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# Peridynamics-informed effect of micro-cracks on topology optimization of lightweight structures

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## Abstract

Most structures are preferred to be light-weighted when they are used in industrial applications such as automotive, aerospace, and naval structures. Classical continuum mechanics (CCM) formulations are commonly adopted to solve the topology optimization problems. However, CCM brings about some restrictions to the modeling, analysis, and solution of complex structures with structural discontinuities, defects, and micro/macro damages. Unlike CCM, peridynamic theory provides a wider range of analysis options because of its nonlocal integration nature, which can eliminate the need for partial derivatives in the equation of motion, thereby being suitable for effective modeling of cracks, damages, etc. This paper presents an application of peridynamics based topology optimization (PD-TO) to study the effect of micro-damages for designing lightweight engineering structures. The PD-TO algorithm used herein is based on the coupling of bond-based method and Optimality Criteria (OC) topology optimization method. The structure is designed by locating various microcracks for investigating the microdamage effect on the optimal topologies. To this end, the PD-TO model is implemented using an in-house MATLAB code, and strain energy density distributions are compared between different topologies. As a result, the importance of including damage regions within the lightweight design optimization stage is revealed.

**Keywords:** Topology optimization, microcracks, optimality criteria, peridynamics, lightweight design.



## 1. Introduction

In contrast to traditional manufacturing processes, additive manufacturing (AM) is layer-by-layer production process that is is faster compared to other methods and can even have higher precision if structures are well designed. In the design process, some expectations are necessary to produce lightweight and stiff structures by reducing defects. These defects, which are commonly observed in the size of microlevel, occur during cooling, solidification and melting steps of AM application for engineering structures. Since coalescence of these micro-defects can easily create macrocracks, it is necessary to apply shape and topology optimization methods [1] for additively manufactured structures by considering the size, orientations and shape of possible microcracks during the optimization of engineering part. The topology optimization approach (TO) finds optimal material deposition/distribution on a predefined design domain. New TO algorithms overcome the problems of early TO methods (i.e., in-stability in results or occurrence of checkerboard pattern) by proposing new mathematical formulations. For instance, Bendsøe [2] proposed a homogenized TO method. Later, Solid Isotropic Material with Penalization technique (SIMP) [3] and Evolutionary Structural Optimization (ESO) [4] as well as bidirectional ESO (BESO) [5] were implemented for higher efficiency and accuracy. One of the most recent TO methods is the continuous density-based approach. The continuous methods can be categorized into two main approaches. First, the proportional approach (PO) [6], wherein the value of the objective function in the previous iteration determines the density of the elements. Next, the optimality criteria approach (OC) [7], this approach satisfies a set of analytically obtained criteria instead of directly optimizing the objective to solve TO problems. During TO, a proper numerical method is needed to perform accurate structural analysis. The most common numerical method is classical continuum mechanics (CCM) formulations in which particles interactions are considered between a particle and its nearest neighbor [8, 9]. Some of CCM assumptions pose a modeling/analysis limitation for structures including damage, discontinuity, internal feature, or defect. Various research efforts have been dedicated to overcoming the limitations of CCM such as Linear Elastic Fracture Mechanics (LFEM) [10] and eXtended Finite Element Method (XFEM) [11]. However, for a complex problem with multiple interacting defects, these methods still become complex, and solving them with traditional finite elements is very complicated or mostly inaccurate especially in the blending regions.

Another approach to perform structural analysis during TO is non-local continuum theories. One of the most common non-local theories that have been extensively studied in the last decade is the reformulation of continuum mechanics, referred to as Peridynamics [12]. In this approach, the material does not necessarily require remaining continuous during the simulations. Thus, it ideally becomes a robust approach for dealing with discontinuities such as cracks, damages, and defects. Overall, these features make PD a viable tool to nucleate damage in the domain of structure without adding any singularity. Silling [13] firstly used PD to simulate complex crack growth in plate structures. Then, the PD has been widely applied to fracture mechanics simulation of various material and structural systems[14, 15]. Few studies have applied PD directly to TO for designing cracked structures. The first study in the literature was conducted by Kefal et al. [16] who combined PD with BESO optimization schemes for structures with/without cracks. Later, the continuous density-based topology optimization method was extended to the gradient-based optimization algorithm [17] for complex engineering problems involving cracks. Recently, the PD-TO algorithm has also been used for multi-material topology optimization [18]. Moreover, a comparative study is performed by Motlagh and Kefal [19] to justify why TO algorithms should substitute peridynamics for the conventional FEM approach in the topology optimization of cracked structures.

