PRICE DISCRIMINATION IN GOODS AND PARKING FEES IN SHOPPING MALLS

by
ZELİHA BEGÜM TUNÇ

Submitted to the Graduate School of Social Sciences
in partial fulfilment of
the requirements for the degree of Master of Arts

Sabancı University
July 2020

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Approved by:

Prof. Eren İnci ....
(Thesis Supervisor)

Assoc. Prof. Sadettin Haluk Çitçi ....

Assoc. Prof. Şerif Aziz Şimşir ....

Date of Approval: July 21, 2020

ABSTRACT

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ZELİHA BEGÜM TUNÇ

ECONOMICS M.A. THESIS, JULY 2020

Thesis Supervisor: Prof. Eren İnci

Keywords: horizontal product differentiation, monopoly pricing, parking fee, price discrimination, shopping mall

This thesis analyzes the pricing strategy of a monopolist shopping mall when it can price discriminate. The mall determines the prices of the goods and parking fees and it can identify different market segments. Customers can visit the mall only by car and they may leave the mall without any purchases. We find that when customers are differentiated with respect to their attitudes towards risk, the mall provides free parking for the most risk-averse customer and not necessarily for the other customers. In all other cases, the mall always provides free parking for all and charges customers more as their likeliness of buying the good decrease.

ÖZET

ALIŞVERİŞ MERKEZLERİNDE ÜRÜN VE PARK ÜCRETLERİNDE FİYAT FARKLILAŞTIRMASI

ZELİHA BEGÜM TUNÇ

EKONOMİ YÜKSEK LİSANS TEZİ, TEMMUZ 2020

Tez Danışmanı: Prof. Dr. Eren İnci

Anahtar Kelimeler: yatay ürün farklılaştırması, tekel fiyatlandırması, park yeri ücreti, fiyat farklılaştırması, alışveriş merkezi

Bu tez, tekel bir alışveriş merkezinin fiyat farklılaştırabildiğinde oluşturduğu fiyatlandırma stratejisini incelemektedir. Alışveriş merkezi, ürünlerin ve park yerinin ücretlerini belirlemektedir ve farklı pazar segmentlerini tanımlayabilmektedir. Tüketiciler, alışveriş merkezine yalnızca arabayla gidebilmektedirler ve ürünü satın almayabilirler. Tüketiciler risk davranışlarına göre ayrıldığında, dengede alışveriş merkezi park yerini riskten en çok kaçınan tüketiciye ücretsiz olarak sağlamaktadır; ancak diğer tüketicileri ücretlendirebilir. Diğer tüm durumlarda, dengede alışveriş merkezi park yerini herkese ücretsiz olarak sunmaktadır ve tüketicilerin bir ürünü satın alma olasılığı azaldıkça, alışveriş merkezi o ürünün fiyatını arttırmaktadır.

ACKNOWLEDGEMENTS

I would like to express my sincerest gratitude to my thesis advisor Prof. Eren İnci for his guidance and continuous support throughout my M.A. studies. I am indebted to him for his constant encouragement, great interest, and insightful suggestions.

I also thank my thesis jury members, Assoc. Prof. Şerif Aziz Şimşir and Assoc. Prof. Sadettin Haluk Çitçi, for examining my thesis and for their valuable comments.

I am deeply grateful to my family and my grandmother for their love and support.

To my family

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1. INTRODUCTION

This thesis is an extension of the base model constructed by Hasker and Inci (2014) and studies the optimal pricing behavior of a monopolist shopping mall under price discrimination and horizontal product differentiation. The mall provides a parking lot to the visitors and decides on the prices of the goods and parking fees. Customers can buy at most one type of good in one visit and they can visit the mall only by car. There are different types of customers and the mall can identify the types of customers to some degree. We examine the equilibrium prices and fees under each class of differentiation. The main contribution of this thesis is that unless customers are differentiated with respect to their degree of risk-aversion, there is always a negative relationship between the price of the good and the probability of buying the good. That is, as customers become more likely to buy the good, they pay less for the good. Furthermore, parking is free for all and the cost of the parking is embedded in the prices of the goods.

The economics of parking studies parking markets from an economic perspective. Parking is one of the most crucial aspects of urban life since it is one of the most used intermediate goods which generates a vast amount of land use. Shopping malls provide a parking lot for its visitors and these parking lots comprise a high percentage of parking space. To my knowledge, Hasker and Inci (2014) are the first to construct the shopping mall parking problem. They show that suburban malls provide free parking in equilibrium and embed the cost of parking in the price of the good. They demonstrate that it is the social optimum in a second-best sense. They construct a model in which a risk-neutral monopolist mall sells one good and customers are strictly risk-averse and they can reach the mall only by car. Customers decide on buying the good only after visiting the mall and there is a probability that they may leave the mall without any purchase. Hence, the mall has an incentive to insure risk-averse customers to some degree by providing free parking. The authors find that the results are robust to the extension of the base model. In particular, the results still hold if the mall decides on the parking lot size, provides vouchers, or prices in a competitive manner. On the other hand, they find that it is optimal for the mall to set a positive parking fee when there are individuals with the intention of using the parking lot but not visiting the mall.

This thesis extends the base model of Hasker and Inci (2014) by allowing the mall to identify different market segments, and hence implement price discrimination and product differentiation. In this setting, since the mall has the ability to separate the markets, and hence the parking lots, the following question arises: How does the mall set the prices of different products and parking fees to different types of customers? It turns out that a crucial instrument in answering this question is the probability of finding (or buying) the good, which is introduced by Hasker and Inci (2014). Moreover, Hasker and Inci (2014) find that their results are independent of the degree of risk aversion. We find that when the mall can identify different market segments, these results do not necessarily hold under certain conditions.

The rest of the thesis is organized as follows. Chapter 2 reviews the literature. Chapter 3 presents Hasker and Inci (2014) model and solves the constrained optimization problem. Chapter 4 sets up the model for two types of customers and derives the equilibrium under price discrimination. Chapter 5 analyzes the model for two types of goods and derives the equilibrium under horizontal product differentiation. Chapter 6 extends the model of Chapter 4 for more than two types of customers. Chapter 7 extends the model of Chapter 5 for more than two types of goods. Chapter 8 concludes.

2. LITERATURE REVIEW

Parking literature mostly focuses on the pricing of parking when there is search externality or congestion externality. There are also studies on minimum and maximum parking requirements, road pricing, and shopping mall parking.

There is a significant amount of work on cruising for parking. Shoup (2006) points out that cruising causes traffic congestion, fuel waste, and pollution. He examines how drivers make the decision between cruising for free curb parking and paying for off-street parking. He finds that cheap curb parking, cheap fuel, and expensive off-street parking are among the factors that make drivers more likely to cruise. He also shows that a positive fee for curb parking can sufficiently decrease cruising.

Arnott and Inci (2006) analyze cruising for parking by constructing a downtown parking model of traffic congestion and saturated on-street parking. They examine the equilibrium outcomes when cruising for parking leads to traffic congestion. They find that the on-street parking fee must be set high until the drivers do not cruise and parking is saturated. They show that this result is robust. Moreover, they find that when the parking fee is fixed, setting the number of on-street parking spaces high until the drivers do not cruise and parking is saturated is the second-best optimal.

Research on spatial competition has been extensive. Arnott and Rowse (1999) present a stochastic model of parking congestion in which congestion is caused by drivers because they disregard their impact on the mean density of vacant parking spaces. They analyze stochastic stationary-state equilibria and find that there may exist multiple equilibria. However, when they examine the social optimum, they find that the optimal parking fee is equal to the congestion externality.

Anderson and de Palma (2004) develop a model of parking congestion in which parking is unassigned and drivers need to search for an on-street parking spot. They indicate that when parking is unpriced, parking spaces that are near to the central business district (CBD) are overused while the distant parking lots are underused. They find that in equilibrium, the prices must be set by taking congestion externality into account and the parking lots near to the CBD must be charged higher. They also point out that drivers do not take into consideration that they increase the search cost of other drivers while searching for a parking spot near to the CBD. The authors also show how to decentralize the optimum.

Anderson and de Palma (2007) provide an extension of the model of Anderson and de Palma (2004) by endogenizing the land use. They find that socially optimum is reached under a monopolistically competitive market, which supports the results of Anderson and de Palma (2004).

Inci and Lindsey (2015) construct a spatial parking model. In their model, there are long-term and short-term parkers and drivers can park either in garages or on the curbside. Curbside parking is scarce and subject to traffic congestion, hence there is a search cost for drivers. Garages compete with each other and with the public curbside parking lot. Since parkers differ in their duration of parking, the parking lots can exercise price discrimination. They characterize the market equilibrium and the social optimum. They find that since privately operated garages exercise market power, the equilibrium is not efficient.

In the literature, the shopping mall parking is a relatively new subject which is introduced by Hasker and Inci (2014). Ersoy, Hasker, and Inci (2016) extend the base model of Hasker and Inci (2014) by analyzing the pricing scheme of a shopping mall when customers make modal choices. Customers choose either car or public transportation to get to the mall. In their model, first, the city decides on the bus fare, then the mall decides on the parking fee and the price of the good. Finally, customers decide to go to the mall or not, and the mode of transportation if they go. They find that in equilibrium, the mall sets the parking fee less than the marginal cost of parking. They extend their analysis by investigating other cases. They find that when the mall provides shuttle service for free or sells multiple goods, parking is still a loss leader.

Inci, Lindsey, and Oz (2018) extend the base model of Hasker and Inci (2014) by investigating the pricing scheme of a retailer when customers choose between valet parking and self-parking. The retail sets the prices of both types of parking as well as the price of the good. As in the model of Hasker and Inci (2014), customers are risk-averse and there is a probability that customers who visit the mall may not find the good they want and leave the store empty-handed. Inci, Lindsey, and Oz (2018) characterize the market equilibrium as well as the social optimum and find that the retailer provides self-parking for free and embeds the cost of self-parking in the price of the good in both the market equilibrium and the social optimum. On the other hand, they find that the price of valet parking may be above or below its cost in equilibrium.

Inan, Inci, and Lindsey (2019) analyze spillover parking generated by a retailer. They indicate that drivers who go to popular areas may prefer parking in the neighborhood to avoid expensive parking fees while causing negative externalities in the region since they generate more traffic. In their model, the retailer decides on the parking lot capacity and sets the parking fee. Some customers walk and others drive. Drivers can use either the parking lot of the retailer or the street to park. The authors investigate different policies in addressing spillover parking and find that the effectiveness of policies depends on congestion, the number of shoppers, and the market power of the retailer.

Guven, Inci, and Russo (Forthcoming) study competition among retailers. They set up a model in which customers are informed about the exact prices and features of the goods only if they go to the mall, and this is costly to the customer. They indicate that since there is a search cost for customers, it is more beneficial for retailers to concentrate under a mall. They point out that the mall can affect the prices of the goods, and hence it can diminish competition between retailers. They show that the concentration of retailers under a mall leads to higher prices. Moreover, they find that the mall uses parking as a loss leader.

Price discrimination has been extensively studied in the literature of industrial organization while it has not been a focal point in parking literature. Even so, there are some crucial studies on price discrimination in parking models. Lindsey and West (1997) study the use of parking coupons by downtown retailers. They analyze the effects of spatial price discrimination in monopolistically competitive markets. In their model, customers are either from downtown or suburban. Suburban customers have more price elastic demand since the travel cost is higher for them and they are closer to suburban shopping centers. Lindsey and West (1997) indicate that discrimination may be exercised in favor of suburban consumers and against downtown consumers. In order to apply price discrimination, parking discount coupons are used since in general, suburban consumers are coupon users while downtown customers are not. They find that if the stores participate in the downtown parking coupon program collectively, the program is beneficial. Otherwise, the stores are better off if they do not participate.

In the parking literature, there are several empirical papers on price discrimination. De Nijs (2012) examines the impact of a large horizontal merger on the price menus of the parking garages. He finds that the presentation of a large horizontal merger causes more discounts on a long duration and more price discrimination. Lin and Wang (2015) examine competition and price discrimination in parking garages. They find that competition limits firms from implementing price discrimination.

