MISOCP-BASED SOLUTION APPROACHES TO THE UNIT COMMITMENT PROBLEM WITH AC POWER FLOWS

by
DENİZ TUNCER

Submitted to the Graduate School of Engineering and Natural Sciences
in partial fulfilment of
the requirements for the degree of Master of Science

Sabancı University
July 2021

MISOCP-BASED SOLUTION APPROACHES TO THE UNIT COMMITMENT PROBLEM WITH AC POWER FLOWS

Approved by:

Date of Approval: July 14, 2021

ABSTRACT

MISOCP-BASED SOLUTION APPROACHES TO THE UNIT COMMITMENT PROBLEM WITH AC POWER FLOWS

DENİZ TUNCER

INDUSTRIAL ENGINEERING M.S. THESIS, JULY 2021

Thesis Supervisor: Asst. Prof. Burak Kocuk

Keywords: unit commitment problem, optimal power flow problem, mixed-integer nonlinear programming, mixed-integer conic programming

Unit Commitment (UC) and Optimal Power Flow (OPF) are fundamental problems in short-term electrical power systems planning. Generally, the UC problem is solved to determine the commitment status of generators. Then, the OPF problem is solved to determine the power generation levels of committed generators. Instead of solving these problems in a serial manner, solving the UC problem and the OPF problem with AC power flows simultaneously as a mixed-integer nonlinear program (MINLP) can yield better results, but there is only a limited number of studies in the literature utilizing such an approach.

Adopting this approach, we develop a base algorithm, in which we solve a mixed-integer second order conic program (MISOCP) relaxation of the UC problem with AC power flows. Then, we solve the multiperiod OPF (MOPF) problem by a local solver, where commitment statuses from the first step are fixed to find a feasible solution to the original MINLP. The second step yields a feasible solution to the original problem. We then assess the quality of the feasible solution by using the lower bound obtained from the first step. In order to obtain better lower bounds, we add some valid inequalities that are originally developed for the OPF problem to the base algorithm, which we call enhanced algorithm.

For the problem instances with small number of buses, the base and the enhanced algorithms are able to provide small optimality gaps for the problem. However, it takes a long time to solve the MISOCP problem in larger instances. In order to

solve the larger instances, we adopt a Lagrangian decomposition method. With the addition of the mentioned valid inequalities, the quality of the lower bound of the Lagrangian subproblems are improved. Thanks to this decomposition method, we obtain feasible solutions to the problem instances that the other algorithms are not able to provide feasible solutions within a reasonable time limit.

ÖZET

ALTERNATİF GÜÇ AKIŞLI BİRİM ATAMA PROBLEMİ'NE İKİNCİ DERECEDEN KONİK KARMA TAMSAYILI PROGRAMLAMA TEMELLİ ÇÖZÜM YAKLAŞIMLARI

DENİZ TUNCER

ENDÜSTRİ MÜHENDİSLİĞİ YÜKSEK LİSANS TEZİ, TEMMUZ 2021

Tez Danışmanı: Dr. Burak KOCUK

Anahtar Kelimeler: birim atama problemi, eniyi güç akışı problemi, karma tamsayılı doğrusal olmayan programlama, karma tamsayılı konik programlama

Kısa dönemli elektrik sistemleri planlamasındaki temel problemlerden ikisi, Birim Tayini (Unit Commitment – UC) ve Eniyi Güç Akışı Problemleridir (Optimal Power Flow – OPF). Genellikle, bu iki problem ardışık olarak çözülür. UC problemi, açık jeneratörlerin çizelgesini elde etmek için her gün veya günaşırı çözülür. Ardından, OPF problemi, bu jeneratör çizelgesine uyumlu elektrik üretim miktarları elde etmek ve elektrik talebini karşılamak için 5-15 dakikalık aralıklarda çözülür. Bu iki problemi ardışık çözmek yerine, aynı anda bir karma tam sayılı doğrusal olmayan program (mixed integer nonlinear program - MINLP) olarak çözmek daha iyi sonuçlar verebilir, fakat literatürde bu yaklaşımı benimseyen az sayıda çalışma vardır.

Bu tezde, iki problemi aynı anda çözme yaklaşımı benimsenerek temel bir algoritma geliştirilmiştir. Bu algoritmanın ilk adımı bir karma tamsayılı ikinci dereceden konik program (mixed integer second order conic program - MISOCP) çözmeye dayanır iken, bu adımdan elde edilen jeneratör çizelgesi sabitlenip bir iç nokta çözücüsü ile çok dönemli eniyi güç akışı problemi çözülür. İlk problem bir gevşetme olup alt sınır sağlarken ikinci problem, MINLP problemi için olurlu çözüm elde eder. Bu iki değer kıyaslanarak olurlu çözümün eniyilik açıklığı hesaplanabilir. Temel algoritmaya ek olarak, eniyi güç akışı problemi için elde edilmiş bazı geçerli eşitsizlikler modele eklenerek daha güçlü alt sınır değerleri elde edilebilir.

Az sayıda jeneratör içeren problem örneklerinde, bu çözüm yöntemleri küçük eniy-

ilik açıklıkları vermektedir. Fakat, daha büyük problem örneklerinde ilk adımda çözülen MISOCP problemini çözmek uzun süre almaktadır. Bu nedenle, Lagrangian ayrıştırma metodu benimsenmiş ve belirtilen geçerli eşitsizlikler de eklenerek Lagrangian alt problemlerinin alt sınır niteliği iyileştirilmiştir. Bu ayrıştırma metodu sayesinde, kısıtlı zamanda olurlu çözüm bulunamayan problem örneklerine olurlu çözümler elde edilmiştir.

ACKNOWLEDGEMENTS

I would first like to thank Dr. Burak Kocuk for greatly helping me with his expertise on the subject. I feel privileged to have written this thesis under his supervision. I would also like to thank my family and my girlfriend for their support for me during my academic life.

This research is conducted as part of a TÜBİTAK project, with project number 119M855. I would also like to thank to TÜBİTAK for their support on young students with projects, and also thank again Dr. Kocuk for accepting me to this project. I would like to further thank the jury members, Prof. Güvenç Şahin and Dr. Murat Elhüseyni for taking part in the thesis jury.

TABLE OF CONTENTS

LIST OF TABLES

LIST OF FIGURES

NOMENCLATURE

Parameters:

$p_{i,t}^d(q_{i,t}^d)$	Active (reactive) energy demand of bus i in period t
$\delta(i)$	Set of neighbors of bus i
$g_{ii}(b_{ii})$	Shunt conductance (susceptance) of bus i
$p_i^{\min}(p_i^{\max})$	Minimum (maximum) active generation limit of generator i
$q_i^{\min}(q_i^{\max})$	Minimum (maximum) reactive generation limit of generator i
Y^{ij}	Admittance matrix of line (i,j)
G_{ij}^{ff} and B_{ij}^{ff}	Real and imaginary parts of Y_{11}^{ij}
G_{ij}^{ft} and B_{ij}^{ft}	Real and imaginary parts of Y_{12}^{ij}
G_{ij}^{tf} and B_{ij}^{tf}	Real and imaginary parts of Y_{21}^{ij}
G_{ij}^{tt} and B_{ij}^{tt}	Real and imaginary parts of Y_{22}^{ij}
$\overline{S}_{ij}$	Maximum power that can pass through line (i,j)
$C_{i,t}(p_{i,t}^g)$	Operational cost of generator i in period t
RU_i (RD_i)	Ramp up (down) rate of generator i
$MinUp_i$ $(MinDw_i)$	Minimum uptime (downtime) of generator i
$f_{i,t}(u_{i,t}, v_{i,t}, w_{i,t})$	Fixed cost of generator i in period t.

Decision Variables:

$\|V_{i,t}\|$	Voltage magnitude of bus i in period t
$\theta_{i,t}$	Voltage phase angle of bus i in period t
$p^g_{i,t}$ ($q^g_{i,t}$)	Active (reactive) power generation amount of generator i in period t
$\overrightarrow{p}_{ij,t}$ ($\overrightarrow{q}_{ij,t}$)	Active (reactive) power through line (i,j) in period t in forward direction
$\overleftarrow{p}_{ij,t}$ ($\overleftarrow{q}_{ij,t}$)	Active (reactive) power through line (i,j) in period t in backward direction
$u_{i,t}$	Commitment status of generator i in period t
$v_{i,t}$	Startup status of generator i in period t
$w_{i,t}$	Shutdown status of generator i in period t .

1. INTRODUCTION

Power systems problems are concerned with generation, transmission and distribution of energy in power networks. In power systems literature, there are short term, medium term and long term decisions. Examples of short term decisions include the Optimal Power Flow (OPF) problem, the Unit Commitment (UC) problem and the Optimal Transmission Switching (OTS) problem. In this thesis, we are interested in two of them: Optimal Power Flow and Unit Commitment problems.

The OPF problem determines the electricity generation amounts in a power grid given the electricity demand at each demand node. While minimizing the electricity generation cost, some physical constraints have to be satisfied. The problem is nonconvex due to alternating current (AC) power flow equations related to these physical constraints. There is also a direct current (DC) approach that simplifies the problem, which disregards the reactive powers and losses in the system. Due to these simplifications, the solution obtained by the DC approach may not satisfy the AC power flow equations and it must be further processed to satisfy the power flow equations. Solving the problem with the AC approach from the beginning may yield better solutions.

The UC problem is generally solved daily, and it determines the generator commitment schedule for the next few days given the electricity demand of the network. While minimizing the total electricity generation cost and operational costs, there are ramping, minimum uptime and downtime constraints for generators that need to be satisfied. Therefore, the problem is a multiperiod problem. Since commitment statuses and turn on-off decisions of generators are represented with binary variables, the problem is a mixed integer program.

In general, the UC problem is solved before the OPF problem. Then, given the generator commitment schedule, the OPF problem is solved. However, we believe that considering the AC power flows while deciding the commitment schedule would be a better approach, leading to more accurate commitment decisions. Therefore, we consider the UC Problem with AC power flows. We solve the UC problem and also add the power flow equations as constraints to make sure the commitment schedule that the UC problem outputs is AC feasible. Generally, obtaining globally optimal solutions for the UC with AC power flows problem is hard since we combine the challenging aspects of the OPF and UC problems. Therefore, we propose three different methods to solve the resulting problem and obtain solutions for which we can compute an optimality gap. Apart from the solution methods we propose, we create problem instances by utilizing publicly available OPF instances since there is a lack of publicly available instances for the UC problem with AC power flows.

The rest of this thesis is organised as follows. In Chapter 2, we review the literature on the OPF problem and present several mathematical programming formulations for the Multiperiod OPF (MOPF) problem. In Chapter 3, we review the literature on the UC problem and introduce the problem formulations. In Chapter 4, we propose three solution methods for the UC problem with AC power flows. All three solution methods are based on solving a mixed integer second order conic program (MISOCP) and then finding a feasible solution to the UC problem with AC power flows based on the solution of the MISOCP. One of these methods depends on the Lagrangian decomposition method in order to be able to solve large instances. In Chapter 5, we present the results of our computational experiments. In Chapter 6, we summarize our conclusions.

2. MULTIPERIOD OPTIMAL POWER FLOW PROBLEM

2.1 Literature Review

Optimal Power Flow (OPF) is a fundamental short term power system problem, along with Unit Commitment (UC) and Optimal Transmission Switching problems. The objective of the OPF problem is to find a minimum cost feasible electricity dispatch in a power network. While minimizing the system-wide production cost, the solution must also obey power flow laws, such as Ohm's Law and Kirchhoff's Law. The problem can be solved for a single time period, as well as being solved for multiple time periods.

Since OPF is a fundamental problem, the related literature is quite rich. For detailed literature reviews, one may see Frank, Steponavičė & Rebennack (2012a,1); Momoh, Adapa & El-Hawary (1999); Momoh, El-Hawary & Adapa (1999). There are different methods for solving the OPF problem. Out of those methods, we focus on nonlinear programming (NLP) methods, linear programming (LP) methods, quadratic programming (QP) methods, semidefinite programming (SDP) relaxations, and second order cone programming (SOCP) relaxations.

NLP methods generally focus on obtaining the stationary points of the problem. They are generally based on Newton's method or interior point methods. Newton's method requires using Lagrangian multipliers as penalty terms for the constraints in the constrained optimization problems (Frank et al., 2012a). Dommel & Tinney (1968) applies a reduced gradient method to the OPF problem. Following their work, Sasson, Viloria & Aboytes (1973) utilized the Newton's method for solving the OPF problem. da Costa (1997) proposes a primal-dual method using Newton's method along with an augmented Lagrangian, exploiting the sparsity of the Hessian matrix. Tognola & Bacher (1999) transform the optimality conditions, which allows the problem to be solvable by an algorithm solely based on the Newton-Raphson method. Jabr, Coonick & Cory (2002); Wu, Debs & Marsten (1993) also propose a primal-dual interior point method. While these methods are good at finding stationary points, they may get stuck at a local optimum point since the OPF problem is nonconvex (Bukhsh, Grothey, McKinnon & Trodden, 2013). Also, since the initial point is important for these methods they may fail to converge to a solution (Kocuk, Dey & Sun, 2016b).

LP based methods use the DC approximation of the AC power flows in the problem. Since an LP can be solved quickly to global optimality, LP based methods are widely used in industry to solve the OPF problem. Bienstock & Munoz (2014); Coffrin & Van Hentenryck (2012); Stott, Jardim & Alsac (2009) all use the LP based methods for solving the OPF problem. LP methods are also used in a successive linear programming (SLP) framework that is first proposed by Griffith & Stewart (1961). Iba, Suzuki, Suzuki & Suzuki (1988) utilize SLP and proposes two fast algorithms. However, the LP methods may not be able to converge to an AC feasible schedule since the reactive powers and losses are completely ignored or approximated.

A QP based method is utilized in Glavitsch & Spoerry (1983), where they propose a QP OPF formulation for the problem. Contaxis, Delkis & Korres (1986) propose a decoupled QP formulation, where they iteratively solve two subproblems using quadratic programming. Kocuk, Dey & Sun (2018) and Chen, Atamturk & Oren (2017) utilize the quadratically constrained quadratic programs (QCQP) in order to solve the AC OPF problem.

SDP relaxation methods have drawn interest due to their strength. SDP problems are efficiently solvable and if the line resistance is small, it might provide optimal OPF solutions (Lavaei & Low, 2012). Jabr (2012) exploits the sparsity in SDP relaxation of the OPF problem and reduces the computational effort of the SDP relaxation. Bai, Weihua, Fujisawa & Wang (2008) and Bai & Wei (2009) construct an SDP relaxation for the problem and develop an interior point algorithm for the SDP relaxation. Madani, Ashraphijuo & Lavaei (2016) use the conic relaxation in order to solve a penalized SDP problem. However, SDP methods may not always be exact or SDP relaxation may be feasible but OPF may be infeasible (Kocuk et al., 2016b).

The SOCP relaxation for the OPF problem is first proposed in Jabr (2006). Jabr (2008) proposes an extended conic quadratic formulation. Kocuk, Dey & Sun (2016a) utilizes the SDP relaxation of the cycles in the cycle basis of the network to obtain SDP cuts to strengthen the SOCP solutions. Currently, SOCP based methods are efficiently solvable with modern solvers.

We reviewed the literature on the OPF problem, whose literature is rich. However, the literature on the multiperiod OPF (MOPF) problem is somewhat limited. Alguacil & Conejo (2000) use the generalized Benders decomposition, in which network constraints are modeled according to a DC approach. Rabiee & Parniani (2013) use the generalized Benders decomposition as well, where they solve a mixed-integer nonlinear program. Demirovic, Tesnjak & Tokic (2006) utilize an LP based interior point algorithm and use the Newton-Raphson method as a corrector. Marley, Molzahn & Hiskens (2017) combine successive QP aproach with the SOCP relaxation of the problem. Schanen, Gilbert, Petra & Anitescu (2018) formulate the problem as a two stage nonlinear program, and solve the problem with an interior point method.

2.2 Formulations

Consider a power network $\mathcal{N} = (\mathcal{B}, \mathcal{L})$ in which the set $\mathcal{B}$ denotes the set of buses and $\mathcal{L}$ denotes the set of transmission lines. Generators are attached to the buses in the network, which are denoted by the set $\mathcal{G} \subseteq \mathcal{B}$. Given the demand of each bus at each time period in a planning horizon $\mathcal{T}$, the multiperiod AC OPF problem aims to determine the amount of energy produced at each generator, and the economic dispatch of the active and reactive energy.

