REFORMULATIONS OF A BI-LEVEL OPTIMIZATION PROBLEM DETECTING COLLUSIONS IN DEREGULATED ELECTRICITY MARKETS

by
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ABSTRACT

REFORMULATIONS OF A BI-LEVEL OPTIMIZATION PROBLEM DETECTING COLLUSIONS IN DEREGULATED ELECTRICITY MARKETS

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Keywords: Deregulated electricity markets, Tacit collusion, Game theory, Bi-level optimization, Reformulations

Main goal of deregulated electricity markets is to provide an environment with perfect competition among generation companies. Tacit collusion is considered as one of the main threats that may disrupt the competition in electricity markets operated by an independent system operator and increase the electricity price. In order to detect collusion opportunities in the market, we present reformulations for a game-theoretic bi-level optimization problem (Aliabadi et al. 2016). There exists no commercial solvers to directly solve a bi-level problem. First, we improve the existing equivalent reformulations of the problem (Çelebi et al. 2019). Then, we propose two new reformulations based on Karush–Kuhn–Tucker (KKT) conditions together with Active Set Theory, and Special Ordered Set (SOS) variables. Four groups of test instances with varying size are used to show and compare the efficiency and effectiveness of the reformulations in detecting collusive opportunities.

ÖZET

SERBESTLEŞMIŞ ELEKTRİK PİYASALARİNDAKİ GİZLİ ANLAŞMALARI TESPİT EDEN İKİ SEVİYELİ BİR OPTİMİZASYON PROBLEMİNİN REFORMÜLASYONLARI

ALI EBADI TORKAYESH

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Anahtar Kelimeler: Serbestleşmiş elektrik piyasası, Gizli anlaşma, Oyun teorisi, İki seviyeli optimizasyon, Reformülasyonlar

Serbestleşmiş elektrik piyasasının temel amacı, üretim şirketleri arasında tam bir rekabet ortamı sağlamaktır. Gizli anlaşmalar, bağımsız bir sistem operatörü tarafından işletilen elektrik piyasalarında rekabeti bozabilecek ve elektrik fiyatını artırabilecek ana tehditlerden biri olarak kabul edilmektedir. Piyasadaki gizli anlaşma fırsatlarını tespit eden oyun teorik bir iki seviyeli optimizasyon problemi (Aliabadi et al. 2016) için reformülasyonlar sunuyoruz . İki seviyeli bir problemi doğrudan çözecek ticari bir çözücü yoktur. Öncelikle problemin mevcut eşdeğer reformülasyonlarını iyileştiriyoruz (Çelebi et al. 2019). Ardından, Aktif Küme Teorisi ile birlikte Karush-Kuhn-Tucker (KKT) koşullarına ve ayrıca Özel Sıralı Küme (SOS) değişkenlerine dayalı iki yeni reformülasyon öneriyoruz. Farklı boyutlarda dört problem grubu kullanarak, gizli anlaşma fırsatlarını tespit etmede reformülasyonların verimliliğini ve etkililiğini göstermek ve karşılaştırmak için bilgisayısal çalışmalar yapıyoruz.

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TABLE OF CONTENTS

LIST OF TABLES

LIST OF FIGURES

LIST OF ABBREVIATONS

1. Introduction

Determining electricity price is one of the important daily issues in many countries, as electricity price is associated with the level of social welfare. In general, electricity markets can be categorized into two groups of regulated electricity markets and deregulated electricity markets. Governments have the major power in regulated electricity markets where a single company applies an exclusive plan to have complete control over the daily tradings. On the other hand, no major company takes control in a deregulated electricity market where power generation companies (GenCos) compete over the electricity price through an auction mechanism.

In order to maximize the social welfare and sustain the competition among GenCos, deregulated electricity markets aim to attain affordable electricity prices. A major challenge in deregulated electricity market is designing a fully competitive market; otherwise, GenCos would be able to use deficiencies of such mechanisms to decrease the level of competition against public welfare. Therefore, one of the core conditions to attain a competitive market is to control and prevent any non-competitive agreement or opportunity (collusion) between GenCos to manipulate the electricity price bids (Chamberlin, 1929). Independent System Operator (ISO), as the decision maker, is responsible for preventing collusion opportunities in the market by regulating the auction mechanism. For this purpose, several restrictive policies can be taken in order to avoid collusion opportunities. However, detection of collusion opportunities is a very hard task for the ISO due to the tacit nature of agreements among GenCos.

Aliabadi et al. (2016) employ a game-theoretic approach to represent the market clearing process of the deregulated electricity markets and develop a bi-level optimization problem under transmission network constraints. However, they propose a complete-enumeration algorithm in order to detect collusion opportunities. Çelebi et al. (2019) solve the bi-level model proposed in Aliabadi et al. (2016) with two new mixed integer programming reformulations. In this study, we first improve the reformulations proposed by Çelebi et al. (2019); then, two new reformulations based on KKT optimality conditions together with Active Set Method (Gümüş & Floudas,

2005) and SOS Type-1 variables (Siddiqui & Gabriel, 2013) are developed for the bi-level problem. For the computational experiments, an iterative algorithm is used to search for collusion opportunities using each reformulation.

The remainder of the thesis is organized as follows. In Section 2, we review the literature of deregulated electricity markets and decision making approaches for market clearing mechanism, analysis of strategic behaviors of GenCos, and detection of tacit collusion. The bi-level problem and market clearing model are presented in Section 3. In Section 4, mathematical reformulations of the bi-level problem are presented. In Section 5, we present our computational study and computational results. Finally, we conclude in Section 6.

2. Literature Review

Studies on collusion opportunities in deregulated electricity markets is a growing sub-field in power systems research. During recent years, researchers have developed preventive mechanisms to attain more collusion-free deregulated markets. One of the important factors that can give insights about the conspiring behaviors in deregulated electricity markets, is the strategic behavior of GenCos which have been analyzed in the literature using conjectural variation models (Ruiz et al. 2010, Ruiz et al. 2012), simulation models (Aliabadi et al. 2017a, Aliabadi et al. 2017b), and optimization-based approaches (Weber & Overbye 1999, Aliabadi et al. 2016, Pineda Morales 2016).

The strategic behavior of a GenCo is reflected in the behavior of the GenCo in a competitive environment through its bidding pattern. Game theory based approaches have been applied widely for different kinds of electricity market problems during recent decades. Conjectural variation models, as one of the well-known game theory techniques, are frequently used to analyze strategic behavior of GenCos. Conjectural variation models are learning based analytical models that are used to estimate the strategic behavior of GenCos considering reactions of rival GenCos in a competitive environment. In one of the very first studies, Song et al. (2004) present a learning method based on conjectural variation to estimate and analyze the strategic bidding performance of generation companies. In order to consider the possible uncertainties and inconsistencies in the electricity market, Liu et al. (2007) apply conjectural variation to analyze strategic bidding performance of GenCos by considering logical inconsistency and possibility of abundant equilibria that can happen, and therefore, have an effect on the strategic bidding performance of GenCos. In order to consider the effect of strategies of rival GenCos, Wang (2009) uses a conjectural variation based Q-learning algorithm to study the bidding strategy performance of GenCos. To consider the uncertainties that may happen in the electricity market in each period of time, Alikhanzadeh & Irving (2011) present an optimization framework for strategic bidding and forecasting process of GenCos using conjectural variation. The proposed methodology investigates how strategic behavior of GenCos changes in response to changes in behaviors of their rival GenCos. Apart from the conjectural variation models, other game theoretic models such as Nash equilibria, Cournot, and Stackelberg games have been extensively applied. In this regard, Dixon et al. (2006) present an experimental study to identify the more efficient strategic behaviors of GenCos using Nash equilibria as collusive, Cournot, and Stackelberg games. To include the effect of strategic behaviors on profit of the GenCos, Ruiz et al. (2010) propose an equilibrium based model to assess electricity markets in terms of profit maximization as well as demand satisfaction. In another study to consider the uncertainty, Benjamin (2016) develops a Nash equilibrium based model for tacit collusion in electricity market with demand uncertainty. The author concludes that increasing number of GenCos has no effect on the equilibrium, while increasing the number of GenCos can have a supporting effect on the collusive equilibria.

Agent-based simulation is another well-known technique employed to analyze strategic behavior of GenCos in deregulated electricity markets using specific types of reinforcement learning algorithms such as Q-learning algorithm. Naghibi-Sistani et al. (2006) utilize a Q-learning algorithm to analyze the strategic behavior of GenCos. They show that GenCos with higher reinforcement learning capability are more likely to adopt the optimal pricing policy in the market. In order to consider the uncertainty of the system that may happen in the future, Botterud et al. (2007) propose a multi-agent simulation model to analyze the generation expansion potential in electricity markets. They use a probabilistic dispatch algorithm in order to calculate electricity prices and profits of GenCos on a case study in Korea. In order to analyze the effects of tacit collusion on strategic behaviors of GenCos, Tellidou & Bakirtzis (2007) develop an agent-based simulation model to analyze the market performance and possibility of tacit collusion through a repeated game where each game denotes an hourly electricity auction. In a similar study to analyze the effects of collusion, Jabbari Zideh & Mohtavipour (2017) present a simulation framework to analyze GenCos' behavior and demand nodes within a transmission network using a learning algorithm, called state-action-reward-state-action, and the standard Boltzmann exploration strategy based on the Q-value for tacit collusion in electricity markets. They use a small network with three nodes to perform a computational experiment with the proposed simulation model. Li & Shi (2012) employ an agent-based simulation model for strategic bidding for a deregulated electricity market of wind power considering the effect of short-term forecasting accuracy of power generation. Using a similar methodology, Aliabadi et al. (2017a) use an agent-based simulation model to analyze the effects of learning and risk aversion on strategic bidding behavior of GenCos as well as to determine locational prices and dispatch quantities. Results show that minor changes in the risk aversion level of even only one GenCo can have dramatic effects on bid offers and profits of all GenCos. Market design and its properties are basic elements that are considered by researchers to analyze the strategic behavior of GenCos in electricity markets using agent-based simulation frameworks. Aliabadi et al. (2017b) utilize an agent-based simulation under a game-theoretical and learning framework to analyze the strategic behavior of GenCos under different types of market-clearing mechanisms. Results show that the market converges to a similar state under most parameter combinations. Recently, Poplavskaya et al. (2020) present an agent-based simulation model to analyze two important parameters, balancing capacity of market and price of balancing for the European electricity market and their effects on the bidding pattern of GenCos. Results for an independent balancing market indicate that having one ISO would reduce the cost of balancing.

Optimization-based approaches have also been employed to study strategic behavior of GenCos in electricity markets. For the first time in the literature, Liu & Hobbs (2013) propose Mathematical Programs with Equilibrium Constraints (MPEC) and Equilibrium Problems with Equilibrium Constraints (EPEC) considering network constraints to model tacit collusion with an objective to maximize the profits of GenCos in a competitive pool-based electricity market which is operated by an ISO. They develop several heuristic algorithms to solve both models with some numerical instances. Bi-level optimization is frequently used to formulate game-theoretic models in electricity markets (Niknam et al. 2013, Kardakos et al. 2014, Yazdani-Damavandi et al. 2017). Kardakos et al. (2014) present a game-theoretic framework to analyze the strategic bidding behaviors of GenCos in order to find the optimal bidding pattern in an electricity market. For this purpose, they propose a bi-level problem under different approaches for network transmission constraints where upper level problem maximizes profits of GenCos and lower level solves a market clearing problem. In order to solve the bi-level problem, they transform the bi-level problem into a single-level MPEC. Karush–Kuhn–Tucker (KKT) conditions and strong duality theory are applied to transform the MPEC model to a mixed integer linear programming (MILP) model. In order to analyze the efficiency of the models, they consider four different types of transmission networks. Nevertheless, these authors have not included the possibility of collusion opportunities within the proposed models as well as the impact of collusion opportunities on profits of GenCos, perfectness of competitions, and competitiveness of the electricity market. In addition to bi-level optimization problems, other optimization tools have been utilized to address the strategic behaviors of GenCos and collusion opportunities. Samadi & Hajiabadi (2019) propose an analytical framework in two stages for evaluation of collusion opportunities in electricity markets. In the first stage, market-clearing problem is formulated as a quadratic problem. In the second stage, profits of GenCos are calculated using a Jacobain matrix which is used to develop several indicators for assessment of collusion opportunities.

Aliabadi et al. (2016) present a game-theoretic framework to determine collusion opportunities in deregulated electricity markets operated by the ISO. A bi-level optimization model is formulated for a strategic bidding problem considering network constraints and maximizing profits of GenCos while solving a market clearing problem. In one of the latest works, Çelebi et al. (2019) propose two reformulations for the bi-level optimization problem in Aliabadi et al. (2016). To the best of our knowledge, no commercial solvers and solution approaches exist to solve a bi-level problem directly; therefore, the proposed reformulations are equivalent single-level problems that can be solved using commercial solvers.

Meta-heuristic algorithms have shown to be reliable to investigate the strategic bidding behavior of GenCos and to detect tacit collusion in electricity markets. Cau & Anderson (2003) apply a genetic algorithm to find out patterns such as collusive behaviors among GenCos. Ma et al. (2005) propose a chance-constrained programming model to investigate bidding strategies of GenCos in electricity markets. In order to solve the proposed mathematical model, a hybrid solution approach is developed using genetic algorithm and Monte Carlo simulation. Soleymani (2011) develops a hybrid solution approach using particle swarm optimization and simulated annealing algorithms to investigate the strategic behavior of GenCos using a game-theoretic framework based on a supply equilibrium model in electricity markets. Moreover, the proposed solution approach is also utilized to analyze the expected strategic behavior of GenCos. As the enumeration algorithm in Aliabadi et al. (2016) fails to solve problems with a large number of GenCos, Esen (2019) develops a genetic algorithm to solve their problem. In a similar work on detection of collusion opportunities, Mohtavipour & Zideh (2019) present an optimization based iterative algorithm to detect collusion opportunities with a simulation model. Sedeh & Ostadi (2020) present a dynamic programming problem to optimize the bidding strategy of GenCos in order to maximize their profits considering the seasonality trend in market clearing process. They use a genetic algorithm to solve the dynamic programming problem where the expected profit of each bidding strategy is calculated using a Monte Carlo simulation model. Ostadi et al. (2020) propose a hybrid framework using the Markowitz model and a genetic algorithm to optimize the bidding pattern of GenCos through maximizing their profits and minimizing the acceptance risk of the offered bids in daily auctions.