3. HASKER-INCI MODEL: ONE TYPE OF GOOD AND CUSTOMER

In the base model of Hasker and Inci (2014), there is a risk-neutral monopolist shopping mall which sells one good at a price $P \geq 0$ that has no marginal cost, $c_{good} = 0$. The mall provides a parking lot to its customers, at a fee t which has a marginal cost $c_{lot} > 0$.

Customers can go to the mall only by car and they can only park at the mall's parking lot. They are strictly risk-averse, have the same utility function $u(.)$, the same initial wealth $w > 0$, and the same reservation value $r > 0$. Each customer have a valuation $v \in [0, \bar{v}]$ for the good which has the cumulative distribution function $F(v)$ and density $f(v)$. It is assumed that $F(v)$ has the standard monotone hazard rate property. Hence the mall's objective function is concave. Moreover, there is a probability $\rho \in (0,1)$ that the customer may find the good. This probability can also be considered as the probability that the customer likes the good enough to buy, that is, $v \geq P$.

A customer decides to go to the mall if and only if the expected utility of going to the mall is greater or equal to the expected utility of not going to the mall. That is, a customer goes to the mall if and only if

$$\rho u(w+v-P-t)+(1-\rho)u(w-t) \geq u(w+r). \tag{3.1}$$

The customer who is indifferent between going to the mall or not has the valuation $\tilde{v}(P,t)$,

$$\tilde{v}(P,t) \equiv u^{-1}\left(\frac{u(w+r)-(1-\rho)u(w-t)}{\rho}\right)-w+P+t. \tag{3.2}$$

Therefore,[1]

$$\tilde{v}_P = 1, \tag{3.3}$$

[1] $\tilde{v}_x$ stands for $\partial\tilde{v}/\partial x$ and u_x stands for $\partial u/\partial x$.

and

$$\tilde{v}_t = 1 + \frac{(1-\rho)}{\rho} \frac{u_t(w-t)}{u_t(w+\tilde{v}-P-t)} \geq \frac{1}{\rho}. \tag{3.4}$$

There are $1-F(\tilde{v})$ customers and the mall's profit is $\Pi(P,t)$,

$$\Pi(P,t) = (1-F(\tilde{v}))(\rho P + t - c_{lot}). \tag{3.5}$$

Hasker and Inci (2014) first solve the unconstrained optimization problem and find that in equilibrium, it must be that $t^* = -r$. That is, the mall gives subsidy to the customers for coming to the mall since they take the risk of not finding the good. Since this is not applicable, the authors solve the constrained optimization problem:

The mall maximizes its profit with respect to P and t, subject to the rationality constraint $\rho P + t - c_{lot} \geq 0$, and non-negativity constraints $P, t \geq 0$.[2] The Lagrangian function is

$$\mathcal{L}(P,t) = (1-F(\tilde{v}))(\rho P + t - c_{lot}) + \lambda_1(\rho P + t - c_{lot}) + \lambda_2 P + \lambda_3 t, \tag{3.6}$$

where λ_1, λ_2, and λ_3 are Lagrangian multipliers.

Taking the first-order conditions with respect to P and t,[3]

$$\mathcal{L}_P = \rho(1-F(\tilde{v})) - f(\tilde{v})\tilde{v}_P(\rho P + t - c_{lot}) + \lambda_1 \rho + \lambda_2. \tag{3.7}$$

$$\mathcal{L}_t = 1 - F(\tilde{v}) - f(\tilde{v})\tilde{v}_t(\rho P + t - c_{lot}) + \lambda_1 + \lambda_3. \tag{3.8}$$

Equating the first-order condition in (3.7) to zero, and using (3.3),

$$\rho(1-F(\tilde{v})) + \lambda_1 \rho + \lambda_2 = f(\tilde{v})(\rho P + t - c_{lot}). \tag{3.9}$$

Notice that the expression $\rho P + t - c_{lot}$ cannot be zero, since $\rho(1-F(\tilde{v}))$ is greater than zero. Therefore, by complementary slackness, $\lambda_1 = 0$. Then, (3.9) becomes

$$1 - F(\tilde{v}) = \frac{f(\tilde{v})(\rho P + t - c_{lot}) - \lambda_2}{\rho}. \tag{3.10}$$

Notice that if $\tilde{v}_t = 1/\rho$, then by (3.4), $\tilde{v} = P$. But then, from the equation (3.2),

[2] Note that taking the individual rationality (IR) constraint into account is not necessary at this point since customers with valuation $\tilde{v}$ or higher are already considered to be going to the mall. However, throughout this thesis, we include IR conditions in the Lagrangian function for the sake of comprehensiveness.

[3] $\mathcal{L}_x$ stands for $\partial \mathcal{L}/\partial x$.

it must be that in equilibrium, $t^* = -r$. Since this is not feasible, it must be that $\tilde{v}_t > 1/\rho$. Hence,

$$1 - F(\tilde{v}) = \frac{f(\tilde{v})(\rho P + t - c_{lot}) - \lambda_2}{\rho} < f(\tilde{v})\tilde{v}_t(\rho P + t - c_{lot}) - \tilde{v}_t\lambda_2. \tag{3.11}$$

Since $\tilde{v}_t\lambda_2 \geq 0$,

$$1 - F(\tilde{v}) < f(\tilde{v})\tilde{v}_t(\rho P + t - c_{lot}). \tag{3.12}$$

Equating the first-order condition in (3.8) to zero and using $\lambda_1 = 0$,

$$1 - F(\tilde{v}) = f(\tilde{v})\tilde{v}_t(\rho P + t - c_{lot}) - \lambda_3. \tag{3.13}$$

Hence, by (3.12), it must be that $\lambda_3 > 0$. But then, by complementary slackness, in equilibrium, $t^* = 0$. Then, since $\rho P + t - c_{lot} > 0$ and $c_{lot} > 0$, P cannot be zero. Then, by complementary slackness, $\lambda_2 = 0$. Therefore, by (3.10), in equilibrium,

$$P^* = \frac{1 - F(\tilde{v})}{f(\tilde{v})} + \frac{c_{lot}}{\rho}. \tag{3.14}$$

Notice that the first term on the right-hand side is the monopoly markup, which is the inverse of the hazard rate. Therefore, by assumption, it decreases as P increases. Hence, the equilibrium price is unique. This solution also shows that the price is determined based on the ratio of the marginal cost of the parking lot to the probability of buying the good. As the probability of buying the good increases, the price decreases. Moreover, in equilibrium, parking is free for every customer and its cost is embedded in the price.

Hasker and Inci (2014) briefly mention the cases where there are different types of customers and each type is interested in only one type of good. In this thesis, we also examine these cases in detail.

In the rest of this thesis, all assumptions in the base model of Hasker and Inci (2014) are retained, except otherwise noted.

4. PRICE DISCRIMINATION MODELS WITH TWO TYPES OF CUSTOMERS

In this chapter, we analyze the pricing strategy of a monopolist mall when it can price discriminate. We assume that the necessary conditions for price discrimination are satisfied. That is, there are different market segments that the mall can identify and the resale of the goods is not allowed.

There are two types of customers, Type 1 and Type 2. Customers are identical within each type. Type 1 customer has a valuation $v_1 \in [0, \bar{v}_1]$ and pays $P_1 \geq 0$ for the good and t_1 for the parking lot. Type 2 customer has a valuation $v_2 \in [0, \bar{v}_2]$ and pays $P_2 \geq 0$ for the good and t_2 for the parking lot.

4.1 First-Degree Price Discrimination

In first-degree price discrimination, the monopoly knows the maximum price each customer is willing to pay and charges them accordingly. Even though first-degree price discrimination is relatively difficult to implement in real life, there are companies that collect data about customers' personal information such as gender, age, district, and past purchases to predict customers' maximum willingness to pay for the good. In this setting, the mall can implement this type of price discrimination by providing an app for its customers that would collect personal data, and then the mall can set a specific price for the good for each customer.

In this section, we analyze the pricing strategy of a monopolist mall when it implements first-degree price discrimination. We assume that there is one type of good.[1] Suppose that the mall can perfectly differentiate the customers and their willingness

[1] The case of two types of goods and two types of customers is examined in Section 4.2.

to pay for the good. Therefore, the mall can set different parking fees and prices for the good for each type of customer.

Type 1 customer goes to the mall if and only if her expected utility of going to the mall is greater or equal to her expected utility of not going to the mall. That is, she goes to the mall if and only if

$$\rho u(w+v_1-P_1-t_1)+(1-\rho)u(w-t_1)\geq u(w+r). \tag{4.1}$$

Type 1 is indifferent between going to the mall and staying at home if she has the valuation $\tilde{v}_1(P_1,t_1)$,

$$\tilde{v}_1(P_1,t_1)\equiv u^{-1}\left(\frac{u(w+r)-(1-\rho)u(w-t_1)}{\rho}\right)-w+P_1+t_1. \tag{4.2}$$

Therefore,[2]

$$\tilde{v}_{1,P_1}=1, \tag{4.3}$$

and

$$\tilde{v}_{1,t_1}=1+\frac{(1-\rho)}{\rho}\frac{u_{t_1}(w-t_1)}{u_{t_1}(w+\tilde{v}_1-P_1-t_1)}\geq\frac{1}{\rho}. \tag{4.4}$$

Notice that if $\tilde{v}_{1,t_1}=1/\rho$, then $\tilde{v}_1=P_1$. But then, from the equation (4.2), it must be that in equilibrium, $t_1^*=-r$. Since this is not feasible, it must be that $\tilde{v}_{1,t_1}>1/\rho$.

Similarly, Type 2 goes to the mall if and only if his expected utility of going to the mall is greater or equal to his expected utility of not going to the mall. That is, he goes to the mall if and only if

$$\rho u(w+v_2-P_2-t_2)+(1-\rho)u(w-t_2)\geq u(w+r). \tag{4.5}$$

Type 2 is indifferent between going to the mall and staying at home if he has the valuation $\tilde{v}_2(P_2,t_2)$,

$$\tilde{v}_2(P_2,t_2)\equiv u^{-1}\left(\frac{u(w+r)-(1-\rho)u(w-t_2)}{\rho}\right)-w+P_2+t_2. \tag{4.6}$$

Therefore,

$$\tilde{v}_{2,P_2}=1, \tag{4.7}$$

and

$$\tilde{v}_{2,t_2}=1+\frac{(1-\rho)}{\rho}\frac{u_{t_2}(w-t_2)}{u_{t_2}(w+\tilde{v}_2-P_2-t_2)}\geq\frac{1}{\rho}. \tag{4.8}$$

[2] $\tilde{v}_{x,y}$ stands for $\partial\tilde{v}_x/\partial y$.

Notice that if $\tilde{v}_{2,t_2} = 1/\rho$, then $\tilde{v}_2 = P_2$. But then, from the equation (4.6), it must be that in equilibrium, $t_2^* = -r$. Since this is not feasible, it must be that $\tilde{v}_{2,t_2} > 1/\rho$.

Therefore, there are $1-F(\tilde{v}_1)$ customers of Type 1 and $1-F(\tilde{v}_2)$ customers of Type 2. The mall's profit is $\Pi(P_1,P_2,t_1,t_2)$, which is the summation of the profits it receives from each type of customer,

$$\begin{aligned}\Pi(P_1,P_2,t_1,t_2) = [1-F(\tilde{v}_1)][\rho(P_1-c_{good})+t_1-c_{lot}] \\ +[1-F(\tilde{v}_2)][\rho(P_2-c_{good})+t_2-c_{lot}].\end{aligned} \tag{4.9}$$

The mall maximizes its profit with respect to P_1, P_2, t_1, and t_2, subject to individual rationality constraints of each type of customer,

$$(IR_1): \quad \rho u(w+v_1-P_1-t_1)+(1-\rho)u(w-t_1) \geq u(w+r), \tag{4.10}$$

$$(IR_2): \quad \rho u(w+v_2-P_2-t_2)+(1-\rho)u(w-t_2) \geq u(w+r), \tag{4.11}$$

and the non-negativity constraints,

$$\rho(P_1-c_{good})+t_1-c_{lot} \geq 0, \quad \rho(P_2-c_{good})+t_2-c_{lot} \geq 0, \quad P_1,P_2,t_1,t_2 \geq 0. \tag{4.12}$$

The Lagrangian function is

$$\begin{aligned}\mathcal{L}(P_1,P_2,t_1,t_2) = {} & [1-F(\tilde{v}_1)][\rho(P_1-c_{good})+t_1-c_{lot}] \\ & +[1-F(\tilde{v}_2)][\rho(P_2-c_{good})+t_2-c_{lot}] \\ & +\lambda_1[\rho u(w+v_1-P_1-t_1)+(1-\rho)u(w-t_1)-u(w+r)] \\ & +\lambda_2[\rho u(w+v_2-P_2-t_2)+(1-\rho)u(w-t_2)-u(w+r)] \\ & +\lambda_3[\rho(P_1-c_{good})+t_1-c_{lot}]+\lambda_4[\rho(P_2-c_{good})+t_2-c_{lot}] \\ & +\lambda_5 P_1+\lambda_6 P_2+\lambda_7 t_1+\lambda_8 t_2\end{aligned} \tag{4.13}$$

where $\lambda_1, \lambda_2, \lambda_3, \lambda_4, \lambda_5, \lambda_6, \lambda_7$, and λ_8 are Lagrangian multipliers.

