ALTERNATIVE FORMULATIONS AND SOLUTION APPROACHES FOR DISTRIBUTION NETWORK DESIGN WITH SEASONALITY

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ABSTRACT

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Keywords: Distribution network design problem, Seasonal variations, Facility location, Routing decisions, Linear-programming-based Constructive Heuristic, Local branching algorithm

In this study, we consider a single-commodity distribution network design problem, which takes seasonal variations in the demand into account. We consider a three-echelon supply chain network design over a planning horizon, consisting of four seasons; products are delivered to outlets from a distribution center through regional depots. We develop alternative mathematical models that have different levels of flexibility while responding to seasonal demand. The problem formulations incorporate decisions related to locations of regional depots, amount of transportation from distribution center to regional depots, and routes used for delivery from regional depots to outlets while the objective function minimizes the total cost due to opening and operating regional depots as well as transportation-related costs. To solve the resulting problems, we first propose a linear-programming-based constructive heuristic approach. Alternatively, we adapt the local branching algorithm to all three models with variations on branching of different binary decision variables. In order to evaluate the efficiency and effectiveness of the proposed heuristics, we solve instances of four sets of problems varying in terms of the problem size. We also evaluate the effect of the truck size used in delivery to outlets on the problem difficulty and also its impact on the solution quality. The results show that the local branching algorithm has mostly demonstrated a better performance in terms of solution quality and computational efficiency compared to other approach.

ÖZET

MEVSİMSEL TALEP VARLIĞINDA DAĞITIM ŞEBEKESİ TASARIMI İÇİN ALTERNATİF FORMÜLASYONLAR VE ÇÖZÜM YAKLAŞIMLARI

Yasaman Karimian Hadi Ardebili

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Talepteki mevsimselliği göz önünde bulunduran tek ürünlü bir dağıtım ağı tasarımı problemi ele alınmaktadır. Dört mevsimden oluşan bir planlama ufku içerisinde, ürünlerin bölgesel depolar aracılığıyla satış noktalarına teslim edildiği, üç seviyeli bir tedarik zinciri ağı tasarımı çalışılmaktadır. Bu problem için, mevsimsel taleplere cevap verirken sahip oldukları esneklikleriyle birbirinden ayrışan alternatif matematiksel modeller geliştiriyoruz. Problem formülasyonları bölgesel depoların yeri, dağıtım merkezinden bölgesel depolara yapılan taşıma miktarları, ve ürünlerin bölgesel depolardan satış noktalarına dağıtımında kullanılan rotalarla ilgili kararları içerirken, amaç fonksiyonu bölgesel depoların kurulması ve işletilmesi ile ilgili maliyetlerin yanında nakliye ile ilgili masrafları da enazlamaktadır. Ortaya çıkan problemin çözümü için, ilk olarak doğrusal-programlama tabanlı sezgisel bir yaklaşım öneriyoruz. Ayrıca, yerel dallanma algoritmasını her üç modele de farklı ikili karar değişkenleri üzerindeki varyasyonlarıyla uyarlıyoruz. Önerilen sezgisellerin etkinlik ve verimliliğini değerlendirmek için, problem büyüklüğü açısından birbirlerinden farklı dört problem kümesindeki örnekleri çözüyoruz. Satış noktalarına teslimatta kullanılan araç büyüklüğünün problemin zorluğundaki ve çözüm kalitesindeki etkisini de değerlendiriyoruz. Sonuçlar tüm çözüm yaklaşımlarının çözüm kalitesi ve bilgisayısal verimlilik açısından iyi çalıştığını gösteriyor.

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TO MY PARENTS
for raising me to believe that anything is possible

AND TO THE ONE
that makes anything possible

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1. INTRODUCTION

Developing a cost-effective distribution network to improve service level provides a strategic privilege for companies to increase competitiveness and enables their system to satisfy the business needs in the long run. One of the main issues in such systems is to determine the network structure and the transportation scheme. The former considers the number of echelons, types of facilities at each echelon, the number of facilities, and their locations. While the latter considers the number and size of vehicles as well as constructing the flow routes. Thus, the design of a distribution network consists of strategic location decisions as well as operational transportation decisions. Strategic decisions affect both the operation costs of the system and its ability to serve customers, directly (Bari (2019); Crainic & Laporte (1997)).

In real-world businesses, many products and appliances face seasonality in demand. According to the National Appliance Repair Report conducted by Puls company, home appliances such as air conditioners and refrigerators have a greater likelihood of breaking down during the Summer months. Therefore, according to this report, demand for repairing or replacing these products increases in Summer. Considering demand as static in such systems with notable seasonal fluctuations can be regarded as a disadvantage since it may result in costly solutions and failing to meet demand during high demand season (Dayarian, Crainic, Gendreau & Rei (2016)). On the other hand, in many systems, the delivery plan may be based on the maximum demand level over horizon. As a result, vehicle utilization can be much less in lower demand seasons and the company may endure a high cost regarding the transportation decisions. Figures 1.1 and 1.2 we present two examples of Google Trends search volumes to demonstrate seasonality.

In this thesis, we address a three-echelon distribution network design problem, which by considering seasonal fluctuation in demand, provides more applicability to the business requirements. We consider a supply chain consisting of a distribution cen-

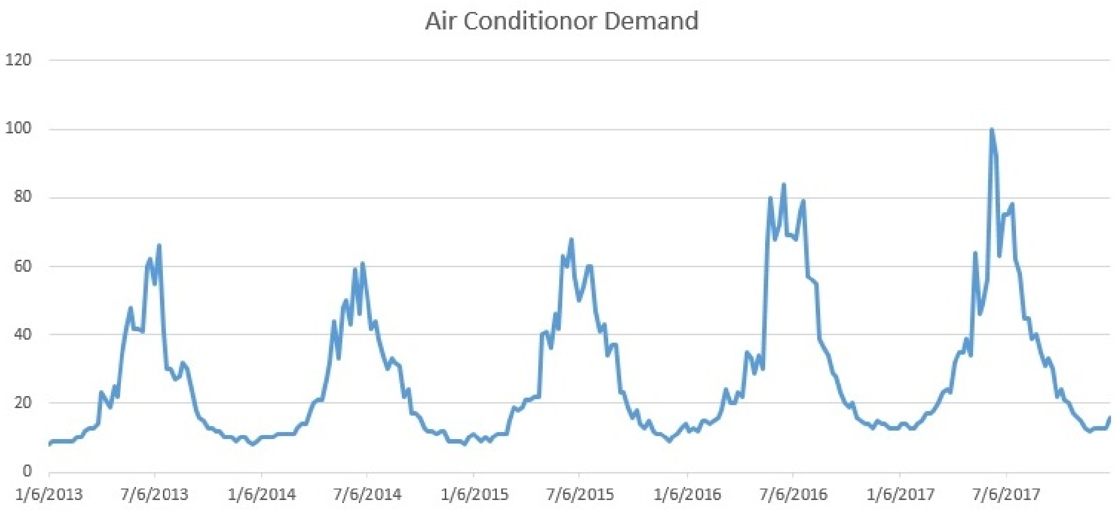

Figure 1.1 The air conditioners Google Trend search volumes over five years

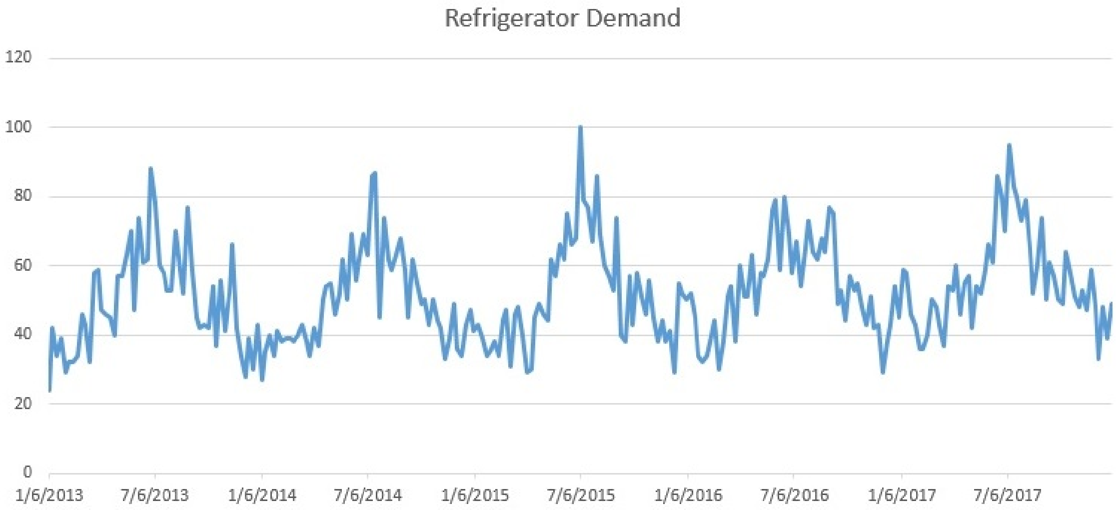

Figure 1.2 The refrigerator Google Trend search volumes over five year

ter (DC), regional depots, and outlets. The inbound transportation of products, from the DC to regional depots, and outbound transportation, from regional depots to outlets, are considered. The decisions include the number and locations of intermediate regional depots, amount of inbound transportation to each depot and selection of routes for outbound transportation. While minimizing the total cost of the distribution network including costs associated with the opening and operating of regional depots, and both inbound and outbound transportation costs, both strategic and tactical decisions are involved.

To formulate this problem, we develop mathematical models. We generate problem instances in four sets, varying in terms of size while each model is solved via a commercial solver, we propose a linear programming based construction heuristic and a local branching algorithm. Our contributions can be summarized as follows:

- Considering seasonal fluctuations in demand, we describe a three-echelon distribution network design problem, which incorporates facility location, transportation quantity, and route selection decisions.
- We develop three mathematical models for the distribution network design problem.
- We propose heuristic algorithms: a linear programming based construction heuristic and variations of the local branching approach.

The rest of this thesis is organized as follows: Chapter 2 includes the literature review. Problem definition and mathematical models are presented in Chapter 3. In Chapter 4, we explain our solution methods. The computational results are given in Chapter 5. Finally, we conclude the main findings in Chapter 6.

2. LITERATURE REVIEW

In this chapter, we present a review on distribution network design problems (DDPs). The distribution network design problem generalizes two combinatorial optimization problems: facility location problems (FLPs) and transportation problems. In the FLPs, customers receive the service by direct shipments from selected warehouses at minimum cost (Daskin (1996); Hakimi (1964)). Transportation problems aim to select a set of minimum cost routes to satisfy customer demands using a fleet of vehicles (Dantzig & Ramser (1959); Toth & Vigo (2014)).

Webb (1968) and Salhi & Rand (1989) evaluated the effect of ignoring the interdependency between routing and location decisions and recognized that it may often lead to sub-optimal solutions. Since then, some studies have focused on this interdependence, in particular location routing problems (LRPs), in the context of DDP. Generally, in these problems, the aim is to decide at minimum cost, which and how many facilities to open while building routes to visit the customers, simultaneously (Casco, Golden & Wasil (1988)). The core of DDP is single-echelon LRPs (Ben Mohamed, Klibi & Vanderbeck (2020)). Maranzana (1964) came up with the first work on this issue.

Due to the high growth of population, especially when the customers are located in a large geographical area, more attention is turned to two-echelon distribution systems. Therefore, LRPs are extended to the two-echelon LRP (2E-LRP) by adding an intermediate echelon standing between warehouses and customers (Ambrosino & Scutella (2005)). First studies, in this area, are conducted by Jacobsen & Madsen (1980) and Madsen (1983) to design a newspaper distribution system.

Given the strategic nature of decisions in two-echelon DDP (2E-DDP), the network must be considered as long term planning problems to fulfill future requirements of business environments efficiently over time (Ben Mohamed et al. (2020)). Albareda-Sambola, Fernández & Nickel (2012) studied a multi-period discrete facility location problem in which location and routing decisions are taken in different time scales. They assumed a discrete facility location problem over a finite time horizon and developed an approximation to solve the problem without missing the privilege to find well-located facilities along with the sets of customers in the time horizon. Tunalıoğlu, Koç & Bektaş (2016) considered a particular case of the mentioned work of Albareda-Sambola et al. (2012) in which all facility decisions are made at the beginning of the planning horizon. They assumed that no location decisions are taken at any point in the time horizon. Besides, capacity constraints for each facility are included in their study. To solve the problem, they proposed an adaptive large neighborhood search metaheuristic. Klibi, Lasalle, Martel & Ichoua (2010) modeled a stochastic location-transportation problem (LTP) over a planning horizon to maximize profit. The decisions about the location of depots, from given potential uncapacitated depots, are made at the beginning of the planning horizon. While to satisfy the uncertain customers' demand, transportation decisions on vehicle size and routes, from a given set of routes, are made daily. They considered outbound transportation costs independent of the load shipped in the vehicle and excluded inbound transportation decisions. They proposed a hierarchical heuristic solution approach based on the sample average approximation (SAA) method to solve the problem. Darvish, Archetti, Coelho & Speranza (2019) addressed an integrated routing problem where a commodity is delivered to customers through a two-echelon supply network. They analyzed two kinds of flexibility in their work. Flexibility in network design and in due dates, which concerns renting a distribution center in any period and serving a customer between the period an order is set and the due date, respectively. They consider outbound and inbound transportation costs and proposed an enhanced parallel exact algorithm based on the interplay between two branch-and-bound algorithms. Ben Mohamed et al. (2020) studied a multi-period, two-echelon supply chain network under stochastic demand. They considered warehouse location, platform allocation, and capacity configurations in their alternative modeling approaches and solve their problem by Benders decomposition approach.

In many systems, the routing plan is implemented repeatedly over a long planning horizon and parameters, such as demand are assumed fixed and known a priori (Dayarian, Crainic, Gendreau & Rei (2015)). However, this assumption is not valid for various real life applications and may result in inferior solutions. Specially, this issue exists in systems with notable seasonal fluctuations in supply/demand over the considered planning horizon (Dayarian et al. (2016)). It should be stated that, since different periods may have different average demand with distinct distributions, seasonal patterns must be recognized and separated from randomness (Gendreau, Jabali & Rei (2016)). The importance of this issue is notable in designing routes over a time horizon which, contains several seasons. In this respect, Dayarian et al. (2015) studied a multi-period vehicle routing problem (VRP) to optimize collecting products from different production locations over a given planning horizon. They considered seasonal fluctuation of the supply in their model and developed a branch and price algorithm to reach the solution. Using this approach, they could solve instances with no more than 60 producers and five periods. However, to have an efficient approach to solve larger problems with several hundred customers, the same authors (Dayarian et al. (2016)) presented an adaptive large-neighborhood search (ALNS) for a multi-period VRP with seasonal fluctuations.

To the best of our knowledge, no work considers three-echelon location transportation problem under seasonality. Considering the importance of this issue, our aim is to provide a realistic definition of a distribution network design problem, present alternative formulations as mixed-integer programming models, solve them using a commercial solver as well as a linear programming based constructive heuristic and variations of the local branching approach, and examine the solution time and quality.