In this section, we will present several formulations for the AC OPF problem. The AC OPF formulation requires defining the voltage V_i at bus i as a complex number. Polar (Section 2.2.1) and rectangular (Section 2.2.2) formulations differ in the way we represent the complex bus voltage. The alternative formulation (Section 2.2.3) gathers some nonconvex expressions and moves into a different variable space, which later can be converted to the original variable space. Then, we present the SOCP relaxation of the AC MOPF problem.

2.2.1 Polar Formulation

In the polar formulation, the bus voltages, V_i's are represented as $V_i = |V_i|(\cos(\theta_i) + i\sin(\theta_i))$, where $|V_i|$ is the voltage magnitude and θ_i is the phase angle.

Multiperiod OPF in polar formulation is defined as (2.1a)–(2.1m).

$$
\begin{aligned}
&\text{(2.1a)} && \min \sum_{t\in\mathcal{T}}\sum_{i\in\mathcal{G}} C_i(p^g_{i,t}) && \\
&\text{(2.1b)} && \text{s.t. } p^g_{i,t} - p^d_{i,t} = g_{ii}|V_{i,t}|^2 + \sum_{j\in\delta(i)} (\overrightarrow{p}_{ij,t} + \overleftarrow{p}_{ij,t}) && i\in\mathcal{B}, t\in\mathcal{T} \\
&\text{(2.1c)} && q^g_{i,t} - q^d_{i,t} = -b_{ii}|V_{i,t}|^2 + \sum_{j\in\delta(i)} (\overrightarrow{q}_{ij,t} + \overleftarrow{q}_{ij,t}) && i\in\mathcal{B}, t\in\mathcal{T} \\
&\text{(2.1d)} && \overrightarrow{p}_{ij,t} = G^{ff}_{ij}|V_{i,t}|^2 + |V_{i,t}||V_{j,t}|[G^{ft}_{ij}\cos(\theta_{i,t}-\theta_{j,t}) && \\
& && \qquad - B^{ft}_{ij}\sin(\theta_{i,t}-\theta_{j,t})] && (i,j)\in\mathcal{L}, t\in\mathcal{T} \\
&\text{(2.1e)} && \overleftarrow{p}_{ij,t} = G^{tt}_{ij}|V_{j,t}|^2 + |V_{i,t}||V_{j,t}|[G^{tf}_{ij}\cos(\theta_{j,t}-\theta_{i,t}) && \\
& && \qquad - B^{tf}_{ij}\sin(\theta_{j,t}-\theta_{i,t})] && (i,j)\in\mathcal{L}, t\in\mathcal{T} \\
&\text{(2.1f)} && \overrightarrow{q}_{ij,t} = -B^{ff}_{ij}|V_{i,t}|^2 - |V_{i,t}||V_{j,t}|[B^{ft}_{ij}\cos(\theta_{i,t}-\theta_{j,t}) && \\
& && \qquad + G^{ft}_{ij}\sin(\theta_{i,t}-\theta_{j,t})] && (i,j)\in\mathcal{L}, t\in\mathcal{T} \\
&\text{(2.1g)} && \overleftarrow{q}_{ij,t} = -B^{tt}_{ij}|V_{i,t}|^2 - |V_{i,t}||V_{j,t}|[B^{tf}_{ij}\cos(\theta_{j,t}-\theta_{i,t}) && \\
& && \qquad + G^{tf}_{ij}\sin(\theta_{j,t}-\theta_{i,t})] && (i,j)\in\mathcal{L}, t\in\mathcal{T} \\
&\text{(2.1h)} && \underline{V}_i \le |V_{i,t}| \le \overline{V}_i && i\in\mathcal{B}, t\in\mathcal{T} \\
&\text{(2.1i)} && (\overrightarrow{p}_{ij,t})^2 + (\overrightarrow{q}_{ij,t})^2 \le \overline{S}^2_{ij} && (i,j)\in\mathcal{L}, t\in\mathcal{T} \\
&\text{(2.1j)} && (\overleftarrow{p}_{ij,t})^2 + (\overleftarrow{q}_{ij,t})^2 \le \overline{S}^2_{ij} && (i,j)\in\mathcal{L}, t\in\mathcal{T} \\
&\text{(2.1k)} && q^{\min}_i \le q^g_{i,t} \le q^{\max}_i && i\in\mathcal{B}, t\in\mathcal{T} \\
&\text{(2.1l)} && p^{\min}_i \le p^g_{i,t} \le p^{\max}_i && i\in\mathcal{B}, t\in\mathcal{T} \\
&\text{(2.1m)} && -RD_i \le p^g_{i,t} - p^g_{i,t-1} \le RU_i && i\in\mathcal{B}, t\in\mathcal{T}.
\end{aligned}
$$

The objective function (2.1a) is a convex quadratic function of real power output of all generators. To be exact, $C_i(p^g_{i,t}) = Qd_i(p^g_{i,t})^2 + Li_i(p^g_{i,t}) + F_i$, where Qd_i is the quadratic cost, Li_i is the linear cost and F_i is the fixed cost of generator i.

Constraints (2.1b) and (2.1c) are real and reactive power balances at bus i, respectively. Constraints (2.1d) and (2.1e) are real power flows from bus i to j and from bus j to i, respectively. Constraints (2.1f) and (2.1g) are reactive power flows from bus i to j and from bus j to i, respectively. Constraint (2.1h) is the voltage magnitude bound at bus i. Constraints (2.1i) and (2.1j) are power flow upper bounds for line (i,j), for forward and backward flow, respectively. Constraints (2.1k) and (2.1l) are bounds for reactive and real power outputs of generator i, respectively. Constraints (2.1m) are the ramping constraints, which ensure that the active power generation amount does not change rapidly from one period to another.

2.2.2 Rectangular Formulation

The complex bus voltage is represented as the expression $V_i = e_i + if_i$ in the rectangular formulation. In this expression, e_i and f_i stand for real and imaginary parts of the complex voltage, respectively.

$$
\begin{aligned}
&(2.2a) && \min \sum_{t\in\mathcal{T}}\sum_{i\in\mathcal{G}} C_i(p^g_{i,t}) && \\
&(2.2b) && \text{s.t. } p^g_{i,t} - p^d_{i,t} = g_{ii}(e^2_{i,t} + f^2_{i,t}) + \sum_{j\in\delta(i)} (\overrightarrow{p}_{ij,t} + \overleftarrow{p}_{ij,t}) && i \in \mathcal{B}, t \in \mathcal{T} \\
&(2.2c) && q^g_{i,t} - q^d_{i,t} = -b_{ii}(e^2_{i,t} + f^2_{i,t}) + \sum_{j\in\delta(i)} (\overrightarrow{q}_{ij,t} + \overleftarrow{q}_{ij,t}) && i \in \mathcal{B}, t \in \mathcal{T} \\
&(2.2d) && \overrightarrow{p}_{ij,t} = G^{ff}_{ij}(e^2_{i,t} + f^2_{i,t}) + G^{ft}_{ij}(e_{i,t}e_{j,t} + e_{i,t}f_{j,t}) && \\
& && \qquad - B^{ft}_{ij}(e_{i,t}f_{j,t} - e_{j,t}f_{i,t}) && (i,j) \in \mathcal{L}, t \in \mathcal{T} \\
&(2.2e) && \overleftarrow{p}_{ij,t} = G^{tt}_{ij}(e^2_{i,t} + f^2_{i,t}) + G^{tf}_{ij}(e_{j,t}e_{i,t} + f_{j,t}e_{i,t}) && \\
& && \qquad - B^{tf}_{ij}(e_{i,t}f_{j,t} - e_{j,t}f_{i,t}) && (i,j) \in \mathcal{L}, t \in \mathcal{T} \\
&(2.2f) && \overrightarrow{q}_{ij,t} = -B^{ff}_{ij}(e^2_{i,t} + f^2_{i,t}) - B^{ft}_{ij}(e_{i,t}e_{j,t} + e_{i,t}f_{j,t}) && \\
& && \qquad - G^{ft}_{ij}(e_{i,t}f_{j,t} - e_{j,t}f_{i,t}) && (i,j) \in \mathcal{L}, t \in \mathcal{T} \\
&(2.2g) && \overleftarrow{q}_{ij,t} = -B^{tt}_{ij}(e^2_{i,t} + f^2_{i,t}) - B^{tf}_{ij}(e_{j,t}e_{i,t} + f_{j,t}e_{i,t}) && \\
& && \qquad + G^{tf}_{ij}(e_{i,t}f_{j,t} - e_{j,t}f_{i,t}) && (i,j) \in \mathcal{L}, t \in \mathcal{T} \\
&(2.2h) && \underline{V}^2_i \leq e^2_{i,t} + f^2_{i,t} \leq \overline{V}^2_i && i \in \mathcal{B}, t \in \mathcal{T} \\
& && (2.1i) - (2.1m). &&
\end{aligned}
$$

By formulating the problem in this manner, sine and cosine functions that are used in the polar formulation are eliminated. However, the formulation still has quadratic nonconvexities and difficult to solve in practice using global solvers.

2.2.3 Alternative Formulation

In this alternative formulation, Gómez Expósito & Romero Ramos (1999) manipulate the nonconvex expressions in the polar and rectangular formulations and define new decision variables. They reformulate the problem with the new variables and the solution obtained by solving the problem in this new variable space can be converted to the original variable space to obtain a feasible solution to the original problem. The new variables are defined as follows (Kocuk et al., 2016a):

$$c_{ii,t} := |V_{i,t}|^2 = e_{i,t}^2 + f_{i,t}^2 \qquad i \in \mathcal{B}, t \in \mathcal{T} \tag{2.3a}$$

$$c_{ij,t} := |V_{i,t}||V_{j,t}|\cos(\theta_{i,t} - \theta_{j,t}) = e_{i,t}e_{j,t} + f_{i,t}f_{j,t} \qquad (i,j) \in \mathcal{L}, t \in \mathcal{T} \tag{2.3b}$$

$$s_{ij,t} := -|V_{i,t}||V_{j,t}|\sin(\theta_{i,t} - \theta_{j,t}) = e_{i,t}f_{j,t} + e_{j,t}f_{i,t} \qquad (i,j) \in \mathcal{L}, t \in \mathcal{T}. \tag{2.3c}$$

Furthermore, we denote the lower (upper) bounds of variables $c_{ii,t}, c_{ij,t}, s_{ij,t}$ as $\underline{c}_{ii,t}, \underline{c}_{ij,t}, \underline{s}_{ij,t}$ ($\overline{c}_{ii,t}, \overline{c}_{ij,t}, \overline{s}_{ij,t}$), respectively. The bounds can be calculated as follows:

$$\underline{c}_{ii,t} := \underline{V}_{i,t}^2, \overline{c}_{ii,t} := \overline{V}_{i,t}^2 \qquad i \in \mathcal{B}, t \in \mathcal{T}$$

$$\overline{c}_{ij,t} = \overline{s}_{ij,t} = \underline{c}_{ij,t} = -\underline{s}_{ij,t} := \overline{V}_{i,t}\overline{V}_{j,t} \qquad (i,j) \in \mathcal{L}, t \in \mathcal{T}.$$

In order to keep the consistency of the relationship between variables, the consistency constraints are also added:

$$(c_{ij,t})^2 + (s_{ij,t})^2 = c_{ii,t}c_{jj,t} \qquad (i,j) \in \mathcal{L}, t \in \mathcal{T} \tag{2.5a}$$

$$s_{ij,t} = \tan(\theta_{j,t} - \theta_{i,t})c_{ij,t} \qquad (i,j) \in \mathcal{L}, t \in \mathcal{T}. \tag{2.5b}$$

Since new variables are defined, constraints (2.1b)-(2.1g) need to be updated as follows:

$$p_{i,t}^g - p_{i,t}^d = g_{ii}c_{ii,t} + \sum_{j \in \delta(i)} (\overrightarrow{p}_{ij,t} + \overleftarrow{p}_{ij,t}) \qquad i \in \mathcal{B}, t \in \mathcal{T} \tag{2.6a}$$

$$q_{i,t}^g - q_{i,t}^d = -b_{ii}c_{ii,t} + \sum_{j \in \delta(i)} (\overrightarrow{q}_{ij,t} + \overleftarrow{q}_{ij,t}) \qquad i \in \mathcal{B}, t \in \mathcal{T} \tag{2.6b}$$

$$\overrightarrow{p}_{ij,t} = G_{ij}^{ff}c_{ii,t} + G_{ij}^{ft}c_{ij,t} - B_{ij}^{ft}s_{ij,t} \qquad (i,j) \in \mathcal{L}, t \in \mathcal{T} \tag{2.6c}$$

$$\overleftarrow{p}_{ij,t} = G_{ij}^{tt}c_{jj,t} + G_{ij}^{tf}c_{ij,t} + B_{ij}^{tf}s_{ij,t} \qquad (i,j) \in \mathcal{L}, t \in \mathcal{T} \tag{2.6d}$$

$$\overrightarrow{q}_{ij,t} = -B_{ij}^{ff}c_{ii,t} - B_{ij}^{ft}c_{ij,t} - G_{ij}^{ft}s_{ij,t} \qquad (i,j) \in \mathcal{L}, t \in \mathcal{T} \tag{2.6e}$$

$$\overleftarrow{q}_{ij,t} = -B_{ij}^{tt}c_{jj,t} - B_{ij}^{tf}c_{ij,t} + G_{ij}^{tf}s_{ij,t} \qquad (i,j) \in \mathcal{L}, t \in \mathcal{T}. \tag{2.6f}$$

Then, alternative formulation is defined as (2.1a),(2.1i)–(2.1l), (2.5) and (2.6). This new formulation is instrumental when building the SOCP relaxation of the MOPF problem given in Section 2.2.4.

2.2.4 SOCP Relaxation

Constraints of the alternative formulation are all convex except for constraints (2.5a) and (2.5b). Constraint (2.5b) is completely relaxed and constraint (2.5a) is relaxed as a conic constraint. Then, the SOCP relaxation of the MOPF problem is obtained as follows:

$$\min \sum_{t \in \mathcal{T}} \sum_{i \in \mathcal{G}} C_i(p^g_{i,t}) \tag{2.7a}$$

$$\text{s.t. } (c_{ij,t})^2 + (s_{ij,t})^2 \leq c_{ii,t} c_{jj,t} \qquad (i,j) \in \mathcal{L}, t \in \mathcal{T} \tag{2.7b}$$

$$(2.1\text{i}) - (2.1\text{l}), (2.6\text{a}) - (2.6\text{f}), (2.1\text{m}).$$

SOCPs can be solved efficiently in practice using commerical interior point solvers such as MOSEK, Gurobi and CPLEX. Hence, solving the SOCP relaxation of the MOPF problem is quite tractable. We will make use of this relaxation to construct an MISOCP relaxation of the UC problem with AC power flows in Chapter 3.

2.2.5 DC Approximation

The DC OPF approximation is a linearization of the AC OPF problem. The DC approximation is obtained by using some physical properties of the power systems, such as small voltage angle differences between the buses. This approach ignores the presence of reactive powers and bus voltage magnitudes. The problem is modeled as a linear program (LP) and therefore can be solved quicker than the nonlinear programming (NLP) problem. However, the main drawback of this approximation is that it completely ignores the reactive powers which may cause errors on stressed networks (Sun & Phan, 2014). Before presenting the formulation, we need to define a new decision variable:

$f_{ij,t}$: Real power flow across line (i,j), $(i,j) \in \mathcal{L}, t \in \mathcal{T}$.

In addition, we define the following sets: $\delta^+(i) := \{j \in \mathcal{B} : (i,j) \in \mathcal{L}\}$ and $\delta^-(i) := \{j \in \mathcal{B} : (j,i) \in \mathcal{L}\}$.

Then, the DC approximation of the AC MOPF is given as follows:

$$\min \sum_{t \in \mathcal{T}} \sum_{i \in \mathcal{G}} C_i(p^g_{i,t}) \tag{2.8a}$$

$$\text{s.t. } p^g_{i,t} - p^d_{i,t} = \sum_{j \in \delta^+(i)} f_{ij,t} - \sum_{j \in \delta^-(i)} f_{ji,t} \qquad i \in \mathcal{B}, t \in \mathcal{T} \tag{2.8b}$$

$$f_{ij,t} = B_{ij}(\theta_{i,t} - \theta_{j,t}) \qquad (i,j) \in \mathcal{L}, t \in \mathcal{T} \tag{2.8c}$$

$$-\overline{S}_{ij} \le f_{ij,t} \le \overline{S}_{ij} \qquad (i,j) \in \mathcal{L}, t \in \mathcal{T} \tag{2.8d}$$

$$(2.1l) – (2.1m).$$

In this formulation, (2.8a) is a convex quadratic function of real power generation of generators. Constraint (2.8b) enforces flow conservation in each bus i. Constraint (2.8c) defines real power across the line (i,j). Constraint (2.8d) enforces an upper bound on the power over line (i,j). We will make use of this formulation to construct an MILP approximation of the UC problem with AC power flows in Chapter 3.

