state rV^u is a constant, as n grows, the last term in inside the integral in (e.4.3) converges uniformly to 1. Thus for sufficiently large n, $\bar{G}(\lambda n)/\left(\lambda^{-k}\bar{G}(n)\right)$ becomes close to 1 as n grows.

Proof 6. *Proof of Proposition 3 In order to prove that Gibrat's law holds, it is sufficient to demonstrate that the product of a function of n and a function of ζ can express the density of firms with employment level n and firm growth $\zeta=\frac{\dot{n}}{n}$.*

Now, note that (d.3) allows us to establish a firm's growth rate in the following form

$$\zeta=\frac{\dot{n}}{n}=\frac{1}{n}(qv(n)-sn)=\frac{1}{n}\left(\frac{q^2}{\gamma}(B(z)-Cn)-sn\right)=\frac{q^2}{\gamma}\frac{B(z)}{n}-\left(s+\frac{q^2C}{\gamma}\right),$$

where

$$B(z)=\left(\zeta+s+\frac{q^2C}{\gamma}\right)\frac{\gamma}{q^2}n \tag{e.5}$$

Substituting for (e.6) into (d.4) presents the following:

$$\frac{g(n)}{x}=\delta n^{-1}\zeta^{-1+\delta/\left(s+q^2C/\gamma\right)}\left(\zeta+s+\frac{q^2C}{\gamma}\right)^{-\delta/\left(s+q^2C/\gamma\right)} \tag{e.6}$$

Next, the combination of (d.2) and (e.5) gives the following value z consistent with employment level and ζ:

$$z=\frac{rV^u}{1-\tau}+j_1\left(\zeta+s+\frac{q^2C}{\gamma}\right)n \tag{e.7}$$

such that

$$j_1=\frac{r+\delta+\sqrt{(r+\delta+2s)^2+\frac{4q^2(1-\phi)(1-\tau)}{\gamma(1-\tau+\phi\tau+\phi)}\sigma}}{2(1-\phi)(1-\tau)}\frac{\gamma}{q^2}$$

is a constant. If we keep n constant, we have that the value of productivity z con-

sistent with ζ such that

$$\frac{\partial z}{\partial \zeta} = j_1 \cdot n \tag{e.8}$$

Next, Assuming that productivity distribution is Pareto, the density of z takes the following form

$$\begin{aligned} f(z) &= \frac{\kappa z_m^\kappa}{(rV^u/\ (1-\tau)+j_1(\zeta+s+q^2C/\gamma)n)^{\kappa+1}} \\ &= \frac{\kappa z_m^\kappa}{c_1^{\kappa+1}}\left(\zeta+s+\frac{q^2C}{\gamma}\right)^{-(\kappa+1)} n^{-(\kappa+1)}\left(1+\frac{rV^u}{j_1(1-\tau)(\zeta+s+q^2C/\gamma)n}\right)^{-(\kappa+1)} \end{aligned} \tag{e.9}$$

The Combination of (e.6), (e.8), and (e.9) gives the density of firms with employment level n and ζ:

$$\frac{g(n)}{x}\cdot f(z)\cdot\left(\frac{\partial z}{\partial \zeta}\right)^{-1} = \frac{\delta\kappa z_m^\kappa}{c_1^{\kappa+2}} n^{-(\kappa+3)}\zeta^{-1+\delta/\left(s+q^2C/\gamma\right)}\left(\zeta+s+\frac{q^2C}{\gamma}\right)^{-(\kappa+1)-\delta/\left(s+q^2C/\gamma\right)}\times$$

$$\left(1+\frac{rV^u}{c_1(1-\tau)(\zeta+s+q^2C/\gamma)n}\right)^{-(\kappa+1)}$$

Notice that the right-hand side is expressed as the product of a function of employment level n, a function of growth rate ζ, and a statement that converges to 1 uniformly in ζ if $n \to \infty$. Thus, Gibrat's law holds asymptotically.

Proof 7. *Proof of Proposition 4*

Since the productivity level is not associated with a firm's destruction rate δ and firms monotonically increase their employment level to their target employment level, a firm's rank in the firm size distribution is directly related to such firm's age, that is, higher a firm's size older firm's age. Thus, the statement regarding the distribution of firm size in Lemma 3 implies the term $c(n;z) \equiv n/n^*(z)$ is the same for any two firms that are the same age. Keep in mind that $c(n;z) \in [0,1)$, higher the firm's age, higher values of $c(n,z)$ firms have.