3. PROBLEM DEFINITION

We consider a distribution network design problem over a planning horizon consisting of four seasons. The network consists of a distribution center (DC), regional depots, and outlets. Without loss of generality, the products are assumed to be aggregated as a single product family since they share the same handling and storage technology (Klibi, Martel & Guitouni (2016)). Through regional depots, the products are sent from DC to outlets. The transportation of products from DC to regional depots are direct (inbound transportation), but from depots to outlets (outbound transportation) selected routes are used. Among the strategic decisions involved in operating and managing such systems, we focus on determining the location and the number of regional depots to open during the whole planning horizon. In addition, we also consider tactical and operational decisions such as the flow of products in different seasons and outbound route selections. Each route originates from a depot and includes a number of outlets. A solution to the distribution network design problem should satisfy the demands of the outlets in the planning horizon in each season. We make the following assumptions:

- There is only one distribution center.
- The number of opened depots faces no limitation; but each depot is associated with an opening and operating cost.
- Open depots are used in the complete planning horizon.
- The locations of DC, regional depots, and outlets are given.
- The inbound transportation cost from the DC to a regional depot is a function of the Euclidean distance between these facilities, the transportation cost per unit volume per unit distance, and the volume of products shipped.
- The demand for each outlet in each season is known.
- Homogeneous vehicle fleet is used.
- More than one route can be used for delivery to an outlet in each season.

- More than one regional depot can cover an outlet.

The main decisions are involved with

- number and location of regional depots to be opened and operated during the planning horizon,
- amount of products transported from DC to the regional depots in each season, and
- selected routes as well as the amount of products transported from regional depots to outlets in each season.

The objective of the distribution network design problem is to minimize the total cost of opening and operating depots, and transportation related costs. We use alternative mathematical modeling approaches in order to solve this problem. Each approach is used to satisfy different purposes. The mathematical models are different from each other with respect to handling of seasonality through transportation decisions. These differences impact the decision variables along with the associated cost structures.

3.1 Mathematical Models

In order to develop the mathematical models, we first present the common notation for sets and parameters in Table 3.1.

Table 3.1 Common notation for all three models

Notations	**Descriptions**
I	set of candidate locations for regional depots
J	set of outlets
S	set of seasons
R	set of routes for outbound transportation considering candidate regional depot locations to outlets
R_i	set of routes that are originating from regional depot i
R_j	set of routes that contain outlet j
J_r	outlets covered in route r
D_{js}	demand of outlet j in season s (in terms of volumetric weight)
Q	capacity of the outbound transportation vehicles
f_i	fixed cost of opening and operating a regional depot at location i
c_i	inbound transportation cost (per unit volume) from the distribution center to regional depot i

In our three models, y_i, w_{is}, and x_{jrs} are the common decision variables. The binary decision variable y_i is equal to 1, if a regional depot is opened at location i; and 0 otherwise. w_{is} represents the amount of product delivered to regional depot i in season s, and x_{jrs} is the amount of product delivered to outlet j through route r in season s. In the upcoming subsections, we will define additional parameters and decision variables as needed for each model.

3.1.1 Model 1: Fixed Route Selection (z_r)

In the first mathematical model, we consider the routing decisions for outbound transportation as fixed throughout the planning horizon. This model would yield a robust solution under seasonal demand and would be appealing for companies that aim operational stability even under seasonality. However, higher outbound transportation costs would be expected due to not using less routes and vehicles in lower demand seasons. In this respect, we define the corresponding decision variable as a set of binary variables z_r, which is equal to 1, if route r is used; 0, otherwise. Parameter c_r is used to denote the outbound transportation cost; it is a function of distance between regional depots and outlets and the capacity of each vehicle. The resulting problem formulation for Model 1 becomes

$$\text{(3.1)} \qquad \text{minimize} \qquad \sum_{i \in I} f_i y_i + \sum_{i \in I} \sum_{s \in S} c_i w_{is} + \sum_{r \in R} c_r z_r$$

$$\text{(3.2)} \qquad \text{subject to} \qquad \sum_{r \in R_j} x_{jrs} = D_{js} \qquad \forall (j,s) \in (J,S)$$

$$\text{(3.3)} \qquad \sum_{r \in R_i} \sum_{j \in J_r} x_{jrs} \leq w_{is} \qquad (i,s) \in (I,S)$$

$$\text{(3.4)} \qquad w_{is} \leq \sum_{j \in J} D_{js} y_i \qquad \forall (i,s) \in (I,S)$$

$$\text{(3.5)} \qquad \sum_{j \in J_r} x_{jrs} \leq Q z_r \qquad \forall (r,s) \in (R,S)$$

$$\text{(3.6)} \qquad x_{jrs} \geq 0 \qquad \forall (j,r,s) \in (J,R_j,S)$$

$$\text{(3.7)} \qquad w_{is} \geq 0 \qquad \forall (i,s) \in (I,S)$$

$$\text{(3.8)} \qquad y_i \in \{0,1\} \qquad \forall i \in I$$

$$\text{(3.9)} \qquad z_r \in \{0,1\} \qquad \forall r \in R$$

The objective function (3.1) minimizes the total cost due to opening and operating regional depots, inbound transportation over all seasons (from DC to regional depots) and outbound transportation (from regional depots to outlets). Demand satisfaction constraint for each outlet in each season is given in (3.2). Constraint (3.3) ensures that in each season, the amount of product, which is transported from DC to regional depots is greater than or equal to the amount of product that is delivered to outlets from regional depots. Constraint (3.4) depicts that the flow of products in each season, should be received from an opened and operated depot. Constraint (3.5) ensures that the number of products delivered to outlets from regional depots should not exceed the capacity of selected route. Finally, (3.6), (3.7), (3.8), and (3.9) are the domain constraints for decision variables.

Solving this problem yields a set of regional depots to open, a set of selected routes, amount of products transported from DC to regional depots in each season, and amount of products transshipped from regional depots to outlets in each season through each route. Both depots and selected routes are used in the whole planning horizon according to this model; they are not season specific.

3.1.2 Model 2: Seasonal Route Selection (z_{rs})

In the second mathematical model, we consider seasonal routing decisions for outbound transportation. This model represents an "ideal approach" as the outbound transportation decisions are changed in response to seasonal changes in demand. While being responsive to seasonal changes, this model would yield the lowest outbound transportation costs and total costs. However, this model is the most computationally challenging one.

We define the corresponding decision binary variable z_{rs}, which is equal to 1, if route r is used in season s. c'_{rs} represents the outbound transportation cost in season s and is a function of distance between regional depots and outlets and the capacity of each vehicle. The relation between the outbound transportation cost terms of Model 1 and Model 2 can be shown as

$$c_r = 4(c'_{rs}) \tag{3.10}$$

When the number of seasons is 4, the resulting problem formulation for Model 2 is a generalization of (3.1)-(3.9) as

$$\text{minimize} \quad \sum_{i \in I} f_i y_i + \sum_{i \in I} \sum_{s \in S} c_i w_{is} + \sum_{r \in R} \sum_{s \in S} c'_{rs} z_{rs} \tag{3.11}$$

$$\text{subject to} \quad (3.2) - (3.4)$$

$$\sum_{j \in J_r} x_{jrs} \leq Q z_{rs} \qquad \forall (r,s) \in (R,S) \tag{3.12}$$

$$(3.6) - (3.8)$$

$$z_{rs} \in \{0,1\} \qquad \forall (r,s) \in (R,S) \tag{3.13}$$

The objective function (3.11) of this model minimizes the total cost of opening and operating regional depots, inbound transportation costs in each season (from DC to regional depots), and outbound transportation costs in each season (from regional depots to outlets). Constraint (3.12) ensures that the number of products delivered to outlets from regional depots do not exceed the capacity of selected route in each season. Finally, (3.13) is the domain constraints for decision variables.

Solving this problem yields a set of regional depots to open, a set of selected routes in each season and amount of products transported in each season. The selected routes are chosen for each season specifically.

3.1.3 Model 3: Amount of Delivered Products (x_{jrs})

In this mathematical model, we do not consider a binary routing decision variable for outbound transportation. As the ideal approach presented in Model 2 is computationally challenging, the objective of this model is to find a lower bound on the total cost by reducing the binary variables associated with route selection in each season. Compared to Model 1, this model is also responsive to seasonal changes. Compared to Model 2, it is less computationally challenging. However, it is expected to underestimate the total cost because outbound transportation cost is a function of the amount of transportation rather than capacity of the vehicle.

The unit transportation cost, c''_{rs} is calculated as

$$c''_{rs} = \frac{c'_{rs}}{Q} \tag{3.14}$$

where c'_{rs} is the seasonal route cost in Model 2. The resulting problem formulation for Model 3 becomes

$$\begin{aligned}
&\text{minimize} && \sum_{i \in I} f_i y_i + \sum_{i \in I} \sum_{s \in S} c_i w_{is} + \sum_{j \in J_r} \sum_{s \in S} c''_{rs} x_{jrs} && (3.15)\\
&\text{subject to} && (3.2) - (3.4) && \\
& && \sum_{j \in J_r} x_{jrs} \leq Q \qquad \forall (r,s) \in (R,S) && (3.16)\\
& && (3.6) - (3.8) &&
\end{aligned}$$

Again, the objective (3.15) of this model is to minimize the total cost of the network, which consists of the fixed cost of opening and operating regional depots, inbound transportation costs in each season (from DC to regional depots), and outbound transportation costs in each season with respect to amount of delivered products (from regional depots to outlets). Constraint (3.16) ensures that the number of products delivered to outlets from regional depots should not exceed the capacity of each vehicle.

Solving this problem leads to a set of opened regional depots, quantities of the transported products from DC to regional depots in each season, and from regional depots to outlets in each season through each route.

These three models are different from each other in formulating the outbound transportation decisions. In Model 1, z_r is independent from seasonality, and the transportation cost is calculated by summing up the fixed cost of using each route. Model 2 is a generalization of Model 1 since it considers seasonality and the routing decision variables are season-dependent and denoted by z_{rs}. In Model 3, x_{jrs} represents the amount of product j delivered to each outlet through each route r in each season s. This model charges the outbound transportation cost for the load on the vehicle, not the vehicle capacity. Although, the real-life businesses mostly consider the number of vehicles and the capacity in their total costs upon a contractual agreement, Model 3 can be also of great use.

To provide an insight on the performance of these models regarding their outbound transportation decisions, we observed the number of vehicles and the percentage of their used capacity in each season. We calculated these values for one problem instance for Model 2 and Model 3 and compered the results. For each season, to calculate the average vehicle utilization we used the formula

$$\frac{\sum_{j \in J_r} x_{jrs}}{nQ} * 100, \tag{3.17}$$

where n is the number of used vehicles.

Table 3.2 summarizes the vehicle utilizations in each season for Model 2 and Model 3. According to this Table, Model 2 yields at least 67% of the vehicle utilization in each season. While Model 3 allows partial flow and uses around 40% of the vehicle capacities in seasons 1, 2, and 4, which are low demand seasons. Moreover, Model 2 selected one route (i.e., vehicle) less than Model 3, overall. In Model 2, eight out of 16 vehicles, and in Model 3, fourteen out of 17 vehicles are used in low demand seasons.

Table 3.2 Vehicle utilizations of Model 2 and Model 3

	Season 1	Season 2	Season 3	Season 4
Model 2	75.7%	81.2%	87.0%	66.6%
Model 3	43.3%	46.4%	80.8%	38.0%

In order to observe how the difference between the outbound transportation decisions affect the overall solutions, we examine the optimal solutions of a problem instance closely when all three models find the same optimal locations for regional depots. For this purpose, let us look at how the values of objective function terms differ from each other. Out of 10 candidate locations, all three models have selected the same

4 regional depots and naturally, their total cost of opening and operating regional depots (first term of the objective function) is the same. However, not only the outbound transportation total cost is different significantly, which is expected due to the nature of decision variables but also the inbound transportation decisions are apparently not identical. In Table 3.3, specific cost of each term in the objective function is presented; *Obj Value* stands for the objective function value of the optimal solution.

Table 3.3 Comparision of the values of objective function terms

	Model 1	Model 2	Model 3
Obj Value	171860.8	160645	143793.6
First term	47525.0	47525.0	47525.0
Second term	66577.1	66084.8	65604.9
Third term	57758.7	47035.1	30663.7

Table 3.4 Selected route by each model

Model 1	5	6	7	8	21	22	24	25	26	30	32	41	47	50	51
Model 2	6	7	8	21	22	25	26	30	41	44	47	50	51		
Model 3	6	7	8	21	22	25	27	28	30	41	42	47	50	51	

To examine the efficiency of our generated routes, we performed an experiment on Model 1 and Model 2. We took one instance randomly and omit one of the routes from optimal solution and observed the result. The objective function value of Model 1 and Model 2 increased by 0.28% and 0.21%, respectively.

4. SOLUTION METHOD

Three-echelon transportation location problem is known as NP-hard (Atamtürk & Zhang (2007)). Hence, the proposed problem in this thesis, which considers seasonality is NP-hard as the seasonality imposes additional complexity to the problem. Regarding this issue, it is not efficient to solve this problem with commercial solvers since they are not able to find good quality solutions in a reasonable time, especially in larger-scale problems. Therefore, in this chapter, we discuss the solution methods applied to solve the problem. First, we explain a linear programming (LP) based construction heuristic. Then, we explain the local branching approach and its implementation on our presented problems.

4.1 A Linear Programming Based Heuristic Approach

We develop an LP-based heuristic solution approach, which iteratively uses the solution of the LP relaxation to build a feasible integer solution.

Initially, the algorithm solves the LP relaxation of the problem and obtain decision variable' values. Then, it finds the candidate regional depot location for which the decision variable y_i has the maximum fractional value in the solution and forces the depot with that location to be opened, in the next iteration the same approach continues iteratively. In each iteration, the algorithm finds the new maximum fractional value of y_i among the remaining candidate locations and adds a constraint to fix its value to 1. The algorithm continues to solve the problem with these additional constraints until all facility location variables are either 0 or 1. Once the facility location variables are fixed accordingly, the algorithm removes the relaxation on the remaining binary variables and solves the mixed-integer programming problem to find a feasible solution.

The steps of this algorithm are illustrated with a flowchart in Figure 4.2. In this flowchart $LIST_1$ is the list of indices, $i \in I$, with $0 < y_i \leq 1$ and $LIST_0$ stands for the list of indices with $y_i = 0, i \in I$. Besides, y_m shows y_i with the largest value.

4.2 Local Branching Algorithm

The local branching algorithm, proposed by Fischetti & Lodi (2003), is a technique to solve mixed-integer programming problems. The method is exact by nature; however, by redefining some control parameters, it becomes a heuristic. In fact, it has been developed to improve the heuristic behavior of a MIP solver as a black box. Many researchers used this approach in their relevant studies such as Fischetti, Polo & Scantamburlo (2004); Hajiyan & Yaghini (2020); Rei, Gendreau & Soriano (2010); Rodríguez-Martín & Salazar-González (2010); Yaghini, Momeni & Sarmadi (2013). We first discuss the main features of the general local branching algorithm.

The following problem and explanations are first presented in Fischetti & Lodi (2003) and are provided here for the sake of completeness. Problem P is a generic MIP with 0-1 variables as:

$$\text{Min}\, C^T x \tag{4.1}$$

$$Ax \geq b \tag{4.2}$$

$$x_j \in \{0,1\} \qquad \forall j \in B \neq \emptyset \tag{4.3}$$

$$x_j \geq 0 \text{ ,} integer \qquad \forall j \in I \tag{4.4}$$

$$x_j \geq 0 \qquad \forall j \in C \tag{4.5}$$

The variable index set is partitioned into (B, I, C), where the set for binary variables is defined by B. Sets I and C are the index set for general integers and continuous variables, respectively, which may be empty sets.

