

Type-3/2 seesaw mechanism


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The type-I seesaw mechanism provides a natural explanation for tiny neutrino masses. The right-handed neutrino masses it requires are, however, too large to keep the Higgs boson mass at its measured value. We show that vector spinors, singlet leptons that are like right-handed neutrinos, generate tiny neutrino masses naturally through the exchange of spin-1/2 and spin-3/2 components. This one-step seesaw mechanism, which we call the type-3/2 seesaw, keeps the Higgs boson mass unchanged at one loop and gives cause therefore to no fine-tuning problem. If the on-shell vector spinor is a pure spin-3/2 particle, then it becomes a potential candidate for hidden dark matter which gets diluted due only to the expansion of the Universe. The type-3/2 seesaw provides a natural framework for the neutrino, Higgs boson, and dark matter sectors, with overall agreement with current experiments and observations.

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I. INTRODUCTION

Neutrino oscillations [1,2] are proof that active neutrinos are massive. Neutrino mass, whose nature is still an open question, is proof that there is new physics beyond the Standard Model (SM) [3–8].

Even though neutrino physics has come of age in the past two decades [9], there is still no telling whether neutrinos are Dirac ($\nu \neq \nu^c$) or Majorana ($\nu = \nu^c$) fermions [8,10–12]. The Dirac masses [13–20] conserve lepton number. The Dirac neutrinos acquire their masses via the electroweak breaking as all the other fermions do, albeit with an unnaturally small Yukawa coupling. The Majorana masses, on the other hand, break lepton number, and arise via the electroweak breaking, with naturally heavy SM-singlet right-handed neutrinos [3–6].

The new physics that generates the tiny neutrino masses has been variously modeled by invoking various fields, symmetries, and scales [9,21,22]. The type-I seesaw mechanism, first and foremost [4], leads to small active neutrino masses [4] via the dimension-5 Weinberg operator [23] induced by the heavy right-handed neutrino mediation. There are also type-II [24–26] and type-III [27] seesaw

mechanisms, which are mediated, respectively, by triplet scalars and triplet fermions. In addition to these tree-level mechanisms, various models [28–39] have been constructed to generate the small neutrino masses radiatively.

In this paper we further the type-I seesaw. We do this by replacing the right-handed neutrino with a vector-spinor field [40] involving both spin-3/2 and spin-1/2 components when it is off shell and only a spin-3/2 component when it is on shell. We call the resulting neutrino mass generation mechanism the “type-3/2 seesaw mechanism.” As will be shown in the sequel, this new mediator leads to important new results for the neutrino, Higgs, and dark matter sectors.

Below, we first summarize the type-I seesaw and discuss its implications for leptogenesis, dark matter, and Higgs boson mass. Next, we turn to the vector spinor and construct the type-3/2 seesaw, with a detailed discussion of its implications for leptogenesis, dark matter, and the Higgs boson mass. We conclude the paper by contrasting the salient features and implications of the type-I and type-3/2 seesaw mechanisms.

II. TYPE-I SEESAW

The right-handed neutrino N , a SM-singlet spin-1/2 neutral fermion [3,4], gives a simple model of the tiny neutrino masses. It couples to the active neutrinos ν_L through the left-handed lepton doublet $L = (\nu_L, \ell_L)$ as

$$y_N \bar{L} H N + \frac{M_N}{2} \bar{N} N + \text{H.c.} \quad (1)$$

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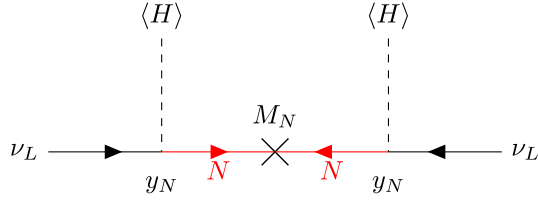


FIG. 1. Type-I seesaw: Heavy right-handed neutrinos N lead to the neutrino Majorana masses m_ν .