To the best of the authors' knowledge, there is no research study on the integration of microcracks in the design of light-weighted structures using PD-TO. Thus, there is a need to justify whether many cracks can interfere with the final topology obtained from PD-TO algorithms (especially for brittle structures). The occurrence of microcracks is inevitable during the manufacturing process, therefore including predefined microcracks within design domain and investigating their orientation and size effect can help for a better design criterion using PD-TO. To this end, the main novelty of this study is to perform PD-TO simulations on a benchmark case including randomly oriented microcracks for minimizing compliance. Furthermore, the strain energy of the benchmark problem is compared for each microcracks inclusion sequence.

## 2. Methods and materials

In this section, we summarize the mathematical formulation of Peridynamics based topology optimization. In PD, each particle $i$ interact with all particles within its support region, named as Horizon with the size of $\delta$ as such: $\{H = \mathbf{x} \in \beta : |\mathbf{x}' - \mathbf{x}| \leq \delta, for\ \delta > 0\}$. Fig. 1 shows a set of particles in a region β and horizon of particle $i$ with the size of δ.

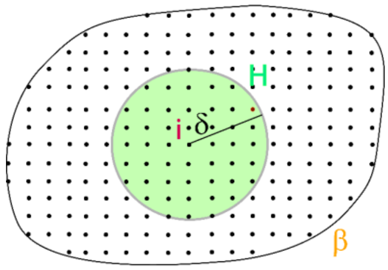


**Fig 1.** Horizon-Family of the material point $i$ for PD model.

According to the bond-based theory, the interaction of particles is defined in a pairwise manner, so the bond forces acting on particles $\mathbf{x}$ and $\mathbf{x}'$ have equal magnitude but with opposite sign $\vec{\mathbf{f}}$=-$\vec{\mathbf{f}}'$. The relative position vector of two material points in deformed configuration can be written as $\mathbf{y}' - \mathbf{y} = (\mathbf{u}' - \mathbf{u}) - (\mathbf{x}' - \mathbf{x})$. Equation of motion for sets of particles is defined as (the reader can refer to [12] for further information):

$$\rho(\mathbf{x})\ddot{\mathbf{u}}(\mathbf{x},t) = \int_H \mathbf{f}(\mathbf{u}' - \mathbf{u}, \mathbf{x}' - \mathbf{x}, t' - t)\mathrm{d}H + \mathbf{b}(\mathbf{x},t) \tag{1}$$

where $\rho(\mathbf{x})$, $\mathbf{b}(\mathbf{x},t)$ and $\ddot{\mathbf{u}}(\mathbf{x},t)$ are density, body force, and acceleration of particle $\mathbf{x}$. Moreover, the micro-potential energy of each bond can be calculated using linear force and displacement relation of the bond as:

$$w = w(\boldsymbol{\xi},\boldsymbol{\eta},t) = \frac{1}{2} f s |\boldsymbol{\xi}| \tag{2}$$

where $\boldsymbol{\xi}$, $\boldsymbol{\eta}$ and $s$ represent relative position vector ($\boldsymbol{\xi} = \mathbf{x}' - \mathbf{x}$), displacement vector ($\boldsymbol{\eta} = \mathbf{u}' - \mathbf{u}$) and stretch between two particles ($s = \frac{|\mathbf{y}'-\mathbf{y}| - |\mathbf{x}'-\mathbf{x}|}{|\mathbf{x}'-\mathbf{x}|}$).