For a feasible solution $\bar{x}$ of (P), consider $\bar{S} := \{j \in B : \bar{x}_j = 1\}$ as the binary support of $\bar{x}$. For a given positive integer parameter k, the *k-OPT* neighborhood $N(\bar{x}, k)$

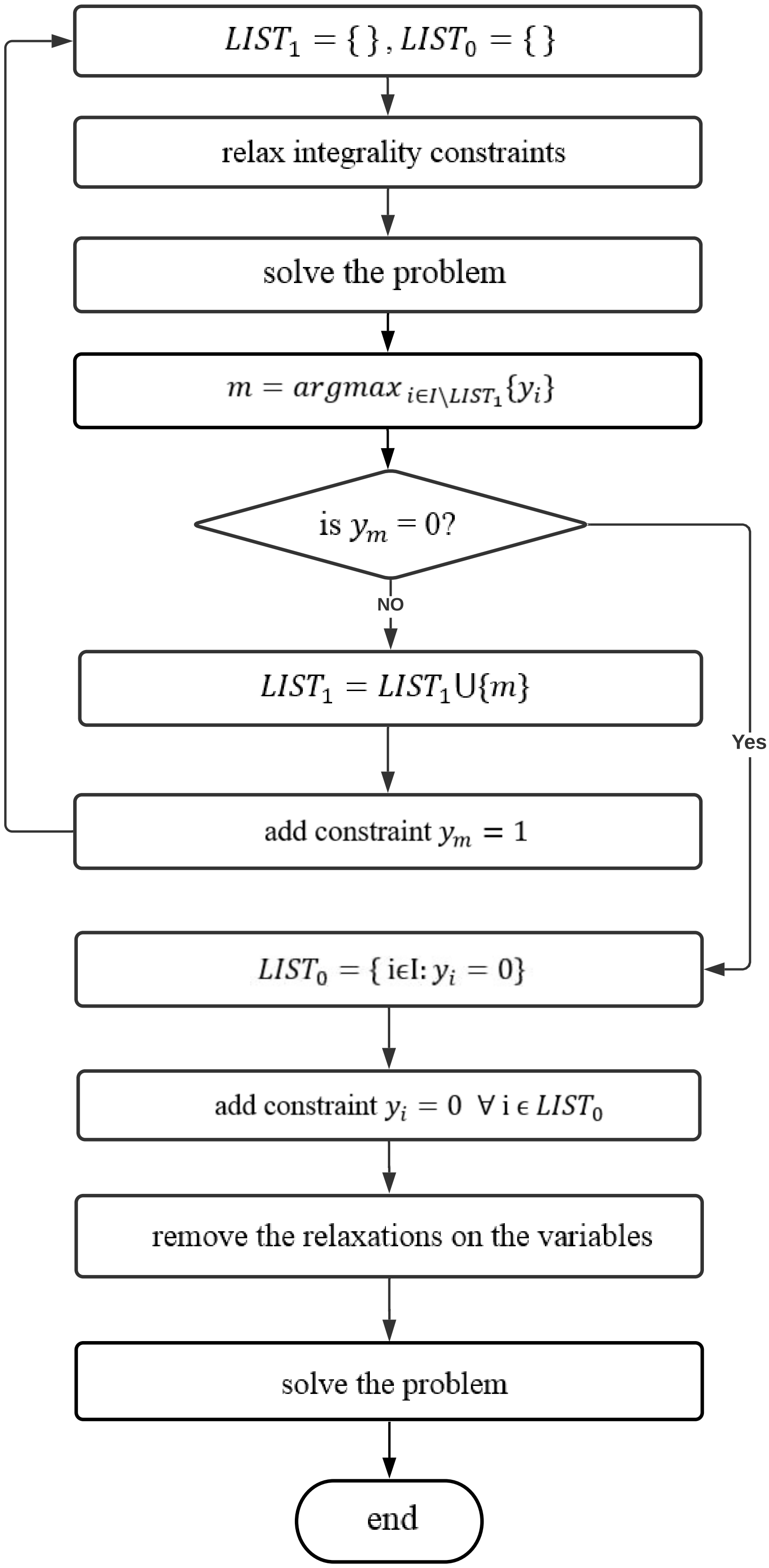

Figure 4.1 The flowchart for the LP-based heuristic solution approach

of $\bar{x}$ is the set of feasible solutions of (P) satisfying the additional *local branching constraint*

$$\Delta(x,\bar{x}) : \sum_{j\in\bar{S}}(1-x_j) + \sum_{j\in B\setminus\bar{S}} x_j \leq k \tag{4.6}$$

In equation (4.6), the two terms on the left-hand side count the number of binary variables. Given the incumbent solution $\bar{x}$, the solution space can be partitioned by means of the disjunction

$$\Delta(x,\bar{x}) \leq k \quad \text{(left branch)} \qquad \textbf{or} \qquad \Delta(x,\bar{x}) \geq k+1 \quad \text{(right branch)} \tag{4.7}$$

The idea is that the neighborhood $N(\bar{x},k)$ of the left-branch sub-problem should be sufficiently large to contain better solutions and yet small enough to be optimized within a short computing time. The neighborhood-size parameter k should be opted as the largest value so that the left-branch sub-problem is much easier to solve than the associated parent problem. According to the computational experiments conducted by Fischetti & Lodi (2003), choosing the value of k is not a problem by itself. In most cases, a value in the range of [10,20] proved to be effective. The method mainly alternates between two phases. The strategic phase in which the local branching cuts define promising solution regions, and the tactical phase in which these regions are explored by a classical branching scheme on the variables using a MIP solver.

In order to put a time limit on each left branch node and the total computation time, two parameters *node time limit* and *total time limit* are used. Since the algorithm starts with a feasible solution, we assume to have a starting incumbent solution $\bar{x}_1$ of (P). The constraint $\Delta(x,\bar{x}_1) \leq k$ is added to create the left branch sub-problem, which is solved with an MIP solver. If $\bar{x}_2$, the solution of this step found within the time limit, is better, it becomes the new incumbent. The process backtracks to the parent node by replacing constraint $\Delta(x,\bar{x}_1) \leq k$ by constraint $\Delta(x,\bar{x}_1) \geq k+1$. Moreover, by adding the cut $\Delta(x,\bar{x}_2) \leq k$ to the model a new left branch node is created. The algorithm solves the model by adding these two constraints to the model at the same time.

On the other hand, it is possible that the solution $\bar{x}_1$ is not improved within the node time limit. In that case, we will go through the intensification step, which means reducing the size of neighborhood $N(\bar{x},k)$, reducing the right-hand side of constraint (4.6). When the MIP solver reports proven infeasibility or when it is not able to reach a feasible solution in the defined node time limit, the diversification method is applied to the local branching scheme. By enlarging the size of neighborhood $N(\bar{x}, k)$, increasing the right-hand side of constraint (4.6), a soft diversification mechanism is incorporated into the algorithm. We consider upper bound for the number of diversifications. Therefore, the algorithm must not exceed the maximum number of diversifications. The main procedure of local branching is an iterative while loop, which continues until either the total time limit or the maximum number of diversifications is met. In what follows we will explain the implementation of the local branching algorithm on proposed models.

4.3 Implementation of Local Branching Algorithm

We now discuss the implementation of local branching algorithm. According to the description in Section 4.2, the local branching algorithm exploits the values of binary decision variables. All three models in Section 3 share the common facility location variable y_i. We discuss the implementation details with local branching on this variable, which is easily generalized for all three models. In our computational experiments, we also adapt the implementation for the other variables z_r and z_{rs}, respectively for Model 1 and Model 2. Indeed, the exact same process is mimicked for these variables.

Given a feasible solution $\bar{y}$, let $\bar{S} = \{i \in I : \bar{y_i} = 1\}$ denote the binary support of $\bar{y}$. For a given value of integer parameter k, the algorithm first solves the left branch sub-problem with added constraint

$$\Delta(y_i, \bar{y_i}) : \sum_{i \in \bar{S}} (1 - y_i) + \sum_{i \in I \setminus \bar{S}} y_i \leq k, \tag{4.8}$$

and finds $\hat{y}$ as a solution within the preset node time limit. The algorithm compares the objective function value of the left branch solution, i.e. $\text{obj}(\hat{y})$ with that of the parent node, i.e. $\text{obj}(\bar{y})$. If there is no improvement, the value of k is reduced by half, and the procedure is repeated. But if the result is improved, i.e. $\text{obj}(\hat{y})$ is less than or equal to $\text{obj}(\bar{y})$, the left branch constraint is replaced by the right branch and a new left branch constraint is added to the problem, simultaneously. As a result, the algorithm continues with constraints $\Delta(y_i, \bar{y_i}) \geq k+1$ and $\Delta(y_i, \hat{y_i}) \leq k$. Given the new solution (y'), in the next step, the objective function value of (y') is compared with that of the previous step $(\hat{y})$. If the result is improved, the left branch constraint $\Delta(y_i, \hat{y}_i) \leq k$ is replaced by the right branch $\Delta(y_i, \bar{y}_i) \geq k+1$, and a new left branch node is created by adding the cut $\Delta(y_i, y_i') \leq k$ to the model. When the MIP solver is not able to reach a feasible solution in the defined node time limit, the diversification method is applied. We also put a limitation on the number of diversification mechanism that the algorithm can perform. This process continues iteratively until either the MIP gap is zero or the total time limit or the maximum number of diversifications is exceeded.

The steps of the algorithm are presented with a flowchart in Figure 4.2.

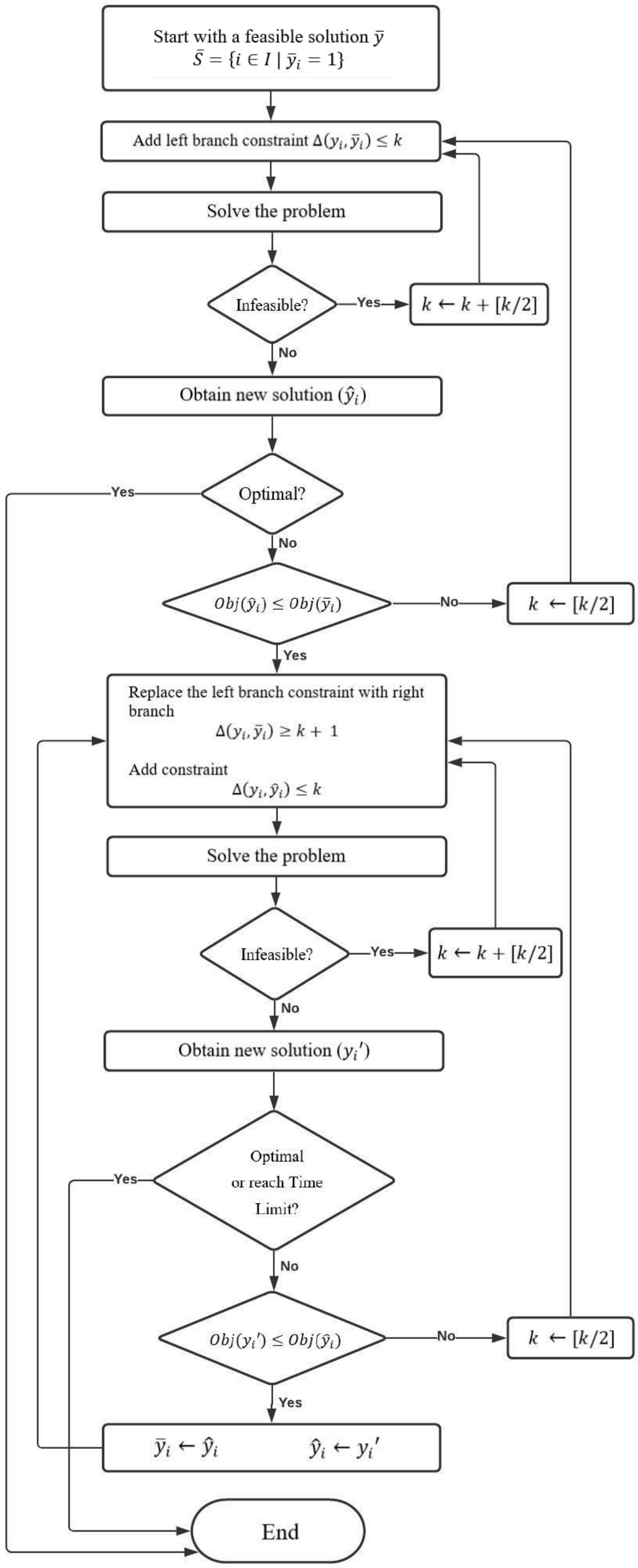

Figure 4.2 Local branching algorithm flowchart

5. COMPUTATIONAL RESULTS

In this chapter, we describe the experimental design carried out and the related computational results. First, we present the data and parameters of each problem sizes. Then, we discuss the results of each problem size for all models and solution approaches. The aim is to evaluate the efficiency and effectiveness of the presented solution approaches.

5.1 Experimental Design

Since the validity of an experiment is intertwined with its construction and execution, its structure is of great importance. To examine the impact of problem size on our formulations, four different sets of problems in terms of the number of outlets, regional depots, and routes are generated:

- 30 outlets, 10 regional depots and 58-91 routes
- 50 outlets, 15 regional depots and 150-213 routes
- 100 outlets, 30 regional depots and 610-786 routes
- 250 outlets, 75 regional depots and 4414-5056 routes

The route sets are generated using the expanded nearest neighbor search algorithm in Ercan (2019). While the locations of the outlets are determined randomly in a 100×100 grid coordinate system, in each problem, the single distribution center is placed at the center of the system. The candidate location of regional depots are determined by the use of a k-means approximation algorithm. The distances between the facilities are calculated as Euclidean distances. In this respect, for each problem size, ten different instances that differ in terms of demands, location of regional depots, and location of outlets are created and solved for each model.

As an illustration, the first instance with 30 outlets and 10 regional depots on the 100×100 grid coordinate system is demonstrated in Figure 5.1.