so that an active neutrino acquires the Majorana mass

$$m_\nu = \frac{y_N^2 \langle H \rangle^2}{2M_N} \quad (2)$$

via the Feynman diagram in Fig. 1. This induced mass, resulting from the Weinberg operator [23], agrees with the experimental data [1,2] for right-handed neutrino masses $M_N \approx 10^{14}$ GeV, Higgs vacuum expectation value $\langle H \rangle \approx 246$ GeV, and Yukawa coupling $y_N \approx \mathcal{O}(1)$. This dynamical mechanism, the type-I seesaw [3–6], generates neutrino Majorana masses m_ν naturally with naturally heavy SM-singlet right-handed neutrinos [7]. Neutrino mixings [9,22] are realized with two or more right-handed neutrinos.

The interactions in Eq. (1) give cause for not only the neutrino masses in Eq. (2) but also the Higgs mass shift [41]

$$(\delta m_h^2)_N = \frac{m_\nu M_N^3}{2\pi^2 \langle H \rangle^2} \log \frac{Q}{M_N} \quad (3)$$

at the renormalization scale $Q \gtrsim M_N$ via the Feynman diagram in Fig. 2. This mass correction, evaluated in dimensional regularization in which quadratic (and quartic) corrections all vanish identically [42], exceeds the Higgs boson mass itself unless $M_N \lesssim 10^7$ GeV [43–45]. This bound is in clear contradiction with the value $M_N \approx 10^{14}$ GeV required by the neutrino masses. This contradiction shows that the right-handed neutrinos generate the active neutrino masses at the expense of an immense fine-tuning of the model parameters entering the Higgs boson mass [46]. This is a serious naturalness problem because the logarithmic correction (3) survives to impede the type-I seesaw [44] even in the supersymmetry [47].

The right-handed neutrinos can decay and annihilate [48] via their interactions with the SM fields in Eq. (1), and

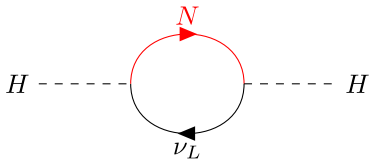


FIG. 2. The active neutrino/right-handed neutrino loop that gives cause to the Higgs boson mass shift in Eq. (3).

therefore can facilitate, for instance, the leptogenesis [49,50]. Thermal leptogenesis requires the lightest right-handed neutrino N_1 to have a mass $M_{N_1} \gtrsim 2 \times 10^9$ GeV in order to produce the requisite asymmetry in the lepton sector. To be able to produce such a massive N_1 thermally, the reheat temperature T_{th} after the inflation must have a value $T_{\text{th}} > M_{N_1}$ [51–53]. It is clear that this leptogenesis value of M_{N_1} is also in contradiction with the bound $M_{N_1} \lesssim 10^7$ GeV imposed by the Higgs boson mass correction in Eq. (3). Even though it is not possible to suppress all the radiative corrections, gravity-mediated softly broken supersymmetry is sometimes incorporated into the thermal leptogenesis to reduce the quadratic corrections from heavy right-handed neutrinos to milder logarithmic ones. This attempt leads, however, to the well-known gravitino problem [54,55]. It turns out that, for gravitinos of masses below a few TeV, the reheat temperature of the Universe should not exceed 10^5 GeV [56,57].

The type-I seesaw does not offer a unique dark matter candidate, though, as an analogous low-energy extension, one can consider incorporating the sterile neutrinos into the setup [58].