Suppose that two firms are the same age and, hence, have the same $c(n,z)$. Let $\bar{c}$ common value of $c(n,z)$ for such firms. Hence, utilizing the statement $\bar{c} = n/\ n^*(z)$ deliver the following:

$$z(1-\tau)-rV^u = \left[(1-\tau+\phi\tau(t))\left(\frac{\gamma s(r+\delta+s)}{q^2(1-\phi)}+\frac{(1-\tau)\sigma}{(1-\tau+\phi\tau+\phi)}\right)\right]\frac{1}{\bar{c}}n \tag{f.1}$$

Next, A firm's growth rate is given by

$$\dot{n}(n;z) = qv(n;z) - sn = \frac{q^2}{\gamma}\left[B(z) - \left(\frac{s\gamma}{q^2} + C\right)n\right]$$

where the second term comes from (d.3). A firm's growth rate turns into that:

(f.2) $$\dot{n}(n;z) = \frac{q^2}{\gamma}\left[\frac{s\gamma}{q^2} + C\right]\left[\frac{1}{\bar{c}} - 1\right]n$$

Note that the wage equation comes from the statement regarding the wage paid by firms in Lemma 3.

$$w(n,z) = \frac{(1-\phi)(rV^u)}{(1-\tau+\phi\tau} + \frac{\phi z}{(1-\tau+\phi\tau)} - \frac{\phi\sigma n}{(1-\tau+\phi\tau+\phi)}.$$

After some algebra, we can write wage as

$$w(n,z) = \frac{(1+\phi(\tau/\ 1-\tau))rV^u}{(1-\tau+\phi\tau)} + \frac{\phi[z-(rV^u/\ 1-\tau)]}{(1-\tau+\phi\tau)} - \frac{\phi\sigma n}{(1-\tau+\phi\tau+\phi)}$$

Finally, Substitute for (f.1) into the wage equation generates that:

(f.3)
$$w(n,z) = \frac{(1+\phi(\tau/\ 1-\tau))rV^u}{(1-\tau+\phi\tau)} + \phi n\left[\frac{\sigma}{(1-\tau+\phi\tau+\phi)}\left[\frac{1}{\bar{c}} - 1\right] + \frac{\gamma s(r+\delta+s)}{q^2(1-\phi)(1-\tau)\bar{c}}\right]$$

Next, the statement associated profit per worker in Lemma 3 gives

$$\frac{\pi(n;z)}{n} = \left[\frac{z(1-\tau)-rV^u}{(1-\tau+\phi\tau)/\ (1-\phi)}\right] - n\sigma\left[\frac{(1-\tau)(1-\phi)}{2(1-\tau+\phi\tau+\phi)}\right].$$

After some algebra and replacing $z(1-\tau)-rV^u$ with the corresponding value in (A.10), we obtain that

(f.4) $$\frac{\pi(n;z)}{n} = (1-\phi)\left[\frac{\sigma(1-\tau)}{(1-\tau+\phi\tau+\phi)}\left[\frac{1}{\bar{c}} - \frac{1}{2}\right] + \frac{\gamma s(r+\delta+s)}{q^2(1-\phi)\bar{c}}\right]$$

because $\bar{c} \in (0,1)$, the coefficient of n in the equations $(f.1)$,$(f.2)$ and $(f.3)$ are positive. This completes the proof.

APPENDIX B

In this appendix, I display the figures to which I refer in the quantitative analysis part.

Figure 3 The flow values of unemployment and employment across proportional unemployment insurance tax