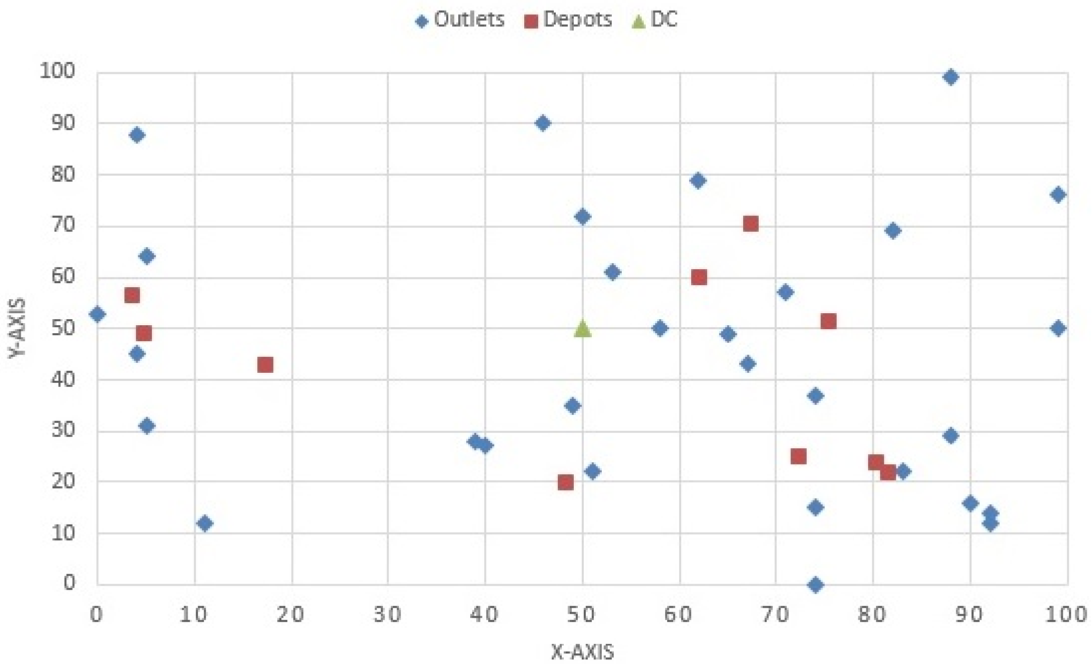

Figure 5.1 Instance 1 with 30 outlets and 10 regional depots

The demand of each outlet in each season is a function of the seasonal demand ratio. In this study, it is assumed that the third season has the highest demand ratio.

The regional depot opening and operating costs are randomly generated between 3000 and 6000.

Both inbound and outbound vehicles traverse the Euclidean distance between two points. The inbound transportation cost is calculated by multiplying the distance between DC and regional depots by the cost of transporting products per unit distance. The outbound transportation cost is calculated by multiplying the length of a route by the cost of transporting products per unit distance. The length of a route is the distance, which starts from a regional depot visiting outlets and returns to the same regional depot. In Model 1 and Model 2, the outbound transportation cost is also multiplied by the capacity of vehicles. However, in Model 3, this cost is multiplied by the number of transferred products.

To evaluate the impact of outbound vehicle capacities on performance of each model, each instance is solved twice with two different outbound vehicle sizes: large capacity (1000) and small capacity (500).

We solve each instance with the commercial solver. Since the commercial solver is not efficient in larger problems, we used the solution methods presented in Chapter 4. Each instance is solved with the LP-based heuristic and local branching algorithm. The local branching algorithm is implemented on each set of the binary variables separately. The value of the integer parameter k is determined according to the size of each problem and the number of binary variables. For most of the instances, especially in larger problems, in most cases we carry out the procedure with two different values for k and compare the results. The computation time limit is defined based on the size of the problem sets as well. Note that, in all the instances the total time limit dedicated to the local branching approach is half of the other two approaches. Our aim is to compare the performance of each modeling approach in terms of objective function value and solution time.

Throughout the experiments, we used GUROBI 7.5.2 on PYTHON 3.7 using an Intel Xeon CPU E5-2640 processor with 2.60 GHz speed, 16 GB RAM, and 64-bit Windows 7 operating system. All the coding for data reading, model preparation, and output generation have been implemented with Python 3.7 with Anaconda Spyder.

The summary of the experiments are presented in Tables A.1-D.6 in Appendices. The results are organized and reported with respect to the modeling approach and the vehicle size. Each table shows the results for 10 instances obtained by one of the models (Model 1, Model 2 and Model 3) with a given vehicle size (small and large). The term *Obj Func* shows the objective function value and *Run Time* depicts the solution run time for each instance. While *Gap* shows the percentage gap between the upper bound and lower bound as reported by the solver, $\Delta(obj)$ is the percentage deviation of the objective function value of each solution approach with respect to that of the commercial solver.

We provide the average percentage deviations of each solution approach for each combination of a model and vehicle capacity in all problems in Tables 5.6-5.8. In these tables *CS* and *LP-H* stands for the commercial solver and the LP-based heuristic, respectively and the rest belongs to local branching approach. In order to avoid mentioning different values of the integer parameter k, we used (1) for smaller value and (2) for larger value.

5.2.1 Small-Scale Problems with 30 Outlets

We solve ten different instances in the set of small scale problems with 30 outlets and 10 regional depots. The total time limit for the commercial solver and the LP-based heuristic is 14400 seconds; it is 7200 seconds for local branching. For all three models and both vehicle capacities, each instance is solved by the commercial solver, LP-based heuristic, and local branching approach. The local branching algorithm is implemented on y_i variables with $k = 5$ for all three models. Moreover, it is implemented on z_r and z_{rs} variables with $k = 5$ and $k = 10$ for Model 1 and Model 2, respectively. The node time limit in the local branching algorithm for all these models is 100 seconds. The results are presented in detail in Tables A.5-A.4 in Appendix A. Note that, in the case of small vehicle capacity, instances 6 and 7 are infeasible to solve.

These instances are our smallest set of instances. As seen in the corresponding Tables in Appendix A, all instances are efficiently solved to optimality with the commercial solver with all three models and both vehicle capacities. The local branching algorithm was successful to reach the optimal solution in all cases. Different values for k hardly make difference in this set of instances. However, in some instances, the LP-based heuristic results show small deviations (less than 2% on average).

Although we put a four-hour time limit for the commercial solver and the LP-based heuristic (LP-H) approach, and a two-hour time limit for the local branching approach, the run time for solving these instances is very small. We provide the average solution run time (in seconds) in Table 5.1, which is the arithmetic mean of solution time for each approach. In this Table, y_i, z_r, z_{rs} represent the local branching implemented on these variable and $k(1)$ and $k(2)$ show the small and large values for k, respectively. According to this table, all instances are solved in less than 4 seconds on average, except Model 2 with small vehicle capacity. This model is the most time consuming among others; however, the LP-based heuristic showed a great improvement in reducing the solution time from 237.77 seconds to 7.40 on average.

5.2.2 Medium-Scale with 50 Outlets

We solve ten different instances in the set of medium scale problems with 50 outlets and 15 regional depots. The total time limit for the commercial solver and the

Table 5.1 Average run time for all three models with large and small vehicle sizes

	CS	LP-H	y_i; $k(1)$	z_r; $k(1)$	z_r; $k(2)$	z_{rs}; $k(1)$	z_{rs}; $k(2)$
Model 1 - Large	0.75	0.86	1.00	1.30	1.11	-	-
Model 1 - Small	1.28	1.86	2.50	1.55	1.54	-	-
Model 2 - Large	2.16	1.19	3.37	-	-	2.47	3.48
Model 2 - Small	237.77	7.40	81.82	-	-	31.55	57.44
Model 3 - Large	0.08	0.46	0.13	-	-	-	-
Model 3 - Small	0.13	0.41	0.20	-	-	-	-

LP-based heuristic is 14400 seconds; it is 7200 seconds for local branching.

The local branching is implemented on:

- Model 1 on y_i and z_r variables with $k = 5$ and $k = 10$
- Model 2 on y_i variables with $k = 5$ and $k = 10$ and z_{rs} variables with $k = 10$ and $k = 20$
- Model 3 on y_i variables with $k = 5$ and $k = 10$

The node time limit in the local branching algorithm for Model 1 and Model 2 is 500 seconds and for Model 3 is 100 seconds. The results are presented in detail in Tables B.1-B.6 in Appendix B. Note that, in all three models with both vehicle sizes, instance 3 is infeasible to solve.

As we can see in the Tables, all instances of Model 1 are solved to optimality with the commercial solver. Although the LP-based heuristic solutions have deviations in some instances (less than 0.9% on average), local branching is successful to reach the exact same solution as the commercial solver. We can see the same pattern for Model 2 with large vehicle capacity and all instances of Model 3.

However, for Model 2 with small vehicle capacity, 8 out of 10 instances reach the 4-hour time limit without finding the optimal solution. Moreover, the LP-based heuristic reach the time limit in some instances and there are deviations in terms of the objective function value (0.73% on average). Even though the local branching has also reached the time limit in some cases it found the exact same solution as the commercial solver. The best performance belongs to the local branching on z_{rs} with $k = 10$ where all instances yield the optimal solution in 432.5 seconds on average.

The average solution times (in seconds) for each model are shown in Tables 5.2, 5.3 and 5.4. As we can see in the corresponding Tables, all of our algorithms outperform the commercial solver in terms of the average run times.

In Model 1 with small vehicle capacity, the average run time for local branching on z_r with $k = 5$ is 22.84 seconds, which is the lowest amongst all other approaches.

Having one set of binary variables (y_i) in its formulation, Model 3 is our quickest to solve. In general, Model 2 has a greater solution time. In all three models, small capacity versions take more time to solve. Model 2 with small vehicle capacity is our most time-consuming model.

Table 5.2 Average run time for Model 1 in medium scale (50 outlets)

	CS	LP-H	y_i, $k(1)$	y_i, $k(2)$	z_r, $k(1)$	z_r, $k(2)$
Model 1 - Large	10.0	4.4	9.2	9.4	13.6	14.2
Model 1 - Small	129.4	32.3	133.0	113.4	22.8	42.6

Table 5.3 Average run time for Model 2 in medium scale (50 outlets)

	CS	LP-H	y_i, $k(1)$	y_i, $k(2)$	z_{rs}, $k(1)$	z_{rs}, $k(2)$
Model 2 - Large	67.5	11.6	162.8	75.1	93.2	136.5
Model 2 - Small	11293.0	5116.4	9714.4	11307.6	432.5	7069.9

Table 5.4 Average run time for Model 3 in medium scale (50 outlets)

	CS	LP-H	y_i, $k = 5$	y_i, $k = 10$
Model 3 - Large	0.3	1.0	0.3	0.3
Model 3 - Small	0.4	1.2	0.4	0.4

5.2.3 Large Scale with 100 Outlets

We solve ten different instances in the set of large scale problems with 100 outlets and 30 regional depots. The results are shown in Tables C.1-C.6 in Appendix C. The total time limit for the commercial solver and the LP-based heuristic approach is 28800 seconds, and for local branching is 14400 seconds.

The local branching algorithm is implemented on all three models on y_i variables with $k = 10$ and $k = 20$. Moreover, it is implemented on z_r and z_{rs} variables with $k = 10$ and $k = 20$ for Model 1 and Model 2, respectively. The node time limit for Model 1 and Model 2 is 1500 seconds, and for Model 3 is 100 seconds.

The average solution time for Model 1 and Model 3 are presented in Table 5.5. Since, Model 2 reaches the time limit in most cases it is excluded from this table.

5.2.3.1 Model 1

Except for the last two, all instances of the large vehicle capacity are solved to optimality with the commercial solver. Although the LP-based heuristic solutions have deviations in some instances (0.88% for large vehicle capacity and 1.11% for small vehicle capacity on average), in the last instance we witness negative deviation, which means this approach finds a better solution compared to the commercial solver. Local branching has yielded the exact same solution as the commercial solver even though it has reached the time limit in some cases. For example, instances 9 and 10 reach the four-hour time limit in local branching on y_i but they yield the same result as the commercial solver. The best performance of this approach is in the case of z_r with $k = 10$, where there is no percentage deviation in the results and the average time limit is 750.09 seconds.

In the case of small vehicle capacity, 6 out of 10 instances reach the 8-hour time limit without reaching the optimal solution. Moreover, the LP-based heuristic reaches the time limit in some instances and there are deviations in terms of the objective function value (0.11% on average). Even though the local branching has reached the time limit in some cases it finds the exact same solution as the commercial solver. The best performance belongs to the local branching on z_r with $k = 20$ where all instances reached the optimal solution in 1169.82 seconds, which is much less than their dedicated time limit.

5.2.3.2 Model 2

Most of the instances in large vehicle capacity and all instances in small vehicle capacity have reached their time limit in all approaches. As it is presented in Figures 5.2 and 5.3, the percentage deviations of local branching is very small (less than 0.71% on average), given that it has half the time to solve instances. Even, in some cases we have negative deviations.

Although most of the cases reach the time limit, for both vehicle sizes local branching on z_{rs} with $k = 40$ has the best performance in terms of percentage deviations (see Table 5.7).

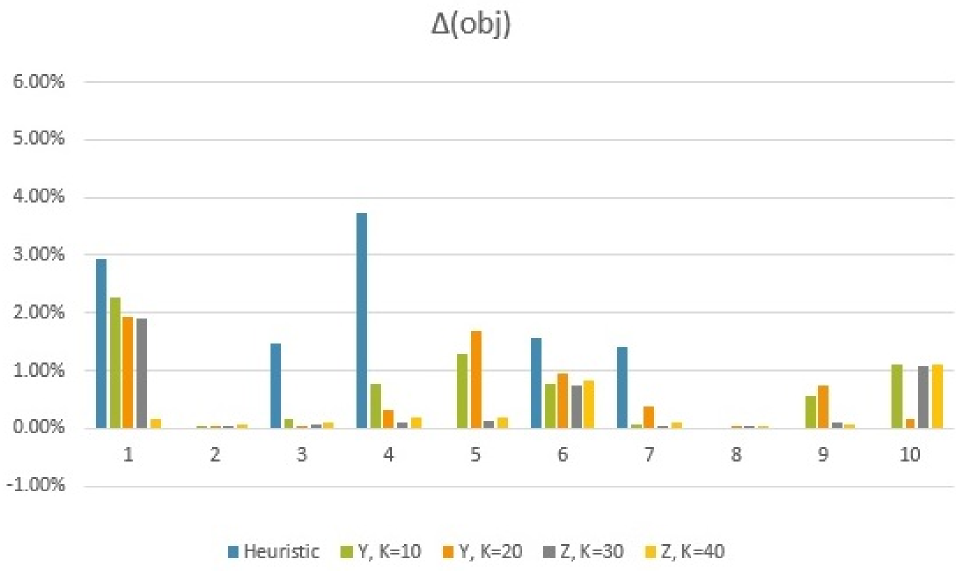

Figure 5.2 Percentage deviation of the result of each solution approach (Model 2 with large vehicle capacity in large scale)

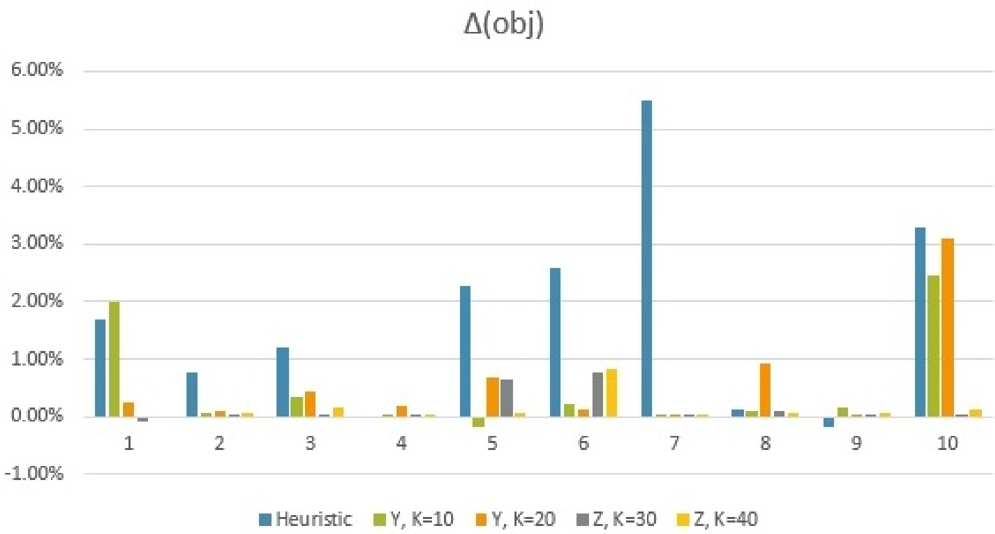

Figure 5.3 Percentage deviation of the result of each solution approach (Model 2 with large vehicle capacity in large scale)

5.2.3.3 Model 3

For both large and small vehicle capacities all instances are solved to optimality with the commercial solver. The local branching approach found the optimal solution with no deviation; however, the LP-based heuristic approach has shown some deviations in results (0.93% for large and 1.79% for small vehicle capacity on average).