The force density vector $f$ can be expressed along the bond direction in the deformed configuration as:

$$f = cs\frac{\mathbf{y}' - \mathbf{y}}{|\mathbf{y}' - \mathbf{y}|}, where\ c = \frac{9E}{\pi h \delta^3} \tag{3}$$

where $E$ is the young modulus of isotropic material, $h$ is the thickness of the domain and $c$ is the bond constant [20]. Integrating this micro-potential for each bond in the horizon of particle x, the strain energy density of each particle can be obtained as:

$$W(\mathbf{x},t) = \frac{1}{2}\int_H w(\boldsymbol{\eta},\boldsymbol{\xi},t)\mathrm{d}H \tag{4}$$

To be able to obtain total strain energy of domain β, one can integrate the strain energy density over the full field domain as:

$$U = \int_\beta W(\mathbf{x},t)\mathrm{d}\beta \tag{5}$$

Here $U$ is the total strain energy that is used as compliance in the topology optimization process. By solving (1) numerically using a meshless method, and following the same methodology as [16], this equation can be modified to :

$$\mathbf{KD} = \mathbf{B} \tag{6}$$

To include local cracks into peridynamics simulation, the ratio of the number of eliminated interactions to the total number of initial interactions is defined as a weighted function $\phi$:

$$\phi(\mathbf{x},t) = 1 - \frac{\int_H \mu(\mathbf{x}' - \mathbf{x}, t)\mathrm{d}V'}{\int_H \mathrm{d}V'} \tag{7}$$

where μ is damage initiation value which is a history-dependent scalar-valued function. During the PD analysis, the stretch, $s_{(i)(j)}(\mathbf{x}_j - \mathbf{x}_i, t)$, between pairs of material points $\mathbf{x}_i$ and $\mathbf{x}_j$ are monitored in each time iteration such that if it exceeds the critical value $s_c$, then the bond is failed, i.e., $\mu = 0$, and not reevaluated in the next time steps. On the other hand, it is set to unity for the intact bonds, $\mu = 1$. For a given material point's horizon, the contributions of $\mu$ lead to the $\phi$ parameter take a value between 0 and 1. If the $\phi = 0$, then all bonds are broken, otherwise at least there is an intact bond in the horizon. The reader can refer to [19, 20] for more information regarding calculation of $\phi$ and $\mu$ functions.

Compliance is an objective function generally used in TO with a target volume fraction. Compliance can be defined from (5), for the general optimization scheme, the constraints are as follows:

$$\min_{\mathrm{k_i}} U(k_i) = \sum_{i=1}^{N} W(\mathbf{x}_i)V_i \;\; s.t. \;\; \begin{cases} \mathbf{KD} = \mathbf{B} \\ \frac{\sum_{i=1}^{N} k_i V_i}{\sum_{i=1}^{N} V_i} = \bar{V} \\ k_{min} \leq k_i \leq 1 \end{cases} \tag{8}$$

where minimization method is subjected to the global displacement solution, updated strain field, and maximum allowable volume constraint $\bar{V}$, i.e., the ratio of the target volume to the total volume of the design space. At each iteration step, the convergence of OC is monitored, and the satisfaction of the target volume constraint is insured as well. Based on error tolerance $\bar{\tau}$, the convergence criteria are defined and compared at each step as:

$$\tau^k = \frac{\sum_{m=k-9}^{k-5}(C^{m+5} - C^m)}{\sum_{m=k-9}^{k-5} C^m} \leq \bar{\tau}, \qquad \text{for } k \geq 10 \tag{9}$$

where $C^m$ represent objective function at iteration $m$.

# 3. Results and discussion

In this section, three benchmark problems are studied to investigate the effect of microcracks on optimum topology. These benchmark problems can easily be manufactured utilizing additive manufacturing method, but some precautions are necessary during design stage to avoid microlevel defects.