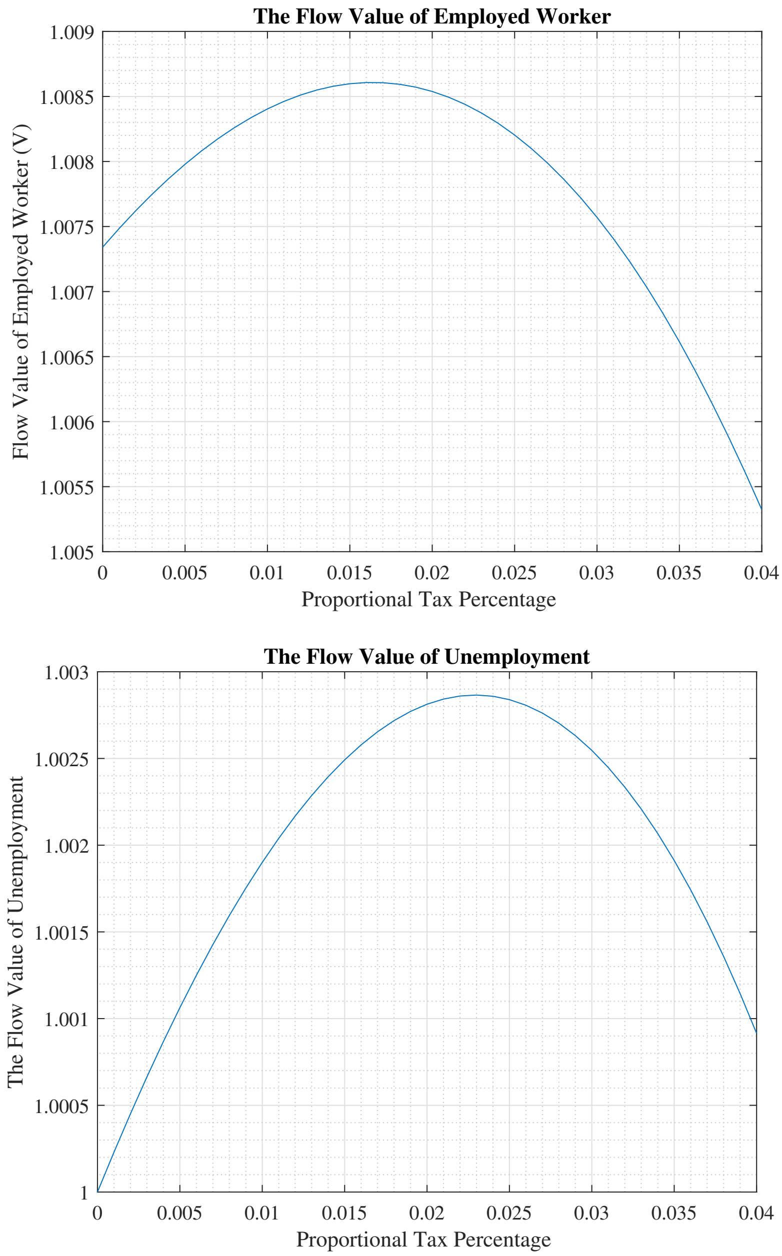


Figure 4 Unemployment rate, average wage level and labor market tightness across proportional unemployment insurance tax

