2022 IEEE/ASME International Conference on
Advanced Intelligent Mechatronics (AIM)

2022 IEEE/ASME International Conference on Advanced Intelligent Mechatronics (AIM) | 978-1-6654-1308-4/22/$31.00 ©2022 IEEE | DOI: 10.1109/AIM52237.2022.9863265

Whole-Body Pace Gait Control Based on Centroidal Dynamics of a Quadruped Robot

Omer Kemal Adak, Beste Bahceci and Kemalettin Erbatur

Abstract— **This paper studies the full-body motion generation of a quadruped robot for pace gait. A motion planning algorithm is designed based on the centroidal dynamics of the robot. The motion planning algorithm generates both position and force reference trajectories. These reference trajectories serve as a guide for the swing motion of feet during the swing phase, while they also serve as a guide for the ground contact forces during the stance phase. A hybrid force-motion control framework is constructed using the operational space formulation (OSF) in order to track generated reference trajectories. We contribute further to the OSF of floating-base robots by decoupling the dynamics of the right and left leg pairs to facilitate pace gait. The proposed motion generation method for pace gait is validated using a full-dynamics simulation environment. The results reveal the competence of the proposed whole-body pace gait control for a quadruped robot.**

I. INTRODUCTION

The field of legged robots has advanced considerably during the last several decades [1], [2], [3], [4], [5]. Legged robots, particularly quadrupedal ones, have an edge over alternative robotic land platforms in rugged terrain. Quadruped robots are expected to be deployed effectively for tasks such as mine neutralization, disaster zone operations, and space applications in the near future.

In nature, quadrupedal animals adapt their gait based on their locomotion speed and the environment's structure [6]. It benefits them by allowing them to maintain their balance while efficiently utilizing their energy. Hence, it is important to have the ability to perform multiple quadrupedal gaits. Hence, we concentrated on pace gait, which has received less attention in the literature than trot gait. Pace gait is a quadrupedal gait pattern in which the lateral legs move in unison, unlike the trot, where diagonal legs move in unison. Pace gait presents several difficulties that must be overcome before it can be successfully performed. When either the right or left legs take off, a small support polygon appears on the ground, which is significantly away from the ground projection of the robot's center of mass (CoM). This distance generates large amounts of angular acceleration around the robot's CoM, which can have a detrimental effect on the robot's balance. Under these circumstances, developing a well-defined motion planning method and designing an effective control algorithm become critical for producing a pace gait.

Numerous quadrupedal locomotion approaches are described in the literature. One of the most popular is the Zero Moment Point (ZMP) criterion. Several researchers have used the ZMP approach to carry out pace gait on their quadruped robots [7], [8]. The Central Pattern Generator (CPG) is another frequently used approach in the literature. This bio-inspired method can produce stable rhythmic references for robot motion. In this approach, pace gait is achieved by designing mutual entrainment (CPG network) between oscillators of robot joints [9], [10]. In addition to these approaches, researchers create a pace gait using the Spring-Loaded Inverted Pendulum (SLIP) approach [11], [12].

The aforementioned approaches have been pioneers in the study of legged robot locomotion for years. However, as technology advances, more effective methods have begun to supplant the aforementioned. Model-based control approaches have been investigated and revealed to be effective on a variety of quadrupedal robots [13]. Whole-Body Control (WBC) is a model-based approach that specifies appropriate joint torques that minimize tracking errors for a variety of desired accelerations while taking the quadruped robot's entire body dynamics into account [13], [14]. Another effective approach is learning-based control, which develops controllers with data-driven strategy. Multiple recent articles describe techniques for developing a quadrupedal locomotion strategy via various trial-and-error tests in a simulation environment and then implementing the outcomes in the actual world [15], [16]. The application of this method takes quite an amount of time at the moment. However, when model-based control approaches are integrated, it has the potential to be effective.

In this work, we present a motion planning method for a quadrupedal pace gait based on centroidal dynamics. An optimization algorithm is developed to determine the most efficient contact forces. A hybrid force-motion control law is constructed using the OSF to track generated force and position references. Lastly, we demonstrate the competence of the proposed whole-body pace gait control method in a simulation environment.

This paper is structured as follows: Section II describes the proposed motion planning method. The control framework is explained in Section III. Section IV presents simulation results. There is a discussion of the proposed method in Section V. Finally, the paper is concluded in Section VI.

This work was granted by The Scientific and Technological Research Council of Turkey (TÜBİTAK) with Project Number: 114E618.

