VEHICLE RELOCATION PROBLEMS IN FREE-FLOATING CAR-SHARING SYSTEMS

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VEHICLE RELOCATION PROBLEMS IN FREE-FLOATING CAR-SHARING SYSTEMS

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ABSTRACT

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Car-sharing systems have attracted plenty of attention for the past few decades as a means to fulfill constantly growing mobility needs especially in urban areas, and to alleviate the difficulties caused by economical and environmental problems due to excessive (and increasing) private car ownership. One of the main challenges faced in car-sharing systems is to maintain a good balance between vehicle supply and user demands by means of relocating the vehicles from regions with excess supply to regions with excess demand. We examine two vehicle relocation problems in a free-floating car-sharing system, which allows users to pick up/drop off vehicles from/to any location of their choice, and pay as they go. Vehicle relocations are typically performed by dedicated personnel, also known as operators, in car-sharing systems. First, we propose an operator-based vehicle relocation model which provides a relocation plan for the vehicles along with a set of routes for the operators consistent with the planned relocation tasks. Second, we formulate a hybrid relocation problem where, in addition to operators, users are also encouraged to participate in repositioning of the vehicles in return for a discount. Both problems are formulated as mixed-integer programs on appropriately defined time-space networks. Furthermore, new sets of instances are generated, and the proposed models are used to obtain solutions to these problem instances. A computational study is conducted to evaluate the operational efficiency of the car-sharing system based on key performance indicators, namely, objective value, number of rejected users and vehicle/operator utilization levels.

ÖZET

SERBEST DOLAŞIMLI ARAÇ PAYLAŞIM SİSTEMLERİNDE ARAÇ YER DEĞİŞTİRME PROBLEMLERİ

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Anahtar Kelimeler: araç yer değiştirme problemi, serbest dolaşımlı araç paylaşım sistemleri, zaman-mekan ağı

Araç paylaşım sistemleri, özellikle kentsel alanlarda sürekli artan hareketlilik ihtiyaçlarını karşılamak ve aşırı (ve artan) özel araç mülkiyeti nedeniyle ekonomik ve çevresel sorunların yol açtığı zorlukları hafifletmek için son birkaç on yıl boyunca büyük ilgi görmüştür. Araç paylaşım sistemlerinde karşılaşılan temel zorluklardan biri, araçların fazla arz olduğu bölgelerden aşırı talep gören bölgelere taşınması vasıtasıyla araç tedariği ve kullanıcı talepleri arasında iyi bir denge sağlamaktır. Kullanıcıların istedikleri herhangi bir yerden araç almalarını/bir yere araç bırakmalarını ve kullandıkları kadar ödeme yapmalarını sağlayan serbest dolaşımlı bir araç paylaşım sisteminde iki araç yer değiştirme sorununu inceliyoruz. Araç paylaşım sistemlerinde, araçların yer değiştirmesi tipik olarak operatör olarak da bilinen özel personel tarafından gerçekleştirilir. İlk olarak, planlanan yer değiştirme görevleriyle tutarlı olarak operatörler için bir dizi rota ile birlikte, taşıtlar için de bir yer değiştirme planı sağlayan operatör tabanlı bir araç yer değiştirme modeli öneriyoruz. İkinci olarak, kullanıcıların indirim karşılığında araç yer değiştirmelerine katılmaya teşvik edildiği karma bir araç yer değiştirme problemi formüle ediyoruz. Her iki problem de uygun şekilde tanımlanmış zaman-mekan ağlarında karma-tamsayılı programlar olarak formüle edilmiştir. Ayrıca yeni örnek kümeleri oluşturulmuş ve önerilen modeller bu sorun örneklerine çözüm bulmak için kullanılmıştır. Araç paylaşım sisteminin operasyonel verimliliğini, temel performans göstergelerine, yani amaç fonksiyonu değerine, reddedilen kullanıcı sayısına ve araç / operatör kullanım seviyelerine, göre değerlendirmek için sayısal bir çalışma yapılmıştır.

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1. INTRODUCTION

Based on a recent study by the United Nations, the proportion of the world's population living in urban areas, currently around 55%, is projected to reach 68% by 2050 (United Nations, 2018). A natural consequence of this is the ever-increasing need for urban mobility. However, constantly growing mobility need has serious economic, social, and environmental implications such as higher costs for road infrastructure and maintenance, traffic congestion, and greenhouse gas emissions, which call for adopting more efficient and sustainable ways to move people and goods around. In response to that, shared mobility systems have emerged, and various alternatives are available such as ride-sharing, car-sharing, and electric scooter/bike sharing. Car-sharing systems are perhaps among the most popular and convenient alternatives of shared mobility. Although it has been a phenomenon since the foundation of the very first known car-sharing system in the late 1940s, car-sharing has grown significantly and enjoys a wide acceptance all around the world in the following decades, especially after Zipcar was launched in 2000. According to Shaheen (2020), as of October 2018, more than 198.000 vehicles were available for about 32 million users in 47 countries.

Unlike traditional car rental services, car-sharing is intended for people that are in the need of a vehicle for a short period time to travel relatively smaller distances. In essence, vehicles distributed across a given service region are available for people to rent for short amounts of time, and pay as they go. Car-sharing systems are quite practical and affordable not only for people who do not own a vehicle and need one from time to time, but they also appeal to people who have a private vehicle of their own yet want to avoid parking hassles (car-sharing operators typically have reserved parking spots/stations across their service region).

Existing car-sharing systems are typically station based and they can be classified into two main categories: two-way and one-way car-sharing systems. In two-way systems, vehicles must be returned to the stations where they are picked up from. In one-way systems, however, users are allowed to return the vehicles to any station of their preference.

In recent years, a new car-sharing business model, known as *free-floating*, has been introduced and launched in several countries around the world. Free-floating car-sharing systems have attracted great attention and started to become more and more popular especially with the advances in modern positioning technologies that enable real-time tracking of vehicles. In contrast to one-way and two-way car-sharing systems, free-floating systems do not involve well-defined stations. Instead, vehicles can be located at any point (that is a legal parking spot) within the service region of the car-sharing operator so that users can take and leave vehicles without visiting a station before or after a trip. This makes it an advantageous alternative among others.

In free-floating car-sharing systems, the proximity of available vehicles to users' requested pick-up locations may have a significant impact on rental decisions of the users. Simply put, if a user struggles finding a vehicle that is (or will soon be) available and close to her origin of request, she will likely choose not to use the car-sharing system and seek other alternatives. According to a survey in Becker et al. (2017), 53% of free-floating car-sharing users stated that they would prefer using public transportation if they could not find vehicles close to their origin locations. During the day, certain areas within the service region experience high demand. This may result in the loss of potential user demand. On the other hand, other areas may have many idle vehicles. Since the size of the car-sharing industry exceeds billions of dollars as outlined in the report by Wadhwani and Saha (2020), increasing the utilization level of the resources by managing the system effectively is crucial in order to facilitate vehicle accessibility and provide reliable service. To achieve this, vehicles should be relocated from areas of excess supply to areas of excess demand so that the vehicle distribution across the service region matches the anticipated demand distribution as closely as possible. Two different strategies can be adopted in vehicle relocation: operator-based and user-based relocations. In the former strategy, vehicles are repositioned within the network by dedicated relocation personnel, whereas in the latter, users are engaged in the relocation operations by means of alternative trip suggestions. In return, users receive an incentive (e.g. a discount) that is determined by the manager of the car-sharing system.

In this thesis, we focus on vehicle relocation problems arising in free-floating car-sharing systems. More specifically, we consider two problems that differ based on the adopted relocation strategy, namely the operator-based vehicle relocation problem and and the hybrid (operator and user-based) vehicle relocation problem. We present multi-commodity network flow formulations for both problems developed using properly defined time-space networks. On these networks, vehicle and operator activities can be represented using flow variables in an integrated manner.

The remainder of this thesis is organized as follows: Chapter 2 reviews the related literature. Chapter 3 formally describes the vehicle relocation problems and provides their mathematical models. These models are then utilized to evaluate the impact of varying problem parameters on the operational efficiency of the car-sharing system under the aforementioned relocation strategies as well as to investigate the benefit of involving users in relocation tasks. New sets of test instances have been generated as described in Chapter 4 and used in the computational experiments for which the results are presented in Chapter 4 along with detailed discussions of our findings. Finally, Chapter 5 provides concluding remarks as well as outlining several directions for future research.

2. LITERATURE REVIEW

Vehicle relocation problems arising in different car-sharing systems has been studied in the literature from various perspectives. Majority of the existing studies consider a deterministic problem framework. Boyaci et al. (2015) propose a multi-objective mixed integer programming (MIP) model which solves a (electric) vehicle relocation problem in a one-way car-sharing system with reservations considering charging constraints. Their model also determines the optimal fleet size as well as the number and the locations of the stations. Due to the large number of relocation variables, the proposed model becomes computationally intractable for problem sizes encountered in practice. The authors present an aggregate model to overcome this difficulty, and perform sensitivity analyses to investigate the effect of parameters such as demand, accessibility distance, and subsidies on the performance of the car-sharing system system. Nourinejad et al. (2015) develop two integrated multi-travelling sales person formulations which simultaneously optimize vehicle relocation and staff rebalancing decisions. Their model suffers from a large number of variables and constraints as well. Although they apply a refinement step, which eliminates redundant variables and constraints, the modified model remains incapable of solving larger instances. Therefore, the authors develop and employ a decomposition based method to solve the problem in which vehicle relocation and staff rebalancing decisions are made sequentially.

Weikl and Bogenberger (2015) develop a six step approach to the vehicle (electric and conventional) relocation problem in free-floating car-sharing systems. First, macroscopic zones, each represented as a collection of (adjacent) microscopic hexagons, are determined. In the second step, vehicle distribution for different periods is obtained using historical data. Vehicle relocations among macroscopic zones are identified using a MIP. After that, two rule based models are used to relocate vehicles among the microscopic hexagons within each macroscopic zone. In the last step, service trips of staff are planned. Bruglieri et al. (2018) work on a multi-objective (electric) vehicle relocation problem involving staff routing decisions. They formulate it as a MIP, and propose an exact solution method, which is not able to solve large instances efficiently. Thus, a two phase heuristic algorithm is developed. In the first phase, an initial set of feasible staff routes is constructed using three different techniques. In the second phase, another MIP is solved to select routes from the initial set constructed in the first phase.

Recently, Folkestad et al. (2020) address vehicle relocation in free-floating car-sharing systems. Unlike other studies, their primary goal is to ensure the relocation of electric vehicles that are in need of charging by staff members, who are provided with service vehicles to travel to the locations where they have to pick up the electric vehicles to be repositioned. Assuming that user demands are known, a MIP is presented to find the routes of staff members and service vehicles as well as vehicle-charging station pairs. To overcome the computational burden when solving real life (large) problem instances, the authors devise a metaheuristic algorithm.

All the studies mentioned above investigate the vehicle relocation problem in a deterministic setting. There are also a number of papers that considered vehicle relocation problems from a stochastic point of view. Nair and Miller-Hooks (2011) propose a stochastic MIP under demand uncertainty. The authors aim to identify a relocation plan that ensures the service quality of the system –measured in terms of user satisfaction– is reliable with a probability of at least p. The vehicles are assumed to be relocated before their service period starts. In some cases, the stochastic MIP may have a nonconvex feasible region. The authors propose two techniques to solve the problem effectively. Fan (2014) also take demand uncertainty into account and develop a multi-stage stochastic model to optimize strategic allocation of vehicles in the system. Several assumptions are made when modeling the problem. More specifically, users are assumed to request vehicles on the day before they need a vehicle, and the vehicles are returned a day later. A vehicle trip from one location to another is therefore assumed to take one day. A complete scenario-tree based approach is used to obtain a solution to the problem and a seven-stage experimental network, i.e., seven days and four locations, is designed to test the proposed solution approach.

More recently, in Benjaafar et al. (2017) and He et al. (2019), the vehicle relocation problem is formulated using stochastic dynamic programming. The above-mentioned assumptions of Fan (2014) are not present in either of these studies. Instead, users are allowed to request vehicles at any time and keep the vehicles for as long as they wish. Moreover, the system provides the users with the flexibility to return the vehicles at any point within a predefined service region. Hence, we can say that Benjaafar et al. (2017) and He et al. (2019) consider a free-floating car-sharing system. Benjaafar et al. (2017) represent the problem in a more general framework by allowing multiple service periods and multiple locations for the optimal policy. They employ approximate dynamic programming to solve the problem. He et al. (2019), on the other hand, seek to find solutions through distributionally robust optimization.

Repoux, Kaspi, et al. (2019) investigate the operational decisions in a reservation based one-way car-sharing system. Two dynamic policies for staff based relocations are provided. The performances of these dynamic relocation policies are evaluated by means of solving a MIP formulation in which it is assumed that the system has knowledge over the future demand. Two policies are also compared using the event based simulation framework of Repoux, Boyacı, et al. (2015). Wang et al. (2019) address the problems of (1) finding the number of vehicles to be relocated, (2) the routes of these vehicles, and (3) the routes of the relocation personnel consistent with the vehicle routes. A probabilistic approach is proposed to assess the station-based relocation needs, i.e., to tackle the first problem. Solutions of the second and the third problems are obtained via an integer linear programming (ILP) model. Warrington and Ruchti (2019) develop a network flow formulation for a vehicle relocation problem arising in shared mobility systems such as bike-sharing, one-way car-sharing, and e-scooter sharing systems. Assuming demand uncertainty, the network flow formulation is converted into a two-stage stochastic program.

Various simulation-based models have also been proposed in the literature. Barth and Todd (1999) study the vehicle relocation problem in one-way car-sharing systems and use a simulation model to assess the operational efficiency of the system. The model mainly focuses on identifying user waiting times and the required number of relocations to satisfy user demand. In another study by Repoux, Boyacı, et al. (2015), an event-based simulation model is introduced to explore the performance of their proposed formulations with a focus on determining vehicle relocations based on user demand (in short term) as well as staff assignments according to the relocation needs. The impact of fleet size and staff level, number of spots per station, minimum battery level, and different strategies related to the relocations are explored using efficiency indicators through event-based simulation.

Many researchers have interpreted vehicle relocation and/or staff rebalancing problems by means of flows in a time-space network. Kek et al. (2009) present a MIP to simultaneously optimize vehicle relocations and staff activities including relocation, travelling, and maintenance. A solution to the MIP model is utilized to determine favorable parameters. The performance of these parameters based on different indicators are evaluated in a simulation framework. Diana Jorge et al. (2012) test a MIP model proposed by Almeida Correia and Antunes (2012) using an agent-based simulation approach. The MIP model aims to choose the locations of car-sharing stations from a set of possible sites. The simulation model focuses on observing how the change in user demand and relocation policy affect the one-way car-sharing system when different scenarios related to the number of stations are considered. D. Jorge et al. (2014) study a slightly different version of the same MIP formulation to decide on the vehicle relocations given the locations of the stations. Multiple relocation policies are tested with a simulation model similar to Diana Jorge et al. (2012). For the vehicle relocation problem, the mathematical model is used to obtain upper bounds whereas the simulation model is employed towards achieving more realistic results.

Krumke et al. (2014) consider a setting with semi-autonomous vehicles, which can be relocated in convoys, and thus, modeled the vehicle relocation problem as a pickup and delivery problem on a time-space network. In particular, they propose a min-cost flow formulation and a max-profit flow formulation, where the former aims to find a least cost set of routes for vehicle convoys while satisfying all the demand, and the latter aims to find a most profitable set of routes for vehicle convoys with a limited relocation budget. Santos and Correia (2015) extend the optimization model proposed by Kek et al. (2009) in a way to allow staff to move with the same vehicle, which is known as trip joining, or to move with an alternative transportation option. Although they are able to find solutions to small problem instances, their model does not scale well to solve large instances. Another time-space network based model is introduced in Carlier et al. (2015), which is an integer program to maximize the satisfied demand in a one-way car sharing system with a limited number of vehicles and a limited number of relocation operations. The authors carry out computational experiments and show that their model can handle randomly generated instances of realistic size.

Ait-Ouahmed et al. (2017) focus on a vehicle relocation and staff repositioning problem with electric vehicles, and introduce a MIP formulation with an integrated objective function and recharging constraints. In order to solve larger instances, the problem is decomposed into two subproblems by considering vehicle routing and staff routing aspects individually. First, a greedy heuristic algorithm is used to construct a solution to the vehicle routing problem, which provides information on the relocation operations to be performed by the staff members. Then, a routing plan for staff is obtained by another greedy heuristic. The authors also propose a tabu search algorithm to obtain better results with respect to objective function value and efficiency. Both algorithms are tested via simulation. For the same problem, Xu et al. (2018) formulate a nonlinear and nonconvex MIP, and derive an equivalent –with respect to the optimal solution– convex program. By assuming elastic demand, vehicle and staff activities, fleet size, and trip pricing are jointly determined. Possible extensions of the model are also studied such as assigning a capacity to each station, and designating the locations of the stations. The authors also suggest that if the service region is categorized properly as suggested in Weikl and Bogenberger (2015), their model is applicable to free-floating car-sharing systems as well.

More recently in Zhao et al. (2018), a MIP formulation is proposed which allows (electric) vehicle and staff activities to be observed dynamically by representing them through different sets of time-space paths. Battery capacity and charging time are regarded among the constraints of the problem. The authors develop a solution approach based on Lagrangian relaxation combined with forward dynamic programming, branch-and-bound, and greedy algorithm. Another recent study by Gambella et al. (2018) addresses the (electric) vehicle relocation problem in a station-based one-way car-sharing system using time-space networks. Initial distributions of vehicles and staff are obtained by a MIP model that maximizes the profit from the satisfied user demand. The overnight relocation and staff activities are scheduled by another MIP model taking into account the solution of the first model. Both exact and heuristic approaches to tackle large-scale instances are described and sensitivity to relocation cost and battery capacity of the electric vehicles are evaluated. Boyacı and Zografos (2019) introduce spatial and/or temporal flexibility of user demand in one-way (electric) car-sharing systems. They consider several operational decisions such as demand acceptance or rejection, vehicle relocation, staff assignment to vehicle relocation operations, and vehicle assignment to user demand. Vehicle assignment decisions are made by a simulation model for the sake of finding solutions quickly. A joint optimization model for staff and vehicles is adopted in case the simulation model produces unsatisfactory solutions.

There are also studies that incorporate user-based vehicle relocation strategies within their framework. Febbraro et al. (2012) model a discrete event scheme which allows users drop off vehicles either at a location of their choice or at a location suggested based on the solution of a MIP. They observe that the number of vehicles and users' acceptance of suggested locations have a significant impact on the demand rejection rate. In a recent study by Febbraro et al. (2019), the discrete event scheme proposed in Febbraro et al. (2012) is enhanced by introducing relocations by staff members as well. Whenever a user rejects an offered drop-off location, a staff member relocates the vehicle to this location. They also add another step to their approach which seeks to optimize the discounts associated with user-based relocations.

As seen above, a number of studies integrating vehicle and operator routing aspects in vehicle relocation problems exists in the literature such as Ait-Ouahmed et al.

(2017), Gambella et al. (2018), Zhao et al. (2018), Xu et al. (2018). Nevertheless, all these studies assume that user requests should be satisfied as soon as they are received. We consider a problem setting where user requests can be covered within a prespecified time frame defined by introducing waiting times that users can tolerate. In terms of operator-based vehicle relocations, this distinguishes our models from the models in other available studies. According to Niels and Bogenberger (2017), in free-floating car-sharing systems, a user is satisfied if there is an available vehicle within 300 to 500 meters of the user's location. In other words, the user may not be able to find a vehicle exactly at her demand point, and may need to walk in order to pick up a nearby vehicle. Another scenario could be that a user finds an available vehicle upon waiting for a certain (and an acceptable) amount of time. Therefore, setting deadlines for meeting user demands has the potential be quite useful for car-sharing systems in practice.

In terms of the adoption of a hybrid relocation strategy, Boyacı and Zografos (2019) is the closest study to ours. They focus on a one-way station-based electric car-sharing system, and investigate the effect of spatial and temporal flexibility of users on the system performance. Their modeling approach is similar to ours in the sense that vehicle and operator movements are formulated on parallel time-space networks using flow variables, and user flexibility is incorporated into the modeling framework through incentives in the form of price discounts. The main difference of our study is in the definition of alternative trips that can be suggested to users and in the incentive scheme employed. Alternative pick-up/drop-off locations that can be suggested to a user should be within a predefined range of the user's origin/destination stations in their case. Same applies to the offered and requested pick-up times. Our alternative trip suggestions, on the other hand, are compiled by jointly considering spatial and temporal aspects of user flexibility without compromising acceptability, that is, we restrict our attention only to what we refer to as *appealing suggestions* for each user. In particular, we assume that a pair of pick-up/drop-off locations can be offered to a user if the total trip duration of the user does not increase (compared to her planned trip) and the user does not have to travel longer than a prespecified threshold between her origin and the suggested pick-up location, and between the suggested drop-off location and her destination in total. Moreover, the earliest possible pick-up time is the user's request arrival time in our problem setting unlike Boyacı and Zografos (2019), who allow picking up vehicles earlier than requested. Our model provides a pick-up time suggestion based on the request arrival time, the total time it takes for the user to travel from her origin to the suggested pick-up location and from the suggested drop-off location to her destination.

In Boyacı and Zografos (2019), the user is given a discount on a per kilometer basis for pick-up/drop-off stations that are different from the requested origin/destination stations, and on an hourly basis for changing the requested pick-up time. We deploy a simpler incentive scheme where users are provided with a fixed percentage discount on their original rental prices depending on whether they are offered pick-up/drop-off locations other than their origin/destination locations or not. We also evaluate a variable percentage discount scenario in our experiments. Finally, Boyacı and Zografos (2019) assume that travel times and driving distances of the users remain constant, whereas in our case, they depend on the suggested pick-up/drop-off locations.

For a comprehensive review of the literature on vehicle relocation problems and solution approaches, we refer the reader to a recent survey by Illgen and Höck (2019). We classify the articles cited above with respect to the type of car-sharing system considered, vehicle type, relocation strategy, focus, and solution approach. The classification of the cited articles can be found in Tables 2.1 and 2.2.

Articles	CS type	Vehicle type	Relocation type	Focus	Solution Approach
Barth and Todd (1999)	OW	E	Platoon	Measures of effectiveness, FS, no. of stations	Simulation
Kek et al. (2009)	OW	C	OB	VR, SR and activities, operating parameters	MIP on a time-space network, heuristic, simulation
Nair and Miller-Hooks (2011)	OW	C	OB	VR	Stochastic MIP, simulation
Almeida Correia and Antunes (2012)	OW	C	-	no., size, and location of depots	MIP
Febbraro et al. (2012)	OW	C	UB	VR, demand acceptance or rejection	IP, simulation
Diana Jorge et al. (2012)	OW	C	OB	Station locations, vehicle distribution	MIP on a time-space network, agent based simulation
D. Jorge et al. (2014)	OW	C	OB	VR	MIP on a time-space network, simulation
Fan (2014)	OW	C	OB	VR	Multi-stage stochastic LP
Krumke et al. (2014)	OW	C	Vehicle platoon	VR, convoy routing	ILP on a time-space network
Repoux, Boyacı, et al. (2015)	OW	E	OB	VR, SR	Event-based simulation
Boyaci et al. (2015)	OW	E	OB	VR, FS, number of stations	Multi-objective MIP
Nourinejad et al. (2015)	OW	C	OB	VR, SR	MIP, decomposition
Weikl and Bogenberger (2015)	FF	C, E	OB	VR, SR, FS, zone categorization	MIP
Santos and Correia (2015)	OW	C	OB	VR, SR	MIP on a time-space network
Carlier et al. (2015)	OW	C	OB	VR	ILP on a time-space network

Table 2.1 Classification of cited articles

Articles	Car-sharing type	Vehicle type	Relocation type	Focus	Solution Approach
Ait-Ouahmed et al. (2017)	OW	E	OB	VR, SR	MIP on a time-space network, greedy and bi-level tabu search algorithms
Benjaafar et al. (2017)	OW or FF	C	OB	VR	Markov decision process
Bruglieri et al. (2018)	OW	E	OB	VR, SR	Multiple objective MIP, heuristics
Xu et al. (2018)	OW	E	OB	VR, SR, FS, trip pricing	MIP
Zhao et al. (2018)	OW	E	OB	VR, SR	MIP on a time-space network, Lagrangian relaxation
Gambella et al. (2018)	OW	E	OB	VR, SR	MIP on a time-space network, heuristics
He et al. (2019)	FF	C	OB	VR	Stochastic dynamic program
Wang et al. (2019)	OW	E	OB	VR, SR	MIP, simulation
Warrington and Ruchti (2019)	OW	-	OB	VR	LP, two-stage stochastic programming
Repoux, Kaspi, et al. (2019)	OW	C	OB	VR, SR	Dynamic relocation algorithms, MIP, simulation
Boyacı and Zografos (2019)	OW	E	UB, OB	Demand acceptance or rejection, VR, staff and vehicle assignment	MIP, simulation
Febbraro et al. (2019)	OW	C	UB, OB	VR, demand acceptance or rejection, optimal discount	IP, Simulation
Folkestad et al. (2020)	FF	E	OB	VR and vehicle assignment to charging stations, SR and service vehicles routing	MIP, metaheuristic

Abrreviations: OW: One-way, FF: Free-floating, E: Electric, C: Convenitonal, OB: Operator-based
UB: User-based, FS: Fleet size, VR: Vehicle relocation, SR: Staff routing

Table 2.2 Classification of cited articles

3. PROBLEM DESCRIPTION and FORMULATIONS

The car-sharing systems that we consider in this thesis consist of three types of entities: vehicles, operators (dedicated personnel of the company providing the car-sharing service), and users. Due to spatial and temporal variations in demand, vehicle distribution across the service region does not always closely match the demand distribution, thereby, necessitating (some) vehicles to be re-positioned to different locations. In this chapter, we formally define two vehicle relocation problem variants within the context of free-floating car-sharing systems, i.e., the operator-based vehicle relocation problem (VR-O) and the operator and user-based (hybrid) vehicle relocation problem (VR-H). For both problem variants, we develop multi-commodity network flow formulations in which the vehicle, operator, and user movements are modeled on a time-space network. Our primary goal is not to develop a solution methodology, but rather to investigate the operational efficiency of a free-floating car-sharing system in its simplest form although our formulations can serve as bases for an effective solution algorithm.