3. Problem Definition

In deregulated electricity markets, an auction problem is solved repeatedly by the ISO where bid prices and available production capacity are submitted by GenCos for a given period of the day-ahead market in a competitive environment. Given the bid offers from the GenCos, the ISO solves a decision-making problem to clear the market in order to maximize the social welfare by minimizing the electricity procurement cost (Aliabadi et al., 2016). The main goal of deregulated electricity markets is to attain affordable electricity prices in a competitive environment. However, the possibility of GenCos conspiring on electricity price would hinder the level of competition and increase tacit collusion opportunities. To address this issue, Aliabadi et al. (2016) develop a game-theoretic bi-level problem to determine collusion opportunities while maximizing the profits of GenCos and minimizing electricity procurement cost through a mathematical formulation that also considers the market clearing process.

Table 3.1 Notation for the DC-OPF problem formulation

Notation	Definition
i	set of nodes $i \in I$
ig	set of generator nodes $ig \in I$
BR	set of transmission line between node i and node j, denoted as (i,j)
P_i^{max}	maximum generation capacity of $GenCo_i$
b_i	bid of $GenCo_i$ submitted to the ISO
D_i	demand at node i
γ_{ij}	negative of the susceptance of the line (1 / reactance of the line) connecting node i to node j
F_{ij}^{max}	power flow limit in the transmission line connecting node i to node j
C_i	pu-adjusted production cost ($/MW) of electricity by $GenCo_i$
P_i	variable for generation amount by $GenCo_i$ at node i
θ_i	voltage angle at node i
LMP_i	locational marginal price of electricity at node i

Two well-known approaches have been frequently employed to formulate the market clearing process; alternating current optimal power flow (AC-OPF) problem and direct current optimal power flow (DC-OPF). AC-OPF problem is a non-linear non-convex optimization problem and approximated by the DC-OPF problem formulation in a linear form. Using notation in Table 3.1, the DC-OPF problem formulation in Aliabadi et al. (2016) becomes

$$
\begin{aligned}
&(3.1\text{a}) && \underset{\{P_i,\theta_i\}}{\text{minimize}} && \sum_i b_i P_i && \\
&(3.1\text{b}) && \text{subject to} && P_i - D_i = \sum_{\forall (i,j)\in BR} \gamma_{ij}(\theta_i - \theta_j) && \forall i, \quad [LMP_i] \\
&(3.1\text{c}) && && P_i \le P_i^{max} && \forall i, \quad [\phi_i] \\
&(3.1\text{d}) && && |\gamma_{ij}(\theta_i - \theta_j)| \le F_{ij}^{max} && \forall (i,j) \in BR, \quad [\psi_{ij}^+, \psi_{ij}^-] \\
&(3.1\text{e}) && && -\pi \le \theta_i \le \pi && \forall i, \\
&(3.1\text{f}) && && P_i \ge 0 \text{ and } \theta_i \text{ free} && \forall i,
\end{aligned}
$$

where variables in the brackets at the end of each constraint represent the dual variables.

The objective function (3.1a) minimizes the total electricity procurement cost. Constraint (3.1b) is the flow balance constraint which ensures the transmission of the excessive generated power of a node to the other connected nodes. Constraint (3.1c) limits the power injection level up to the capacity of the corresponding power producer at each node. Constraint (3.1d) controls the maximum allowed flow in each line of the transmission network. Constraint (3.1e) restricts the voltage angle at each node by upper and lower bounds. Constraint (3.1f) represents the bounds for decision variables.

According to the solution of the DC-OPF problem in (3.1a)-(3.1e), profit for $GenCo_i$ can be calculated as

$$r_i = P_i(LMP_i - C_i)$$

where LMP_i is obtained from the dual of DC-OPF problem.

In a real day-ahead electricity market, the ISO repetitively clears the market for each time period and then determines assigned power of each GenCo and electricity price at each node. Aliabadi et al. (2016) develop a game-theoretic framework for understanding collusion among GenCos in electricity markets. The market is modeled as a game where the set of submitted bids by GenCos ($b_1 \in B_1, ..., b_n \in B_n$) is considered as the state of the game and profits of GenCos are calculated based on the solution of the DC-OPF problem. Among all possible states, a collusive state is defined as an equilibrium where the profit is greater than that at any Nash equilibrium (r_i^*) for all GenCos, and thus, GenCos have no incentive to deviate. Aliabadi et al. (2016) defines Nash equilibrium as a bidding strategy where any $GenCo_i$ are not able to make a better payoff than the payoff of Nash equilibrium by selecting another bid until all other GenCos do not change their bids. In this regard, all states with positive profits are considered as suspicious to be collusive; however, only those suspicious states are considered as real collusive states where their profits are strictly greater than that at any Nash equilibrium (r_i^*). Mathematically, if there exists a state where $r_i^{Collusive} > r_i^* \quad \forall i \in ig$, while r_i^* representing the best Nash payoff for $GenCo_i$, this state is considered as collusive according to Aliabadi et al. (2016).

In order to identify collusive states, Aliabadi et al. (2016) propose a bi-level problem maximizing the profits of all GenCos at the upper level, while minimizing the electricity procurement cost through the DC-OPF problem at the lower level. In the original problem, each GenCo maximizes its own profit; however, they approximate and simplify the objective function by maximizing the minimum of the profits of all GenCos. The proposed bi-level problem by Aliabadi et al. (2016) is

$$
\begin{aligned}
&(3.2a) && \underset{\{b_i,\lambda\}}{\text{maximize}} \quad \lambda && \\
&(3.2b) && \text{subject to} \quad \lambda \le P_i(LMP_i - C_i) && \forall i, \\
&(3.2c) && \qquad \underset{\{P_i,\theta_i\}}{\text{minimize}} \sum_i b_i P_i && \\
&(3.2d) && \qquad \text{subject to } P_i - D_i = \sum_{\forall (i,j)\in BR} \gamma_{ij}(\theta_i - \theta_j) && \forall i, \quad [LMP_i] \\
&(3.2e) && \qquad\qquad P_i \le P_i^{max} && \forall i, \quad [\phi_i] \\
&(3.2f) && \qquad\qquad |\gamma_{ij}(\theta_i - \theta_j)| \le F_{ij}^{max} && \forall (i,j) \in BR, [\psi_{ij}^+, \psi_{ij}^-] \\
&(3.2g) && \qquad\qquad -\pi \le \theta_i \le \pi && \forall i, \\
&(3.2h) && \qquad\qquad P_i \ge 0 \text{ and } \theta_i \text{ free} && \forall i,
\end{aligned}
$$

where b_i denoting the bid submitted by $GenCo_i$ is now a decision variable.

In the bi-level problem, the upper level objective function (3.2a) maximizes the profits of all GenCos using an auxiliary variable λ which is enforced by constraint (3.2b) to the minimum profit among all GenCos. In the lower level problem, the DC-OPF problem is solved according to the upper level bid decisions by GenCos.

In the next step, the bi-level problem is to be transformed to single-level so that it can be be solved. The techniques that transform bi-level problem into a single-level require dual information from the lower level problem (3.2c)-(3.2h). Thus, Çelebi et al. (2019) formulate the dual problem as

$$\text{(3.3a)} \qquad \text{maximize} \quad \sum_i D_i LMP_i - \sum_i P_i^{max} \phi_i - \sum_{\forall (i,j) \in BR} F_{ij}^{max} (\psi_{ij}^{+} + \psi_{ij}^{-})$$

$$\text{(3.3b)} \qquad \text{subject to} \quad LMP_i - \phi_i \leq b_i \qquad \forall i$$

$$\text{(3.3c)} \qquad \sum_{(i,j) \in BR} \gamma_{ij} (LMP_j - LMP_i) + \sum_{(i,j) \in BR} \gamma_{ij} (\psi_{ij}^{-} - \psi_{ij}^{+}) + \sum_{(j,i) \in BR} \gamma_{ji} (\psi_{ji}^{+} - \psi_{ji}^{-}) = 0 \quad \forall i$$

$$\text{(3.3d)} \qquad LMP_i \quad free \qquad \forall i$$

$$\text{(3.3e)} \qquad \phi_i \geq 0 \qquad \forall i$$

$$\text{(3.3f)} \qquad \psi_{ij}^{+}, \psi_{ij}^{-} \geq 0 \qquad \forall (i,j) \in BR$$

where decision variables LMP_i, LMP_j, ψ_{ij}^{+}, ψ_{ij}^{-}, and ϕ_i are defined in the original primal problem formulation for DC-OPF in (3.1a)-(3.1f) as the dual variables.

In next section, we discuss the importance of techniques for transforming the bi-level problem to a single-level problem. Next, four reformulations are presented to transform the bi-level problem to a single-level problem based on different approaches.

4. Reformulations of the Bi-level Problem

Developing reformulations of a bi-level problem and transforming it to a single-level problem is an essential task in solving such problems. We are unable to solve bi-level problems directly and there exist no commercially available solvers for such problems. Therefore, reformulations become very important techniques in transforming a bi-level problem into single-level so that it can be potentially solved with commercial solvers.

4.1 Reformulations with MPEC model

In order to solve the bi-level problem, we may reformulate the bi-level problem as a MPEC using dual formulation and complementary constraints. MPEC is one of the well-known approaches to solve bi-level problems (Dempe 2003, Luo et al. 1996). MPEC is employed in non-linear programming with variational inequality or complementary constraints (Li et al. 2018, Ye et al.2016, Baumrucker & Biegler 2010, Hobbs et al. 2000). Unlike the bi-level problems, MPEC is a two-level optimization problem with an upper-level problem and a lower-level complementary problem (Pineda et al. 2018). The bi-level problem is equivalent to an MPEC if the lower-level problem can be reformulated by KKT optimality conditions (Gabriel et al. 2012, Pineda et al. 2018). An MPEC is formulated in the following general form as

(4.1a) $$\text{minimize} \quad f(x,y,z)$$

(4.1b) $$\text{subject to} \quad h(x,y,z) = 0$$

(4.1c) $$g(x,y,z) \geq 0$$

(4.1d) $$0 \leq x \perp y \geq 0$$

Accordingly, the bi-level problem in (3.2a)-(3.2h) can be rewritten as an MPEC as

$$\text{(4.2a)} \quad \underset{\{b_i,\lambda\}}{\text{maximize}} \quad \lambda$$

$$\text{(4.2b)} \quad \text{subject to} \quad \lambda \leq P_i(LMP_i - C_i) \qquad \forall i \in ig$$

$$\text{(4.2c)} \quad P_i \geq 0 \perp b_i - LMP_i + \phi_i \geq 0 \qquad \forall i \in ig$$

$$\text{(4.2d)} \quad \theta_i \text{ free} \perp \sum_{(i,j)\in BR} \gamma_{ij}(LMP_j - LMP_i) + \sum_{(i,j)\in BR} \gamma_{ij}(\psi_{ij}^- - \psi_{ij}^+) + \sum_{(j,i)\in BR} \gamma_{ji}(\psi_{ji}^+ - \psi_{ji}^-) = 0, \qquad \forall i$$

$$\text{(4.2e)} \quad LMP_i \text{ free} \perp P_i - D_i = \sum_{ij\in BR} \gamma_{ij}(\theta_i - \theta_j) \qquad \forall i$$

$$\text{(4.2f)} \quad \phi_i \geq 0 \perp P_i \leq P_i^{max} \qquad \forall i \in ig$$

$$\text{(4.2g)} \quad \psi_{ij}^+ \geq 0 \perp \gamma_{ij}(\theta_i - \theta_j) \leq F_{ij}^{max} \qquad \forall (i,j) \in BR$$

$$\text{(4.2h)} \quad \psi_{ij}^- \geq 0 \perp -\gamma_{ij}(\theta_i - \theta_j) \leq F_{ij}^{max} \qquad \forall (i,j) \in BR$$

$$\text{(4.2i)} \quad -\pi \leq \theta_i \leq \pi \qquad \forall i$$

Objective function (4.2a) and constraint (4.2b) belong to the upper level of the bi-level problem in (3.2a) - (3.2h). Constraint (4.2c) shows the complementarity between (3.3b) in the dual problem and (3.2h) in the bi-level problem. Constraint (4.2d) shows the complementarity between constraint (3.3c) and constraint (3.2h). Constraint (4.2e) shows the complementarity between constraint (3.3d) and constraint (3.2d). Complementarity between constraint (3.3e) in the dual problem and constraint (3.2e) in the bi-level model is shown in constraint (4.2f). Constraints (4.2g) and (4.2h) are complementary constraints related to (3.3f) and (3.2f). Constraint (4.2i) restricts the voltage angle at each node by upper and lower bounds.

One of the main issues with MPEC models is related to their feasible regions that are not necessarily convex or connected topological space. A well-known approach to solve MPEC models is to reformulate them as MILP. Next, new MILP reformulations are presented for the MPEC model in (4.2a)-(4.2i).

4.1.1 Reformulation1: MILP model based on FAM Method

Çelebi et al. (2019) use KKT conditions for lower level problem to reformulate the MPEC problem in (4.2a)-(4.2i) as a MILP. They propose a reformulation based on the methodology presented in Fortuny-Amat & McCarl (1981) in order to convert the MPEC model in (4.2a)-(4.2i) to a MINLP. Fortuny-Amat & McCarl (1981) use binary variables to address the issues related to complementary slackness conditions. General form of this method is formulated as below where z_j is a binary variable and M is a sufficiently big parameter.

$$y_j \leq M z_j \qquad j \in J \tag{4.3a}$$

$$x_j \leq (1 - z_j) M \qquad j \in J \tag{4.3b}$$

where (4.3a) and (4.3b) are two constraints that replace the complementarity between y_j and x_j in an MPEC.