Taking the first-order conditions with respect to P_1, P_2, t_1, and t_2,

$$\begin{aligned}\mathcal{L}_{P_1} = {} & \rho[1-F(\tilde{v}_1)]-f(\tilde{v}_1)\tilde{v}_{1,P_1}[\rho(P_1-c_{good})+t_1-c_{lot}] \\ & +\lambda_1\rho u_{P_1}(w+v_1-P_1-t_1)(\tilde{v}_{1,P_1}-1)+\lambda_3\rho+\lambda_5.\end{aligned} \tag{4.14}$$

$$\begin{aligned}\mathcal{L}_{P_2} = {} & \rho[1-F(\tilde{v}_2)]-f(\tilde{v}_2)\tilde{v}_{2,P_2}[\rho(P_2-c_{good})+t_2-c_{lot}] \\ & +\lambda_2\rho u_{P_2}(w+v_2-P_2-t_2)(\tilde{v}_{2,P_2}-1)+\lambda_4\rho+\lambda_6.\end{aligned} \tag{4.15}$$

$$\begin{aligned}\mathcal{L}_{t_1} &= 1 - F(\tilde{v}_1) - f(\tilde{v}_1)\tilde{v}_{1,t_1}[\rho(P_1 - c_{good}) + t_1 - c_{lot}] \\ &\quad + \lambda_1[\rho u_{t_1}(w + v_1 - P_1 - t_1)(\tilde{v}_{1,t_1} - 1) - (1-\rho)u_{t_1}(w - t_1)] \\ &\quad + \lambda_3 + \lambda_7.\end{aligned} \tag{4.16}$$

$$\begin{aligned}\mathcal{L}_{t_2} &= 1 - F(\tilde{v}_2) - f(\tilde{v}_2)\tilde{v}_{2,t_2}[\rho(P_2 - c_{good}) + t_2 - c_{lot}] \\ &\quad + \lambda_2[\rho u_{t_2}(w + v_2 - P_2 - t_2)(\tilde{v}_{2,t_2} - 1) - (1-\rho)u_{t_2}(w - t_2)] \\ &\quad + \lambda_4 + \lambda_8.\end{aligned} \tag{4.17}$$

To find the equilibrium parking fee and price of the good for Type 1, first we equate the first-order condition in (4.14) to zero and use $\tilde{v}_{1,P_1} = 1$,

$$\rho[1 - F(\tilde{v}_1)] + \lambda_3\rho + \lambda_5 = f(\tilde{v}_1)[\rho(P_1 - c_{good}) + t_1 - c_{lot}]. \tag{4.18}$$

Notice that the expression $\rho(P_1 - c_{good}) + t_1 - c_{lot}$ cannot be zero, since $\rho[1 - F(\tilde{v}_1)]$ is greater than zero. Then, it must be that $\lambda_3 = 0$, since by complementary slackness condition, $\lambda_3[\rho(P_1 - c_{good}) + t_1 - c_{lot}] = 0$. Therefore, by (4.18),

$$1 - F(\tilde{v}_1) = \frac{f(\tilde{v}_1)[\rho(P_1 - c_{good}) + t_1 - c_{lot}] - \lambda_5}{\rho}. \tag{4.19}$$

Since $\tilde{v}_{1,t_1} > 1/\rho$,

$$\begin{aligned}1 - F(\tilde{v}_1) &= \frac{f(\tilde{v}_1)[\rho(P_1 - c_{good}) + t_1 - c_{lot}] - \lambda_5}{\rho} \\ &< f(\tilde{v}_1)\tilde{v}_{1,t_1}[\rho(P_1 - c_{good}) + t_1 - c_{lot}] - \tilde{v}_{1,t_1}\lambda_5.\end{aligned} \tag{4.20}$$

Since $\tilde{v}_{1,t_1}\lambda_5 \geq 0$,

$$1 - F(\tilde{v}_1) < f(\tilde{v}_1)\tilde{v}_{1,t_1}[\rho(P_1 - c_{good}) + t_1 - c_{lot}]. \tag{4.21}$$

Equating the first-order condition in (4.16) to zero,

$$\begin{aligned}1 - F(\tilde{v}_1) &= f(\tilde{v}_1)\tilde{v}_{1,t_1}[\rho(P_1 - c_{good}) + t_1 - c_{lot}] \\ &\quad - \lambda_1[\rho u_{t_1}(w + v_1 - P_1 - t_1)(\tilde{v}_{1,t_1} - 1) - (1-\rho)u_{t_1}(w - t_1)] - \lambda_7.\end{aligned} \tag{4.22}$$

Then, by (4.4), (4.22) becomes

$$1 - F(\tilde{v}_1) = f(\tilde{v}_1)\tilde{v}_{1,t_1}[\rho(P_1 - c_{good}) + t_1 - c_{lot}] - \lambda_7. \tag{4.23}$$

Then, by (4.21), $\lambda_7 > 0$. Hence, by complementary slackness condition, $t_1^* = 0$. Then, it must be that $P_1 > 0$, since the expression $\rho(P_1 - c_{good}) + t_1 - c_{lot}$ cannot be negative. Therefore, by complementary slackness condition, $\lambda_5 = 0$ and by (4.19), in equilibrium,

$$P_1^* = \frac{1-F(\tilde{v}_1)}{f(\tilde{v}_1)} + \frac{c_{lot}}{\rho} + c_{good}, \quad \text{and} \quad t_1^* = 0. \tag{4.24}$$

To find the equilibrium parking fee and price of the good for Type 2, similar steps are followed:

Equating the first-order condition in (4.15) to zero and using $\tilde{v}_{2,P_2} = 1$,

$$\rho[1-F(\tilde{v}_2)] + \lambda_4\rho + \lambda_6 = f(\tilde{v}_2)[\rho(P_2 - c_{good}) + t_2 - c_{lot}]. \tag{4.25}$$

Notice that the expression $\rho(P_2 - c_{good}) + t_2 - c_{lot}$ cannot be zero, since $\rho[1-F(\tilde{v}_2)]$ is greater than zero. Then, it must be that $\lambda_4 = 0$, since by complementary slackness condition, $\lambda_4[\rho(P_2 - c_{good}) + t_2 - c_{lot}] = 0$. Therefore, by (4.25),

$$1-F(\tilde{v}_2) = \frac{f(\tilde{v}_2)[\rho(P_2 - c_{good}) + t_2 - c_{lot}] - \lambda_6}{\rho}. \tag{4.26}$$

Since $\tilde{v}_{2,t_2} > 1/\rho$,

$$\begin{aligned} 1-F(\tilde{v}_2) &= \frac{f(\tilde{v}_2)[\rho(P_2 - c_{good}) + t_2 - c_{lot}] - \lambda_6}{\rho} \\ &< f(\tilde{v}_2)\tilde{v}_{2,t_2}[\rho(P_2 - c_{good}) + t_2 - c_{lot}] - \tilde{v}_{2,t_2}\lambda_6. \end{aligned} \tag{4.27}$$

Since $\tilde{v}_{2,t_2}\lambda_6 \geq 0$,

$$1-F(\tilde{v}_2) < f(\tilde{v}_2)\tilde{v}_{2,t_2}[\rho(P_2 - c_{good}) + t_2 - c_{lot}]. \tag{4.28}$$

Equating the first-order condition in (4.17) to zero,

$$\begin{aligned} 1-F(\tilde{v}_2) = {} & f(\tilde{v}_2)\tilde{v}_{2,t_2}[\rho(P_2 - c_{good}) + t_2 - c_{lot}] \\ & - \lambda_2[\rho u_{t_2}(w + v_2 - P_2 - t_2)(\tilde{v}_{2,t_2} - 1) - (1-\rho)u_{t_2}(w - t_2)] - \lambda_8. \end{aligned} \tag{4.29}$$

Then, by (4.8), (4.29) becomes

$$1-F(\tilde{v}_2) = f(\tilde{v}_2)\tilde{v}_{2,t_2}[\rho(P_2 - c_{good}) + t_2 - c_{lot}] - \lambda_8. \tag{4.30}$$

Then, by (2.28), $\lambda_8 > 0$. Hence, by complementary slackness condition, $t_2^* = 0$. Then, it must be that $P_2 > 0$, since the expression $\rho(P_2 - c_{good}) + t_2 - c_{lot}$ cannot be negative. Therefore by complementary slackness condition, $\lambda_6 = 0$ and by (4.26), in equilibrium,

$$P_2^* = \frac{1 - F(\tilde{v}_2)}{f(\tilde{v}_2)} + \frac{c_{lot}}{\rho} + c_{good}, \quad \text{and} \quad t_2^* = 0. \tag{4.31}$$

Under first-degree price discrimination where the mall can perfectly differentiate between Type 1 and Type 2 customers, the mall can set two prices, P_1^* and P_2^*, for the same good. We find that in equilibrium, these prices are unique. Also, parking is free for both Type 1 and Type 2 customers. Moreover, the cost of the parking lot is embedded in the prices of the good.

The prices of the good depend positively on the ratio of the marginal cost of the parking lot to the probability of finding the good and depend negatively on the probability of finding the good. For a fixed marginal cost of the good and the parking lot, as the probability of finding the good decreases, the equilibrium prices of the good increase. Hence, as the good becomes more difficult to find, customers pay more for the good.

Since ρ can be thought of as the probability of buying the good, these equilibrium prices also mean that as customers become less likely to buy the good, they pay more for the good. Hence, the mall charges the customers more as their likeliness of buying the good decreases and charges them less as they get more likely to buy the good. This could be interpreted as the following: the mall aims to attract customers with a higher probability of buying the good by offering them a lower price while making customers with a lower probability of buying the good pay more for the good.

Therefore, the following proposition is established:

Proposition 1. Under first-degree price discrimination where the mall can perfectly differentiate between Type 1 and Type 2, the prices are unique and cover the cost of the parking lot. Moreover, as customers become less (more) likely to buy the good, they pay more (less) for the good.

4.2 Second-Degree Price Discrimination

In second-degree price discrimination, the mall knows that there are different types of customers but does not know which customer is which type. Hence, the mall offers the goods in different quality or quantity. For instance, there may be higher-income and lower-income customers and the mall may sell luxury and regular watches or service the customers in two restaurants such that one has a nice view and comfortable seats while the other one does not.

In this section, we analyze the pricing strategy of the mall when it implements second-degree price discrimination. We assume that there are two types of customers, Type 1 and Type 2, and two goods, Good 1 and Good 2. Good 1 has a marginal cost $c_{good_1} > 0$ and the probability of finding it is $\rho_1 \in (0,1)$ while Good 2 has a marginal cost $c_{good_2} > 0$ and the probability of finding it is $\rho_2 \in (0,1)$. The marginal costs of the parking lots for Good 1 and Good 2 buyers are $c_{lot_1} > 0$ and $c_{lot_2} > 0$, respectively.[3] Customers who are interested in Good 1 pay t_1 while customers who are interested in Good 2 pay t_2 for the parking lot. The mall can provide parking vouchers to implement this.

In this section, we assume that the mall cannot distinguish the type of customers. Therefore, while solving for the optimal prices, the mall also imposes incentive compatibility conditions in order to make customers reveal their true types. Following Stiglitz (1977), we examine this case for two situations: (i) Two types are differing only in the probability of finding (or buying) the good, and (ii) two types are differing in their attitudes towards risk.