Table 5.5 The average run time for Models 1 and 3 in large scale (100 outlets)

	CS	LP-H	y_i, $k(1)$	y_i, $k(2)$	z_r, $k(1)$	z_r, $k(2)$
Model 1 - Large	10363.48	584.78	6387.89	4983.50	750.09	1517.30
Model 1 - Small	18128.80	16106.03	8101.07	7880.13	3240.38	1169.82
Model 3 - Large	3.85	9.73	9.16	9.29	-	-
Model 3 - Small	7.44	10.81	12.39	11.88	-	-

5.2.4 Large Scale with 250 Outlets

We solve five instances in the set of large scale problems with 250 outlets and 75 regional depots. The results are shown in Tables D.1-D.6 in Appendix D. The term *LB* in these Tables refers to local branching. The total time limit for the commercial solver and the LP-based heuristic is 57600 seconds; it is 28800 seconds for local branching. Each instance is solved by the commercial solver and the LP-based heuristic for both vehicle capacities.

The local branching algorithm is implemented on Model 1 and Model 2 on y_i variables with $k = 20$. Moreover, it is applied on z_r and z_{rs} variables with $k = 100$ and $k = 200$ for Model 1 and Model 2, respectively. The node time limit for these models is 2000 seconds. For Model 3, the implementation is done on y_i variables with $k = 10$ and with node time limit of 300 seconds.

All the instances for both vehicle capacities in Model 1 and Model 2 reach their time limit in all approaches. As we can see in Figures 5.4-5.7, the LP-based heuristic approach have negative deviations in some cases, which means it found a better solution. Except for instance 2, the deviations of local branching is small, given that it has half of the time to solve. For Model 1 with large vehicle capacity, local branching on y_i has the lowest deviations (3.99%). For Model 2 with both vehicle capacities, local branching on z_{rs} with $k = 100$ has shown the best performance with percentage deviations of 2.71% and 1.67% for large and small vehicle sizes, respectively.

On the other hand, all instances of Model 3 are solved to optimality with the commercial solver. Moreover, the local branching approach found the optimal solution with no deviation; however, the LP-based heuristic has shown some deviations in results (0.93% on average).

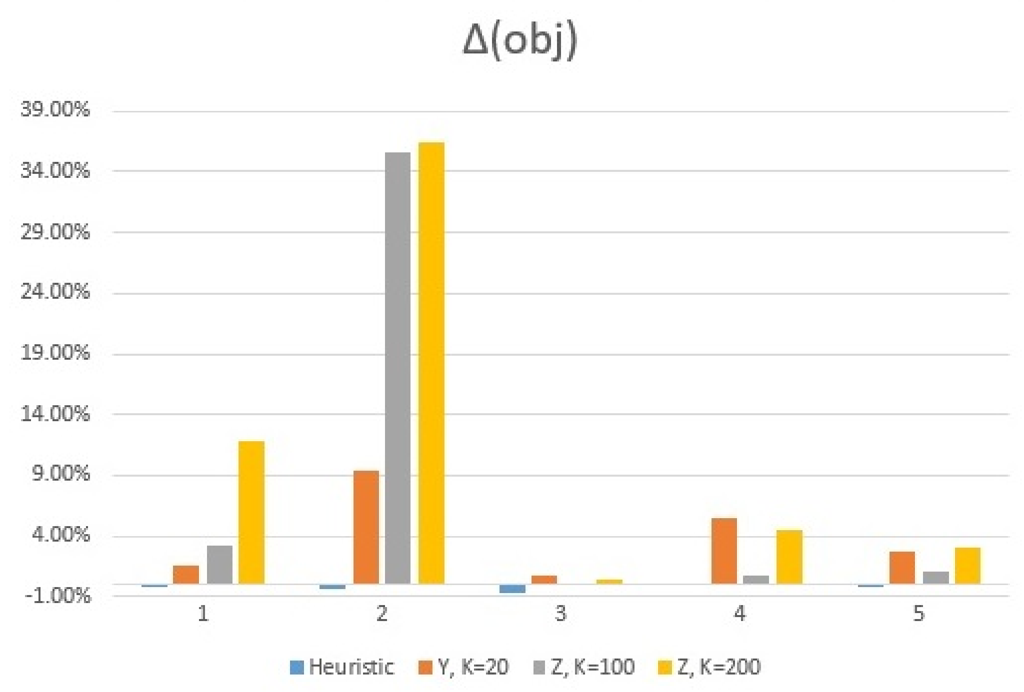

Figure 5.4 Percentage deviation of the result of each solution approach with that of the commercial solver for Model 1 with large vehicle capacity

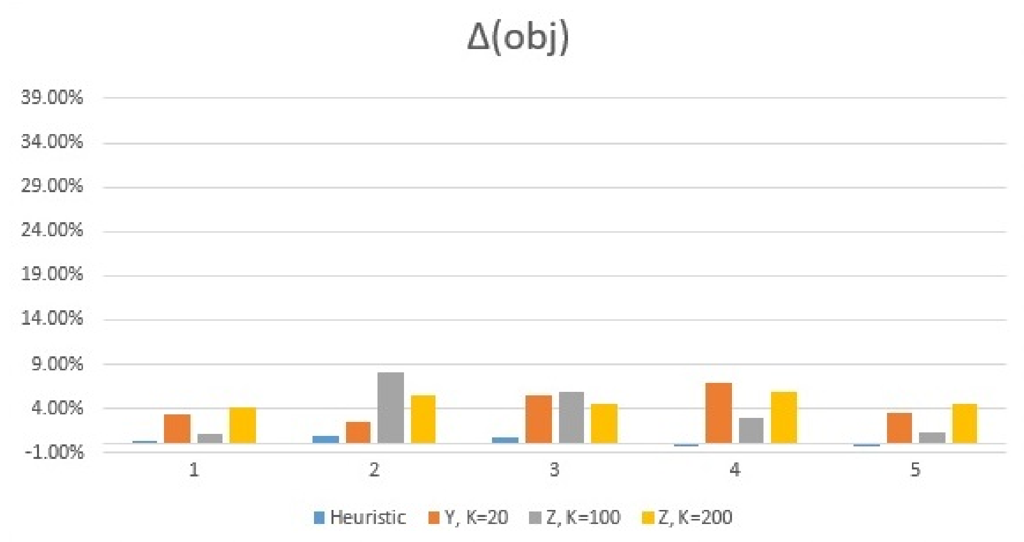

Figure 5.5 Percentage deviation of the result of each solution approach with that of the commercial solver for Model 1 with small vehicle capacity

5.2.5 General Analysis

As we can see from Tables 5.6-5.8 in all problem sizes, Model 3 has the least deviations compared to others. The local branching approach results have shown no deviations for this model and the LP-based heuristic has at most 2.38% deviations.

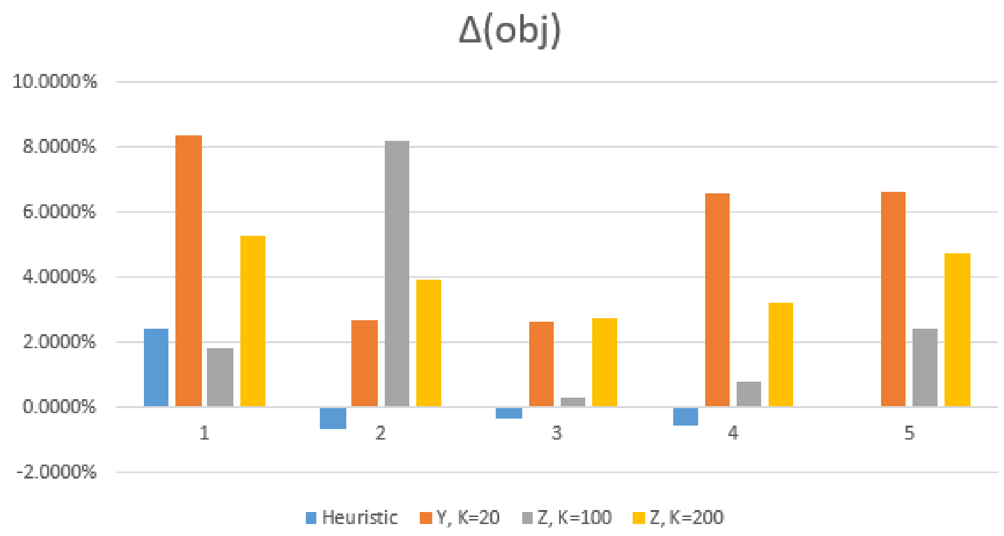

Figure 5.6 Percentage deviation of the result of each solution approach with that of the commercial solver for Model 2 large vehicle capacity

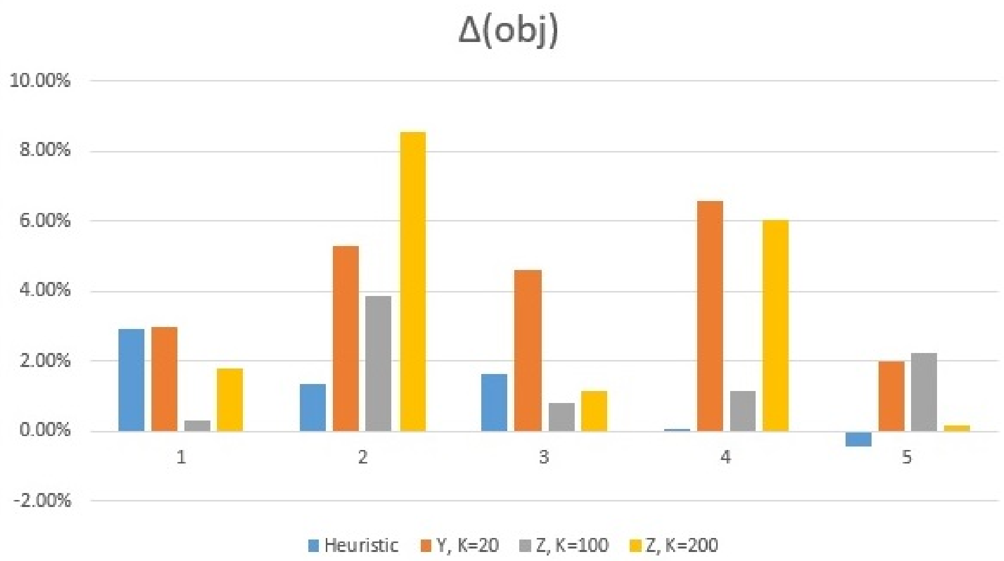

Figure 5.7 Percentage deviation of the result of each solution approach with that of the commercial solver for Model 2 small vehicle capacity

Moreover, the LP-based heuristic performance improves as the problem size increases. In the largest scale of our problem sets, this approach shows the least amount of deviations compared to other approaches. Even we have negative average deviation in Model 1.

In general, in the sets of small and medium scale problems, local branching does not show deviations in results. The highest deviation of this approach can be seen in the large scale with 250 outlets with 11.25% on average. Note that this approach always has half the time of the commercial solver and LP-based heuristic.

Table 5.6 The average percentage deviations for Model 1

	Model 1 - Large vehicle Capacity					Model 1 - Small vehicle Capacity				
Problem sizes	LP-H	$y_i, k(1)$	$y_i, k(2)$	$z_r, k(1)$	$z_r, k(2)$	LP-H	$y_i, k(1)$	$y_i, k(2)$	$z_r, k(1)$	$z_r, k(2)$
30 outlets	1.68%	0.00%	-	0.00%	0.00%	0.87%	0.00%	-	0.00%	0.00%
50 outlets	0.50%	0.00%	0.00%	0.00%	0.00%	0.71%	0.00%	0.00%	0.00%	0.00%
100 outlets	0.88%	0.00%	0.00%	0.00%	0.00%	0.11%	1.24%	0.00%	0.00%	0.00%
250 outlets	-0.21%	3.99%	-	8.17%	11.25%	0.34%	4.41%	-	3.91%	4.92%

Table 5.7 The average percentage deviations for Model 2

	Model 2 - Large vehicle Capacity					Model 2 - Small vehicle Capacity				
Problem sizes	LP-H	$y_i, k(1)$	$y_i, k(2)$	$z_{rs}, k(1)$	$z_{rs}, k(2)$	LP-H	$y_i, k(1)$	$y_i, k(2)$	$z_{rs}, k(1)$	$z_{rs}, k(2)$
30 outlets	1.78%	0.00%	-	0.00%	0.00%	2.31%	0.00%	-	0.00%	0.00%
50 outlets	0.89%	0.00%	0.00%	0.00%	0.00%	0.73%	0.00%	0.00%	0.00%	0.02%
100 outlets	1.11%	0.71%	0.63%	0.42%	0.28%	1.73%	0.52%	0.59%	0.16%	0.15%
250 outlets	0.19%	5.38%	-	2.72%	3.99%	1.12%	4.30%	-	1.67%	3.55%

Table 5.8 The average percentage deviations for Model 3

	Model 3 - Large vehicle Capacity			Model 3 - Small vehicle Capacity		
Problem sizes	LP-H	LB on y(1)	LB on y(2)	LP-H	LB on y(1)	LB on y(2)
30 outlets	1.69%	0.00%	-	2.38%	0.00%	-
50 outlets	0.54%	0.00%	0.00%	0.39%	0.00%	0.00%
100 outlets	0.93%	0.00%	0.00%	1.79%	0.00%	0.00%
250 outlets	0.93%	0.01%	0.00%	0.00%	0.00%	-

In Table 5.9, we present the number of instances solved to optimality with the commercial solver for all three models and both vehicle capacities. All instances of small scale with all three models are solved to optimality. We see this pattern in medium scale except for Model 2 with small vehicle capacity. As the problem size grows, the number of instances solve to optimality decreases. In general, small vehicle capacity instances are harder to solve.

In all problem sizes with both vehicle capacities all instances in Model 3 yield the optimal solution.

Table 5.9 Number of problems solved to optimality with commercial solver

	Large Capacity			Small Capacity		
Problem sizes	Model 1	Model 2	Model 3	Model 1	Model 2	Model 3
30 outlets	10	10	10	8	8	8
50 outlets	9	9	9	9	2	9
100 outlets	8	3	10	4	0	10
250 outlets	0	0	5	0	0	5

In order to observe the performance of each solution approach we compare the number of instances solved to optimality for each model by each approach. The results are presented in Tables 5.10, 5.11, and 5.12 for Model 1, 2, and 3, respectively.

For Model 1 and Model 3, the local branching approach has a similar performance as the commercial solver in all problem sizes. The same pattern exists for Model 2 except in large scale (100 outlets) with large vehicle capacity.

In most cases, even in small scales, the LP-based heuristic found less optimal solutions.