In our first reformulation named as Reformulation1, four binary variables, w_i, x_i, y_{ij}, and z_{ij}, corresponding to the complementary constraints are introduced. The resulting formulation becomes

$$\underset{\{b_i,\lambda\}}{\text{maximize}} \quad \lambda \tag{4.4a}$$

$$\text{subject to} \quad \lambda \leq P_i(LMP_i - C_i) \qquad \forall i \in ig \tag{4.4b}$$

$$P_i \geq 0 \qquad \forall i \tag{4.4c}$$

$$P_i \leq M_{1i}(1 - w_i) \qquad \forall i \tag{4.4d}$$

$$b_i - LMP_i + \phi_i \leq M_{2i} w_i \qquad \forall i \in ig \tag{4.4e}$$

$$b_i - LMP_i + \phi_i \geq 0 \qquad \forall i \in ig \tag{4.4f}$$

$$\sum_{(i,j)\in BR} \gamma_{ij}(LMP_j - LMP_i) + \sum_{(i,j)\in BR} \gamma_{ij}(\psi_{ij}^- - \psi_{ij}^+) + \sum_{(j,i)\in BR} \gamma_{ji}(\psi_{ji}^+ - \psi_{ji}^-) = 0 \quad \forall i \tag{4.4g}$$

$$P_i - D_i = \sum_{(i,j)\in BR} \gamma_{ij}(\theta_i - \theta_j) \qquad \forall i \tag{4.4h}$$

$$\phi_i \geq 0 \qquad \forall i \tag{4.4i}$$

$$\phi_i \leq M_{3i}(1 - x_i) \qquad \forall i \tag{4.4j}$$

$$P_i \geq P_i^{max} - M_{4i} x_i \qquad \forall i \in ig \tag{4.4k}$$

$$P_i \leq P_i^{max} \qquad \forall i \in ig \tag{4.4l}$$

$$\psi_{ij}^+ \geq 0 \qquad \forall (i,j) \in BR \tag{4.4m}$$

$$\psi_{ij}^+ \leq M_{5ij}(1 - y_{ij}) \qquad \forall (i,j) \in BR \tag{4.4n}$$

$$\gamma_{ij}(\theta_i - \theta_j) \geq F_{ij}^{max} - M_{6ij} y_{ij} \qquad \forall (i,j) \in BR \tag{4.4o}$$

$$\gamma_{ij}(\theta_i - \theta_j) \leq F_{ij}^{max} \qquad \forall (i,j) \in BR \tag{4.4p}$$

$$\psi_{ij}^- \geq 0 \qquad \forall (i,j) \in BR \tag{4.4q}$$

(4.4r)	$\psi^{-}_{ij} \leq M_{7ij}(1-z_{ij})$	$\forall(i,j) \in BR$
(4.4s)	$\gamma_{ij}(\theta_i-\theta_j) \leq -F^{max}_{ij} + M_{8ij}z_{ij}$	$\forall(i,j) \in BR$
(4.4t)	$\gamma_{ij}(\theta_i-\theta_j) \geq -F^{max}_{ij}$	$\forall(i,j) \in BR$
(4.4u)	$-\pi \leq \theta_i \leq \pi$	$\forall i$
(4.4v)	$w_i, x_i \in \{0,1\}$	$\forall i$
(4.4w)	$y_{ij}, z_{ij} \in \{0,1\}$	$\forall(i,j) \in BR$

Objective function (4.4a) and constraint (4.4b) belong to the upper level problem of the bi-level problem. Using FAM method described in (4.3a)-(4.3b), constraints (4.4c)-(4.4f) are associated with complementary constraint (4.2c). Constraints (4.4i)-(4.4l) are associated with complementary constraint (4.2f). Constraints (4.4m)-(4.4p) are related to complementary constraint (4.2g). Constraints (4.4q)-(4.4t) are related to complementary constraint (4.2h). Constraints (4.4v) and (4.4w) represent the binary variables.

Proposition 4.1.1. *When constraint (4.4b) is replaced by $\lambda \leq P_i(b_i - C_i)$, in the resulting constraint, b_i provides a lower bound for LMP_i value.*

Proof. The proposed Reformulation1 is a non-linear program due to constraint (4.4b) where two continuous variables P_i and LMP_i are multiplied. In order to facilitate the linearization the constraint, LMP_i is replaced with b_i. If we consider constraint (3.2b) in the bi-level model and constraint (3.3b) in the dual of lower level and their complementary problems in the MPEC model,

$$P_i(b_i - LMP_i + \phi_i) = 0$$
$$(P_i - P^{max}_i)\phi_i = 0$$

so,

$$P_i > 0 \rightarrow b_i = LMP_i - \phi_i$$
$$P_i \neq P^{max}_i \rightarrow LMP_i = b_i, \phi_i = 0$$

LMP_i value is either $LMP_i = b_i$ or $LMP_i > b_i$. □

Following the replacement, we utilize the approach considered by Jin et al. (2013), Pozo et al. (2012), and Kazempour et al. (2013) to linearize the non-linear term that includes the multiplication of two continuous variables P_i and b_i. First, we represent b_i by Bid_{ik} which is comprised of possible discrete values b_i and newly defined B_{ik} which is a binary variable such that $b_i = \sum_{k \in K} Bid_{ik} B_{ik}$ where $\sum_{k \in K} B_{ik} =$

1. Thereafter, we introduce a new auxiliary variable $V_{ik} = P_i B_{ik}$ to the model. Linearized model after resolving the non-linearity due to multiplication of P_i and b_i is represented below.

$$\underset{\{B_{ik},\lambda\}}{\text{maximize}} \quad \lambda \tag{4.5a}$$

$$\text{subject to} \quad \lambda \leq \sum_{k \in K} \text{Bid}_{ik} V_{ik} - P_i C_i \qquad \forall i \tag{4.5b}$$

$$b_i = \sum_{k \in K} \text{Bid}_{ik} B_{ik} \qquad \forall i \tag{4.5c}$$

$$\sum_{k \in K} B_{ik} = 1 \qquad \forall i \tag{4.5d}$$

$$V_{ik} \leq P_i^{max} B_{ik} \qquad \forall i \text{ and } k \tag{4.5e}$$

$$V_{ik} \leq P_i \qquad \forall i \text{ and } k \tag{4.5f}$$

$$V_{ik} \geq P_i - P_i^{max}[1 - B_{ik}] \qquad \forall i \text{ and } k \tag{4.5g}$$

$$V_{ik} \geq 0 \qquad \forall i \text{ and } k \tag{4.5h}$$

$$P_i \geq 0 \qquad \forall i \tag{4.5i}$$

$$P_i \leq M_{1i}(1 - w_i) \qquad \forall i \tag{4.5j}$$

$$b_i - LMP_i + \phi_i \leq M_{2i} w_i \qquad \forall i \tag{4.5k}$$

$$b_i - LMP_i + \phi_i \geq 0 \qquad \forall i \tag{4.5l}$$

$$\sum_{(i,j) \in BR} \gamma_{ij}(LMP_j - LMP_i) + \sum_{(i,j) \in BR} \gamma_{ij}(\psi_{ij}^- - \psi_{ij}^+) + \sum_{(j,i) \in BR} \gamma_{ji}(\psi_{ji}^+ - \psi_{ji}^-) = 0 \qquad \forall i \tag{4.5m}$$

$$P_i - D_i = \sum_{(i,j) \in BR} \gamma_{ij}(\theta_i - \theta_j) \qquad \forall i \tag{4.5n}$$

$$\phi_i \geq 0 \qquad \forall i \tag{4.5o}$$

$$\phi_i \leq M_{3i}(1 - x_i) \qquad \forall i \tag{4.5p}$$

$$P_i \geq P_i^{max} - M_{4i} x_i \qquad \forall i \tag{4.5q}$$

$$P_i \leq P_i^{max} \qquad \forall i \tag{4.5r}$$

$$\psi_{ij}^+ \geq 0 \qquad \forall (i,j) \in BR \tag{4.5s}$$

$$\psi_{ij}^+ \leq M_{5i}(1 - y_{ij}) \qquad \forall (i,j) \in BR \tag{4.5t}$$

$$\gamma_{ij}(\theta_i - \theta_j) \geq F_{ij}^{max} - M_{6ij} y_{ij} \qquad \forall (i,j) \in BR \tag{4.5u}$$

$$\gamma_{ij}(\theta_i - \theta_j) \leq F_{ij}^{max} \qquad \forall (i,j) \in BR \tag{4.5v}$$

$$\psi_{ij}^- \geq 0 \qquad \forall (i,j) \in BR \tag{4.5w}$$

$$\psi_{ij}^- \leq M_7(1 - z_{ij}) \qquad \forall (i,j) \in BR \tag{4.5x}$$

$$\gamma_{ij}(\theta_i - \theta_j) \leq -F_{ij}^{max} + M_8 z_{ij} \qquad \forall (i,j) \in BR \tag{4.5y}$$

$$\gamma_{ij}(\theta_i - \theta_j) \geq -F_{ij}^{max} \qquad \forall (i,j) \in BR \tag{4.5z}$$

(4.5aa) $-\pi \leq \theta_i \leq \pi \quad \forall i$

(4.5ab) $B_{ik} \in \{0,1\} \quad \forall i$ and k

(4.5ac) $w_i, x_i \in \{0,1\} \quad \forall i$

(4.5ad) $y_{ij}, z_{ij} \in \{0,1\} \quad \forall (i,j) \in BR$

Constraints (4.5b)-(4.5h) and (4.5ab) are new constraints derived through linearization process. Big M parameters in the model are defined as $M_{1i} = P_i^{max}$, $M_{2i} = 100$, $M_{3i} = 100$, $M_{4i} = P_i^{max}$, $M_{5ij} = 100$, $M_{6ij} = 2F_{ij}^{max}$, $M_{7ij} = 100$, and $M_{8ij} = 2F_{ij}^{max}$.

4.1.2 An Improved-Reformulation1

We now want to take a closer look at Reformulation1 proposed by Çelebi et al. (2019) and improve it using several observations. For this purpose, we present Propositions 4.1.2, 4.1.3, and 4.1.4 in order to eliminate the redundant constraints and to add valid inequalities.

Proposition 4.1.2. *For a given transmission link (i,j)* $\in BR$, $F_{ij}^{max} - \gamma_{ij}(\theta_i - \theta_j) \geq 0$ *and* $F_{ij}^{max} + \gamma_{ij}(\theta_i - \theta_j) \geq 0$ *are redundant when* $M_{6ij} = M_{8ij} = 2F_{ij}^{max}$ *in* $\gamma_{ij}(\theta_i - \theta_j) \geq F_{ij}^{max} - M_{6ij}y_{ij}$ *and* $\gamma_{ij}(\theta_i - \theta_j) \leq -F_{ij}^{max} + M_8 z_{ij}$ *constraints.*

Proof. Consider constraints (4.5u) - (4.5v) and constraints (4.5y) - (4.5z) from the Reformulation1.

(4.6a) $F_{ij}^{max} - \gamma_{ij}(\theta_i - \theta_j) \leq M_{6ij}y_{ij}$

(4.6b) $F_{ij}^{max} - \gamma_{ij}(\theta_i - \theta_j) \geq 0$

(4.6c) $F_{ij}^{max} + \gamma_{ij}(\theta_i - \theta_j) \leq M_{8ij}z_{ij}$

(4.6d) $F_{ij}^{max} + \gamma_{ij}(\theta_i - \theta_j) \geq 0$

According to possible values of y_{ij} and z_{ij}, four cases could occur:

1.1 $y_{ij} = 0,\ z_{ij} = 0$: It is not possible as summation of (4.6a) and (4.6c) yields $2F_{ij}^{max} \leq 0$ which cannot happen. Either y_{ij} or z_{ij} must be positive in the formulation.

1.2 $y_{ij} = 1,\ z_{ij} = 0$: Constraint (4.6a) results in $\gamma_{ij}(\theta_i - \theta_j) \geq -F_{ij}^{max}$ and constraint (4.6c) yields $\gamma_{ij}(\theta_i - \theta_j) \leq -F_{ij}^{max}$. It follows that $\gamma_{ij}(\theta_i - \theta_j) = -F_{ij}^{max}$ which holds for constraints (4.6b) and (4.6d).

1.3 $y_{ij} = 0$, $z_{ij} = 1$: Constraint (4.6a) leads to $\gamma_{ij}(\theta_i - \theta_j) \geq F_{ij}^{max}$ and constraint (4.6c) constitutes $\gamma_{ij}(\theta_i - \theta_j) \leq F_{ij}^{max}$. It results in $\gamma_{ij}(\theta_i - \theta_j) = F_{ij}^{max}$ that is feasible for constraints (4.6b) and (4.6d).

1.4 $y_{ij} = 1$, $z_{ij} = 1$: Constraints (4.6a) and (4.6c) produce constraint (4.6d) and (4.6b), respectively.

□

Proposition 4.1.3. *Constraint $P_i \leq P_i^{max}$ is redundant given that $M_{1i} = M_{4i} = P_i^{max}$ in $P_i \leq M_{1i}(1 - w_i)$ and $P_i \geq P_i^{max} - M_{4i}x_i$ constraints.*

Proof. According to possible values of w_i, and x_i, four cases could occur:

2.1 $w_i = 0$, $x_i = 0$:

(4.7a) $$P_i \leq P_i^{max}$$

(4.7b) $$P_i \geq P_i^{max}$$

leads to $P_i = P_i^{max}$ which holds for $P_i \leq P_i^{max}$.

2.2 $w_i = 0$, $x_i = 1$:

(4.8a) $$P_i \leq P_i^{max}$$

(4.8b) $$P_i \geq 0$$

always satisfy $P_i \leq P_i^{max}$.

2.3 $w_i = 1$, $x_i = 1$:

(4.9a) $$P_i \leq 0$$

(4.9b) $$P_i \geq 0$$

produces $P_i = 0$ holds for $P_i \leq P_i^{max}$.

2.4 $w_i = 1$, $x_i = 0$:

(4.10a) $$P_i \leq 0$$

(4.10b) $$P_i \geq P_i^{max}$$

is infeasible so could not happen. □

Proposition 4.1.4. *According to Propositions 4.1.2 and 4.1.3,* $y_{ij} + z_{ij} \geq 1$ *and* $w_i \leq x_i$ *are valid inequalities.*

Proof. In Proposition 4.1.2, $y_{ij} = 0, z_{ij} = 0$ case is impossible. It is cut off by $y_{ij} + z_{ij} \geq 1$ constraint. We eliminate $w_i = 1, x_i = 0$ case in Proposition 4.1.3 by $w_i \leq x_i$ inequality. □

Applying the proposed propositions on the Reformulation1, we develop a new formulation which is called Improved-Reformulation1.