4.2.1 Types are differing in probability of buying the good

In this model, customers differ only in their probabilities of finding (or buying) the good. For instance, there can be fastidious and easily pleased customers who differ in probability of buying the good. The mall knows that customers have different types but cannot identify the type of a given customer.

Type 1 customer goes to the mall if

$$\rho_1 u(w+v_1-P_1-t_1)+(1-\rho_1)u(w-t_1) \geq u(w+r). \tag{4.32}$$

Type 1 customer who is indifferent between going to the mall or not has a valuation

[3]For instance, the parking lot of a certain type of good's buyers may need a larger space, hence it may have a higher marginal cost.

$\tilde{v}_1(P_1, t_1)$,

$$\tilde{v}_1(P_1, t_1) \equiv u^{-1}\left(\frac{u(w+r)-(1-\rho_1)u(w-t_1)}{\rho_1}\right) - w + P_1 + t_1. \tag{4.33}$$

Therefore,[4]

$$\tilde{v}_{1,P_1} = 1, \tag{4.34}$$

and

$$\tilde{v}_{1,t_1} = 1 + \frac{(1-\rho_1)}{\rho_1}\frac{u_{t_1}(w-t_1)}{u_{t_1}(w+\tilde{v}_1-P_1-t_1)} \geq \frac{1}{\rho_1}. \tag{4.35}$$

Notice that if $\tilde{v}_{1,t_1} = 1/\rho_1$, then $\tilde{v}_1 = P_1$. But then, from the equation (4.33), it must be that in equilibrium, $t_1^* = -r$. Since this is not feasible, it must be that $\tilde{v}_{1,t_1} > 1/\rho_1$.

Similarly, Type 2 customer goes to the mall if

$$\rho_2 u(w+v_2-P_2-t_2) + (1-\rho_2)u(w-t_2) \geq u(w+r). \tag{4.36}$$

Type 2 customer who is indifferent between going to the mall or not has a valuation $\tilde{v}_2(P_2, t_2)$,

$$\tilde{v}_2(P_2, t_2) \equiv u^{-1}\left(\frac{u(w+r)-(1-\rho_2)u(w-t_2)}{\rho_2}\right) - w + P_2 + t_2. \tag{4.37}$$

Therefore,[5]

$$\tilde{v}_{2,P_2} = 1, \tag{4.38}$$

and

$$\tilde{v}_{2,t_2} = 1 + \frac{(1-\rho_2)}{\rho_2}\frac{u_{t_2}(w-t_2)}{u_{t_2}(w+\tilde{v}_2-P_2-t_2)} \geq \frac{1}{\rho_2}. \tag{4.39}$$

Notice that if $\tilde{v}_{2,t_2} = 1/\rho_2$, then $\tilde{v}_2 = P_2$. But then, from the equation (4.37), it must be that in equilibrium, $t_2^* = -r$. Since this is not feasible, it must be that $\tilde{v}_{2,t_2} > 1/\rho_2$.

The mall's profit is $\Pi(P_1, P_2, t_1, t_2)$, which is the summation of the profits it receives from each type of customer,

$$\begin{aligned}\Pi(P_1, P_2, t_1, t_2) = {} & [1-F(\tilde{v}_1)]\left[\rho_1\left(P_1 - c_{good_1}\right) + t_1 - c_{lot_1}\right] \\ & + [1-F(\tilde{v}_2)]\left[\rho_2\left(P_2 - c_{good_2}\right) + t_2 - c_{lot_2}\right].\end{aligned} \tag{4.40}$$

[4] Note that $\tilde{v}_{1,P_2} = 0$ and $\tilde{v}_{1,t_2} = 0$.

[5] Note that $\tilde{v}_{2,P_1} = 0$ and $\tilde{v}_{2,t_1} = 0$.

The mall maximizes its profit with respect to P_1, P_2, t_1, and t_2, subject to individual rationality constraints of each type,

$$(IR_1): \quad \rho_1 u(w+v_1-P_1-t_1)+(1-\rho_1)u(w-t_1) \geq u(w+r), \tag{4.41}$$

$$(IR_2): \quad \rho_2 u(w+v_2-P_2-t_2)+(1-\rho_2)u(w-t_2) \geq u(w+r), \tag{4.42}$$

and incentive compatibility constraints of each type,

$$\begin{aligned}(IC_1): \quad & \rho_1 u(w+v_1-P_1-t_1)+(1-\rho_1)u(w-t_1) \\ & \geq \rho_2 u(w+v_2-P_2-t_2)+(1-\rho_2)u(w-t_2),\end{aligned} \tag{4.43}$$

$$\begin{aligned}(IC_2): \quad & \rho_2 u(w+v_2-P_2-t_2)+(1-\rho_2)u(w-t_2) \\ & \geq \rho_1 u(w+v_1-P_1-t_1)+(1-\rho_1)u(w-t_1),\end{aligned} \tag{4.44}$$

and the non-negativity constraints,

$$\begin{aligned}\rho_1(P_1-c_{good_1})+t_1-c_{lot_1} \geq 0, \quad \rho_2(P_2-c_{good_2})+t_2-c_{lot_2} \geq 0, \\ P_1, P_2, t_1, t_2 \geq 0.\end{aligned} \tag{4.45}$$

The Lagrangian function is

$$\begin{aligned}\mathcal{L}(P_1,P_2,t_1,t_2) = {} & [1-F(\tilde{v}_1)][\rho_1(P_1-c_{good_1})+t_1-c_{lot_1}] \\ & +[1-F(\tilde{v}_2)][\rho_2(P_2-c_{good_2})+t_2-c_{lot_2}] \\ & +\lambda_1[\rho_1 u(w+v_1-P_1-t_1)+(1-\rho_1)u(w-t_1)-u(w+r)] \\ & +\lambda_2[\rho_2 u(w+v_2-P_2-t_2)+(1-\rho_2)u(w-t_2)-u(w+r)] \\ & +\lambda_3[\rho_1 u(w+v_1-P_1-t_1)+(1-\rho_1)u(w-t_1) \\ & -\rho_2 u(w+v_2-P_2-t_2)-(1-\rho_2)u(w-t_2)] \\ & +\lambda_4[\rho_2 u(w+v_2-P_2-t_2)+(1-\rho_2)u(w-t_2) \\ & -\rho_1 u(w+v_1-P_1-t_1)-(1-\rho_1)u(w-t_1)] \\ & +\lambda_5[\rho_1(P_1-c_{good_1})+t_1-c_{lot_1}] \\ & +\lambda_6[\rho_2(P_2-c_{good_2})+t_2-c_{lot_2}] \\ & +\lambda_7 P_1+\lambda_8 P_2+\lambda_9 t_1+\lambda_{10} t_2\end{aligned} \tag{4.46}$$

where $\lambda_1, \lambda_2, \lambda_3, \lambda_4, \lambda_5, \lambda_6, \lambda_7, \lambda_8, \lambda_9$, and λ_{10} are Lagrangian multipliers.

Taking the first-order conditions with respect to P_1, P_2, t_1, and t_2,

$$\begin{aligned}\mathcal{L}_{P_1} &= \rho_1[1-F(\tilde{v}_1)] - f(\tilde{v}_1)\tilde{v}_{1,P_1}[\rho_1(P_1-c_{good_1})+t_1-c_{lot_1}] \\ &\quad + (\lambda_1+\lambda_3-\lambda_4)\rho_1 u_{P_1}(w+v_1-P_1-t_1)(\tilde{v}_{1,P_1}-1)+\lambda_5\rho_1+\lambda_7.\end{aligned} \tag{4.47}$$

$$\begin{aligned}\mathcal{L}_{P_2} &= \rho_2[1-F(\tilde{v}_2)] - f(\tilde{v}_2)\tilde{v}_{2,P_2}[\rho_2(P_2-c_{good_2})+t_2-c_{lot_2}] \\ &\quad + (\lambda_2+\lambda_4-\lambda_3)\rho_2 u_{P_2}(w+v_2-P_2-t_2)(\tilde{v}_{2,P_2}-1)+\lambda_6\rho_2+\lambda_8.\end{aligned} \tag{4.48}$$

$$\begin{aligned}\mathcal{L}_{t_1} &= 1-F(\tilde{v}_1) - f(\tilde{v}_1)\tilde{v}_{1,t_1}[\rho_1(P_1-c_{good_1})+t_1-c_{lot_1}] \\ &\quad + (\lambda_1+\lambda_3-\lambda_4)[\rho_1 u_{t_1}(w+v_1-P_1-t_1)(\tilde{v}_{1,t_1}-1)-(1-\rho_1)u_{t_1}(w-t_1)] \\ &\quad +\lambda_5+\lambda_9.\end{aligned} \tag{4.49}$$

$$\begin{aligned}\mathcal{L}_{t_2} &= 1-F(\tilde{v}_2) - f(\tilde{v}_2)\tilde{v}_{2,t_2}[\rho_2(P_2-c_{good_2})+t_2-c_{lot_2}] \\ &\quad + (\lambda_2+\lambda_4-\lambda_3)[\rho_2 u_{t_2}(w+v_2-P_2-t_2)(\tilde{v}_{2,t_2}-1)-(1-\rho_2)u_{t_2}(w-t_2)] \\ &\quad +\lambda_6+\lambda_{10}.\end{aligned} \tag{4.50}$$

To find the equilibrium parking fee and price of the good for Type 1, first we equate $\mathcal{L}_{P_1}$ to zero and use $\tilde{v}_{1,P_1}=1$,

$$1-F(\tilde{v}_1) = \frac{f(\tilde{v}_1)[\rho_1(P_1-c_{good_1})+t_1-c_{lot_1}]-\lambda_5\rho_1-\lambda_7}{\rho_1}. \tag{4.51}$$

Notice that the expression $\rho_1(P_1-c_{good_1})+t_1-c_{lot_1}$ cannot be zero. Thus, by complementary slackness, $\lambda_5=0$. Therefore, (4.51) becomes

$$1-F(\tilde{v}_1) = \frac{f(\tilde{v}_1)[\rho_1(P_1-c_{good_1})+t_1-c_{lot_1}]-\lambda_7}{\rho_1}. \tag{4.52}$$

Since $\tilde{v}_{1,t_1} > 1/\rho_1$,

$$\begin{aligned}1-F(\tilde{v}_1) &= \frac{f(\tilde{v}_1)[\rho_1(P_1-c_{good_1})+t_1-c_{lot_1}]-\lambda_7}{\rho_1} \\ &< f(\tilde{v}_1)\tilde{v}_{1,t_1}[\rho_1(P_1-c_{good_1})+t_1-c_{lot_1}]-\lambda_7\tilde{v}_{1,t_1}.\end{aligned} \tag{4.53}$$

Since $\lambda_7\tilde{v}_{1,t_1} \geq 0$,

$$1-F(\tilde{v}_1) < f(\tilde{v}_1)\tilde{v}_{1,t_1}[\rho_1(P_1-c_{good_1})+t_1-c_{lot_1}]. \tag{4.54}$$

Equating $\mathcal{L}_{t_1}$ to zero and using (4.35),

$$1 - F(\tilde{v}_1) = f(\tilde{v}_1)\tilde{v}_{1,t_1}[\rho_1(P_1 - c_{good_1}) + t_1 - c_{lot_1}] - \lambda_9. \tag{4.55}$$

Then, by (4.54), λ_9 cannot be zero. Then, by complementary slackness, $t_1^* = 0$. Then, P_1 cannot be zero since the expression $\rho_1(P_1 - c_{good_1}) + t_1 - c_{lot_1}$ cannot be negative. Hence, by complementary slackness, $\lambda_7 = 0$. Then, by (4.52), in equilibrium,

$$P_1^* = \frac{1 - F(\tilde{v}_1)}{f(\tilde{v}_1)} + \frac{c_{lot_1}}{\rho_1} + c_{good_1}, \quad \text{and} \quad t_1^* = 0. \tag{4.56}$$

To find the equilibrium parking fee and price of the good for Type 2, similar steps are followed:

Equating $\mathcal{L}_{P_2}$ to zero and using $\tilde{v}_{2,P_2} = 1$,

$$1 - F(\tilde{v}_2) = \frac{f(\tilde{v}_2)[\rho_2(P_2 - c_{good_2}) + t_2 - c_{lot_2}] - \lambda_6\rho_2 - \lambda_8}{\rho_2}. \tag{4.57}$$