Table 5.10 Number of problems in Model 1 solved to optimality with other approaches

	Model 1-Large capacity					Model 1-Small capacity				
Problem sizes	LP-H	$y_i, k(1)$	$y_i, k(2)$	$z_r, k(1)$	$z_r, k(2)$	LP-H	$y_i, k(1)$	$y_i, k(2)$	$z_r, k(1)$	$z_r, k(2)$
30 outlets	5	10	-	10	10	4	8	-	8	8
50 outlets	1	9	9	9	9	5	9	9	9	9
100 outlets	3	8	8	8	8	1	4	4	4	4
250 outlets	0	0	-	0	0	0	0	-	0	0

Table 5.11 Number of problems in Model 2 solved to optimality with other approaches

	Model 2-Large Capacity					Model-Small Capacity				
Problem sizes	LP-H	$y_i, k(1)$	$y_i, k(2)$	$z_{rs}, k(1)$	$z_{rs}, k(2)$	LP-H	$y_i, k(1)$	$y_i, k(2)$	$z_{rs}, k(1)$	$z_{rs}, k(2)$
30 outlets	5	10	-	10	10	2	8	-	8	8
50 outlets	4	9	9	0	9	2	2	2	2	2
100 outlets	3	1	0	0	0	0	0	0	0	0
250 outlets	0	0	-	0	0	0	0	-	0	0

Table 5.12 Number of problems in Model 3 solved to optimality with other approaches

	Model 3-Large Capacity			Model 3-Large Capacity		
Problem sizes	LP-H	$y_i, k(1)$	$y_i, k(2)$	LP-H	$y_i, k(1)$	$y_i, k(2)$
30 outlets	5	10	-	2	8	-
50 outlets	5	9	9	7	9	9
100 outlets	4	10	10	2	10	10
250 outlets	1	4	-	5	5	-

The small scale problems are the only problem sets where all instances for all three models reach the optimal solution. We calculated the average objective function values for both large and small capacity vehicles and compared them.

The results show that the objective function value of Model 2 is 5.6% and 9.3% less than Model 1 for large and small vehicle capacity, respectively. As we expected, since Model 1 does not use fewer routes (i.e., vehicles) in low demand seasons, it has a more costly objective function. Therefore, using the so-called "ideal approach" yields economic advantages.

The objective function value of Model 3 for the large and small vehicle capacity is 9.9% and 3.0% less than that of the Model 2. As we anticipated, since in Model 3 the outbound transportation cost is a function of the amount of transported products, this model underestimates the total cost. This underestimation is larger when it comes to the large vehicle capacity instances.

6. CONCLUSION

In this thesis, we study a single commodity three-echelon distribution network design problem, which takes real-life aspects, such as seasonal fluctuations in demands into account. We develop three alternative mixed-integer linear programming models that have different levels of flexibility while responding to seasonal demand. Products are delivered to outlets from a distribution center through regional depots. Each model includes transshipment amounts, facility location, and routing decisions. The objective is to minimize the total cost, which consists of opening and operating costs of regional depots, inbound and outbound transportation costs. We also evaluate the effect of the vehicle size used in delivery to outlets on the problem difficulty and also its impact on the solution quality.

Our models are different in terms of formulating the outbound transportation decisions. In Model 1, the routing decisions for outbound transportation are considered as fixed throughout the planning horizon. This model would yield a robust solution under seasonal demand and well suits the company that prefers operational stability even under seasonality. However, higher outbound cost would be a consequence of using this model. Model 2 mainly uses the same approach considering seasonal routing decisions for outbound transportation throughout the planning horizon. This model reflects the "ideal approach" as the outbound transportation decisions are changed in response to seasonal fluctuations in demand. Using this model would yield the lowest outbound transportation costs and total costs. Model 3 finds a lower bound on the total cost by reducing the outbound binary variables in each season. This model is also responsive to seasonal changes and less computationally challenging. However, since this model charges the outbound transportation cost for the load on the vehicle not the vehicle capacity it is expected to underestimate the total cost. Since this model does not consider vehicle selection cost it also allows partial flow through each selected route. Although, the real-life businesses mostly consider the number of vehicles and their capacity in their total costs, upon a contractual agreement model 3 can be also of great use, especially due to its short solution time even in large-scale problems. In general, all instances of model 3 are solved to optimality, even in the largest set of problems (250 outlets).

To solve the resulting problems, we propose a LP-based constructive heuristic approach. Alternatively, we adapt the local branching algorithm to all three different models with variations branching on different binary decision variables. In order to evaluate the efficiency and effectiveness of the proposed heuristics, we solve instances of four sets of problems varying in terms of the problem size. The total time limit granted to the local branching approach is half of the other approaches. For each instance, we observed the solution run time, objective function value, and percentage deviations of the results of each solution approach with respect to that of the commercial solver.

The results show that using large vehicles leads the solution approaches to perform well in terms of solution quality and computational efficiency. According to our experiments, all instances of small and medium-size problems for all three models have yielded the optimal solution with commercial solver, except for model 2 with small vehicle capacity in the medium scale. As the problem size increases, the number of instances solve to optimality decrease. The local branching approach has mostly shown the same performance as the commercial solver in terms of the number of optimal solutions but in less computational time.

In most cases, even in small scales, the LP-based heuristic found less optimal solutions. However, the performance of this approach improves as the problem size increases. In the largest set of problems, this approach shows the least amount of deviations compared to other approaches.

In future studies, instead of dedicating half the time to the local branching approach, dynamic stopping criteria can be applied. Besides, additional assumptions such as multiple DCs and multiple products can be considered. Opening regional depots can depend on the season. Other solution approaches can be performed to solve the model.

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APPENDIX A: Results of Small Scale Instances (30 Outlets)

Table A.1 Model 1 with large vehicle capacity

	Commercial Solver			**LP-Based Heuristic**			**Local Branching on y_i; $k=5$**			**Local Branching on z_r; $k=5$**			**Local Branching on z_r; $k=10$**		
No.	Obj Func	Run Time	Gap	Obj Func	Run Time	Δ(obj)	Obj Func	Run Time	Δ(obj)	Obj Func	Run Time	Δ(obj)	Obj Func	Run Time	Δ(obj)
1	171860.8	0.79	0%	171860.8	1.05	0.0%	171860.8	0.88	0%	171860.8	1.75	0%	171860.8	0.89	0%
2	202295.6	0.33	0%	202295.6	0.71	0.0%	202295.6	0.34	0%	202295.6	0.38	0%	202295.6	0.43	0%
3	239504.6	0.34	0%	240877.4	0.65	0.6%	239504.6	0.33	0%	239504.6	0.55	0%	239504.6	0.39	0%
4	156778.1	2.42	0%	158981.8	1.10	1.4%	156778.1	2.83	0%	156778.1	3.05	0%	156778.1	2.89	0%
5	185870.7	1.07	0%	185870.7	1.00	0.0%	185870.7	2.33	0%	185870.7	2.12	0%	185870.7	2.61	0%
6	177013.5	0.60	0%	179212.0	0.80	1.2%	177013.5	0.92	0%	177013.5	0.94	0%	177013.5	0.83	0%
7	181395.3	0.38	0%	181395.3	0.88	0.0%	181395.3	0.40	0%	181395.3	0.58	0%	181395.3	0.54	0%
8	158537.9	0.70	0%	162975.5	0.88	2.8%	158537.9	0.80	0%	158537.9	2.16	0%	158537.9	0.99	0%
9	195411.9	0.42	0%	195411.9	0.58	0.0%	195411.9	0.56	0%	195411.9	0.67	0%	195411.9	0.79	0%
10	187470.3	0.49	0%	207630.8	0.94	10.8%	187470.3	0.58	0%	187470.3	0.83	0%	187470.3	0.76	0%

Table A.2 Model 1 with small vehicle capacity

	Commercial Solver			LP-Based Haeuristic			Local Branching on y_i; $k=5$			Local Branching on z_r; $k=5$			Local Branching on z_r; $k=10$		
No.	Obj Func	Run Time	Gap	Obj Func	Run Time	Δ(obj)	Obj Func	Run Time	Δ(obj)	Obj Func	Run Time	Δ(obj)	Obj Func	Run Time	Δ(obj)
1	181139.2	0.91	0%	183684.0	1.37	1.4%	181139.2	2.02	0%	181139.2	1.04	0%	181139.2	1.00	0%
2	217459.5	0.87	0%	217459.5	0.93	0.0%	217459.5	0.92	0%	217459.5	0.81	0%	217459.5	0.81	0%
3	251579.0	0.39	0%	251579.0	1.15	0.0%	251579.0	0.47	0%	251579.0	0.45	0%	251579.0	0.43	0%
4	169748.7	1.99	0%	169748.7	1.86	0.0%	169748.7	4.40	0%	169748.7	2.27	0%	169748.7	2.69	0%
5	196617.7	1.32	0%	202143.0	2.65	2.8%	196617.7	2.62	0%	196617.7	1.82	0%	196617.7	1.44	0%
8	159830.2	3.74	0%	164248.2	4.78	2.8%	159830.2	7.55	0%	159830.2	4.57	0%	159830.2	4.52	0%
9	194745.6	0.72	0%	194745.6	1.13	0.0%	194745.6	1.65	0%	194745.6	1.02	0%	194745.6	1.00	0%
10	191054.9	0.29	0%	191074.7	0.98	0.0%	191054.9	0.33	0%	191054.9	0.41	0%	191054.9	0.47	0%

Table A.3 Model 2 with large vehicle capacity

	Commercial Solver			LP-Based Haeuristic			Local Branching on y_i; $k = 5$			Local Branching on z_{rs}; $k = 5$			Local Branching on z_{rs}; $k = 10$		
No.	Obj Func	Run Time	Gap	Obj Func	Run Time	Δ(obj)	Obj Func	Run Time	Δ(obj)	Obj Func	Run Time	Δ(obj)	Obj Func	Run Time	Δ(obj)
1	160645.0	2.76	0%	160645.0	1.59	0.0%	160645.0	3.59	0%	160645.0	2.67	0%	160645.0	4.81	0%
2	187702.7	0.25	0%	187702.7	0.87	0.0%	187702.7	0.32	0%	187702.7	0.52	0%	187702.7	0.33	0%
3	228077.3	0.51	0%	228077.3	1.05	0.0%	228077.3	0.61	0%	228077.3	0.94	0%	228077.3	0.77	0%
4	144680.4	1.96	0%	148618.8	0.86	2.7%	144680.4	3.88	0%	144680.4	4.54	0%	144680.4	3.88	0%
5	173489.6	6.90	0%	176818.3	1.77	1.9%	173489.6	10.77	0%	173489.6	3.80	0%	173489.1	8.77	0%
6	170044.8	1.19	0%	170726.7	1.24	0.4%	170044.8	2.34	0%	170044.8	2.58	0%	170044.8	3.16	0%
7	170455.2	1.59	0%	170455.2	1.41	0.0%	170455.2	2.37	0%	170455.2	3.21	0%	170455.2	2.94	0%
8	155569.7	2.32	0%	160208.9	0.78	3.0%	155569.7	3.24	0%	155569.7	2.14	0%	155569.7	3.66	0%
9	187843.9	1.91	0%	187843.9	1.28	0.0%	187843.9	3.27	0%	187852.5	2.19	0%	187843.9	3.60	0%
10	178463.6	2.17	0%	195914.2	1.06	9.8%	178463.6	3.32	0%	178463.6	2.08	0%	178463.6	2.87	0%

Table A.4 Model 2 with small vehicle capacity

	Commercial Solver			**LP-Based Haeuristic**			**Local Branching on** y_i**;** $k=5$			**Local Branching on** z_{rs}**;** $k=5$			**Local Branching on** z_{rs}**;** $k=10$		
1	164226.5	344.14	0%	165479.3	8.80	0.8%	164226.5	189.26	0%	164226.5	61.44	0%	164226.5	88.11	0%
2	199050.3	4.73	0%	200667.2	1.38	0.8%	199050.3	6.96	0%	199050.3	6.64	0%	199050.3	6.80	0%
3	233252.8	6.26	0%	233252.8	6.95	0.0%	233252.8	9.28	0%	233252.8	9.44	0%	233252.8	9.57	0%
4	153134.2	1337.93	0%	155464.9	2.93	1.5%	153134.2	242.29	0%	153134.2	62.57	0%	153134.2	115.36	0%
5	179959.0	171.16	0%	186475.2	33.43	3.6%	179959.0	160.04	0%	179959.0	64.18	0%	179959.0	191.03	0%
8	144622.5	11.39	0%	148983.1	1.63	3.0%	144622.5	15.42	0%	144622.5	16.47	0%	144622.5	15.99	0%
9	178900.9	2.38	0%	178900.9	2.32	0.0%	178900.9	3.15	0%	178900.9	3.47	0%	178900.9	3.51	0%
10	174948.0	24.14	0%	190230.6	1.74	8.7%	174948.0	28.15	0%	174948.0	28.22	0%	174948.0	29.14	0%

Table A.5 Model 3 with large vehicle capacity

	Commercial Solver			**LP-Based Haeuristic**			**Local Branching on y_i; $k = 5$**		
No.	Obj Func	Run Time	Gap	Obj Func	Run Time	Δ(obj)	Obj Func	Run Time	Δ(obj)
1	143793.6	0.10	0%	143793.6	0.32	0.0%	143793.6	0.13	0%
2	173626.6	0.04	0%	173626.6	0.33	0.0%	173626.6	0.08	0%
3	211656.8	0.08	0%	211656.8	0.58	0.0%	211656.8	0.13	0%
4	132065.5	0.11	0%	133353.9	0.49	1.0%	132065.5	0.13	0%
5	158188.1	0.08	0%	163183.2	0.56	3.2%	158188.1	0.19	0%
6	154407.5	0.16	0%	155094.1	0.43	0.4%	154407.5	0.13	0%
7	153995.0	0.05	0%	153995.0	0.57	0.0%	153995.0	0.17	0%
8	136182.9	0.05	0%	140057.5	0.45	2.8%	136182.9	0.10	0%
9	168334.7	0.09	0%	168334.7	0.40	0.0%	168334.7	0.12	0%
10	165699.4	0.07	0%	181440.9	0.50	9.5%	165699.4	0.16	0%

Table A.6 Model 3 with small vehicle capacity

	Commercial Solver			**LP-Based Haeuristic**			**Local Branching on y_i; $k=5$**		
No.	Obj Func	Run Time	Gap	Obj Func	Run Time	Δ(obj)	Obj Func	Run Time	Δ(obj)
1	158064.1	0.20	0%	159883.8	0.42	1.2%	158064.1	0.34	0%
2	193055.9	0.06	0%	194366.5	0.39	0.7%	193055.9	0.17	0%
3	226388.1	0.17	0%	226388.1	0.36	0.0%	226388.1	0.24	0%
4	149452.0	0.10	0%	151616.3	0.42	1.4%	149452.0	0.09	0%
5	174867.1	0.19	0%	181581.0	0.65	3.8%	174867.1	0.26	0%
8	139782.5	0.08	0%	143762.5	0.29	2.8%	139782.5	0.11	0%
9	173720.7	0.14	0%	173720.7	0.41	0.0%	173720.7	0.22	0%
10	170304.0	0.09	0%	185803.0	0.36	9.1%	170304.0	0.24	0%