4.1.3 Reformulation2: MILP model based on Strong-Duality conditions

Çelebi et al. (2019) transform the bi-level problem into a single-level problem using strong duality approach. They use the primal-dual constraints of the DC-OPF problem in (3.1a) - (3.1f) to transform the lower level of the bi-level problem to a set of constraints as follows.

$$\max_{\{b_i,\lambda\}} \quad \lambda \tag{4.11a}$$

$$\text{subject to} \quad \lambda \leq P_i(LMP_i - C_i) \quad \forall i \tag{4.11b}$$

$$P_i \geq 0 \quad \forall i \tag{4.11c}$$

$$b_i - LMP_i + \phi_i \geq 0 \quad \forall i \tag{4.11d}$$

$$\sum_{(i,j)\in BR} \gamma_{ij}(LMP_j - LMP_i) + \sum_{(i,j)\in BR} \gamma_{ij}(\psi_{ij}^- - \psi_{ij}^+) + \sum_{(j,i)\in BR} \gamma_{ji}(\psi_{ji}^+ - \psi_{ji}^-) = 0 \quad \forall i \tag{4.11e}$$

$$P_i - D_i = \sum_{(i,j)\in BR} \gamma_{ij}(\theta_i - \theta_j) \quad \forall i \tag{4.11f}$$

$$\phi_i \geq 0 \quad \forall i \tag{4.11g}$$

$$P_i \leq P_i^{max} \quad \forall i \tag{4.11h}$$

$$\psi_{ij}^+ \geq 0 \quad \forall (i,j) \in BR \tag{4.11i}$$

$$\gamma_{ij}(\theta_i - \theta_j) \leq F_{ij}^{max} \quad \forall (i,j) \in BR \tag{4.11j}$$

$$\psi_{ij}^- \geq 0 \quad \forall (i,j) \in BR \tag{4.11k}$$

$$\gamma_{ij}(\theta_i - \theta_j) \geq -F_{ij}^{max} \quad \forall (i,j) \in BR \tag{4.11l}$$

$$-\pi \leq \theta_i \leq \pi \qquad \forall i \tag{4.11m}$$

$$\sum_i b_i P_i = \sum_i D_i LMP_i - \sum_i P_i^{max} \phi_i - \sum_{(i,j)\in BR} F_{ij}^{max}(\psi_{ij}^+ + \psi_{ij}^-) \tag{4.11n}$$

Constraint (4.11n) is derived based on the strong duality condition for the lower level problem of the DC-OPF. Reformulation2 in (4.11a) - (4.11n) is a non-linear model due to (4.11b) and (4.11n) constraints. In order to resolve the issue related to the non-linearity of constraints (4.11b) and (4.11n), we replace P_i with LMP_i and linearize the model in the same way that is done in the Reformulation1 through new variables and parameters as V_{ik}, B_{ik} and Bid_{ik} that includes possible discrete values of b_i variable. Linearized form of Reformulation2 becomes

$$\underset{\{b_i,\lambda\}}{\text{maximize}} \quad \lambda \tag{4.12a}$$

$$\text{subject to} \quad \lambda \leq \sum_{k\in K} \text{Bid}_{ik} V_{ik} - P_i C_i \qquad \forall i \tag{4.12b}$$

$$b_i = \sum_{k\in K} \text{Bid}_{ik} B_{ik} \qquad \forall i \tag{4.12c}$$

$$\sum_{k\in K} B_{ik} = 1 \qquad \forall i \tag{4.12d}$$

$$V_{ik} \leq P_i^{max} B_{ik} \qquad \forall i \text{ and } k \tag{4.12e}$$

$$V_{ik} \leq P_i \qquad \forall i \text{ and } k \tag{4.12f}$$

$$V_{ik} \geq P_i - P_i^{max}(1 - B_{ik}) \qquad \forall i \text{ and } k \tag{4.12g}$$

$$V_{ik} \geq 0 \qquad \forall i \text{ and } k \tag{4.12h}$$

$$P_i \geq 0 \qquad \forall i \tag{4.12i}$$

$$b_i - LMP_i + \phi_i \geq 0 \qquad \forall i \tag{4.12j}$$

$$\phi_i \geq 0 \qquad \forall i \tag{4.12k}$$

$$P_i \leq P_i^{max} \qquad \forall i \tag{4.12l}$$

$$\sum_{(i,j)\in BR} \gamma_{ij}(LMP_j - LMP_i) + \sum_{(i,j)\in BR} \gamma_{ij}(\psi_{ij}^- - \psi_{ij}^+) + \sum_{(j,i)\in BR} \gamma_{ji}(\psi_{ji}^+ - \psi_{ji}^-) = 0 \quad \forall i \tag{4.12m}$$

$$P_i - D_i = \sum_{(i,j)\in BR} \gamma_{ij}(\theta_i - \theta_j) \qquad \forall i \tag{4.12n}$$

$$\psi_{ij}^+ \geq 0 \qquad \forall (i,j) \in BR \tag{4.12o}$$

$$\gamma_{ij}(\theta_i - \theta_j) \leq F_{ij}^{max} \qquad \forall (i,j) \in BR \tag{4.12p}$$

$$\psi_{ij}^- \geq 0 \qquad \forall (i,j) \in BR \tag{4.12q}$$

$$\gamma_{ij}(\theta_i - \theta_j) \geq -F_{ij}^{max} \qquad \forall (i,j) \in BR \tag{4.12r}$$

(4.12s) $-\pi \le \theta_i \le \pi \quad \forall i$

(4.12t) $LMP_i \quad \text{free} \quad \forall i$

(4.12u) $B_{ik} \in \{0,1\} \quad \forall i \text{ and } k$

(4.12v)
$$\sum_i \sum_{k\in K} \text{Bid}_{ik} V_{ik} = \sum_i D_i LMP_i - \sum_i P_i^{max}\phi_i - \sum_{ij} F_{ij}^{max}(\psi_{ij}^{+} + \psi_{ij}^{-})$$

Constraints (4.12b) - (4.12h) are linearized constraints for (4.11b), and (4.12t) is linearized form of the constraint (4.11n).

4.1.4 Reformulation3: MILP model based on KKT conditions and Active Set Method

We propose another reformulation to transform the bi-level problem to a single-level problem using KKT conditions together with Active Set Method in Gümüş & Floudas (2005). They employ KKT conditions when the lower problem is convex. In order to use these conditions, we need to ascertain a new formulation for the bi-level problem in (3.2a) - (3.2h). In order to apply this method, right hand sides of the equality constraints and less than or equal to inequality constraints in the bi-level problem must be zero. With these changes, the bi-level problem is formulated as

(4.13a) $\underset{\{b_i,\lambda\}}{\text{maximize}} \quad \lambda$

(4.13b) subject to $\lambda - P_i(LMP_i - C_i) \le 0 \quad \forall i,$

(4.13c) $\underset{\{P_i,\theta_i\}}{\text{minimize}} \sum_i b_i P_i$

(4.13d) subject to $P_i - D_i - \sum_{\forall (i,j)\in BR} \gamma_{ij}(\theta_i - \theta_j) = 0 \quad \forall i, \quad [LMP_i]$

(4.13e) $P_i - P_i^{max} \le 0 \quad \forall i, \quad [\phi_i]$

(4.13f) $|\gamma_{ij}(\theta_i - \theta_j)| - F_{ij}^{max} \le 0 \quad \forall (i,j) \in BR, \quad [\psi_{ij}^{+}, \psi_{ij}^{-}]$

(4.13g) $-\pi \le \theta_i \le \pi \quad \forall i,$

(4.13h) $-P_i \le 0 \quad \forall i, \quad [\alpha_i]$

One of the main difference of Reformulation3 and Reformulation1 is that in addition to dual variables that were defined earlier, we define a dual variable as α_i associated with constraint (4.13h). Next, we need to use KKT conditions on the lower level of the modified bi-level problem in (4.13a) - (4.13h). For a constraint $g(x,y) \leq 0$, the following procedure is executed:

Step 1. Let s be a slack variable so that $s+g(x,y)=0$. Since $s=-g(x,y)$, thus, $s \geq 0$, $\forall j$ holds.

Step 2. Let $u \geq 0$ be the dual variable of constraint $g(x,y) \leq 0$.

Step 3. The complementary slackness conditions can be written as $u.s=0$.

Step 4. A KKT stationary condition $u(-g(x,y))=0$ is added to replace constraint $g(x,y)$ to the lower level.

Accordingly, Reformulation3 becomes

$$\text{(4.14a)} \qquad \text{maximize} \quad \lambda$$

$$\text{(4.14b)} \qquad \text{subject to} \quad \lambda - P_i(LMP_i - C_i) \leq 0 \qquad \forall i$$

$$\text{(4.14c)} \qquad \sum_{(i,j)\in BR} \gamma_{ij}(LMP_j - LMP_i) + \sum_{(i,j)\in BR} \gamma_{ij}(\psi_{ij}^- - \psi_{ij}^+) + \sum_{(j,i)\in BR} \gamma_{ji}(\psi_{ji}^+ - \psi_{ji}^-) = 0 \qquad \forall i$$

$$\text{(4.14d)} \qquad P_i - \sum_{(i,j)\in BR} \gamma_{ij}(\theta_i - \theta_j) = D_i \qquad \forall i$$

$$\text{(4.14e)} \qquad -b_i + LMP_i - \phi_i + \alpha_i = 0 \qquad \forall i$$

$$\text{(4.14f)} \qquad \phi_i(P_i^{max} - P_i) = 0 \qquad \forall i$$

$$\text{(4.14g)} \qquad \psi_{ij}^+\left(F_{ij}^{max} - \gamma_{ij}(\theta_i - \theta_j)\right) = 0 \qquad \forall (i,j) \in BR$$

$$\text{(4.14h)} \qquad \psi_{ij}^-\left(F_{ij}^{max} + \gamma_{ij}(\theta_i - \theta_j)\right) = 0 \qquad \forall (i,j) \in BR$$

$$\text{(4.14i)} \qquad \alpha_i P_i = 0 \qquad \forall i$$

$$\text{(4.14j)} \qquad -\pi \leq \theta_i \leq \pi \qquad \forall i$$

$$\text{(4.14k)} \qquad P_i, \alpha_i \geq 0 \qquad \forall i$$

$$\text{(4.14l)} \qquad \psi_{ij}^+, \psi_{ij}^- \geq 0 \qquad \forall (i,j) \in BR$$

$$\text{(4.14m)} \qquad LMP_i \quad free \qquad \forall i$$

Constraint (4.14i) and (4.14k) are new constraints related to (4.13h). Active set Method in Gümüş & Floudas (2005) is then applied to resolve the non-convexity caused by KKT complementary slackness conditions in (4.14f) - (4.14h). Based on this method, we replace a constraint in the form of $u.s=0$ by defining a new binary variable v and a sufficiently big parameter M as follows.

$$u - Mv \leq 0, \qquad j \in J \tag{4.15a}$$
$$s - M(1-v) \leq 0, \qquad j \in J \tag{4.15b}$$
$$u, s \geq 0, \qquad j \in J \tag{4.15c}$$
$$v \in \{0,1\} \tag{4.15d}$$

As we take a look at the model (4.14a) - (4.14m), we also observe the non-linearity constraints (4.14b), (4.14f) - (4.14i). The same method which was applied to linearize the non-linear terms in Reformulation1 and Reformulation2 is also used here. As a matter of fact, we formulate a linearized version as

$$\text{maximize} \quad \lambda \tag{4.16a}$$
$$\text{subject to } \lambda \leq \sum_{k \in K} \text{Bid}_{ik} V_{ik} - P_i C_i \qquad \forall i \tag{4.16b}$$
$$b_i = \sum_{k \in K} \text{Bid}_{ik} B_{ik} \qquad \forall i \tag{4.16c}$$
$$\sum_{k \in K} B_{ik} = 1 \qquad \forall i \tag{4.16d}$$
$$V_{ik} \leq P_i^{max} B_{ik} \qquad \forall i \text{ and } k \tag{4.16e}$$
$$V_{ik} \leq P_i \qquad \forall i \text{ and } k \tag{4.16f}$$
$$V_{ik} \geq P_i - P_i^{max}[1 - B_{ik}] \qquad \forall i \text{ and } k \tag{4.16g}$$
$$V_{ik} \geq 0 \qquad \forall i \text{ and } k \tag{4.16h}$$
$$P_i - \sum_{ij \in BR} \gamma_{ij}(\theta_i - \theta_j) = D_i \qquad \forall i \tag{4.16i}$$
$$-P_i \leq 0 \qquad \forall i \tag{4.16j}$$
$$\sum_{ij \in BR} \gamma_{ij}(LMP_j - LMP_i) + \sum_{ij \in BR} \gamma_{ij}(\psi_{ij}^- - \psi_{ij}^+) + \sum_{ji \in BR} \gamma_{ji}(\psi_{ji}^+ - \psi_{ji}^-) = 0 \qquad \forall i \tag{4.16k}$$
$$-b_i + LMP_i - \phi_i + \alpha_i = 0 \qquad \forall i \tag{4.16l}$$
$$\phi_i - M_{1i} v_i^1 \leq 0 \qquad \forall i \tag{4.16m}$$
$$P_i^{max} - P_i - M_{2i}(1 - v_i^1) \leq 0 \qquad \forall i \tag{4.16n}$$
$$\psi_{ij}^+ - M_{3ij} v_{ij}^2 \leq 0 \qquad \forall (i,j) \in BR \tag{4.16o}$$
$$-\gamma_{ij}(\theta_i - \theta_j) + F_{ij}^{max} - M_{4ij}(1 - v_{ij}^2) \leq 0 \qquad \forall (i,j) \in BR \tag{4.16p}$$
$$\psi_{ij}^- - M_{5ij} v_{ij}^3 \leq 0 \qquad \forall (i,j) \in BR \tag{4.16q}$$

(4.16r)	$\gamma_{ij}(\theta_i - \theta_j) + F_{ij}^{max} - M6_{ij}(1 - v_{ij}^3) \leq 0$	$\forall (i,j) \in BR$
(4.16s)	$\alpha_i - M_{7i} v_i^4 \leq 0$	$\forall i$
(4.16t)	$P_i - M_{8i}(1 - v_i^4) \leq 0$	$\forall i$
(4.16u)	$v_i^1 + v_i^4 \leq 1$	$\forall i$
(4.16v)	$v_{ij}^2 + v_{ij}^3 \leq 1$	$\forall (i,j) \in BR$
(4.16w)	$P_i \leq P_i^{max}$	$\forall i$
(4.16x)	$\gamma_{ij}(\theta_i - \theta_j) \leq F_{ij}^{max}$	$\forall (i,j) \in BR$
(4.16y)	$\gamma_{ij}(\theta_i - \theta_j) \geq -F_{ij}^{max}$	$\forall (i,j) \in BR$
(4.16z)	$-\pi \leq \theta_i \leq \pi$	$\forall i$
(4.16aa)	$v_i^1, v_i^4 \in \{0,1\}$	$\forall i$
(4.16ab)	$v_{ij}^2, v_{ij}^3 \in \{0,1\}$	$\forall (i,j) \in BR$
(4.16ac)	$P_i, \alpha_i \geq 0$	$\forall i$
(4.16ad)	LMP_i free	$\forall i$
(4.16ae)	$B_{ik} \in \{0,1\}$	$\forall i$ and k
(4.16af)	$V_{ik} \geq 0$	$\forall i$ and k
(4.16ag)	$\psi_{ij}^+, \psi_{ij}^- \geq 0$	$\forall (i,j) \in BR$

Constraints (4.16b)-(4.16h) are linearized form of constraint (4.14b). Constraints (4.16m)-(4.16n) are associated with constraint (4.14f). Constraints (4.16o)-(4.16p) are associated with constraint (4.14g). Constraints (4.16q)-(4.16r) are associated with constraint (4.14h). Constraints (4.16s)-(4.16t) are associated with constraint (4.14i). Constraints (4.16u)-(4.16v), and constraints (4.16aa)-(4.16ab) are related with constraints (4.15c)-(4.15d) of the active set method. Big M parameters are defined as $M_{1i} = 100$, $M_{2i} = P_i^{max}$, $M_{3ij} = 100$, $M_{4ij} = 2F_{ij}^{max}$,$M_{5ij} = 100$, $M_{6ij} = 2F_{ij}^{max}$, $M_{7i} = P_i^{max}$, and $M_{8i} = P_i^{max}$.