The authors are with the Faculty of Engineering and Natural Sciences, Sabanci University, Istanbul, Turkey (omerkemal,bestebahceci,erbatur)@sabanciuniv .edu

II. MOTION PLANNING

Motion planning is a critical component of a legged robot's locomotion. Since legged robots have an unactuated body, their actuated joints must work in harmony to produce stable body motion. Centroidal momentum dynamics has recently gained significant attention and success in the motion planning of legged robots [17], [18]. In this section, we describe how we utilize centroidal momentum dynamics to develop stable motion planning algorithm.

A. Centroidal Dynamics

The motion equations of a n-degrees of freedom (DoF) floating-base robot are

$$M(q)\ddot{q}+C(q,\dot{q})+G(q)=S^T\tau+J(q)^T F_c, \tag{1}$$

where $M(q)\in\mathbb{R}^{(n+6)\times(n+6)}$ is the inertia matrix, $C(q,\dot{q})\in\mathbb{R}^{(n+6)}$ is the Coriolis and centrifugal forces, $G(q)\in\mathbb{R}^{(n+6)}$ is the gravitational effect, $F_c\in\mathbb{R}^{12}$ is the vector of contact forces (The quadruped robot's feet were regarded as point feet.), $J(q)\in\mathbb{R}^{(12)\times(n+6)}$ is the contact Jacobian, $S=\begin{bmatrix}0_{n\times 6} & I_{n\times n}\end{bmatrix}$ is the selection matrix of the actuated joints, $\tau\in\mathbb{R}^n$ is the vector of joint torques. The generalized coordinates of the robot are denoted by $q=\begin{bmatrix}x_b^T & q_j^T\end{bmatrix}^T$. It contains position and orientation of the body ($x_b\in SE(3)$), and joint positions of the robot ($q_j\in\mathbb{R}^n$).

Mansard proposed a mapping into the null space of the robot's motion equations with a singular value decomposition. The concept was to clearly split the robot's motion and actuation states. This mapping enables the utilization of the decoupled ones, which have a lower dimension, rather than retaining the entire set of generalized coordinates [19]. Using this concept, we decompose (1) into unactuated body dynamics (with the subscript b) and actuated joint dynamics (with the subscript j).

$$M_b(q)\ddot{q}+C_b(q,\dot{q})+G_b(q)=J_b(q)^T F_c, \tag{2a}$$

$$M_j(q)\ddot{q}+C_j(q,\dot{q})+G_j(q)=\tau+J_j(q)^T F_c. \tag{2b}$$

Here, (2a) is inferred as the system's Newton–Euler equations [20], and it quantifies how the robot's momentum changes in response to contact forces. Assuming that the robot is capable of producing sufficient torque at all times, the robot states and contact forces that meet (2a) also meet (2b) [21]. The centroidal dynamics expressed at the robot CoM is

$$\begin{bmatrix}\dot{l}\\ \dot{k}\end{bmatrix}=\begin{bmatrix}m\vec{g}+\sum_{i=1}^4(\vec{F})\\ \sum_{i=1}^4(\vec{P}\times\vec{F})\end{bmatrix}, \tag{3}$$

where $\dot{l}$ and $\dot{k}$ are linear and angular momentum rates, respectively. m is the mass of the robot, P is the distance between foot locations and CoM of the body and F is the external contact forces.

B. Robot Model

The detailed quadruped model employed in simulations consists of 18 DoF, with 3 DoF on each leg. The remaining 6 DoF are the position and orientation of the floating base. Euler angles are utilized for describing robot body orientation. Three successive rotations around the body frame are selected in the order of roll, pitch, and yaw. Every DoF on the legs is rotational. Each leg has an adduction/abduction (a/a) joint on the hip, flexion/extension (f/e) joints on the hip and knee. There is an illustration of the detailed quadruped model used in simulations in Figure 1.

C. Reference Generation

A floating base robot's motion is described as a hybrid dynamic system [22]. It consists of swing and stance phases with varying dynamics. Throughout the swing phase, the foot mostly follows an elliptical trajectory until contact is achieved. However, during the stance phase, the robot's dynamics alter as a consequence of the inclusion of contact forces from the environment. These dynamics are linked to each other by instantaneous events such as touchdowns and take-offs. Due to the hybrid dynamics of a floating-base robot's locomotion, it is more effective to generate references for each phase separately.