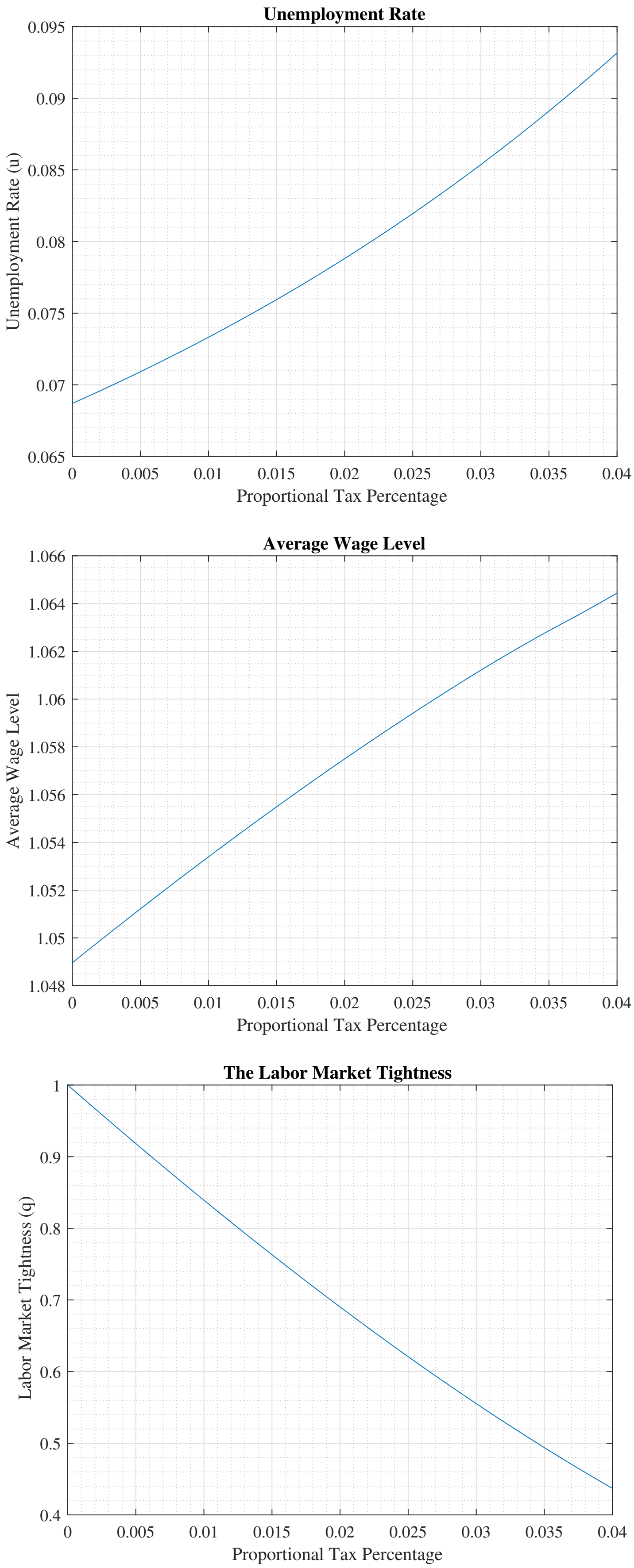


Figure 5 Social welfare across proportional unemployment insurance tax