In order to develop an improved version of Reformulation3, some redundant constraints are eliminated using propositions 4.1.2 and 4.1.3 which eliminates constraints (4.16w)-(4.16y). The new formulation derived after applying propositions is called Improved-Reformulation3.

4.1.5 Reformulation4: MILP model based on SOS type 1 variables

One of the well-known techniques to transform a bi-level problem to a single-level MILP is to use SOS variables which is applied by reformulating the complementary conditions. SOS type 1 refers to a set of variables; in such a set, only one of the variable can take a positive value (Siddiqui Gabriel 2013, Pineda et al. 2018). The general form of SOS type 1 variables is defined as follow.

$$v^1 = u$$
$$v^2 = g(x,y)$$

where v^1 and v^2 are SOS type 1 variables. Furthermore, SOS type 1 variables can be applied for Reformulation1 where SOS type 1 variables are utilized to express the complementary conditions. Using this technique, problem formulation in (4.2a) - (4.2i) can be alternatively reformulated as

$$\text{(4.17a)} \quad \underset{\{b_i,\lambda\}}{\text{maximize}} \quad \lambda$$

$$\text{(4.17b)} \quad \text{subject to} \quad \lambda \leq \sum_{k\in K} \text{Bid}_{ik} V_{ik} - P_i C_i \qquad \forall i$$

$$\text{(4.17c)} \quad b_i = \sum_{k\in K} \text{Bid}_{ik} B_{ik} \qquad \forall i$$

$$\text{(4.17d)} \quad \sum_{k\in K} B_{ik} = 1 \qquad \forall i$$

$$\text{(4.17e)} \quad V_{ik} \leq P_i^{max} B_{ik} \qquad \forall i \text{ and } k$$

$$\text{(4.17f)} \quad V_{ik} \leq P_i \qquad \forall i \text{ and } k$$

$$\text{(4.17g)} \quad V_{ik} \geq P_i - P_i^{max}[1 - B_{ik}] \qquad \forall i \text{ and } k$$

$$\text{(4.17h)} \quad V_{ik} \geq 0 \qquad \forall i \text{ and } k$$

$$\text{(4.17i)} \quad \sum_{ij\in BR} \gamma_{ij}(LMP_j - LMP_i) + \sum_{ij\in BR} \gamma_{ij}(\psi_{ij}^- - \psi_{ij}^+) + \sum_{ji\in BR} \gamma_{ji}(\psi_{ji}^+ - \psi_{ji}^-) = 0 \qquad \forall i$$

$$\text{(4.17j)} \quad P_i - D_i = \sum_{ij\in BR} \gamma_{ij}(\theta_i - \theta_j) \qquad \forall i$$

$$\text{(4.17k)} \quad P_i \leq P_i^{max} \qquad \forall i$$

$$\text{(4.17l)} \quad b_i - LMP_i + \phi_i \geq 0 \qquad \forall i$$

$$\text{(4.17m)} \quad v_i^{11} = P_i \qquad \forall i$$

$$\text{(4.17n)} \quad v_i^{12} = b_i - LMP_i + \phi_i \qquad \forall i$$

$$\text{(4.17o)} \quad v_i^{21} = \phi_i \qquad \forall i$$

$$\text{(4.17p)} \quad v_i^{22} = P_i^{max} - P_i \qquad \forall i$$

(4.17q)	$v_{ij}^{31} = \psi_{ij}^{+}$	$\forall i$
(4.17r)	$v_{ij}^{32} = F_{ij}^{max} - \gamma_{ij}(\theta_i - \theta_j)$	$\forall i$
(4.17s)	$v_{ij}^{41} = \psi_{ij}^{-}$	$\forall i$
(4.17t)	$v_{ij}^{42} = F_{ij}^{max} + \gamma_{ij}(\theta_i - \theta_j)$	$\forall i$
(4.17u)	$\gamma_{ij}(\theta_i - \theta_j) \leq F_{ij}^{max}$	$\forall (i,j) \in BR$
(4.17v)	$\gamma_{ij}(\theta_i - \theta_j) \geq -F_{ij}^{max}$	$\forall (i,j) \in BR$
(4.17w)	$-\pi \leq \theta_i \leq \pi$	$\forall i$
(4.17x)	$v_i^{11}, v_i^{12} \quad SOS1$	$\forall i$
(4.17y)	$v_i^{21}, v_i^{22} \quad SOS1$	$\forall i$
(4.17z)	$v_{ij}^{31}, v_{ij}^{32} \quad SOS1$	$\forall i, j$
(4.17aa)	$v_{ij}^{41}, v_{ij}^{42} \quad SOS1$	$\forall i, j$
(4.17ab)	$P_i \geq 0$	$\forall i$
(4.17ac)	LMP_i free	$\forall i$
(4.17ad)	$B_{ik} \in \{0, 1\}$	$\forall i$ and k
(4.17ae)	$V_{ik} \geq 0$	$\forall i$ and k
(4.17af)	$\psi_{ij}^{+}, \psi_{ij}^{-} \geq 0$	$\forall ij \in BR$

In the proposed reformulation4 in (4.17a) - (4.17af), we have four SOS type 1 variables which have produced new SOS type 1 constraints (4.17m) - (4.17t). Constraints (4.17m) - (4.17n) are associated with constraint (4.2c). Constraints (4.17o) - (4.17p) are related to constraint (4.2f). Constraints (4.17q) - (4.17r) are associated with constraint (4.2g), and constraints (4.17s) - (4.17t) are associated with constraint (4.2h).

4.2 Comparison of Reformulations

Four reformulations are proposed to transform the bi-level problem. Based on FAM method, KKT conditions were used for the Reformulation1. In Reformulation2, strong-duality conditions were used for the lower level problem. In a similar way to what Reformulation1 was derived, Reformulation3 was formulated based on KKT conditions together with Active Set Method. In Reformulation4, we use SOS variables.

As we compare the proposed reformulations based on the number of variables and constraints, Reformulation2 has the least number of constraints and binary variables. Reformulation1 and Reformulation3 have almost the same number of constraints as they are based on KKT conditions. Reformulation4 has lower number of constraints in comparison to Reformulation1 and Reformulation3, but has higher number of constraints in comparison to Reformulation2. The main advantage of SOS variables is that no large constant is required. Moreover, both improved-Reformulation1 and Improved-Reformulation3 decrease the size of Reformulation1 and Reformulation3 in terms of number of constraints, respectively.

5. Computational Study

To illustrate performance and efficiency of the proposed reformulations to detect collusion opportunities, several test instances are generated in four size groups as small, medium, medium-plus, and large. The most important challenge to generate the test instances is to ensure that there is at least one collusive state.

5.1 Test Instances

For each size group, the transmission network is fixed, i.e., the nodes, GenCos, the transmission links, demands, and production costs are given. The parameters that are randomly generated are P_i^{max} and F_{ij}^{max}. In order to generate test instances, we solve the DC-OPF problem using the total enumeration algorithm for each randomly generated P_i^{max} and F_{ij}^{max} values and check whether the generated instance has any collusive states. Instance generation process is a computationally challenging task since each generated instance has to be solved with DC-OPF problem. As the size of instance increases, DC-OPF takes longer time to be solved. For small instances, it took averagely 145 minutes to generate one instance with at least one collusive state. For medium instances, the instance generation process averagely took 360 minutes for an instance with at least one collusive state. However, the instance generation process gets more challenging as the size of instances increases. For example, instance generation process averagely took 3 weeks to generate a medium-plus instance with at least one collusive state. For large instances, the instance generation process took approximately 6 weeks to generate an instance with at least one collusive state. After completing the instance generation process, we filter instances by eliminating those which have alternative solutions when we solve the DC-OPF problem.

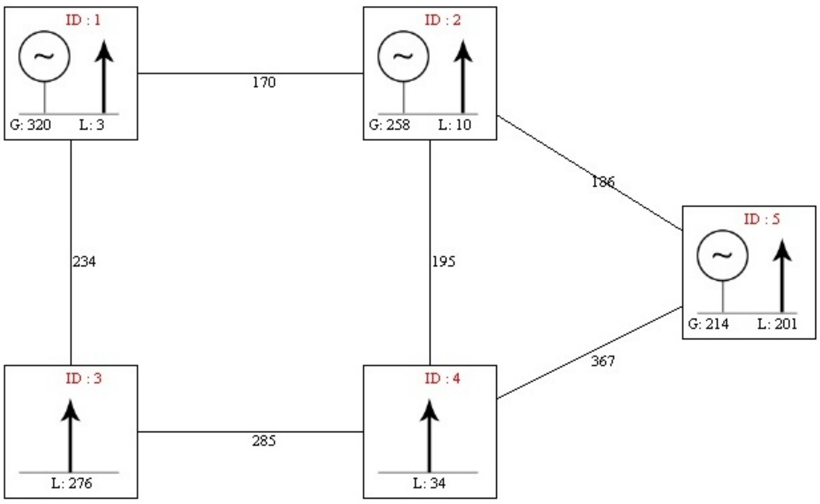
5.1.1 Small instances

Small instances are generated based on the transmission grid in Aliabadi et al. (2016). Table 5.1 represents the important parameters of these instances while Figure 1 shows the network with five nodes and six transmission arcs where three of them are GenCos. Production costs (C_i) and offered bids (B_i) are represented in Table 5.1. $GenCo_1$ and $GenCo_2$ have seven and $GenCo_3$ has five distinct bid offers. Therefore, in total, we have 7*7*5=245 bid-offer states in the market. Demand of each node is presented in Figure 5.1.

Table 5.1 Small network parameters

	C_i($/MW)	B_i($/MW)
GenCo-1	20	{22,27,32,37,42,47,52}
GenCo-2	20	{21,26,31,36,41,46,51}
GenCo-5	30	{30,35,40,45,50}

Figure 5.1 Small network grid



Using the small transmission grid network, we generate ten instances with different P_i^{max} and F_{ij}^{max} values which are presented in Table 5.2. The number of Nash states and collusive states found with the total-enumeration of all states solving the DC-OPF problem as suggested in (4.2a) - (4.2i) are also shown in Table 5.2. We also report the running time of the enumeration algorithm in Table 5.2.

Table 5.2 Small network instances

Instance	P_i^{max}	F_{ij}^{max}	Nash states	Collusive states	Time
1	1. 320, 2. 258, 5. 214	1.2. 170, 1.3. 234, 2.4. 195, 2.5. 186, 3.4. 285, 4.5. 367	2	5	13.29
2	1. 262, 2. 495, 5. 249	1.2. 156, 1.3. 466, 2.4. 171, 2.5. 423, 3.4. 207, 4.5. 26	2	5	12.56
3	1. 371, 2. 202, 5. 362	1.2. 124, 1.3. 348, 2.4. 432, 2.5. 473, 3.4. 300, 4.5. 401	6	5	11.64
4	1. 363, 2. 450, 5. 415	1.2. 80, 1.3. 224, 2.4. 251, 2.5. 222, 3.4. 380, 4.5. 300	1	6	11.77
5	1. 353, 2. 260, 5. 346	1.2. 76, 1.3. 227, 2.4. 239, 2.5. 306, 3.4. 150, 4.5. 208	2	7	12.46
6	1. 345, 2. 259, 5. 272	1.2. 103, 1.3. 369, 2.4. 375, 2.5. 253, 3.4. 198, 4.5. 59	3	9	12.15
7	1. 355, 2. 326, 5. 236	1.2. 100, 1.3. 226, 2.4. 492, 2.5. 443, 3.4. 128, 4.5. 35	3	6	12.16
8	1. 340, 2. 270, 5. 234	1.2. 170, 1.3. 234, 2.4. 195, 2.5. 186, 3.4. 285, 4.5. 367	2	5	12.14
9	1. 234, 2. 302, 5. 232	1.2. 367, 1.3. 210, 2.4. 167, 2.5. 324, 3.4. 264, 4.5. 185	4	3	11.78
10	1. 234, 2. 487, 5. 327	1.2. 264, 1.3. 450, 2.4. 462, 2.5. 478, 3.4. 306, 4.5. 31	2	13	11.89

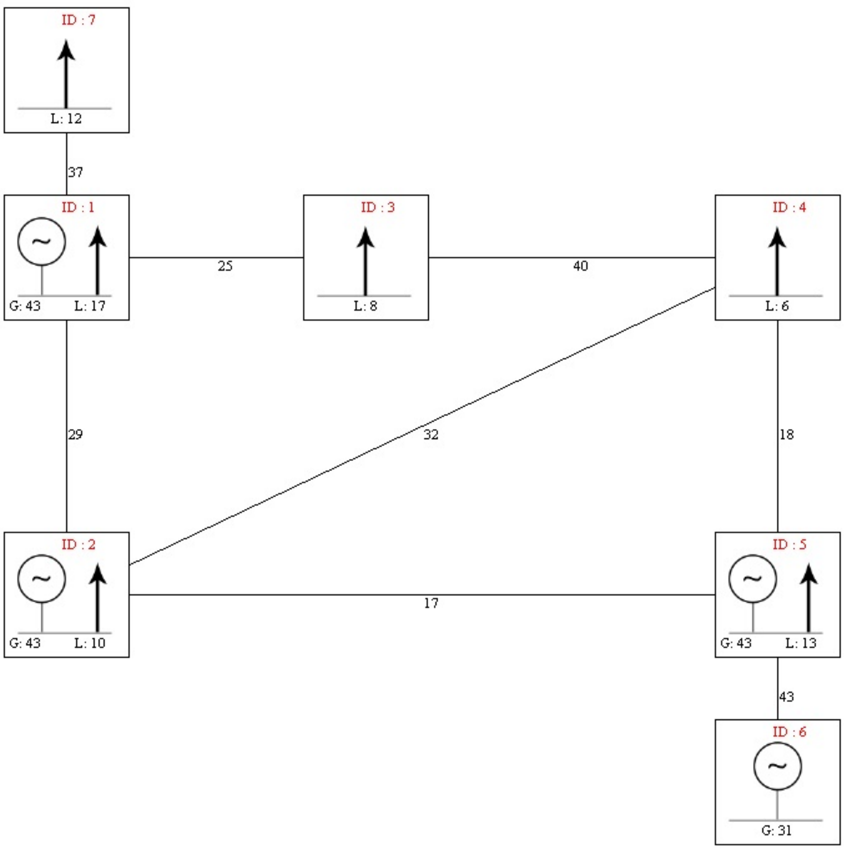
5.1.2 Medium instances

In order to increase the size of market, we add two more nodes to the network grid, one of which also becomes a GenCo. According to Table 5.3, $GenCo_1$ and $GenCo_2$ have seven, $GenCo_5$ has five, and the new $GenCo_6$ has nine distinct bid offers. In total, there exist 7*7*5*9=2205 bid-offer states in the medium-size market. Production costs (C_i), offered bids (B_i), and node demands are presented in Table 5.3 and Figure 5.2.