In free-floating car-sharing systems, the service region does not involve well-defined stations at which the vehicles should be picked-up or returned. In essence, users are free to drop-off the cars at any location within the service region that qualifies as a legitimate parking spot. Moreover, a user does not have to make a reservation in advance, she can just make a pick-up request whenever she needs a car. At the time of her request, she does not have to reveal her destination, nor the return time of the vehicle at that destination. This highly dynamic and stochastic nature of free-floating car-sharing systems makes it quite challenging to model and solve the vehicle relocation problem efficiently. However, companies providing this type of car-sharing service collect plenty of data at the user level by continuously tracking their vehicles. Based on this data, they can make predictions about the timing and the origin-destination locations of future requests. Furthermore, users do not mind providing this information if they are asked to do so. According to the results of a survey in Herrmann et al. (2014), around 89 % of the respondents are willing to provide the information regarding their destinations before they book a vehicle.

With this motivation, we study the VR-O and the VR-H in a deterministic setting where origin-destination locations and the request arrival time are assumed to be known for each user. We present the detailed descriptions of the VR-O and the VR-H along with our assumptions, and provide our mathematical formulations in the sequel.

3.1 The operator-based vehicle relocation problem (VR-O)

In the operator based vehicle relocation problem, re-positioning tasks are performed only by the operators. We represent this problem on a time-space network obtained by discretizing a finite planning horizon into T periods. We assume that user requests within this planning horizon, characterized by an origin-destination pair as well as a request arrival time for each user, are known beforehand. Moreover, since a user would not be willing to wait (or walk) for an extended period of time to pick up a car, we construct a deadline for every user by adding a constant slack (specified in terms of number of periods) to the user's request arrival time. If a vehicle cannot be supplied to the user before this deadline, we consider the user's request to be rejected. Otherwise, we assume that the user picks up a vehicle and drives directly to her destination. Given the user requests, and the initial locations of the vehicles and the operators across the service region, the VR-O aims to identify a set of vehicle and operator paths to provide service to a set of user requests in a way to strike a balance between minimizing relocation related costs and maximizing revenue from the satisfied requests. We use the following notation to formulate the VR-O.

Sets and Parameters

- T: number of time-steps/periods in the planning horizon (depends on the time discretization scheme)
- $N = \{1, .., n\}$: locations of interest in the service region (initial vehicle/operator locations, users' origin-destination locations)
- $A = \{(i,j) : i,j \in N, i \neq j\}$: set of links connecting the locations in N
- t_{ij} : travel time associated with arc $(i,j) \in A$ (in terms of number of time-steps)

- V: set of vehicles
- U: set of users
- O: set of operators
- $\mathcal{G} = (\mathcal{N}, \mathcal{A})$: a time-expanded network with $\mathcal{N} = \{(i,t) : i \in N, t \in \{0,\ldots,T\}\} \cup \{(0,0),(n+1,T),(r^*,T)\}$ and $\mathcal{A} = \mathcal{A_T} \cup \mathcal{A_R} \cup \mathcal{A_W} \cup \mathcal{A_D} \cup \mathcal{A_S}$, where
  - $(0,0)$: artificial source node
  - $(n+1,T)$: artificial sink node
  - $\mathcal{A_T}$: set of travelling arcs, i.e., arcs of the form $((i,t),(j,t+t_{ij}))$ for $(i,j) \in A$ and $0 \le t \le T - t_{ij}$
  - $\mathcal{A_R}$: set of relocation arcs, same as the set of travelling arcs, but defined separately in order to differentiate between the two cases where a vehicle is being relocated by an operator, and where it is picked up by a user and moved to a different location
  - $\mathcal{A_W}$: set of waiting arcs, i.e., arcs of the form $((i,t),(i,t+1))$ for $(i,t),(i,t+1) \in \mathcal{N}$
  - $\mathcal{A_D}$: set of dummy arcs, flow over which represents rejection of users
  - $\mathcal{A_S}$: set of artificial arcs connecting the artificial source node $(0,0)$ to the nodes $(i,0)$, and the nodes (i,T) to the artificial sink node $(n+1,T)$, for $i \in \mathcal{N}$.
- c_a: the cost associated with an operator using the arc $a \in \mathcal{A_T} \cup \mathcal{A_R}$ (proportional to the travel time t_{ij} where i and j are the source and the target locations of the arc a)
- p_u: profit obtained if user u is served; penalty incurred if the user is rejected (proportional to the travel time t_{ij} where i and j are the origin and the destination of user u)
- o_u, d_u: origin and destination locations of user u
- $[e_u, l_u]$: time window associated with user u (e_u is the time index at which user u makes a rental request, and $l_u = e_u + t_{o_u,d_u} + s_u$, where s_u is the number of time steps that the user is willing to wait)
- $\mathcal{A}^u = \mathcal{A}^u_\mathcal{T} \cup \mathcal{A}^u_\mathcal{W} \cup \mathcal{A}^u_\mathcal{D}$, where $\mathcal{A}^u_\mathcal{T}, \mathcal{A}^u_\mathcal{W}, \mathcal{A}^u_\mathcal{D}$ are the sets of travelling, waiting, and dummy arcs that can be used by user u, respectively, i.e.,

- $\mathcal{A}^u_{\mathcal{T}} = \{((o_u,t),(d_u,t+t_{o_u,d_u})) : t \in \{e_u,\ldots,e_u+s_u\}\}$
- $\mathcal{A}^u_{\mathcal{W}} = \{((o_u,t),(o_u,t+1)) : t \in \{e_u,\ldots,e_u+s_u-1\}\} \cup \{((d_u,t),(d_u,t+1)) : t \in \{e_u+t_{o_u,d_u},\ldots,l_u-1\}\}$
- $\mathcal{A}^u_{\mathcal{D}} = \{((o_u,e_u+s_u),(r^*,T))\}$, where (r^*,T) is a dummy node which absorbs the rejected user flow.

- O_i: number of operators initially available at location $i \in N$
- V_i: number of vehicles initially available at location $i \in N$

Decision Variables

$$x^v_a = \begin{cases} 1 & \text{if vehicle } v \text{ uses arc } a \\ 0 & \text{otherwise} \end{cases} \qquad \text{for } a \in \mathcal{A}, v \in V$$

$$y^u_a = \begin{cases} 1 & \text{if user } u \text{ uses arc } a \\ 0 & \text{otherwise} \end{cases} \qquad \text{for } a \in \mathcal{A},\ u \in U$$

$$z^o_a = \begin{cases} 1 & \text{if operator } o \text{ uses an arc } a \\ 0 & \text{otherwise} \end{cases} \qquad \text{for } a \in \mathcal{A},\ o \in O$$

As indicated above, the time-space network contains $T+1$ timed copies of each location in the set N as well as the artificial source and sink nodes. Figure 3.1 depicts an example time-space network, which is defined by three locations and $T=3$ periods. All vehicle and operator paths originate from the (artificial) source node and end at the (artificial) sink node using (artificial) black arcs. The travel time between locations 1 and 2 is two time steps, whereas it takes one time step and three time steps to travel between locations 2 and 3, and between locations 1 and 3, respectively. A unit flow per each rejected user request is sent to the dummy node through the dummy arcs shown in blue. To make the example network simple, we only show dummy arcs from the last timed copy of each location. However, in our original network, we define dummy arcs from multiple timed copies of the origin location of each user to the dummy node after reaching the maximum waiting time. In order to represent activities of vehicles, operators, and users, different sets of arcs (waiting ($\rightarrow$), relocation ($\rightarrow$), and travelling ($\dashrightarrow$) arcs) are defined.

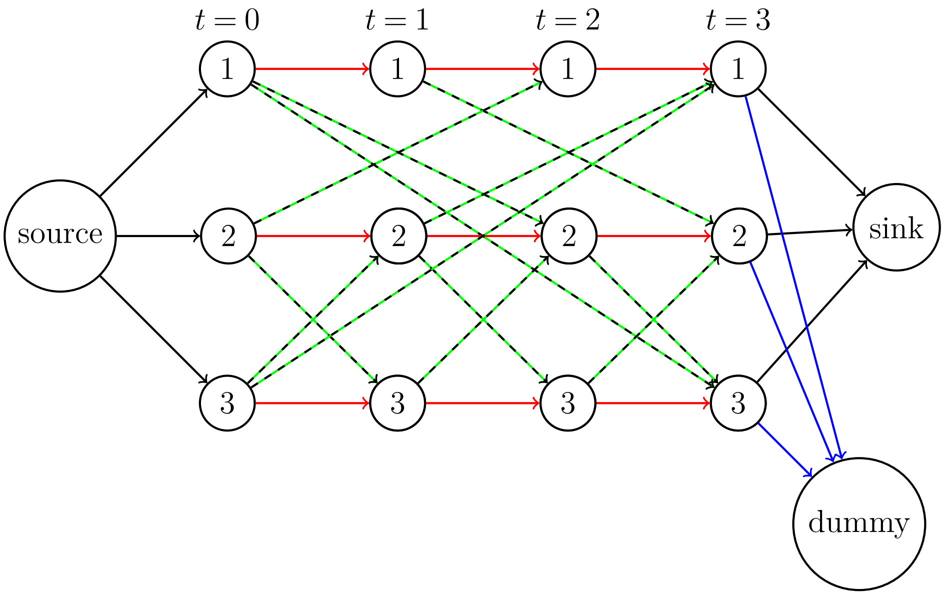


Figure 3.1 An example of a time-space network

For a given node $(i,t) \in \mathcal{N}$, we use $\delta^-(i,t)$ to denote the set of all incoming arcs of (i,t) and $\delta^+(i,t)$ to denote the set of all outgoing arcs of (i,t). Finally, for a vector $\alpha \in \mathbb{R}^{|S|}$ and $S' \subseteq S$, we let $\alpha(S') = \sum_{s \in S'} \alpha_s$. The VR-O can then be formulated as follows.

$$\begin{aligned}
\mathbf{min} & \sum_{a \in \mathcal{A}_{\mathcal{R}}} \sum_{v \in V} c_a x_a^v + \sum_{a \in \mathcal{A}_{\mathcal{T}}} \sum_{o \in O} c_a z_a^o \\
& - \sum_{a \in \mathcal{A}_{\mathcal{T}}} \sum_{u \in U} p_u y_a^u + \sum_{a \in \mathcal{A}_{\mathcal{D}}} \sum_{u \in U} p_u y_a^u && && (3.1) \\
\mathbf{s.t.} & x^v(\delta^+(0,0)) = 1 && v \in V && (3.2) \\
& \sum_{v \in V} x^v(\delta^-(i,0)) = V_i && i \in N && (3.3) \\
& x^v(\delta^+(i,t)) - x^v(\delta^-(i,t)) = 0 && i \in N, t \in \{0,\ldots,T\}, v \in V && (3.4) \\
& y^u(\delta^+(o_u,e_u)) = 1 && u \in U && (3.5) \\
& y^u(\delta^-(d_u,l_u)) + y^u(\mathcal{A}_{\mathcal{D}}^u) = 1 && u \in U && (3.6) \\
& y^u(\delta^+(o_u,t)) - y^u(\delta^-(o_u,t)) = 0 && t = e_u+1,\ldots,e_u+s_u, u \in U && (3.7) \\
& y^u(\delta^+(d_u,t)) - y^u(\delta^-(d_u,t)) = 0 && t = e_u+t_{o_u,d_u},\ldots,l_u-1, u \in U && (3.8) \\
& z^o(\delta^+(0,0)) = 1 && o \in O && (3.9) \\
& \sum_{o \in O} z^o(\delta^-(i,0)) = O_i && i \in N && (3.10) \\
& z^o(\delta^+(i,t)) - z^o(\delta^-(i,t)) = 0 && i \in N, t \in \{0,\ldots,T\}, o \in O && (3.11) \\
& \sum_{o \in O} z_a^o = \sum_{v \in V} x_a^v && a \in \mathcal{A}_{\mathcal{R}} && (3.12)
\end{aligned}$$

$$\sum_{u \in U} y_a^u = \sum_{v \in V} x_a^v \qquad a \in \mathcal{A}_{\mathcal{T}} \tag{3.13}$$

$$\sum_{a \in \mathcal{A}_{\mathcal{D}}} \Big(\sum_{v \in V} x_a^v + \sum_{o \in O} z_a^o\Big) = 0 \tag{3.14}$$

$$\sum_{a \in \mathcal{A} \setminus \mathcal{A}^u} y_a^u = 0 \qquad u \in U \tag{3.15}$$

$$x_a^v \in \{0,1\} \qquad a \in \mathcal{A}, v \in V \tag{3.16}$$

$$y_a^u \in \{0,1\} \qquad a \in \mathcal{A}, u \in U \tag{3.17}$$

$$z_a^o \in \{0,1\} \qquad a \in \mathcal{A}, o \in O \tag{3.18}$$

The objective function (3.1) minimizes the relocation cost (incurred by fuel cost of the relocated vehicles plus the travelling cost of operators) and the rejection penalty (incurred by rejected requests) minus the reward gained from the satisfied requests. Constraints (3.2) ensure that one unit of flow emanates from the artificial source node for each vehicle. The vehicles are distributed to their initial locations in the time-space network through the constraints (3.3). The flow conservation for the vehicles is assured with the constraints (3.4). Constraints (3.5) ensure that the unit flow associated with a given user u emanates from location o_u at time e_u, and is eventually absorbed either by the node (d_u, l_u) (if the user is served), or by the dummy node (if the user is rejected) due to constraints (3.6). For every user, flow balance equations at the user's origin and destination locations are given by (3.7) and (3.8), respectively. Operator flow constraints are imposed by the equations (3.9)–(3.11), which are analogous to the vehicle flow constraints in (3.2)–(3.4). Constraints (3.12) guarantee that the number of vehicles is equal to the number of operators on each relocation arc since the vehicles cannot relocate themselves. Similarly, the number of vehicles should be equal to the number of users on each travelling arc as indicated by the constraints (3.13). As the dummy arcs can only carry (rejected) user flow by definition, constraint (3.14) sets the total vehicle and operator flow on these arcs equal to zero. Moreover, a given user u cannot have positive flow on any arc that does not belong to the set $\mathcal{A}^u$ due to constraints (3.15). Finally, (3.16)–(3.18) specify the domain restrictions for the variables.

Solving the formulation (3.1)–(3.18) produces a path in the time-space network per each vehicle, operator, and user. However, we assume that all vehicles and operators are identical except possibly for their initial locations. Even considering different initial locations, only the number of vehicles/operators available at a particular location matters, not the individual vehicles/operators present at that location. Hence, the VR-O can also be modeled and solved using aggregate vehicle and operator flows, which leads to a formulation with a fewer variables and constraints. If one needs to identify the paths corresponding to each vehicle and operator separately, the solution produced by the aggregate formulation can easily be decomposed into individual vehicle and operator paths by solving (3.2)–(3.18) with additional constraints specifying the total vehicle and operator flow values.

Letting $x_a = \sum_{v\in V} x_a^v$ and $z_a = \sum_{o\in O} z_a^o$, we obtain the following integer programming formulation for the VR-O:

$$
\begin{aligned}
\mathbf{min} & \sum_{a\in\mathcal{A}_\mathcal{R}} c_a x_a + \sum_{a\in\mathcal{A}_\mathcal{T}} c_a z_a && \\
& - \sum_{a\in\mathcal{A}_\mathcal{T}} \sum_{u\in U} p_u y_a^u + \sum_{a\in\mathcal{A}_\mathcal{D}} \sum_{u\in U} p_u y_a^u && (3.19)\\
\mathbf{s.t.} & x(\delta^+(0,0)) = |V| && (3.20)\\
& x(\delta^-(i,0)) = V_i & i\in N \quad & (3.21)\\
& x(\delta^+(i,t)) - x(\delta^-(i,t)) = 0 & i\in N, t\in\{0,\dots,T\} \quad & (3.22)\\
& y^u(\delta^+(o_u,e_u)) = 1 & u\in U \quad & (3.23)\\
& y^u(\delta^-(d_u,l_u)) + y^u(\mathcal{A}_\mathcal{D}^u) = 1 & u\in U \quad & (3.24)\\
& y^u(\delta^+(o_u,t)) - y^u(\delta^-(o_u,t)) = 0 & t = e_u+1,\dots,e_u+s_u, u\in U \quad & (3.25)\\
& y^u(\delta^+(d_u,t)) - y^u(\delta^-(d_u,t)) = 0 & t = e_u+t_{o_u,d_u},\dots,l_u-1, u\in U \quad & (3.26)\\
& z(\delta^+(0,0)) = |O| && (3.27)\\
& z(\delta^-(i,0)) = O_i & i\in N \quad & (3.28)\\
& z(\delta^+(i,t)) - z(\delta^-(i,t)) = 0 & i\in N, t\in\{0,\dots,T\} \quad & (3.29)\\
& z_a = x_a & a\in\mathcal{A}_\mathcal{R} \quad & (3.30)\\
& \sum_{u\in U} y_a^u = x_a & a\in\mathcal{A}_\mathcal{T} \quad & (3.31)\\
& \sum_{a\in\mathcal{A}_\mathcal{D}} (x_a + z_a) = 0 && (3.32)\\
& \sum_{a\in\mathcal{A}\setminus\mathcal{A}^u} y_a^u = 0 & u\in U \quad & (3.33)\\
& x_a \in \mathbb{Z}_+ & a\in\mathcal{A} \quad & (3.34)\\
& y_a^u \in \{0,1\} & a\in\mathcal{A}, u\in U \quad & (3.35)\\
& z_a \in \mathbb{Z}_+ & a\in\mathcal{A} \quad & (3.36)
\end{aligned}
$$

3.2 The hybrid vehicle relocation problem (VR-H)

In the hybrid vehicle relocation problem, re-positioning tasks can be performed by both the operators and the users. Suppose that we have the option to provide the users with some discount in order to incentivize them to relocate the vehicles they rent. We consider the following two options, which can be offered separately or simultaneously:

- Offer an alternative pick-up location with 50% discount on the original rental price
- Offer an alternative drop-off location with 50% discount on the original rental price

The percentage discount can be altered according to the choice of the car-sharing provider. For a given user, we restrict our attention only to *appealing suggestions*, i.e., to the alternative pick-up/drop-off locations which are within a reasonable distance of the actual origin/destination of the user (offered locations must be reasonably close so that the user is willing to walk to/from those locations), and which do not increase the total trip duration of the user. Therefore, we assume that a user will accept the incentive (if offered any), and benefit from a discounted price. In particular, when only one of the options is offered, the user receives a 50% discount whereas when both options are offered, the user gets to rent a vehicle for free. Note that, there may, and in most cases will, be multiple alternatives that can be offered to a user. Due to our assumption that the users are willing to accept any alternative (appealing) suggestion, and the fact that every incentive incurs a cost (reduction in profit) for the rental company, it is important to determine which users should be provided with an alternative trip suggestion, and which pick-up/drop-off location(s) should be offered to those users as well as the timing of the suggested trip (defined by the pick-up time of the vehicle).

To model user-based relocations on our time-expanded network, we introduce the following (additional) notation and extend some of our existing definitions:

- τ_{ij}: time it takes to walk from location i to location j (in terms of number of time-steps)
- a_{ij}^u: binary parameter which indicates whether a trip from location i to location

j is acceptable for user u or not. Mathematically, we let

$$a^u_{ij} = \begin{cases} 1 & \text{if } e_u + \tau_{o_u,i} + t_{ij} + \tau_{j,d_u} \leq l_u \\ 0 & \text{otherwise} \end{cases} \qquad i,j \in N, u \in U$$

- $S_u = \{(i,j) : a^u_{ij} = 1, \tau_{o_u,i} + \tau_{j,d_u} \leq s_u\} \cup \{(o_u, r^*)\}$: set of pick-up/drop-off location pairs that can be offered to user u. Note that the pairs (i,j) with $i = o_u$ and/or $j = d_u$ are included in S_u, and the pair (o_u, r^*) is added to S_u to model the case where the user is rejected service.
- $P_u = i \in N : (i,j) \in S_u$: set of possible pick-up locations for user u
- $D_u = i \in N : (j,i) \in S_u, i \neq r^*$: set of possible drop-off locations for user u
- $lpt^i_u = l_u - \min_{j:(i,j)\in S_u}\{t_{ij} + \tau_{j,d_u}\}$: latest possible pick-up time from location i considering all possible drop-off locations for user u
- $edt^j_u = e_u + \min_{i:(i,j)\in S_u, j\neq r^*}\{\tau_{o_u,i} + t_{ij}\}$: earliest possible drop-off time at location j considering all possible pick-up locations for user u
- p^u_{ij}: profit obtained when the user is offered $(i,j) \in S_u$ as the pair of pick-up and drop-off locations, in particular:

$$p^u_{ij} = \begin{cases} p_u & \text{if } (i,j) = (o_u, d_u) \\ -p_u & \text{if } (i,j) = (o_u, r^*) \\ 0 & \text{if } i \neq o_u \text{ and } j \neq d_u \\ 0.5 * p_u & \text{otherwise} \end{cases} \qquad \text{for } u \in U, (i,j) \in S_u$$

- $\mathcal{A}^u = \mathcal{A}^u_{\mathcal{T}} \cup \mathcal{A}^u_{\mathcal{W}} \cup \mathcal{A}^u_{\mathcal{D}}$, where $\mathcal{A}^u_{\mathcal{T}}, \mathcal{A}^u_{\mathcal{W}}, \mathcal{A}^u_{\mathcal{D}}$ are the sets of traveling, waiting, and dummy arcs that can be used by user u, respectively, i.e.,
  - $\mathcal{A}^u_{\mathcal{T}} = \{((i,t),(j,t+t_{ij})) : (i,j) \in S_u, j \neq r^*, t \in \{e_u + \tau_{o_u,i}, \ldots, l_u - \tau_{j,d_u} - t_{ij}\}\}$
  - $\mathcal{A}^u_{\mathcal{W}} = \{((i,t),(i,t+1)) : i \in P_u, t \in \{e_u + \tau_{o_u,i}, \ldots, lpt^i_u - 1\}\} \cup \{((j,t),(j,t+1)) : j \in D_u, t \in \{edt^j_u, \ldots, l_u - \tau_{j,d_u} - 1\}\}$
  - $\mathcal{A}^u_{\mathcal{D}} = \{((o_u, e_u + s_u),(r^*, T))\}$, where (r^*, T) is a dummy node which absorbs the rejected user flow.

Decision Variables

$$x_a = \text{the number of vehicles on arc } a \qquad \text{for } a \in \mathcal{A}$$

$$z_a = \text{the number of operators on arc } a \qquad \text{for } a \in \mathcal{A}$$

$$y_a^u = \begin{cases} 1 & \text{if user } u \text{ uses arc } a \\ 0 & \text{otherwise} \end{cases} \qquad \text{for } a \in \mathcal{A},\ u \in U$$

$$w_{ij}^u = \begin{cases} 1 & \text{if pair } (i,j) \text{ is selected for user } u \\ 0 & \text{otherwise} \end{cases} \qquad \text{for } u \in\ U,\ (i,j) \in S_u$$

Below we present the hybrid vehicle relocation model, where users –in addition to the operators– are also employed in vehicle relocation operations through fare discounts.

$$\mathbf{min} \sum_{a \in \mathcal{A}_\mathcal{R}} c_a x_a + \sum_{a \in \mathcal{A}_\mathcal{T}} c_a z_a - \sum_{u \in U} \sum_{(i,j) \in S_u} p_{ij}^u w_{ij}^u \tag{3.37}$$

$$\mathbf{s.t.} x(\delta^+(0,0)) = |V| \tag{3.38}$$

$$x(\delta^-(i,0)) = V_i \qquad i \in N \tag{3.39}$$

$$x(\delta^+(i,t)) - x(\delta^-(i,t)) = 0 \qquad i \in N, t \in \{0,\dots,T\} \tag{3.40}$$

$$y^u(\delta^+(i, e_u + \tau_{o_u,i})) = \sum_{j:(i,j) \in S_u} w_{ij}^u \qquad i \in P_u, u \in U \tag{3.41}$$

$$y^u(\delta^-(j, l_u - \tau_{j,d_u})) = \sum_{i:(i,j) \in S_u} w_{ij}^u \qquad j \in D_u, u \in U \tag{3.42}$$

$$y^u(\mathcal{A}_\mathcal{D}^u) = w_{o_u,r^*}^u \qquad u \in U \tag{3.43}$$

$$y^u(\delta^+(i,t)) - y^u(\delta^-(i,t)) = 0 \qquad \begin{matrix} t = e_u + \tau_{o_u,i} + 1, \dots, lpt_i^u \\ \text{i} \in P_u, u \in U \end{matrix} \tag{3.44}$$

$$y^u(\delta^+(j,t)) - y^u(\delta^-(j,t)) = 0 \qquad \begin{matrix} t = edt_u^j, \dots, l_u - \tau_{j,d_u} - 1 \\ \text{j} \in D_u, u \in U \end{matrix} \tag{3.45}$$

$$\sum_{(i,j) \in S_u} w_{ij}^u = 1 \qquad u \in U \tag{3.46}$$

$$z(\delta^+(0,0)) = |O| \tag{3.47}$$

$$z(\delta^-(i,0)) = O_i \qquad i \in N \tag{3.48}$$

$$z(\delta^+(i,t)) - z(\delta^-(i,t)) = 0 \qquad i \in N, t \in \{0,\dots,T\} \tag{3.49}$$

$$z_a = x_a \qquad a \in \mathcal{A}_\mathcal{R} \tag{3.50}$$

$$\sum_{u \in U} y_a^u = x_a \qquad a \in \mathcal{A}_\mathcal{T} \tag{3.51}$$

$$\sum_{a \in \mathcal{A}_\mathcal{D}} (x_a + z_a) = 0 \tag{3.52}$$

$$\sum_{a \in \mathcal{A} \setminus \mathcal{A}^u} y_a^u = 0 \qquad u \in U \tag{3.53}$$

$$\sum_{a \in \mathcal{A}_\mathcal{T}^u} y_a^u \leq 1 \qquad u \in U \tag{3.54}$$

$$x_a \in \mathbb{Z}_+ \qquad a \in \mathcal{A} \tag{3.55}$$

$$y_a^u \in \{0,1\} \qquad a \in \mathcal{A}, u \in U \tag{3.56}$$

$$z_a \in \mathbb{Z}_+ \qquad a \in \mathcal{A} \tag{3.57}$$

$$w_{ij}^u \in \{0,1\} \qquad u \in U, (i,j) \in S_u \tag{3.58}$$

The objective function (3.37) aims to minimize the cost incurred (1) by operators traveling and relocating vehicles, and (2) by discounts offered to users. Constraints (3.38) guarantee that all vehicle flow originates from the artificial source node. The initial vehicle distribution across the network is defined by the constraints (3.39). Vehicle flow conservation constraints are given by (3.40). Constraints (3.47)–(3.49) serve the same purpose for operators. Constraints (3.41) ensure that for each user, the corresponding path in the time-space network starts from the node defined by the pick-up location offered to the user and the earliest time at which the user is ready to pick up a vehicle at that location –considering the walking time if the suggested pick-up location is different from the user's origin. Due to (3.42), the user's path in the time-space network should end at the node defined by the drop-off location offered to the user and the latest time by which the user arrives at that location –considering the walking time if the suggested drop-off location is different from the user's destination. Constraints (3.43) indicate that if a user is rejected service, then the unit flow associated with the user is sent to the dummy node. For every user, flow conservation constraints considering all candidate pick-up and drop-off locations and times are given by the equations (3.44) and (3.45). Constraints (3.46) assure that only one pair of origin-destination locations, among all candidates, should be offered to each user. Constraints (3.50) and (3.51) equate the number of vehicles with the number of operators on each relocation arc, and the number of vehicles with the number of users on each travelling arc, respectively. Constraint (3.52) sets the vehicle and operator flow equal to zero on dummy arcs. Moreover, a given user u cannot have positive flow on any arc that does not belong to the set $\mathcal{A}^u$ due to constraints (3.53). Constraints (3.54) guarantee that each user should use at most one travelling arc. Finally, (3.55)–(3.58) impose domain restrictions on the variables.