3. UNIT COMMITMENT PROBLEM WITH AC POWER FLOWS

3.1 Literature Review

Since Unit Commitment (UC) is a fundamental power systems problem, the literature on the problem is very rich. For comprehensive literature reviews, one can see Abdou & Tkiouat (2018); Bhardwaj, Kamboj, Shukla, Singh & Khurana (2012); Padhy (2004). Since the problem is NP-Hard (Guan, Zhai & Papalexopoulos, 2003), it is quite challenging to solve the problem to global optimality. Over the years, many different approaches for the problem are developed. The methods we consider can be listed as Lagrangian relaxation, mixed integer linear programming, Benders decomposition, semidefinite programming, conic relaxations and metaheuristics.

One of the traditionally used methods for the UC problem is the Lagrangian Relaxation (LR) method since it decomposes the problem and is efficiently solvable in general. Muckstadt & Koenig (1977) propose a branch and bound algorithm where they use a Lagrangian method and they decompose the problem into problems with a single generator. Murillo-Sanchez & Thomas (1999) utilize the LR method and then solve single period OPF problems. Ongsakul & Petcharaks (2004) relax the power balance and spinning reserve constraints and utilizes a two state dynamic programming model to obtain heuristic solutions to the problem. Dubost, Gonzalez & Lemaréchal (2005) utilize LR to find a lower bound and a primal relaxed solution, where the solution is used in a heuristic resolution method. Drawback of these methods is that the solution obtained by solving LR dual problems might not be primal feasible for the UC problem (Beltran & Heredia, 2002). One of the traditionally used methods for the UC problem is the Lagrangian Relaxation (LR) method since it decomposes the problem and is efficiently solvable in general. Muckstadt & Koenig (1977) propose a branch and bound algorithm where they use a Lagrangian method and they decompose the problem into problems with a single generator.

Murillo-Sanchez & Thomas (1999) utilize the LR method and then solve single period OPF problems. Ongsakul & Petcharaks (2004) relax the power balance and spinning reserve constraints and utilizes a two state dynamic programming model to obtain heuristic solutions to the problem. Dubost et al. (2005) utilize LR to find a lower bound and a primal relaxed solution, where the solution is used in a heuristic resolution method. Drawback of these methods is that the solution obtained by solving LR dual problems might not be primal feasible for the UC problem (Beltran & Heredia, 2002).

Mixed integer linear program (MILP) based methods depend on the DC approach for the power flow equations and is the state of the art for the UC problem. Solving an MILP is commonly used for solving the UC problem. MILP is an approximation of the original UC problem. Antonio, Gentile & Lacalandra (2009); Carrion & Arroyo (2006); Morales-España, Latorre & Ramos (2013); Ostrowski, Anjos & Vannelli (2012); Rajan & Takriti (2005) all propose formulations that tighten the MILP formulation. Nanou, Psarros & Papathanassiou (2021) propose incorporating the AC power flow constraints into the MILP model as piecewise linear constraints.

The Benders Decomposition (BD) method is another approach that is commonly used to solve the UC problem. Fu, Shahidehpour & Li (2006); Ma & Shahidehpour (1999); Sifuentes & Vargas (2007) utilize the BD method. They have a master problem and nonlinear subproblems, from which they obtain Benders cuts for the master problem. Paredes, Martins, Soares & Ye (2021) solve a mixed-integer problem where they linearize the power generation constraints. In subproblems, they solve the SDP relaxation of the AC OPF problem, which provide cuts for the master problem. The drawback of the BD approach is that it converges slowly and is computationally expensive (Watkins & McKinney, 1998).

Semidefinite Programming (SDP) and conic relaxation methods are also considered. Bai et al. (2008) propose an SDP relaxation for the AC OPF problem, which is useful while solving the UC considering AC power flows. This relaxation is useful for solving the UC with AC power flows. Lavaei & Low (2012) show that a global optimum for OPF problem can be found by using the SDP relaxation under some restrictive assumptions. Fattahi, Ashraphijuo, Lavaei & Atamturk (2016) propose a formulation for the UC with AC power flows problem, which is a single convex problem, whose global minimum can be found efficiently. However, Kocuk et al. (2016b) show that SDP relaxations may not always be exact.

Liu, Laird, Scott, Watson & Castillo (2018) follow a similar approach to ours in the study. They propose solving a master MISOCP problem and NLP subproblems which are multiperiod OPF problems. However, their approach is not able to produce solutions within a timeframe required for real world operations.

There are also metaheuristic methods utilized for the problem. For instance, Ting, Rao & Chu Kiong (2006) use hybrid particle swarm optimization for the problem. Ji, Yuan, Chen & Tian (2014) propose a gravitational search algorithm that considers the wind uncertainty.

3.2 Formulation

The aim of the UC problem is to determine the committed generators over a time horizon given the electricity demand of the power system. The objective functions of the UC problem is to minimize system-wide electricity generation cost, fixed and startup costs of generators. While satisfying the electricity demand, the UC problem aims to find a feasible commitment schedule that considers minimum up/down time and ramping constraints of generators as well. Generally, the commitment schedule is given to the OPF problem and OPF is solved to obtain the economic power dispatch afterwards. However, considering both UC and OPF problems simultaneously can yield a better solution.

In the literature, there are studies that consider a system-wide total electricity demand. However, in order to use the AC power flow approach, we consider the individual demands of each bus. Since we want to find a solution to the UC problem that also satisfies AC power flows, we replace the power flow and balance constraints in the original UC problem with AC power flow and balance constraints. We also assume that demand is deterministic, therefore the capacity reserve constraints are omitted. Similar to the MOPF formulation, we consider a set of generators at a power network $\mathcal{N} = (\mathcal{B}, \mathcal{L})$, where $\mathcal{B}$ denotes the set of buses. The set $\mathcal{G} \subseteq \mathcal{B}$ denotes the generators. Let $\mathcal{T}$ denote the set of time periods.

Since we have difficulty in finding problem instances, we create our own instances and the procedure to create the instances is explained in Section 5.1. For the instances, the history of generators is needed. Instead of creating history, we assume that the demand repeats itself daily. Hence, our planning horizon is cyclic. With this assumption, we are able to create instances that do not require the history of the generators. In order to represent the cyclic time horizon, we define some parameters:

$$t' = \begin{cases} t-1, & \text{if } t \neq 1 \\ 24, & \text{if } t = 1 \end{cases}$$

$$t'' = \begin{cases} t+1, & \text{if } t \neq 24 \\ 1, & \text{if } t = 24 \end{cases}$$

Index sets for the minimum uptime/downtime constraints are defined as follows:

$$\begin{aligned} T^{up}_{i,t} &:= \{x : x = (t - MinUp_i + j)\text{mod}\,24 + 1, \\ &\qquad j \in \mathbb{Z}^+_0, 0 \leq j \leq MinUp_i - 1\} && \forall g \in \mathcal{G}, \forall t \in \mathcal{T} \\ T^{dw}_{i,t} &:= \{x : x = (t - MinDw_i + j)\text{mod}\,24 + 1, \\ &\qquad j \in \mathbb{Z}^+_0, 0 \leq j \leq MinDw_i - 1\} && \forall g \in \mathcal{G}, \forall t \in \mathcal{T}. \end{aligned}$$

Then, the Unit Commitment Problem with AC power flows can be formulated as follows:

$$\begin{aligned}
&\text{(3.1a)} && \min \sum_{t \in \mathcal{T}} \sum_{i \in \mathcal{G}} (f_{i,t}(u_{i,t}, v_{i,t}, w_{i,t}) + c_{i,t}(p^g_{i,t})) \\
&\text{(3.1b)} && \text{s.t. } u_{i,t'} - u_{i,t} = v_{i,t} - w_{i,t} && i \in \mathcal{G}, t \in \mathcal{T} \\
&\text{(3.1c)} && v_{i,t} - u_{i,t} \leq 0 && i \in \mathcal{G}, t \in \mathcal{T} \\
&\text{(3.1d)} && w_{i,t} + u_{i,t} \leq 1 && i \in \mathcal{G}, t \in \mathcal{T} \\
&\text{(3.1e)} && \sum_{\tau \in T^{up}_{i,t}} v_{i,\tau} \leq u_{i,t} && i \in \mathcal{G}, t \in \mathcal{T} \\
&\text{(3.1f)} && \sum_{\tau \in T^{dw}_{i,t}} w_{i,\tau} \leq 1 - u_{i,t} && i \in \mathcal{G}, t \in \mathcal{T} \\
&\text{(3.1g)} && -RD_i \leq p^g_{i,t} - p^g_{i,t'} \leq RU_i && i \in \mathcal{G}, t \in \mathcal{T} \\
&\text{(3.1h)} && p^{min}_i u_{i,t} \leq p^g_{i,t} \leq p^{max}_i u_{i,t} && i \in \mathcal{G}, t \in \mathcal{T} \\
&\text{(3.1i)} && q^{min}_i u_{i,t} \leq q^g_{i,t} \leq q^{max}_i u_{i,t} && i \in \mathcal{G}, t \in \mathcal{T} \\
&\text{(3.1j)} && u_{i,t}, v_{i,t}, w_{i,t} \in \{0,1\} && i \in \mathcal{G}, t \in \mathcal{T} \\
& && (2.6\text{a}) - (2.6\text{f}), (2.5\text{a}) - (2.5\text{b}).
\end{aligned}$$

The first component of the objective function (3.1a) is a linear function of the binary variables: commitment status, startup status and shutdown status of the generators.

To be exact, $f_{i,t}(u_{i,t}, v_{i,t}, w_{i,t}) = F_i u_{i,t} + StUp_i v_{i,t} + ShDw_i w_{i,t}$, where F_i is the fixed cost, $StUp_i$ is the startup cost, and $ShDw_i$ is the shut down cost of the generator i. The second component of the objective function is a function of active power generation of generators, and is a quadratic function. Constraints (3.1b), (3.1c) and (3.1d) are the logical constraints for the relation between on-off statuses and turn on-off actions of generators. Constraints (3.1e) and (3.1f) enforce minimum uptime and downtime requirements for generator i, respectively. Constraint (3.1g) is the ramping constraint for generator i. Constraints (3.1h) and (3.1i) are the limits of real and reactive power outputs of generator i, considering the on-off status of the generator. Constraints (2.6a)-(2.6f), (2.5a)-(2.5b) account for the AC power flow equations.

3.2.1 MISOCP Relaxation

In this section, we formulate an MISOCP relaxation for the UC problem. In order to obtain the MISOCP relaxation, constraint (2.5a) is replaced with (2.7b) and constraint (2.5b) is completely omitted. Then, the MISOCP relaxation for the UC problem with AC power flows is formulated as:

$$\begin{aligned} \text{(3.2)} \qquad \min\ & \sum_{t \in \mathcal{T}} \sum_{i \in \mathcal{G}} (f_{i,t}(u_{i,t}, v_{i,t}, w_{i,t}) + c_{i,t}(p^g_{i,t})) \\ \text{s.t. } & (3.1\text{b}) - (3.1\text{j}) \\ & (2.6\text{a}) - (2.6\text{f}), (2.7\text{b}). \end{aligned}$$

3.2.2 DC Approximation

Similar to the MOPF problem, a DC approximation for the UC problem can be constructed. Assuming a linear objective function, the resulting problem is an MILP. The output of the MILP solution needs to be further processed for AC power flows and a feasible commitment schedule is obtained. The formulation for the DC approximation of UC problem is given as follows:

$$\min \sum_{t \in \mathcal{T}} \sum_{i \in \mathcal{G}} (f_{i,t}(u_{i,t}, v_{i,t}, w_{i,t}) + c_{i,t}(p^g_{i,t})) \tag{3.3}$$
$$\text{s.t. } (3.1b) - (3.1h), (3.1j)$$
$$(2.8b) - (2.8d).$$

3.2.3 Valid Inequalities

For the UC problem, valid inequalities proposed by Knueven, Ostrowski & Watson (2020) can be added to the formulation in order to solve the problem quickly and to small optimality gaps. They propose the valid inequalities by conducting a literature search and combining the different representations of the variables and constraints in order to obtain a formulation, which they call 'tight formulation'. We will now present how to obtain this formulation. To present the tight formulation, there are some additional decision variables and parameters that have to be defined.

New decision variables are defined as follows:

$p'_{i,t}$: Power generation amount above minimum for generator i at time period t
$\overline{p}_{i,t}$: Maximum power available for generator i at time period t
$\overline{p}'_{i,t}$: Maximum power available above minimum for generator i at time period t.

The relationship between those variables are stated as follows:

$$p_{i,t} = p'_{i,t} + p_i^{min} u_{i,t}$$
$$\overline{p}_{i,t} = \overline{p}'_{i,t} + p_i^{min} u_{i,t}$$
$$\overline{p}'_{i,t} = p'_{i,t}$$

New parameters are defined for each generator i as follows:

Then, the tight formulation is obtained as follows:

$$
\begin{aligned}
T_i^{RU} = & \quad \left\lfloor \frac{p_i^{max} - SU_i}{RU_i} \right\rfloor \\
T_i^{RD} = & \quad \left\lfloor \frac{p_i^{max} - SU_i}{RD_i} \right\rfloor \\
K_i^{SD}(t) = & \quad \min\{T_i^{RD}, UT_i - 1, T - t - 1\} \\
K_i^{SU}(t) = & \quad \min\{T_i^{RU}, UT_i - 2 - [K_i^{SD(t)}]^+, t - 1\}.
\end{aligned}
$$

- Replace the ramping constraints (3.1g) for all $i \in \mathcal{G}, \forall t \in \mathcal{T}$ with:

$$
\begin{aligned}
\overline{p}'_{i,t} - p'_{i,t-1} &\leq (SU_i - p_i^{min} - RU_i)v_{i,t} + RU_i u_{i,t} \\
p'_{i,t-1} - p'_{i,t} &\leq (SD_i - p_i^{min} - RD_i)w_{i,t} + RD_i u_{i,t-1}.
\end{aligned}
$$

- If $MinUp_i = 1$ for a generator i, add the following constraints for all $i \in \mathcal{G}, \forall t \in \mathcal{T}$:

$$
\begin{aligned}
p'_{i,t} + r_{i,t} &\leq (p_i^{max} - p_i^{min})u_{i,t} - (p_i^{max} - SU_i)v_{i,t} - [SU_i - SD_i]^+ w_{i,t+1} \\
p'_{i,t} + r_{i,t} &\leq (p_i^{max} - p_i^{min})u_{i,t} - (p_i^{max} - SD_i)w_{i,t+1} - [SD_i - SU_i]^+ v_{i,t}.
\end{aligned}
$$

- In order to tighten the ramping constraints, add the following constraints for all $i \in \mathcal{G}, \forall t \in \mathcal{T}$:

$$
\begin{aligned}
\overline{p}_{i,t} \leq p_i^{max} u_{i,t} - (p_i^{max} - SD_i)w_{i,t+1} & \\
- \sum_{j=0}^{min\{UT_i - 2, T_i^{RU}\}} & (p_i^{max} - SU_i - jRU_i)v_{i,t-j}.
\end{aligned}
$$

- If $T_i^{RU} > UT_i - 2$, add the following constraints for all $i \in \mathcal{G}, t \in \mathcal{T}$:

$$
\overline{p}_{i,t} \leq p_i^{max} u_{i,t} - \sum_{j=0}^{min\{UT_i - 1, T_i^{RU}\}} (p_i^{max} - SU_i - jRU_i)v_{i,t-j}.
$$

- If $K_i^{SD}(t) > 0$, add the following constraints for all $i \in \mathcal{G}, t \in \mathcal{T}$:

$$
\begin{aligned}
p_{i,t} \leq p_i^{max} u_{i,t} - \sum_{j=0}^{K_i^{SD}(t)} (p_i^{max} - (SD_i + jRD_i))w_{i,t+1+j} & \\
- \sum_{j=0}^{K_i^{SU}(t)} (p_i^{max} - (SU_i + jRU_i))v_{i,t-j}. &
\end{aligned}
$$

- For the relationship between the new variables, the following constraints are added for all $i \in \mathcal{G}, t \in \mathcal{T}$:

$$p'_{i,t} \leq \overline{p}'_{i,t}.$$

By making these changes in the formulation (3.1), the tight formulation can be obtained. The performance of this formulation is presented in Section 5.2.