Notice that the expression $\rho_2(P_2 - c_{good_2}) + t_2 - c_{lot_2}$ cannot be zero. Thus, by complementary slackness, $\lambda_6 = 0$. Therefore, (4.57) becomes

$$1 - F(\tilde{v}_2) = \frac{f(\tilde{v}_2)[\rho_2(P_2 - c_{good_2}) + t_2 - c_{lot_2}] - \lambda_8}{\rho_2}. \tag{4.58}$$

Since $\tilde{v}_{2,t_2} > 1/\rho_2$,

$$\begin{aligned} 1 - F(\tilde{v}_2) &= \frac{f(\tilde{v}_2)[\rho_2(P_2 - c_{good_2}) + t_2 - c_{lot_2}] - \lambda_8}{\rho_2} \\ &< f(\tilde{v}_2)\tilde{v}_{2,t_2}[\rho_2(P_2 - c_{good_2}) + t_2 - c_{lot_2}] - \lambda_8\tilde{v}_{2,t_2}. \end{aligned} \tag{4.59}$$

Since $\lambda_8\tilde{v}_{2,t_2} \geq 0$,

$$1 - F(\tilde{v}_2) < f(\tilde{v}_2)\tilde{v}_{2,t_2}[\rho_2(P_2 - c_{good_2}) + t_2 - c_{lot_2}]. \tag{4.60}$$

Equating $\mathcal{L}_{t_2}$ to zero and using (4.39),

$$1 - F(\tilde{v}_2) = f(\tilde{v}_2)\tilde{v}_{2,t_2}[\rho_2(P_2 - c_{good_2}) + t_2 - c_{lot_2}] - \lambda_{10}. \tag{4.61}$$

Then, by (4.60), λ_{10} cannot be zero. Then, by complementary slackness, $t_2^* = 0$. Then, P_2 cannot be zero since the expression $\rho_2(P_2 - c_{good_2}) + t_2 - c_{lot_2}$ cannot be negative. Hence, by complementary slackness, $\lambda_8 = 0$. Then, by (4.58), in equilibrium,

$$P_2^* = \frac{1 - F(\tilde{v}_2)}{f(\tilde{v}_2)} + \frac{c_{lot_2}}{\rho_2} + c_{good_2}, \quad \text{and} \quad t_2^* = 0. \tag{4.62}$$

Therefore, under second-degree price discrimination where the types of customers have the same utility functions but different probabilities of finding (or buying) the goods, parking is free for both Type 1 and Type 2, and the cost of the parking lot is embedded in the prices of Good 1 and Good 2.

Moreover, the prices of Good 1 and Good 2 are unique. For each good, the price of the good depends negatively on the probability of finding the good. For a fixed marginal cost of good and parking lot, as the probability of finding (or buying) the good decreases, the equilibrium price of that good increases. Hence, as customers become less likely to buy the good, they pay more for that good. Thus, the mall aims to attract customers with a higher probability of buying the good by offering them a lower price while making customers with a lower probability of buying the good pay more for the good.

Therefore, the following proposition is established:

Proposition 2. Under second-degree price discrimination where customers differ in their probabilities of finding the good, the prices of Good 1 and Good 2 are unique and cover the costs of the parking lots. Moreover, as customers become less (more) likely to buy the good, they pay more (less) for the good.

In this model, the types are not observable to the mall. However, the mall can set the prices such that every customer reveals their own types. The equilibrium prices of the goods are not affected by the probability of finding the other good. This makes sense since the marginal customer's valuation of a good does not depend on the price of the other good. In addition, in this section we allow the costs of the goods and parking lots to be different for each type. Even though this does not change the analysis, we show that in equilibrium, the price of a good does not depend on the cost of the other good or the cost of the parking lot reserved for the other type of customer.

4.2.2 Types are differing in their attitudes towards risk

In this section, we study the pricing strategy of the mall when customers have different utility functions and degree of risk-aversion. Factors such as gender, age, marital status, and health condition may affect a person's degree of risk-aversion. For instance, parents can be more risk-averse than other people. In this model, the mall knows that customers have different attitudes towards risk but cannot identify the type of a given customer.

In our model, Type 1 and Type 2 customers have utility functions u_1 and u_2, respectively. Without loss of generality, we let Type 1 customer be more risk-averse than Type 2 customer. Thus, $u_1 = h \circ u_2$, for some concave function h.

Type 1 goes to the mall if

$$\rho_1 u_1(w + v_1 - P_1 - t_1) + (1 - \rho_1) u_1(w - t_1) \geq u_1(w + r). \tag{4.63}$$

The marginal customer of Type 1 who is indifferent between going to the mall or not has a valuation $\tilde{v}_1(P_1, t_1)$,

$$\tilde{v}_1(P_1, t_1) \equiv u_1^{-1}\left(\frac{u_1(w+r) - (1-\rho_1)u_1(w-t_1)}{\rho_1}\right) - w + P_1 + t_1. \tag{4.64}$$

Therefore,[6,7]

$$\tilde{v}_{1,P_1} = 1, \tag{4.65}$$

and

$$\tilde{v}_{1,t_1} = 1 + \frac{(1-\rho_1)}{\rho_1} \frac{u_{1,t_1}(w - t_1)}{u_{1,t_1}(w + \tilde{v}_1 - P_1 - t_1)} \geq \frac{1}{\rho_1}. \tag{4.66}$$

Notice that if $\tilde{v}_{1,t_1} = 1/\rho_1$, then $\tilde{v}_1 = P_1$. But then, from the equation (4.64), it must be that in equilibrium, $t_1^* = -r$. Since this is not feasible, it must be that $\tilde{v}_{1,t_1} > 1/\rho_1$.

Similarly, Type 2 goes to the mall if

$$\rho_2 u_2(w + v_2 - P_2 - t_2) + (1 - \rho_2) u_2(w - t_2) \geq u_2(w + r). \tag{4.67}$$

The marginal customer of Type 2 who is indifferent between going to the mall or not has a valuation $\tilde{v}_2(P_2, t_2)$,

$$\tilde{v}_2(P_2, t_2) \equiv u_2^{-1}\left(\frac{u_2(w+r) - (1-\rho_2)u_2(w-t_2)}{\rho_2}\right) - w + P_2 + t_2. \tag{4.68}$$

[6] $u_{x,y}$ stands for $\partial u_x / \partial y$

[7] Note that $\tilde{v}_{1,P_2} = 0$ and $\tilde{v}_{1,t_2} = 0$.

Therefore,[8]

$$\tilde{v}_{2,P_2} = 1, \tag{4.69}$$

and

$$\tilde{v}_{2,t_2} = 1 + \frac{(1-\rho_2)}{\rho_2} \frac{u_{2,t_2}(w-t_2)}{u_{2,t_2}(w+\tilde{v}_2-P_2-t_2)} \geq \frac{1}{\rho_2}. \tag{4.70}$$

Notice that if $\tilde{v}_{2,t_2} = 1/\rho_2$, then $\tilde{v}_2 = P_2$. But then, from the equation (4.68), it must be that in equilibrium, $t_2^* = -r$. Since this is not feasible, it must be that $\tilde{v}_{2,t_2} > 1/\rho_2$.

The mall's profit is $\Pi(P_1, P_2, t_1, t_2)$, which is the summation of the profits it receives from each type of customer,

$$\begin{aligned} \Pi(P_1, P_2, t_1, t_2) = & [1 - F(\tilde{v}_1)] \left[\rho_1 \left(P_1 - c_{good_1}\right) + t_1 - c_{lot_1}\right] \\ & + [1 - F(\tilde{v}_2)] \left[\rho_2 \left(P_2 - c_{good_2}\right) + t_2 - c_{lot_2}\right]. \end{aligned} \tag{4.71}$$

The mall maximizes its profit with respect to P_1, P_2, t_1, and t_2, subject to individual rationality constraints of each type,

$$(IR_1): \quad \rho_1 u_1(w + v_1 - P_1 - t_1) + (1-\rho_1) u_1(w - t_1) \geq u_1(w + r), \tag{4.72}$$

$$(IR_2): \quad \rho_2 u_2(w + v_2 - P_2 - t_2) + (1-\rho_2) u_2(w - t_2) \geq u_2(w + r), \tag{4.73}$$

and incentive compatibility constraints of each type,

$$\begin{aligned} (IC_1): \quad & \rho_1 u_1(w + v_1 - P_1 - t_1) + (1-\rho_1) u_1(w - t_1) \\ & \geq \rho_2 u_1(w + v_2 - P_2 - t_2) + (1-\rho_2) u_1(w - t_2), \end{aligned} \tag{4.74}$$

$$\begin{aligned} (IC_2): \quad & \rho_2 u_2(w + v_2 - P_2 - t_2) + (1-\rho_2) u_2(w - t_2) \\ & \geq \rho_1 u_2(w + v_1 - P_1 - t_1) + (1-\rho_1) u_2(w - t_1), \end{aligned} \tag{4.75}$$

and the non-negativity constraints,

$$\begin{aligned} \rho_1(P_1 - c_{good_1}) + t_1 - c_{lot_1} \geq 0, \quad \rho_2(P_2 - c_{good_2}) + t_2 - c_{lot_2} \geq 0, \\ P_1, P_2, t_1, t_2 \geq 0. \end{aligned} \tag{4.76}$$

[8] Note that $\tilde{v}_{2,P_1} = 0$ and $\tilde{v}_{2,t_1} = 0$.

The Lagrangian function is

$$
\begin{aligned}
\mathcal{L}(P_1,P_2,t_1,t_2) = {} & [1-F(\tilde{v}_1)][\rho_1(P_1-c_{good_1})+t_1-c_{lot_1}] \\
& +[1-F(\tilde{v}_2)][\rho_2(P_2-c_{good_2})+t_2-c_{lot_2}] \\
& +\lambda_1[\rho_1 u_1(w+v_1-P_1-t_1)+(1-\rho_1)u_1(w-t_1)-u_1(w+r)] \\
& +\lambda_2[\rho_2 u_2(w+v_2-P_2-t_2)+(1-\rho_2)u_2(w-t_2)-u_2(w+r)] \\
& +\lambda_3[\rho_1 u_1(w+v_1-P_1-t_1)+(1-\rho_1)u_1(w-t_1) \\
& -\rho_2 u_1(w+v_2-P_2-t_2)-(1-\rho_2)u_1(w-t_2)] \\
& +\lambda_4[\rho_2 u_2(w+v_2-P_2-t_2)+(1-\rho_2)u_2(w-t_2) \\
& -\rho_1 u_2(w+v_1-P_1-t_1)-(1-\rho_1)u_2(w-t_1)] \\
& +\lambda_5[\rho_1(P_1-c_{good_1})+t_1-c_{lot_1}] \\
& +\lambda_6[\rho_2(P_2-c_{good_2})+t_2-c_{lot_2}] \\
& +\lambda_7 P_1+\lambda_8 P_2+\lambda_9 t_1+\lambda_{10} t_2
\end{aligned}
\tag{4.77}
$$

where $\lambda_1,\lambda_2,\lambda_3,\lambda_4,\lambda_5,\lambda_6,\lambda_7,\lambda_8,\lambda_9$, and λ_{10} are Lagrangian multipliers.