Table 5.3 Medium network parameters

	C_i(\$/MW)	B_i(\$/MW)
GenCo-1	20	{21,26,31,36,41,46,51}
GenCo-2	20	{22,27,32,37,42,47,52}
GenCo-5	30	{33,38,43,48,53}
GenCo-6	10	{14,19,24,29,34,39,44,49,54}

Figure 5.2 Medium network grid



According to parameters of the medium network, ten instances are generated again by varying P_i^{max} and F_{ij}^{max} values. Generated instances, parameter values and problem characteristics are presented in Table 5.4. The number of Nash states and collusive states as well as the running time of the enumeration algorithm are also presented in Table 5.4.

Table 5.4 Medium network instances

Instance	P_i^{max}	F_{ij}^{max}	Nash states	Collusive states	Time
1	1. 43, 2. 43, 5. 43, 6. 31	1.2. 29, 1.3. 25, 1.7. 37, 2.4. 32, 2.5. 17, 3.4. 40, 4.5. 18, 5.6. 43	10	28	157.54
2	1. 21, 2. 23, 5. 28, 6. 23	1.2. 17, 1.3. 29, 1.7. 37, 2.4. 26, 2.5. 18, 3.4. 23, 4.5. 42, 5.6. 43	12	81	129.45
3	1. 36, 2. 34, 5. 30, 6. 31	1.2. 20, 1.3. 15, 1.7. 30, 2.4. 32, 2.5. 8, 3.4. 17, 4.5. 11, 5.6. 6	4	4	145.11
4	1. 36, 2. 46, 5. 33, 6. 41	1.2. 16, 1.3. 36, 1.7. 13, 2.4. 25, 2.5. 8, 3.4. 8, 4.5. 44, 5.6. 7	2	52	153.21
5	1. 24, 2. 32, 5. 39, 6. 37	1.2. 29, 1.3. 25, 1.7. 37, 2.4. 32, 2.5. 17, 3.4. 40, 4.5. 18, 5.6. 43	3	29	160.45
6	1. 62, 2. 25, 5. 33, 6. 47	1.2. 47, 1.3. 45, 1.7. 17, 2.4. 13, 2.5. 12, 3.4. 45, 4.5. 31, 5.6. 6	1	9	156.23
7	1. 62, 2. 45, 5. 45, 6. 44	1.2. 29, 1.3. 25, 1.7. 37, 2.4. 32, 2.5. 17, 3.4. 40, 4.5. 18, 5.6. 43	5	16	148.52
8	1. 80, 2. 42, 5. 31, 6. 32	1.2. 69, 1.3. 99, 1.7. 35, 2.4. 24, 2.5. 22, 3.4. 34, 4.5. 21, 5.6. 45	10	38	123.25
9	1. 25, 2. 69, 5. 74, 6. 57	1.2. 56, 1.3. 63, 1.7. 65, 2.4. 33, 2.5. 16, 3.4. 48, 4.5. 18, 5.6. 46	5	19	124.58
10	1. 55, 2. 31, 5. 15, 6. 69	1.2. 23, 1.3. 79, 1.7. 68, 2.4. 64, 2.5. 59, 3.4. 88, 4.5. 37, 5.6. 97	40	54	129.85

5.1.3 Medium-plus instances

In order to generate instances that are closer to real-life cases, we take the structure of medium network grid and increase the number of bid offers by each GenCo. New bid offers of each GenCo are presented in Table 5.5. Based on Table 5.5, $GenCo_1$ and $GenCo_2$ have sixteen, $GenCo_5$ has eleven, and $GenCo_6$ has sixteen distinct bid offers. In total, there exist 16*16*11*16=45056 bid-offer states in medium-size market. The increase in bid offers lead to a great increase in bid-offer options in the market.

Table 5.5 Medium-plus network parameters

	C_i($/MW)	B_i($/MW)
GenCo-1	20	{21,23,25,27,29,31,33,35,37,39,41,43,45,47,49,51}
GenCo-2	20	{22,24,26,28,30,32,34,36,38,40,42,44,46,48,50,52}
GenCo-5	30	{33,35,37,39,41,43,45,47,49,51,53}
GenCo-6	10	{14,16,18,20,22,24,26,28,30,32,34,36,38,40,42,44}

Table 5.6 Medium-plus network instances

Instances	P_i^{max}	F_{ij}^{max}	Nash states	Collusive states	Time
1	1. 132, 2. 291, 5. 31, 6. 85	1.2. 278, 1.3. 68, 1.7. 92, 2.4. 185, 2.5. 45, 3.4. 13, 4.5. 281, 5.6. 24	11	1316	2350.51
2	1. 61, 2. 34, 5. 24, 6. 21	1.2. 278, 1.3. 68, 1.7. 92, 2.4. 185, 2.5. 45, 3.4. 13, 4.5. 281, 5.6. 24	4	438	2228.56
3	1. 33, 2. 54, 5. 48, 6. 26	1.2. 29, 1.3. 14, 1.7. 27, 2.4. 8, 2.5. 42, 3.4. 10, 4.5. 16, 5.6. 35	12	715	2349.37
4	1. 29, 2. 52, 5. 30, 6. 31	1.2. 29, 1.3. 14, 1.7. 27, 2.4. 8, 2.5. 42, 3.4. 10, 4.5. 16, 5.6. 35	11	844	2289.42
5	1. 31, 2. 54, 5. 21, 6. 33	1.2. 29, 1.3. 14, 1.7. 27, 2.4. 8, 2.5. 42, 3.4. 10, 4.5. 16, 5.6. 35	5	903	2140.23
6	1. 27, 2. 41, 5. 43, 6. 28	1.2. 29, 1.3. 14, 1.7. 27, 2.4. 8, 2.5. 42, 3.4. 10, 4.5. 16, 5.6. 35	20	561	2514.78

According to parameters of the medium transmission grid network, six instances are generated again by varying P_i^{max} and F_{ij}^{max} values. Generated six medium-plus instances and their related parameters are represented in Table 5.5. The number of Nash states and collusive states as well as the running time of the enumeration algorithm are also presented in Table 5.6. As we observe, the running time of the enumeration algorithm has increased significantly as the number of offered bids increased.

5.1.4 Large instances

In order to further extend the transmission grid network, we construct a new grid network with nine nodes, five of which are GenCos (Figure 5.3). Table 5.7 presents the parameters of the large test instances. $GenCo_1$ has twelve, $GenCo_2$ has ten, $GenCo_5$ has five, $GenCo_6$ has ten, and the new $GenCo_9$ has twelve distinct bid offers. In total, there exist 12*10*5*10*12=72000 bid-offer states in large-size market.

Table 5.7 Large network parameters

	C_i(\$/MW)	B_i(\$/MW)
GenCo-1	20	{21,26,31,36,41,46,51,56,61,66,71,76}
GenCo-2	20	{22,27,32,37,42,47,52,57,62,67}
GenCo-5	30	{33,38,43,48,53}
GenCo-6	10	{14,19,24,29,34,39,44,49,54,59}
GenCo-9	25	{35,40,45,50,55,60,65,70,75,80,85,90}

According to parameters of the large network, three instances are generated by varying P_i^{max} and F_{ij}^{max} values. Generated three large instances and their related parameters are presented in Table 5.8. The number of Nash states and collusive states as well as the running time of the enumeration algorithm are also presented in Table 5.8. In comparison to the previous test instances, the running time of large instances has increased dramatically due to the increase in number of offered bids and number of GenCos.

Figure 5.3 Large network grid

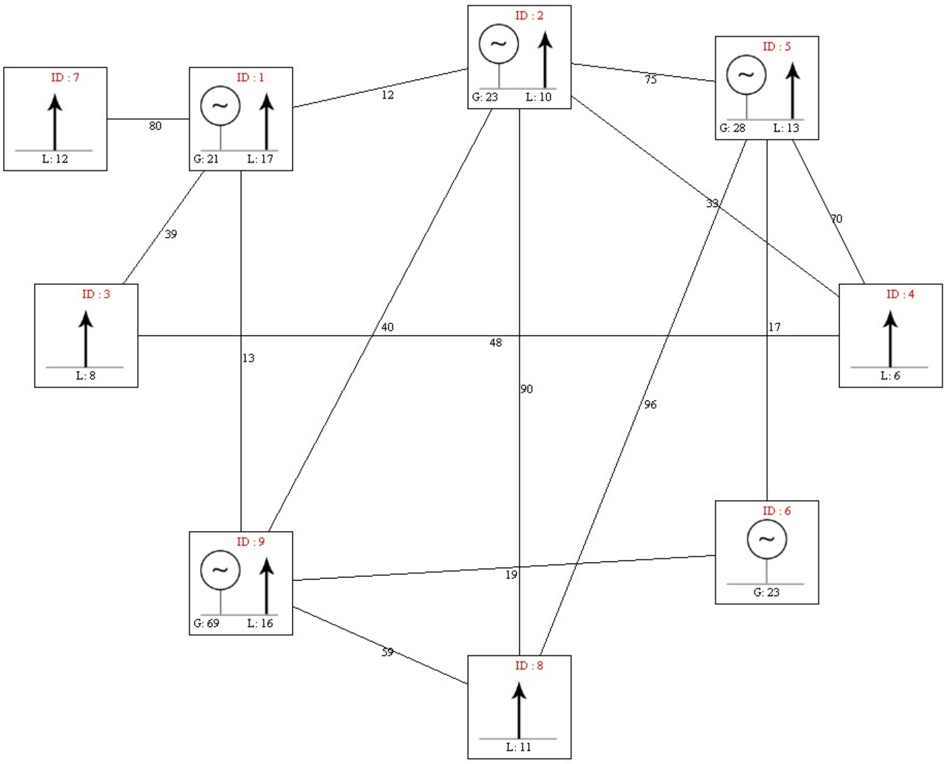
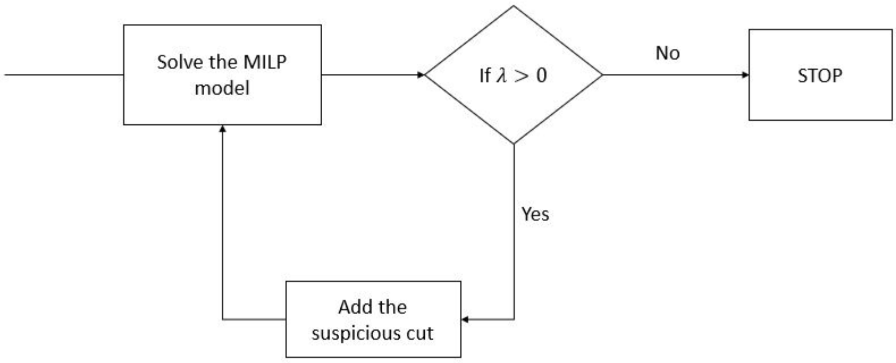


Table 5.8 Large network instances

Instances	P_i^{max}	F_{ij}^{max}	Nash states	Collusive states	Time
1	1. 38, 2. 35, 5. 43, 6. 80 , 9. 69	1.2. 12 , 1.3. 39, 1.7. 80, 1.9. 13, 2.4. 33, 2.5. 75, 2.8. 90, 2.9. 40, 3.4. 48, 4.5. 70, 5.6. 17, 5.8. 96, 6.9. 19, 8.9. 59,	1	855	4220.03
2	1. 84, 2. 69, 5. 38, 6. 29, 9. 30	1.2. 12 , 1.3. 39, 1.7. 80, 1.9. 13, 2.4. 33, 2.5. 75, 2.8. 90, 2.9. 40, 3.4. 48, 4.5. 70, 5.6. 17, 5.8. 96, 6.9. 19, 8.9. 59,	1	43	5344.37
3	1. 32, 2. 51, 5. 85, 6. 32, 9. 23	1.2. 12 , 1.3. 39, 1.7. 80, 1.9. 13, 2.4. 33, 2.5. 75, 2.8. 90, 2.9. 40, 3.4. 48, 4.5. 70, 5.6. 17, 5.8. 96, 6.9. 19, 8.9. 59,	40	5	4438.56

In order to test both efficiency and effectiveness of the proposed reformulations in detecting collusive opportunities, we develop a search algorithm called as the iterative algorithm to solve reformulations using all generated test instances. As r_i^* values are not known and cannot be determined by solving reformulations; therefore, a state found from solving the reformulations cannot be completely guaranteed to be collusive. In this regard, feasible solutions, states, found by the reformulations are considered to be suspicious of being collusive.