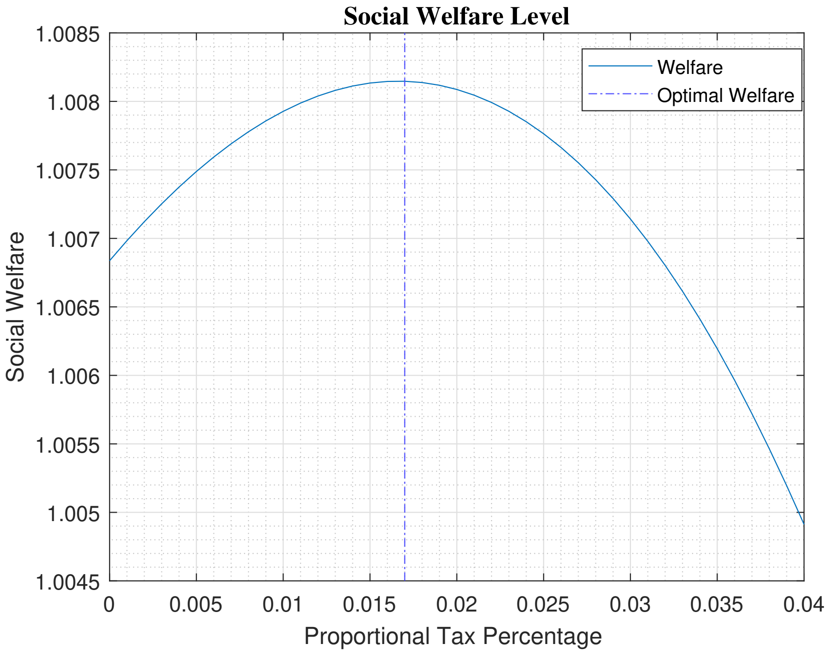
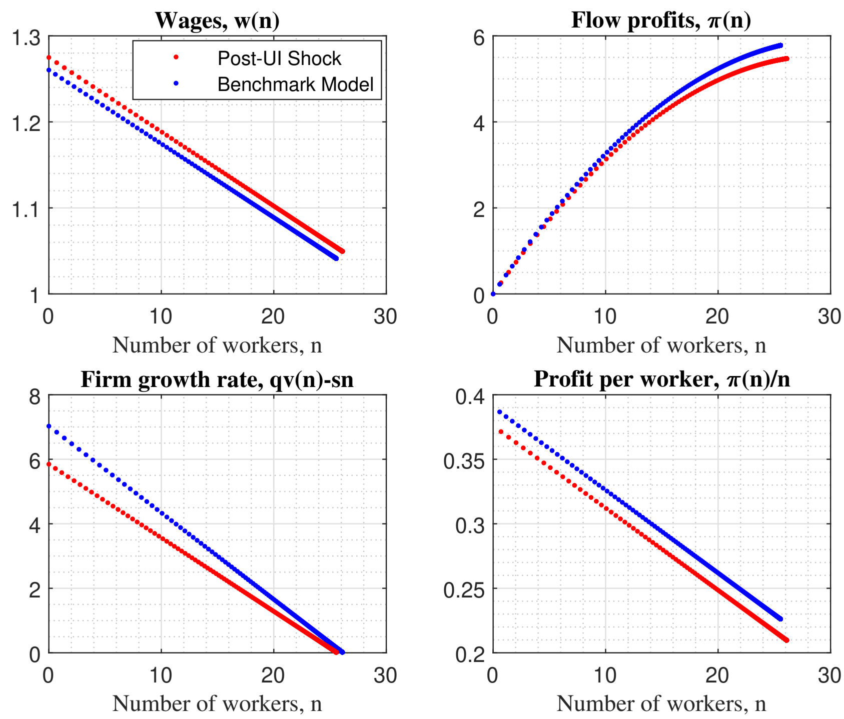
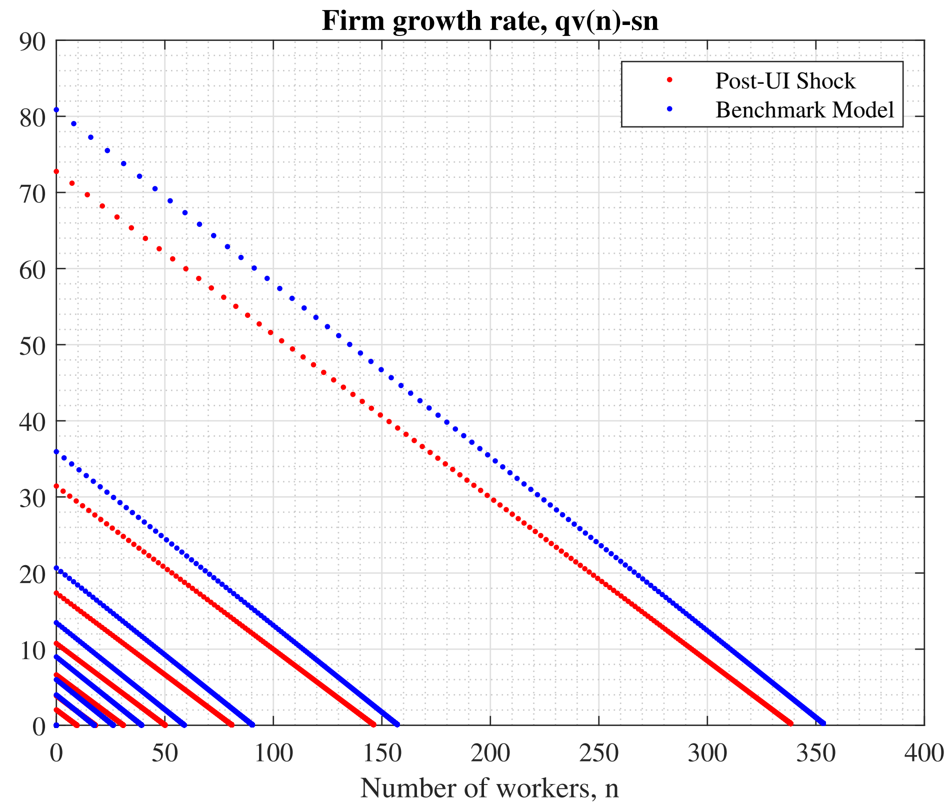