III. TYPE-3/2 SEESAW

Having discussed the type-I seesaw and its physics implications, we now develop a new approach in which we envision the right-handed neutrino as the spin-1/2 component of a SM-singlet, neutral vector spinor ψ_μ of mass M_ψ . This Rarita-Schwinger field [40] decomposes as $[(1, \frac{1}{2}) \oplus (0, \frac{1}{2})] \oplus [(\frac{1}{2}, 1) \oplus (\frac{1}{2}, 0)]$ under the Lorentz group. Namely, it involves spin-3/2 and spin-1/2 components. As pointed out by Demir *et al.* [59], it directly couples to the Higgs and lepton doublets via the Lagrangian

$$y_\psi \bar{L} H \gamma^\mu \psi_\mu + y_\psi \bar{\psi}_\mu \gamma^\mu H^\dagger L + \bar{\psi}_\mu \Lambda^{\mu\nu} \psi_\nu, \quad (4)$$

in which the kinetic operator [40]

$$\Lambda^{\mu\nu} = \eta^{\mu\nu} (\not{p} - M_\psi) - (\gamma^\mu p^\nu + p^\mu \gamma^\nu) + \gamma^\mu \not{p} \gamma^\nu + M_\psi \gamma^\mu \gamma^\nu \quad (5)$$

differs from the Dirac operator by the terms on the second line. The Lagrangian (4) leads to the equation of motion

$$\Lambda_{\mu\nu} \psi^\nu = y_\psi \gamma_\mu H^\dagger L, \quad (6)$$

which is in agreement with the vector-spinor description in [60,61]. For a free field, that is, for an on-shell vector spinor $\psi_{(\text{free})}^\mu(p)$ satisfying $p^2 = M_\psi^2$, one gets the equation of motion $\Lambda_{\mu\nu} \psi_{(\text{free})}^\nu = 0$. This homogeneous equation can be consistently split into three distinct parts,

$$(\not{p} - M_\psi)\psi_{(\text{free})}^\mu = 0, \quad (7)$$

$$\gamma_\mu \psi_{(\text{free})}^\mu = 0, \quad (8)$$

$$p_\mu \psi_{(\text{free})}^\mu = 0, \quad (9)$$

as follows from the kinetic structure in Eq. (5) as particular choices for its individual terms. Here the point is that Eqs. (8) and (9) eliminate the spin-1/2 component of the vector spinor $\psi_{(\text{free})}^\mu$ to yield an on-shell pure spin-3/2 field.

For an off-shell vector spinor $\psi^\mu(p)$ satisfying $p^2 \neq M_\psi^2$, the description is given by its propagator $S^{\mu\nu}$ [60,61] as

$$S^{\mu\nu} = -\frac{i\eta^{\mu\nu}}{\not{p} - M_\psi} + \frac{i}{\not{p} - M_\psi} \left(\frac{\gamma^\mu \gamma^\nu}{3} + \frac{(\gamma^\mu p^\nu - p^\mu \gamma^\nu)}{3M_\psi} + \frac{2p^\mu p^\nu}{3M_\psi^2} \right), \quad (10)$$

where the second line stands for deviation from the spin-1/2 propagator that is in parallel with Eq. (5). This propagator is simply the inverse of the kinetic structure in Eq. (5), namely, $S^{\mu\alpha} \Lambda_{\alpha\nu} = \delta_\nu^\mu$. This propagator holds in the entire momentum and spin space [62]. In other words, this vector-spinor propagator involves propagations of both the spin-3/2 and spin-1/2 components. It certainly relates ψ_μ to its source in Eq. (6), but it does not satisfy the individual motion equations (7)–(9).

It is useful to contrast the vector-spinor above with the gravitino [63,64]—the gauge field of supergravity which acquires mass by swallowing the Goldstino field [65]. The gravitino is a spin-3/2 field and therefore obeys Eqs. (7)–(9) when it is on shell and off shell. The gravitino propagator is constructed in [63] by imposing Eq. (7)–(9) [with Eqs. (8) and (9) kind of being gauge conditions] so that as a propagator it satisfies Eqs. (7)–(9).

The vector spinor ψ_μ differs from the gravitino. Its propagator in Eq. (10) propagates both the spin-3/2 and spin-1/2 components and does not obey the motion equations (7)–(9). It can be reduced to a spin-3/2 particle when it is on shell, that is, when it obeys the individual motion equations (7)–(9). Its electric neutrality ensures that no problems arise with local causality [66,67].