Using the formulations presented in this chapter, we perform computational experiments to analyse the operational efficiency of free-floating car-sharing systems through several performance metrics. The results of our experiments are provided in the next chapter.

4. COMPUTATIONAL EXPERIMENTS

This chapter presents the results of our computational experiments, which have been conducted to (1) investigate the operational efficacy of a free-floating car-sharing system under two vehicle relocation strategies, assuming a deterministic problem framework, and (2) assess the benefits of crowdsourcing (part of) the relocation tasks by means of incentivizing users. To this end, we employ the mathematical formulations developed in the previous chapter.

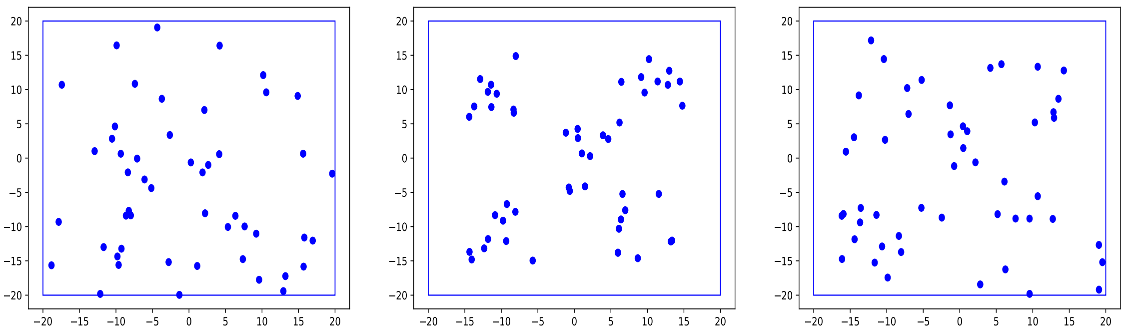
4.1 Generation of test instances

We generated new test instances to use in our experiments due to the lack of benchmark instances in the literature –most of the existing studies focusing on vehicle relocation problems perform tests using real data gathered from car-sharing providers in the industry and such data sets are not made publicly available due to confidentiality issues.

Three sets of instances have been generated, differing based on the geographical distribution of the locations representing the service network: uniform (U), clustered (C), and a combination of uniform and clustered (UC) instances. In what follows, our instance generation scheme is described in detail along with our parameter choices.

4.1.1 Locations and travel times

For each instance used in our computational study, the service region is represented by a network consisting of 50 randomly selected points inside the two-dimensional box defined by $-20 \leq x \leq 20$ and $-20 \leq y \leq 20$. In uniform instances, these 50 locations are uniform randomly distributed across the two-dimensional box, whereas in clustered instances, they are selected in a way that they form five non-overlapping clusters, each being a subset of the box. In combined instances, some locations are selected in a way to form clusters while the others are uniform randomly distributed. The generated locations for each instance can be found in the Figure 4.1.



(a) Locations in uniform instances

(b) Locations in clustered instances

(c) Locations in combined instances

Figure 4.1 The Generated Locations

Based on these locations, pairwise Euclidean distances are computed first. Assuming that the travel time and cost are directly proportional to the distance traveled, the distance values are also regarded as the travel times (in minutes) and the travel costs between locations. In all experiments, a planning horizon of two hours is considered, and it is discretized into 12 equal time steps of 10 minutes when constructing the time-space network. Travel times are converted into time steps so that they are represented properly in the time-space network. In particular, to obtain the number of time steps needed to travel from one location to another in the time-space network, the corresponding travel time is divided by the length of a time step, i.e., 10 minutes, and rounded up. As a consequence, some travel times, and therefore, costs of the solutions to VR-O or VR-H may be overestimated, but will never be underestimated. Hence, it is important to note here that solving the models (3.19)–(3.36) and (3.37)–(3.57) on the time-space network constructed with the aforementioned time discretization scheme will provide upper bounds on the true optimal values. One can increase the time granularity of the network through a finer discretization in order to get closer to optimal solutions at the expense of having to solve the problems on significantly larger networks. The steps of the procedure used for generating the locations and calculating the travel times between different locations (in terms of time steps) are outlined in Algorithm 1.

Algorithm 1 Location Generation & Travel Time Calculation

Given: a two-dimensional box, geography (geo), number of locations (n), time step size (t)
$locations = \emptyset$
Initialize $travel\_times$ and $travel\_time\_steps$ to be empty dictionaries
for $i = 1, 2, \ldots, n$ **do**
    Randomly generate a location loc_i (a pair of coordinates) within the two-dimensional box w.r.t. the given geography geo (U, C, or UC)
    $locations \leftarrow locations \cup \{loc_i\}$
end for
for $i = 1, 2, \ldots, n$ **do**
    for $j = 1, 2, \ldots, n$ **do**
        Initialize $dist_{ij}$ to be the Euclidean distance between loc_i and loc_j
        $travel\_times \leftarrow travel\_times \cup ((i, j) \rightarrow dist_{ij})$
        $travel\_time\_steps \leftarrow travel\_time\_steps \cup ((i, j) \rightarrow \lceil dist_{ij}/t \rceil)$
    end for
end for
return $locations, travel\_times, travel\_time\_steps$

4.1.2 User requests

A user request is characterized by a pair of origin-destination locations, arrival time of the request (specified in terms of time index), and the number of time steps that the user is willing to wait for picking up a vehicle after making the rental request. For each user, an origin-destination pair is randomly selected from the set of 50 locations generated earlier to represent the service network such that the travel time between the selected locations is at least 10 minutes. The motivation behind this restriction is that rentals of shorter duration are expected to take place relatively less frequently (given the fact that a vehicle may not be available for pick up immediately and that it may be faster for a user to just use other means of transport instead of waiting for a vehicle only to make a short trip). However, it should be noted here that the travel time between an origin-destination pair does not necessarily indicate the rental duration since a user might make a detour, or drive to a distant location, and then come back to the starting point (or to a nearby location) to drop off the vehicle. We do not account for such cases due to the way in which the time-space network is constructed although it can easily be modified to do so. In particular, varying rental durations for a given origin-destination pair can be incorporated into our modeling framework by introducing additional arcs to the time-space network corresponding to different rental durations. Moreover, if the origin is the same as the destination for some users, copies of the nodes associated with those locations can be created. For the sake of simplicity, we impose a minimum travel time restriction of 10 minutes when generating our instances.

According to a survey conducted by Herrmann et al. (2014) with a number of users of a free-floating car-sharing system, around 95% of the participants stated that they are willing to wait for up to half an hour to pick up a vehicle. Hence, we assume that users will tolerate waiting for a maximum of three time steps. Considering that this tolerance level will also depend on the planned rental durations of users, the number of time steps that a user can be kept waiting at her origin location is adjusted based on the travel time between the origin and destination locations of that user. More specifically, it is presumed that each user will tolerate a waiting time of no more than the minimum of the travel time between her origin and destination, and three time steps. This means, if the user's travel time is less than three time steps, the allowable waiting time of the user is set to her travel time; otherwise, it is set to three time steps. Request arrival times of users are taken to be randomly generated integers within the planning horizon of 12 time steps, indicating, for each user, the time index at which a rental request is made. For example, if the request arrival time of a user is two, it means that the user's request is assumed to be received at the end of the second time step. For a given user u, the request arrival time e_u is generated in a way to ensure that the closing of the time window l_u does not exceed T (which is 12 in our experiments) so that there is a chance to serve the user within the planning horizon based on the availability of vehicles close to the user's origin.

The steps of the procedure used for generating the parameters related to user requests are provided in Algorithm 2.

Algorithm 2 Generation of Parameters for User Requests

Given : output of Algorithm 1 (*locations*, *travel_times*, *travel_time_steps*), number of users ($users$)
Initialize OD_pairs, $waiting\_time\_steps$, and $arrival\_times$ to be empty dictionaries
for $u = 1, 2, \ldots, users$ **do**
  do
    Randomly select two distinct locations i and j from *locations*
  while $travel\_times[i, j] < 10$
  $o_u \leftarrow i$, $d_u \leftarrow j$
  $OD\_pairs \leftarrow OD\_pairs \cup (u \rightarrow (o_u, d_u))$
  $waiting\_time\_steps \leftarrow waiting\_time\_steps \cup (u \rightarrow min\{3, travel\_time\_steps[o_u, d_u]\})$
  Randomly generate an integer k between 0 and $T - travel\_time\_steps[o_u, d_u] - waiting\_time\_steps[u]$
  $arrival\_times \leftarrow arrival\_times \cup (u \rightarrow k)$
end for
return $OD\_pairs, waiting\_time\_steps, arrival\_times$

4.1.3 Starting locations of vehicles and operators

Positions of the vehicles at the beginning of the planning horizon are chosen randomly from among a subset of the 50 locations defining the service network, particularly, among the locations that lie inside the circle with a radius of 10 units centered at the origin $(0, 0)$. If no such locations exist, we increment the radius of the circle by one unit and repeat the same steps until at least four distinct locations are identified.

We limit our selection of the initial operator locations to four randomly chosen points among the very same subset of locations defined above (i.e., the ones that coincide with the circle of radius 10, centered at the origin). For each operator, a starting location is then designated by randomly selecting one of these four points.

The procedure used for generating the starting locations of vehicles and operators is presented in Algorithm 3.

Algorithm 3 Starting Locations of Vehicles and Operators

Given : *locations* returned by Algorithm 1, number of vehicles (v), number of operators (o), radius(r)
Initialize $st\_loc, st\_loc\_v, st\_loc\_o, base\_o$ to be empty lists
while the length of st_loc is less than 4 **do**
    for i in *locations* **do**
        if i lies inside the circle with a radius of r centered at the origin $(0,0)$ and i is not in st_loc **then**
            $st\_loc \leftarrow st\_loc \cup \{i\}$
        end if
    end for
    $r \leftarrow r+1$
end while
$st\_loc\_v \leftarrow$ Randomly choose a location v times from st_loc with replacement
$base\_o \leftarrow$ Randomly choose four locations from st_loc without replacement
$st\_loc\_o \leftarrow$ Randomly choose a location o times from $base\_o$ with replacement
return st_loc, st_loc_v, $base\_o$, st_loc_o

4.2 Experimental setup and results

This section presents the results of our computational experiments performed by solving the mathematical formulations developed for VR-O and VR-H in Chapter 3. The models were implemented using the Gurobi Python interface, and the procedures used for generating test instances were also implemented in Python programming language. All computational experiments were carried out on a 64-bit machine with Intel Xeon E5-2640 v3 processor at 2.60 GHz using 12.4 GB of RAM, and executed in single thread mode. Run time is measured in seconds. As mentioned earlier, three sets of instances (i.e., uniform (U), clustered (C), and a combination of uniform and clustered (UC)) were generated using the methods described in the previous section.

Each test set contains small, medium and large instances with varying numbers of users, vehicles, and operators. The numbers of vehicles and operators are adjusted based on the number of users. For each instance class (U, C, or UC), Table 4.1 demonstrates the numbers of vehicles and operators considered for different numbers of users. Every combination (of the number of vehicles and the number of operators) in each row corresponds to an instance configuration, e.g., there are 15 configurations with 10 users for a given instance class. Moreover, we generate five different versions of each instance configuration by varying the user-related parameters (OD pairs, request arrival times, and allowable waiting times), which makes a total of 75 instances involving 10 users, per instance class. We grouped the instances with 10, 15, or 20 users as small-size instances, those with 30, 40, or 50 users as medium-size instances, and the others (with 75 and 100 users) as large-size instances. Considering all three test sets, a total of 1800 instances were used in our experiments. Both formulations (3.19)-(3.36) and (3.37)–(3.57) were solved for these 1800 instances.

Since users may be offered alternative pick-up and/or drop-off locations in the hybrid vehicle relocation model, we also need an estimate of the time it takes for users to walk (or travel by other means of transport such as biking etc.) from one location to another. We compute the time it takes for a user to travel from her actual origin to the suggested pick-up location, or from the suggested drop-off location to her actual destination, without a vehicle of the car-sharing provider, by multiplying the distance traveled with four, and then dividing the resulting number by 10 and rounding up to obtain its time-step equivalent.

# of users	# of vehicles	# of operators
10	2, 3, 4, 5, 10	1, 2, 3
15	2, 3, 4, 5, 10, 15	1, 2, 3
20	2, 3, 4, 5, 10, 15, 20	1, 2, 3
30	5, 10, 15, 20	3, 5, 7
40	5, 10, 15, 20	3, 5, 7
50	5, 10, 15, 20	3, 5, 7
75	10, 20, 30, 40, 50	5, 10, 15
100	10, 20, 30, 40, 50	5, 10, 15

Table 4.1 Parameters for the number of users, vehicles, and operators

Gurobi is capable of solving all instances to optimality within a reasonable amount of time under the above settings. For each instance and relocation model, we report the run time as well as several statistics used as performance indicators to measure the operational efficiency under a given relocation strategy, i.e., the objective value, vehicle and operator utilization, the number of rejected users, and in the case of the hybrid relocation model, statistics regarding origin/destination changes for users. The vehicle and operator utilization values are calculated using equations 4.1 and 4.2, i.e., by taking the ratio of the total time spent by the vehicles traveling (with users) to the total time spent by the vehicles, and the ratio of the total time spent by the operators relocating the vehicles to the total time spent by the operators, respectively.

$$\textit{Vehicle Utilization} = \frac{\textit{Total travel time of (all) vehicles}}{\textit{The number of vehicles} \times \textit{The length of planning horizon}} \tag{4.1}$$

$$\textit{Operator Utilization} = \frac{\textit{Total relocation time of (all) operators}}{\textit{The number of operators} \times \textit{The length of planning horizon}} \tag{4.2}$$

Moreover, for the hybrid relocation model, changes in pick-up/drop-off locations of users are also examined by *partitioning* the users into four categories as follows:

- users that are offered a pick-up location different from their origin
- users that are offered a drop-off location different from their destination
- users that are offered a pick-up location different from their origin, and a drop-off location different from their destination
- users that are offered their origin and destination as pick-up and drop-off locations

The number of users in each of these four categories is calculated according to the w_{ij}^u values and reported in order to demonstrate users' contribution to relocation operations.

Our results for the instance class U are summarized in Tables 4.3–4.8, and the abbreviations used when presenting the results are listed in Table 4.2. As mentioned earlier, we experimented with five versions of each instance configuration obtained by changing the set of users while keeping everything else (the service network and the starting locations of vehicles and operators) constant. We report the results averaged out over these five instances for a particular configuration.

In addition, we create scatter plots to better illustrate how the performance indicators behave when parameter values are varied. Because we observe similar trends for instance classes U, C, and UC in general, we only provide the plots for the instance class U. Note that to be able to evaluate the impact of increasing the number of users/vehicles/operators on system performance, we keep the existing users/vehicles/operators as is and introduce new ones on top of those when generating our instances. For each version of a given instance configuration, we can say that smaller instances are derived from larger ones by omitting a subset of the users/vehicles/operators.

We discuss our findings based on the results of our computations on the uniform instance set in the following subsections. Similar conclusions are drawn from the tests with other instance sets (see Appendix A for detailed results) regarding the behavior of the performance indicators with changes in parameter values and relocation strategy. If we make a comparison based on the network geography, both models yield superior results on clustered instances among others in terms of the objective value and the number of rejected users. This can be explained by shorter travel times between the location pairs belonging to the same cluster, making it easier for users and operators to access the vehicles positioned inside their cluster. Hence, clustering user requests and positioning resources (vehicles and operators) accordingly may improve the overall performance of free-floating car-sharing systems.

Notation	Definition
# of V/O/U	The number of vehicles/operators/users
OV	Objective value
T(s)	Run time (in seconds)
VU	Vehicle utilization
OU	Operator utilization
#RU	The number of rejected users
OC	The number of users with a pick-up location different from their origin
DC	The number of users with a drop-off location different from their destination
ODC	The number of users with a pick-up location different from their origin and a drop-off location different from their destination
NC	The number of users with a pick-up location same as their origin and a drop-off location same as their destination

Table 4.2 Abbreviations used in the tables

# of V/O/U	OV	T(s)	VU	OU	#RU
2/1/10	158.21	142.7	0.31	0.21	6.8
2/2/10	126.51	60.46	0.37	0.16	6
2/3/10	121.32	54.65	0.39	0.11	5.8
3/1/10	142.57	191.12	0.22	0.19	6.6
3/2/10	81.3	71.11	0.34	0.21	4.8
3/3/10	53.59	71.44	0.36	0.14	4.4
4/1/10	123.59	191.12	0.19	0.21	6.2
4/2/10	42.67	88.7	0.29	0.21	4.2
4/3/10	13.18	82.75	0.33	0.17	3.2
5/1/10	102.01	189.74	0.16	0.17	6
5/2/10	12.9	65.75	0.25	0.19	3.6
5/3/10	-24.34	79.21	0.28	0.16	2.8
10/1/10	83.91	315.73	0.08	0.19	5.8
10/2/10	-6.75	66.51	0.13	0.19	3.4
10/3/10	-46.88	41.63	0.15	0.18	2.2
2/1/15	254.34	166.59	0.37	0.17	11.2
2/2/15	231.92	64.57	0.41	0.12	10.8
2/3/15	226.07	56.14	0.43	0.09	10.4
3/1/15	234.41	219.3	0.28	0.2	10.8
3/2/15	172.7	76.41	0.39	0.19	9
3/3/15	147.2	67.86	0.42	0.16	8.6
4/1/15	214.13	225	0.23	0.26	10.6
4/2/15	117.13	83.39	0.35	0.23	7.6
4/3/15	83.59	84.63	0.39	0.18	7
5/1/15	180.66	221.74	0.2	0.24	9.8
5/2/15	63.38	77.93	0.31	0.22	7
5/3/15	23.47	81.82	0.36	0.21	5.8
10/1/15	110.2	350.09	0.12	0.22	8.8
10/2/15	7.21	66.17	0.17	0.21	6
10/3/15	-64.3	48.31	0.21	0.21	4.2
15/1/15	105.5	362.83	0.08	0.22	8.8
15/2/15	-0.27	70.43	0.11	0.21	6
15/3/15	-72.14	41.63	0.14	0.22	4
2/1/20	377.3	192.81	0.39	0.17	15.8
2/2/20	355.76	69.58	0.42	0.12	15.4
2/3/20	351.53	57.75	0.42	0.07	15.2
3/1/20	344.37	261.47	0.3	0.22	15.2
3/2/20	283.52	68.81	0.4	0.16	13.4
3/3/20	263.11	72.48	0.42	0.12	13
4/1/20	321.98	249.48	0.25	0.23	15.2
4/2/20	206.62	83.33	0.38	0.21	12.2
4/3/20	177.25	79.87	0.41	0.17	11.4
5/1/20	265.06	266.06	0.23	0.25	14
5/2/20	151.72	80.23	0.34	0.21	11
5/3/20	107.39	85.33	0.39	0.19	9.8
10/1/20	142.89	402.99	0.16	0.23	11.8
10/2/20	27.98	52.83	0.21	0.23	8.8
10/3/20	-58.23	39.11	0.26	0.24	6.4
15/1/20	128	425.73	0.11	0.24	11.4
15/2/20	10	48.44	0.14	0.23	8.8
15/3/20	-77.92	44.41	0.17	0.24	6
20/1/20	128	408.57	0.08	0.24	11.4
20/2/20	-2.9	50.07	0.11	0.24	8.2
20/3/20	-88.22	46.39	0.13	0.23	6

Table 4.3 Results of operator-based relocation problem for small-size instances in the instance class U

# of V/O/U	OV	T(s)	VU	OU	#RU
5/3/30	307.79	115.17	0.45	0.17	18.6
5/5/30	267.6	394.11	0.49	0.12	18
5/7/30	258.17	524.93	0.49	0.09	18.2
10/3/30	60.75	74.13	0.32	0.24	13.4
10/5/30	-90.6	263.44	0.39	0.21	9.4
10/7/30	-141.62	381.17	0.41	0.16	8.8
15/3/30	29.28	44.19	0.22	0.24	13
15/5/30	-154.06	270.72	0.28	0.22	8.2
15/7/30	-257.91	418.74	0.31	0.21	5
20/3/30	10.47	53.66	0.16	0.23	13
20/5/30	-194.95	51.94	0.22	0.23	7.2
20/7/30	-331.64	306.32	0.25	0.21	4.2
5/3/40	512.65	113.04	0.51	0.15	26.8
5/5/40	482.2	436.91	0.53	0.11	26.4
5/7/40	477.2	541.66	0.53	0.08	26.4
10/3/40	159	53.03	0.38	0.22	19.8
10/5/40	30.03	256.39	0.45	0.2	16.4
10/7/40	-16	406.08	0.47	0.16	15.8
15/3/40	95.25	62.8	0.27	0.22	18.8
15/5/40	-119.35	258.5	0.34	0.23	13.6
15/7/40	-237.3	421.65	0.38	0.21	10.4
20/3/40	86.06	36.22	0.21	0.23	18.8
20/5/40	-165.81	82.38	0.26	0.24	12.6
20/7/40	-345.92	355.64	0.3	0.22	8.6
5/3/50	728.04	97.72	0.56	0.15	35.6
5/5/50	682.21	443.58	0.58	0.09	35.2
5/7/50	676.75	576.54	0.58	0.06	35.2
10/3/50	277.53	57.82	0.44	0.2	26.8
10/5/50	149	257.37	0.5	0.17	23.6
10/7/50	113.69	401.45	0.52	0.14	23.4
15/3/50	183.28	63.76	0.31	0.22	25
15/5/50	-80.7	245.6	0.39	0.23	18.8
15/7/50	-218.37	432.48	0.43	0.2	15.8
20/3/50	157.09	67.62	0.24	0.21	24.8
20/5/50	-150.94	80.48	0.31	0.22	18
20/7/50	-375.37	369.18	0.35	0.22	12.8

Table 4.4 Results of operator-based relocation problem for medium-size instances in the instance class U

# of V/O/U	OV	T(s)	VU	OU	#RU
10/5/75	648.09	263.24	0.55	0.14	45.2
10/10/75	619.47	191.61	0.56	0.08	44
10/15/75	614.64	154.31	0.57	0.06	44.4
20/5/75	-7.71	87.02	0.4	0.22	31.6
20/10/75	-403.93	395.49	0.48	0.17	23.4
20/15/75	-504.11	321.52	0.5	0.13	21.8
30/5/75	-240	323.27	0.29	0.23	27.6
30/10/75	-788.24	425.95	0.38	0.23	15.2
30/15/75	-1004	394.85	0.4	0.18	10
40/5/75	-364.92	85.18	0.23	0.25	25.6
40/10/75	-917.78	291.13	0.29	0.23	12.8
40/15/75	-1108.21	246.53	0.31	0.18	8.4
50/5/75	-393.39	88.16	0.19	0.24	25.4
50/10/75	-949.53	264.87	0.24	0.23	12.2
50/15/75	-1143.74	245.48	0.25	0.18	7.4
10/5/100	1215.97	274.09	0.6	0.12	68.4
10/10/100	1201.44	207.47	0.6	0.07	68.4
10/15/100	1197.57	157.79	0.6	0.04	68.4
20/5/100	281.01	108.47	0.47	0.2	49.2
20/10/100	-21.7	421.92	0.53	0.16	42.8
20/15/100	-79.25	327.48	0.55	0.11	41.2
30/5/100	-121.67	404.00	0.36	0.24	41.2
30/10/100	-770.8	522.15	0.45	0.21	27
30/15/100	-1007.65	436.92	0.49	0.17	22.8
40/5/100	-296.97	103.17	0.29	0.26	38
40/10/100	-1013.18	315.32	0.36	0.22	22.6
40/15/100	-1367.77	278.97	0.4	0.21	14
50/5/100	-348.02	90.81	0.23	0.25	37.2
50/10/100	-1098.27	308.07	0.3	0.25	21
50/15/100	-1449.66	254.30	0.33	0.21	12.8

Table 4.5 Results of operator-based relocation problem for large-size instances in the instance class U