A quadruped robot's stability is mostly determined by its body motion. In the pace gait, the body is subjected to a significant amount of linear and angular accelerations. As a result, the body's balance may be compromised. Considering reaction forces are critical for body stability, the vertical contact force is primarily responsible for balancing the robot's body against gravity's effects. In this study, we aim to generate the desired roll motion of the robot's body by planning contact forces. The desired roll motion is selected as a trigonometric sinusoidal function that is suitable to pace gait and has periodicity. In (2a), the body dynamics of the quadruped robot are decoupled. We define roll selection matrix $S_{roll}=\begin{bmatrix}0_{1\times 3} & I_{1\times 1} & 0_{1\times 14}\end{bmatrix}$ to separate roll motion from decoupled body dynamics.

$$\ddot{q}_{roll}=S_{roll}M_b^{\dagger}(J_b^T F_c-C_b-G_b). \tag{4}$$

Here, $M_b^{\dagger}=M^{-1}M_b^T(M_bM^{-1}M_b^T)^{-1}$ is dynamically consistent generalized inverse of decoupled body inertia.

The limit cycle behavior of legged robots is one of the criteria for gait stability documented in the literature [23]. A periodic motion is sought in every individual gait cycle (the sum of the stance and swing periods of a single leg) in order to preserve the pace gait's limit cycle behavior. If periodicity is attained, motion remains stable over a timespan of time. In order to achieve periodicity in all directions of motion, the integral of centroidal dynamics must be zero for each gait cycle.

$$\int_0^T(m\vec{g}+\sum_{i=1}^4(\vec{F}))dt=\vec{0}, \tag{5a}$$

$$\int_0^T(\sum_{i=1}^4(\vec{P}\times\vec{F}))dt=\vec{0}, \tag{5b}$$

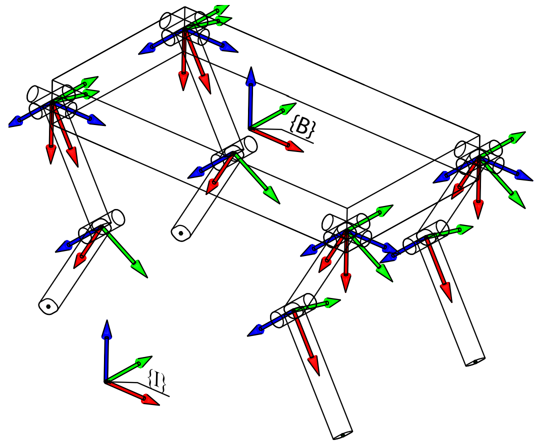

Fig. 1. The quadruped model utilized in the simulations. The x, y, and z axes are represented with red, green, and blue arrows, respectively.

Appropriate contact force reference trajectories are established through the formulation and solution of an optimization problem. The objective is to design contact forces that achieve the desired body roll motion while adhering to the zero momentum change constraint. We define a matrix $D_{roll} = \begin{bmatrix} 1_{6\times3} & 0_{6\times1} & 1_{6\times2} \end{bmatrix}^T$ to subtract roll momentum from entire momentum equations. A discrete optimization technique to plan suitable contact forces is designed as follows,

$$\begin{aligned} \min_{\mathbf{F_c}} \quad & \sum_{i=1}^{i_t} |\ddot{q}_{ref_i} - \ddot{q}_{roll_i}(\mathbf{F_c})|^2, \\ \text{s.t.} \quad & \sum_{i=1}^{i_t} D_{roll} H = \vec{0}, \\ & |F_{c_{i+1}} - F_{c_i}| \leq \delta, \\ & F_{T_i} \leq \mu_f F_{N_i}, \end{aligned} \tag{6}$$

where i_t is the ratio between the gait cycle duration and sampling time. H is a vector that includes linear and angular momenta. $\ddot{q}_{ref}$ and $\ddot{q}_{roll}$ are the reference roll acceleration and actual roll acceleration of the body, respectively. δ is a positive small number. For the optimization process, discretization is applied to all quantities within a single gait cycle duration. The first constraint implies no momentum change (roll motion is excluded) during one gait cycle, while the second constraint prevents large fluctuations in planned contact forces and ensures continuity. The final constraint is set to eliminate the possibility of friction. Additional information related to contact force planning is provided in [24].

In a hybrid motion, high impact forces between phase transitions can be problematic. With the assistance of the smooth transition between swing and stance dynamics, the control of the hybrid system becomes easier. Therefore, the swing phase reference trajectory of the foot is constructed by a polynomial that have zero arriving velocity and acceleration to the floor. Additional information is supplied in [25].

III. CONTROL FRAMEWORK

Operating a system with hybrid dynamics is a difficult endeavor. Establishing a robust and stable control law is crucial. The control framework of a system must be appropriate for the task at hand and compatible with the motion planning algorithm. As a result, the establishment of a control framework is critical for achieving successful operations. Our motion planning algorithm takes into account both the position and force references of the robot's feet. As a result, we have chosen to employ the OSF [26] in our control algorithm in order to track these references.