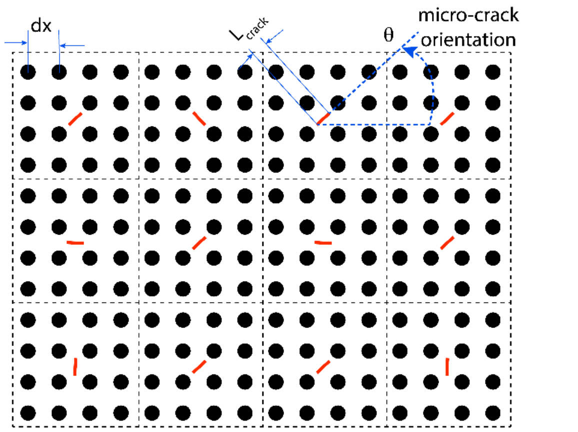


**Fig 2.** Schematic of microcrack in a design domain, black circles represent the particles in PD, the grid of 4 by 4 is located between dashed lines; red lines are microcracks with the size of $L_{crack}$ and orientation of θ.

To this end, for each benchmark scenario, microcracks are included in the design domain between a grid of 4 by 4 of particles. Hence, at least a microcrack with an orientation of θ and length of $L_{crack}$ exists. A schematic of these microcracks in a partially selected domain can be seen from Fig. 2.

## *3.1. A Simply supported beam with microcracks of size dx/2*

First geometry is a simply supported beam with the length to the width ratio of is 2:1 where $L = 1$m as depicted in Fig. 3. Embedded microcracks with length of $\frac{dx}{2}$ are utilized in the first case study. A downward point load is applied with the magnitude of F = 200N from the middle of the top side of the structure. The beam is fixed from the left bottom corner and simply supported on the right. For PD discretization, the solution domain is uniformly divided into 80 by 40 particles and $\bar{V} = 0.5$ is the target volume ratio. By extracting the optimum topologies from each microcracks orientation sets, strain energy and final topology are compared in Fig. 4.

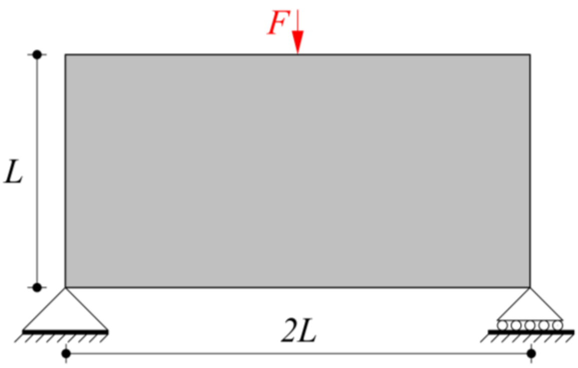


**Fig 3.** Design domain of the simply supported beam.

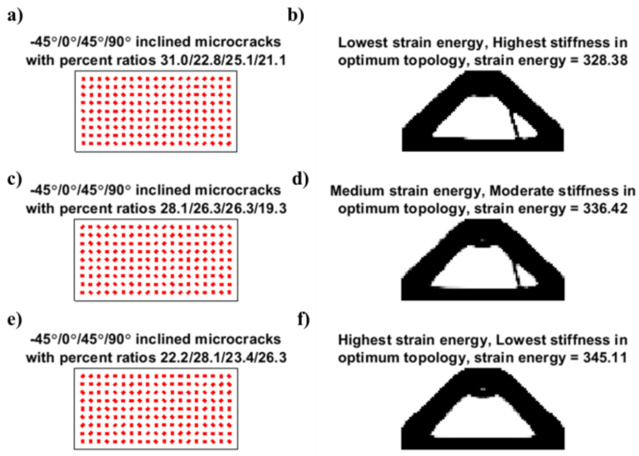


**Fig 4.** A simply supported beam with microcracks ($L_{crack} = dx/2$), a) microcracks orientation for achieving minimum strain energy, b) optimum topology for minimum strain energy, c) microcracks orientation for achieving medium strain energy, d) optimum topology for medium strain energy, e) microcracks orientation for achieving maximum strain energy, f) optimum topology for maximum strain energy.