4. SOLUTION METHODS

In this section, we present three different algorithms to solve the UC problem with AC power flows. In these algorithms, we make use of the fact that if we knew the generator commitment decisions, the UC problem would reduce to an MOPF problem, and then can be solved by an interior point solver, for instance IPOPT (Wächter & Biegler, 2006), in an efficient manner. In the algorithms, we first find a lower bound for the solution using global solver Gurobi (Gurobi Optimization, 2021) and then find a feasible solution, from which we can calculate the optimality gap.

4.1 Base Algorithm

In the base algorithm, we first solve the MISOCP relaxation (3.2) of the UC problem with AC power flows with the commercial solver Gurobi. The solution of (3.2) provides us a lower bound on the UC with AC power flows problem (3.1) and a candidate commitment schedule. When the commitment schedule (binary variables) is fixed in the UC problem, the problem simply becomes an MOPF problem. The MOPF problem is solved with IPOPT and a feasible solution is found. Then, optimality gap for the solution is calculated to measure the performance of the algorithm. A brief summary of the algorithm is given in Algorithm 1.

Algorithm 1: Base Algorithm

Result: Lower and upper bounds for UC with AC power flows

1. Solve the MISOCP relaxation (3.2) of the problem.
2. Obtain the commitment schedule.
3. Solve the MOPF problem described in Section 2.2.3 with the commitment schedule.
4. Calculate the optimality gap.

4.2 Enhanced Algorithm

Base algorithm is able to quickly produce solutions for the UC problem with small optimality gaps. For obtaining even smaller optimality gaps, it may be useful to improve the lower bound obtained by the MISOCP problem (3.2). To improve lower bounds, we use two methods originally derived for the OPF problem (Kocuk et al., 2016a) and adapt it to the MOPF problem. We will now briefly explain the two methods that are used in this algorithm.

4.2.1 Arctangent Envelopes

Constraint (2.5a) includes an arctangent function that is nonconvex. For convexifying this constraint, four linear envelopes can be utilized. We will now explain how to find these envelopes.

For each line (i,j), let $\theta_{i,j} = \theta_j - \theta_i$. For simplicity, if i and j indices are dropped, the equation can be written as $\theta = \arctan(s/c)$, with $(c,s) \in [\underline{c},\overline{c}] \times [\underline{s},\overline{s}]$.

Assuming $\underline{c} > 0$, the four corners of the box constraints are defined in the (c,s,θ) space as:

$$z^1 = (\underline{c},\overline{s},\arctan(\overline{s}/\underline{c})), \qquad z^2 = (\overline{c},\overline{s},\arctan(\overline{s}/\overline{c})),$$
$$z^3 = (\overline{c},\underline{s},\arctan(\underline{s}/\overline{c})), \qquad z^4 = (\underline{c},\underline{s},\arctan(\underline{s}/\underline{c})).$$

Then, the following can be computed:

- $\theta = \gamma_1 + \alpha_1 c + \beta_1 s$, the plane that passes through $\{z^1, z^2, z^3\}$,
- $\theta = \gamma_2 + \alpha_2 c + \beta_2 s$, the plane that passes through $\{z^1, z^3, z^4\}$,
- $\theta = \gamma_3 + \alpha_3 c + \beta_3 s$, the plane that passes through $\{z^1, z^2, z^4\}$,
- $\theta = \gamma_4 + \alpha_4 c + \beta_4 s$, the plane that passes through $\{z^2, z^3, z^4\}$.

We can find the valid inequalities that approximate the upper envelope of the arctangent constraint by following the steps below:

- Find $\Delta\gamma_k$ by solving the optimization problem:

$\Delta\gamma_k = \max\{\arctan(s/c) - (\gamma_k + \alpha_k c + \beta_k s) : c \in [\underline{c}, \overline{c}],\ s \in [\underline{s}, \overline{s}]\}$, for $k = 1,2$

- Let $\gamma_k' = \gamma_k + \Delta\gamma_k$,
- Add constraint $\gamma_k' + \alpha_k c + \beta_k s \geq \arctan(s/c)$, for $k = 1,2$.

For the valid inequalities that approximate the lower envelope of the arctangent constraint, we can follow a similar procedure:

- Find $\Delta\gamma_k$ by solving the optimization problem:
  $\Delta\gamma_k = \max\{(\gamma_k + \alpha_k c + \beta_k s) - \arctan(s/c) : c \in [\underline{c}, \overline{c}],\ s \in [\underline{s}, \overline{s}]\}$, for $k = 3,4$
- Let $\gamma_k' = \gamma_k - \Delta\gamma_k$,
- Add constraint $\gamma_k' + \alpha_k c + \beta_k s \leq \arctan(s/c)$, for $k = 3,4$.

For solving the optimization problems mentioned above, the Karush-Kuhn-Tucker points are enumerated. By evaluating the objective function value at each KKT point, the point that yields the maximum objective function value is used.

4.2.2 SDP Separation

In the literature, the SDP relaxation of the OPF problem is utilized for solving the OPF problem. Since utilizing the SDP relaxation of the whole problem is computationally expensive, Kocuk et al. (2016a) suggest obtaining a cycle basis and applying SDP to the cycles of the cycle basis in order to obtain cutting planes. Referring to the rectangular formulation (2.2), we have the following system of equations:

$$
\begin{aligned}
c_{ij} &= e_i e_j + f_i f_j = W_{ij} + W_{i'j'} && (i,j) \in \mathcal{L} \\
s_{ij} &= e_i f_j + e_j f_i = W_{ij'} - W_{ji'} && (i,j) \in \mathcal{L} \\
c_{ii} &= e_i^2 + f_i^2 = W_{ii} + W_{i'i'} && i \in \mathcal{B}.
\end{aligned}
$$

From this system of equations, if solutions of all W variables satisfy the relationship $W = [e;f][e;f]^T \succeq 0$, then they are also feasible for the UC problem. However, since this relaxation is computationally expensive for the entire problem, we solve SDP relaxations for the cycles in the cycle basis of the network. For a given cycle C, we consider the matrix $\tilde{W} \in \mathbb{R}^{2|C| \times 2|C|}$, which is a submatrix of W and consider the following set, where $z = (c, s)$ denotes the variables of the cycle C:

$$S := \{z \in \mathbb{R}^{2|C|} : \exists \tilde{W} \in \mathbb{R}^{2|C| \times 2|C|} \text{s.t.} \; -z_l + A_l \bullet \tilde{W} = 0 \; \forall l \in L, \tilde{W} \succeq 0\}.$$

Given a feasible solution of (3.2) $z^* = (c^*, s^*)$, for the set S, where e is a vector with all entries equal to 1, the separation problem is formulated as follows:

$$\begin{aligned} v^* := &\min\{-\alpha^T z^*\} \\ \text{s.t.} &\sum_{l \in L} \lambda_l A_l \succeq 0 \\ &\alpha + \lambda = 0 \\ &-e \leq \alpha \leq e. \end{aligned} \tag{4.2}$$

If we solve this SDP and $v^* \leq 0$, the inequality ${\alpha^*}^T z \leq 0$ can be added to the OPF formulation as a constraint, where α^* is an optimal solution. Applying this procedure to the cycles in the cycle basis numerous times, we can obtain a stronger dual value. This method works especially well in the congested networks.

4.2.3 Applying Enhanced Algorithm

We explain the two methods that are used in the enhanced algorithm in Sections 4.2.1 and 4.2.2. The enhanced algorithm starts with solving the continuous relaxation of the MISOCP (3.2). This problem might include the arctangent envelope constraints added for each line in the network. Then, given the solution of the continuous relaxation of (3.2), the SDP separation problem (4.2) is solved in order to obtain cuts. Having added the cuts, the continuous relaxation is resolved. This procedure is repeated five times, where in the last iteration we solve the MISOCP problem (3.2) with all of the cuts obtained. The solution of this problem provides a lower bound on the UC with AC power flows problem. From the same solution, we obtain the generator commitment schedule and solve the MOPF problem described in Section 2.2.3. An explanation of the enhanced algorithm is presented in

Algorithm 2.

Algorithm 2: Enhanced Algorithm

Result: Lower and upper bounds for UC problem with AC power flows

1. Compute a cycle basis.
2. For each edge, add the arctangent constraints, explained in Section 4.2.1
3. Solve the continuous relaxation of the MISOCP (3.2).
4. For i=1 to 5
    1. Solve the separation problem (4.2) for each cycle in the cycle basis in parallel.
    2. Add the cuts obtained from the separation problem and resolve the continuous relaxation of MISOCP (3.2).
5. Solve the MISOCP problem with the cuts obtained.
6. Obtain the generator commitment decisions.
7. Solve the problem described in Section 2.2.3.
8. Calculate the optimality gap.

4.3 Decomposition Method

The base and enhanced algorithms depend on solving the MISOCP problem (3.2). This problem can be solved quickly for instances with small number of buses. However, for some instances with more than 100 buses, problem (3.2) typically takes more than one hour to solve. For example, for the 118 bus instance, we solve problem (3.2) with one hour time limit. At the end of time limit, the candidate commitment schedule we obtain is sent to IPOPT and MOPF described in Section 2.2.3 is solved. However, the solver returns a local infeasible point, which may be caused by solver error or the candidate commitment schedule being inaccurate. Therefore, we had to come up with a method that would quicken the solution of the MISOCP problem.

In order to solve the MISOCP, we propose a decomposition with respect to the time horizon of the problem. Assume that we divide the time horizon of the problem (3.2) into B many blocks. Then, for the ramping constraints (3.1g), logical constraints (3.1b) and minimum up-downtime constraints (3.1e)-(3.1f) we make several modifications. For these constraints, if there are variables whose time period belong to more than one block, we relax these constraints. For instance, assume that we divide the time horizon into 4 equally sized parts. The first two subproblems we solve have the time horizons 1 to 6 and 7 to 12, respectively. Then, the ramping constraint (3.1g) for $t = 7$ includes the variables with the time index $t = 7$ and $t = 6$. Therefore, we relax the ramping constraint with the time index $t = 7$. When we apply the same logic to the above mentioned constraints, the blocks become independent of each other, hence, they can be solved separately. The way to express the constraints and the objective function of each block are explained in detail in (4.3) and (4.4).

The method starts with solving the continuous relaxation of the MISOCP problem having added the arctangent envelopes explained in Section 4.2.1. This continuous relaxation is solved together with the cuts obtained from the SDP separation problem explained in Section 4.2.2. After five iterations of solving the continuous relaxation, we obtain the optimal dual variable values for the constraints that are going to be relaxed. Then, for each relaxed constraint, its Lagrangian multiplier is assigned as the optimal dual variable value of the constraint. Then, the Lagrangian relaxed MISOCP problem (4.4) is solved for each block. The sum of the lower bounds from each block provides a lower bound on the original MINLP. This lower bound is expected to be weaker than solution of the 24-period MISOCP (3.2) since we relax some constraints. However, the lower bound obtained is at least as strong as the lower bound obtained by solving the continuous relaxation of the MISOCP since we use the optimal duals from the continuous relaxation.

However, the schedule obtained by solving each block separately may not satisfy the minimum uptime-downtime relations and ramping relations for the original time horizon. Therefore, given the output of each block, we solve another MISOCP, which we call a restricted MISOCP. The restricted MISOCP is the same as (3.2) except for the three types of additional constraints. The first type of additional constraint enforces a generator to be turned on in all of the time horizon, if all blocks decide that the generator is always turned on for their respective time horizon. The second type of additional constraint enforces a generator to be turned off, if the generator is not turned on in any of the blocks. The third type of constraint enforces that if a generator is turned on in a time period in any block, that generator needs to be turned on in that period. The additional constraints are added and the restricted MISOCP is solved. The solution of the restricted MISOCP yields the candidate commitment schedule and is sent to IPOPT. Then, IPOPT solves the MOPF problem described in Section 2.2.3 and finds a feasible solution for the UC with AC power flows problem. Then, the subgradient algorithm is applied and Lagrangian multipliers are updated. After obtaining new Lagrangian multipliers, we solve the Lagrangian relaxation of the MISOCP relaxation and solve the restricted MISOCP, then solve the MOPF again. The decomposition algorithm is given in

Algorithm 3.

Let us now give a formal description of our approach. We assume that the 24 periods are divided into blocks with respect to time indices. Each block b has a time horizon $\mathcal{T}_b$, which consists of consecutive time indices. We also assume for any b, $\mathcal{T}_b$ cannot contain the time indices 1 and 24 at the same time.

For simplicity in the formulations, we define the following parameters for each generator $i \in \mathcal{G}$, time index $t \in \mathcal{T}$ and block b:

$$
\begin{aligned}
&\underline{t_b} := \min(\mathcal{T}_b)\\
&\overline{t_b} := \max(\mathcal{T}_b)\\
&T_{i,t}^{intu} := T_{i,t}^{up} \cap (\mathcal{T} \setminus \mathcal{T}_b)\\
&T_{i,t}^{intd} := T_{i,t}^{dw} \cap (\mathcal{T} \setminus \mathcal{T}_b).\\
&T_{i,r1} := \{t : T_{i,t}^{up} \cap \mathcal{T}_b \neq \emptyset, T_{i,t}^{up} \not\subseteq \mathcal{T}_b\}\\
&T_{i,r2} := \{t : T_{i,t}^{intu} \neq \emptyset\}\\
&T_{i,r3} := \{t : T_{i,t}^{dw} \cap \mathcal{T}_b \neq \emptyset, T_{i,t}^{dw} \not\subseteq \mathcal{T}_b\}\\
&T_{i,r4} := \{t : T_{i,t}^{intd} \neq \emptyset\}.
\end{aligned}
$$

We denote the Lagrangian multipliers for the constraints that will be relaxed as follows:

$\lambda_{i,t}^{up}$:	Lagrangian multiplier for minimum uptime constraint
$\lambda_{i,t}^{dw}$:	Lagrangian multiplier for minimum downtime constraint
$\lambda_{i,t}^{log}$:	Lagrangian multiplier for the logical constraint
$\lambda_{i,t}^{ru}$:	Lagrangian multiplier for the ramp up constraint
$\lambda_{i,t}^{rd}$:	Lagrangian multiplier for the ramp down constraint.