# of V/O/U	OV	T(s)	VU	OU	#RU	OC	DC	ODC	NC
2/1/10	118.28	140.33	0.4	0.18	5.8	1.2	0.6	0	2.4
2/2/10	89.47	54.62	0.49	0.14	4.8	1	1.8	0	2.4
2/3/10	85.34	50.26	0.49	0.1	4.8	1	1.6	0	2.6
3/1/10	92.13	192.93	0.3	0.19	5.6	1.2	0.8	0	2.4
3/2/10	42.38	69.45	0.4	0.18	4.2	1.2	1	0	3.6
3/3/10	28.82	77.37	0.44	0.16	3	1.2	1.6	0	4.2
4/1/10	60.94	212.18	0.27	0.26	4.6	1.6	0.8	0	3
4/2/10	1.61	73	0.34	0.19	3	1.4	1.4	0	4.2
4/3/10	-22.23	66.48	0.38	0.15	2.2	2	1.6	0	4.2
5/1/10	33.84	218.48	0.24	0.28	3.6	2.4	1.2	0	2.8
5/2/10	-32.46	77.86	0.3	0.21	2.4	1.6	1.2	0	4.8
5/3/10	-58.9	59.61	0.32	0.17	1.8	1	1.2	0	6
10/1/10	18.87	282.29	0.12	0.3	3.6	2	1	0	3.4
10/2/10	-55.41	60.24	0.16	0.24	1.8	1.6	0.8	0	5.8
10/3/10	-82.7	49.17	0.17	0.18	1.2	1.6	0.8	0	6.4
2/1/15	217.34	176.27	0.45	0.15	10.2	1.4	1	0	2.4
2/2/15	201.52	62.51	0.53	0.13	9.6	1.4	1.2	0	2.8
2/3/15	195.97	56.85	0.52	0.09	9.6	1.2	1.2	0	3
3/1/15	162.53	246.19	0.39	0.19	9.2	1.6	1	0	3.2
3/2/15	125.97	70.08	0.44	0.15	8	1.4	1.4	0	4.2
3/3/15	111.48	64.98	0.47	0.11	7.6	1.4	1.6	0	4.4
4/1/15	117.37	234.85	0.34	0.2	8	2.2	1.2	0	3.6
4/2/15	57.62	94.98	0.42	0.17	6.2	1.6	2	0	5.2
4/3/15	35.02	69.57	0.45	0.15	5.6	1.8	2.8	0	4.8
5/1/15	68.26	236.14	0.31	0.23	6.8	2.6	1.6	0	4
5/2/15	3.32	82.25	0.38	0.23	5	1.6	2	0	6.4
5/3/15	-35.51	62.48	0.41	0.19	4	2.2	1.6	0	7.2
10/1/15	-6.65	359.58	0.18	0.23	5.4	2.8	1	0	5.8
10/2/15	-86.64	58.9	0.22	0.26	3.2	2.4	1.6	0	7.8
10/3/15	-130.88	43.41	0.24	0.22	2.4	1.4	1.8	0	9.4
15/1/15	-7.81	370.16	0.12	0.24	5.6	3	0.6	0	5.8
15/2/15	-91.23	56.76	0.15	0.26	3.2	2	1.8	0	8
15/3/15	-140.8	38.24	0.16	0.21	2	2	1.6	0	9.4
2/1/20	344.92	189.49	0.46	0.17	15	1.2	1	0	2.8
2/2/20	318.68	65.95	0.52	0.1	14	1.8	1.8	0	2.4
2/3/20	315.38	61.54	0.53	0.08	14	1.2	2.2	0	2.6
3/1/20	275.72	274.04	0.4	0.17	13.2	1.8	1.8	0	3.2
3/2/20	234.88	81.23	0.48	0.14	12.2	1.6	1.8	0.2	4.2
3/3/20	220.45	70.52	0.52	0.11	11.6	2	3	0.2	3.2
4/1/20	189.21	259.4	0.39	0.19	11.6	2.8	1.8	0	3.8
4/2/20	130.43	75.14	0.47	0.17	10.2	1.2	2.4	0.2	6
4/3/20	100.32	72.63	0.51	0.13	9	2.2	3.6	0.2	5
5/1/20	122.23	283.76	0.37	0.26	9.4	3.8	2.2	0.2	4.4
5/2/20	49.16	70.58	0.43	0.18	8.2	2	2.8	0.4	6.6
5/3/20	19.41	69.48	0.46	0.16	7.4	2.6	2	0.4	7.6
10/1/20	-14.79	414.07	0.23	0.25	7	3.4	3	0.2	6.4
10/2/20	-106.33	80.02	0.28	0.25	4.8	4	2.6	0	8.6
10/3/20	-168.4	82.41	0.31	0.23	3.2	3.6	3.2	0	10
15/1/20	-35.98	388.74	0.16	0.21	7	3.2	2.2	0.6	7
15/2/20	-121.98	69.35	0.19	0.23	4.6	4	2.2	0	9.2
15/3/20	-186	44.56	0.21	0.22	3	4	2.8	0	10.2
20/1/20	-43.27	378.01	0.12	0.21	6.8	2.8	2.4	0.6	7.4
20/2/20	-130.82	35.28	0.14	0.22	4.6	3	2.2	0.2	10
20/3/20	-197.8	57.14	0.16	0.22	2.8	3.6	2.4	0.2	11

Table 4.6 Results of hybrid relocation problem for small-size instances in the instance class U

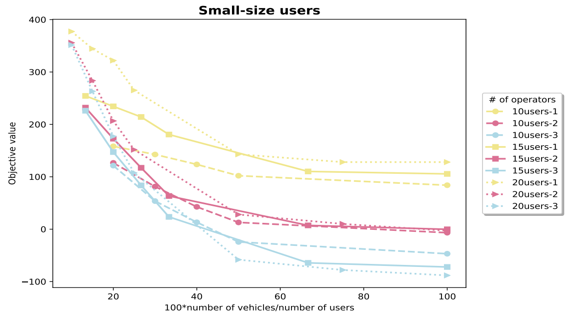
# of V/O/U	OV	T(s)	VU	OU	#RU	OC	DC	ODC	NC
5/3/30	199.13	78.76	0.55	0.14	14.8	3.6	3.6	0.4	7.6
5/5/30	149.87	401.53	0.59	0.1	14.2	2.8	3.4	0.6	9
5/7/30	140.66	555.02	0.59	0.07	14.2	2.8	3.6	0.6	8.8
10/3/30	-128.98	84.16	0.41	0.22	7.4	5.2	4.8	0	12.6
10/5/30	-231.57	229.57	0.46	0.18	5	5.8	5.4	0.2	13.6
10/7/30	-267.05	383.46	0.48	0.15	4.4	5.2	5	0.2	15.2
15/3/30	-178.23	79.71	0.29	0.24	6.4	6.6	4.2	0	12.8
15/5/30	-309.02	253.37	0.34	0.21	3.2	5.4	4.8	0	16.6
15/7/30	-353.94	410.25	0.34	0.19	2.6	2.8	4.2	0	20.4
20/3/30	-199.98	83.82	0.22	0.25	6	5.4	5	0	13.6
20/5/30	-343.94	90.91	0.25	0.21	2.8	5.2	4.2	0	17.8
20/7/30	-433.22	277.57	0.27	0.18	1.8	3.2	3.8	0	21.2
5/3/40	392.99	108.68	0.61	0.14	23	3.8	4.8	0.4	8
5/5/40	366.09	399.45	0.62	0.09	22	4.8	5	0.6	7.6
5/7/40	360.62	535.34	0.62	0.07	22	4.6	5.2	0.6	7.6
10/3/40	-77.81	77.1	0.5	0.2	13	6.2	7.2	0.2	13.4
10/5/40	-167.91	223.09	0.55	0.18	10.6	7.2	7	0.2	15
10/7/40	-191.46	380.6	0.55	0.13	10.6	6.4	6.8	0.4	15.8
15/3/40	-217.08	92.66	0.37	0.24	10	7.6	6.8	0.2	15.4
15/5/40	-362.92	251.53	0.42	0.22	6.4	7.8	6.2	0.2	19.4
15/7/40	-432.37	393.34	0.44	0.18	5.2	6.6	6.4	0.2	21.6
20/3/40	-245.65	74.86	0.28	0.23	9.8	7.2	6.6	0.2	16.2
20/5/40	-421.1	98.98	0.33	0.24	5.2	7.4	6.2	0.2	21
20/7/40	-544.03	293.42	0.35	0.21	3	6.4	5.4	0.2	25
5/3/50	596.36	116.29	0.64	0.09	31.4	4.4	4.6	0.4	9.2
5/5/50	571.33	414.12	0.65	0.06	31	4.6	4.6	0.4	9.4
5/7/50	567.6	552.48	0.65	0.05	30.4	5.4	4.8	0.4	9
10/3/50	7.79	100.09	0.57	0.19	18.4	8.4	8	0.4	14.8
10/5/50	-72.31	244.78	0.6	0.14	16.6	8.4	8.4	0.6	16
10/7/50	-84.61	409.44	0.61	0.11	16.4	8	8.4	0.6	16.6
15/3/50	-229.73	114.99	0.44	0.24	13.8	7.6	9	0.8	18.8
15/5/50	-376.97	246.09	0.49	0.21	10.2	7.8	8.8	1	22.2
15/7/50	-440.03	387.32	0.51	0.19	9	8.2	7.4	0.6	24.8
20/3/50	-281.05	120.32	0.34	0.23	12.8	10	7	0.8	19.4
20/5/50	-486.54	108.51	0.39	0.24	8	8.2	7.8	0.8	25.2
20/7/50	-637.98	301.45	0.42	0.22	5.2	8	6.8	0.4	29.6

Table 4.7 Results of hybrid relocation problem for medium-size instances in the instance class U

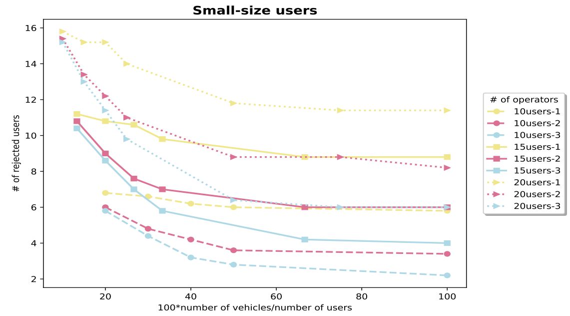
# of V/O/U	OV	T(s)	VU	OU	#RU	OC	DC	ODC	NC
10/5/75	388.11	270.58	0.65	0.1	36.4	9	8.6	0.6	20.4
10/10/75	380.16	237.65	0.65	0.05	36.4	9.2	8.8	0.6	20
10/15/75	378.11	224.98	0.65	0.03	36.8	8.8	8.4	0.6	20.4
20/5/75	-491.76	190.68	0.52	0.22	17.8	16.4	10.6	0.6	29.6
20/10/75	-730.17	477.61	0.56	0.15	13.4	13.6	11.2	0.2	36.6
20/15/75	-783.48	392.17	0.57	0.11	12.4	13.2	11.4	0.4	37.6
30/5/75	-788.57	354.94	0.39	0.25	12.6	15.6	11.6	0.4	34.8
30/10/75	-1147.4	454.67	0.43	0.21	5.6	11.2	8.8	0.2	49.2
30/15/75	-1278.74	358.70	0.45	0.17	3.4	8.6	7.6	0.4	55
40/5/75	-884.97	135.27	0.3	0.26	10.8	14.6	12.8	0.4	36.4
40/10/75	-1235.9	276.05	0.33	0.21	4.6	10.2	9.4	0.2	50.6
40/15/75	-1365.02	240.29	0.34	0.17	2.8	6.6	7.2	0	58.4
50/5/75	-899.43	135.52	0.24	0.26	10.4	14.6	12.6	0.4	37
50/10/75	-1265.97	263.63	0.27	0.22	3.8	11.6	8.6	0.2	50.8
50/15/75	-1393.21	231.34	0.27	0.16	2.8	6	7.4	0	58.8
10/5/100	942.56	296.26	0.69	0.06	58.4	10.6	11.6	0.4	19
10/10/100	939.05	273.09	0.69	0.03	58.2	10.6	11.2	0.6	19.4
10/15/100	937.73	276.06	0.69	0.02	58.2	10.4	11.4	0.6	19.4
20/5/100	-280.19	221.51	0.6	0.16	32.8	17.4	14.6	0.6	34.6
20/10/100	-440.12	510.63	0.63	0.11	30.4	15.4	16.6	0.4	37.2
20/15/100	-472.49	434.27	0.63	0.08	29.2	15	16.8	0.6	38.4
30/5/100	-931.82	450.28	0.49	0.23	19.6	22.2	15.6	0.4	42.2
30/10/100	-1279.76	565.43	0.53	0.19	13.8	16.8	15.4	0.2	53.8
30/15/100	-1403.58	492.25	0.55	0.15	11.8	15.2	13.6	0.6	58.8
40/5/100	-1131.67	182.58	0.39	0.26	15.6	22.6	15.6	0.8	45.4
40/10/100	-1553.78	357.94	0.43	0.22	8.2	16.8	13.4	0.2	61.4
40/15/100	-1733.18	348.61	0.45	0.19	5.6	11.8	11	0	71.6
50/5/100	-1167.11	170.66	0.31	0.27	15.6	20.8	15	0.4	48.2
50/10/100	-1617.04	338.46	0.35	0.23	7.4	16.4	13.2	0	63
50/15/100	-1796.42	313.77	0.36	0.19	4.8	11.2	11	0	73

Table 4.8 Results of hybrid relocation problem for large-size instances in the instance class U

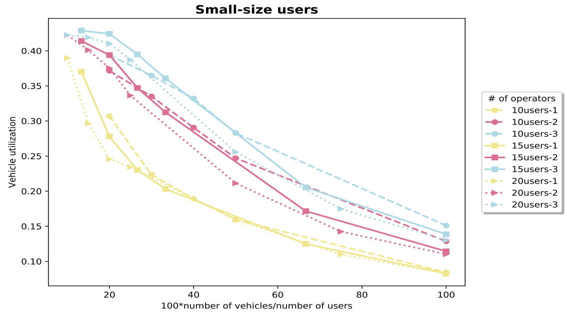
4.2.1 The effect of changing the fleet size



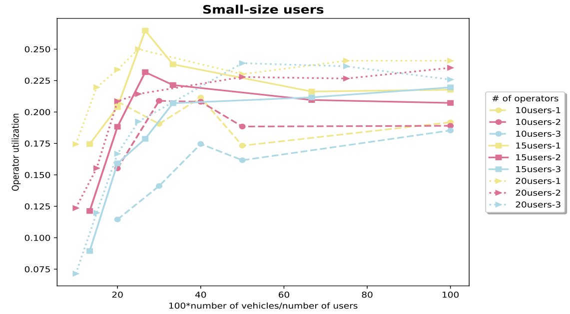
(a) Objective value



(b) No of rejected user

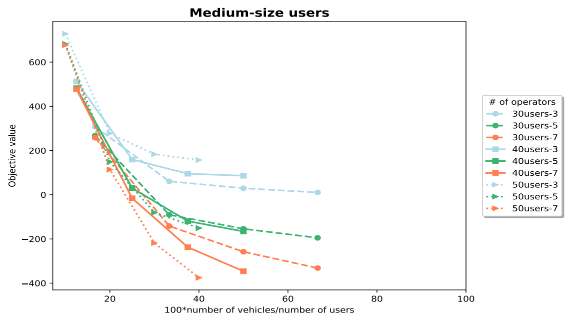


(c) Vehicle utilization

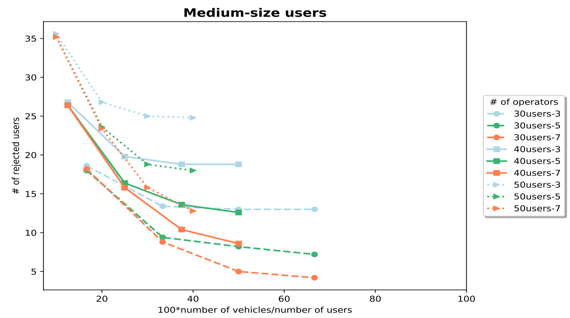


(d) Operator utilization

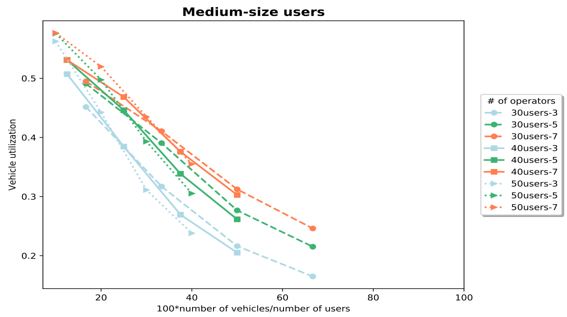
Figure 4.2 Operator-based model plots for small-size users (Fixed # of operators)



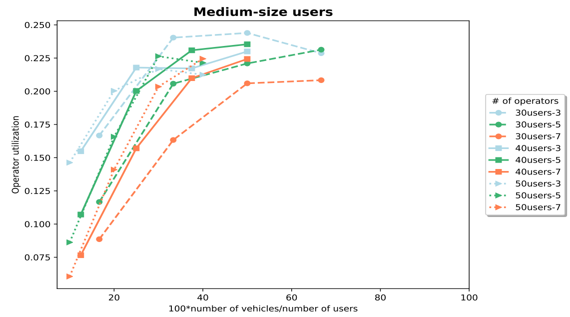
(a) Objective value



(b) No of rejected user

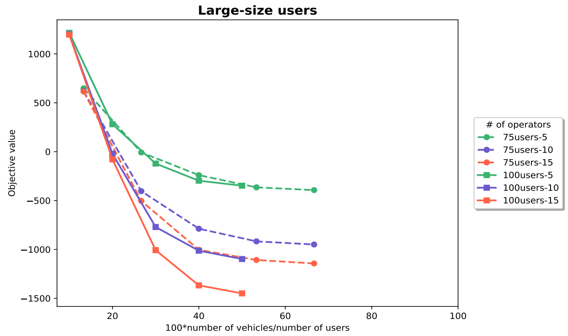


(c) Vehicle utilization

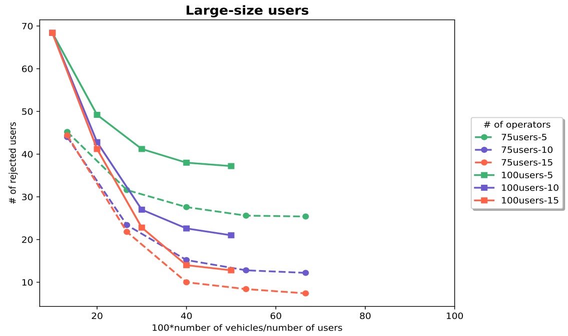


(d) Operator utilization

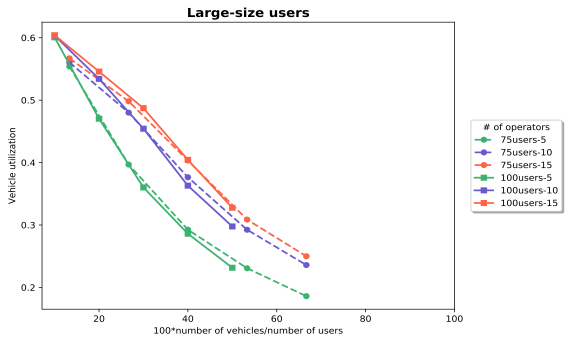
Figure 4.3 Operator-based model plots for medium-size users (Fixed number of operators)



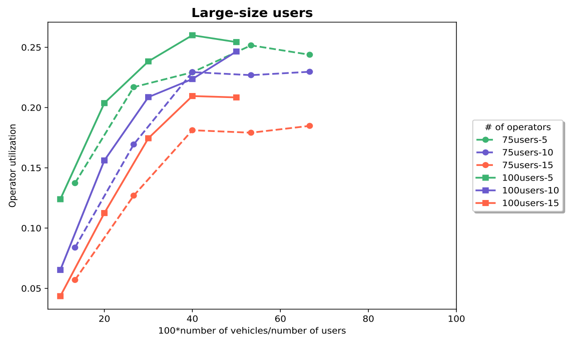
(a) Objective value



(b) No of rejected user

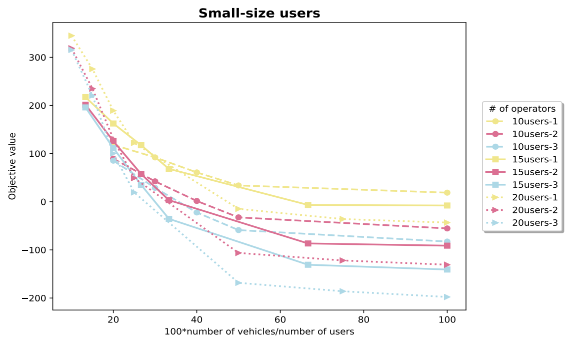


(c) Vehicle utilization

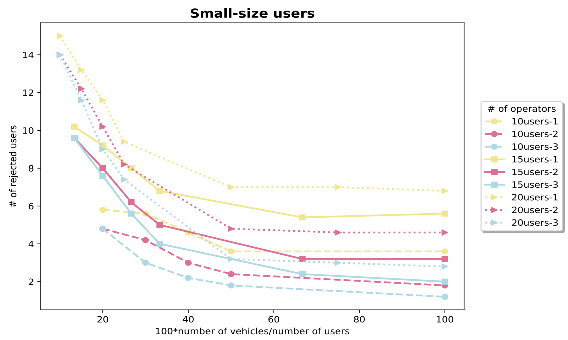


(d) Operator utilization

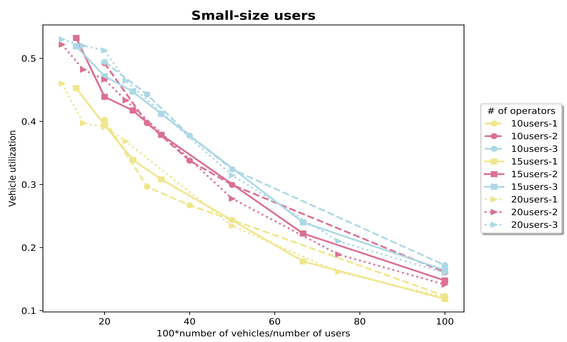
Figure 4.4 Operator-based model plots for large-size users (Fixed number of operators)



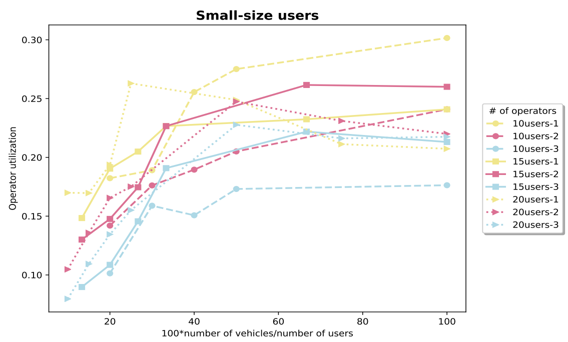
(a) Objective value



(b) No of rejected user

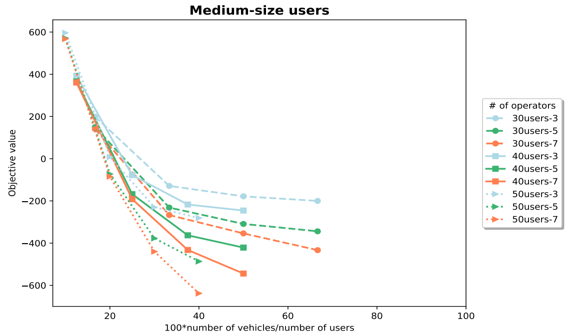


(c) Vehicle utilization

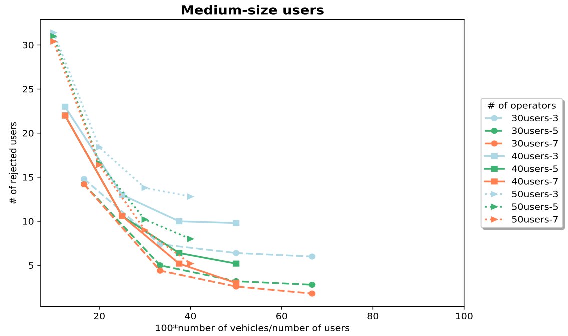


(d) Operator utilization

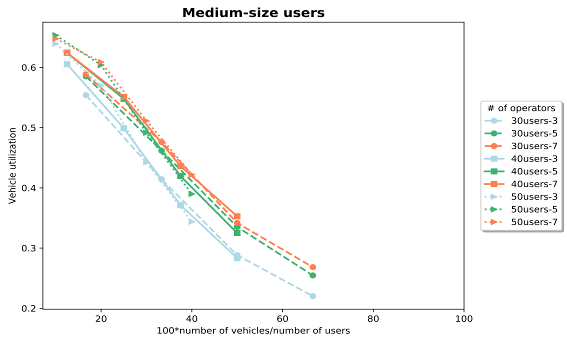
Figure 4.5 Hybrid model plots for small-size users (Fixed number of operators)



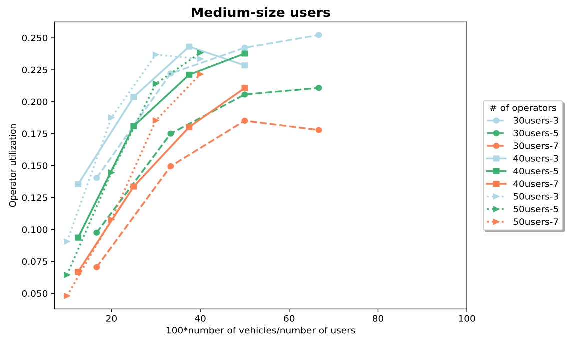
(a) Objective value



(b) No of rejected user

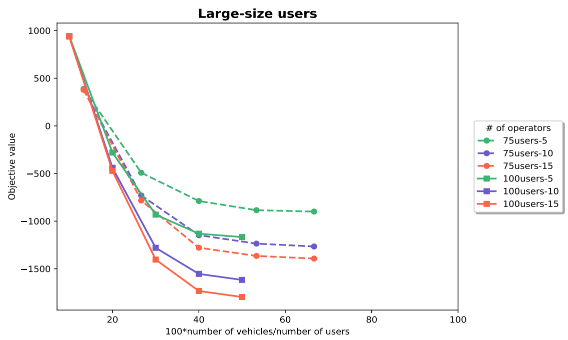


(c) Vehicle utilization

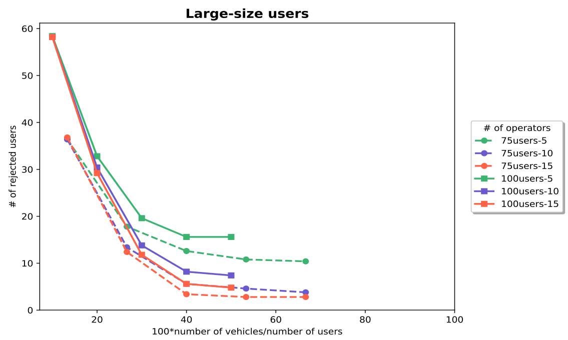


(d) Operator utilization

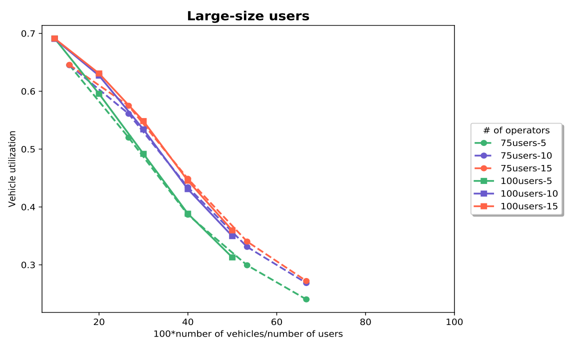
Figure 4.6 Hybrid model plots for medium-size users (Fixed number of operators)