The right and left leg pairs move antagonistically during the pace gait. For instance, these pairs do not execute the stance phase concurrently. Force control is employed during the stance phase to stabilize the body. During the swing phase, motion control is utilized to move the robot's feet forward. As a result, the swing and stance phases each have their controller. Furthermore, the right and left leg pairs' OSFs are decoupled from the overall robot dynamics, allowing for the execution of two distinct control laws.

In this section, the proposed control framework is explained in detail. First, the OSF of the quadruped robot in pace gait is derived, followed by the explanation of motion and force control laws. There is an information of swing and stance phase coordination and the presentation of the final form of the hybrid force-motion control law.

A. Pace Gait Operational Space Formulation

The OSF of the floating-base systems is examined in [27]. In this study, we contribute further to the OSF of floating-base robots by decoupling the dynamics of the right and left leg pairs to facilitate pace gait. In order to avoid the possibility of singularity within the OSF, the contact Jacobian in (1) is separated into the right (J_r) and left (J_l) contact Jacobians.

$$M\ddot{q} + C + G = S^T\tau + J_{cr}^T F_{cr} + J_{cl}^T F_{cl}. \tag{7}$$

The generalized coordinates are updated in accordance with the selection of the right and left legs' joints.

$$q_{(r,l)} = S_{(r,l)} q. \tag{8}$$

Here S is the joint selection matrix. r and l subscripts stand for right and left leg pairs, respectively. By applying these selection matrices, the generalized dynamics can be mapped to both right and left joint dynamics.

$$M_r\ddot{q}_r + C_r + G_r = \tau_r + J_r^T F_{cr}, \tag{9a}$$

$$M_l\ddot{q}_l + C_l + G_l = \tau_l + J_l^T F_{cl}. \tag{9b}$$

The relation among motion equations and leg joint dynamics is,

$$\begin{aligned} M_{(r,l)} &= \left(S_{(r,l)} M^{-1} S_{(r,l)}^T\right)^{-1}, \\ S_{(r,l)}^{\dagger} &= M^{-1} S_{(r,l)}^T M_{(r,l)}, \\ C_{(r,l)} &= \left(S_{(r,l)}^{\dagger}\right)^T C, \\ G_{(r,l)} &= \left(S_{(r,l)}^{\dagger}\right)^T G, \\ \tau_{(r,l)} &= \left(S_{(r,l)}^{\dagger}\right)^T \left(S^T \tau\right), \\ J_{(r,l)} &= J_{(cr,cl)} S_{(r,l)}^{\dagger}. \end{aligned} \tag{10}$$

$M_{(r,l)}$ is the inertia matrix, $C_{(r,l)}$ is the Coriolis and centrifugal effects, $G_{(r,l)}$ is the gravity effect, $J_{(r,l)}$ are the Jacobian of the leg joint space. $S_{(r,l)}^{\dagger}$ is the generalized inverse of the leg joint selection matrix. $\tau_{(r,l)}$ is the joint torques of the right and left leg pairs.

Since the dynamic equations are mapped to the actuated joint space, end-effector dynamics for a floating-base robot can be obtained by multiplying (9a) and (9b) with the generalized inverse of Jacobian transpose ($(J_{(r,l)}^{\dagger})^T$):

$$(J_r^{\dagger})^T \left(M_r \ddot{q}_r + C_r + G_r = \tau_r + J_r^T F_{cr}\right), \tag{11a}$$

$$(J_l^{\dagger})^T \left(M_l \ddot{q}_l + C_l + G_l = \tau_l + J_l^T F_{cl}\right), \tag{11b}$$

after derivations these equations are become,

$$\Lambda_r(q_r)\ddot{x}_{er} + \mu_r(q_r, \dot{q}_r) + p_r(q_r) - F_{cr} = F_{er}, \tag{12a}$$

$$\Lambda_l(q_l)\ddot{x}_{el} + \mu_l(q_l, \dot{q}_l) + p_l(q_l) - F_{cl} = F_{el}, \tag{12b}$$

where $J_{(r,l)}^{\dagger} = M_{(r,l)}^{-1} J_{(r,l)}^T \Lambda_{(r,l)}$ is the dynamically consistent generalized inverse of actuated joint space Jacobian that minimizes the instantaneous kinetic energy of the robot [26].

$$\begin{aligned} \Lambda_{(r,l)} &= (J_{(r,l)} M_{(r,l)}^{-1} J_{(r,l)}^T)^{-1} \\ \mu_{(r,l)} &= (J_{(r,l)}^{\dagger})^T C_{(r,l)} - \Lambda_{(r,l)} \dot{J}_{(r,l)} \dot{q}_{(r,l)}, \\ p_{(r,l)} &= (J_{(r,l)}^{\dagger})^T G_{(r,l)}. \end{aligned} \tag{13}$$

Here Λ, μ, and p are the operational space inertia, Coriolis and centrifugal effects, and gravity term, respectively. $F_e = J^T \tau$ is the end effector force, and x_e is the position of the end effector in the task space.