Some of the constraints that contain multiple time indices are not relaxed since their time indices are a subset of time horizon of their blocks. Hence, we update the constraints as follows:

$$\sum_{\tau \in T_{i,t}^{up}} v_{i,\tau} \leq u_{i,t} \qquad i \in \mathcal{G}, t \in \mathcal{T}_b : T_{i,t}^{up} \subseteq \mathcal{T}_b \tag{4.3a}$$

$$\sum_{\tau \in T_{i,t}^{dw}} w_{i,\tau} \leq 1 - u_{i,t} \qquad i \in \mathcal{G}, t \in \mathcal{T}_b : T_{i,t}^{dw} \subseteq \mathcal{T}_b \tag{4.3b}$$

$$u_{i,T_t} - u_{i,t} + v_{i,t} - w_{i,t} = 0 \qquad i \in \mathcal{G}, t \in \mathcal{T}_b \setminus \{\underline{t_b}\} \tag{4.3c}$$

$$-RD_i \leq p_{i,t}^g - p_{i,T_t}^g \qquad i \in \mathcal{G}, t \in \mathcal{T}_b \setminus \{\underline{t_b}\} \tag{4.3d}$$

$$p_{i,t}^g - p_{i,T_t}^g \leq RU_i \qquad i \in \mathcal{G}, t \in \mathcal{T}_b \setminus \{\underline{t_b}\}. \tag{4.3e}$$

Then, for any block b, the Lagrangian subproblem is defined as follows:

$$\begin{aligned}
\min \; & \sum_{t \in \mathcal{T}_b} \sum_{i \in \mathcal{G}} (f_{i,t}(u_{i,t}, v_{i,t}, w_{i,t}) + c_{i,t}(p_{i,t}^g)) \\
& + \sum_{i \in \mathcal{G}} \lambda_{i,\underline{t_b}}^{ru} (p_{i,\underline{t_b}}^g - RU_i) + \sum_{i \in \mathcal{G}} \lambda_{i,\overline{t_b}''}^{ru} (-p_{i,\overline{t_b}}^g) \\
& + \sum_{i \in \mathcal{G}} \lambda_{i,\underline{t_b}}^{rd} (p_{i,\underline{t_b}}^g + RD_i) + \sum_{i \in \mathcal{G}} \lambda_{i,\overline{t_b}''}^{rd} (-p_{i,\overline{t_b}}^g) \\
& + \sum_{i \in \mathcal{G}} \lambda_{i,\underline{t_b}}^{log} (-u_{i,\underline{t_b}} + v_{i,\underline{t_b}} - w_{\underline{t_b}}) + \sum_{i \in \mathcal{G}} \lambda_{i,\overline{t_b}''}^{log} (u_{\overline{t_b}}) \\
& + \sum_{t \in T_{i,r1}} \sum_{i \in \mathcal{G}} \lambda_{i,t}^{up} (\sum_{k \in T_{i,t}^{intu}} (v_{i,k}) - u_{i,t}) + \sum_{t \in T_{i,r2}} \sum_{i \in \mathcal{G}} \lambda_{i,t}^{up} (\sum_{k \in T_{i,t}^{intu}} (v_{i,k})) \\
& + \sum_{t \in T_{i,r3}} \sum_{i \in \mathcal{G}} \lambda_{i,t}^{dw} (\sum_{k \in T_{i,t}^{intd}} (w_{i,k}) + u_{i,t} - 1) + \sum_{t \in T_{i,r4}} \sum_{i \in \mathcal{G}} \lambda_{i,t}^{up} (\sum_{k \in T_{i,t}^{intd}} (w_{i,k})) \\
\text{s.t. } & (4.3a) - (4.3e), (3.1h) - (3.1j), (2.6a) - (2.6f), (2.7b).
\end{aligned} \tag{4.4}$$

Algorithm 3: Decomposition Algorithm

Result: Lower and upper bounds for UC with AC power flows

1. Compute a cycle basis.
2. For each edge, add the arctangent constraints, explained in Section 4.2.1
3. Solve the continuous relaxation of the MISOCP (3.2).
4. For i=1 to 5
    1. Solve the separation problem (4.2) for each cycle in the cycle basis in parallel.
    2. Add the cuts obtained from the separation problem and resolve the continuous relaxation of MISOCP (3.2).
5. Obtain the Lagrangian multipliers from the previous step.
6. For i=1 to 5
    1. For each block b, solve the Lagrangian Relaxation problem (4.4).
    2. Obtain the commitment decisions for the original time horizon, solve the restricted MISOCP.
    3. Solve the MOPF described in Section 2.2.3 in order to find a feasible solution to the UC with AC power flows problem.
    4. Update the Lagrangian Multiplier values with the subgradient algorithm.
7. Take the best lower bound and the best feasible solution as the upper bound from the previous step.
8. Calculate the optimality gap.

5. COMPUTATIONAL RESULTS

In this chapter, we present our computational results. In Section 5.1, we explain the procedure used to construct the UC problem instances from AC OPF instances. In Section 5.2, we present the preliminary experiments comparing the OPF formulations and the UC formulations. In Section 5.3, we give detailed computational results for MISOCP based methods and DC based methods. In Section 5.3.2, we present the computational results for the decomposition approach.

5.1 Instance Creation

We had difficulty in finding realistic problem instances for the UC problem with AC power flows. Therefore, we decided to construct our own problem instances based on NESTA AC OPF instances (Coffrin, Gordon & Scott, 2019). To be able to convert the AC OPF instance to an AC UC instance, we need the following parameters for each generator $i \in \mathcal{G}$: ramp-up/down rate, minimum up/down time, startup cost and fixed cost. Also, for each bus $i \in \mathcal{B}$, we need the demand for 24 periods.

For each bus $i \in \mathcal{B}$, we randomly assign one of the three real demand profiles taken from (Kazarlis, Bakirtzis & Petridis, 1996), (Grigg, Wong, Albrecht, Allan, Bhavaraju, Billinton, Chen, Fong, Haddad, Kuruganty, Li, Mukerji, Patton, Rau, Reppen, Schneider, Shahidehpour & Singh, 1999) and (Castillo, Laird, Silva-Monroy, Watson & O'Neill, 2016) to the bus i. We assume that peak demand of the demand profile is equal to the demand of the AC OPF instance. For the reactive demand, we take the demand profile from (Castillo et al., 2016). The demand profiles are given in the Table 5.1.

Table 5.1 Demand profiles for creation of the problem instances

Time period	Real Profile 1	Real Profile 2	Real Profile 3	Max Real Profile	Reactive Profile
1	0.68	0.57	0.67	0.68	0.68
2	0.64	0.64	0.63	0.64	0.65
3	0.61	0.68	0.60	0.68	0.62
4	0.60	0.71	0.59	0.71	0.60
5	0.60	0.75	0.59	0.75	0.61
6	0.62	0.78	0.60	0.78	0.63
7	0.67	0.82	0.74	0.82	0.68
8	0.74	0.85	0.86	0.86	0.69
9	0.80	0.88	0.95	0.95	0.73
10	0.84	0.92	0.96	0.96	0.81
11	0.89	0.97	0.96	0.97	0.89
12	0.92	1.00	0.95	1.00	0.92
13	0.94	0.92	0.95	0.95	0.95
14	0.95	0.88	0.95	0.95	0.95
15	0.97	0.85	0.93	0.97	0.97
16	0.99	0.78	0.94	0.99	1.00
17	1.00	0.71	0.99	1.00	1.00
18	0.96	0.78	1.00	1.00	0.96
19	0.96	0.85	1.00	1.00	0.96
20	0.92	0.92	0.96	0.96	0.93
21	0.92	0.85	0.91	0.92	0.93
22	0.88	0.78	0.83	0.88	0.91
23	0.78	0.71	0.73	0.78	0.77
24	0.76	0.64	0.63	0.76	0.76

For the determination of the above mentioned parameters for the generators, we randomly assign each generator i a generator type, and then calculate the parameters based on their types. The calculation of the parameters are described as follows: For each generator $i \in \mathcal{G}$, we assign three types, Type 1, 2 and 3.

- If generator i is of Type 1, then $RU_i = RD_i = \max\{p_i^{min}, \frac{p_i^{max}}{2}\}$, $MinUp_i = MinDw_i = 2$.
- If generator i is of Type 2, then $RU_i = RD_i = \max\{p_i^{min}, \frac{p_i^{max}}{3}\}$, $MinUp_i = MinDw_i = 3$.
- If generator i is of Type 3, then $RU_i = RD_i = \max\{p_i^{min}, \frac{p_i^{max}}{5}\}$, $MinUp_i =$

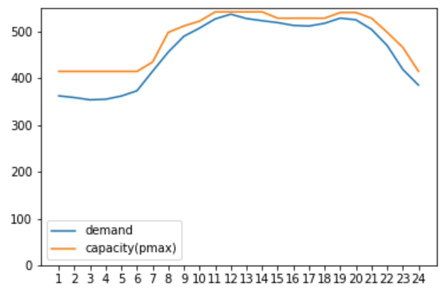
$MinDw_i = 4$.

We also experiment with different cost combinations for the fixed cost and the startup cost of the generators: cheap-cheap, cheap-expensive, expensive-cheap and expensive-expensive. For instance, cheap-expensive cost combination refers to cheap fixed cost and expensive startup cost. Below we explain how we calculate these parameters, assuming $ShDw_i = 0$:

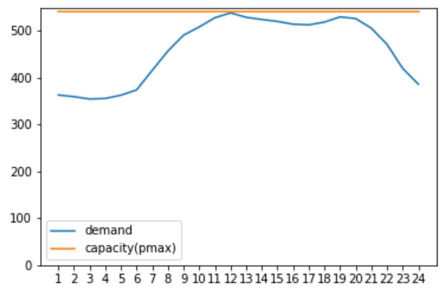
- For cheap-cheap cost combination, $F_i = 5Li_i$ and $StUp_i = 25Li_i$.
- For cheap-expensive cost combination, $F_i = 5Li_i$ and $StUp_i = 100Li_i$.
- For expensive-cheap cost combination, $F_i = 10Li_i$ and $StUp_i = 50Li_i$.
- For expensive-expensive cost combination, $F_i = 10Li_i$ and $StUp_i = 100Li_i$.

In order to verify the meaningfulness of these combinations, we solve an instance with different cost combinations and examine the generator turn on and off decisions. Figure 5.1 shows the total demand and the total generation capacity of the system.

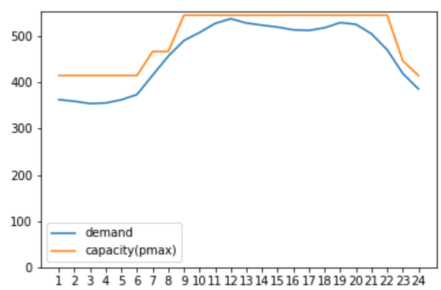
Figure 5.1 Total demand and the total generation capacity of the committed generators for the case29edin instance for different cost combinations.



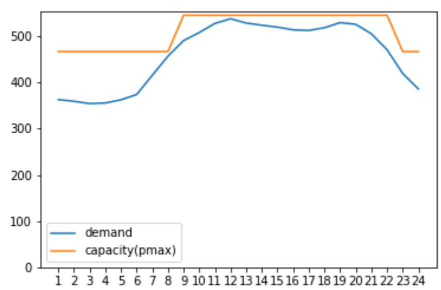
(a) Cheap fixed cost, cheap startup cost combination



(b) Cheap fixed cost, expensive startup cost combination



(c) Expensive fixed cost, cheap startup cost combination



(d) Expensive fixed cost, expensive startup cost combination

When we examine Subfigure 5.1a, we see that the capacity follows the demand trend closely, which is expected since both fixed and startup costs are cheap. When we examine Subfigure 5.1b and the solution of the problem, we see that the set of committed generators do not change. Since the fixed cost is cheap and startup cost is expensive, the solution does not include any turn on decisions for a generator, which is also expected. Subfigures 5.1c and 5.1d demonstrate that there are generator turn on and off decisions, which is expected compared to Subfigure 5.1b.

5.2 Preliminary Experiments

In this section, we present the results of the preliminary experiments, which we carried out on a laptop computer with 1.6GHz processor and 8 GB of RAM. We utilize the global solver Gurobi and local solver IPOPT.

5.2.1 OPF Formulations

In Section 2.2, we have presented different formulations for the OPF formulations. We conducted computational experiments in order to decide which formulations to use. We gave each problem 300 seconds of time limit. The results are presented in Table 5.2.

Table 5.2 Results for different OPF formulations

OPF Formulations	Solver		3lmbd	6ww	14ieee	29edin
Polar	IPOPT	LB	-	-	-	-
		UB	58.12	31.43	2.44	298.95
		Time (s)	0.16	0.17	0.18	0.93
Rectangular	Gurobi	LB	58.12	31.26	-	-
		UB	58.12	31.43	-	-
		Time (s)	5.24	300	300	300
SOCP Relaxation	Gurobi	LB	57.36	31.24	2.43	298.53
		UB	-	-	-	-
		% Gap	1.30	0.60	0.40	0.14
		Time (s)	0.02	0.03	0.03	0.16

Out of the three methods, the polar and rectangular formulations are exact formulations. Since the problem is solved with a local solver in the polar formulation, it can only provide an upper bound. Rectangular formulation is solved with a conic solver, therefore it yields both a lower and an upper bound for the problem. The SOCP relaxation is solved with a global solver, but since it is a relaxation, it can only provide a lower bound. The optimality gap for the SOCP relaxation is calculated by comparing its objective value with the objective value of the polar formulation. Out of the exact formulations, the polar formulation can be solved faster compared to rectangular formulation since it uses a local solver. For finding lower bound, we see that the SOCP relaxation provides optimality gaps less than 1.30%.

5.2.2 MISOCP Relaxation vs. Tight Formulation

In this section, we compare the two formulations for the UC problem with AC power flows. MISOCP is the problem (3.2). M-Tight is the modified tight formulation, which is obtained by applying the procedure explained in Section 3.2.3 to the MISOCP relaxation (3.2). We make the comparison between two formulations according to the results presented in Table 5.3.

Table 5.3 Results for MISOCP and M-Tight formulations

Instance	Method	LB	LB Time (s)	UB
case14	MISOCP	45.61	3.17	45.65
case14	M-Tight	45.61	3.52	45.65
case24	MISOCP	12116.24	1.02	12117.23
case24	M-Tight	12005.41	8.05	12117.23
case29	MISOCP	5298.03	21.35	5320.61
case29	M-Tight	5296.78	28.17	5326.19
case30	MISOCP	31.27	4.45	36.08
case30	M-Tight	31.27	9.03	36.08
case39	MISOCP	17710.92	8.03	17861.33
case39	M-Tight	17701.31	10.69	local inf

If the MOPF described in Section 2.2.3 is not feasible, we denote it by ‘local inf’. Table 5.3 shows that the M-Tight formulation does not outperform the MISOCP formulation. It generally takes more time to obtain a lower bound with M-Tight formulation than the MISOCP formulation, and also the feasible solution obtained by solving the MISOCP is not worse. Therefore, we decided to use the MISOCP formulation for solving the problem.

5.3 Detailed Computational Experiments

In this section, we present the results of the computational experiments for different methods used in order to solve the UC problem with AC power flows. For the following sections, the experiments are carried out in a desktop workstation with 3.7 GHz processor and 32 GB of RAM.

5.3.1 Comparison of MISOCP Based Methods and DC

For this subsection, the commitment decisions are found by solving either MISOCP relaxation of the original MINLP or the MILP approximation of the original MINLP. If the DC approach is used, the solution of the problem does not provide a lower bound to the original problem. However, for MISOCP based methods we are able to obtain the lower bound of the MISOCP problem. While solving the instances, we use 0.1% optimality gap tolerance.

In Tables 5.4-5.15, `DC` refers to solving the problem (3.3). For MISOCP based methods, `MISOCP` refers to solving the MISOCP relaxation (3.2). `MISOCP + ARC` and `MISOCP + SDP` refer to the problem (3.2) with arctangent constraints, and SDP cuts, respectively. `MISOCP++` refers to solving the problem (3.2) with both arctangent constraints and SDP cuts. All five methods are followed by a solution of the MOPF problem described in Section 2.2.3. 'LB time' is the sum of the solution times of SDP problems and solution time of MISOCP variants and 'UB time' is the solution time of the MOPF problem. Also, %gap is calculated by the formula: $\frac{(UB-LB)}{UB} \times 100$.

The `DC` method does not provide a lower bound for the problem. If the problem (3.3) is infeasible, we denote it by 'inf'. Also, if IPOPT fails to provide a feasible solution, we denote it by 'local inf'. Out of the 96 types of instances that we have attempted to solve, for 7 of them we were not able to find a feasible UC solution with the methods we have proposed. For only 2 types of those instances, the `DC` approach was able to find a feasible solution to the UC problem that satisfies AC power flows. However, the DC method was not successful to provide a feasible UC solution in 56 types of problems out of the 96 types of problems. The method fails in a big portion of the cases with small angle difference operating conditions. Although it is very fast, the DC method may also lead to suboptimal commitment schedules as seen in Tables 5.13 and 5.15, for case57ieee. In any of the instances, the DC method was not able to provide an AC feasible UC solution that has better objective value than the MISOCP based approaches. Therefore, we conclude that using MISOCP based methods rather than DC based methods are more accurate.

Let us now compare the MISOCP based approaches among each other. In general, applying SDP cuts to the problem instances increase the solution time of the MISOCP problems. For typical operating conditions, the average optimality gaps for the instances that can provide an upper bound, the average optimality gaps for MISOCP, MISOCP + ARC, MISOCP + SDP and MISOCP++ are 0.09, 0.09, 0.01 and 0.02, respectively. The average optimality gaps for the methods in congested operating conditions, in which there is increased electricity demand for each bus, are 0.33, 0.33, 0.16, and 0.16, respectively. The average optimality gaps for the methods in the small angle difference conditions are 0.40, 0.20, 0.11 and 0.06, respectively. On average, the MISOCP++ method provide the best optimality gap except for the congested operating conditions. However, the difference in average gap between MISOCP++ and MISOCP + ARC is small, hence it can be explained by optimality tolerances.

Let us now make a comparison of cost combinations for the MISOCP based methods. We are able to find AC feasible solutions for 74, 92, 74 and 81 problems out of 96 problems, for cheap-cheap, cheap-expensive, expensive-cheap and expensive-expensive cost combinations, respectively. Therefore, we can conclude that since cheap-expensive instances generally yield no or few number of commitment status change for generators, are easier to solve. To support the claim, we see that we are not able to find an AC feasible solution for almost one fourth of the problems with the expensive-cheap cost combination.