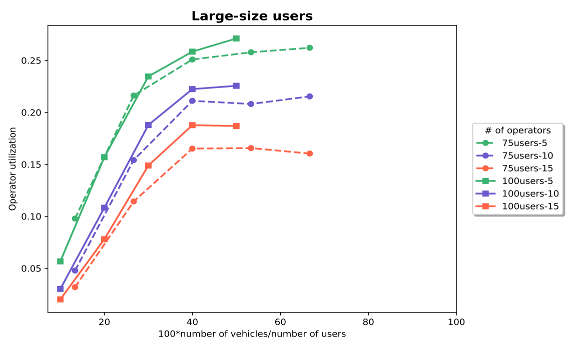
(a) Objective value



(b) No of rejected user



(c) Vehicle utilization



(d) Operator utilization

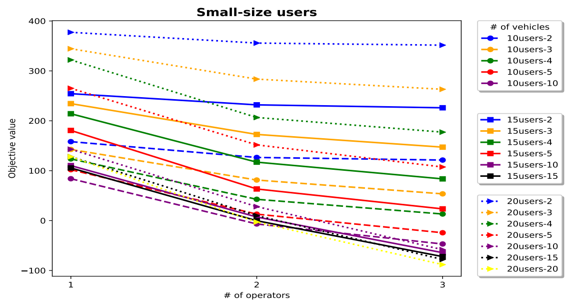
Figure 4.7 Hybrid model plots for large-size users (Fixed number of operators)

Figures 4.2–4.7 demonstrate the effect of increasing the number of vehicles on the objective function value, the number of rejected users, vehicle utilization, and operator utilization for a fixed number of operators and a fixed number of users. The first three performance indicators exhibit an overall decreasing trend, whereas the operator utilization tends to increase with increasing number of vehicles. Since more user requests can be satisfied when a larger number of vehicles is available, the number of rejected users decrease, resulting in an improvement in the objective function values. We observe that although introducing more vehicles to the car-sharing system benefits the users, it may lead to a significant reduction in vehicle utilization rates, which implies that the vehicles are idle most of the time. Considering each vehicle has to be maintained by the company to meet certain standards and user expectations, marginal profit earned by serving a few more users may not be enough to offset the additional expenses incurred by including more vehicles in the car-sharing system with low utilization rates. Making an efficient use of the existing resources is therefore more sustainable instead of having plenty of resources that mostly stay idle. Briefly, when the numbers of operators and users are fixed, our results show a trade-off between demand satisfaction and vehicle utilization, and the fleet size can be adjusted in a way to achieve a targeted service level while keeping the utilization at an acceptable rate. Managing the fleet effectively (assignment of available vehicles to users, repositioning of the vehicles, scheduling the vehicle refueling/maintenance activities etc.) is a key to increase user satisfaction and vehicle occupancy levels simultaneously.

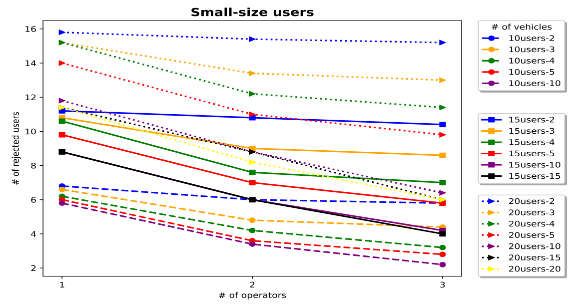
Operator utilization tends to increase in general although we also observe a reduction in some rare cases. The increased utilization can be explained simply by the fact that a larger number of vehicles translates into more vehicles per operator, and possibly more relocation tasks to be performed within the planning horizon. Despite that, the relocation tasks performed by the operators can change in a way to take less time in total, which in turn, decreases their utilization on average. As an example, in the case of 10 users, four vehicles, and two operators (for the operator-based relocation model), the operators perform five relocations which take 57.5 minutes in total. On the other hand, when the number of vehicles is increased to five, the operator performs six relocations which take 43.78 minutes in total. Even though the total number of relocation tasks handled by the operator increases, the amount of time required decreases. As a result, operator utilization decreases from 19% to 18%. Similarly, for the hybrid relocation model, when the number of vehicles is increased from three to four in the presence of 15 users and two operators, five relocation operations take place in a total of 50.72 minutes compared to four relocation operations which take 57.62 minutes when there are three vehicles. As a consequence, operator utilization, drops down from 24% to 21%. We should also note here that depending on the locations of the vehicles, it is also possible that relocation needs decline when more vehicles are available to serve a given number of user requests.

Overall, we conclude that for a given number of users and operators, increasing the fleet size up to a certain point leads to remarkable improvements in the number of rejected users and the objective value at the expense of deteriorating vehicle utilization levels. For example, considering the uniform problem instances with 50 users, seven operators and 15 vehicles, providing an additional five vehicles for service yields nearly a 71% reduction in the objective value and a 19% increase in the number of requests served when only operator-based relocations are allowed. In the hybrid relocation case, the objective value decreases by 45%, and the number of users served increases by 42%. It should be noted however that enlarging the fleet beyond a certain size results only in marginal gains in terms of these two performance measures, while significantly lowering the average vehicle utilization rate. This indicates the importance of sizing the fleet so as to achieve a good balance between demand satisfaction and vehicle utilization levels. Flooding the service region with vehicles when the demand is relatively stable and no additional operators are available does not benefit car-sharing providers as it would be overly expensive to maintain a fleet with majority of the vehicles being idle most of the time.

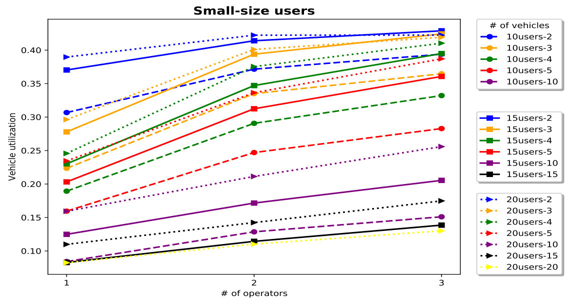
4.2.2 The effect of changing the staff level



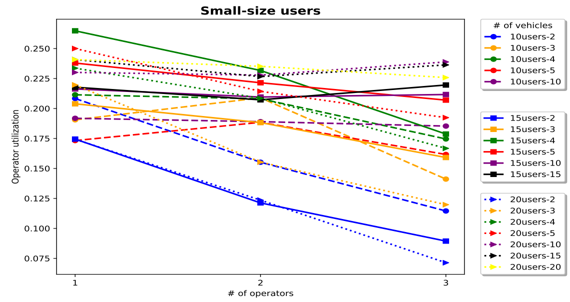
(a) Objective value



(b) No of rejected user

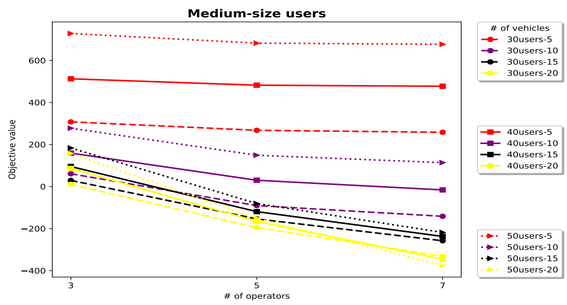


(c) Vehicle utilization

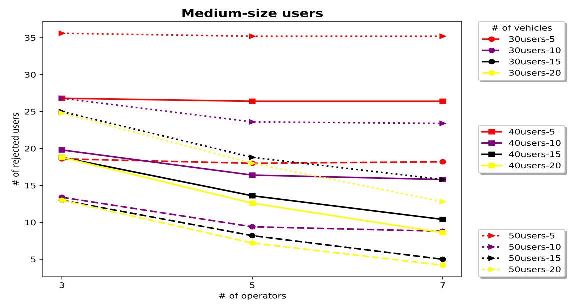


(d) Operator utilization

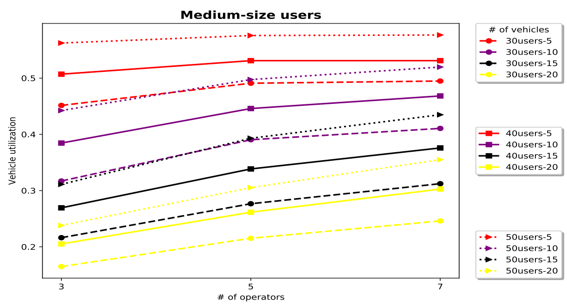
Figure 4.8 Operator-based model plots for small-size users (Fixed number of vehicles)



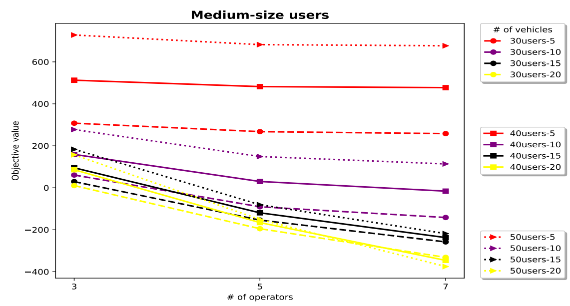
(a) Objective value



(b) No of rejected user

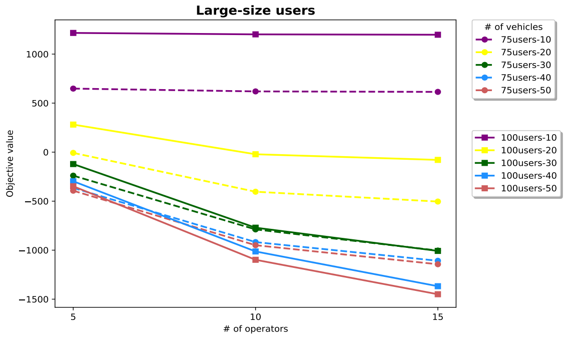


(c) Vehicle utilization

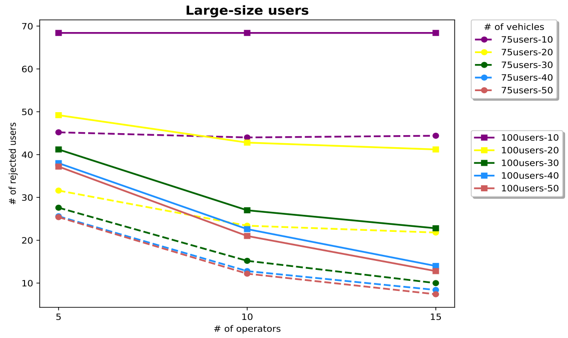


(d) Operator utilization

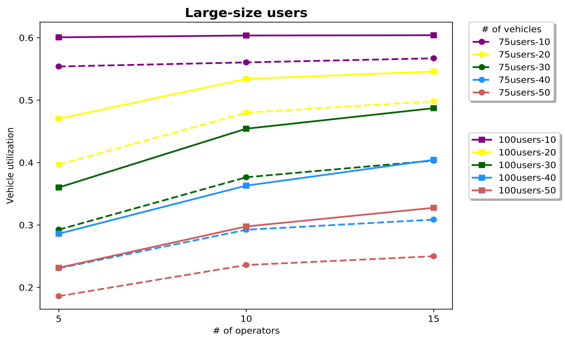
Figure 4.9 Operator-based model plots for medium-size users (Fixed number of vehicles)



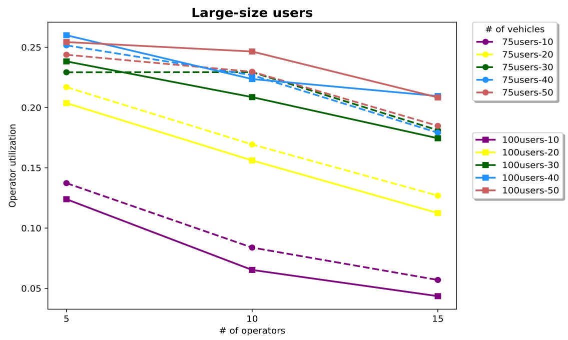
(a) Objective value



(b) No of rejected user

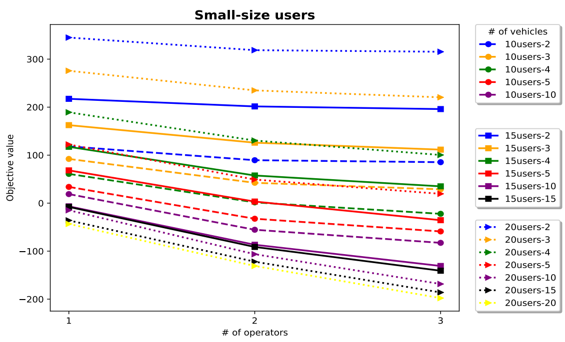


(c) Vehicle utilization

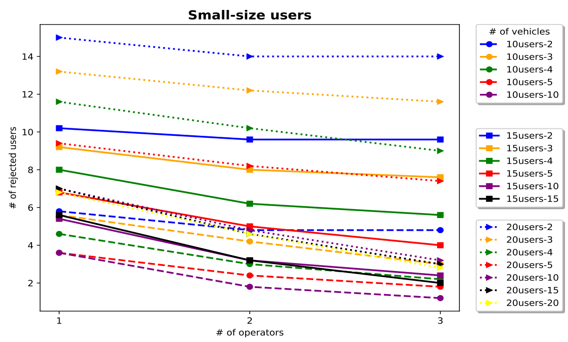


(d) Operator utilization

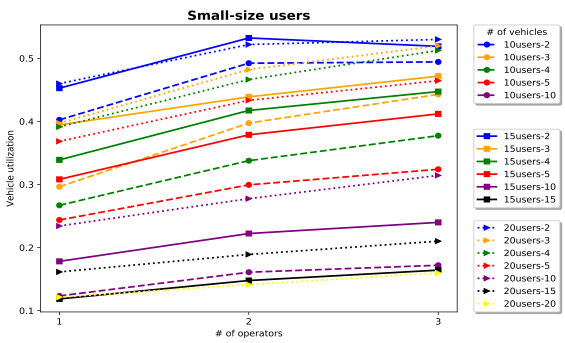
Figure 4.10 Operator-based model plots for large-size users (Fixed number of vehicles)



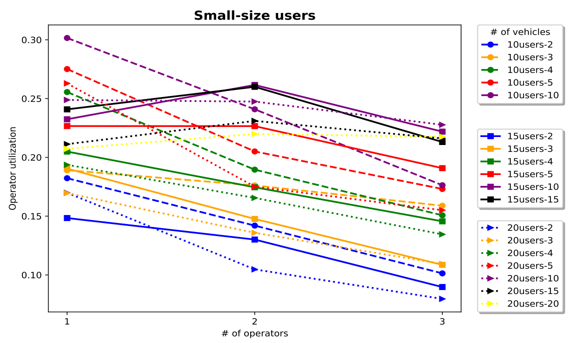
(a) Objective value



(b) No of rejected user

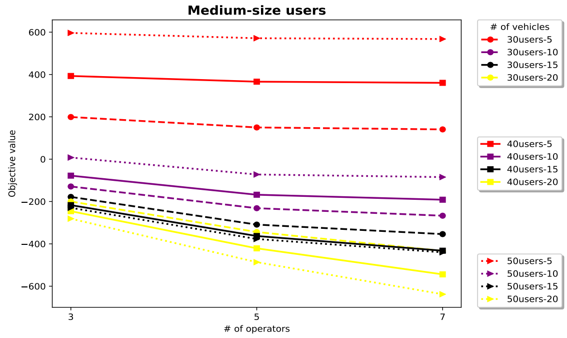


(c) Vehicle utilization

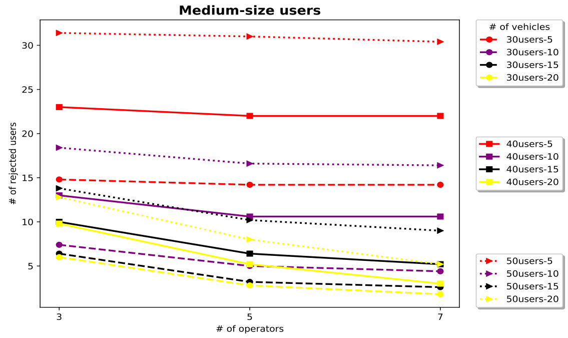


(d) Operator utilization

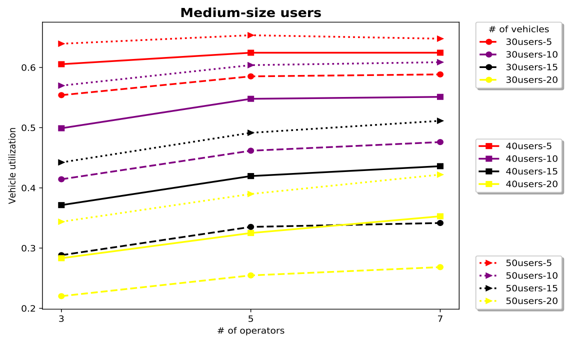
Figure 4.11 Hybrid model plots for small-size users (Fixed number of vehicles)



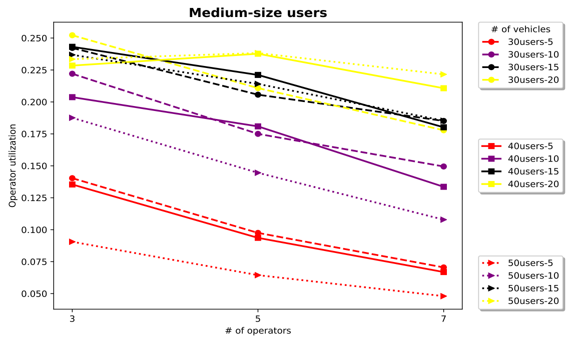
(a) Objective value



(b) No of rejected user

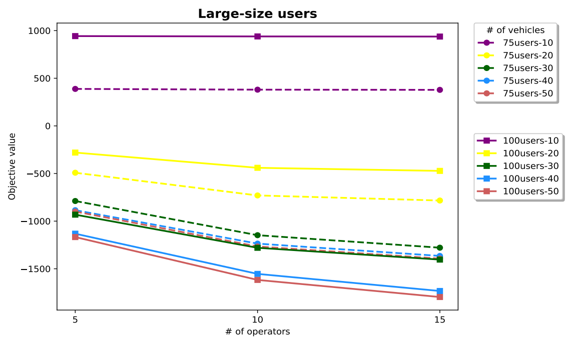


(c) Vehicle utilization

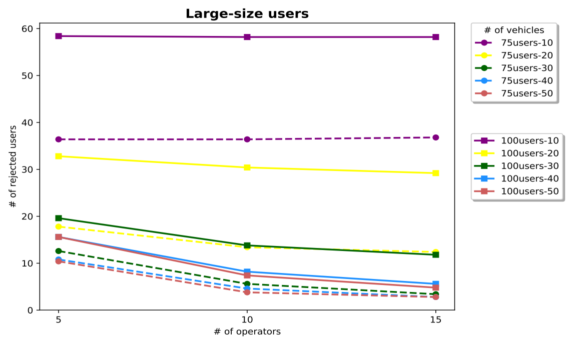


(d) Operator utilization

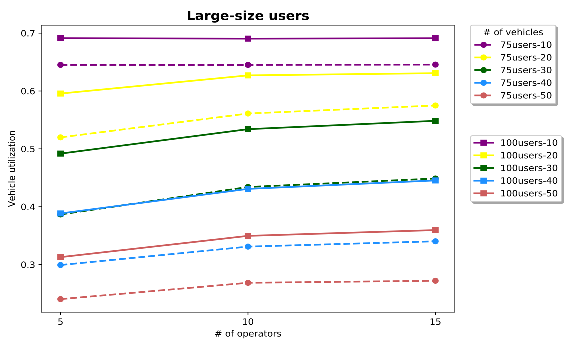
Figure 4.12 Hybrid model plots for medium-size users (Fixed number of vehicles)



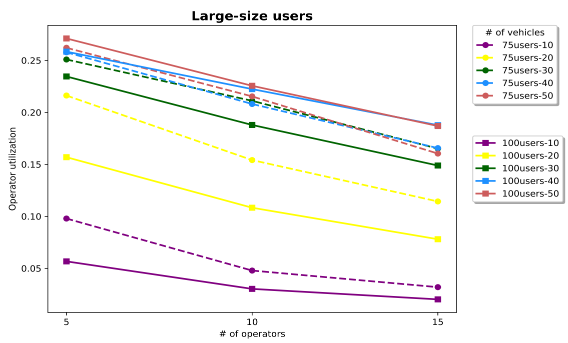
(a) Objective value



(b) No of rejected user



(c) Vehicle utilization



(d) Operator utilization

Figure 4.13 Hybrid model plots for large-size users (Fixed number of vehicles)

Figures 4.8–4.13 depict the relationship between the staff level and different performance indicators. For a fixed number of vehicles and a fixed number of users, the objective value and the number of rejected users demonstrate a declining trend as the number of operators increase, similar to what we observed earlier in subsection 4.2.1. This can be attributed to the fact that having more operators dedicated to repositioning tasks facilitates a better matching of the supply (of vehicles) with the demand, making it easier/faster for users to access the vehicles. As a consequence, vehicle utilization level tends to become larger as shown in the associated plots.

The improvements in the observed values of these three performance indicators are more evident for those instances in which the vehicle-to-operator ratio, that is, the number of vehicles per operator, is higher. For example, considering the operator-based model and the group of instances involving 50 users and three operators, hiring an additional four operators leads to 339% reduction in the objective value in the presence of 20 vehicles compared to only a %7 reduction when there are five vehicles. The hybrid relocation model seems to yield relatively more robust results with varying staff levels because the car-sharing system does not rely on operators for relocation tasks. Regarding the same instance group with 50 users, increasing the number of operators from three to seven improves the objective value by approximately 126% and 4% for 20 vehicles and five vehicles, respectively. Our objective function does not involve a fixed cost term for hiring an operator. Therefore, we can not observe the trade-off between the number of operators and the objective function. Especially for the operator-based relocation model, when there is a fixed cost, increasing the number of operators may not be that useful to the objective function. It is also worth mentioning that under the hybrid relocation strategy, increasing the number of operators contributes to the objective value by having fewer users change their pick-up/drop-off locations and employing more operators in relocation tasks. This is a result of the discount scheme used in our model and the fact that our objective function does not involve a fixed cost term for operators; it may be less costly to have an operator reposition a vehicle rather than offering a discount to a user to do it, depending on the profit associated with that user.

Operator utilization level has a tendency to decrease despite a few exceptional cases. With the deployment of additional operators, the number of vehicles per operator gets smaller, thereby reducing the workload per operator. Note that reduced workload does not necessarily imply that a particular operator will be employed less (or less frequently). Rather, it implies that more resources are available to fulfill the same relocation needs. Because we use the average utilization as a measure, operator utilization rate exhibits a decreasing pattern in general. Especially when hiring more operators does not result in a significant change with respect to the (number of) user requests served, performing similar (or sometimes even the same) relocation tasks with a larger number of operators leads to a lower operator utilization on the average. As mentioned earlier, there are some cases where an increase in operator utilization is reported. This is observed to happen in a few small/medium size instances where the number of operators is incremented from one to two in small size instances, and from three to five in medium size instances. For these instance groups, the corresponding curves that represent the number of rejected users show a relatively more obvious decrease, hence, having one or two more operators contributes towards increasing the efficiency of the car-sharing system.

We conclude that hiring more operators benefits the overall system performance as long as it yields a considerable increase in vehicle occupancy, and by extension, service level. As in the case of fleet size, the staff level may be determined based on the objective value, the number of user requests served, and the vehicle utilization ratio while keeping the operator utilization at a reasonable rate. Since including too many operators in the system may cause (at least some) operators to mostly remain idle, finding an optimal threshold for the number of operators considering the fleet size and user demands can impact the service quality of the system significantly.