Figure 5.4 Iterative algorithm



In the reformulations, λ value represents the minimum of profits of each $GenCo_i$ (r_i). Based on the definition of the collusive state, the profit of each $GenCo_i$ is strictly greater than its profits at any Nash state ($r_i > r_i^*$). This means that all GenCos in a collusive state definitely have non-zero profits. The iterative algorithm solves the reformulations and continues finding new solutions as long as $\lambda > 0$ which means that the algorithm continues to solve the reformulation model as long as it finds a state that its minimum profit (λ) is positive. In order to avoid finding the same recently found suspicious states in subsequent iterations, a constraint is added to the model. Constraints added in each iteration are named as suspicious cuts. Using the suspicious cuts, we eliminate suspicious states which are found in previous iterations by restricting the associated binary variables B_{ik}. In other words, suspicious cut hinders sum of the associate binary variables to be 1 at the same time. Mathematically, the suspicious cut for all b_i in solution space is expressed as

$$\textstyle\sum_i B_{ik} \leq n - 1 \quad \forall\, b_i \in \text{suspicious solution}$$

where n denotes the number of GenCos. Iterative algorithm continues to solve the model until it finds a solution with zero profit ($\lambda = 0$) or the problem becomes infeasible (Figure 5.4).

Without knowing true r_i^* values, it is not exactly known which suspicious solutions are exactly real collusive states. However, based on the definition of the collusive state, we know that all real collusive states are definitely included among suspicious solutions that are found using the iterative algorithm for reformulations. In order to measure the performance of each reformulation, we compare the results of each reformulation with results of enumeration algorithm that were obtained using DC-OPF problem. For the purpose of comparison, Esen (2019) introduced two performance measures as found ratio, and coverage ratio. Found ratio which is now renamed as detection accuracy (DA) denotes the accuracy of the algorithm represented as the ratio of the number of real collusive states to the total number of suspicious states found, is expressed as

$$\text{Detection accuracy} = \frac{\#\text{ of collusive states found}}{\#\text{ of suspicious states found}}$$

While coverage ratio (CR) denotes the sensitivity of the algorithm represented as the ratio of the number of collusive states found by a reformulation to the total number of collusive states. Coverage ratio is calculated as

$$\text{Coverage ratio} = \frac{\#\text{ of collusive states found}}{\#\text{ of real collusive states}}$$

Using the two performance measures defined above, results of reformulations for each test instance are presented based on the number of suspicious solutions (SS), DA, CR, the running time, and the first λ value. All reformulations are solved with Python 3.7 using GUROBI 9.0.2 solver. Computational tests are conducted on an Intel Xeron(R) CPU E5-2640 processor with 2.60 GHz speed and 16 GB RAM, with 64-bit Windows 7 operating system.

Table 5.9 presents the results of reformulations for small test instances. CR values are 1 for all the small instances which means that the proposed reformulations are successful in detecting all collusive states in each instance. Based on the definition of DA by Esen (2019), it is known that DA has a negative relationship with number of SS such that high number of SS would lead to lower DA value. Small instance 7 has the highest number of states that are suspicious of being collusive. On the other hand, small instance 3 has the lowest number of states which are suspicious of being collusive. DA of each reformulation is strongly affected by the number of solutions that are suspicious of being collusive; therefore, lower number of suspicious solutions would increase the value of DA ratio. In the last column of Table 5.9, first λ value of each reformulation is reported. Results show that λ value derived by all reformulations are the same in all small instances. Figure 5.5 presents the performance of reformulations in terms of their running time for iterative algorithm. As the running time of small test instances is very low, reformulations have very similar performances; however, Reformulation2 has shown better performance in most of the instances. Improved-Reformulation1 and Improved-Reformulation3 have shown better performances in some of the instances in comparison to Reformulation1 and Reformulation3, respectively.

Table 5.9 Results for small instances

Instances	Reformulation	SS	DA	CR	Time	λ
1	Reformulation1	66	7.5%	100%	10.13	16.64
	Improved-Reformulation1	66	7.5%	100%	10.89	16.64
	Reformulation2	66	7.5%	100%	14.53	16.64
	Reformulation3	66	7.5%	100%	10.91	16.64
	Improved-Reformulation3	66	7.5%	100%	12.71	16.64
	Reformulation4	66	7.5%	100%	11.42	16.64
2	Reformulation1	87	5.7%	100%	14.75	27.35
	Improved-Reformulation1	87	5.7%	100%	25.12	27.35
	Reformulation2	87	5.7%	100%	22.16	27.35
	Reformulation3	87	5.7%	100%	15.72	27.35
	Improved-Reformulation3	87	5.7%	100%	16.6	27.35
	Reformulation4	87	5.7%	100%	15.58	27.35
3	Reformulation1	14	35.7%	100%	2.22	13.8
	Improved-Reformulation1	14	35.7%	100%	4.42	13.8
	Reformulation2	14	35.7%	100%	3.83	13.8
	Reformulation3	14	35.7%	100%	2.71	13.8
	Improved-Reformulation3	14	35.7%	100%	3.89	13.8
	Reformulation4	14	35.7%	100%	3.66	13.8
4	Reformulation1	40	15%	100%	6.52	19.39
	Improved-Reformulation1	40	15%	100%	10.92	19.39
	Reformulation2	40	15%	100%	6.54	19.39
	Reformulation3	40	15%	100%	5.99	19.39
	Improved-Reformulation3	40	15%	100%	9.27	19.39
	Reformulation4	40	15%	100%	9.05	19.39
5	Reformulation1	80	8.8%	100%	15.21	26.19
	Improved-Reformulation1	80	8.8%	100%	15.72	26.19
	Reformulation2	80	8.8%	100%	14.34	26.19
	Reformulation3	80	8.8%	100%	16.09	26.19

Table 5.9 Results for small instances

Instances	Reformulation	SS	DA	CR	Time	λ
	Improved-Reformulation3	80	8.8%	100%	17.56	26.19
	Reformulation4	80	8.8%	100%	16.54	26.19
6	Reformulation1	68	13.2%	100%	12.77	16.89
	Improved-Reformulation1	68	13.2%	100%	15.72	16.89
	Reformulation2	68	13.2%	100%	12.63	16.89
	Reformulation3	68	13.2%	100%	12.93	16.89
	Improved-Reformulation3	68	13.2%	100%	14.61	16.89
	Reformulation4	68	13.2%	100%	12.75	16.89
7	Reformulation1	99	6.1%	100%	18.32	17.14
	Improved-Reformulation1	99	6.1%	100%	20.43	17.14
	Reformulation2	99	6.1%	100%	15.83	17.14
	Reformulation3	99	6.1%	100%	21.93	17.14
	Improved-Reformulation3	99	6.1%	100%	20.22	17.14
	Reformulation4	99	6.1%	100%	18.45	17.14
8	Reformulation1	66	7.6%	100%	13.58	17.2
	Improved-Reformulation1	66	7.6%	100%	11.93	17.2
	Reformulation2	66	7.6%	100%	7.53	17.2
	Reformulation3	66	7.6%	100%	19.52	17.2
	Improved-Reformulation3	66	7.6%	100%	16.68	17.2
	Reformulation4	66	7.6%	100%	15.59	17.2
9	Reformulation1	56	5.4%	100%	10.36	17.98
	Improved-Reformulation1	56	5.4%	100%	9.61	17.98
	Reformulation2	56	5.4%	100%	7.97	17.98
	Reformulation3	56	5.4%	100%	15.53	17.98
	Improved-Reformulation3	56	5.4%	100%	15.98	17.98
	Reformulation4	56	5.4%	100%	12.35	17.98
10	Reformulation1	87	14.9%	100%	15.26	33.94
	Improved-Reformulation1	87	14.9%	100%	14.85	33.94
	Reformulation2	87	14.9%	100%	12.38	33.94
	Reformulation3	87	14.9%	100%	14.78	33.94
	Improved-Reformulation3	87	14.9%	100%	17.49	33.94
	Reformulation4	87	14.9%	100%	9.48	33.94

Considering the running time of the total enumeration algorithm with the DC-OPF problem for small instances in Table 5.2, we observe that in most of instances reformulations perform better than the total enumeration algorithm. In instances 1, 3, and 4, running time of all reformulations is better than those of the total enumeration. Only Improved-Reformulation1 and Reformulation2 perform well with instance 8. In instance 9, Reformulation2 again performs better than other reformulations and total enumeration. In instance 10, Reformulation4 performs better than all reformulations and total enumeration in terms of the running time. In the rest of instances, reformulations fail to perform better than the total enumeration.

Figure 5.5 CPU time of reformulations for small instance

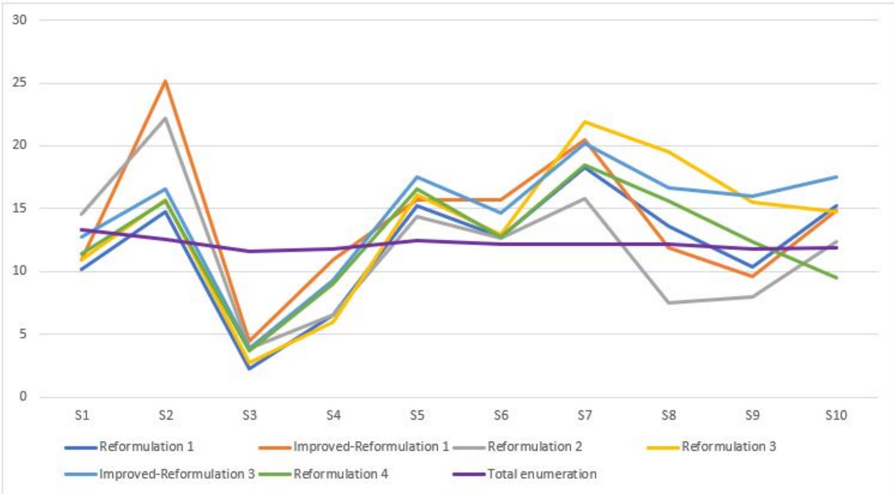


Table 5.10 presents the results of reformulations for medium test instances. In all instances except instance 4, the CR value is 1 which means that reformulations detect all collusive states. Reformulation4 fails to completely detect all collusive states in instance 4 but still most of the collusive states are covered and only one collusive state is missed. Instances 1, 7, 8, and 9 have the lowest suspicious solution among all medium instances. DA results in Table 5.10 show that these cases have obtained better DA values. In the last column of Table 5.10, first λ values of each reformulation is reported. Results show that λ values derived by all reformulations are same in all medium instances and there is no difference in λ value of reformulations in each test instance. Figure 5.6 presents the performance of reformulations in terms of their running time using iterative algorithm. As the size of problem increases in medium instances, the running time of medium test instances increases as well. Reformulation3 and Reformulation4 have shown better performances among all reformulations. Unlike small instances, Reformulation2 does not show a good performance for medium instances. Except in 3 instances, Improved-Reformulation1 shows better performance in comparison to Reformulation1. However, Improved-Reformulation3 does not show better performance in comparison to Reformulation3, as the Reformulations3 yields better running time values in most of the instances.

Table 5.10 Results for medium instances

Instances	Reformulation	SS	DA	CR	Time	λ
1	Reformulation1	38	73.7%	100%	8.92	1.55
	Improved-Reformulation1	38	73.7%	100%	7.52	1.55
	Reformulation2	38	73.7%	100%	15.72	1.55
	Reformulation3	38	73.7%	100%	7.64	1.55
	Improved-Reformulation3	38	73.7%	100%	8.06	1.55
	Reformulation4	38	73.7%	100%	8.61	1.55
2	Reformulation1	398	20.4%	100%	106.25	2.35
	Improved-Reformulation1	398	20.4%	100%	97.48	2.35
	Reformulation2	398	20.4%	100%	95.28	2.35
	Reformulation3	398	20.4%	100%	99.14	2.35
	Improved-Reformulation3	398	20.4%	100%	129.03	2.35
	Reformulation4	398	20.4%	100%	107.05	2.35
3	Reformulation1	570	0.7%	100%	147.91	1.99
	Improved-Reformulation1	570	0.7%	100%	132.59	1.99
	Reformulation2	570	0.7%	100%	271.15	1.99
	Reformulation3	570	0.7%	100%	142.94	1.99
	Improved-Reformulation3	570	0.7%	100%	179.56	1.99
	Reformulation4	570	0.7%	100%	136.04	1.99
4	Reformulation1	927	5.6%	100%	419.75	2.03
	Improved-Reformulation1	927	5.6%	100%	401.11	2.03
	Reformulation2	927	5.6%	100%	563.24	2.03
	Reformulation3	927	5.6%	100%	321.26	2.03
	Improved-Reformulation3	927	5.6%	100%	414.29	2.03
	Reformulation4	927	5.5%	98.1%	405.98	2.03
5	Reformulation1	271	10.7%	100%	60.59	2.05
	Improved-Reformulation1	271	10.7%	100%	57.56	2.05
	Reformulation2	271	10.7%	100%	72.52	2.05
	Reformulation3	271	10.7%	100%	78.59	2.05
	Improved-Reformulation3	271	10.7%	100%	92.37	2.05
	Reformulation4	271	10.7%	100%	69.09	2.05
6	Reformulation1	398	2.3%	100%	378.54	2.04
	Improved-Reformulation1	398	2.3%	100%	370.41	2.04
	Reformulation2	398	2.3%	100%	319.63	2.04
	Reformulation3	398	2.3%	100%	242.39	2.04
	Improved-Reformulation3	398	2.3%	100%	238.54	2.04
	Reformulation4	398	2.3%	100%	180.71	2.04

Table 5.10 Results for medium instances

Instances	Reformulation	SS	DA	CR	Time	λ
7	Reformulation1	38	42.1%	100%	12.31	1.01
	Improved-Reformulation1	38	42.1%	100%	12.18	1.01
	Reformulation2	38	42.1%	100%	10.95	1.01
	Reformulation3	38	42.1%	100%	8.08	1.01
	Improved-Reformulation3	38	42.1%	100%	10.47	1.01
	Reformulation4	38	42.1%	100%	12.35	1.01
8	Reformulation1	38	100%	100%	13.14	1.23
	Improved-Reformulation1	38	100%	100%	15.19	1.23
	Reformulation2	38	100%	100%	9.54	1.23
	Reformulation3	38	100%	100%	9.38	1.23
	Improved-Reformulation3	38	100%	100%	9.61	1.23
	Reformulation4	38	100%	100%	13.88	1.23
9	Reformulation1	28	67.9%	100%	6.56	0.13
	Improved-Reformulation1	28	67.9%	100%	9.87	0.13
	Reformulation2	28	67.9%	100%	15.87	0.13
	Reformulation3	28	67.9%	100%	9.64	0.13
	Improved-Reformulation3	28	67.9%	100%	9.27	0.13
	Reformulation4	28	67.9%	100%	11.07	0.13
10	Reformulation1	89	60.7%	100%	11.88	0.39
	Improved-Reformulation1	89	60.7%	100%	13.35	0.39
	Reformulation2	89	60.7%	100%	27.68	0.39
	Reformulation3	89	60.7%	100%	23.81	0.39
	Improved-Reformulation3	89	60.7%	100%	23.35	0.39
	Reformulation4	89	60.7%	100%	17.54	0.39

Considering the running time of the total enumeration for medium instances in Table 5.4, we observe that reformulations perform very well, as the running times of all reformulations are lower than those of total enumeration in instances 1, 2, 5, 7, 8, 9, and 10. However, reformulations do not perform very well in instances 4, and 6. In instance 3, all reformulations except Reformulation2 and Improved-Reformulation3 perform better than total enumeration.