We proposed SDP cuts for improving the lower bound. However, we see in Table 5.4 for case30ieee, without adding SDP cuts, we are not able to obtain an AC feasible solution, which means that SDP cuts are also useful for finding an AC feasible schedule while improving the lower bound. We expect MISOCP++ to increase the lower bound values and reduce the optimality gap. However, in some cases, for example in Table 5.5 for case39epri, the MISOCP method provided the same feasible solution for the UC problem, but optimality gap is smaller than the MISOCP method. Such cases are explicable because we solve the MISOCP problems to 0.1% optimality gaps.

Table 5.4 Results for modified NESTA instances with typical operating conditions and cheap-cheap cost combination

case	method	LB Time	UB Time	UB	% Gap
case6ww	DC	0.01	0.78	4417.38	N/A
	MISOCP	0.18	0.77	4417.38	0.02
	MISOCP + ARC	0.32	0.78	4417.38	0.02
	MISOCP + SDP	4.43	0.76	4417.38	0.00
	MISOCP++	4.92	0.79	4417.38	0.00
case9wscc	DC	0.01	0.77	1563.87	N/A
	MISOCP	0.17	0.70	1563.87	0.00
	MISOCP + ARC	0.21	0.68	1563.87	0.00
	MISOCP + SDP	2.46	0.67	1563.87	0.00
	MISOCP++	2.23	0.67	1563.87	0.00
case14ieee	DC	0.01	3.80	local inf	N/A
	MISOCP	3.29	1.76	194.90	0.03
	MISOCP + ARC	2.29	1.64	194.89	0.02
	MISOCP + SDP	8.56	1.64	194.89	0.00
	MISOCP++	9.62	1.66	194.89	0.00
case24ieeerts	DC	0.06	4.64	155165.66	N/A
	MISOCP	1.08	3.63	155161.46	0.00
	MISOCP + ARC	1.29	3.61	155161.46	0.00
	MISOCP + SDP	13.52	3.50	155161.46	0.00
	MISOCP++	13.35	3.51	155161.46	0.00
case30as	DC	0.03	2.38	1832.06	N/A
	MISOCP	0.78	2.37	1832.06	0.00
	MISOCP + ARC	1.13	2.29	1832.06	0.00
	MISOCP + SDP	12.52	2.30	1832.06	0.00
	MISOCP++	12.87	2.36	1832.06	0.00
case30ieee	DC	0.05	4.40	local inf	N/A
	MISOCP	10.98	6.11	local inf	N/A
	MISOCP + ARC	8.25	6.74	local inf	N/A
	MISOCP + SDP	16.96	2.93	224.09	0.02
	MISOCP++	24.94	4.66	224.09	0.02
case39epri	DC	12.06	4.73	local inf	N/A
	MISOCP	315.68	5.56	28251.45	0.04
	MISOCP + ARC	453.55	5.58	28251.45	0.04
	MISOCP + SDP	283.97	5.51	28251.45	0.06
	MISOCP++	365.58	5.64	28251.45	0.04
case57ieee	DC	0.20	24.38	local inf	N/A
	MISOCP	37.86	6.55	538.88	0.04
	MISOCP + ARC	70.56	7.27	538.88	0.03
	MISOCP + SDP	49.41	6.79	538.88	0.01
	MISOCP++	110.14	7.89	538.88	0.01

Table 5.5 Results for modified NESTA instances with typical operating conditions and cheap-expensive cost combination

case	method	LB Time	UB Time	UB	% Gap
case6ww	DC	0.01	0.88	4417.38	N/A
	MISOCP	0.17	0.74	4417.38	0.02
	MISOCP + ARC	0.24	0.82	4417.38	0.02
	MISOCP + SDP	4.45	0.76	4417.38	0.00
	MISOCP++	5.02	0.73	4417.38	0.00
case9wscc	DC	0.01	0.70	1563.87	N/A
	MISOCP	0.16	0.74	1563.87	0.00
	MISOCP + ARC	0.21	0.67	1563.87	0.00
	MISOCP + SDP	1.93	0.68	1563.87	0.00
	MISOCP++	2.23	0.68	1563.87	0.00
case14ieee	DC	0.01	3.66	local inf	N/A
	MISOCP	1.57	1.67	227.14	0.10
	MISOCP + ARC	2.43	1.54	227.13	0.05
	MISOCP + SDP	7.75	1.73	227.13	0.00
	MISOCP++	8.93	1.24	227.13	0.01
case24ieeerts	DC	0.06	4.58	155165.66	N/A
	MISOCP	0.96	3.56	155161.46	0.00
	MISOCP + ARC	1.39	3.55	155161.46	0.00
	MISOCP + SDP	15.29	3.51	155161.46	0.00
	MISOCP++	14.24	3.51	155161.46	0.00
case30as	DC	0.02	2.37	1832.06	N/A
	MISOCP	0.84	2.32	1832.06	0.00
	MISOCP + ARC	1.19	2.38	1832.06	0.00
	MISOCP + SDP	12.07	2.35	1832.06	0.00
	MISOCP++	13.81	2.30	1832.06	0.00
case30ieee	DC	0.04	5.08	local inf	N/A
	MISOCP	6.87	3.14	235.46	2.14
	MISOCP + ARC	8.17	2.82	235.45	2.11
	MISOCP + SDP	13.79	3.71	235.44	0.02
	MISOCP++	20.66	2.92	235.45	0.02
case39epri	DC	0.31	4.07	local inf	N/A
	MISOCP	39.82	4.57	28336.21	0.03
	MISOCP + ARC	68.02	4.72	28336.21	0.12
	MISOCP + SDP	54.82	4.65	28336.21	0.03
	MISOCP++	73.21	4.55	28336.21	0.12
case57ieee	DC	0.08	5.85	542.19	N/A
	MISOCP	14.45	5.84	542.08	0.08
	MISOCP + ARC	21.60	5.97	542.08	0.05
	MISOCP + SDP	33.32	5.77	542.08	0.02
	MISOCP++	39.59	5.95	542.08	0.06

Table 5.6 Results for modified NESTA instances with typical operating conditions and expensive-cheap cost combination

case	method	LB Time	UB Time	UB	% Gap
case6ww	DC	0.01	0.80	8357.58	N/A
	MISOCP	0.17	0.75	8357.58	0.01
	MISOCP + ARC	0.24	0.85	8357.58	0.01
	MISOCP + SDP	4.41	0.75	8357.58	0.00
	MISOCP++	4.95	0.76	8357.58	0.00
case9wscc	DC	0.01	0.77	2427.87	N/A
	MISOCP	0.22	0.67	2427.87	0.00
	MISOCP + ARC	0.21	0.68	2427.87	0.00
	MISOCP + SDP	1.82	0.67	2427.87	0.00
	MISOCP++	2.10	0.73	2427.87	0.00
case14ieee	DC	0.01	3.69	local inf	N/A
	MISOCP	3.23	1.65	334.86	0.01
	MISOCP + ARC	2.55	1.70	334.86	0.02
	MISOCP + SDP	7.65	1.99	334.86	0.00
	MISOCP++	10.00	1.64	334.86	0.00
case24ieeerts	DC	0.06	4.58	300860.66	N/A
	MISOCP	1.07	3.55	300856.46	0.00
	MISOCP + ARC	1.28	3.52	300856.46	0.00
	MISOCP + SDP	13.67	3.58	300856.46	0.00
	MISOCP++	17.43	3.54	300856.46	0.00
case30as	DC	0.02	2.34	3512.06	N/A
	MISOCP	0.83	2.38	3512.06	0.00
	MISOCP + ARC	1.04	2.36	3512.06	0.00
	MISOCP + SDP	11.69	2.30	3512.06	0.00
	MISOCP++	13.33	2.36	3512.06	0.00
case30ieee	DC	0.05	4.35	local inf	N/A
	MISOCP	3.67	6.02	local inf	N/A
	MISOCP + ARC	6.05	6.03	local inf	N/A
	MISOCP + SDP	17.46	3.14	411.52	0.00
	MISOCP++	25.45	2.85	411.52	0.00
case39epri	DC	7.04	4.67	local inf	N/A
	MISOCP	343.97	6.48	37517.39	0.03
	MISOCP + ARC	434.75	6.52	37517.39	0.03
	MISOCP + SDP	189.58	6.53	37517.39	0.12
	MISOCP++	508.83	6.30	37517.39	0.12
case57ieee	DC	0.17	24.15	local inf	N/A
	MISOCP	56.06	7.51	849.07	0.02
	MISOCP + ARC	114.08	8.24	849.07	0.02
	MISOCP + SDP	57.25	8.30	849.07	0.03
	MISOCP++	93.10	7.62	849.07	0.01

Table 5.7 Results for modified NESTA instances with typical operating conditions and expensive-expensive cost combination

case	method	LB Time	UB Time	UB	% Gap
case6ww	DC	0.01	0.76	8357.58	N/A
	MISOCP	0.23	0.74	8357.58	0.01
	MISOCP + ARC	0.24	0.82	8357.58	0.01
	MISOCP + SDP	4.32	0.81	8357.58	0.00
	MISOCP++	5.05	0.76	8357.58	0.00
case9wscc	DC	0.00	0.71	2427.87	N/A
	MISOCP	0.16	0.66	2427.87	0.00
	MISOCP + ARC	0.21	0.74	2427.87	0.00
	MISOCP + SDP	1.79	0.73	2427.87	0.00
	MISOCP++	2.09	0.75	2427.87	0.00
case14ieee	DC	0.01	3.59	local inf	N/A
	MISOCP	3.51	1.63	358.90	0.01
	MISOCP + ARC	3.52	1.62	358.90	0.01
	MISOCP + SDP	7.54	1.59	358.90	0.00
	MISOCP++	9.95	1.64	358.90	0.00
case24ieeerts	DC	0.06	4.51	300860.66	N/A
	MISOCP	0.97	3.61	300856.46	0.00
	MISOCP + ARC	1.28	3.67	300856.46	0.00
	MISOCP + SDP	16.08	3.52	300856.46	0.00
	MISOCP++	13.46	3.53	300856.46	0.00
case30as	DC	0.02	2.34	3512.06	N/A
	MISOCP	0.74	2.40	3512.06	0.00
	MISOCP + ARC	1.13	2.30	3512.06	0.00
	MISOCP + SDP	11.82	2.29	3512.06	0.00
	MISOCP++	13.73	2.35	3512.06	0.00
case30ieee	DC	0.06	5.16	local inf	N/A
	MISOCP	3.57	6.03	local inf	N/A
	MISOCP + ARC	15.43	9.26	local inf	N/A
	MISOCP + SDP	18.12	2.92	434.23	0.00
	MISOCP++	22.90	2.99	434.22	0.00
case39epri	DC	2.12	4.65	37926.61	N/A
	MISOCP	48.01	4.51	37926.61	0.03
	MISOCP + ARC	91.49	4.45	37926.61	0.03
	MISOCP + SDP	62.13	4.46	37926.61	0.02
	MISOCP++	94.07	4.48	37926.61	0.11
case57ieee	DC	0.10	5.56	867.40	N/A
	MISOCP	45.01	5.71	867.29	0.03
	MISOCP + ARC	38.02	5.85	867.28	0.06
	MISOCP + SDP	41.32	5.86	867.29	0.02
	MISOCP++	70.13	5.70	867.28	0.01

Table 5.8 Results for modified NESTA instances with congested operating conditions and cheap-cheap cost combination

case	method	LB Time	UB Time	UB	% Gap
case6ww	DC	0.01	0.85	421.97	N/A
	MISOCP	0.21	0.87	421.97	0.21
	MISOCP + ARC	0.27	0.88	421.97	0.21
	MISOCP + SDP	4.39	0.87	421.97	0.05
	MISOCP++	4.99	4.99	421.97	0.04
case9wscc	DC	0.00	0.86	434.30	N/A
	MISOCP	0.17	0.76	434.30	0.08
	MISOCP + ARC	0.29	0.79	434.30	0.08
	MISOCP + SDP	1.84	0.76	434.30	0.00
	MISOCP++	2.12	0.80	434.30	0.00
case14ieee	DC	0.01	2.98	local inf	N/A
	MISOCP	1.95	3.37	local inf	N/A
	MISOCP + ARC	1.69	3.38	local inf	N/A
	MISOCP + SDP	7.34	2.01	200.90	0.04
	MISOCP++	10.02	1.96	200.90	0.00
case24ieeerts	DC	0.04	4.29	11336.22	N/A
	MISOCP	0.82	3.60	11335.96	1.23
	MISOCP + ARC	2.72	3.53	11335.96	1.22
	MISOCP + SDP	13.75	3.54	11335.96	0.77
	MISOCP++	14.47	3.57	11335.96	0.73
case30as	DC	0.02	2.46	795.26	N/A
	MISOCP	0.80	2.45	795.26	0.31
	MISOCP + ARC	1.28	2.35	795.26	0.31
	MISOCP + SDP	11.98	2.35	795.26	0.21
	MISOCP++	13.80	2.40	795.26	0.21
case30ieee	DC	0.04	7.76	local inf	N/A
	MISOCP	2.90	5.86	local inf	N/A
	MISOCP + ARC	6.59	4.58	local inf	N/A
	MISOCP + SDP	17.89	3.20	215.64	0.07
	MISOCP++	17.73	3.17	215.64	0.06
case39epri	DC	0.56	6.49	local inf	N/A
	MISOCP	176.65	7.64	local inf	N/A
	MISOCP + ARC	256.11	6.85	local inf	N/A
	MISOCP + SDP	266.21	7.63	local inf	N/A
	MISOCP++	178.82	7.59	local inf	N/A
case57ieee	DC	0.16	15.81	local inf	N/A
	MISOCP	32.69	6.85	610.45	0.07
	MISOCP + ARC	120.11	6.93	610.45	0.05
	MISOCP + SDP	61.38	6.93	610.45	0.02
	MISOCP++	114.48	6.80	610.45	0.02

Table 5.9 Results for modified NESTA instances with congested operating conditions and cheap-expensive cost combination

case	method	LB Time	UB Time	UB	% Gap
case6ww	DC	0.01	0.92	421.97	N/A
	MISOCP	0.20	0.89	421.97	0.21
	MISOCP + ARC	0.27	0.81	421.97	0.21
	MISOCP + SDP	4.41	0.86	421.97	0.05
	MISOCP++	5.24	0.84	421.97	0.05
case9wscc	DC	0.00	0.80	434.30	N/A
	MISOCP	0.17	0.83	434.30	0.08
	MISOCP + ARC	0.23	0.77	434.30	0.08
	MISOCP + SDP	1.95	0.77	434.30	0.00
	MISOCP++	2.17	0.80	434.30	0.00
case14ieee	DC	0.01	2.82	local inf	N/A
	MISOCP	2.12	1.53	214.07	0.13
	MISOCP + ARC	1.80	1.52	214.06	0.08
	MISOCP + SDP	6.92	1.52	214.06	0.03
	MISOCP++	8.45	1.36	214.06	0.02
case24ieeerts	DC	0.04	4.16	11336.22	N/A
	MISOCP	0.93	3.57	11335.96	1.23
	MISOCP + ARC	2.61	3.54	11335.96	1.22
	MISOCP + SDP	12.90	3.54	11335.96	0.75
	MISOCP++	15.07	3.55	11335.96	0.74
case30as	DC	0.02	2.42	795.26	N/A
	MISOCP	0.90	2.34	795.26	0.31
	MISOCP + ARC	1.27	2.42	795.26	0.31
	MISOCP + SDP	13.32	2.40	795.26	0.18
	MISOCP++	15.73	2.37	795.26	0.18
case30ieee	DC	0.03	7.55	local inf	N/A
	MISOCP	6.58	4.79	local inf	N/A
	MISOCP + ARC	4.96	4.33	local inf	N/A
	MISOCP + SDP	21.60	2.97	273.31	0.00
	MISOCP++	31.36	3.37	273.31	0.00
case39epri	DC	0.12	4.96	2211.70	N/A
	MISOCP	18.63	4.73	2211.70	1.34
	MISOCP + ARC	27.18	4.78	2211.70	1.34
	MISOCP + SDP	32.23	4.75	2211.70	1.30
	MISOCP++	49.16	4.73	2211.70	1.30
case57ieee	DC	0.08	5.82	632.60	N/A
	MISOCP	17.35	10.92	632.30	0.05
	MISOCP + ARC	30.25	11.00	632.30	0.05
	MISOCP + SDP	29.06	6.85	632.31	0.05
	MISOCP++	49.77	11.04	632.30	0.02