B. Control Algorithm

The hybrid dynamic system's motion and force control are decoupled using resolved-acceleration control. During the swing phase, the inverse dynamics control law based on acceleration is used to control motion in the operation space.

$$\begin{aligned} F_{e_m} &= \hat{\Lambda} \ddot{x}_{e_d} + \hat{\mu} + \hat{p}, \\ \ddot{x}_{e_d} &= \left(K_{P_m} e_m + K_{I_m} \int_{t_0}^{t} e_m(\tau)\, d\tau + K_{D_m} \dot{e}_m\right). \end{aligned} \tag{14}$$

Here x_{e_d} is the desired swing trajectory of the foot and K_{P_m}, K_{D_m}, and K_{I_m} are positive-definite motion control matrix gains. $e_m = x_{e_d} - x_e$ is the error between the desired and the actual position of the foot in the operational space. F_{e_m} is the motion control force in the operational space.

The objective of the stance phase is to keep track of planned contact force references. (15) presents a direct force control law for tracking predetermined trajectories in (6).

$$\begin{aligned} F_{e_f} &= \hat{\Lambda} \ddot{x}_e + \hat{\mu} + \hat{p} - F_c - F_d, \\ F_d &= \left(K_{P_f} e_f + K_{I_f} \int_{t_0}^{t} e_f(\tau)\, d\tau + K_{D_f} \dot{e}_f\right). \end{aligned} \tag{15}$$

Here K_{P_f}, K_{I_f}, and K_{D_f} are are positive-definite force control matrix gains. $e_f = F_r - F_c$ is the error between planned contact forces and actual contact forces. F_d is the additional force to track desired contact forces and F_{e_f} is force control force in the operational space.

It is advantageous to define the transition law between the force and motion control algorithms in order to cope with the hybrid dynamics of the system. When the foot touches the ground, the gait phase transitions from swing to stance. After contact is detected, the force control system is activated, and the stance phase begins. When the time reaches the duration of the stance phase, the transition to the swing phase occurs. After entering the swing phase, the motion control is activated. The gait phase transition ensures coordination between motion and force control. A transition parameter is defined for the phase transition.

$$t_p = \begin{cases} I, & stance\ \ phase, \\ 0, & swing,\ \ phase. \end{cases} \tag{16}$$

Both control laws are concatenated into a single control law after defining the gait phase transition parameter.

$$F_e = \hat{\Lambda}\left(t_p \ddot{x}_e + (I - t_p)\ddot{x}_{e_d}\right) + \hat{\mu} + \hat{p} - t_p\left(F_c + F_d\right). \tag{17}$$

Here F_e is the hybrid force-motion control force in the operational space. Finally, the proposed hybrid force-motion controller in (17) is applied on each right and left pair of legs.

$$\begin{aligned} F_r &= \hat{\Lambda}_r\left(t_p \ddot{x}_{er} + (I - t_p)\ddot{x}_{er_d}\right) + \hat{\mu}_r + \hat{p}_r - t_p\left(F_{cr} + F_{r_d}\right), \\ F_l &= \hat{\Lambda}_l\left(t_p \ddot{x}_{el} + (I - t_p)\ddot{x}_{el_d}\right) + \hat{\mu}_l + \hat{p}_l - t_p\left(F_{cl} + F_{l_d}\right). \end{aligned} \tag{18}$$

The following equation calculates the controller torques applied to the leg joints:

$$\begin{aligned} \tau_r &= J_r^T F_r, \\ \tau_l &= J_l^T F_l. \end{aligned} \tag{19}$$

The control framework's overall structure is depicted in Figure 2.

IV. SIMULATION RESULTS

A simulation environment is utilized to validate the proposed method for full-motion generation of a quadruped robot performing a pace gait. In simulations, the robot body measures 1 m in length and weighs 40 kg; each leg link measures 0.4 m in length and weighs 2.5 kg. The simulation environment is built in MATLAB & Simulink. Detailed explanations of the simulation environment can be found in [28].