Similar to the case of right-handed neutrinos, the ψ_μ couplings in Eq. (4) lead to the active neutrino masses

$$m_\nu = \frac{2y_\psi^2 \langle H \rangle^2}{9M_\psi} \quad (11)$$

via the Feynman diagram in Fig. 3 with the exchange of spin-3/2 and spin-1/2 components. This result, derived by using the ψ_μ propagator in Eq. (10), agrees with the experimental data [1,2] for $M_\psi \approx 10^{14}$ GeV and $y_\psi \approx \mathcal{O}(1)$. This new mechanism, which is what we term a type-3/2 seesaw, generates the neutrino Majorana masses naturally

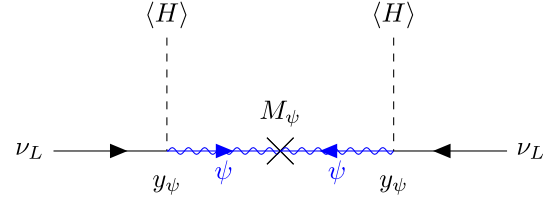


FIG. 3. Type-3/2 seesaw. Heavy vector-spinor neutrinos ψ_μ lead to the neutrino Majorana masses m_ν .

with a naturally heavy SM-singlet vector spinor ψ_μ . Neutrino mixings [9,22] can be realized with two or more ψ_μ fields.

If the vector spinor ψ_μ obeys the three equations of motion (7)–(9), then it becomes a spin-3/2 field when it is on shell and, in this case, in contrast to the right-handed neutrinos, it can neither decay nor annihilate despite its couplings in Eq. (4) to the SM fields. The reason is that in such processes ψ_μ is on its mass shell as an isolated physical particle and for an on-shell ψ_μ , namely, for a ψ_μ satisfying Eqs. (7)–(9), the $H - L - \psi_\mu$ vertex vanishes identically [60,61]. (This vertex vanishes for both on-shell and off-shell gravitinos [63].) This means that the scattering processes with on-shell ψ_μ (its decays, annihilations, and productions) all vanish. This on-shell nullity of ψ_μ has three important implications. First, ψ_μ , unlike the right-handed neutrinos, cannot facilitate leptogenesis simply because there is no decay channel to transfer the lepton number in ψ_μ to active neutrinos and charged leptons [68]. Needless to say, baryogenesis can occur via another mechanism such as Affleck-Dine [69] baryogenesis, and this makes the theory exempt from the gravitino problem [70] in gravity-mediated softly broken supersymmetry [65].

Second, if on-shell ψ_μ is a spin-3/2 particle obeying Eqs. (7)–(9), then it is an everlasting particle. It was around, is around, and will be around to participate in certain processes in a hidden or invisible way [59]. Its density falls with the volume of the Universe as it dilutes due only to the expansion of the Universe. It can therefore form dark matter [71] if it leads to flat rotation curves, structure formation, and other relevant phenomena. The setup in Eq. (4) can be modified to obtain a detectable and more conventional dark matter candidate. One possibility is to invoke higher-dimension operators [72], but the $H - L - \psi_\mu$ coupling in Eq. (4) must still be taken into account when ψ_μ is off shell.

Third, when it is off shell ψ_μ reveals itself via its spin-3/2 and spin-1/2 components [59] by inducing the neutrino Majorana masses as in Eq. (11), altering certain SM scattering amplitudes such as $hh \rightarrow \bar{\nu}_L \nu_L$ [59], and facilitating loop corrections to certain SM masses and couplings. Its loop with the active neutrinos, depicted in Fig. 4, is expected to shift the Higgs boson mass as in Eq. (3) but, to one's surprise, it gives actually zero contribution,