Taking the first-order conditions with respect to P_1,P_2,t_1, and t_2,

$$
\begin{aligned}
\mathcal{L}_{P_1} = {} & \rho_1[1-F(\tilde{v}_1)]-f(\tilde{v}_1)\tilde{v}_{1,P_1}[\rho_1(P_1-c_{good_1})+t_1-c_{lot_1}] \\
& +(\lambda_1+\lambda_3)\,\rho_1 u_{1,P_1}(w+v_1-P_1-t_1)(\tilde{v}_{1,P_1}-1) \\
& -\lambda_4\rho_1 u_{2,P_1}(w+v_1-P_1-t_1)(\tilde{v}_{1,P_1}-1)+\lambda_5\rho_1+\lambda_7.
\end{aligned}
\tag{4.78}
$$

$$
\begin{aligned}
\mathcal{L}_{P_2} = {} & \rho_2[1-F(\tilde{v}_2)]-f(\tilde{v}_2)\tilde{v}_{2,P_2}[\rho_2(P_2-c_{good_2})+t_2-c_{lot_2}] \\
& +(\lambda_2+\lambda_4)\,\rho_2 u_{2,P_2}(w+v_2-P_2-t_2)(\tilde{v}_{2,P_2}-1) \\
& -\lambda_3\rho_2 u_{1,P_2}(w+v_2-P_2-t_2)(\tilde{v}_{2,P_2}-1)+\lambda_6\rho_2+\lambda_8.
\end{aligned}
\tag{4.79}
$$

$$
\begin{aligned}
\mathcal{L}_{t_1} = {} & 1-F(\tilde{v}_1)-f(\tilde{v}_1)\tilde{v}_{1,t_1}[\rho_1(P_1-c_{good_1})+t_1-c_{lot_1}] \\
& +(\lambda_1+\lambda_3)\,[\rho_1 u_{1,t_1}(w+v_1-P_1-t_1)(\tilde{v}_{1,t_1}-1)-(1-\rho_1)u_{1,t_1}(w-t_1)] \\
& -\lambda_4[\rho_1 u_{2,t_1}(w+v_1-P_1-t_1)(\tilde{v}_{1,t_1}-1)-(1-\rho_1)u_{2,t_1}(w-t_1)] \\
& +\lambda_5+\lambda_9.
\end{aligned}
\tag{4.80}
$$

$$
\begin{aligned}
\mathcal{L}_{t_2} = {} & 1-F(\tilde{v}_2)-f(\tilde{v}_2)\tilde{v}_{2,t_2}[\rho_2\left(P_2-c_{good_2}\right)+t_2-c_{lot_2}] \\
& +(\lambda_2+\lambda_4)\,[\rho_2 u_{2,t_2}(w+v_2-P_2-t_2)(\tilde{v}_{2,t_2}-1)-(1-\rho_2)u_{2,t_2}(w-t_2)] \\
& -\lambda_3[\rho_2 u_{1,t_2}(w+v_2-P_2-t_2)(\tilde{v}_{2,t_2}-1)-(1-\rho_2)u_{1,t_2}(w-t_2)] \\
& +\lambda_6+\lambda_{10}.
\end{aligned}
\tag{4.81}
$$

First, we find the equilibrium parking fee and price of the good for Type 1. Equating $\mathcal{L}_{P_1}$ to zero and using $\tilde{v}_{1,P_1} = 1$,

$$1 - F(\tilde{v}_1) = \frac{f(\tilde{v}_1)[\rho_1(P_1 - c_{good_1}) + t_1 - c_{lot_1}] - \lambda_5 \rho_1 - \lambda_7}{\rho_1}. \tag{4.82}$$

Notice that the expression $\rho_1(P_1 - c_{good_1}) + t_1 - c_{lot_1}$ cannot be zero since $1 - F(\tilde{v}_1)$ is strictly positive. Thus, by complementary slackness, λ_5 is zero. Then, (4.82) becomes

$$1 - F(\tilde{v}_1) = \frac{f(\tilde{v}_1)[\rho_1(P_1 - c_{good_1}) + t_1 - c_{lot_1}] - \lambda_7}{\rho_1}. \tag{4.83}$$

Since $\tilde{v}_{1,t_1} > 1/\rho_1$,

$$\begin{aligned} 1 - F(\tilde{v}_1) &= \frac{f(\tilde{v}_1)[\rho_1(P_1 - c_{good_1}) + t_1 - c_{lot_1}] - \lambda_7}{\rho_1} \\ &< f(\tilde{v}_1)\tilde{v}_{1,t_1}[\rho_1(P_1 - c_{good_1}) + t_1 - c_{lot_1}] - \lambda_7 \tilde{v}_{1,t_1}. \end{aligned} \tag{4.84}$$

Since $\lambda_7 \tilde{v}_{1,t_1} \geq 0$,

$$1 - F(\tilde{v}_1) < f(\tilde{v}_1)\tilde{v}_{1,t_1}[\rho_1(P_1 - c_{good_1}) + t_1 - c_{lot_1}]. \tag{4.85}$$

Equating $\mathcal{L}_{t_1}$ to zero and using (4.66),

$$\begin{aligned} 1 - F(\tilde{v}_1) = {} & f(\tilde{v}_1)\tilde{v}_{1,t_1}[\rho_1(P_1 - c_{good_1}) + t_1 - c_{lot_1}] \\ & + \lambda_4[\rho_1 u_{2,t_1}(w + v_1 - P_1 - t_1)(\tilde{v}_{1,t_1} - 1) - (1 - \rho_1)u_{2,t_1}(w - t_1)] \\ & - \lambda_9. \end{aligned} \tag{4.86}$$

Again by (4.66),

$$\begin{aligned} & \rho_1 u_{2,t_1}(w + v_1 - P_1 - t_1)(\tilde{v}_{1,t_1} - 1) - (1 - \rho_1)u_{2,t_1}(w - t_1) \\ & = (1 - \rho_1)\left[u_{2,t_1}(w + v_1 - P_1 - t_1)\frac{u_{1,t_1}(w - t_1)}{u_{1,t_1}(w + \tilde{v}_1 - P_1 - t_1)} - u_{2,t_1}(w - t_1)\right]. \end{aligned} \tag{4.87}$$

We will show that the right-hand side of (4.87) is strictly positive. That is, $u_{1,t_1}(w - t_1)/u_{1,t_1}(w + \tilde{v}_1 - P_1 - t_1) > u_{2,t_1}(w - t_1)/u_{2,t_1}(w + \tilde{v}_1 - P_1 - t_1)$.[9]

[9]Notice that $u_{1,t_1}(w - t_1)/u_{1,t_1}(w + \tilde{v}_1 - P_1 - t_1)$ is the marginal rate of substitution of Type 1 customer while $u_{2,t_1}(w - t_1)/u_{2,t_1}(w + \tilde{v}_1 - P_1 - t_1)$ is the marginal rate of substitution of Type 2 customer.

Since $u_1 = h \circ u_2$ for some concave function h,[10]

$$\frac{u_{1,t_1}(w-t_1)}{u_{1,t_1}(w+\tilde{v}_1-P_1-t_1)} = \frac{h_{t_1}(u_2(w-t_1))u_{2,t_1}(w-t_1)}{h_{t_1}(u_2(w+\tilde{v}_1-P_1-t_1))u_{2,t_1}(w+\tilde{v}_1-P_1-t_1)}. \quad (4.88)$$

Since u_2 is an increasing function and $\tilde{v}_1 > P_1$, $u_2(w+\tilde{v}_1-P_1-t_1) > u_2(w-t_1)$. Also, since h is a concave function, its derivative is a decreasing function. Therefore, $h_{t_1}(u_2(w-t_1)) > h_{t_1}(u_2(w+\tilde{v}_1-P_1-t_1))$. Since the derivative of the function u_2 is positive,

$$\frac{h_{t_1}(u_2(w-t_1))u_{2,t_1}(w-t_1)}{h_{t_1}(u_2(w+\tilde{v}_1-P_1-t_1))u_{2,t_1}(w+\tilde{v}_1-P_1-t_1)} > \frac{u_{2,t_1}(w-t_1)}{u_{2,t_1}(w+\tilde{v}_1-P_1-t_1)}. \quad (4.89)$$

Therefore, (4.87) is strictly positive.

Hence, by (4.85), λ_9 is strictly positive. By complementary slackness, $t_1^* = 0$. Then, since the expression $\rho_1(P_1 - c_{good_1}) + t_1 - c_{lot_1}$ cannot be negative, $P_1 > 0$. Hence, by complementary slackness, $\lambda_7 = 0$. Then, by (4.83), in equilibrium,

$$P_1^* = \frac{1-F(\tilde{v}_1)}{f(\tilde{v}_1)} + \frac{c_{lot_1}}{\rho_1} + c_{good_1} \quad \text{and} \quad t_1^* = 0. \quad (4.90)$$

We now find the equilibrium price of the good and the parking fee for Type 2 customer. Equating $\mathcal{L}_{P_2}$ to zero and using $\tilde{v}_{2,P_2} = 1$,

$$1 - F(\tilde{v}_2) = \frac{f(\tilde{v}_2)[\rho_2(P_2 - c_{good_2}) + t_2 - c_{lot_2}] - \lambda_6\rho_2 - \lambda_8}{\rho_2}. \quad (4.91)$$

Notice that the expression $\rho_2(P_2 - c_{good_2}) + t_2 - c_{lot_2}$ cannot be zero since $1 - F(\tilde{v}_2)$ is strictly positive. Thus, by complementary slackness, λ_6 is zero. Then, (4.91) becomes

$$1 - F(\tilde{v}_2) = \frac{f(\tilde{v}_2)[\rho_2(P_2 - c_{good_2}) + t_2 - c_{lot_2}] - \lambda_8}{\rho_2}. \quad (4.92)$$

Since $\tilde{v}_{2,t_2} > 1/\rho_2$,

$$\begin{aligned} 1 - F(\tilde{v}_2) &= \frac{f(\tilde{v}_2)[\rho_2(P_2 - c_{good_2}) + t_2 - c_{lot_2}] - \lambda_8}{\rho_2} \\ &< f(\tilde{v}_2)\tilde{v}_{2,t_2}[\rho_2(P_2 - c_{good_2}) + t_2 - c_{lot_2}] - \lambda_8\tilde{v}_{2,t_2}. \end{aligned} \quad (4.93)$$

Since $\lambda_8\tilde{v}_{2,t_2} \geq 0$,

$$1 - F(\tilde{v}_2) < f(\tilde{v}_2)\tilde{v}_{2,t_2}[\rho_2(P_2 - c_{good_2}) + t_2 - c_{lot_2}]. \quad (4.94)$$

[10] h_x stands for $\partial h/\partial x$.

Equating $\mathcal{L}_{t_2}$ to zero and using (4.70),

$$\begin{aligned}1-F(\tilde{v}_2) &= f(\tilde{v}_2)\tilde{v}_{2,t_2}[\rho_2\left(P_2-c_{good_2}\right)+t_2-c_{lot_2}] \\ &\quad +\lambda_3[\rho_2 u_{1,t_2}(w+v_2-P_2-t_2)(\tilde{v}_{2,t_2}-1)-(1-\rho_2)u_{1,t_2}(w-t_2)] \\ &\quad -\lambda_{10}.\end{aligned} \tag{4.95}$$

Again by (4.70),

$$\begin{aligned}&\rho_2 u_{1,t_2}(w+v_2-P_2-t_2)(\tilde{v}_{2,t_2}-1)-(1-\rho_2)u_{1,t_2}(w-t_2) \\ &=(1-\rho_2)\left[u_{1,t_2}(w+v_2-P_2-t_2)\frac{u_{2,t_2}(w-t_2)}{u_{2,t_2}(w+\tilde{v}_2-P_2-t_2)}-u_{1,t_2}(w-t_2)\right].\end{aligned} \tag{4.96}$$

We will show that the right-hand side of (4.96) is strictly negative. That is, $u_{1,t_2}(w-t_2)/u_{1,t_2}(w+\tilde{v}_2-P_2-t_2) > u_{2,t_2}(w-t_2)/u_{2,t_2}(w+\tilde{v}_2-P_2-t_2)$.

Since $u_1 = h\circ u_2$ for some concave function h,

$$\frac{u_{1,t_2}(w-t_2)}{u_{1,t_2}(w+\tilde{v}_2-P_2-t_2)}=\frac{h_{t_2}(u_2(w-t_2))u_{2,t_2}(w-t_2)}{h_{t_2}(u_2(w+\tilde{v}_2-P_2-t_2))u_{2,t_2}(w+\tilde{v}_2-P_2-t_2)}. \tag{4.97}$$

Since u_2 is an increasing function and $\tilde{v}_2 > P_2$, $u_2(w+\tilde{v}_2-P_2-t_2) > u_2(w-t_2)$. Since h is a concave function, its derivative is a decreasing function. Therefore, $h_{t_2}(u_2(w-t_2)) > h_{t_2}(u_2(w+\tilde{v}_2-P_2-t_2))$. Since the derivative of the function u_2 is positive,

$$\frac{h_{t_2}(u_2(w-t_2))u_{2,t_2}(w-t_2)}{h_{t_2}(u_2(w+\tilde{v}_2-P_2-t_2))u_{2,t_2}(w+\tilde{v}_2-P_2-t_2)} > \frac{u_{2,t_2}(w-t_2)}{u_{2,t_2}(w+\tilde{v}_2-P_2-t_2)}. \tag{4.98}$$

Hence, (4.96) is strictly negative. Then, combining (4.94) and (4.95), λ_3 and λ_{10} cannot be both zero at the same time. Therefore, there are infinitely many optimal solutions of the parking fee t_2 and the price of Good 2. These solutions include the cases where the mall charges a strictly positive parking fee as well as the case where it charges zero parking fee.