4.2.3 The effect of variations in demand

# of V/O/U	OV	T(s)	VU	OU	#RU
5/3/10	-24.34	79.21	0.28	0.16	2.8
5/3/15	23.47	81.82	0.36	0.21	5.8
5/3/20	107.39	85.33	0.39	0.19	9.8
5/3/30	307.79	115.17	0.45	0.17	18.6
5/3/40	512.65	113.04	0.51	0.15	26.8
5/3/50	728.04	97.72	0.56	0.15	35.6
10/3/10	-46.88	41.63	0.15	0.19	2.2
10/3/15	-64.3	48.31	0.21	0.21	4.2
10/3/20	-58.23	39.11	0.26	0.24	6.4
10/3/30	60.75	74.13	0.32	0.24	13.4
10/3/40	159	53.03	0.38	0.22	19.8
10/3/50	277.53	57.82	0.44	0.2	26.8
10/5/30	-90.6	263.44	0.39	0.21	9.4
10/5/40	30.03	256.39	0.45	0.2	16.4
10/5/50	149	257.37	0.5	0.17	23.6
10/5/75	648.09	263.24	0.55	0.14	45.2
10/5/100	1215.97	274.09	0.6	0.12	68.4
15/3/15	-72.14	41.63	0.14	0.22	4
15/3/20	-77.92	44.41	0.17	0.24	6
15/3/30	29.28	44.19	0.22	0.24	13
15/3/40	95.25	62.8	0.27	0.22	18.8
15/3/50	183.28	63.76	0.31	0.22	25
20/5/30	-194.95	51.94	0.22	0.23	7.2
20/5/40	-165.81	82.38	0.26	0.24	12.6
20/5/50	-150.94	80.48	0.31	0.22	18
20/5/75	-7.71	87.02	0.4	0.22	31.6
20/5/100	281.01	108.47	0.47	0.2	49.2

Table 4.9 Results of operator-based relocation problem regarding the growth of user demand (Fixed number of vehicles and operators)

Tables 4.9 and 4.10 present the results obtained by varying the number of user requests while keeping the numbers of vehicles and operators constant for the VR-O and the VR-H, respectively. In Table 4.9, we see that all the performance indicators except operator utilization have a tendency to increase. This is not surprising given that we have the same amount of resources, i.e., a fixed number of vehicles and a fixed number of operators. Hence, after a certain point, these resources will fall short of covering the growing demand as a result of which more users will be rejected. Moreover, with an increased number of requests, vehicle utilization is also expected to get higher because there will be a larger set of alternatives regarding the assignment of vehicles to users, that is, a vehicle that was not used before is now more likely to be assigned to a user, or similarly, a vehicle that was mostly idle before may now serve additional users. Improving the vehicle utilization level will have a positive impact on the objective value. On the other hand, achieving this may require performing more relocation operations, hence, bringing additional cost.

We observe that under the operator-based relocation strategy, the increase in the total rejection penalty and relocation cost typically exceeds the improvement resulting from the profit gained by making a more effective use of the vehicles, and consequently, the objective value deteriorates. Nevertheless, we cannot draw the same conclusion for the objective value under the hybrid relocation strategy based on the results in Table 4.10. For the first and third instance groups (five vehicles/three operators and 10 vehicles/five operators), the objective value consistently gets worse whereas we observe the opposite for the fourth instance group (15 vehicles/three operators) except the decrease in 30 users as the number of requests increase. For the other two groups, the objective value is improved to a certain extent, but then starts becoming worse.

Even though the involvement of additional users contributes to relocation operations and facilitates serving more requests, the reward gained from the satisfied user requests does not always compensate for the cost associated with rejected users and relocation discount, resulting in an increase in the objective value. We observe this to be the case especially when there is a significant reduction in the operator utilization ratio. Diminished use of operators implies that most of the relocation tasks are undertaken by the users, and therefore, we speculate that the increase in the objective value is mainly due to the discounts offered to users taking part in relocation operations. In other cases, i.e., when the operator utilization remains at a fairly steady level, growing demand can yield remarkable improvements in the objective value as a result of (more) users being involved in repositioning the vehicles.

Similar to the operator-based model, vehicles are used more effectively as the number of requests increase in the case of the hybrid model. This arises from the fact that the vehicles are easier to access across the entire service network since users can also relocate them.

Finally, it is expected that the increase in the number of users necessitates more relocation operations implying higher operator utilization (especially under the operator-based relocation strategy). However, our results do not indicate a particular trend in that respect. Even though a larger number of relocations is observed to take place in general with more user requests, it does not always lead to an increase in the amount of time required for these relocation operations. Furthermore, regardless of the relocation strategy, when there are more user requests to be fulfilled, the set of users served in an optimal solution may be altered substantially in a way to acquire a larger amount of profit. Accordingly, the number of operator-based relocations as well as their total duration may increase or decrease.

# of V/O/U	OV	T(s)	VU	OU	#RU	OC	DC	ODC	NC
5/3/10	-58.9	59.61	0.32	0.17	1.8	1	1.2	0	6
5/3/15	-35.51	62.48	0.41	0.19	4	2.2	1.6	0	7.2
5/3/20	19.41	69.48	0.46	0.16	7.4	2.6	2	0.4	7.6
5/3/30	199.13	78.76	0.55	0.14	14.8	3.6	3.6	0.4	7.6
5/3/40	392.99	108.68	0.61	0.14	23	3.8	4.8	0.4	8
5/3/50	596.36	116.29	0.64	0.09	31.4	4.4	4.6	0.4	9.2
10/3/10	-82.7	49.17	0.17	0.18	1.2	1.6	0.8	0	6.4
10/3/15	-130.88	43.41	0.24	0.22	2.4	1.4	1.8	0	9.4
10/3/20	-168.4	82.41	0.31	0.23	3.2	3.6	3.2	0	10
10/3/30	-128.98	84.16	0.41	0.22	7.4	5.2	4.8	0	12.6
10/3/40	-77.81	77.1	0.5	0.2	13	6.2	7.2	0.2	13.4
10/3/50	7.79	100.09	0.57	0.19	18.4	8.4	8	0.4	14.8
10/5/30	-231.57	229.57	0.46	0.18	5	5.8	5.4	0.2	13.6
10/5/40	-167.91	223.09	0.55	0.18	10.6	7.2	7	0.2	15
10/5/50	-72.31	244.78	0.6	0.14	16.6	8.4	8.4	0.6	16
10/5/75	388.11	270.58	0.65	0.1	36.4	9	8.6	0.6	20.4
10/5/100	942.56	296.26	0.69	0.06	58.4	10.6	11.6	0.4	19
15/3/15	-140.8	38.24	0.16	0.21	2	2	1.6	0	9.4
15/3/20	-186	44.56	0.21	0.22	3	4	2.8	0	10.2
15/3/30	-178.23	79.71	0.29	0.24	6.4	6.6	4.2	0	12.8
15/3/40	-217.08	92.66	0.37	0.24	10	7.6	6.8	0.2	15.4
15/3/50	-229.73	114.99	0.44	0.24	13.8	7.6	9	0.8	18.8
20/5/30	-343.94	90.91	0.25	0.21	2.8	5.2	4.2	0	17.8
20/5/40	-421.1	98.98	0.33	0.24	5.2	7.4	6.2	0.2	21
20/5/50	-486.54	108.51	0.39	0.24	8	8.2	7.8	0.8	25.2
20/5/75	-491.76	190.68	0.52	0.22	17.8	16.4	10.6	0.6	29.6
20/5/100	-280.19	221.51	0.6	0.16	32.8	17.4	14.6	0.6	34.6

Table 4.10 Results of hybrid relocation problem regarding the growth of user demand (Fixed number of vehicles and operators)

4.2.4 Comparison between operator-based and hybrid relocation strategies

Next, we compare the results obtained with the operator-based relocation model and the hybrid relocation model to investigate how the adopted relocation strategy impacts the operational efficiency of the car-sharing system measured by the performance indicators defined earlier. Recall that the main difference between these two models is that in the former, repositioning of the vehicles is carried out only by operators, whereas in the latter, users are also engaged in relocation activities by picking up/dropping off the vehicles at locations different from their origin/destination in exchange for a discount. Even if the numbers of vehicles and operators in the service network remain the same, we observe that the values of the performance indicators change significantly depending on the relocation strategy. In particular, our results clearly demonstrate the advantages of involving the users in relocation operations in terms of the number of requests served, the objective value, the vehicle and operator utilization levels.

Table 4.11 reports the values of our performance indicators averaged out over all uniform instances including the same number of users to be able to assess the benefits of allowing user-based relocations in addition to employing operators. For a given number of users and a given performance indicator, the average values are presented first for the operator-based model (VR-O), then for the hybrid model (VR-H). Followed by these two numbers, the percentage difference with respect to the operator-based value is provided. Negative (positive) percentages refer to a decrease (increase) in the associated quantity obtained by adopting the hybrid strategy. As an example, for the instances with 100 users, the average objective value decreases by 199%, the average number of rejected users decreases by 36%, the average vehicle utilization ratio increases by 18%, and the average operator utilization ratio decreases by 11%.

The flexibility arising from the sheer existence of alternative service options for users enables making more effective use of the vehicles, which contributes towards higher vehicle utilization levels, lower rejection rates and increased profitability. Consequently, we observe a considerable improvement in the objective values. On the other hand, adoption of the hybrid relocation strategy has a negative effect on operator utilization. This is not surprising due to the diminishing need for operators when users participate in relocating the vehicles. Moreover, having a user reposition a vehicle may be more convenient and less expensive than deploying an operator for the same task, especially when no operators are available nearby.

	OV		**%**	**#RU**		**%**	**VU**		**%**	**OU**		**%**
#U	**VR-O**	**VR-H**	**change**	**VR-O**	**VR-H**	**change**	**VR-O**	**VR-H**	**change**	**VR-O**	**VR-H**	**change**
10	65.58	21.33	-67	4.79	3.49	-27	0.26	0.32	23	0.18	0.19	6
15	113.06	44.27	-61	8.13	6.2	-24	0.28	0.34	21	0.2	0.19	-5
20	162.63	62.64	-61	11.45	8.55	25	0.27	0.35	30	0.2	0.18	-10
30	-19.73	-163.02	-726	11.42	6.9	-40	0.33	0.4	21	0.19	0.18	-5
40	79.83	-128.38	-261	17.87	11.73	34	0.38	0.47	24	0.19	0.18	-5
50	178.52	-72.18	-140	24.58	16.93	-31	0.43	0.53	23	0.17	0.16	-6
75	-396.22	-741.21	-87	23.67	14	-41	0.37	0.44	19	0.18	0.17	-6
100	-245.26	-732.52	-199	38.33	24.64	-36	0.44	0.52	18	0.18	0.16	-11

Table 4.11 Comparison between operator-based and hybrid relocation model based on average values

4.2.5 An alternative user incentive scheme

Incentive scheme used in the hybrid relocation model determines the degree of discount offered to users, and hence, is likely to have a considerable impact on the profitability of the car-sharing system. In our formulation of the VR-H, we considered a simple incentive scheme where a user is given 50% discount on the original rental price if the user is offered either an alternative pick-up location or an alternative drop-off location, and a free rental if the user is offered both an alternative pick-up location and an alternative drop-off location. To explore how the performance of the system changes when a different incentive scheme is adopted, we now consider total walking distances of users when offering discounts to the users. First, for every user u and every pair of candidate pick-up/drop-off locations $(i,j) \in S_u$, we calculate the total of the (walking) distances from the user's actual origin o_u to i and from j to the user's actual destination d_u. Second, we apply a normalization step to bring these distance values into the range [0,1] by assigning 0 to the minimum distance and 1 to the maximum distance for each user. We then use these normalized values as discount scores, in other words, the percentage discount offered to user u for picking up a vehicle at i and dropping it off at j such that $(i,j) \in S_u$ will be equal to the corresponding normalized distance.

To find out the effect of this walking distance based incentive scheme on overall system performance, an additional set of experiments was performed on the uniform instances containing 100 users. The average values of performance indicators are reported in Table 4.12 where "fixed" and "variable" in the first column refer to our original incentive scheme and the walking distance based incentive scheme, respectively.

Based on the results, we observe slightly worse results with respect to the objective value, the vehicle utilization ratio and the service level when a discount based on the distance that a user has to walk before picking up or after dropping off a vehicle is offered. Even though the alternative incentive scheme ensures a relatively more fair distribution of reward among the users, the discount level generally increases in pairs of candidate pick-up/drop-off locations $(i,j) \in S_u$ for each user. As a consequence, user-based relocations become more expensive and less users are employed in relocation tasks (see the increase in the values under the column NC). Higher discount level affects the number of served users inversely. It is more profitable to the system to reject some users instead of relocating vehicles to serve them. This, in return, decreases the average vehicle occupancy rates. Moreover, less user involvement in relocation operations increases the dependence on operators, and thus, the average operator utilization levels.

Incentive Type	# of V/O/U	OV	T(s)	VU	OU	#RU	OC	DC	ODC	NC
fixed	10/5/100	942.55	296.26	0.69	0.06	58.4	10.6	11.6	0.4	19
variable	10/5/100	998.02	892.32	0.69	0.07	60.6	6.6	9.8	1.2	21.8
fixed	10/10/100	939.73	273.09	0.69	0.03	58.2	10.6	11.2	0.6	19.4
variable	10/10/100	992.73	1261.65	0.69	0.03	60.4	6.8	9.6	1	22.2
fixed	10/15/100	937.73	276.06	0.69	0.02	58.2	10.4	11.4	0.6	19.4
variable	10/15/100	992.21	538.82	0.69	0.02	60.4	7	9.6	1	22
fixed	20/5/100	-280.18	221.5	0.59	0.16	32.8	17.4	14.6	0.6	34.6
variable	20/5/100	-187.71	422.6	0.58	0.18	34.8	13.6	13	0.6	38
fixed	20/10/100	-440.11	510.63	0.63	0.11	30.4	15.4	16.6	0.4	37.2
variable	20/10/100	-357.51	719.94	0.62	0.12	32.6	10.8	14.4	0.4	41.8
fixed	20/15/100	-472.49	433.27	0.63	0.08	29.2	15	16.8	0.6	38.4
variable	20/15/100	-392.38	588.9	0.62	0.08	32.4	10	12.8	0.6	44.2
fixed	30/5/100	-931.82	450.28	0.49	0.23	19.6	22.2	15.6	0.4	42.2
variable	30/5/100	-802.21	442.36	0.48	0.24	22.6	15.8	12.8	1	47.8
fixed	30/10/100	-1279.76	565.43	0.53	0.19	13.8	16.8	15.4	0.2	53.8
variable	30/10/100	-1176.53	607.57	0.52	0.19	16.2	12.2	11.6	0.8	59.2
fixed	30/15/100	-1403.58	492.25	0.55	0.15	11.8	15.2	13.6	0.6	58.8
variable	30/15/100	-1307.97	518.52	0.54	0.16	13.8	10.8	11	1	63.4
fixed	40/5/100	-1131.67	182.58	0.39	0.26	15.6	22.6	15.6	0.8	45.4
variable	40/5/100	-985.88	607.57	0.38	0.27	18.6	16.8	12.2	1.4	51
fixed	40/10/100	-1553.78	357.94	0.43	0.22	8.2	16.8	13.4	0.2	61.4
variable	40/10/100	-1447.43	385.93	0.43	0.24	10.6	13	11.2	0.4	64.8
fixed	40/15/100	-1733.18	348.61	0.44	0.19	5.6	11.8	11	0	71.6
variable	40/15/100	-1657.13	376.56	0.44	0.2	6.6	9.8	9.4	0.2	74
fixed	50/5/100	-1167.11	170.65	0.31	0.27	15.6	20.8	15	0.4	48.2
variable	50/5/100	-1021.12	246.1	0.31	0.28	18.2	15.4	12	2	52.4
fixed	50/10/100	-1617.04	338.45	0.35	0.22	7.4	16.4	13.2	0	63
variable	50/10/100	-1508.47	400.54	0.35	0.24	9.4	12.6	12.2	0.2	65.6
fixed	50/15/100	-1796.41	313.77	0.36	0.19	4.8	11.2	11	0	73
variable	50/15/100	-1721.49	315.27	0.36	0.2	5.8	7.6	9.8	0.2	76.6

Table 4.12 Results of hybrid relocation problem regarding different incentive types

4.2.6 Increasing the time granularity of the network

As mentioned earlier, we discretized the planning horizon using a time step length of 10 minutes. Due to the rounding process in our calculation of the travel times in terms of number of time steps, some travel times are overestimated. Hence, the discretization scheme adopted in our experiments yields approximate solutions to the VR-O and the VR-H. The effect of overestimated travel times on solution quality can be reduced by refining the time-space network, i.e., using shorter time steps. This will enlarge the search space, and likely increase the computational effort required to solve the models. However, working on a finer network, one can expect to obtain solutions that are closer to optimal. To this end, we explore the effect of increasing the granularity of our time-expanded networks by reducing the time step length to five minutes. We perform experiments using the hybrid relocation model on the uniform instances with 100 users to observe the impact of changing the discretization scheme. Average values of the performance indicators are reported in Table 4.13.

Discretization Scheme	# of V/O/U	OV	T(s)	VU	OU	#RU	OC	DC	ODC	NC
10 min	10/5/100	942.55	296.26	0.69	0.06	58.4	10.6	11.6	0.4	19
5 min	10/5/100	825.1	2144.6	0.73	0.07	57.4	7.2	11.2	0.6	23.6
10 min	10/10/100	939.73	273.09	0.69	0.03	58.2	10.6	11.2	0.6	19.4
5 min	10/10/100	808.3	748.86	0.73	0.04	57.4	8	9.6	0.2	24.8
10 min	10/15/100	937.73	276.06	0.69	0.02	58.2	10.4	11.4	0.6	19.4
5 min	10/15/100	807	726.4	0.73	0.03	57	8.2	9.8	0.2	24.8
10 min	20/5/100	-280.18	221.5	0.59	0.16	32.8	17.4	14.6	0.6	34.6
5 min	20/5/100	-483.43	862.09	0.63	0.19	29.2	15	14.4	0.2	41.2
10 min	20/10/100	-440.11	510.63	0.63	0.11	30.4	15.4	16.6	0.4	37.2
5 min	20/10/100	-671.55	3066.41	0.66	0.12	26.4	12.2	14.4	0.4	46.6
10 min	20/15/100	-472.49	433.27	0.63	0.08	29.2	15	16.8	0.6	38.4
5 min	20/15/100	-713.46	2355.5	0.66	0.09	25.4	12.4	14.4	0.4	47.4
10 min	30/5/100	-931.82	450.28	0.49	0.23	19.6	22.2	15.6	0.4	42.2
5 min	30/5/100	-1105.99	1753.94	0.5	0.26	17	19.6	13.4	0.2	49.8
10 min	30/10/100	-1279.76	565.43	0.53	0.19	13.8	16.8	15.4	0.2	53.8
5 min	30/10/100	-1469	2802.9	0.55	0.22	11	12.8	12.6	0.2	63.4
10 min	30/15/100	-1403.58	492.25	0.55	0.15	11.8	15.2	13.6	0.6	58.8
5 min	30/15/100	-1607.45	2511.6	0.57	0.18	7.6	10.2	12.4	0	69.8
10 min	40/5/100	-1131.67	182.58	0.39	0.26	15.6	22.6	15.6	0.8	45.4
5 min	40/5/100	-1244	705.58	0.39	0.28	15	19.4	12.4	0	53.2
10 min	40/10/100	-1553.78	357.94	0.43	0.22	8.2	16.8	13.4	0.2	61.4
5 min	40/10/100	-1654.84	1790.29	0.43	0.24	7.2	13.6	9.6	0	69.6
10 min	40/15/100	-1733.18	348.61	0.44	0.19	5.6	11.8	11	0	71.6
5 min	40/15/100	-1805.97	1341.46	0.45	0.2	4.2	6.8	10	0	79
10 min	50/5/100	-1167.11	450.28	0.31	0.27	15.6	20.8	15	0.4	48.2
5 min	50/5/100	-1268.82	566.35	0.31	0.3	14.2	18.4	13.4	0	54
10 min	50/10/100	-1617.04	338.45	0.35	0.22	7.4	16.4	13.2	0	63
5 min	50/10/100	-1703.11	1583.21	0.35	0.24	6.6	12	11.4	0	70
10 min	50/15/100	-1796.41	313.77	0.36	0.19	4.8	11.2	11	0	73
5 min	50/15/100	-1850.05	974.87	0.36	0.2	4.2	6	8.4	0	81.4

Table 4.13 Results of hybrid relocation problem regarding different time discretization schemes

Decreasing the time step length from 10 minutes to five minutes leads to a better approximation of the travel times, giving rise to a larger number of paths that users, vehicles, and operators can take in the time-space network. As a result, alternative opportunities will emerge for serving the demand within the planning horizon, i.e., some solutions that were deemed infeasible before (in the 10 minute discretization scenario) will now be feasible. Thus, when shorter time steps are used, the objective value is expected to be improved, or in the worst case, remain unchanged. This fact is supported by our results, that is, we observe an average reduction of 20% in the (average) objective values based on our experiments. Our results also suggest a slight growth in the average number of served user requests as well as vehicle and operator utilization levels. Nevertheless, these improvements are achieved at the cost of a notable increase in solution times although they remain within an acceptable range.

Finally, we remark that in a problem setting where all travel times are integer (in minutes), our formulations can be used to produce optimal solutions with a time step length of one minute although dealing with such large time-space networks is typically more challenging and computationally expensive. In that case, an efficient solution approach manipulating a proper family of partially time expanded networks may be developed instead of using an off-the-shelf solver on the fully time expanded network (see Boland et al. (2017)).

5. CONCLUSION

In this thesis, we consider two vehicle relocation problems encountered in free-floating car-sharing systems and propose multi-commodity flow formulations based on time-space networks. First, we address an operator-based vehicle relocation problem, namely the VR-O, in which vehicles are repositioned only by dedicated staff. In this problem, the goal is to decide on (1) which user requests to be served given their time windows and (2) the routes of the vehicles and the operators during a finite planning horizon. We propose two formulations for the VR-O. The first formulation treats each vehicle, operator, and user as separate commodities, and is capable of producing detailed solutions, i.e., a route/path per each individual commodity in the time-space network. Assuming that all vehicles and operators are identical, we derive a second, and a more compact, formulation for the VR-O by using aggregate vehicle and operator flow variables. Since the aggregate formulation involves significantly fewer variables and constraints, it can be solved more efficiently compared to the first model.

Later, we study a hybrid relocation problem, namely the VR-H, in which users are incentivized to take part in relocation tasks in addition to the operators. In particular, each user is offered a pick-up location and a drop-off location, among a set of alternatives, which may not be the same as the user's origin and/or destination. In case the suggested trip is different from the user's requested trip, the user benefits from a discounted price depending on the duration of her requested trip. Although these discounts lead to a reduction in the profit gained from certain users, they may help achieving better vehicle utilization levels, thereby increasing overall profitability of the car-sharing system. Hence, which users to provide with an incentive to change their pick-up and drop-off locations and by how much, should also be determined in the VR-H. The aggregate formulation proposed for the VR-O is extended to cover these additional decisions.

Most of the existing data sets used for computational testing purposes in the car-sharing literature are not publicly available. Therefore, we generate new sets of instances of different geographies, i.e., clustered, uniformly distributed, and a combination of clustered and uniformly distributed. The results obtained by solving these test instances with our mathematical models are analyzed thoroughly. We evaluate different vehicle, operator, and user related parameter configurations to gain deeper insights into the operational performance of free-floating car-sharing systems. We examine the effect of different parameter configurations on several performance indicators, i.e., objective value, number of rejected users, vehicle and operator utilization levels.

We aim to evaluate the operational performance of a given car-sharing provider with our models. The difficulties arising from the deterministic and static assumptions in our models can be dealt by using the proposed models in a rolling horizon framework. By solving the models with a shorter planning horizon on a rolling basis, the computational challenges with longer planning horizons can be handled easily. If a longer planning horizon is preferred, the length of a time step can be increased and then for certain areas (a subset of locations in the network), the network can be refined to obtain better solutions. Additionally, in case of changes in given input information, the models can be re-solved based on the updated input information.

Our computational results show that adopting a hybrid relocation strategy over an operator-based one can remarkably increase the profitability of the car-sharing system by attaining higher service levels. For small-size instances (10, 15, 20 users), the number of rejected users decreases on the average by 27%, 24%, 25%, respectively. For medium-size instances (30, 40, 50 users), we obtain an average 39%, 34%, 31% reduction in the same quantity, which drops down by 41% and 36% in case of large-size instances (75, 100 users), respectively. It is worth noting that, a significant decrease in the objective value results from smaller user rejection rates in all instance sets.

It is also suggested by our results that it is particularly important to decide carefully on the size of the vehicle fleet and the staff level to be maintained with respect to given demand. Hence, car-sharing companies should devote a sufficient amount of time and effort to examine alternative scenarios with varying parameter configurations to achieve better performance.

We also carry out two additional experiments, one by changing the time discretization scheme and another by considering a different incentive mechanism. We are able to experimentally show that the choice of time granularity of the network plays an important role in improving the performance of the car-sharing system. We also observe that the choice of user incentive scheme is likely to contribute to the results.

Last but not least, we observe better results for both models regarding objective value and the number of rejected users in clustered instances compared to other network geographies. Hence, clustering user demands and positioning resources strategically across the network may provide a worthwhile opportunity to further enhance the overall performance of car-sharing systems.

The relocations in our current models are designed to satisfy user demands in the planning period. The final distribution of vehicles and operators does not necessarily reflect the optimal distribution for the next planning period. At the end of the planning horizon, vehicles and operators may be positioned in an anticipation of future demand. This can be achieved by adding two sets of constraints to our models, one for the vehicles and one for the operators. In the long term, distributing vehicles to their optimized final locations (equivalently, the optimized starting locations for the next planning period) taking into account the user demands for the next period could be useful for the car-sharing systems.

This thesis helps us enhance our understanding of vehicle relocation problem in free-floating car-sharing systems under different relocation strategies. This study is not specifically designed to explore the effect of users' behaviour. We assume that users will accept picking up/dropping off vehicles close to their origin/destination locations when they are provided with an incentive. However, this assumption may not always hold in real life. Even though a user accepts the trip suggested to her, she may change her mind while en-route and decide to keep the vehicle for a longer period of time, or leave the vehicle at a different location etc. She may even cancel the trip completely. Therefore, to produce more robust solutions to vehicle relocation problems, it would be interesting to develop a modeling approach or a solution framework incorporating different user behaviors.

Additionally, we assume that we have perfect knowledge of the demand, i.e., the demands are considered to be static and deterministic. This is a limiting assumption for a real life free-floating car-sharing system, which is highly dynamic and stochastic in nature. Hence, another direction for future research could be to devise a method that is capable of finding solutions at the operational level under stochastic (and possibly also dynamically arriving) user requests as well as to examine the impact of stochasticity of demand on the performance of car-sharing systems. Another limiting assumption could be deterministic travel times. Depending on the day or the hour, travelling from one location to another location may take different amounts of time. Alternatively, the time travelled in opposite directions can be different. Considering the presence of uncertainties in real life, the possibility of travel time variability warrants further investigation.

Finally, autonomous vehicles will likely transform and (re-)shape urban mobility in the not-so-distant future. As a promising mode of transportation, these vehicles may also have an enormous impact on the future of car-sharing industry. Hence, it is important to further investigate the opportunities of autonomous (or semi-autonomous) vehicle integration in car-sharing systems.