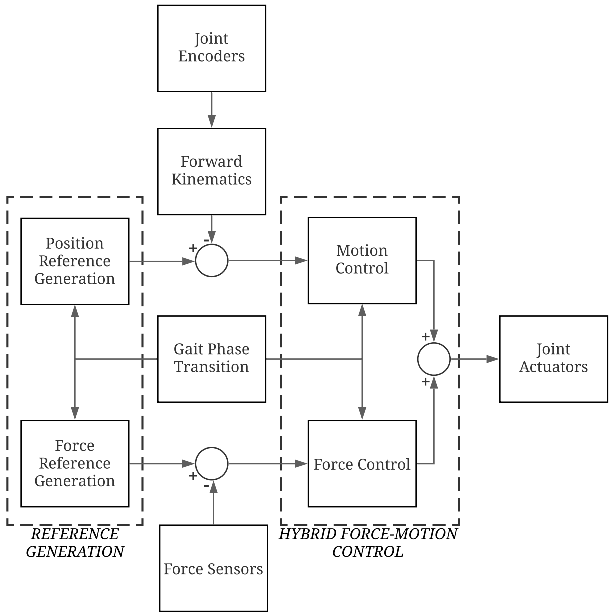

Fig. 2. Control framework of the hybrid force-motion controller.

Figure 3 illustrates the simulation results for a hybrid force-motion controller in a single gait cycle. The shared result belongs to the left front foot of the robot. Since our constraint-based contact model does not require ground penetration, it performed better than our previous spring-damper-based contact model [25]. Due to scaling in Figure 3, the force reference trajectory appears to be constant, however, it actually follows an elliptical orbit. Although the force controller exhibits some chattering as a result of the impact force during touchdown, it performs admirably. The pre-design of a reference trajectory with zero ground-reaching velocity and acceleration has a considerable contribution to the performance of the force controller. The body roll motion during the five-step simulation is depicted in Figure 4. Although the reference trajectory is slightly exceeded by the effect of the impulse force, the result is quite successful considering that the robot body is underactuated. Besides this, we are more concerned with achieving stability and periodicity than with precise reference tracking for the body's roll motion. In addition to the simulation results, the animation screenshots of the simulation environment are shared in Figure 5. The simulations are animated using MATLAB & Simscape.

V. DISCUSSION

It is critical not to disturb the balance of the trunk in order to ensure stable locomotion in legged robots. Since legged robot systems are underactuated, the balance of the trunk is achieved through the coordinated motion of the leg joints. These coordination rules are defined by the motion planning algorithm. Due to the natural instability of the pace gait (the robot falls if the motion does not proceed), it is advantageous to incorporate force references into the motion planning algorithm. Thus, the trunk balance is accomplished more effectively while the robot is in the stance phase.

Motion planning alone is insufficient for quadruped robots to produce a stable pace gait. Additionally, a stable and robust control algorithm must be constructed to track the desired reference trajectories generated by the motion planning algorithm. Since our motion planning algorithm incorporates both force and position references, which are defined on the robot's feet, our controller employs the OSF. A hybrid force-motion controller is operating in the task space in order to perform successful locomotion. Although implementing active force control is a challenging task, the outcomes convince us that force control should be integrated into quadruped locomotion.

VI. CONCLUSION AND FUTURE WORK

In this paper, we propose a full locomotion generation technique for a quadruped robot in pace gait. Firstly, a motion

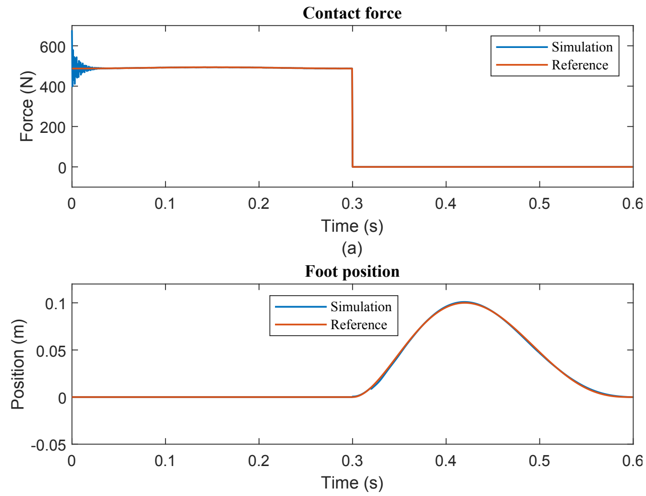

Fig. 3. Simulation results of the hybrid force-motion controller in one gait cycle ((a) Force control (b) Position control in vertical direction).