If the mall wants to implement the solution where it charges the less risk-averse customer with a strictly positive parking fee, then IC_1 must hold with equality. That is, the mall must make Type 1 customers -more risk-averse customers- indifferent between buying Good 1 and Good 2. In that case, $P_2^* = (1-F(\tilde{v}_2))/f(\tilde{v}_2) + \lambda_8/(\rho_2 f(\tilde{v}_2)) + (-t_2^* + c_{lot_2})/\rho_2 + c_{good_2}$. Then, the price of Good 2 may be zero or strictly positive. If the mall wants to sell Good 2 for free, then it must charge the parking lot $t_2^* > c_{lot_2} + \rho_2 c_{good_2}$. Then, the upper limit for t_2^* will be set by IC_1. If it sets $P_2^* > 0$, then it must set $t_2^* < \rho_2\left([1-F(\tilde{v}_2)]/f(\tilde{v}_2) + c_{good_2}\right) + c_{lot_2}$.

On the other hand, if the mall implements a solution where $t_2^* = 0$, then $P_2^* = (1 - F(\tilde{v}_2))/f(\tilde{v}_2) + \lambda_8/(\rho_2 f(\tilde{v}_2)) + (c_{lot_2}/\rho_2) + c_{good_2}$. And since $P_2^* > 0$, and hence $\lambda_8 = 0$, it must be that $P_2^* = [1 - F(\tilde{v}_2)]/f(\tilde{v}_2) + (c_{lot_2}/\rho_2) + c_{good_2}$. In that case, P_2^* is unique.

Under second-degree price discrimination where customers have different attitudes towards risk and probabilities of finding (or buying) the goods, parking is free for Type 1, who is more risk-averse than Type 2. Thus, the mall has an incentive to insure the more risk-averse customer for the risk of not finding the good. The cost of the parking lot of Type 1 is embedded in the prices of Good 1. Moreover, the price of Good 1 is unique and depends negatively on the probability of finding the good. As Type 1 customers become less likely to buy the good, they pay more for that good.

Even though the mall can provide free parking for Type 2 as well, it is not the only optimal solution for the mall. Hence, the mall is flexible in embedding the cost of parking lot reserved for Type 2 customers into the price of Good 2.

If the mall chooses to provide parking for free for Type 2 as well, then the optimal price it can set is unique. In that case, the price of Good 2 depends negatively on the probability of finding the good. As Type 2 customers become less likely to buy the good, they pay more for that good.

On the other hand, if the mall chooses to charge the parking lot of Type 2 with a positive fee, then there are infinitely many optimal prices for Good 2 it can set, including selling Good 2 for free. In that case, the mall must set the prices such that Type 1 customers are indifferent between revealing their true types and behaving as if they are Type 2. The negative relationship between the price of the good and the probability of buying the good still holds except when the price is zero.

Therefore, the following proposition is established:

Proposition 3. Under second-degree price discrimination where customers differ in their degree of risk-aversion, the price of the good that the more risk-averse customer buys is unique and covers the costs of the parking lots. Moreover, as the more risk-averse customer becomes less (more) likely to buy the good, she pays more (less) for the good. On the other hand, these results may not hold for the less risk-averse customer.

The results also show that the equilibrium prices of the goods are not affected by the probability of finding the other good, the cost of the other good, and the cost of the parking lot reserved for the other type of customer.

4.3 Third-Degree Price Discrimination

In third-degree price discrimination, the mall can separate the customers into groups and charge each group differently. For instance, the mall can offer children or student discounts. In this type of price discrimination, the mall can verify the groups of the customers, for instance by asking for an identity card.

In this section, we analyze the pricing strategy of a monopolist mall when it implements third-degree price discrimination. In this model, there are two groups of customers, Type 1 and Type 2, who are interested in the same certain good but have different utility functions. Type 1 and Type 2 customers decide to buy the good with $\rho_1 \in (0,1)$ and $\rho_2 \in (0,1)$ probabilities, respectively. Since the mall can distinguish the types of the customers, it can charge different prices and parking fees. Also, we assume that the marginal cost of the good is the same for both groups since there is one good. However, we assume that the marginal cost of the parking lot may be different from each other. We let c_{lot_1} and c_{lot_2} be the marginal costs of the parking lots of Type 1 and Type 2 customers, respectively.

Type 1 goes to the mall if

$$\rho_1 u_1(w+v_1-P_1-t_1)+(1-\rho_1)u_1(w-t_1) \geq u_1(w+r). \tag{4.99}$$

The marginal customer of Type 1 who is indifferent between going to the mall or not has a valuation $\tilde{v}_1(P_1,t_1)$,

$$\tilde{v}_1(P_1,t_1) \equiv u_1^{-1}\left(\frac{u_1(w+r)-(1-\rho_1)u_1(w-t_1)}{\rho_1}\right)-w+P_1+t_1. \tag{4.100}$$

Therefore,[11]

$$\tilde{v}_{1,P_1} = 1, \tag{4.101}$$

and

$$\tilde{v}_{1,t_1} = 1+\frac{(1-\rho_1)}{\rho_1}\frac{u_{1,t_1}(w-t_1)}{u_{1,t_1}(w+\tilde{v}_1-P_1-t_1)} \geq \frac{1}{\rho_1}. \tag{4.102}$$

Notice that if $\tilde{v}_{1,t_1} = 1/\rho_1$, then $\tilde{v}_1 = P_1$. But then, from the equation (4.100), it must be that in equilibrium, $t_1^* = -r$. Since this is not feasible, it must be that $\tilde{v}_{1,t_1} > 1/\rho_1$.

[11]Note that $\tilde{v}_{1,P_2} = 0$ and $\tilde{v}_{1,t_2} = 0$.

Type 2 goes to the mall if

$$\rho_2 u_2(w+v_2-P_2-t_2)+(1-\rho_2)u_2(w-t_2)\geq u_2(w+r). \tag{4.103}$$

The marginal customer of Type 2 who is indifferent between going to the mall or not has a valuation $\tilde{v}_2(P_2,t_2)$,

$$\tilde{v}_2(P_2,t_2)\equiv u_2^{-1}\left(\frac{u_2(w+r)-(1-\rho_2)u_2(w-t_2)}{\rho_2}\right)-w+P_2+t_2. \tag{4.104}$$

Therefore,[12]

$$\tilde{v}_{2,P_2}=1, \tag{4.105}$$

and

$$\tilde{v}_{2,t_2}=1+\frac{(1-\rho_2)}{\rho_2}\frac{u_{2,t_2}(w-t_2)}{u_{2,t_2}(w+\tilde{v}_2-P_2-t_2)}\geq\frac{1}{\rho_2}. \tag{4.106}$$

Notice that if $\tilde{v}_{2,t_2}=1/\rho_2$, then $\tilde{v}_2=P_2$. But then, from the equation (4.104), it must be that in equilibrium, $t_2^*=-r$. Since this is not feasible, it must be that $\tilde{v}_{2,t_2}>1/\rho_2$.

The mall's profit is $\Pi(P_1,P_2,t_1,t_2)$, which is the summation of the profits it receives from each type of customer,

$$\begin{aligned}\Pi(P_1,P_2,t_1,t_2)=&[1-F(\tilde{v}_1)]\left[\rho_1\left(P_1-c_{good}\right)+t_1-c_{lot_1}\right]\\&+[1-F(\tilde{v}_2)]\left[\rho_2\left(P_2-c_{good}\right)+t_2-c_{lot_2}\right].\end{aligned} \tag{4.107}$$

The mall maximizes its profit with respect to P_1,P_2,t_1, and t_2, subject to individual rationality constraints of each type,

$$(IR_1):\quad \rho_1 u_1(w+v_1-P_1-t_1)+(1-\rho_1)\,u_1(w-t_1)\geq u_1(w+r), \tag{4.108}$$

$$(IR_2):\quad \rho_2 u_2(w+v_2-P_2-t_2)+(1-\rho_2)\,u_2(w-t_2)\geq u_2(w+r), \tag{4.109}$$

and the non-negativity constraints,

$$\begin{aligned}\rho_1(P_1-c_{good})+t_1-c_{lot_1}\geq 0,\quad \rho_2(P_2-c_{good})+t_2-c_{lot_2}\geq 0,\\ P_1,P_2,t_1,t_2\geq 0.\end{aligned} \tag{4.110}$$

[12]Note that $\tilde{v}_{2,P_1}=0$ and $\tilde{v}_{2,t_1}=0$.

The Lagrangian function is

$$\begin{aligned}\mathcal{L}(P_1,P_2,t_1,t_2) = {} & [1-F(\tilde{v}_1)][\rho_1(P_1-c_{good})+t_1-c_{lot_1}] \\ & +[1-F(\tilde{v}_2)][\rho_2(P_2-c_{good})+t_2-c_{lot_2}] \\ & +\lambda_1[\rho_1 u_1(w+v_1-P_1-t_1)+(1-\rho_1)u_1(w-t_1)-u_1(w+r)] \\ & +\lambda_2[\rho_2 u_2(w+v_2-P_2-t_2)+(1-\rho_2)u_2(w-t_2)-u_2(w+r)] \\ & +\lambda_3[\rho_1(P_1-c_{good})+t_1-c_{lot_1}]+\lambda_4[\rho_2(P_2-c_{good})+t_2-c_{lot_2}] \\ & +\lambda_5 P_1+\lambda_6 P_2+\lambda_7 t_1+\lambda_8 t_2\end{aligned} \tag{4.111}$$

where $\lambda_1,\lambda_2,\lambda_3,\lambda_4,\lambda_5,\lambda_6,\lambda_7$, and λ_8 are Lagrangian multipliers.

Taking the first-order conditions with respect to P_1,P_2,t_1, and t_2,

$$\begin{aligned}\mathcal{L}_{P_1} = {} & \rho_1[1-F(\tilde{v}_1)]-f(\tilde{v}_1)\tilde{v}_{1,P_1}[\rho_1(P_1-c_{good})+t_1-c_{lot_1}] \\ & +\lambda_1\rho_1 u_{1,P_1}(w+v_1-P_1-t_1)(\tilde{v}_{1,P_1}-1)+\lambda_3\rho_1+\lambda_5.\end{aligned} \tag{4.112}$$

$$\begin{aligned}\mathcal{L}_{P_2} = {} & \rho_2[1-F(\tilde{v}_2)]-f(\tilde{v}_2)\tilde{v}_{2,P_2}[\rho_2(P_2-c_{good})+t_2-c_{lot_2}] \\ & +\lambda_2\rho_2 u_{2,P_2}(w+v_2-P_2-t_2)(\tilde{v}_{2,P_2}-1)+\lambda_4\rho_2+\lambda_6.\end{aligned} \tag{4.113}$$

$$\begin{aligned}\mathcal{L}_{t_1} = {} & 1-F(\tilde{v}_1)-f(\tilde{v}_1)\tilde{v}_{1,t_1}[\rho_1(P_1-c_{good})+t_1-c_{lot_1}] \\ & +\lambda_1[\rho_1 u_{1,t_1}(w+v_1-P_1-t_1)(\tilde{v}_{1,t_1}-1)-(1-\rho_1)u_{1,t_1}(w-t_1)] \\ & +\lambda_3+\lambda_7.\end{aligned} \tag{4.114}$$

$$\begin{aligned}\mathcal{L}_{t_2} = {} & 1-F(\tilde{v}_2)-f(\tilde{v}_2)\tilde{v}_{2,t_2}[\rho_2(P_2-c_{good})+t_2-c_{lot_2}] \\ & +\lambda_2[\rho_2 u_{2,t_2}(w+v_2-P_2-t_2)(\tilde{v}_{2,t_2}-1)-(1-\rho_2)u_{2,t_2}(w-t_2)] \\ & +\lambda_4+\lambda_8.\end{aligned} \tag{4.115}$$