In addition, as shown in Fig. 4, first two micro-crack sets have an extra support in the optimum topology, leading the design domain to have lower strain energy as compared to the third case. Furthermore, moderate level of strain energy is achieved when the ratios of -45˚, 45˚ and 0˚ inclined micro-cracks are nearly same and relatively higher than 90˚ cracks. Hence, it can be indicated that increasing the number of micro-cracks aligned parallel to the loading direction tends to reduce the stiffness of the brittle structures.

### *3.2. A Cantilever beam with microcracks of size $dx/2$*

The second geometry studied here is a cantilever beam. Length of the randomly oriented microcracks is chosen to be dx/2 for this problem. The beam is fully restricted along the left side and a concentrated force $F = 200N$ is applied at the center of right edge as shown in Fig. 5. The structure is discretized into 80 and 40 particles along the x and y directions, respectively. Volume ratio is assigned as $\bar{V} = 0.5$.

After running 25 cases with randomly orientated microcracks, optimum topologies with minimum, medium and maximum strain energy are extracted. For micro-crack orientations of -45˚/0˚/45˚/90˚, the 30.99%/22.81%/25.15%/21.05% ratio generates the lowest strain energy among all sets of microcracks orientations simulated in the analysis. However, the micro-crack percent distribution of 22.22%/28.07% /23.39%/26.32% has the highest value of strain energy. This comparison can be examined by the optimum topologies shown in Fig. 6. Additionally, medium strain energy is achieved by setting the distribution ratio of 45˚/0˚/45˚/90˚ inclined micro-cracks to 22.81% /22.22%/28.65%/26.32%, respectively, as clearly illustrated in Fig. 6.

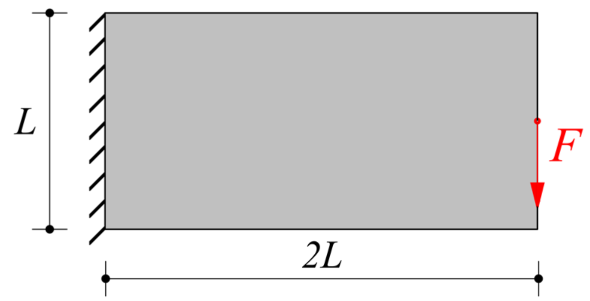


**Fig 5.** Design domain of the cantilever beam.

Despite two problems have different boundary condition, the results obtained for two different benchmark problems demonstrated that the strain energy values of the 25 micro-crack orientation scenarios are ranked similarly from the smallest to the largest. Strain energy varies from 1713.77 to 1777.71 for this case study (Fig. 6). As it can be seen, even with a small variation of microcracks orientations (microcracks may occur during the manufacturing process with different size and orientations) optimum topologies varies case to case. In addition, it can be observed, like the previous case, when the strain energy is minimum, extra support appears in the design domain. This extra support makes the optimum topology have lower strain energy.

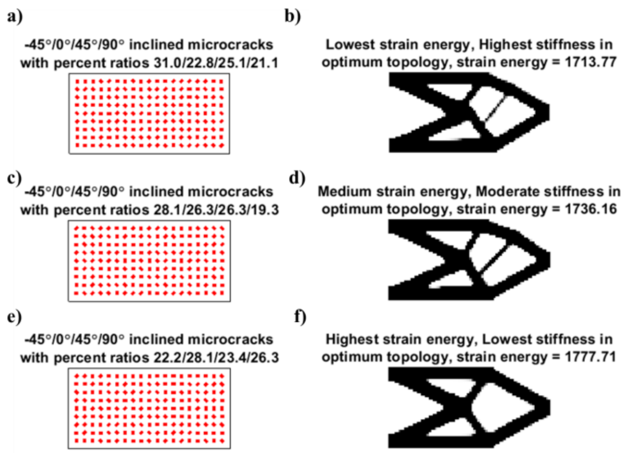


**Fig 6.** Cantilever beam with microcracks ($L_{crack} = dx/2$), a) microcracks orientation for achieving minimum strain energy, b) optimum topology for minimum strain energy, c) microcracks orientation for achieving medium strain energy, d) optimum topology for medium strain energy, e) microcracks orientation for achieving maximum strain energy, f) optimum topology for maximum strain energy.