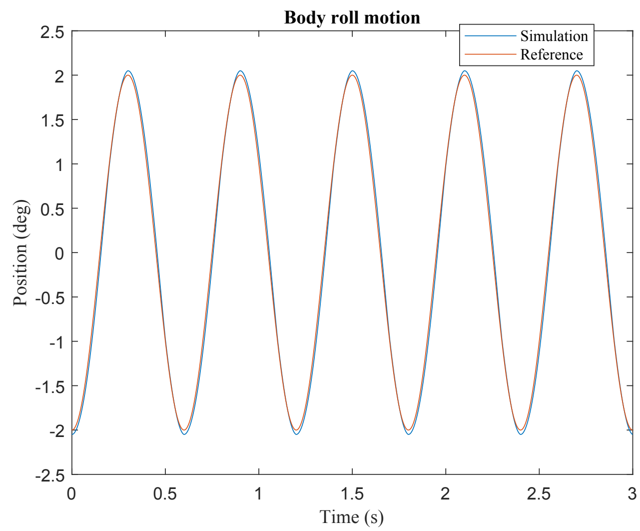

Fig. 4. Body roll motion in five-step simulation.

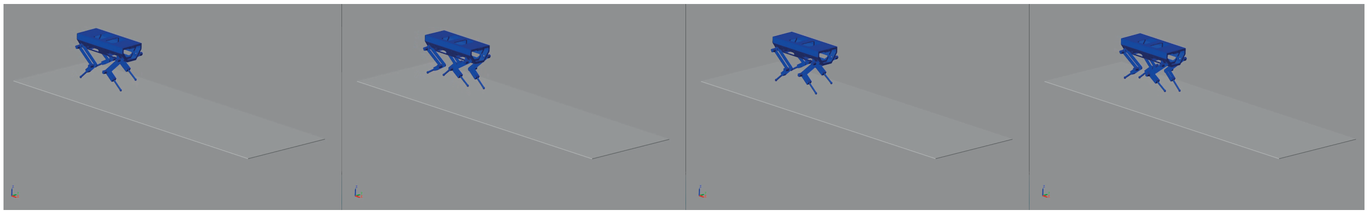

Fig. 5. Animation of the simulation environment.

planning algorithm based on the centroidal dynamics of the quadruped robot is developed. The motion planning algorithm generates both force and position references. The desired reference trajectories are then tracked using an inverse dynamics control algorithm. Since reference trajectories are defined at the robot's feet, the OSF is applied to ensure a mapping between the configuration space and the task space. A hybrid force-motion controller is designed in the task space and mapped to configuration space using the OSF in order to obtain joint torques. Finally, the simulation results demonstrated that the proposed whole-body pace gait control is capable of performing stable quadruped locomotion. In terms of future directions, we aim at implementing our method using an actual quadruped robot and integrating learning algorithms into certain aspects of the method.

REFERENCES

[1] Y. Sakagami, R. Watanabe, C. Aoyama, S. Matsunaga, and N. Higaki et al., "The intelligent ASIMO: System overview and integration," IEEE/RSJ international conference on intelligent robots and systems (IROS), vol.3, pp. 2478-2483, 2002.

[2] M. Raibert, K. Blankespoor, G. Nelson, and R. Playter, "Bigdog, the rough-terrain quadruped robot," 17th World Congress of the International Federation of Automatic Control (IFAC), pp. 10822-10825, 2008.

[3] M. Neunert, F. Farshidian, A.W. Winkler, and J. Buchli, "Design of HyQ–a hydraulically and electrically actuated quadruped robot," Proceedings of the Institution of Mechanical Engineers, Part I: Journal of Systems and Control Engineering, vol. 225, no. 6, pp. 831-849, 2011.

[4] M. Hutter, C. Gehring, D. Jud, A. Lauber, and C. Bellicoso et al., "Anymal-a highly mobile and dynamic quadrupedal robot," IEEE/RSJ International Conference on Intelligent Robots and Systems (IROS), pp. 38-44, 2016.

[5] B. Katz, J. Di Carlo, and S. Kim, "Mini cheetah: A platform for pushing the limits of dynamic quadruped control," IEEE International Conference on Robotics and Automation (ICRA), pp. 6295-6301, 2019.

[6] R.M. Alexander, "Principles of animal locomotion," Princeton University Press, 2013.

[7] T. Akbas, S.E. Eskimez, S. Ozel, O.K. Adak, and K.C. Fidan et al., "Zero moment point based pace reference generation for quadruped robots via preview control," IEEE International Workshop on Advanced Motion Control (AMC), pp. 1-7, 2012.

[8] A.W. Winkler, F. Farshidian, D. Pardo, M. Neunert and J. Buchli, "Fast trajectory optimization for legged robots using vertex-based zmp constraints," IEEE Robotics and Automation Letters, vol. 2, no. 4, pp. 2201-2208, 2017.