Figure 5.6 CPU time of reformulations for medium instances

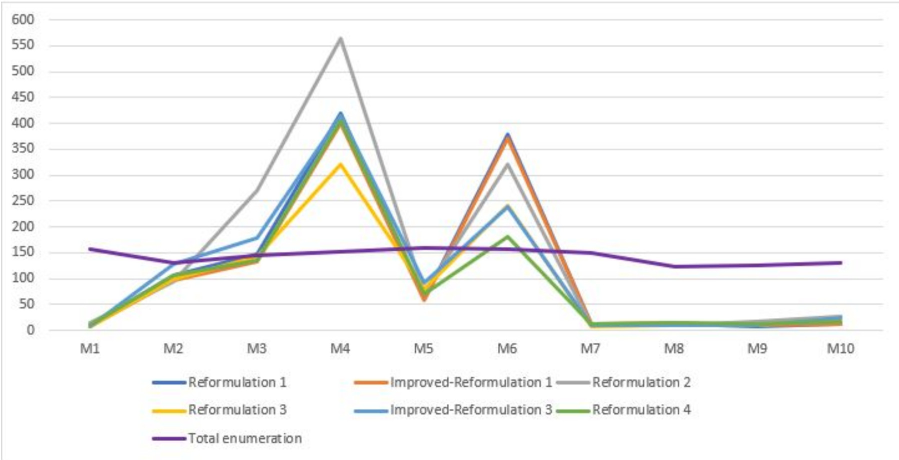


Table 5.11 presents the results of reformulations for medium-plus instances. In all instances, reformulations detect all collusive states; therefore the CR value is 1 in all of them. High number of bid-offer states in medium-plus instances has increased the possibility of having many solutions which are suspicious of being collusive (Table 5.11). In the last column of Table 5.11, first λ value of each reformulation is reported. Results show that λ values derived by all reformulations are same in all medium-plus instances and there is no difference in λ of reformulations in each instance. Figure 5.7 presents the performance of reformulations in terms of their running time using iterative algorithm. In comparison to market size of the medium instances, the size of problem increases more than 20 times in medium-plus instances which dramatically affects the running time of instances. Reformulation4 shows better performance in most of the instances, followed by Improved-Reformulation3 and Reformulation3. Reformulation2 shows the worst performance among all reformulations. In some instances, running time of Reformulation4 is approximately 3 to 5 times better than Reformulation2. Improved-Reformulation1 just shows better performances in two instances 5 and 6 in comparison to Reformulation1. Improved-Reformulation3 also does not show better performance in all instances in comparison to Reformulation3; however, Improved-Reformulation3 shows outstanding performance in instance 6 where it yields best value in comparison to all reformulations.

Table 5.11 Results for medium-plus instances

Instances	Reformulation	SS	DA	CR	Time	λ
1	Reformulation1	1840	71.5%	100%	1614.27	2
	Improved-Reformulation1	1840	71.5%	100%	1916.35	2
	Reformulation2	1840	71.5%	100%	6124.41	2
	Reformulation3	1840	71.5%	100%	1488.46	2
	Improved-Reformulation3	1840	71.5%	100%	1327.96	2
	Reformulation4	1840	71.5%	100%	1130.57	2
2	Reformulation1	3359	13%	100%	4697.24	1.37
	Improved-Reformulation1	3365	13%	100%	5181.54	1.37
	Reformulation2	3359	13%	100%	9341.49	1.37
	Reformulation3	3359	13%	100%	5300.58	1.37
	Improved-Reformulation3	3359	13%	100%	6095.11	1.37
	Reformulation4	3359	13%	100%	3304.56	1.37
3	Reformulation1	1208	59.2%	100%	751.58	2.74
	Improved-Reformulation1	1208	59.2%	100%	1163.91	2.74
	Reformulation2	1208	59.2%	100%	1883.48	2.74
	Reformulation3	1208	59.2%	100%	968.51	2.74
	Improved-Reformulation3	1208	59.2%	100%	806.51	2.74
	Reformulation4	1208	59.2%	100%	789.95	2.74
4	Reformulation1	1633	51.7%	100%	1361.36	2.16
	Improved-Reformulation1	1633	51.7%	100%	1545.82	2.16
	Reformulation2	1633	51.7%	100%	4525.21	2.16
	Reformulation3	1633	51.7%	100%	1114.58	2.16
	Improved-Reformulation3	1633	51.7%	100%	1123.06	2.16
	Reformulation4	1633	51.7%	100%	890.59	2.16
5	Reformulation1	1633	55.3%	100%	1211.84	1.96
	Improved-Reformulation1	1633	55.3%	100%	1025.26	1.96
	Reformulation2	1633	55.3%	100%	7096.12	1.96
	Reformulation3	1633	55.3%	100%	614.2	1.96
	Improved-Reformulation3	1633	55.3%	100%	752.67	1.96
	Reformulation4	1633	55.3%	100%	701.56	1.96
6	Reformulation1	1416	39.6%	100%	1152.57	2.51
	Improved-Reformulation1	1416	39.6%	100%	1029.18	2.51
	Reformulation2	1416	39.6%	100%	5981.75	2.51
	Reformulation3	1416	39.6%	100%	796.46	2.51
	Improved-Reformulation3	1416	39.6%	100%	271.32	2.51
	Reformulation4	1416	39.6%	100%	374.59	2.51

Considering the running time of the total enumeration for medium-plus instances in Table 5.6, we observe that all reformulations perform better than the total enumeration in only instance 3. However, the reformulations show outstanding performance in other instances. For example, in instances 1, 4, 5, and 6, all reformulations perform very well except for Reformulation2 which solves the model in high time in comparison to other reformulations. Reformulations does not perform well only in instance 2 where none of them get better running times in comparison to the total enumeration.

Figure 5.7 CPU time of reformulations for medium-plus instances

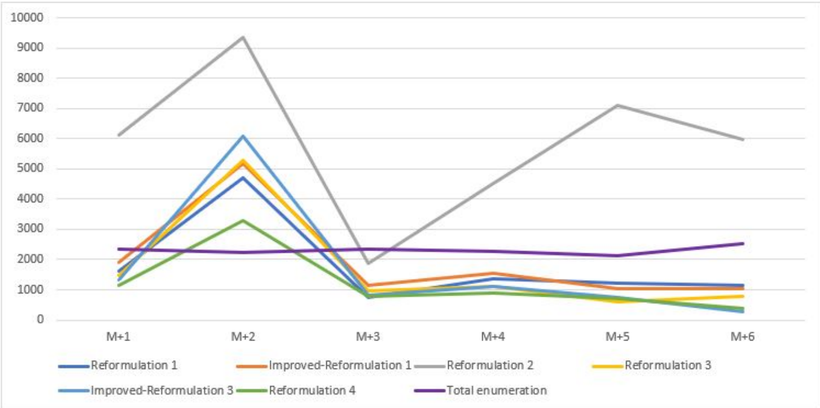


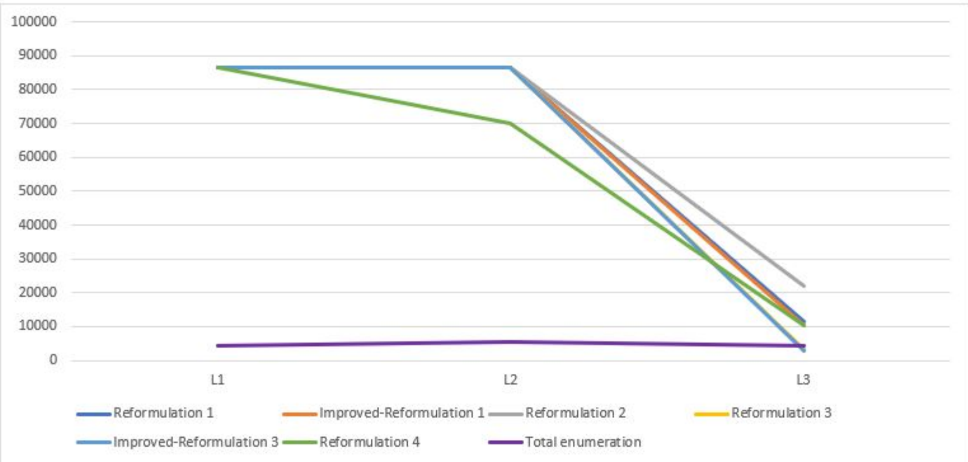
Table 5.12 presents the results of reformulations for large instances. As the size of problem increases significantly in large instances, we decide to limit the running time of reformulations to one day or equally 86400 seconds. In instance 1, only Improved-Reformulation3 succeeds to detect all collusive states. It also finds highest number of suspicious solutions among all reformulations. In instance 2, all reformulations except Reformulation1 find all collusive states. Unlike other reformulations, Reformulation4 detects all collusive states in less than a day, for almost 70145.45 seconds. In instance 3, all reformulations succeed in detecting collusive states in less than a day. Improved-Reformulation3 shows the best performance to detect collusive states in very short time among all reformulations. According to Table 5.12, we observe that unlike small, medium, and medium-plus instances, reformulations find different number of suspicious solutions. Based on this observation, results indicate that Reformulation4 has the highest DA value in instance1. In instances 2 and 3, we observe that DA values of all reformulations are better than DA value of Reformulation2, while other reformulations obtain same DA values. Figure 5.8 illustrates the performance of reformulations based on their running time for large instances. Similar to previous results, first λ values for each large instance is given in the last column of Table 5.12 where we observe that all reformulations obtain same λ values in each instance.

Considering the running time of the total enumeration for large instances in Table 5.8, none of the reformulations perform better than total enumeration in instance 1, and 2. Only Reformulation3 and Improved-Reformulation3 perform better than the total enumeration in instance 3.

Table 5.12 Results for large instances

Instances	Reformulation	SS	DA	CR	Time	λ
1	Reformulation1	3548	11.2%	46.5%	86400	2.81
	Improved-Reformulation1	3492	11%	44.9%	86400	2.81
	Reformulation2	4378	11.6%	59.2%	86400	2.81
	Reformulation3	4870	11.5%	65.6%	86400	2.81
	Improved-Reformulation3	7660	11.2%	100%	86400	2.81
	Reformulation4	4328	12.8%	64.6%	86400	2.81
2	Reformulation1	3316	1%	76.7%	86400	2.80
	Improved-Reformulation1	4106	1%	100%	86400	2.80
	Reformulation2	4730	0.9%	100%	86400	2.80
	Reformulation3	4221	1%	100%	86400	2.80
	Improved-Reformulation3	4221	1%	100%	86400	2.80
	Reformulation4	4221	1%	100%	70145.45	2.80
3	Reformulation1	1421	0.4%	100%	11513	2.12
	Improved-Reformulation1	1421	0.4%	100%	10374	2.12
	Reformulation2	1434	0.3%	100%	22110	2.12
	Reformulation3	1421	0.4%	100%	3160	2.12
	Improved-Reformulation3	1422	0.4%	100%	2850	2.12
	Reformulation4	1421	0.4%	100%	10466	2.12

Figure 5.8 CPU time of reformulations for large instances



6. Conclusions

The main goal of deregulated electricity markets is to provide a completely competitive environment in order to attain affordable electricity prices and finally maximize the social welfare. Collusion is the most important threat for the competitiveness of an electricity market where GenCos agree to conspire and restrict the competition. In order to detect collusion opportunities in deregulated electricity market, we present several reformulations for a game-theoretic based bi-level problem in Ali-abadi et al. (2016). First, we present two reformulations in Çelebi et al. (2019); then, we improve one of the reformulations in Çelebi et al. (2019) using several propositions in order to eliminate the redundant constraints. Second, we present two new reformulations for the bi-level problem based on KKT conditions together with Active Set Method, and SOS variables. In first new reformulation, we use KKT conditions and Active Set Method to derive a new reformulation which is named as Reformulation3. We improve Reformulation3 by eliminating the redundant constraints and name it as Improved-Reformulation3. In the second new reformulation, we use SOS variables to reformulate the bi-level problem which is named as Reformulation4.

In order to show the quality of solutions and performances of the reformulations, we present several test instances in four size groups of small, medium, medium-plus, and large. Two measures as DA and CR are defined to show the performances of reformulations in detection of collusion opportunities. Results indicate that Reformulation1 and Reformulation2 proposed by Çelebi et al. (2019) only works well for small-size instances and it does not perform well in medium, medium-plus and large instance in terms of computational time among all reformulations. Reformulation4, Reformulation3, and Improved-Reformulation3 have shown better performances in most of test instances among all reformulations, respectively. As we compare the performance of the reformulations with total enumeration algorithm, we observe that Reformulation4, Improved-Reformulation3, and Reformulation3 have shown better performances in most of test instances among all reformulations, respectively.

This work can be extended in several directions. One of the main limitations of the proposed MILP reformulations in this thesis is related to the fact that such models do not perform efficiently as the market size increases. In other words, the running time of test instances increases significantly as their size increases. Therefore one may use heuristic or metaheuristic algorithms to solve the reformulations in computationally efficient and reasonable time for real-life practices. Similarly, heuristic or metaheuristic algorithms can be used as the total enumeration algorithm might not be able to perform well in real-life practices with large number of bids. Based on the observations from the reformulations and the computational study, although Reformulation2 has less complications, it does not perform well as the market size increases in medium, medium-plus and large test instances. For this issue, one may develop a new reformulation based on an improved version of Reformulation2 which can be able to perform well even in larger instances. Another extension may include another decision making problem to prevent collusion opportunities focusing on capacity of transmission lines and bid options which will finally lead to a tri-level problem. One may use different types of market clearing paradigms for the same problem in this thesis in order to analyze the effects of market clearing mechanisms on detecting collusion opportunities. In another study, one may consider the uncertainty in some of the parameters like demand, production cost while preventing the collusion opportunities.

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