### *3.3. A Simply supported beam with microcracks of size dx*

The last case study includes the same simply supported beam used in the first case study with similar constraint boundary and loading conditions. However, to demonstrate the effect of microcracks size, the microcracks length is increased to two-times longer of the previous case (i.e., dx). Fig. 7 represents the cases which have minimum, medium and maximum strain energy from 25 various micro-crack orientation sets.

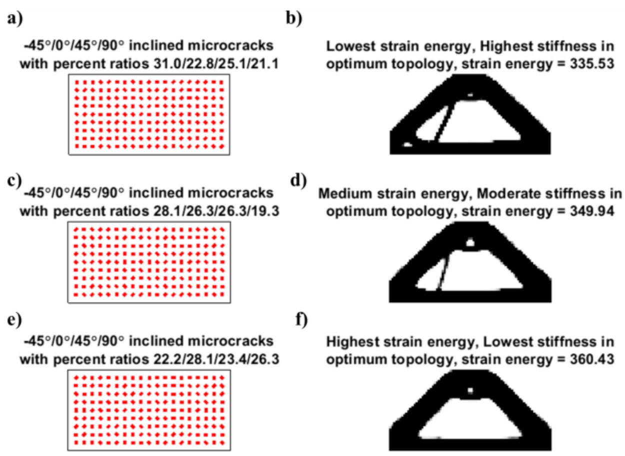


**Fig 7.** Simply supported beam with sets of microcracks ($L_{crack} = dx$), a) microcracks orientation for achieving minimum strain energy, b) optimum topology for minimum strain energy, c) microcracks orientation for achieving medium strain energy, d) optimum topology for medium strain energy, e) microcracks orientation for achieving maximum strain energy, f) optimum topology for maximum strain energy.

These selected cases are again observed at the same micro-crack ratio rank. Therefore, it can be concluded that crack size does not affect the ranking of the cases with respect to the strain energy. On the other hand, after the strain energy values of all scenarios are examined, it is clearly seen that the strain energy increases in each scenario. Furthermore, including longer microcracks may predict further manufacturing defects which are inevitable. Lastly note that the optimum case with the lowest strain energy has an extra support element like previous cases, which is just shifted from right to left hand side.

# 4. Conclusions

During additive manufacturing process, various microlevel size defects may occur, and if they are not properly considered at the design stage, they are likely to cause macro-damages with the engineering part. Thus, necessary safety measures must be taken by considering predefined microcracks in TO process to prevent further development of crack propagation of additively manufactured structures. This paper investigates the inclusion of microcracks in designing lightweight structures utilizing continuous density-based PD-TO. Generally, including defects in the design domain may cause computational complexity which cannot be easily solved via conventional finite elements. Here, on the other hand, it was shown PD-TO is generally a better tool for handling discontinuity such as cracks, defects, and damages. Three benchmark cases are investigated via 25 orientation case sets. Each sets includes microcracks' orientation sets wherein each microcrack orientation has a random value between $-45°$ to $90°$ (sub-step $45°$). After extracting optimum topology for each case sets, the strain energy is calculated, and it is shown a variation of microcrack orientation has a direct effect on optimum topology and obtained strain energy value. All in all, the PD-TO provides the better support for these structures without adding extra complexity, hence, it can be indicated that it is a robust methodology for analyzing the effect of microcracks in topology optimization. For future studies, this method can be extended to include randomly oriented crack with different size in stress/strain hotspots to reduce strain energy.

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## Author's statement

Conflict of interest: Authors state no conflict of interest. Informed consent: Informed consent has been obtained from all individuals included in this study. Ethical approval: The research related to human use complies with all the relevant national regulations, institutional policies and was performed in accordance with the tenets of the Helsinki Declaration and has been approved by the authors' institutional review board or equivalent committee.

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