APPENDIX B: Results of Medium Scale Instances (50 Outlets)

Table B.1 Model 1 with large vehicle capacity

	Commercial Solver			**Heuristic**			**Local branching on y_i, $k=5$**			**Local branching on y_i, $k=10$**			**Local branching on z_r, $k=5$**			**Local branching on z_r, $k=10$**		
1	275660.0	15.84	0%	276204.5	7.25	0.2%	275660.0	20.54	0%	275660.0	20.99	0%	275660.0	17.51	0%	275660.0	18.40	0%
2	304323.9	5.97	0%	304625.7	2.85	0.1%	304323.9	5.88	0%	304323.9	5.93	0%	304323.9	7.37	0%	304323.9	7.02	0%
4	255738.5	7.18	0%	255738.5	3.40	0.0%	255738.5	3.55	0%	255738.5	3.79	0%	255738.5	10.44	0%	255738.5	10.30	0%
5	295414.9	4.42	0%	301988.0	4.79	2.2%	295414.9	6.58	0%	295414.9	6.66	0%	295414.9	6.83	0%	295414.9	6.72	0%
6	242223.6	5.06	0%	242754.6	2.50	0.2%	242223.6	3.87	0%	242223.6	4.04	0%	242223.6	6.64	0%	242223.6	6.02	0%
7	275810.9	15.72	0%	277038.4	2.44	0.4%	275810.9	21.26	0%	275810.9	21.47	0%	275810.9	19.22	0%	275810.9	20.11	0%
8	329291.1	2.17	0%	329577.8	1.67	0.1%	329291.1	1.98	0%	329291.1	1.93	0%	329291.1	3.03	0%	329291.1	3.45	0%
9	284550.2	12.86	0%	286469.4	2.69	0.7%	284550.2	10.43	0%	284550.2	10.86	0%	284550.2	18.65	0%	284550.2	17.61	0%
10	300810.4	21.07	0%	302492.7	11.59	0.6%	300810.4	8.93	0%	300810.4	9.05	0%	300810.4	32.62	0%	300810.4	37.78	0%

Table B.2 Model 1 with small vehicle capacity

	Commercial Solver			**Heuristic**			**Local branching on y_i, $k=5$**			**Local branching on y_i, $k=10$**			**Local branching on z_r, $k=5$**			**Local branching on z_r, $k=10$**		
No.	Obj Func	Run Time	Gap	Obj Func	Run Time	Δ(obj)	Obj Func	Run Time	Δ(obj)	Obj Func	Run Time	Δ(obj)	Obj Func	Run Time	Δ(obj)	Obj Func	Run Time	Δ(obj)
1	292763.4	959.10	0%	292763.4	207.98	0.0%	292763.4	1103.68	0%	292763.4	929.53	0%	292763.4	34.21	0%	292763.4	131.40	0%
2	333536.0	49.34	0%	333536.0	29.23	0.0%	333536.0	2.03	0%	333536.0	1.68	0%	333536.0	32.87	0%	333536.0	38.08	0%
4	262615.3	2.38	0%	265880.6	3.73	1.2%	262615.3	2.53	0%	262615.3	2.22	0%	262615.3	3.06	0%	262615.3	3.25	0%
5	314524.6	52.76	0%	314986.7	9.84	0.1%	314524.6	18.18	0%	314524.6	17.15	0%	314524.6	32.95	0%	314524.6	50.26	0%
6	260126.8	34.19	0%	264882.5	4.48	1.8%	260126.8	16.00	0%	260126.8	15.37	0%	260126.8	32.01	0%	260126.8	47.02	0%
7	300933.7	22.03	0%	300933.7	6.47	0.0%	300933.7	17.60	0%	300933.7	16.92	0%	300933.7	29.90	0%	300933.7	27.03	0%
8	351693.0	2.19	0%	362932.5	3.56	3.2%	351693.0	2.28	0%	351693.0	2.16	0%	351693.0	2.47	0%	351693.0	2.20	0%
9	287951.3	41.08	0%	287951.3	22.17	0.0%	287951.3	33.12	0%	287951.3	34.06	0%	287951.3	36.36	0%	287951.3	82.94	0%
10	317159.0	1.19	0%	317159.0	2.82	0.0%	317159.0	1.23	0%	317159.0	1.08	0%	317159.0	1.73	0%	317159.0	1.50	0%

Table B.3 Model 2 with large vehicle capacity

	Commercial Solver			**Heuristic**			**Local branching on y_i, $k=5$**			**Local branching on y_i, $k=10$**			**Local branching on z_{rs}, $k=10$**			**Local branching on z_{rs}, $k=20$**		
No.	Obj Func	Run Time	Gap	Obj Func	Run Time	Δ(obj)	Obj Func	Run Time	Δ(obj)	Obj Func	Run Time	Δ(obj)	Obj Func	Run Time	Δ(obj)	Obj Func	Run Time	Δ(obj)
1	253241.3	170.68	0%	253241.3	9.57	0.0%	253241.3	318.54	0%	253241.3	114.22	0%	253241.3	114.51	0%	253241.3	185.86	0%
2	280243.0	218.73	0%	280770.9	4.82	0.2%	280243.0	335.01	0%	280243.0	112.42	0%	280243.0	236.54	0%	280243.0	418.25	0%
4	235932.4	17.56	0%	245289.5	10.38	4.0%	235932.4	41.48	0%	235932.4	13.12	0%	235932.4	23.40	0%	235932.4	25.43	0%
5	272520.8	25.24	0%	279437.0	16.24	2.5%	272520.8	133.80	0%	272520.8	115.48	0%	272520.8	89.88	0%	272520.8	151.97	0%
6	222331.8	40.92	0%	222648.7	17.54	0.1%	222331.8	231.42	0%	222331.8	122.63	0%	222331.8	119.05	0%	222331.8	95.53	0%
7	255250.3	13.40	0%	258139.0	15.75	1.1%	255250.3	38.73	0%	255250.3	14.23	0%	255250.3	22.86	0%	255250.3	22.12	0%
8	310778.2	32.56	0%	310778.2	7.02	0.0%	310778.2	117.46	0%	310778.2	82.03	0%	310778.2	72.30	0%	310778.2	122.47	0%
9	268891.0	41.72	0%	268891.0	6.61	0.0%	268891.0	50.88	0%	268891.0	27.61	0%	268891.0	36.17	0%	268891.0	35.42	0%
10	280119.3	46.27	0%	280119.3	16.89	0.0%	280119.3	197.59	0%	280119.3	74.40	0%	280119.3	124.09	0%	280119.3	171.11	0%

Table B.4 Model 2 with small vehicle capacity

	Commercial Solver			Heuristic			Local branching on y_i, $k=5$			Local branching on y_i, $k=10$			Local branching on z_{rs}, $k=10$			Local branching on z_{rs}, $k=20$		
No.	Obj Func	Run Time	Gap	Obj Func	Run Time	Δ(obj)	Obj Func	Run Time	Δ(obj)	Obj Func	Run Time	Δ(obj)	Obj Func	Run Time	Δ(obj)	Obj Func	Run Time	Δ(obj)
1	264347.4	14400.0	0.8%	266406.1	14400.0	0.8%	264347.4	7200.0	0%	264347.4	7200.0	0%	264348.5	1019.1	0%	264595.0	7200.0	0%
2	303371.8	33.0	0.0%	303371.8	5.4	0.0%	303371.8	77.0	0%	303371.8	78.0	0%	303371.8	42.6	0%	303371.8	77.5	0%
4	231532.2	14400.0	0.2%	237748.3	30.1	2.7%	231532.2	261.9	0%	231532.2	7200.0	0%	231532.2	160.4	0%	231532.2	1086.6	0%
5	285991.3	14400.0	0.5%	287196.5	14400.0	0.4%	285991.3	7200.0	0%	286006.4	7200.0	0%	285991.3	372.3	0%	286017.2	5774.2	0%
6	231299.4	14400.0	0.5%	236705.9	2732.5	2.3%	231299.4	7200.0	0%	231325	7200.0	0%	231299.4	318.1	0%	231361.9	7200.0	0%
7	271382.4	14400.0	0.2%	272318.8	14400.0	0.3%	271382.4	7200.0	0%	271382.4	7200.0	0%	271382.4	1056.8	0%	271382.4	8916.3	0%
8	320235.2	14400.0	0.4%	320235.2	15.2	0.0%	320235.2	7200.0	0%	320235.2	7200.0	0%	320235.2	323.5	0%	320336.9	3528.0	0%
9	260207.2	14400.0	0.1%	260207.2	35.6	0.0%	260207.2	13811.9	0%	260207.2	7200.0	0%	260207.2	408.2	0%	260307.9	7200.0	0%
10	285823.3	803.8	0.0%	285823.3	28.7	0.0%	285823.3	1279.0	0%	285823.3	890.2	0%	285823.3	191.1	0%	285823.3	1046.2	0%

Table B.5 Model 3 with large vehicle capacity

	Commercial Solver			**Heuristic**			**Local branching on y_i, $k = 5$**			**Local branching on y_i, $k = 10$**		
No.	Obj Func	Run Time	Gap	Obj Func	Run Time	Δ(obj)	Obj Func	Run Time	Δ(obj)	Obj Func	Run Time	Δ(obj)
1	238308.5	0.31	0%	238308.5	0.93	0.0%	238308.5	0.29	0%	238308.5	0.36	0%
2	261375.6	0.17	0%	261545.4	0.76	0.1%	261375.6	0.21	0%	261375.6	0.18	0%
4	214232.9	0.22	0%	219497.7	1.37	2.5%	214232.9	0.27	0%	214232.9	0.25	0%
5	250435.7	0.26	0%	255965.1	0.97	2.2%	250435.7	0.32	0%	250435.7	0.24	0%
6	194242.4	0.21	0%	194245.9	0.99	0.0%	194242.4	0.24	0%	194242.4	0.20	0%
7	233151.9	0.53	0%	233151.9	1.17	0.0%	233151.9	0.57	0%	233151.9	0.50	0%
8	288583.7	0.22	0%	288828.1	0.80	0.1%	288583.7	0.27	0%	288583.7	0.21	0%
9	247306.1	0.27	0%	247306.1	0.94	0.0%	247306.1	0.42	0%	247306.1	0.46	0%
10	260908.3	0.16	0%	260908.3	0.80	0.0%	260908.3	0.20	0%	260908.3	0.28	0%

Table B.6 Model 3 with small vehicle capacity

	Commercial Solver			**Heuristic**			**Local branching on** y_i, $k=5$			**Local branching on** y_i, $k=10$		
No.	Obj Func	Run Time	Gap	Obj Func	Run Time	Δ(obj)	Obj Func	Run Time	Δ(obj)	Obj Func	Run Time	Δ(obj)
1	259011.5	0.52	0%	259011.5	1.16	0.0%	259011.5	0.42	0%	259011.5	0.52	0%
2	296299.7	0.20	0%	296299.7	1.52	0.0%	296299.7	0.14	0%	296299.7	0.14	0%
4	223874.0	0.48	0%	223874.0	1.46	0.0%	223874.0	0.41	0%	223874.0	0.36	0%
5	275886.8	0.56	0%	276389.7	1.14	0.2%	275886.8	0.51	0%	275886.8	0.48	0%
6	221326.5	0.62	0%	228711.3	1.37	3.3%	221326.5	0.51	0%	221326.5	0.52	0%
7	262595.7	0.42	0%	262595.7	1.58	0.0%	262595.7	0.30	0%	262595.7	0.28	0%
8	313233.1	0.62	0%	313233.1	0.85	0.0%	313233.1	0.54	0%	313233.1	0.67	0%
9	254172.9	0.31	0%	254172.9	0.96	0.0%	254172.9	0.32	0%	254172.9	0.35	0%
10	280370.1	0.11	0%	280370.1	0.96	0.0%	280370.1	0.12	0%	280370.1	0.12	0%

APPENDIX C: Results of Large Scale Instances (100 Outlets)

Table C.1 Model 1 with large vehicle capacity

	Commercial Solver			**LP-Based Haeuristic**			**Local branching on y_i, $k = 10$**			**Local branching on y_i, $k = 20$**			**Local branching on z_r, $k = 10$**			**Local branching on z_r, $k = 20$**		
No.	Obj Func	Run Time	Gap	Obj Func	Run Time	Δ(obj)	Obj Func	Run Time	Δ(obj)	Obj Func	Run Time	Δ(obj)	Obj Func	Run Time	Δ(obj)	Obj Func	Run Time	Δ(obj)
1	437535.0	7627.6	0.0%	447111.7	392.8	2.2%	437535.0	1529.6	0%	437535.0	1617.4	0%	437535.0	276.7	0%	437535.0	1416.9	0%
2	384177.0	621.7	0.0%	384177.0	179.9	0.0%	384177.0	918.5	0%	384177.0	571.3	0%	384177.0	646.4	0%	384177.0	545.3	0%
3	395787.9	2554.1	0.0%	398340.8	798.6	0.6%	395787.9	14400.0	0%	395787.9	1041.3	0%	395845.0	2159.6	0%	395787.9	1137.6	0%
4	386566.2	1749.2	0.0%	398920.4	92.8	3.2%	386566.2	207.2	0%	386566.2	157.0	0%	386566.2	262.2	0%	386566.2	188.8	0%
5	393123.1	1305.5	0.0%	393123.1	881.8	0.0%	393123.1	1031.3	0%	393123.1	933.7	0%	393123.1	216.8	0%	393123.1	634.0	0%
6	414773.0	3094.2	0.0%	421607.7	479.2	1.6%	414773.0	1288.2	0%	414773.0	1073.6	0%	414773.0	940.1	0%	414773.0	1178.4	0%
7	437532.4	27206.3	0.0%	443592.2	481.3	1.4%	437532.4	14401.5	0%	437532.4	14400.0	0%	437532.4	515.4	0%	437532.4	2191.3	0%
8	403820.5	1876.3	0.0%	403820.5	578.5	0.0%	403820.5	1302.6	0%	403820.5	1240.7	0%	403820.5	770.8	0%	403820.5	1029.3	0%
9	423425.8	28800.0	0.2%	423425.8	746.8	0.0%	423425.8	14400.0	0%	423425.8	14400.0	0%	423425.8	731.5	0%	423482.4	4239.4	0%
10	432093.1	28800.0	0.6%	431141.7	1216.1	-0.2%	432093.1	14400.0	0%	432093.1	14400.0	0%	432093.1	981.4	0%	432093.1	2612.1	0%