Table 5.10 Results for modified NESTA instances with congested operating conditions and expensive-cheap cost combination

case	method	LB Time	UB Time	UB	% Gap
case6ww	DC	0.01	0.85	800.41	N/A
	MISOCP	0.20	0.86	800.41	0.11
	MISOCP + ARC	0.27	0.81	800.41	0.11
	MISOCP + SDP	4.47	0.86	800.41	0.03
	MISOCP++	5.14	0.88	800.41	0.03
case9wscc	DC	0.00	0.87	747.13	N/A
	MISOCP	0.18	0.83	747.13	0.05
	MISOCP + ARC	0.23	0.76	747.13	0.05
	MISOCP + SDP	1.86	0.76	747.13	0.00
	MISOCP++	2.12	0.76	747.13	0.00
case14ieee	DC	0.01	2.80	local inf	N/A
	MISOCP	1.25	3.37	local inf	N/A
	MISOCP + ARC	1.79	3.37	local inf	N/A
	MISOCP + SDP	9.74	2.00	338.20	0.00
	MISOCP++	9.09	2.03	338.20	0.02
case24ieeerts	DC	0.04	3.81	21559.14	N/A
	MISOCP	0.92	3.48	21558.88	0.65
	MISOCP + ARC	2.59	3.54	21558.88	0.64
	MISOCP + SDP	14.07	3.57	21558.88	0.41
	MISOCP++	14.69	3.55	21558.88	0.39
case30as	DC	0.02	2.37	1482.55	N/A
	MISOCP	0.90	2.43	1482.55	0.17
	MISOCP + ARC	1.20	2.43	1482.55	0.17
	MISOCP + SDP	13.24	2.39	1482.55	0.12
	MISOCP++	14.20	2.39	1482.55	0.13
case30ieee	DC	0.04	7.57	local inf	N/A
	MISOCP	2.83	6.25	local inf	N/A
	MISOCP + ARC	5.68	7.94	local inf	N/A
	MISOCP + SDP	18.97	3.22	348.40	0.00
	MISOCP++	23.01	3.28	348.40	0.01
case39epri	DC	0.86	6.37	local inf	N/A
	MISOCP	140.66	7.62	local inf	N/A
	MISOCP + ARC	233.74	7.59	local inf	N/A
	MISOCP + SDP	157.20	7.64	local inf	N/A
	MISOCP++	232.35	7.63	local inf	N/A
case57ieee	DC	0.16	15.76	local inf	N/A
	MISOCP	37.00	6.84	943.42	0.04
	MISOCP + ARC	66.99	6.92	943.42	0.04
	MISOCP + SDP	45.88	7.01	943.42	0.01
	MISOCP++	80.75	5.83	943.43	0.05

Table 5.11 Results for modified NESTA instances with congested operating conditions and expensive-expensive cost combination

case	method	LB Time	UB Time	UB	% Gap
	`DC`	0.01	0.90	800.41	N/A
	`MISOCP`	0.20	0.87	800.41	0.11
case6ww	`MISOCP + ARC`	0.27	0.84	800.41	0.11
	`MISOCP + SDP`	4.49	0.80	800.41	0.03
	`MISOCP++`	5.08	0.85	800.41	0.02
	`DC`	0.00	0.80	747.13	N/A
	`MISOCP`	0.23	0.77	747.13	0.05
case9wscc	`MISOCP + ARC`	0.23	0.81	747.13	0.05
	`MISOCP + SDP`	1.79	0.82	747.13	0.00
	`MISOCP++`	2.07	0.85	747.13	0.00
	`DC`	0.01	2.80	local inf	N/A
	`MISOCP`	1.42	3.38	local inf	N/A
case14ieee	`MISOCP + ARC`	3.11	3.37	local inf	N/A
	`MISOCP + SDP`	7.50	1.35	366.51	0.07
	`MISOCP++`	9.28	3.42	local inf	N/A
	`DC`	0.04	4.19	21559.14	N/A
	`MISOCP`	0.92	3.56	21558.88	0.65
case24ieeerts	`MISOCP + ARC`	2.59	3.65	21558.88	0.64
	`MISOCP + SDP`	12.79	3.54	21558.88	0.41
	`MISOCP++`	14.95	3.50	21558.88	0.38
	`DC`	0.02	2.40	1482.55	N/A
	`MISOCP`	0.80	2.44	1482.55	0.17
case30as	`MISOCP + ARC`	1.27	2.33	1482.55	0.17
	`MISOCP + SDP`	12.37	2.34	1482.55	0.10
	`MISOCP++`	15.18	2.39	1482.55	0.10
	`DC`	0.04	7.55	local inf	N/A
	`MISOCP`	5.14	4.05	local inf	N/A
case30ieee	`MISOCP + ARC`	5.82	6.59	local inf	N/A
	`MISOCP + SDP`	19.06	3.23	386.86	0.00
	`MISOCP++`	22.94	3.64	386.87	0.01
	`DC`	0.43	5.98	local inf	N/A
	`MISOCP`	71.63	8.65	local inf	N/A
case39epri	`MISOCP + ARC`	64.26	8.69	local inf	N/A
	`MISOCP + SDP`	69.88	8.64	local inf	N/A
	`MISOCP++`	145.60	8.65	local inf	N/A
	`DC`	0.13	15.78	local inf	N/A
	`MISOCP`	30.25	6.57	987.14	0.04
case57ieee	`MISOCP + ARC`	54.54	11.12	987.13	0.06
	`MISOCP + SDP`	43.84	11.12	987.13	0.01
	`MISOCP++`	72.40	10.91	987.13	0.01

Table 5.12 Results for modified NESTA instances with small angle difference operating conditions and cheap-cheap cost combination

case	method	LB Time	UB Time	UB	% Gap
case6ww	DC	inf			N/A
	MISOCP	0.16	0.69	4417.40	0.02
	MISOCP + ARC	0.33	0.68	4417.40	0.01
	MISOCP + SDP	4.55	0.68	4417.40	0.00
	MISOCP++	5.31	0.74	4417.40	0.00
case9wscc	DC	inf			N/A
	MISOCP	0.17	0.76	1582.36	0.52
	MISOCP + ARC	0.21	0.82	1582.36	0.35
	MISOCP + SDP	1.85	0.81	1582.36	0.02
	MISOCP++	2.18	0.80	1582.36	0.03
case14ieee	DC	inf			N/A
	MISOCP	2.04	2.45	local inf	N/A
	MISOCP + ARC	3.28	3.72	local inf	N/A
	MISOCP + SDP	7.18	1.78	215.08	0.04
	MISOCP++	8.91	1.78	215.08	0.06
case24ieeerts	DC	inf			N/A
	MISOCP	3.12	4.78	157448.37	0.71
	MISOCP + ARC	6.21	4.78	157448.37	0.34
	MISOCP + SDP	17.52	4.78	157448.37	0.63
	MISOCP++	18.45	4.80	157448.37	0.25
case30as	DC	inf			N/A
	MISOCP	0.76	2.60	1843.64	0.48
	MISOCP + ARC	1.23	2.51	1843.64	0.17
	MISOCP + SDP	14.26	2.51	1843.64	0.03
	MISOCP++	15.66	2.56	1843.64	0.02
case30ieee	DC	inf			N/A
	MISOCP	5.36	5.09	local inf	N/A
	MISOCP + ARC	8.70	9.05	local inf	N/A
	MISOCP + SDP	20.63	3.34	224.09	0.02
	MISOCP++	25.70	3.34	224.09	0.02
case39epri	DC	1.58	4.57	local inf	N/A
	MISOCP	375.16	7.06	local inf	N/A
	MISOCP + ARC	1437.22	5.69	local inf	N/A
	MISOCP + SDP	514.97	8.18	local inf	N/A
	MISOCP++	1360.79	5.71	local inf	N/A
case57ieee	DC	0.13	6.60	606.39	N/A
	MISOCP	12.25	8.37	local inf	N/A
	MISOCP + ARC	57.79	8.52	local inf	N/A
	MISOCP + SDP	30.72	10.39	local inf	N/A
	MISOCP++	72.24	8.48	local inf	N/A

Table 5.13 Results for modified NESTA instances with small angle difference operating conditions and cheap-expensive cost combination

case	method	LB Time	UB Time	UB	% Gap
case6ww	DC	inf			N/A
	MISOCP	0.16	0.67	4417.40	0.02
	MISOCP + ARC	0.34	0.69	4417.40	0.01
	MISOCP + SDP	4.48	0.69	4417.40	0.00
	MISOCP++	5.07	0.72	4417.40	0.00
case9wscc	DC	inf			N/A
	MISOCP	0.16	0.77	1582.36	0.52
	MISOCP + ARC	0.28	0.76	1582.36	0.35
	MISOCP + SDP	1.92	0.76	1582.36	0.02
	MISOCP++	2.24	0.76	1582.36	0.03
case14ieee	DC	inf			N/A
	MISOCP	1.08	1.51	local inf	N/A
	MISOCP + ARC	1.58	1.34	227.13	0.10
	MISOCP + SDP	6.94	1.50	227.14	0.02
	MISOCP++	8.48	1.48	227.13	0.01
case24ieeerts	DC	inf			N/A
	MISOCP	3.02	4.84	157448.37	0.71
	MISOCP + ARC	6.24	4.79	157448.37	0.34
	MISOCP + SDP	16.36	4.86	157448.37	0.61
	MISOCP++	18.56	4.80	157448.37	0.25
case30as	DC	inf			N/A
	MISOCP	0.77	2.60	1843.64	0.48
	MISOCP + ARC	1.23	2.50	1843.64	0.17
	MISOCP + SDP	12.43	2.50	1843.64	0.02
	MISOCP++	16.68	2.56	1843.64	0.02
case30ieee	DC	inf			N/A
	MISOCP	5.20	2.91	235.44	1.02
	MISOCP + ARC	5.68	3.14	235.44	0.73
	MISOCP + SDP	17.50	3.08	235.44	0.00
	MISOCP++	20.59	3.11	235.44	0.01
case39epri	DC	0.13	3.57	29687.04	N/A
	MISOCP	36.50	5.89	local inf	N/A
	MISOCP + ARC	90.78	3.42	29687.04	0.04
	MISOCP + SDP	46.46	3.39	29687.04	0.14
	MISOCP++	68.63	3.38	29687.04	0.13
case57ieee	DC	0.11	6.39	642.30	N/A
	MISOCP	7.08	6.21	545.44	0.67
	MISOCP + ARC	17.56	6.37	545.44	0.30
	MISOCP + SDP	21.76	6.55	545.44	0.34
	MISOCP++	33.88	6.43	545.44	0.19

Table 5.14 Results for modified NESTA instances with small angle difference operating conditions and expensive-cheap cost combination

case	method	LB Time	UB Time	UB	% Gap
case6ww	DC	inf			N/A
	MISOCP	0.16	0.69	8357.60	0.01
	MISOCP + ARC	0.26	0.76	8357.60	0.00
	MISOCP + SDP	4.50	0.68	8357.60	0.00
	MISOCP++	5.13	0.74	8357.60	0.00
case9wscc	DC	inf			N/A
	MISOCP	0.16	0.83	2446.36	0.34
	MISOCP + ARC	0.21	0.76	2446.36	0.23
	MISOCP + SDP	1.93	0.75	2446.36	0.02
	MISOCP++	2.23	0.76	2446.36	0.02
case14ieee	DC	inf			N/A
	MISOCP	1.96	2.36	local inf	N/A
	MISOCP + ARC	3.19	3.16	local inf	N/A
	MISOCP + SDP	8.04	1.78	379.08	0.02
	MISOCP++	9.46	1.64	379.09	0.00
case24ieeerts	DC	inf			N/A
	MISOCP	3.13	4.82	303143.37	0.37
	MISOCP + ARC	6.16	4.89	303143.37	0.18
	MISOCP + SDP	16.28	4.81	303143.37	0.33
	MISOCP++	18.24	4.84	303143.37	0.13
case30as	DC	inf			N/A
	MISOCP	0.76	2.60	3523.64	0.25
	MISOCP + ARC	1.22	2.50	3523.64	0.09
	MISOCP + SDP	13.66	2.50	3523.64	0.01
	MISOCP++	13.88	2.56	3523.64	0.01
case30ieee	DC	inf			N/A
	MISOCP	8.06	5.84	local inf	N/A
	MISOCP + ARC	6.33	5.28	local inf	N/A
	MISOCP + SDP	16.42	3.18	411.52	0.09
	MISOCP++	25.67	3.25	411.52	0.01
case39epri	DC	2.44	4.42	local inf	N/A
	MISOCP	373.42	10.84	local inf	N/A
	MISOCP + ARC	821.12	8.09	local inf	N/A
	MISOCP + SDP	629.09	8.23	local inf	N/A
	MISOCP++	1193.13	8.05	local inf	N/A
case57ieee	DC	0.13	6.39	994.37	N/A
	MISOCP	20.89	9.36	local inf	N/A
	MISOCP + ARC	52.06	8.52	local inf	N/A
	MISOCP + SDP	33.90	8.45	local inf	N/A
	MISOCP++	84.33	8.45	local inf	N/A

Table 5.15 Results for modified NESTA instances with small angle difference operating conditions and expensive-expensive cost combination

case	method	LB Time	UB Time	UB	% Gap
case6ww	DC	inf			N/A
	MISOCP	0.16	0.69	8357.60	0.01
	MISOCP + ARC	0.27	0.77	8357.60	0.00
	MISOCP + SDP	4.48	0.67	8357.60	0.00
	MISOCP++	5.16	0.70	8357.60	0.00
case9wscc	DC	inf			N/A
	MISOCP	0.16	0.82	2446.36	0.34
	MISOCP + ARC	0.21	0.75	2446.36	0.23
	MISOCP + SDP	1.84	0.75	2446.36	0.01
	MISOCP++	2.25	0.80	2446.36	0.04
case14ieee	DC	inf			N/A
	MISOCP	1.90	4.33	local inf	N/A
	MISOCP + ARC	2.28	2.58	local inf	N/A
	MISOCP + SDP	7.50	1.81	403.13	0.09
	MISOCP++	10.48	1.58	403.13	0.01
case24ieeerts	DC	inf			N/A
	MISOCP	3.16	4.89	303143.37	0.37
	MISOCP + ARC	6.43	4.84	303143.37	0.18
	MISOCP + SDP	16.46	4.83	303143.37	0.33
	MISOCP++	20.26	4.96	303143.37	0.13
case30as	DC	inf			N/A
	MISOCP	0.76	2.60	3523.64	0.25
	MISOCP + ARC	1.22	2.58	3523.64	0.09
	MISOCP + SDP	16.12	2.56	3523.64	0.01
	MISOCP++	17.52	2.50	3523.64	0.01
case30ieee	DC	inf			N/A
	MISOCP	7.77	3.26	434.23	0.53
	MISOCP + ARC	6.66	2.85	434.23	0.40
	MISOCP + SDP	18.78	2.59	434.22	0.00
	MISOCP++	20.19	3.15	434.22	0.00
case39epri	DC	0.92	4.21	local inf	N/A
	MISOCP	90.64	5.88	local inf	N/A
	MISOCP + ARC	440.98	5.05	41346.06	0.15
	MISOCP + SDP	445.15	8.09	local inf	N/A
	MISOCP++	674.61	5.03	41346.06	0.04
case57ieee	DC	0.11	6.37	1036.22	N/A
	MISOCP	13.19	6.48	870.65	0.43
	MISOCP + ARC	39.34	6.47	870.65	0.23
	MISOCP + SDP	29.85	6.42	870.65	0.23
	MISOCP++	49.60	6.29	870.65	0.16

5.3.2 Decomposition Approach

In this subsection, we present our computational results for the decomposition algorithm described in Section 4.3. While finding the lower bound, unless there was a numerical problem with the SDP separation procedure, we have used the strengthened MISOCP method by adding arctangent envelopes and SDP cuts. The results presented in Tables 5.16 and 5.17 are for the typical operating conditions and cheap-expensive cost combination. Therefore, they may be compared with the results in Table 5.5. In the following cases, we solve the MISOCP problems to 1% optimality tolerance.