BIBLIOGRAPHY

Ait-Ouahmed, Amine, Didier Josselin, and Fen Zhou (2017). "Relocation optimization of electric cars in one-way car-sharing systems: modeling, exact solving and heuristics algorithms". In: *International Journal of Geographical Information Science* 32.2, pp. 367–398. DOI: 10.1080/13658816.2017.1372762. URL: https://doi.org/10.1080/13658816.2017.1372762.

Almeida Correia, Gonçalo Homem de and António Pais Antunes (2012). "Optimization approach to depot location and trip selection in one-way carsharing systems". In: *Transportation Research Part E: Logistics and Transportation Review* 48.1, pp. 233–247. DOI: 10.1016/j.tre.2011.06.003. URL: https://doi.org/10.1016/j.tre.2011.06.003.

Barth, Matthew and Michael Todd (1999). "Simulation model performance analysis of a multiple station shared vehicle system". In: *Transportation Research Part C: Emerging Technologies* 7.4, pp. 237–259. ISSN: 0968-090X. DOI: https://doi.org/10.1016/S0968-090X(99)00021-2. URL: http://www.sciencedirect.com/science/article/pii/S0968090X99000212.

Becker, Henrik, Francesco Ciari, and Kay W. Axhausen (2017). "Comparing carsharing schemes in Switzerland: User groups and usage patterns". In: *Transportation Research Part A: Policy and Practice* 97, pp. 17–29. DOI: 10.1016/j.tra.2017.01.004. URL: https://doi.org/10.1016/j.tra.2017.01.004.

Benjaafar, Saif, Xiang Li, and Xiaobo Li (2017). "Inventory Repositioning in On-Demand Product Rental Networks". In: *SSRN Electronic Journal*. DOI: 10.2139/ssrn.2942921. URL: https://doi.org/10.2139/ssrn.2942921.

Boland, Natashia, Mike Hewitt, Luke Marshall, and Martin Savelsbergh (2017). "The continuous-time service network design problem". In: *Operations Research* 65.5, pp. 1303–1321.

Boyaci, Burak, Konstantinos G. Zografos, and Nikolas Geroliminis (2015). "An optimization framework for the development of efficient one-way car-sharing systems." In: *European Journal of Operational Research* 240.3, pp. 718–733. URL: http://dblp.uni-trier.de/db/journals/eor/eor240.html#BoyaciZG15.

Boyacı, Burak and Konstantinos G. Zografos (2019). "Investigating the effect of temporal and spatial flexibility on the performance of one-way electric carsharing systems". In: *Transportation Research Part B: Methodological* 129, pp. 244–272. DOI: 10.1016/j.trb.2019.09.003. URL: https://doi.org/10.1016/j.trb.2019.09.003.

Bruglieri, Maurizio, Ferdinando Pezzella, and Ornella Pisacane (2018). "A two-phase optimization method for a multiobjective vehicle relocation problem in electric carsharing systems". In: *Journal of Combinatorial Optimization* 36.1, pp. 162–193. DOI: 10.1007/s10878-018-0295-5. URL: https://doi.org/10.1007/s10878-018-0295-5.

Carlier, Aurélien, Alix Munier-Kordon, and Witold Klaudel (2015). "Mathematical Model for the Study of Relocation Strategies in One-way Carsharing Systems". In: *Transportation Research Procedia* 10, pp. 374–383. DOI: 10.1016/j.trpro.2015.09.087. URL: https://doi.org/10.1016/j.trpro.2015.09.087.

Fan, Wei (David) (2014). "Optimizing Strategic Allocation of Vehicles for One-Way Car-sharing Systems Under Demand Uncertainty". In: *Journal of the Transportation Research Forum*. DOI: 10.5399/osu/jtrf.53.3.4252. URL: https://doi.org/10.5399/osu/jtrf.53.3.4252.

Febbraro, Angela Di, Nicola Sacco, and Mahnam Saeednia (2012). "One-Way Carsharing". In: *Transportation Research Record: Journal of the Transportation Research Board* 2319.1, pp. 113–120. DOI: 10.3141/2319-13. URL: https://doi.org/10.3141/2319-13.

— (2019). "One-Way Car-Sharing Profit Maximization by Means of User-Based Vehicle Relocation". In: *IEEE Transactions on Intelligent Transportation Systems* 20.2, pp. 628–641. DOI: 10.1109/tits.2018.2824119. URL: https://doi.org/10.1109/tits.2018.2824119.

Folkestad, Carl Axel, Nora Hansen, Kjetil Fagerholt, Henrik Andersson, and Giovanni Pantuso (2020). "Optimal charging and repositioning of electric vehicles in a free-floating carsharing system". In: *Computers & Operations Research* 113, p. 104771. DOI: 10.1016/j.cor.2019.104771. URL: https://doi.org/10.1016/j.cor.2019.104771.

Gambella, Claudio, Enrico Malaguti, Filippo Masini, and Daniele Vigo (2018). "Optimizing relocation operations in electric car-sharing". In: *Omega* 81, pp. 234–245. DOI: 10.1016/j.omega.2017.11.007. URL: https://doi.org/10.1016/j.omega.2017.11.007.

He, Long, Zhenyu Hu, and Meilin Zhang (2019). "Robust Repositioning for Vehicle Sharing". In: *Manufacturing & Service Operations Management*. DOI: 10.1287/msom.2018.0734. URL: https://doi.org/10.1287/msom.2018.0734.

Herrmann, Sascha, Frederik Schulte, and Stefan Vo (2014). "Increasing Acceptance of Free-Floating Car Sharing Systems Using Smart Relocation Strategies: A Survey Based Study of car2go Hamburg”. In: *Lecture Notes in Computer Science*. Springer International Publishing, pp. 151–162. DOI: 10.1007/978-3-319-11421-7_10. URL: https://doi.org/10.1007/978-3-319-11421-7_10.

Illgen, Stefan and Michael Höck (2019). “Literature review of the vehicle relocation problem in one-way car sharing networks”. In: *Transportation Research Part B: Methodological* 120, pp. 193–204. DOI: 10.1016/j.trb.2018.12.006. URL: https://doi.org/10.1016/j.trb.2018.12.006.

Jorge, D., G. H. A. Correia, and C. Barnhart (2014). “Comparing Optimal Relocation Operations With Simulated Relocation Policies in One-Way Carsharing Systems”. In: *IEEE Transactions on Intelligent Transportation Systems* 15.4, pp. 1667–1675. ISSN: 1558-0016. DOI: 10.1109/TITS.2014.2304358.

Jorge, Diana, Gonçalo Correia, and Cynthia Barnhart (2012). “Testing the Validity of the MIP Approach for Locating Carsharing Stations in One-way Systems”. In: *Procedia - Social and Behavioral Sciences* 54, pp. 138–148. DOI: 10.1016/j.sbspro.2012.09.733. URL: https://doi.org/10.1016/j.sbspro.2012.09.733.

Kek, Alvina G.H., Ruey Long Cheu, Qiang Meng, and Chau Ha Fung (2009). “A decision support system for vehicle relocation operations in carsharing systems”. In: *Transportation Research Part E: Logistics and Transportation Review* 45.1, pp. 149–158. DOI: 10.1016/j.tre.2008.02.008. URL: https://doi.org/10.1016/j.tre.2008.02.008.

Krumke, Sven O., Alain Quilliot, Annegret K. Wagler, and Jan-Thierry Wegener (2014). “Relocation in Carsharing Systems Using Flows in Time-Expanded Networks”. In: *Experimental Algorithms*. Springer International Publishing, pp. 87–98. DOI: 10.1007/978-3-319-07959-2_8. URL: https://doi.org/10.1007/978-3-319-07959-2_8.

Nair, Rahul and Elise Miller-Hooks (2011). “Fleet Management for Vehicle Sharing Operations”. In: *Transportation Science* 45.4, pp. 524–540. DOI: 10.1287/trsc.1100.0347. URL: https://doi.org/10.1287/trsc.1100.0347.

Niels, Tanja and Klaus Bogenberger (2017). “Booking Behavior of Free-Floating Carsharing Users”. In: *Transportation Research Record: Journal of the Transportation Research Board* 2650.1, pp. 123–132. DOI: 10.3141/2650-15. URL: https://doi.org/10.3141/2650-15.

Nourinejad, Mehdi, Sirui Zhu, Sina Bahrami, and Matthew J. Roorda (2015). “Vehicle relocation and staff rebalancing in one-way carsharing systems”. In: *Transportation Research Part E: Logistics and Transportation Review* 81, pp. 98–113. ISSN: 1366-5545. DOI: https://doi.org/10.1016/j.tre.2015.06.012. URL: http://www.sciencedirect.com/science/article/pii/S1366554515001349.

Repoux, Martin, Burak Boyacı, and Nikolas Geroliminis (2015). “Simulation and optimization of one-way car-sharing systems with variant relocation policies”. In:

Repoux, Martin, Mor Kaspi, Burak Boyacı, and Nikolas Geroliminis (2019). “Dynamic prediction-based relocation policies in one-way station-based carsharing systems with complete journey reservations”. In: *Transportation Research Part B: Methodological* 130, pp. 82–104. DOI: 10.1016/j.trb.2019.10.004. URL: https://doi.org/10.1016/j.trb.2019.10.004.

Santos, Gonçalo and Gonçalo Correia (2015). “A MIP Model to Optimize Real Time Maintenance and Relocation Operations in One-way Carsharing Systems”. In: *Transportation Research Procedia* 10, pp. 384–392. DOI: 10.1016/j.trpro.2015.09.088. URL: https://doi.org/10.1016/j.trpro.2015.09.088.

Shaheen, Susan (2020). “Innovative Mobility: Carsharing Outlook; Carsharing Market Overview, Analysis, and Trends - Spring 2020”. In: DOI: 10.7922/G2125QWJ. URL: https://escholarship.org/uc/item/61q03282.

United Nations (2018). *68% of the world population projected to live in urban areas by 2050, says UN*. URL: https://www.un.org/development/desa/en/news/population/2018-revision-of-world-urbanization-prospects.html.

Wadhwani, Preeti and Prasenjit Saha (2020). *Car Sharing Market Size By Model (P2P, Station-Based, Free-Floating), Business Model (Round Trip, One Way), Application, Industry Analysis Report, Regional Outlook, Application Potential, Price Trend, Competitive Market Share & Forecast, 2020-2026*. Last accessed 10 July 2020. URL: https://www.gminsights.com/industry-analysis/carsharing-market.

Wang, Ling, Qi Liu, and Wanjing Ma (2019). “Optimization of dynamic relocation operations for one-way electric carsharing systems”. In: *Transportation Research Part C: Emerging Technologies* 101, pp. 55–69. DOI: 10.1016/j.trc.2019.01.005. URL: https://doi.org/10.1016/j.trc.2019.01.005.

Warrington, Joseph and Dominik Ruchti (2019). “Two-stage stochastic approximation for dynamic rebalancing of shared mobility systems”. In: *Transportation Research Part C: Emerging Technologies* 104, pp. 110–134. DOI: 10.1016/j.trc.2019.04.021. URL: https://doi.org/10.1016/j.trc.2019.04.021.

Weikl, Simone and Klaus Bogenberger (2015). “A practice-ready relocation model for free-floating carsharing systems with electric vehicles – Mesoscopic approach and field trial results”. In: *Transportation Research Part C: Emerging Technologies* 57, pp. 206–223. DOI: 10.1016/j.trc.2015.06.024. URL: https://doi.org/10.1016/j.trc.2015.06.024.

Xu, Min, Qiang Meng, and Zhiyuan Liu (2018). “Electric vehicle fleet size and trip pricing for one-way carsharing services considering vehicle relocation and personnel assignment”. In: *Transportation Research Part B: Methodological* 111, pp. 60–82. DOI: 10.1016/j.trb.2018.03.001. URL: https://doi.org/10.1016/j.trb.2018.03.001.

Zhao, Meng et al. (2018). “An integrated framework for electric vehicle rebalancing and staff relocation in one-way carsharing systems: Model formulation and Lagrangian relaxation-based solution approach”. In: *Transportation Research Part B: Methodological* 117, pp. 542–572. DOI: 10.1016/j.trb.2018.09.014. URL: https://doi.org/10.1016/j.trb.2018.09.014.

APPENDIX A

Table A.1 Results of operator-based relocation problem for small-size instances in the instance class C

# of V/O/U	OV	T(s)	VU	OU	#RU
2/1/10	116.73	391.67	0.27	0.18	6.6
2/2/10	83.95	396.99	0.36	0.13	5.6
2/3/10	77.14	372.98	0.36	0.1	5.6
3/1/10	101.18	397.8	0.22	0.22	6
3/2/10	45.29	418.34	0.31	0.19	4.4
3/3/10	22.47	311.72	0.34	0.15	4
4/1/10	81.76	438.64	0.18	0.25	5.6
4/2/10	-6.67	518.77	0.29	0.22	3.4
4/3/10	-36.78	517.77	0.33	0.22	2.2
5/1/10	58.13	498	0.16	0.28	5
5/2/10	-34.64	541.38	0.26	0.24	2.6
5/3/10	-78.39	495.85	0.3	0.22	1.2
10/1/10	18.06	424.82	0.09	0.29	4.4
10/2/10	-68.84	533.29	0.14	0.26	1.8
10/3/10	-113.11	491.14	0.17	0.22	0.2
2/1/15	197.49	432.14	0.36	0.19	10.8
2/2/15	159.48	492.59	0.44	0.12	9.8
2/3/15	151.9	389.34	0.46	0.1	9.4
3/1/15	164.14	471.76	0.29	0.22	10
3/2/15	94.21	403.39	0.4	0.22	8
3/3/15	69.02	281.81	0.43	0.16	7.4
4/1/15	143.93	478.83	0.24	0.25	9.4
4/2/15	20.49	506.67	0.37	0.24	6.4
4/3/15	-18.87	534.68	0.42	0.19	5.4
5/1/15	120.72	599.84	0.21	0.28	9
5/2/15	-13.37	647.61	0.32	0.22	5.8
5/3/15	-75.04	577.16	0.37	0.19	4
10/1/15	58.92	669.86	0.12	0.25	7.4
10/2/15	-95.04	634.2	0.19	0.24	3.8
10/3/15	-161.26	647.38	0.22	0.23	1.8
15/1/15	21.54	507.73	0.09	0.29	6.8
15/2/15	-118.84	693.7	0.13	0.23	3.6
15/3/15	-190.11	416.88	0.16	0.24	1.2
2/1/20	316.32	579.16	0.38	0.16	15.6
2/2/20	267	515.78	0.49	0.12	14.4
2/3/20	263.12	403.12	0.49	0.07	14.4
3/1/20	265.4	651	0.32	0.22	14.4
3/2/20	194.28	509.36	0.42	0.2	12.6
3/3/20	167.6	365.95	0.47	0.13	12.4
4/1/20	231.37	564.45	0.27	0.25	13.6
4/2/20	95.65	684.97	0.43	0.24	10.2
4/3/20	47.82	577.33	0.46	0.17	9.6
5/1/20	190.45	558.15	0.24	0.25	12.8
5/2/20	33.24	729.97	0.38	0.25	8.8
5/3/20	-27.73	748.28	0.43	0.21	7.4
10/1/20	97.7	623.59	0.15	0.27	10.8
10/2/20	-98.37	656.46	0.23	0.27	6.2
10/3/20	-188.01	688.94	0.28	0.25	3.4
15/1/20	55.16	566.59	0.11	0.31	10
15/2/20	-148.15	630.55	0.17	0.25	5.4
15/3/20	-238.39	416.67	0.2	0.26	2.6
20/1/20	53.48	645.95	0.08	0.29	10
20/2/20	-151.95	718.82	0.13	0.26	5.2
20/3/20	-247.19	456.84	0.15	0.26	2.4

Table A.2 Results of operator-based relocation problem for medium-size instances in the instance class C

# of V/O/U	OV	T(s)	VU	OU	#RU
5/3/30	86.87	809.07	0.51	0.16	15.6
5/5/30	57.75	470.32	0.53	0.12	14.8
5/7/30	52.4	361.54	0.54	0.09	14.6
10/3/30	-182.89	763.59	0.35	0.24	9.2
10/5/30	-319.63	612.53	0.42	0.21	5
10/7/30	-378.9	368.31	0.44	0.18	3.4
15/3/30	-300.93	453.28	0.26	0.24	6.6
15/5/30	-455.83	431.13	0.31	0.23	2
15/7/30	-523.47	288.82	0.33	0.18	0
20/3/30	-319.85	398.54	0.2	0.24	6.2
20/5/30	-474.9	380.3	0.24	0.23	1.6
20/7/30	-541.45	241.07	0.25	0.18	0
5/3/40	322.23	854.85	0.54	0.16	24.8
5/5/40	283.54	471.01	0.57	0.12	24
5/7/40	274.04	364.35	0.58	0.09	23.8
10/3/40	-47.89	882.1	0.41	0.25	16.8
10/5/40	-217.7	662.29	0.48	0.22	12.8
10/7/40	-285.66	440.86	0.5	0.17	11.2
15/3/40	-229.27	445.05	0.31	0.25	13
15/5/40	-451.85	412.42	0.38	0.28	7
15/7/40	-554.66	356.37	0.42	0.25	3.8
20/3/40	-258.87	413.51	0.24	0.24	12.4
20/5/40	-499.61	428.97	0.29	0.26	6.4
20/7/40	-621.75	296.46	0.32	0.23	2.8
5/3/50	523.22	838.6	0.56	0.14	34
5/5/50	483.47	501.23	0.6	0.11	33
5/7/50	480.54	423.87	0.6	0.08	33
10/3/50	83.03	881.48	0.45	0.23	24.8
10/5/50	-103.95	690.92	0.52	0.19	21
10/7/50	-171.4	484.52	0.54	0.15	18.6
15/3/50	-120.59	488.42	0.34	0.26	20.6
15/5/50	-411.62	493.07	0.43	0.24	13.2
15/7/50	-554.06	375.9	0.47	0.21	9.8
20/3/50	-166.16	443.09	0.27	0.24	19.6
20/5/50	-501.35	454.24	0.34	0.23	11.4
20/7/50	-688.19	338.29	0.38	0.23	6.6

Table A.3 Results of operator-based relocation problem for large-size instances in the instance class C

# of V/O/U	OV	T(s)	VU	OU	#RU
10/5/75	314.59	623.74	0.59	0.15	41.4
10/10/75	250.59	329.38	0.61	0.08	40.6
10/15/75	249.20	326.18	0.61	0.06	40.6
20/5/75	-449.14	535.10	0.44	0.24	24.8
20/10/75	-836.32	356.80	0.52	0.2	15.6
20/15/75	-930.10	300.51	0.54	0.14	13.6
30/5/75	-700.32	508.52	0.32	0.26	20.2
30/10/75	-1221.96	335.64	0.4	0.22	7.2
30/15/75	-1404.92	262.92	0.42	0.18	1.6
40/5/75	-777.66	438.85	0.25	0.24	19.2
40/10/75	-1294.59	277.18	0.3	0.22	5.6
40/15/75	-1467.05	222.79	0.32	0.18	0.4
50/5/75	-837.57	498.22	0.2	0.25	17.8
50/10/75	-1342.71	195.28	0.25	0.22	4.6
50/15/75	-1501.03	123.84	0.26	0.18	0.2
10/5/100	840.28	772.99	0.62	0.1	64.8
10/10/100	791.09	450.59	0.64	0.07	63.8
10/15/100	789.38	429.73	0.64	0.04	63.6
20/5/100	-184.87	538.88	0.5	0.21	42.6
20/10/100	-530.30	318.26	0.56	0.15	36.2
20/15/100	-600.86	288.47	0.58	0.11	34.4
30/5/100	-681.22	478.11	0.39	0.26	32.8
30/10/100	-1325.66	344.07	0.48	0.23	17
30/15/100	-1543.02	301.40	0.51	0.19	11.6
40/5/100	-827.85	467.11	0.3	0.27	30
40/10/100	-1540.87	274.32	0.38	0.24	12.4
40/15/100	-1842.08	253.98	0.42	0.21	4
50/5/100	-934.47	500.13	0.25	0.27	28
50/10/100	-1631.93	265.95	0.31	0.25	10.6
50/15/100	-1916.66	203.51	0.34	0.21	2.2

Table A.4 Results of hybrid relocation problem for small-size instances in the instance class C

# of V/O/U	OV	T(s)	VU	OU	#RU	OC	DC	ODC	NC
2/1/10	85.64	247.42	0.36	0.22	5	1.8	0.2	0	3
2/2/10	61.86	184.02	0.39	0.14	5.2	1	0	0	3.8
2/3/10	59.82	213.93	0.39	0.1	4.8	1.2	0.4	0	3.6
3/1/10	50.18	335.99	0.29	0.23	4.4	2	0	0	3.6
3/2/10	4.26	318.29	0.35	0.18	3.2	2.4	0.2	0	4.2
3/3/10	-6.33	174.85	0.37	0.14	3	1.8	0.4	0	4.8
4/1/10	31.2	512.59	0.23	0.25	4	2.2	0	0	3.8
4/2/10	-51.09	408.24	0.33	0.2	2	2.4	0.2	0	5.4
4/3/10	-75.43	201.3	0.35	0.18	1.4	1.8	0.2	0	6.6
5/1/10	-13.24	430.13	0.23	0.31	2.6	3.8	0	0	3.6
5/2/10	-81.11	304.63	0.3	0.24	1	2.4	0.2	0	6.4
5/3/10	-100.46	308.44	0.31	0.21	0.6	1.6	0.2	0	7.6
10/1/10	-58.65	401.2	0.13	0.29	1.8	3.8	0.2	0	4.2
10/2/10	-114.09	380.81	0.16	0.24	0.4	2	0.4	0	7.2
10/3/10	-132.83	435.2	0.17	0.19	0	1.6	0.2	0	8.2
2/1/15	146.01	438.69	0.48	0.16	9.2	2.4	1.2	0	2.2
2/2/15	118.53	208.08	0.53	0.12	8.6	1.6	1.4	0	3.4
2/3/15	115.93	176.49	0.53	0.09	8.6	1.4	1.2	0	3.8
3/1/15	88.44	396.41	0.4	0.22	7.4	3.6	0.8	0	3.2
3/2/15	39.78	377.63	0.46	0.15	6.4	2.6	1.8	0	4.2
3/3/15	19.81	208.55	0.49	0.14	6	2.2	1.4	0	5.4
4/1/15	43.4	467.36	0.34	0.25	6	4.8	0.8	0	3.4
4/2/15	-41.5	482.04	0.43	0.2	4	4.6	1.2	0	5.2
4/3/15	-76.47	368.97	0.48	0.18	3.2	3.2	1.8	0	6.8
5/1/15	-7.52	568.07	0.32	0.25	5.2	4.2	0.6	0.2	4.8
5/2/15	-95.49	432.84	0.38	0.22	3.2	3.6	1.6	0	6.6
5/3/15	-130.91	383.89	0.43	0.21	1.8	3.4	1.4	0	8.4
10/1/15	-87.5	502.77	0.19	0.31	3	6.4	0	0.2	5.4
10/2/15	-165.44	492.43	0.23	0.27	0.8	5.4	0.8	0	8
10/3/15	-198.24	565.64	0.24	0.22	0.2	3.6	0.6	0	10.6
15/1/15	-118.06	464.59	0.13	0.3	2.6	6.2	0.2	0	6
15/2/15	-193.6	451.59	0.15	0.25	0.6	5	0.4	0	9
15/3/15	-219.57	380.29	0.16	0.2	0	3.6	0.4	0	11
2/1/20	233.79	293.37	0.57	0.11	13.4	3	1.2	0.4	2
2/2/20	218.63	126.41	0.59	0.07	13	2.2	1.8	0.2	2.8
2/3/20	218.63	105.09	0.59	0.05	13	2.2	1.8	0.2	2.8
3/1/20	151.86	350.2	0.49	0.16	11.6	4.4	0.4	0	3.6
3/2/20	112.25	359.28	0.55	0.14	10.6	2.8	2.2	0	4.4
3/3/20	101.27	173.31	0.55	0.09	10.4	3.2	2.2	0	4.2
4/1/20	93	451.87	0.44	0.23	9.4	5.6	1.4	0	3.6
4/2/20	10.3	458.3	0.51	0.19	7.8	4.4	2.4	0	5.4
4/3/20	-15.74	376.6	0.53	0.14	6.8	4.4	2.4	0	6.4
5/1/20	20.91	612.63	0.39	0.24	7.8	6.4	1.6	0	4.2
5/2/20	-76.71	216.99	0.47	0.22	5.2	5	2.4	0	7.4
5/3/20	-113.58	195.27	0.5	0.19	4.2	4.4	2.6	0	8.8
10/1/20	-110.52	601.31	0.25	0.29	4.6	8.2	1.6	0	5.6
10/2/20	-215.66	486.98	0.29	0.29	2	6	1	0	11
10/3/20	-261.85	451.07	0.31	0.25	0.8	5.6	0.6	0	13
15/1/20	-157.47	597.41	0.18	0.32	3.6	7.6	1.4	0	7.4
15/2/20	-260.77	529.1	0.2	0.26	1.2	5.8	1	0	12
15/3/20	-296.63	365.53	0.22	0.24	0.2	4.6	1	0	14.2
20/1/20	-158.62	608.72	0.13	0.32	3.6	7.8	1.2	0	7.4
20/2/20	-263.67	511.96	0.15	0.26	1.2	5.8	0.8	0	12.2
20/3/20	-300.94	344.13	0.16	0.22	0.2	5.4	0.6	0	13.8

Table A.5 Results of hybrid relocation problem for medium-size instances in the instance class C