Figure 6 The effect of the unemployment insurance on wages, firms' profit and growth rate under homogeneous productivity



In each figure, x-axis refers to employment level n and y-axis corresponds to variable of interest given an employment level.

Figure 7a The effect of unemployment insurance on wages, growth under heterogeneous productivity



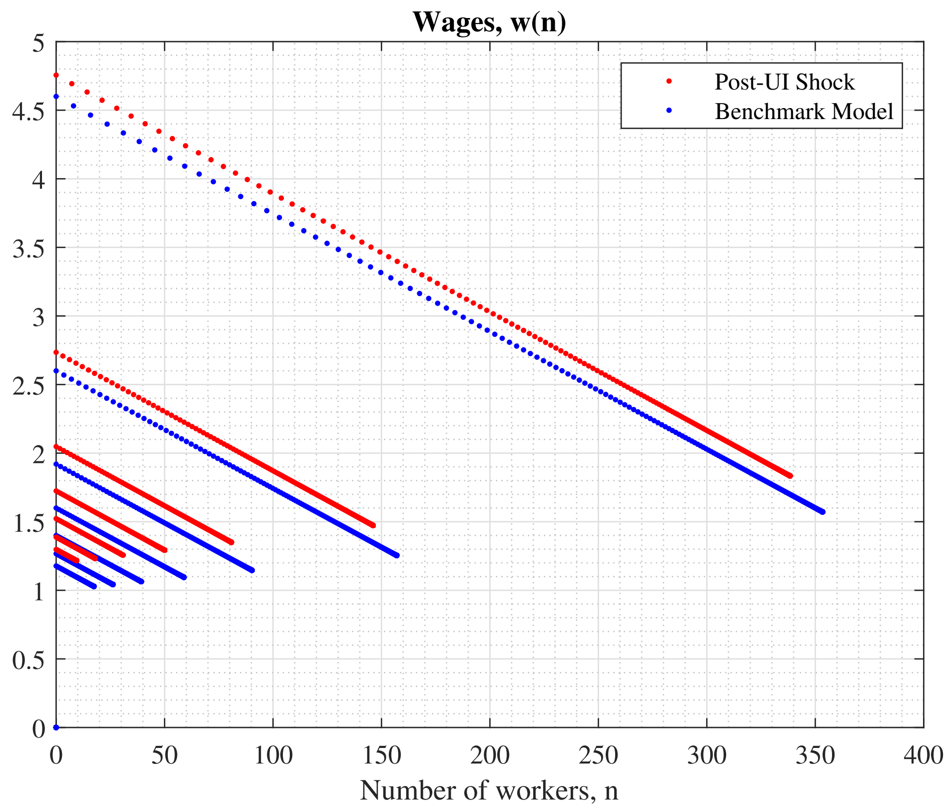
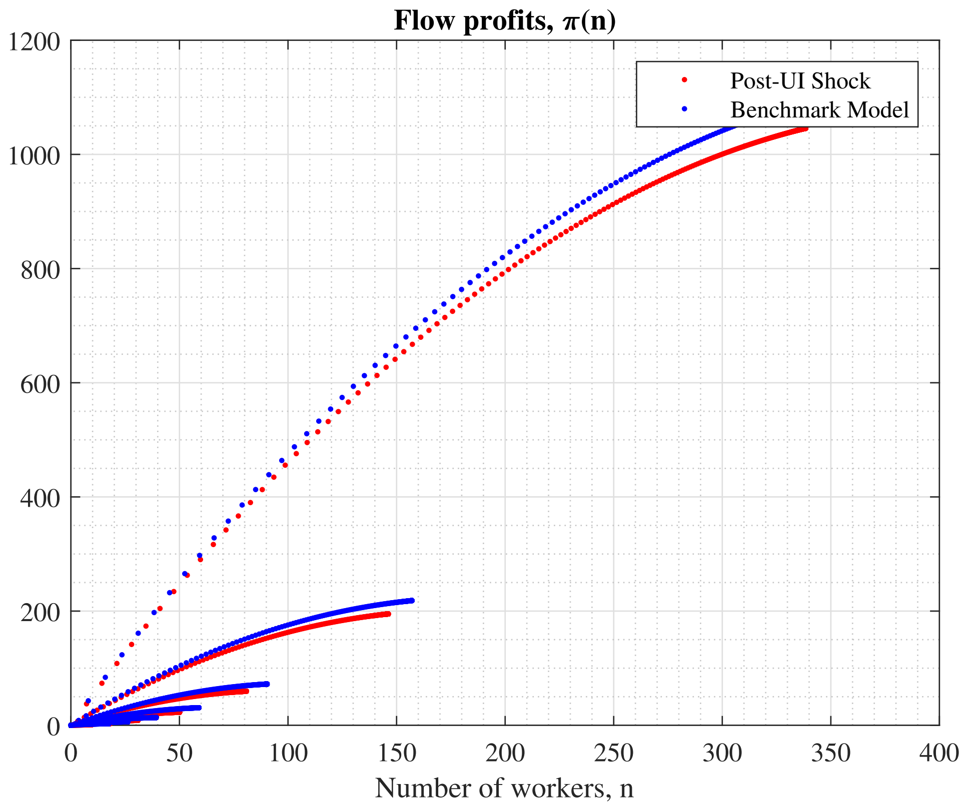


Figure 7b The effect of unemployment insurance on profit and profit per worker under heterogeneous productivity