[9] S. Rutishauser, A. Sprowitz, L. Righetti, and A.J. Ijspeert, "Passive compliant quadruped robot using central pattern generators for locomotion control," IEEE RAS & EMBS International Conference on Biomedical Robotics and Biomechatronics, pp. 710-715, 2008.

[10] L. Righetti and A.J. Ijspeert, "Pattern generators with sensory feedback for the control of quadruped locomotion," IEEE International Conference on Robotics and Automation (ICRA), pp. 819-824, 2008.

[11] Y. He, S. Guo, L. Shi, S. Pan, and P. Guo, "Dynamic gait analysis of a multi-functional robot with bionic springy legs," IEEE International Conference on Mechatronics and Automation, pp. 689-694, 2016.

[12] D. Gong, P. Wang, S. Zhao, L. Du, and Y. Duan, "Bionic quadruped robot dynamic gait control strategy based on twenty degrees of freedom," IEEE/CAA Journal of Automatica Sinica, vol. 5, no. 1, pp. 382-388, 2017.

[13] D. Kim, J. Di Carlo, B. Katz, G. Bledt, and S. Kim, "Highly dynamic quadruped locomotion via whole-body impulse control and model predictive control," arXiv preprint arXiv:1909.06586, 2019.

[14] D. Kang, S. Zimmermann, and S. Coros, "Animal Gaits on Quadrupedal Robots Using Motion Matching and Model-Based Control," IEEE/RSJ International Conference on Intelligent Robots and Systems (IROS), pp. 8500-8507, 2021.

[15] J. Tan, T. Zhang, E. Coumans, A. Iscen, and Y. Bai et al., "Sim-to-real: Learning agile locomotion for quadruped robots," arXiv preprint arXiv:1804.10332, 2018.

[16] J. Hwangbo, J. Lee, A. Dosovitskiy, D. Bellicoso, and V. Tsounis et al., "Learning agile and dynamic motor skills for legged robots," Science Robotics, vol. 4, no. 26, 2019.

[17] B. Ponton, A. Herzog, A. Del Prete, S. Schaal, and L. Righetti, "On time optimization of centroidal momentum dynamics," IEEE International Conference on Robotics and Automation (ICRA), pp. 5776-5782, 2018.

[18] Y.C. Lin, B. Ponton, L. Righetti and D. Berenson, "Efficient humanoid contact planning using learned centroidal dynamics prediction," IEEE International Conference on Robotics and Automation (ICRA), pp. 5280-5286, 2019.

[19] N. Mansard, "A dedicated solver for fast operational-space inverse dynamics," IEEE International Conference on Robotics and Automation (ICRA), pp. 4943-4949, 2012.

[20] P.B. Wieber, "Holonomy and nonholonomy in the dynamics of articulated motion," Fast motions in biomechanics and robotics, 411-425, 2006.

[21] A. Herzog, N. Rotella, S. Mason, F. Grimminger and S. Schaal et al., "Momentum control with hierarchical inverse dynamics on a torque-controlled humanoid," Autonomous Robots, vol. 40, no. 3, pp. 473-491, 2016.

[22] A.J. Van Der Schaft and J.M. Schumacher, "An introduction to hybrid dynamical systems," vol. 251, London: Springer, 2000.

[23] R. Katoh and M. Mori, "Control method of biped locomotion giving asymptotic stability of trajectory," Automatica, vol. 20, no. 4, pp. 405-414, 1984.

[24] O.K. Adak and K. Erbatur, "Bound gait reference generation of a quadruped robot via contact force planning," International Journal of Mechanical Engineering and Robotics Research, vol. 11, no. 3, pp. 129-137, 2022.

[25] O.K. Adak, B. Bahceci, and K. Erbatur, "Hybrid Force-Motion Control for One-Legged Robot in Operational Space," IEEE/ASME International Conference on Advanced Intelligent Mechatronics (AIM), pp. 905-910, 2021.

[26] O. Khatib, "A unified approach for motion and force control of robot manipulators: The operational space formulation," IEEE Journal on Robotics and Automation, vol. 3, no. 1, pp. 43-53, 1987.

[27] L. Sentis and O. Khatib, "Control of free-floating humanoid robots through task prioritization," IEEE International Conference on Robotics and Automation (ICRA), pp. 1718-1723, 2005.

[28] O.K. Adak, B. Bahceci and K. Erbatur, "Modeling of a Quadruped Robot with Spine Joints and Full-Dynamics Simulation Environment Construction," arXiv preprint arXiv:2203.09622, 2022.