Table 5.16 Results for modified NESTA instances with decomposition approach after 1 iteration

Instance	B	After 1 iteration				
		LB Time	UB time	LB	UB	%Gap
case39epri	4	34.72	4.42	27856.96	29675.32	6.13
case39epri	6	28.64	4.40	27819.19	29675.32	6.25
case57ieee	4	36.12	7.39	541.94	542.08	0.03
case57ieee	6	33.92	7.38	541.94	542.08	0.03
case89pegase	4	26.73	96.47	2336.00	2337.23	0.05
case89pegase	6	26.52	88.92	2336.00	2337.23	0.05
case118ieee	4	1351.24	24.13	1550.08	1878.93	17.50
case118ieee	6	542.61	34.41	1558.35	2232.58	30.20
case500goc	4	744.13	384.71	626725.03	626791.30	0.01
case500goc	6	717.93	387.07	626501.81	626791.30	0.05

Table 5.17 Results for modified NESTA instances with decomposition approach after 5 iterations

Instance	B	After 5 iterations			
		Total Time	LB	UB	%Gap
case39epri	4	155.14	27935.24	29675.32	5.86
case39epri	6	123.05	27856.37	29675.32	6.13
case57ieee	4	162.80	541.94	542.08	0.03
case57ieee	6	144.54	541.94	542.08	0.03
case89pegase	4	526.81	2336.00	2337.23	0.05
case89pegase	6	492.62	2336.00	2337.23	0.05
case118ieee	4	4958.19	1551.65	1722.95	9.94
case118ieee	6	2057.51	1558.35	1961.92	20.57
case500goc	4	2891.60	626726.40	626791.30	0.01
case500goc	6	2891.40	626604.45	626791.30	0.03

Table 5.16 presents the results after only one iteration of the decomposition approach. In Table 5.17, the subgradient method is utilized for five iterations. 'B' represents how many equally sized subproblems are solved. For example, if number of blocks is 4, the Lagrangian relaxation is solved for time periods 1 to 6, 7 to 12, 13 to 18 and 19 to 24. For Table 5.16, 'LB time' is the sum of the solution times of SDP separation and Lagrangian subproblems. 'UB time' is the sum of the solution times of Restricted MISOCP and MOPF problems.

As expected, as the number of blocks increase, the lower bound found for the problem is not better than the solution with fewer number of blocks. However, as the size of the suproblems increase, the solution time for the subproblems also increases.

For case39epri, after one iteration of solving the decomposed problems, we achieve 6.13 and 6.25% optimality gaps for $B = 4$ and $B = 6$, respectively. After five iterations, these optimality gaps are improved to 5.86 and 6.13%. These numbers may be improved by allowing more subgradient iterations to update the Lagrangian multipliers.

For case118ieee, we are not able to obtain a feasible solution for the UC problem from the `MISOCP++` method in one hour. However, using the decomposition method, we are able to find a feasible solution to the problem in about 10 minutes. In addition, five iterations are able to improve the optimality gap from 17.50% to 9.94%. We also note that 89pegase instance has problems with SDP, therefore we do not utilize the SDP procedure and solve the Lagrangian subproblems with arctangent constraints.

For the cases case57ieee, case89pegase and case500goc, the decomposition method is able to find feasible solutions with less than 0.05% optimality gaps. However the instances, case39epri and case118ieee are harder to optimize to less than 1% optimality gap, and we have 5.86 and 9.94% optimality gaps, respectively. The solution time can be improved by solving the subproblems in parallel, which we did not implement. Therefore, there may still be room for more subgradient iterations, and possibly lower optimality gaps.

6. CONCLUSION

In this thesis, we have studied the UC problem with AC power flow equations. We have proposed three algorithms for the solution of the UC with AC power flow equations problem that are MISOCP based, and compared those methods with the classical DC based approach. Out of the three algorithms we propose for solving the problem, two of the algorithms are based on solving a 24-period MISOCP relaxation of the problem and then solving a MOPF to find a feasible solution. For instances up to 57 buses, the two methods which we call base algorithm and enhanced algorithm were able to produce feasible solutions to the problem within 2% optimality gap. For solving larger instances, we have developed a decomposition method based on the Lagrangian relaxation method. The decomposition method enabled us to solve instances with up to 500 buses. Apart from the proposed solution methods for the problems, since there was a lack of publicly available problem instances, we have constructed our own UC with AC power flows instances based on publicly available AC OPF instances. Future work may include strengthening the decomposition method and solving instances with more than 500 buses.

BIBLIOGRAPHY

Abdou, I. & Tkiouat, M. (2018). Unit commitment problem in electrical power system: A literature review. *International Journal of Electrical and Computer Engineering (IJECE)*, *8*, 1357.

Alguacil, N. & Conejo, A. (2000). Multiperiod optimal power flow using benders decomposition. *IEEE Transactions on Power Systems*, *15*(1), 196–201.

Antonio, F., Gentile, C., & Lacalandra, F. (2009). Tighter approximated milp formulations for unit commitment problems. *Power Systems, IEEE Transactions on*, *24*, 105 – 113.

Bai, X. & Wei, H. (2009). Semi-definite programming-based method for security-constrained unit commitment with operational and optimal power flow constraints. *Generation, Transmission and Distribution, IET*, *3*, 182 – 197.

Bai, X., Weihua, H., Fujisawa, K., & Wang, Y. (2008). Semidefinite programming for optimal power flow problems. *International Journal of Electrical Power and Energy Systems*, *30*, 383–392.

Beltran, C. & Heredia, F.-J. (2002). Unit commitment by augmented lagrangian relaxation: testing two decomposition approaches. *Journal of optimization theory and applications*, *112*(2), 295–314.

Bhardwaj, A., Kamboj, V. K., Shukla, V. K., Singh, B., & Khurana, P. (2012). Unit commitment in electrical power system-a literature review. In *2012 IEEE International Power Engineering and Optimization Conference Melaka, Malaysia*, (pp. 275–280).

Bienstock, D. & Munoz, G. (2014). On linear relaxations of opf problems.

Bukhsh, W., Grothey, A., McKinnon, K., & Trodden, P. (2013). Local solutions of the optimal power flow problem. *Power Systems, IEEE Transactions on*, *28*, 4780–4788.

Carrion, M. & Arroyo, J. (2006). A computationally efficient mixed-integer linear formulation for the thermal unit commitment problem. *IEEE Transactions on Power Systems*, *21*(3), 1371–1378.

Castillo, A., Laird, C., Silva-Monroy, C., Watson, J.-P., & O'Neill, R. (2016). The unit commitment problem with ac optimal power flow constraints. *IEEE Transactions on Power Systems*, *31*, 1–14.

Chen, C., Atamturk, A., & Oren, S. S. (2017). A spatial branch-and-cut method for nonconvex qcqp with bounded complex variables.

Coffrin, C., Gordon, D., & Scott, P. (2019). Nesta, the nicta energy system test case archive.

Coffrin, C. & Van Hentenryck, P. (2012). A linear-programming approximation of ac power flows. *INFORMS Journal on Computing*, *26*.

Contaxis, G. C., Delkis, C., & Korres, G. (1986). Decoupled optimal load flow using linear or quadratic programming. *IEEE Transactions on Power Systems*, *1*(2), 1–7.

da Costa, G. (1997). Optimal reactive dispatch through primal-dual method. *IEEE Transactions on Power Systems*, *12*(2), 669–674.

Demirovic, N., Tesnjak, S., & Tokic, A. (2006). Hot start and warm start in lp based interior point method and it's application to multiperiod optimal power flows.

In *2006 IEEE PES Power Systems Conference and Exposition*, (pp. 699–704).

Dommel, H. W. & Tinney, W. F. (1968). Optimal power flow solutions. *IEEE Transactions on Power Apparatus and Systems*, *PAS-87*(10), 1866–1876.

Dubost, L., Gonzalez, R., & Lemaréchal, C. (2005). A primal-proximal heuristic applied to the french unit-commitment problem. *Math. Program.*, *104*(1), 129–151.

Fattahi, S., Ashraphijuo, M., Lavaei, J., & Atamturk, A. (2016). Conic relaxation of the unit commitment problem. *Energy*, *134*.

Frank, S., Steponavičė, I., & Rebennack, S. (2012a). Optimal power flow: a bibliographic survey i. *Energy Systems*, *3*.

Frank, S., Steponavičė, I., & Rebennack, S. (2012b). Optimal power flow: a bibliographic survey ii. *Energy Systems*, *3*.

Fu, Y., Shahidehpour, M., & Li, Z. (2006). Ac contingency dispatch based on security-constrained unit commitment. *IEEE Transactions on Power Systems*, *21*(2), 897–908.

Glavitsch, H. & Spoerry, M. (1983). Quadratic loss formula for reactive dispatch. *IEEE Transactions on Power Apparatus and Systems*, *PAS-102*(12), 3850–3858.

Gómez Expósito, A. & Romero Ramos, E. (1999). Reliable load flow technique for radial distribution networks. *IEEE Transactions on Power Systems*, *14*(3), 1063–1069.

Griffith, R. E. & Stewart, R. A. (1961). A nonlinear programming technique for the optimization of continuous processing systems. *Management Science*, *7*(4), 379–392.

Grigg, C., Wong, P., Albrecht, P., Allan, R., Bhavaraju, M., Billinton, R., Chen, Q., Fong, C., Haddad, S., Kuruganty, S., Li, W., Mukerji, R., Patton, D., Rau, N., Reppen, D., Schneider, A., Shahidehpour, M., & Singh, C. (1999). The ieee reliability test system-1996. a report prepared by the reliability test system task force of the application of probability methods subcommittee. *IEEE Transactions on Power Systems*, *14*(3), 1010–1020.

Guan, X., Zhai, Q., & Papalexopoulos, A. (2003). Optimization based methods for unit commitment: Lagrangian relaxation versus general mixed integer programming. In *2003 IEEE Power Engineering Society General Meeting (IEEE Cat. No.03CH37491)*, volume 2, (pp. 1095–1100 Vol. 2).

Gurobi Optimization, L. (2021). Gurobi optimizer reference manual.

Iba, K., Suzuki, H., Suzuki, K.-I., & Suzuki, K. (1988). Practical reactive power allocation/operation planning using successive linear programming. *IEEE Transactions on Power Systems*, *3*(2), 558–566.

Jabr, R. (2006). Radial distribution load flow using conic programming. *IEEE Transactions on Power Systems*, *21*(3), 1458–1459.

Jabr, R., Coonick, A., & Cory, B. (2002). A primal-dual interior point method for optimal power flow dispatching. *IEEE Power Engineering Review*, *22*, 55–55.

Jabr, R. A. (2008). Optimal power flow using an extended conic quadratic formulation. *IEEE Transactions on Power Systems*, *23*(3), 1000–1008.

Jabr, R. A. (2012). Exploiting sparsity in sdp relaxations of the opf problem. *IEEE Transactions on Power Systems*, *27*(2), 1138–1139.

Ji, B., Yuan, X., Chen, Z., & Tian, H. (2014). Improved gravitational search algorithm for unit commitment considering uncertainty of wind power. *Energy*, *67*.

Kazarlis, S., Bakirtzis, A., & Petridis, V. (1996). A genetic algorithm solution to the unit commitment problem. *IEEE Transactions on Power Systems*, *11*(1), 83–92.

Knueven, B., Ostrowski, J., & Watson, J.-P. (2020). On mixed-integer programming formulations for the unit commitment problem. *INFORMS Journal on Computing*, *32*(4), 857–876.

Kocuk, B., Dey, S., & Sun, X. (2016a). Strong socp relaxations for the optimal power flow problem. *Operations Research*, *64*.

Kocuk, B., Dey, S., & Sun, X. (2018). Matrix minor reformulation and socp-based spatial branch-and-cut method for the ac optimal power flow problem. *Mathematical Programming Computation*, *10*, 557–596.

Kocuk, B., Dey, S. S., & Sun, X. A. (2016b). Inexactness of sdp relaxation and valid inequalities for optimal power flow. *IEEE Transactions on Power Systems*, *31*(1), 642–651.

Lavaei, J. & Low, S. H. (2012). Zero duality gap in optimal power flow problem. *IEEE Transactions on Power Systems*, *27*(1), 92–107.

Liu, J., Laird, C. D., Scott, J. K., Watson, J. P., & Castillo, A. (2018). Global solution strategies for the network-constrained unit commitment problem with ac transmission constraints. *IEEE Transactions on Power Systems*, *34*(2).

Ma, H. & Shahidehpour, S. (1999). Unit commitment with transmission security and voltage constraints. *IEEE Transactions on Power Systems*, *14*(2), 757–764.

Madani, R., Ashraphijuo, M., & Lavaei, J. (2016). Promises of conic relaxation for contingency-constrained optimal power flow problem. *IEEE Transactions on Power Systems*, *31*(2), 1297–1307.

Marley, J. F., Molzahn, D. K., & Hiskens, I. A. (2017). Solving multiperiod opf problems using an ac-qp algorithm initialized with an socp relaxation. *IEEE Transactions on Power Systems*, *32*(5), 3538–3548.

Momoh, J., Adapa, R., & El-Hawary, M. (1999). A review of selected optimal power flow literature to 1993. i. nonlinear and quadratic programming approaches. *IEEE Transactions on Power Systems*, *14*(1), 96–104.

Momoh, J., El-Hawary, M., & Adapa, R. (1999). A review of selected optimal power flow literature to 1993. ii. newton, linear programming and interior point methods. *IEEE Transactions on Power Systems*, *14*(1), 105–111.

Morales-España, G., Latorre, J. M., & Ramos, A. (2013). Tight and compact milp formulation of start-up and shut-down ramping in unit commitment. In *2013 IEEE Power Energy Society General Meeting*, (pp. 1–1).

Muckstadt, J. A. & Koenig, S. A. (1977). An application of lagrangian relaxation to scheduling in power-generation systems. *Operations Research*, *25*(3), 387–403.

Murillo-Sanchez, C. & Thomas, R. (1999). Thermal unit commitment with nonlinear power flow constraints. In *IEEE Power Engineering Society. 1999 Winter Meeting (Cat. No.99CH36233)*, volume 1, (pp. 484–489 vol.1).

Nanou, S. I., Psarros, G. N., & Papathanassiou, S. A. (2021). Network-constrained unit commitment with piecewise linear ac power flow constraints. *Electric Power Systems Research*, *195*, 107125.

Ongsakul, W. & Petcharaks, N. (2004). Unit commitment by enhanced adaptive lagrangian relaxation. *IEEE Transactions on Power Systems*, *19*(1), 620–628.

Ostrowski, J., Anjos, M., & Vannelli, A. (2012). Tight mixed integer linear programming formulations for the unit commitment problem. *IEEE Transactions on Power Systems - IEEE TRANS POWER SYST*, *27*, 39–46.

Padhy, N. (2004). Unit commitment-a bibliographical survey. *IEEE Transactions on Power Systems*, *19*(2), 1196–1205.

Paredes, M., Martins, L., Soares, S., & Ye, H. (2021). Benders' decomposition of the unit commitment problem with semidefinite relaxation of ac power flow constraints. *Electric Power Systems Research*, *192*, 106965.

Rabiee, A. & Parniani, M. (2013). Voltage security constrained multi-period optimal reactive power flow using benders and optimality condition decompositions. *IEEE Transactions on Power Systems*, *28*(2), 696–708.

Rajan, D. & Takriti, S. (2005). Ibm research report minimum up/down polytopes of the unit commitment problem with start-up costs.

Sasson, A. M., Viloria, F., & Aboytes, F. (1973). Optimal load flow solution using the hessian matrix. *IEEE Transactions on Power Apparatus and Systems*, *PAS-92*(1), 31–41.

Schanen, M., Gilbert, F., Petra, C. G., & Anitescu, M. (2018). Toward multiperiod ac-based contingency constrained optimal power flow at large scale. In *2018 Power Systems Computation Conference (PSCC)*, (pp. 1–7).

Sifuentes, W. & Vargas, A. (2007). Hydrothermal scheduling using benders decomposition: Accelerating techniques. *Power Systems, IEEE Transactions on*, *22*, 1351 – 1359.

Stott, B., Jardim, J., & Alsac, O. (2009). Dc power flow revisited. *Power Systems, IEEE Transactions on*, *24*, 1290 – 1300.

Sun, X. & Phan, D. (2014). *Some Optimization Models and Techniques for Electric Power System Short-term Operations*, (pp. 1–17).

Ting, T., Rao, M., & Chu Kiong, L. (2006). A novel approach for unit commitment problem via an effective hybrid particle swarm optimization. *Power Systems, IEEE Transactions on*, *21*, 411 – 418.

Tognola, G. & Bacher, R. (1999). Unlimited point algorithm for opf problems. *IEEE Transactions on Power Systems*, *14*(3), 1046–1054.

Watkins, D. W. & McKinney, D. C. (1998). Decomposition methods for water resources optimization models with fixed costs. *Advances in Water Resources*, *21*(4), 283–295.

Wächter, A. & Biegler, L. (2006). On the implementation of an interior-point filter line-search algorithm for large-scale nonlinear programming. *Mathematical programming*, *106*, 25–57.

Wu, Y.-C., Debs, A., & Marsten, R. E. (1993). A direct nonlinear predictor-corrector primal-dual interior point algorithm for optimal power flows. *IEEE Transactions on Power Systems*, *9*, 876–883.