# of V/O/U	OV	T(s)	VU	OU	#RU	OC	DC	ODC	NC
5/3/30	-21.51	636.22	0.62	0.13	10.8	5.4	4.6	0.4	8.8
5/5/30	-46.38	406.26	0.62	0.09	10.4	5.6	4.8	0.2	9
5/7/30	-47.99	427.16	0.62	0.06	10.4	5.6	4.6	0.2	9.2
10/3/30	-352.14	528.44	0.44	0.21	2.4	9.4	4.2	0	14
10/5/30	-432.47	595.14	0.47	0.17	0.8	6.6	4.2	0	18.4
10/7/30	-461.03	410.76	0.48	0.14	0.4	5.8	3.8	0	20
15/3/30	-442.42	381.34	0.31	0.22	1	8.6	3	0	17.4
15/5/30	-517.88	383.44	0.33	0.18	0	4.6	2.2	0	23.2
15/7/30	-551.21	269.88	0.33	0.15	0	2.4	1.4	0	26.2
20/3/30	-456.41	394.24	0.23	0.24	1.2	6.6	2.2	0	20
20/5/30	-532.79	356.27	0.25	0.18	0	3.8	1.4	0	24.8
20/7/30	-563.9	247.8	0.25	0.15	0	2.4	0.6	0	27
5/3/40	163.85	656.76	0.67	0.1	19	7.6	5.2	0.2	8
5/5/40	133.2	406.14	0.69	0.07	18.6	7.2	5.4	0	8.8
5/7/40	130.65	363.57	0.69	0.05	18.6	7.2	5.4	0	8.8
10/3/40	-297.17	590.31	0.52	0.21	8	11.8	5	0.4	14.8
10/5/40	-405.23	678.33	0.56	0.16	6	9.4	5.4	0.6	18.6
10/7/40	-448.59	425.86	0.58	0.14	5	8.6	6	0.4	20
15/3/40	-497.24	413.39	0.4	0.24	3.6	13.8	3.6	0.2	18.8
15/5/40	-610.25	422.32	0.43	0.21	1.2	9.6	4.8	0	24.4
15/7/40	-665.62	321.61	0.45	0.19	0.4	6.2	3.6	0	29.8
20/3/40	-537.18	380.79	0.31	0.26	3.2	10.4	4	0.2	22.2
20/5/40	-644.84	426.65	0.33	0.22	0.8	7.6	4	0	27.6
20/7/40	-701.51	312.82	0.34	0.19	0	5.8	2.4	0	31.8
5/3/50	342.37	573.03	0.7	0.08	27.8	8.4	6.6	0	7.2
5/5/50	316.09	507.63	0.72	0.06	27.4	7.8	6	0.2	8.6
5/7/50	313.78	391.63	0.72	0.04	27.4	7.8	5.6	0.4	8.8
10/3/50	-236.74	831.35	0.6	0.16	13.8	13.2	7	0.6	15.4
10/5/50	-334.46	696.64	0.63	0.12	12.2	12	6.6	0.8	18.4
10/7/50	-379.46	498.82	0.64	0.11	11	10.8	9	0.4	18.8
15/3/50	-531.56	514.18	0.48	0.22	7.4	16.2	4.4	0.6	21.4
15/5/50	-662.5	476.02	0.51	0.2	4	14	4.8	0.2	27
15/7/50	-736.48	346.87	0.53	0.17	2.2	13.8	4.4	0.4	29.2
20/3/50	-605.8	515.17	0.37	0.27	5	16.2	4.4	0.6	23.8
20/5/50	-747.79	459.63	0.4	0.22	2	14.6	4.6	0	28.8
20/7/50	-825.39	342.56	0.41	0.2	1	10.4	3.2	0	35.4

Table A.6 Results of hybrid relocation problem for large-size instances in the instance class C

# of V/O/U	OV	T(s)	VU	OU	#RU	OC	DC	ODC	NC
10/5/75	14.09	689.76	0.72	0.09	30.2	16.6	11.2	0.6	16.4
10/10/75	-17.77	443.07	0.73	0.06	29.8	14.2	11.8	0.6	18.6
10/15/75	-18.48	428.12	0.73	0.03	29.8	13.4	12.6	0.6	18.6
20/5/75	-949.76	502.58	0.55	0.18	8.2	20.6	11.2	0.8	34.2
20/10/75	-1127.33	355.90	0.59	0.14	4.6	18.4	10.2	0.4	41.4
20/15/75	-1183.01	338.53	0.6	0.1	3.4	16.8	10.4	0.2	44.2
30/5/75	-1166.95	470.48	0.4	0.25	4.2	19.6	7.6	0	43.6
30/10/75	-1364.71	364.10	0.42	0.18	1.2	10.8	5.8	0	57.2
30/15/75	-1456.01	316.76	0.43	0.15	0	5	3.8	0	66.2
40/5/75	-1213.99	518.17	0.3	0.25	3.2	19.6	7.2	0	45
40/10/75	-1410.03	354.08	0.32	0.19	0.4	9.8	4	0	60.8
40/15/75	-1497.20	332.17	0.32	0.15	0	3.8	1.6	0	69.6
50/5/75	-1253.66	541.60	0.24	0.25	3	18.2	7.2	0	46.6
50/10/75	-1445.23	243.39	0.26	0.2	0.2	7.8	4.4	0	62.6
50/15/75	-1526.75	189.85	0.26	0.15	0	3.4	1.6	0	70
10/5/100	503.04	820.71	0.74	0.05	50.8	19.2	14.8	0.2	15
10/10/100	478.16	533.24	0.75	0.03	50	18.6	15.6	0.2	15.6
10/15/100	477.80	538.58	0.75	0.02	50	18.6	15.8	0.2	15.4
20/5/100	-789.05	627.02	0.63	0.15	22	28.2	15.8	0.2	33.8
20/10/100	-959.24	475.07	0.66	0.11	17.8	26.4	16.6	0.6	38.6
20/15/100	-1005.23	426.51	0.67	0.08	17.8	25.2	15.8	0.6	40.6
30/5/100	-1404.97	658.95	0.5	0.23	7.8	30.4	11.8	0.4	49.6
30/10/100	-1664.33	549.46	0.54	0.19	3.8	21.8	9	0.6	64.8
30/15/100	-1791.16	530.96	0.55	0.15	2	18.8	8.4	0	70.8
40/5/100	-1526.79	610.08	0.39	0.25	6.4	27	10.8	0.4	55.4
40/10/100	-1800.84	458.98	0.41	0.21	2	17.8	5.4	0.2	74.6
40/15/100	-1935.76	404.15	0.42	0.18	0.2	9.6	3.6	0	86.6
50/5/100	-1591.71	645.80	0.31	0.26	6	25.4	10.2	0.4	58
50/10/100	-1857.22	300.15	0.33	0.22	1.6	14.4	6	0	78
50/15/100	-1987.37	267.42	0.34	0.18	0	7.2	4.2	0	88.6

Table A.7 Results of operator-based relocation problem for small-size instances in the instance class UC

# of V/O/U	OV	T(s)	VU	OU	#RU
2/1/10	162.96	353.06	0.26	0.19	7.2
2/2/10	115.49	221.14	0.4	0.17	6.2
2/3/10	110.22	223.26	0.4	0.11	6
3/1/10	141.5	348.3	0.21	0.23	6.8
3/2/10	70.3	204.43	0.33	0.23	4.8
3/3/10	48.19	208.52	0.39	0.18	4
4/1/10	107.51	392.78	0.19	0.22	6
4/2/10	29.63	248.23	0.28	0.19	4.2
4/3/10	-4.59	310.36	0.34	0.2	2.6
5/1/10	93.28	354.3	0.17	0.25	5.8
5/2/10	3.84	286.34	0.25	0.23	3.4
5/3/10	-43.79	280.02	0.31	0.2	2.2
10/1/10	58.25	355.89	0.09	0.25	5
10/2/10	-37.3	219.17	0.14	0.23	2.8
10/3/10	-94.76	204.52	0.17	0.21	0.8
2/1/15	231.07	397.99	0.39	0.22	10.8
2/2/15	192.91	213.03	0.48	0.15	10
2/3/15	186.34	216.64	0.48	0.09	10
3/1/15	215.14	441.03	0.28	0.26	10.4
3/2/15	142.84	242.55	0.4	0.21	8.4
3/3/15	111.93	211.97	0.45	0.17	7.6
4/1/15	185.87	464.39	0.24	0.3	9.6
4/2/15	87.6	277.93	0.35	0.22	7.4
4/3/15	35.88	257.21	0.42	0.21	5.8
5/1/15	169.73	509.06	0.2	0.22	9.6
5/2/15	48.99	301.87	0.3	0.22	7
5/3/15	-13.93	275.7	0.37	0.22	4.8
10/1/15	104.75	442.42	0.12	0.22	8.6
10/2/15	-25.03	231.1	0.18	0.26	5.4
10/3/15	-102.6	195.6	0.22	0.24	3.2
15/1/15	77.63	448.39	0.09	0.24	8.2
15/2/15	-56.67	522.98	0.13	0.25	4.8
15/3/15	-151.8	414.27	0.16	0.24	2.4
2/1/20	367.25	494.62	0.39	0.21	15.4
2/2/20	318.49	210.37	0.5	0.12	15
2/3/20	316.53	214.19	0.5	0.08	14.8
3/1/20	341.56	526.2	0.31	0.23	15
3/2/20	257.4	212.25	0.43	0.19	13.2
3/3/20	219.09	195.09	0.49	0.13	12.8
4/1/20	314.82	571.98	0.26	0.26	14.4
4/2/20	183.64	265.88	0.4	0.2	12
4/3/20	130.72	282.23	0.46	0.18	10.4
5/1/20	296.18	592.43	0.21	0.24	14.2
5/2/20	146.09	317.42	0.34	0.2	11
5/3/20	58.04	294.74	0.43	0.2	9
10/1/20	191.4	527.33	0.14	0.25	12.4
10/2/20	33.36	257.61	0.22	0.27	8.6
10/3/20	-74.49	213.05	0.27	0.25	6
15/1/20	143.74	516.23	0.11	0.28	11.6
15/2/20	-23.08	586.6	0.16	0.28	8.2
15/3/20	-148.09	428.88	0.2	0.28	4.8
20/1/20	143.52	550.83	0.08	0.29	11.6
20/2/20	-25.75	112.31	0.12	0.31	7.8
20/3/20	-157.19	222.56	0.15	0.29	4.6

Table A.8 Results of operator-based relocation problem for medium-size instances in the instance class UC

# of V/O/U	OV	T(s)	VU	OU	#RU
5/3/30	275.01	290.4	0.46	0.18	18.2
5/5/30	216.12	275.44	0.52	0.13	17.4
5/7/30	205.51	208.26	0.52	0.1	17.2
10/3/30	58.96	229.39	0.32	0.27	13
10/5/30	-130.13	606.4	0.4	0.22	9.2
10/7/30	-211.37	480.12	0.44	0.2	7.2
15/3/30	-35.08	490.4	0.24	0.29	11
15/5/30	-254.88	492.58	0.31	0.26	6.2
15/7/30	-360.91	451.91	0.34	0.23	3.2
20/3/30	-50.99	254.43	0.18	0.29	10.8
20/5/30	-274.41	313.22	0.24	0.27	5.4
20/7/30	-384.33	298.44	0.26	0.23	2.6
5/3/40	462.86	299.74	0.51	0.16	26.8
5/5/40	407.4	252.66	0.54	0.1	26
5/7/40	400.31	217.48	0.54	0.07	25.6
10/3/40	154.95	224.67	0.37	0.25	19.6
10/5/40	-64.23	556.19	0.47	0.2	16
10/7/40	-152.65	451.41	0.51	0.17	14
15/3/40	-4.9	485.59	0.29	0.3	16.2
15/5/40	-260.23	498.45	0.37	0.25	11.2
15/7/40	-407.34	500.11	0.42	0.24	6.8
20/3/40	-35.56	252.59	0.22	0.31	16.2
20/5/40	-290.3	347.07	0.28	0.27	10.6
20/7/40	-458	336.07	0.32	0.26	5.8
5/3/50	666.93	308.38	0.54	0.13	35.2
5/5/50	627.72	266.87	0.57	0.09	34.8
5/7/50	621.87	238.82	0.57	0.06	34.8
10/3/50	243.74	257.75	0.43	0.22	27.6
10/5/50	58.83	515.74	0.51	0.19	23.4
10/7/50	-0.72	488.66	0.53	0.14	22.8
15/3/50	19.13	562.52	0.34	0.27	23
15/5/50	-265.17	584.18	0.43	0.24	17
15/7/50	-399.9	546.3	0.47	0.22	13.2
20/3/50	-29.4	298.16	0.27	0.28	22
20/5/50	-335.22	403.57	0.34	0.27	15
20/7/50	-521	361.72	0.38	0.25	10

Table A.9 Results of operator-based relocation problem for large-size instances in the instance class UC

# of V/O/U	OV	T(s)	VU	OU	#RU
10/5/75	510.26	582.36	0.56	0.14	44.8
10/10/75	451.20	481.66	0.59	0.08	43.4
10/15/75	449.37	421.74	0.59	0.05	43.4
20/5/75	-217.40	459.41	0.42	0.27	29.8
20/10/75	-589.07	385.13	0.51	0.2	21.8
20/15/75	-718.50	311.22	0.53	0.15	19.6
30/5/75	-490.95	611.64	0.32	0.28	24.2
30/10/75	-1036.55	387.18	0.4	0.26	11
30/15/75	-1269.92	207.33	0.44	0.21	4.8
40/5/75	-569.72	599.85	0.24	0.3	23
40/10/75	-1144.87	283.19	0.31	0.26	9
40/15/75	-1372.44	178.80	0.33	0.21	3
50/5/75	-606.26	503.67	0.2	0.28	22.2
50/10/75	-1213.32	300.63	0.25	0.24	7.8
50/15/75	-1435.57	158.82	0.27	0.2	1.8
10/5/100	840.28	772.99	0.61	0.13	64.8
10/10/100	791.09	450.59	0.62	0.06	63.8
10/15/100	789.38	429.73	0.62	0.04	63.6
20/5/100	-184.87	538.88	0.5	0.24	42.6
20/10/100	-530.30	318.26	0.56	0.16	36.2
20/15/100	-600.86	288.47	0.58	0.11	34.4
30/5/100	-681.22	478.11	0.39	0.28	32.8
30/10/100	-1325.66	344.07	0.48	0.24	17
30/15/100	-1543.02	301.40	0.51	0.19	11.6
40/5/100	-827.85	467.11	0.31	0.28	30
40/10/100	-1540.87	274.32	0.38	0.26	12.4
40/15/100	-1842.08	253.98	0.42	0.22	4
50/5/100	-934.47	500.13	0.25	0.27	28
50/10/100	-1631.93	265.95	0.31	0.25	10.6
50/15/100	-1916.66	203.51	0.34	0.22	2.2

Table A.10 Results of hybrid relocation problem for small-size instances in the instance class UC

# of V/O/U	OV	T(s)	VU	OU	#RU	OC	DC	ODC	NC
2/1/10	112.07	329.63	0.4	0.25	5.6	1.8	1	0	1.6
2/2/10	74.61	141.32	0.49	0.18	4.8	1	1	0	3.2
2/3/10	67.02	137.35	0.49	0.12	4.8	1	1	0	3.2
3/1/10	83.86	340.62	0.31	0.26	4.6	2.2	1	0	2.2
3/2/10	31.97	260.29	0.4	0.23	3.4	1.8	1	0	3.8
3/3/10	2.5	113.11	0.43	0.16	2.8	1.4	1	0	4.8
4/1/10	50.1	339.53	0.26	0.25	4.2	2.2	1	0	2.6
4/2/10	-9.33	223.16	0.35	0.25	2.6	2	0.8	0	4.6
4/3/10	-38.06	296.22	0.39	0.2	1.8	1.2	0.8	0	6.2
5/1/10	25.96	431.17	0.23	0.26	3.6	2.6	0.8	0	3
5/2/10	-46.6	295.52	0.31	0.25	1.6	2.2	1.2	0	5
5/3/10	-79.2	295.42	0.34	0.21	1	1	0.8	0	7.2
10/1/10	-12.97	394.72	0.12	0.24	3.2	2	1	0	3.8
10/2/10	-89.18	246.4	0.17	0.2	1	2.8	0.8	0	5.4
10/3/10	-126.4	188.9	0.18	0.18	0.2	1.8	0.8	0	7.2
2/1/15	194.05	425.88	0.46	0.19	9.8	1	1.4	0	2.8
2/2/15	157.69	172.8	0.52	0.13	9	0.8	1	0	4.2
2/3/15	154.76	180.84	0.54	0.09	9	0.6	1.4	0	4
3/1/15	162.66	455.65	0.37	0.26	9	1.4	1.2	0	3.4
3/2/15	97.22	237.65	0.45	0.2	7.2	1.6	1.4	0	4.8
3/3/15	71.81	184.13	0.5	0.16	6.8	1	1.6	0	5.6
4/1/15	108.13	489.42	0.32	0.25	7.6	2	1.8	0	3.6
4/2/15	34.18	207.28	0.41	0.24	5.6	2	1.2	0	6.2
4/3/15	6.84	246.24	0.44	0.18	5.2	1	1.2	0	7.6
5/1/15	71.53	535.07	0.28	0.26	7	2.8	1.4	0	3.8
5/2/15	-15.88	275.05	0.37	0.27	4.4	2.8	1.2	0	6.6
5/3/15	-54.47	303.33	0.41	0.21	3.6	2	1	0	8.4
10/1/15	2.2	458.89	0.17	0.3	5.2	3.8	1	0	5
10/2/15	-93.45	266.08	0.22	0.25	2.8	3.2	2	0	7
10/3/15	-151.69	170.58	0.25	0.24	1	3	1.6	0	9.4
15/1/15	-35.77	378.08	0.12	0.28	4.8	3.2	2	0	5
15/2/15	-129.12	516.04	0.15	0.26	2.2	4.2	1.2	0.2	7.2
15/3/15	-190.07	338.3	0.17	0.24	0.4	3	1.2	0	10.4
2/1/20	291.37	520.46	0.56	0.11	13.6	2.6	1.2	0.2	2.4
2/2/20	263.03	208.84	0.61	0.09	13.2	1.8	1.2	0	3.8
2/3/20	263.03	202.42	0.61	0.06	13.2	1.8	1.2	0	3.8
3/1/20	221.94	566.52	0.48	0.2	12.4	2.4	1.6	0.2	3.4
3/2/20	171.56	253.5	0.55	0.13	11.2	2.6	1.6	0.2	4.4
3/3/20	156.94	186.54	0.57	0.11	10.6	2.8	1.2	0.2	5.2
4/1/20	161.34	588.95	0.43	0.24	11	3.2	1.2	0.2	4.4
4/2/20	85.15	263.52	0.5	0.18	9	3.8	1.4	0.2	5.6
4/3/20	63.22	230.95	0.54	0.16	8.8	2.6	1.8	0	6.8
5/1/20	106.68	567.33	0.38	0.24	9.6	4.6	1.4	0.4	4
5/2/20	22.3	313.63	0.47	0.19	7.6	5.2	1.2	0.2	5.8
5/3/20	-15.68	322.93	0.5	0.16	6.8	4.8	1.4	0.2	6.8
10/1/20	3.27	610.2	0.23	0.27	7.6	4.8	2.4	0	5.2
10/2/20	-113.3	270.5	0.29	0.27	4.4	5.8	1.6	0	8.2
10/3/20	-181.02	214.44	0.33	0.27	2.4	5.2	1.4	0	11
15/1/20	-41.71	612.8	0.17	0.32	6.2	6.4	1.4	0.4	5.6
15/2/20	-158.81	573.14	0.21	0.28	3	6.8	2	0.4	7.8
15/3/20	-235.16	416.07	0.23	0.26	1.8	4	1.8	0	12.4
20/1/20	-45.58	493.62	0.12	0.32	6.2	6.2	1.4	0.2	6
20/2/20	-166.68	145.3	0.15	0.29	3.2	6.8	1.6	0.2	8.2
20/3/20	-240.49	215.97	0.17	0.28	1.8	3.4	2	0	12.8

Table A.11 Results of hybrid relocation problem for medium-size instances in the instance class UC

# of V/O/U	OV	T(s)	VU	OU	#RU	OC	DC	ODC	NC
5/3/30	158.92	365.09	0.57	0.17	14.2	5	1.8	0.6	8.4
5/5/30	122.14	320.3	0.59	0.1	13.8	4.6	3	0.2	8.4
5/7/30	115.94	294.92	0.59	0.08	13.2	4.8	2.8	0.2	9
10/3/30	-139.53	242.92	0.41	0.24	7.4	7.6	2	0.2	12.8
10/5/30	-243.64	589.26	0.46	0.2	5.4	6.4	3.4	0	14.8
10/7/30	-290.69	498.94	0.48	0.16	4.4	4.8	3.6	0	17.2
15/3/30	-240.58	556.14	0.31	0.28	4.8	9.4	2.6	0	13.2
15/5/30	-358.19	524.23	0.34	0.23	2.4	6.8	2.8	0.2	17.8
15/7/30	-420.56	458.98	0.36	0.2	1.2	5.6	1.8	0	21.4
20/3/30	-251.09	275.16	0.23	0.28	5.2	8.4	2.6	0	13.8
20/5/30	-378.35	334.02	0.26	0.24	2	6.6	2.6	0	18.8
20/7/30	-443.8	309.94	0.27	0.21	0.6	5.2	1.6	0	22.6
5/3/40	311.91	334.14	0.64	0.11	22.2	7.6	2.2	0.2	7.8
5/5/40	293.05	324.31	0.63	0.07	22	6	2.8	0	9.2
5/7/40	288.72	269.6	0.63	0.05	22	6	2.8	0	9.2
10/3/40	-144.83	265.38	0.51	0.19	11.8	10	3.8	0.2	14.2
10/5/40	-244.49	595.65	0.55	0.17	9.8	10	3.6	0.4	16.2
10/7/40	-286.86	459.74	0.56	0.13	9.2	7.4	4.8	0.4	18.2
15/3/40	-335.98	613.5	0.39	0.26	7.8	10.6	3.8	0.2	17.6
15/5/40	-471.29	526.54	0.43	0.24	4.4	7.6	5	0.2	22.8
15/7/40	-541.61	537.26	0.46	0.2	2.6	8	3.8	0.2	25.4
20/3/40	-361.58	333.77	0.3	0.29	6.8	12	3.6	0.2	17.4
20/5/40	-509.87	347.72	0.34	0.25	3	9.4	4	0	23.6
20/7/40	-588.69	363.42	0.35	0.22	1.6	8	3	0	27.4
5/3/50	521.1	331.29	0.66	0.1	31.2	6.8	2.2	0.2	9.6
5/5/50	502.18	306.49	0.68	0.07	30.8	6.2	3.2	0.2	9.6
5/7/50	499.71	266.87	0.68	0.05	30.8	6.2	3.2	0.2	9.6
10/3/50	-40.64	262.84	0.56	0.17	18.4	11.8	4.8	0	15
10/5/50	-127.53	577.6	0.59	0.14	16.8	11.2	4.8	0.2	17
10/7/50	-160.79	501.02	0.61	0.12	16	10.8	5.2	0.2	17.8
15/3/50	-353.86	603.37	0.46	0.26	11.6	13.4	4.6	0.2	20.2
15/5/50	-483.74	645.53	0.51	0.21	8.8	13.8	5.2	0	22.2
15/7/50	-553.57	526.57	0.52	0.18	7.8	9.8	5.8	0	26.6
20/3/50	-422.97	335.66	0.36	0.29	9.6	15.6	4.6	0	20.2
20/5/50	-586.53	419.96	0.4	0.25	5.6	14.4	6	0	24
20/7/50	-688.25	393.44	0.43	0.23	3.2	12.6	4.4	0	29.8

Table A.12 Results of hybrid relocation problem for large-size instances in the instance class UC

# of V/O/U	OV	T(s)	VU	OU	#RU	OC	DC	ODC	NC
10/5/75	254.33	537.02	0.67	0.09	36.4	10.6	8.8	0.4	18.8
10/10/75	224.84	503.90	0.69	0.06	35.4	11	9.6	0.2	18.8
10/15/75	223.85	442.97	0.69	0.04	35.4	11	9.6	0.2	18.8
20/5/75	-620.11	496.85	0.52	0.24	16.8	16	9.2	0	33
20/10/75	-828.79	397.65	0.57	0.16	12.4	13.8	9.2	0.2	39.4
20/15/75	-917.14	338.16	0.58	0.12	11.4	11.4	8.2	0	44
30/5/75	-922.23	601.92	0.39	0.29	10	18.4	6.6	0.2	39.8
30/10/75	-1225.60	336.73	0.44	0.23	3.2	13.6	7	0	51.2
30/15/75	-1363.43	219.61	0.45	0.18	1	7.8	5.8	0	60.4
40/5/75	-1021.25	572.95	0.3	0.28	8.8	17.8	6.8	0	41.6
40/10/75	-1308.28	316.13	0.33	0.23	2.6	10.8	5.8	0	55.8
40/15/75	-1437.70	204.48	0.34	0.18	0.4	6.4	4.6	0	63.6
50/5/75	-1070.65	560.63	0.24	0.29	8	16.6	6.6	0	43.8
50/10/75	-1366.14	299.12	0.27	0.22	2	10	5.4	0	57.6
50/15/75	-1487.49	185.32	0.27	0.18	0	5.4	3.4	0	66.2
10/5/100	748.62	599.81	0.71	0.06	57	13.2	10.2	0.2	19.4
10/10/100	729.11	530.59	0.72	0.04	56.8	11.6	11.8	0.2	19.6
10/15/100	729.11	483.29	0.72	0.03	57	10.6	11.8	0.2	20.4
20/5/100	-497.09	520.45	0.6	0.17	30.4	20	14.6	0	35
20/10/100	-653.85	459.43	0.63	0.11	27.4	16	15.2	0	41.4
20/15/100	-723.05	389.60	0.65	0.09	26	17.2	13.4	0.4	43
30/5/100	-1094.42	657.47	0.49	0.27	18.2	22	9.6	0.4	49.8
30/10/100	-1417.40	398.57	0.53	0.2	10.2	21.2	10.8	0	57.8
30/15/100	-1590.16	256.56	0.56	0.16	7	20.2	9.4	0	63.4
40/5/100	-1260.39	680.24	0.38	0.28	14.6	24.2	9.6	0	51.6
40/10/100	-1627.54	374.44	0.42	0.23	6.4	17.8	8.2	0	67.6
40/15/100	-1815.47	228.56	0.44	0.19	3	11.8	7	0	78.2
50/5/100	-1371.77	640.15	0.31	0.28	13.2	23.4	8.4	0	55
50/10/100	-1729.31	356.00	0.34	0.22	5.4	17.2	7.6	0	69.8
50/15/100	-1905.50	199.14	0.35	0.19	2.6	10.2	5.6	0	81.6