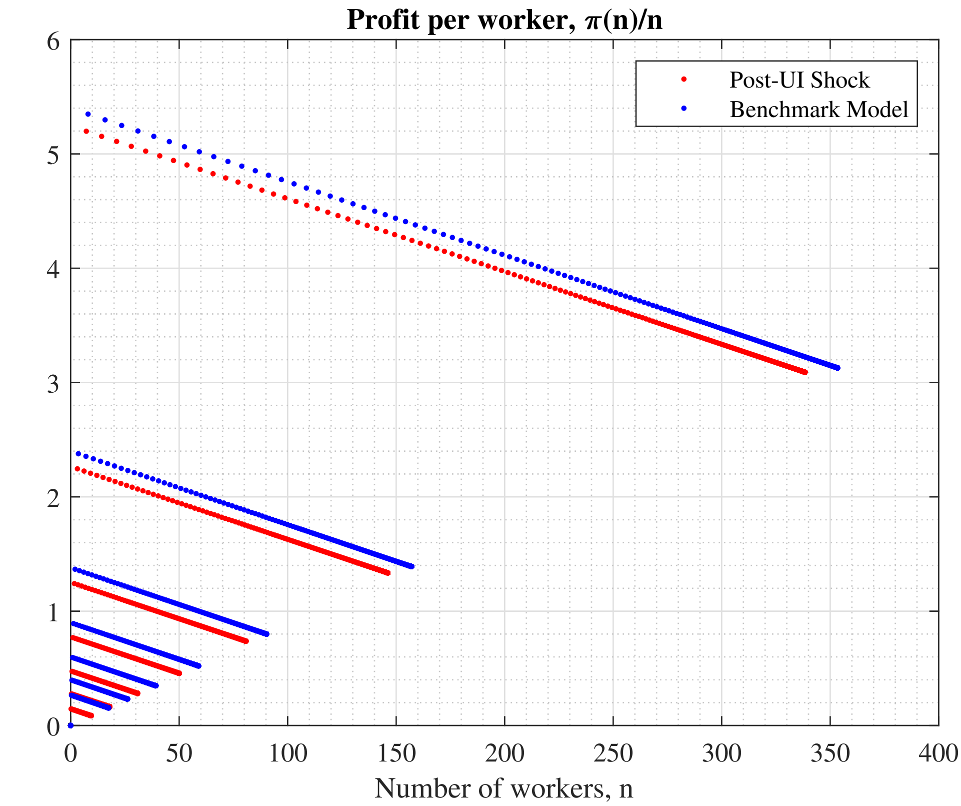
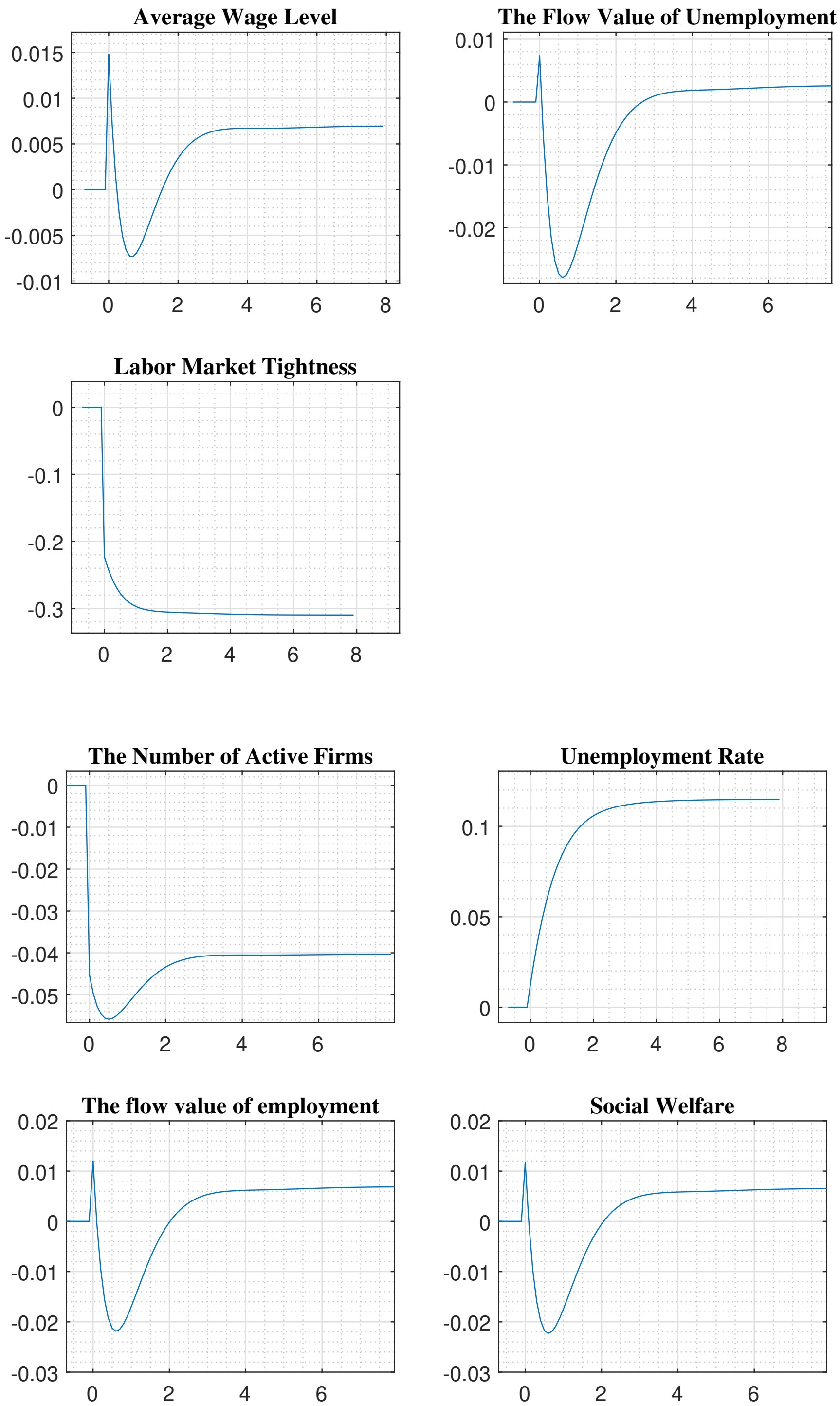
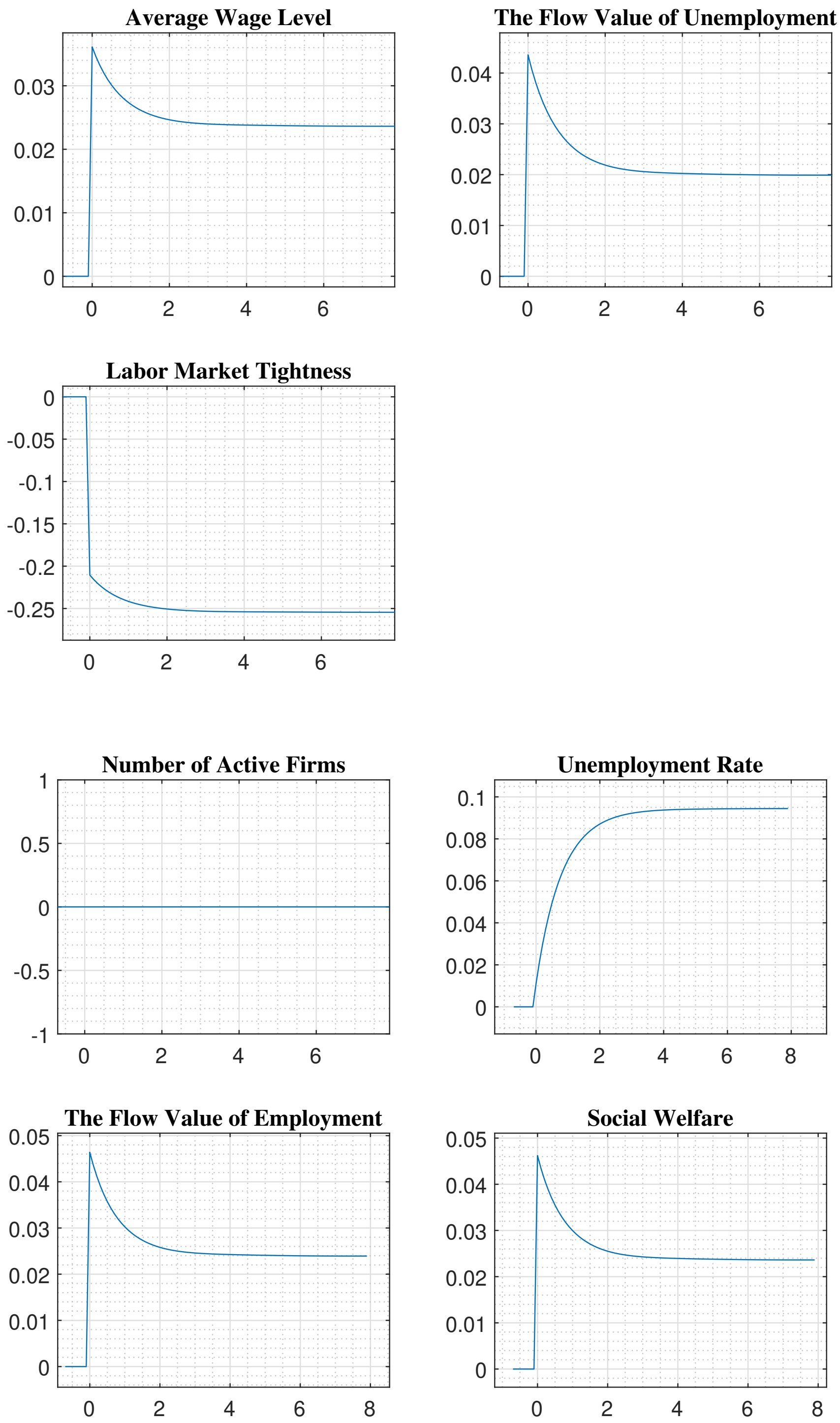


Figure 8 Impulse response to the introduction of unemployment insurance



In each figure, y-axis corresponds to log deviations of variables interest from the initial steady-state and x-axis refers to quarters following the shock.

Figure 9 Impulse response to the introduction of unemployment insurance under fixed number of active firms



In each figure, y-axis corresponds to log deviations of variables interest from the initial steady-state and x-axis refers to quarters following the shock.