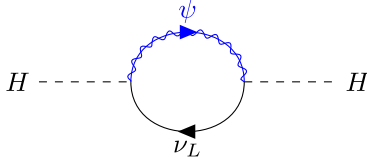


FIG. 4. The vector spinor/active neutrino loop that does not lead to any Higgs boson mass shift.

$$(\delta m_h^2)_{\psi_\mu} = 0, \quad (12)$$

at one loop. This follows from the fact that the Feynman diagram in Fig. 4 evaluates to zero,

$$\begin{aligned} & -i \frac{y_\psi^2}{2} \int \frac{d^4 p}{(2\pi)^4} \frac{1}{p^2(p^2 - M_\psi^2)} \text{Tr} \left[P_R \gamma_\alpha (\not{p} + M_\psi) \left\{ \eta^{\alpha\beta} - \frac{\gamma^\alpha \gamma^\beta}{3} - \frac{2p^\alpha p^\beta}{3M_\psi^2} - \frac{(\gamma^\alpha p^\beta - p^\alpha \gamma^\beta)}{3M_\psi} \right\} \gamma_\beta \not{p} P_R \right] \\ & = -iy_\psi^2 M_\psi^2 \lim_{\epsilon \rightarrow 0} Q^\epsilon \left(\frac{M_\psi^2}{4\pi} \right)^{2-\frac{\epsilon}{2}} \left[\Gamma\left(-1 + \frac{\epsilon}{2}\right) - \left(-2 + \frac{\epsilon}{2}\right) \Gamma\left(-2 + \frac{\epsilon}{2}\right) \right] = 0, \end{aligned} \quad (13)$$

which confirms the nullity of $\delta^4(0)$ in dimensional regularization [42]. This zero Higgs mass shift, parametrized by the ψ_μ momentum p_α , the right projector $P_R = (1 + \gamma_5)/2$, and the renormalization scale Q , is evaluated in the same regularization scheme as the Higgs mass shift in Eq. (3). This ensures that the active neutrinos and the Higgs boson can acquire their measured masses with no contradiction concerning the scale of M_ψ . The naturalness (fine-tuning) problem impeding the type-I seesaw simply does not exist in the type-3/2 seesaw, owing to Eq. (12).

IV. CONCLUSION

In this paper we have shown that the SM-singlet vector spinor [40] gives rise to a new one-step seesaw mechanism. This new mechanism, the type-3/2 seesaw, exhibits physically important features not found in the type-I seesaw. The salient features are as follows:

- (a) Tiny neutrino Majorana masses arise naturally (in agreement with the type-I seesaw).
- (b) The Higgs boson acquires its mass without fine-tuning (in disagreement with the type-I seesaw).

TABLE I. A comparative summary of the implications of the type-I and type-3/2 seesaw mechanisms.

	Type-I seesaw	Type-3/2 seesaw
ν oscillations	Yes	Yes
Stable Higgs mass	No	Yes
Dark matter	No ^a	Yes
Leptogenesis	Yes	No ^b

^aNote that the type-I seesaw does not result in any distinct dark matter candidates.

^bRemember that baryogenesis must occur via mechanisms other than leptogenesis.

- (c) If the on-shell vector spinor is a spin-3/2 field, then baryogenesis is not sourced by leptogenesis (in disagreement with the type-I seesaw), and this makes the model impervious to the gravitino problem in the type-I seesaw.
- (d) If the on-shell vector spinor is a spin-3/2 field, dark matter can exist as an everlasting field hidden in the SM spectrum (in disagreement with the type-I seesaw).

These points are tabulated in Table I in a comparative fashion. In view of them, one can conclude that the type-3/2 seesaw proposed in this paper can open up a novel approach to neutrino and dark sector phenomenology, with its inherent naturalness in both the neutrino and Higgs sectors. The minimal structure considered in this paper can be extended to make the spin-3/2 dark matter detectable. This can be done in various ways such as using higher-dimension operators as in [72,73] or adding new fields, such as a scalar field coupling to $\bar{\psi}_\mu \psi^\mu$ [74].

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