Table C.2 Model 1 with small vehicle capacity

	Commercial Solver			**LP-Based Haeuristic**			**Local branching on y_i, $k = 10$**			**Local branching on y_i, $k = 20$**			**Local branching on z_r, $k = 10$**			**Local branching on z_r, $k = 20$**		
No.	Obj Func	Run Time	Gap	Obj Func	Run Time	Δ(obj)	Obj Func	Run Time	Δ(obj)	Obj Func	Run Time	Δ(obj)	Obj Func	Run Time	Δ(obj)	Obj Func	Run Time	Δ(obj)
1	466148.5	28800.0	0.3%	466391.3	28800.0	0.1%	466148.5	14400.0	0%	466148.5	14400.0	0%	466148.5	4871.7	0%	466148.5	3421.2	0%
2	398818.0	28800.0	0.2%	398818.0	28800.0	0.0%	398818.0	13024.8	0%	398818.0	14400.0	0%	398818.0	3586.5	0%	398818.0	1898.1	0%
3	412246.0	28800.0	0.1%	412513.8	28800.0	0.1%	412246.0	14400.0	0%	412246.0	14400.0	0%	412246.0	3370.9	0%	412246.0	1821.3	0%
4	415400.0	3861.2	0.0%	427995.2	636.1	3.0%	415400.0	1199.9	0%	415400.0	837.2	0%	415400.0	1002.3	0%	415400.0	440.9	0%
5	420291.6	28800.0	0.1%	429622.7	28800.0	2.2%	420291.6	14400.0	0%	420291.6	4810.0	0%	420291.6	2531.8	0%	420291.6	1086.7	0%
6	444243.2	589.6	0.0%	454282.3	284.7	2.3%	444243.2	225.5	0%	444243.2	153.1	0%	444243.2	136.1	0%	444243.2	146.8	0%
7	453502.7	738.3	0.0%	475205.1	442.4	4.8%	453502.7	99.3	0%	453502.7	69.3	0%	453502.7	68.5	0%	453502.7	76.4	0%
8	438437.8	28800.0	0.3%	438503.3	14420.7	0.0%	438437.8	14401.3	0%	438437.8	14400.0	0%	438437.8	14400.0	0%	438437.8	2050.1	0%
9	440268.9	28800.0	0.2%	440268.9	28800.0	0.0%	440268.9	7819.5	0%	440268.9	14400.0	0%	440268.9	1914.2	0%	440268.9	408.9	0%
10	448737.4	3298.8	0.0%	448737.4	1276.4	0.0%	448737.4	1040.3	0%	448737.4	931.7	0%	448737.4	521.8	0%	448737.4	347.7	0%

Table C.3 Model 2 with large vehicle capacity

	Commercial Solver			**LP-Based Haeuristic**			**Local branching on y_i, $k = 10$**			**Local branching on y_i, $k = 20$**			**Local branching on z_{rs}, $k = 10$**			**Local branching on z_{rs}, $k = 20$**		
1	400596.3	28800	0.9%	412365.8	28800	2.9%	409659.0	14400	2.3%	408315.8	14400	1.9%	408196.9	14400	1.9%	401311.4	14400	0.2%
2	346097.8	15332	0.0%	346097.8	4245	0.0%	346201.5	14400	0.0%	346201.5	14400	0.0%	346226.7	14400	0.0%	346300.0	14400	0.1%
3	356329.6	28800	0.3%	361589.1	28800	1.5%	356938.1	14400	0.2%	356504.0	14400	0.0%	356578.1	14400	0.1%	356678.3	14400	0.1%
4	353106.8	28800	0.4%	366278.2	1612	3.7%	355848.9	14400	0.8%	354254.0	14400	0.3%	353469.7	14400	0.1%	353816.5	14400	0.2%
5	357325.2	16413	0.0%	357325.2	5034	0.0%	361974.1	14400	1.3%	363347.9	14400	1.7%	357756.9	14400	0.1%	358000.2	14400	0.2%
6	379365.2	28800	0.6%	385344.0	28800	1.6%	382351.7	14400	0.8%	383029.8	14400	1.0%	382215.1	14400	0.8%	382483.3	14400	0.8%
7	392484.5	28800	0.7%	398062.3	28800	1.4%	392783.2	14400	0.1%	393917.7	14400	0.4%	392689.7	14400	0.1%	392856.0	14400	0.1%
8	361487.6	17527	0.0%	361487.6	3481	0.0%	361487.6	14400	0.0%	361645.7	14400	0.0%	361538.4	14400	0.0%	361533.2	14400	0.0%
9	384214.0	28800	0.7%	384214.0	28800	0.0%	386382.5	14400	0.6%	387085.0	14400	0.7%	384653.9	14400	0.1%	384517.1	14400	0.1%
10	383761.3	28800	0.8%	383761.3	28800	0.0%	387994.8	14400	1.1%	384359.7	14400	0.2%	387846.5	14400	1.1%	388015.8	14400	1.1%

Table C.4 Model 2 with small vehicle capacity

	Commercial Solver			LP-Based Haeuristic			Local branching on y_i, $k = 10$			Local branching on y_i, $k = 20$			Local branching on z_{rs}, $k = 10$			Local branching on z_{rs}, $k = 20$		
No.	Obj Func	Run Time	Gap	Obj Func	Run Time	Δ(obj)	Obj Func	Run Time	Δ(obj)	Obj Func	Run Time	Δ(obj)	Obj Func	Run Time	Δ(obj)	Obj Func	Run Time	Δ(obj)
1	406219.6	28800	1.0%	413073.1	28800	1.7%	414278.7	14400	2.0%	407226.0	14400	0.2%	405860.2	14400	-0.1%	406217.4	14400	0.0%
2	341698.4	28800	0.4%	344361.7	28800	0.8%	341926.2	14400	0.1%	342035.1	14400	0.1%	341715.7	14400	0.0%	341897.6	14400	0.1%
3	355166.4	28800	0.4%	359476.1	28800	1.2%	356380.5	14400	0.3%	356741.5	14400	0.4%	355203.7	14400	0.0%	355705.7	14400	0.2%
4	362351.7	28800	0.2%	362351.7	28800	0.0%	362451.3	14400	0.0%	362992.8	14400	0.2%	362509.0	14400	0.0%	362489.4	14400	0.0%
5	365683.7	28800	1.3%	374021.4	28800	2.3%	365069.0	14400	-0.2%	368224.4	14400	0.7%	368058.3	14400	0.6%	365974.2	14400	0.1%
6	388277.9	28800	0.6%	398320.4	28800	2.6%	389124.7	14400	0.2%	388819.8	14400	0.1%	391305.8	14400	0.8%	391510.2	14400	0.8%
7	394566	28800	0.3%	416295.1	28800	5.5%	394668.9	14400	0.0%	394594.9	14400	0.0%	394716.8	14400	0.0%	394664.8	14400	0.0%
8	379705.8	28800	0.5%	380241.0	28800	0.1%	380052.7	14400	0.1%	383202.9	14400	0.9%	380105.4	14400	0.1%	380011.8	14400	0.1%
9	383619.3	28800	1.0%	382925.8	28800	-0.2%	384289.3	14400	0.2%	383795.8	14400	0.0%	383672.0	14400	0.0%	383841.5	14400	0.1%
10	388624.5	28800	1.1%	401426.6	28800	3.3%	398216.6	14400	2.5%	400688.9	14400	3.1%	388766.7	14400	0.0%	389182.1	14400	0.1%

Table C.5 Model 3 with large vehicle capacity

	Commercial Solver			LP-Based Haeuristic			Local branching on y_i, $k=10$			Local branching on y_i, $k=20$		
No.	Obj Func	Run Time	Gap	Obj Func	Run Time	Δ(obj)	Obj Func	Run Time	Δ(obj)	Obj Func	Run Time	Δ(obj)
1	369870.2	6.92	0%	377372.0	8.39	2.0%	369870.2	10.59	0%	369870.2	11.22	0%
2	310263.6	2.50	0%	310969.1	11.90	0.2%	310263.6	7.78	0%	310263.6	7.88	0%
3	332314.4	5.87	0%	335963.6	7.87	1.1%	332314.4	13.08	0%	332314.4	12.93	0%
4	314866.7	2.63	0%	325413.4	9.96	3.3%	314866.7	7.24	0%	314866.7	7.57	0%
5	324660.0	2.77	0%	324660.0	8.17	0.0%	324660.0	7.90	0%	324660.0	7.40	0%
6	350583.8	5.56	0%	355284	10.79	1.3%	350583.8	11.32	0%	350583.8	11.83	0%
7	368497.3	1.84	0%	373095.3	9.71	1.2%	368497.3	6.65	0%	368497.3	6.66	0%
8	329504.1	1.42	0%	329504.1	9.86	0.0%	329504.1	5.70	0%	329504.1	5.56	0%
9	357193.6	6.47	0%	357193.6	12.01	0.0%	357193.6	12.60	0%	357193.6	13.41	0%
10	358210.8	2.54	0%	358210.8	8.65	0.0%	358210.8	8.79	0%	358210.8	8.41	0%

Table C.6 Model 3 with small vehicle capacity

	Commercial Solver			**LP-Based Haeuristic**			**Local branching on y_i, $k=10$**			**Local branching on y_i, $k=20$**		
No.	Obj Func	Run Time	Gap	Obj Func	Run Time	Δ(obj)	Obj Func	Run Time	Δ(obj)	Obj Func	Run Time	Δ(obj)
1	395690.2	12.84	0%	402967.1	10.95	1.8%	395690.2	18.60	0%	395690.2	18.20	0%
2	329381.4	5.77	0%	331843.9	12.78	0.7%	329381.4	10.34	0%	329381.4	10.09	0%
3	345613.8	9.39	0%	349373.4	7.17	1.1%	345613.8	12.63	0%	345613.8	11.45	0%
4	347840.5	9.59	0%	347840.5	11.98	0.0%	347840.5	14.45	0%	347840.5	14.14	0%
5	351808.6	8.59	0%	362072.8	11.03	2.9%	351808.6	16.83	0%	351808.6	16.50	0%
6	379499.7	6.24	0%	387682.3	12.94	2.2%	379499.7	10.17	0%	379499.7	10.52	0%
7	384066.2	3.51	0%	405681.7	8.17	5.6%	384066.2	7.84	0%	384066.2	6.89	0%
8	367976.8	4.24	0%	368714.4	10.52	0.2%	367976.8	9.65	0%	367976.8	10.32	0%
9	373389.9	8.63	0%	373389.9	13.70	0.0%	373389.9	11.29	0%	373389.9	9.81	0%
10	379208.0	5.58	0%	391703.7	8.90	3.3%	379208.0	12.08	0%	379208.0	10.88	0%

APPENDIX D: Results of Large Scale Instances (250 Outlets)

Table D.1 Model 1 with large vehicle capacity

	CS		**Heuristic**		**LB on y_i; $k=20$**		**LB on z_r; $k=100$**		**LB on z_r; $k=200$**	
No.	Obj Func	Gap	Obj Func	Δ(obj)	Obj Func	Δ(obj)	Obj Func	Δ(obj)	Obj Func	Δ(obj)
1	763805.0	2.6%	763707.7	0.0%	776472.5	1.7%	788122.7	3.2%	854675.8	11.9%
2	745886.6	1.4%	742716.2	-0.4%	815435.9	9.3%	1011879.5	35.7%	1017398	36.4%
3	909292.5	3.3%	903541.2	-0.6%	916083.0	0.7%	910547.7	0.1%	912940.8	0.4%
4	823094.9	0.9%	823607.2	0.1%	868167.6	5.5%	829089.9	0.7%	859803.8	4.5%
5	822409.2	0.6%	822220.6	0.0%	844981.3	2.7%	831918.1	1.2%	848016.4	3.1%

Table D.2 Model 1 with small vehicle capacity

	CS		**Heuristic**		**LB on y_i; $k=20$**		**LB on z_r; $k=100$**		**LB on z_r; $k=200$**	
No.	Obj Func	Gap	Obj Func	Δ(obj)	Obj Func	Δ(obj)	Obj Func	Δ(obj)	Obj Func	Δ(obj)
1	813093.2	1.8%	815224.2	0.3%	840800.2	3.4%	822779.8	1.2%	846351.1	4.1%
2	791942.0	0.3%	798898.6	0.9%	812253.9	2.6%	857026.2	8.2%	835643.3	5.5%
3	949749.2	2.0%	956187.8	0.7%	1002449.7	5.5%	1006046.1	5.9%	992710.7	4.5%
4	881256.4	0.5%	880044.2	-0.1%	941963.0	6.9%	906524.9	2.9%	933175.3	5.9%
5	877016.5	0.3%	876975.2	0.0%	908823.0	3.6%	888885.8	1.4%	917356	4.6%

Table D.3 Model 2 with large vehicle capacity

	CS		**Heuristic**		**LB on y_i; $k = 20$**		**LB on z_{rs}; $k = 100$**		**LB on z_{rs}; $k = 200$**	
No.	Obj Func	Gap	Obj Func	Δ(obj)	Obj Func	Δ(obj)	Obj Func	Δ(obj)	Obj Func	Δ(obj)
1	659928.4	3.3%	676084.7	2.4%	715045.16	8.4%	672188.6	1.9%	694742.2	5.3%
2	640182.5	5.6%	635969	-0.7%	657596.38	2.7%	692691.6	8.2%	665268.7	3.9%
3	790660.2	4.2%	788192.1	-0.3%	811343.85	2.6%	793132.5	0.3%	812608.4	2.8%
4	714396.2	2.8%	710631.4	-0.5%	761399.49	6.6%	720128.1	0.8%	737367.4	3.2%
5	709159.9	1.7%	709313	0.0%	756272.25	6.6%	726197.6	2.4%	743030	4.8%

Table D.4 Model 2 with small vehicle capacity

	CS		**Heuristic**		**LB on y_i; $k = 20$**		**LB on z_{rs}; $k = 100$**		**LB on z_{rs}; $k = 200$**	
No.	Obj Func	Gap	Obj Func	Δ(obj)	Obj Func	Δ(obj)	Obj Func	Δ(obj)	Obj Func	Δ(obj)
1	674386.9	3.2%	694252.1	2.9%	694653.35	3.0%	676366.2	0.3%	686515.3	1.8%
2	645194.9	1.2%	654018.8	1.4%	679475.92	5.3%	670106.7	3.9%	700340.6	8.5%
3	806513.9	1.0%	819619.9	1.6%	843574.93	4.6%	813077.9	0.8%	815799	1.2%
4	737729.6	3.0%	738289.1	0.1%	786203.66	6.6%	746097.1	1.1%	782509.9	6.1%
5	732954.9	1.6%	729930.8	-0.4%	747588.92	2.0%	749514.7	2.3%	734321.8	0.2%

Table D.5 Model 3 with large vehicle capacity

	Commercial Solver			Heuristic			Local branching on y_i; $k = 10$		
No.	Obj Func	Run Time	Gap	Obj Func	Run Time	Δ(obj)	Obj Func	Run Time	Δ(obj)
1	608125.4	502.99	0%	624961.1	353.18	2.8%	608125.4	315.85	0.0%
2	583129.0	256.27	0%	586063.0	268.46	0.5%	583129.0	236.45	0.0%
3	745327.6	1121.80	0%	755092.3	259.63	1.3%	745862.9	321.77	0.1%
4	671399.2	533.78	0%	671399.2	261.52	0.0%	671399.2	195.09	0.0%
5	665844.6	225.41	0%	666138.8	259.90	0.0%	665844.6	152.88	0.0%

Table D.6 Model 3 with small vehicle capacity

	Commercial Solver			Heuristic			Local branching on y_i; $k = 10$		
No.	Obj Func	Run Time	Gap	Obj Func	Run Time	Δ(obj)	Obj Func	Run Time	Δ(obj)
1	656141.7	888.49	0%	675735.6	266.51	0%	656141.7	303.88	0%
2	624376.3	389.77	0%	635284.7	293.49	0%	624376.3	148.39	0%
3	790197.4	511.89	0%	803659.7	273.45	0%	790197.4	269.32	0%
4	722003.3	286.28	0%	725716.0	288.41	0%	722003.3	218.72	0%
5	711612.5	161.38	0%	711981.4	293.27	0%	711612.5	155.09	0%