We first find the equilibrium price of the good and the parking fee for Type 1 customer. Equating the first-order condition in (4.112) to zero and using $\tilde{v}_{1,P_1}=1$,

$$1-F(\tilde{v}_1)=\frac{f(\tilde{v}_1)[\rho_1(P_1-c_{good})+t_1-c_{lot_1}]-\lambda_3\rho_1-\lambda_5}{\rho_1}. \tag{4.116}$$

Notice that the expression $\rho_1(P_1-c_{good})+t_1-c_{lot_1}$ cannot be zero since otherwise $1-F(\tilde{v}_1)$ is negative. Thus, by complementary slackness, $\lambda_3=0$. Then, (4.116) becomes

$$1 - F(\tilde{v}_1) = \frac{f(\tilde{v}_1)[\rho_1(P_1 - c_{good}) + t_1 - c_{lot_1}] - \lambda_5}{\rho_1}. \tag{4.117}$$

Since $\tilde{v}_{1,t_1} > 1/\rho_1$,

$$\begin{aligned} 1 - F(\tilde{v}_1) &= \frac{f(\tilde{v}_1)[\rho_1(P_1 - c_{good}) + t_1 - c_{lot_1}] - \lambda_5}{\rho_1} \\ &< f(\tilde{v}_1)\tilde{v}_{1,t_1}[\rho_1(P_1 - c_{good}) + t_1 - c_{lot_1}] - \lambda_5\tilde{v}_{1,t_1}. \end{aligned} \tag{4.118}$$

Since $\lambda_5\tilde{v}_{1,t_1} \geq 0$,

$$1 - F(\tilde{v}_1) < f(\tilde{v}_1)\tilde{v}_{1,t_1}[\rho_1(P_1 - c_{good}) + t_1 - c_{lot_1}]. \tag{4.119}$$

Equating the first-order condition in (4.114) to zero and using (4.102),

$$1 - F(\tilde{v}_1) = f(\tilde{v}_1)\tilde{v}_{1,t_1}[\rho_1(P_1 - c_{good}) + t_1 - c_{lot_1}] - \lambda_7. \tag{4.120}$$

Then, by (4.119), it must be that $\lambda_7 > 0$. Thus, by complementary slackness, $t_1^* = 0$. Then, it cannot be that $P_1 = 0$ since the expression $\rho_1(P_1 - c_{good}) + t_1 - c_{lot_1}$ cannot be negative. Therefore, by complementary slackness, $\lambda_5 = 0$. Then, by (4.117), in equilibrium,

$$P_1^* = \frac{1 - F(\tilde{v}_1)}{f(\tilde{v}_1)} + \frac{c_{lot_1}}{\rho_1} + c_{good}, \quad \text{and} \quad t_1^* = 0. \tag{4.121}$$

Similarly, for Type 2 customers, equating the first-order condition in (4.113) to zero and using $\tilde{v}_{2,P_2} = 1$,

$$1 - F(\tilde{v}_2) = \frac{f(\tilde{v}_2)[\rho_2(P_2 - c_{good}) + t_2 - c_{lot_2}] - \lambda_4\rho_2 - \lambda_6}{\rho_2}. \tag{4.122}$$

Notice that the expression $\rho_2(P_2 - c_{good}) + t_2 - c_{lot_2}$ cannot be zero since otherwise $1 - F(\tilde{v}_2)$ is negative. Thus, by complementary slackness, $\lambda_4 = 0$. Then, (4.122) becomes

$$1 - F(\tilde{v}_2) = \frac{f(\tilde{v}_2)[\rho_2(P_2 - c_{good}) + t_2 - c_{lot_2}] - \lambda_6}{\rho_2}. \tag{4.123}$$

Since $\tilde{v}_{2,t_2} > 1/\rho_2$,

$$\begin{aligned} 1 - F(\tilde{v}_2) &= \frac{f(\tilde{v}_2)[\rho_2(P_2 - c_{good}) + t_2 - c_{lot_2}] - \lambda_6}{\rho_2} \\ &< f(\tilde{v}_2)\tilde{v}_{2,t_2}[\rho_2(P_2 - c_{good}) + t_2 - c_{lot_2}] - \lambda_6\tilde{v}_{2,t_2}. \end{aligned} \tag{4.124}$$

Since $\lambda_6\tilde{v}_{2,t_2} \geq 0$,

$$1 - F(\tilde{v}_2) < f(\tilde{v}_2)\tilde{v}_{2,t_2}[\rho_2(P_2 - c_{good}) + t_2 - c_{lot_2}]. \tag{4.125}$$

Equating the first-order condition in (4.115) to zero and using (4.106),

$$1 - F(\tilde{v}_2) = f(\tilde{v}_2)\tilde{v}_{2,t_2}[\rho_2(P_2 - c_{good}) + t_2 - c_{lot_2}] - \lambda_8. \tag{4.126}$$

Then, by (4.125), it must be that $\lambda_8 > 0$. Thus, by complementary slackness, $t_2^* = 0$. Then, it cannot be that $P_2 = 0$ since the expression $\rho_2(P_2 - c_{good}) + t_2 - c_{lot_2}$ cannot be negative. Therefore, by complementary slackness, $\lambda_6 = 0$. Then, by (4.123), in equilibrium,

$$P_2^* = \frac{1 - F(\tilde{v}_2)}{f(\tilde{v}_2)} + \frac{c_{lot_2}}{\rho_2} + c_{good}, \quad \text{and} \quad t_2^* = 0. \tag{4.127}$$

Therefore, under third-degree price discrimination where the types of customers have the different utility functions and probabilities of buying the good, parking is free for both Type 1 and Type 2, and the costs of the parking lots are embedded in the prices of the good.

We find that the equilibrium prices, P_1^* and P_2^*, are unique. P_1^* depends negatively on Type 1's probability of buying the good while P_2^* depends negatively on Type 2's probability of buying the good. As customers become less likely to buy the good, they pay more for the good. Hence, the mall aims to attract customers with a higher probability of buying the good by offering them a lower price while making customers with a lower probability of buying the good pay more for the good.

Therefore, the following proposition is established:

Proposition 4. Under third-degree price discrimination where the types are observable to the mall, the prices are unique and cover the costs of the parking lots. Moreover, as customers become less (more) likely to buy the good, they pay more (less) for the good.

Also, since there is one good, one may consider ρ to be the same regardless of the type of the customer. In that case, the analysis would not change.

Moreover, notice that the essential difference between this type of price discrimination and the first type is that in this model, the types of the customer differentiate with respect to their utility functions. However, as the results show, assigning different utility functions for the two different groups of customers does not change the equilibrium outcomes.

5. HORIZONTAL PRODUCT DIFFERENTIATION MODEL WITH TWO TYPES OF GOODS

In horizontal product differentiation, the goods are not superior to each other and customers decide to buy the goods based on their subjective preferences such as color and taste rather than the quality of the goods. For instance, the mall may sell two types of coffee beans: Arabica and Robusta, and customers purchase a product based on their preferences on sweeter or stronger taste.

In this chapter, we analyze the pricing strategy of a monopolist mall when there is horizontal product differentiation. We assume that there are two types of customers, Type 1 and Type 2, who are only interested in Good 1 and Good 2, respectively. Since customers make their purchases based on their preferences, they have an incentive to reveal their true types. That is why the maximization problem does not need to include the incentive compatibility constraints.

Since customers self-select their types, the mall can perfectly distinguish the types and it can separate the parking lot for each type. It is also assumed that the goods and the parking lots are costly. The mall decides on the prices of each good and the parking fee for each type of customer.

Type 1 goes to the mall if

$$\rho_1 u(w+v_1-P_1-t_1)+(1-\rho_1)u(w-t_1)\geq u(w+r). \tag{5.1}$$

Type 1 is indifferent between going to the mall and staying at home if she has the valuation $\tilde{v}_1(P_1,t_1)$,

$$\tilde{v}_1(P_1,t_1)\equiv u^{-1}\left(\frac{u(w+r)-(1-\rho_1)u(w-t_1)}{\rho_1}\right)-w+P_1+t_1. \tag{5.2}$$

Therefore,

$$\tilde{v}_{1,P_1}=1, \tag{5.3}$$

and

$$\tilde{v}_{1,t_1} = 1 + \frac{(1-\rho_1)}{\rho_1} \frac{u_{t_1}(w-t_1)}{u_{t_1}(w+\tilde{v}_1 - P_1 - t_1)} \geq \frac{1}{\rho_1}. \tag{5.4}$$

Notice that if $\tilde{v}_{1,t_1} = 1/\rho_1$, then $\tilde{v}_1 = P_1$. But then, from the equation (5.2), it must be that in equilibrium, $t_1^* = -r$. Since this is not feasible, it must be that $\tilde{v}_{1,t_1} > 1/\rho_1$.

Similarly, Type 2 goes to the mall if

$$\rho_2 u(w+v_2-P_2-t_2) + (1-\rho_2)u(w-t_2) \geq u(w+r). \tag{5.5}$$

Type 2 is indifferent between going to the mall and staying at home if he has the valuation $\tilde{v}_2(P_2,t_2)$,

$$\tilde{v}_2(P_2,t_2) \equiv u^{-1}\left(\frac{u(w+r)-(1-\rho_2)u(w-t_2)}{\rho_2}\right) - w + P_2 + t_2. \tag{5.6}$$

Therefore,

$$\tilde{v}_{2,P_2} = 1, \tag{5.7}$$

and

$$\tilde{v}_{2,t_2} = 1 + \frac{(1-\rho_2)}{\rho_2} \frac{u_{t_2}(w-t_2)}{u_{t_1}(w+\tilde{v}_2 - P_2 - t_2)} \geq \frac{1}{\rho_2}. \tag{5.8}$$

Notice that if $\tilde{v}_{2,t_2} = 1/\rho_2$, then $\tilde{v}_2 = P_2$. But then, from the equation (5.6), it must be that in equilibrium, $t_2^* = -r$. Since this is not feasible, it must be that $\tilde{v}_{2,t_2} > 1/\rho_2$.

The mall's profit is $\Pi(P_1,P_2,t_1,t_2)$, which is the summation of the profits it receives from each type of customer:

$$\begin{aligned}\Pi(P_1,P_2,t_1,t_2) = {} & [1-F(\tilde{v}_1)][\rho_1(P_1 - c_{good_1}) + t_1 - c_{lot_1}] \\ & + [1-F(\tilde{v}_2)][\rho_2(P_2 - c_{good_2}) + t_2 - c_{lot_2}].\end{aligned} \tag{5.9}$$

The mall maximizes its profit with respect to P_1, P_2, t_1, and t_2, subject to individual rationality constraints of each type,

$$(IR_1): \quad \rho_1 u(w+v_1-P_1-t_1) + (1-\rho_1)u(w-t_1) \geq u(w+r), \tag{5.10}$$

$$(IR_2): \quad \rho_2 u(w+v_2-P_2-t_2) + (1-\rho_2)u(w-t_2) \geq u(w+r), \tag{5.11}$$

and the non-negativity constraints,

$$\begin{aligned}\rho_1(P_1 - c_{good_1}) + t_1 - c_{lot_1} \geq 0, \quad \rho_2(P_2 - c_{good_2}) + t_2 - c_{lot_2} \geq 0,\\ P_1, P_2, t_1, t_2 \geq 0.\end{aligned} \tag{5.12}$$

The Lagrangian function is

$$\begin{aligned}\mathcal{L}(P_1, P_2, t_1, t_2) = {} & [1 - F(\tilde{v}_1)][\rho_1(P_1 - c_{good_1}) + t_1 - c_{lot_1}]\\ & + [1 - F(\tilde{v}_2)][\rho_2(P_2 - c_{good_2}) + t_2 - c_{lot_2}]\\ & + \lambda_1[\rho_1 u(w + v_1 - P_1 - t_1) + (1 - \rho_1)u(w - t_1) - u(w + r)]\\ & + \lambda_2[\rho_2 u(w + v_2 - P_2 - t_2) + (1 - \rho_2)u(w - t_2) - u(w + r)]\\ & + \lambda_3[\rho_1(P_1 - c_{good_1}) + t_1 - c_{lot_1}] + \lambda_4[\rho_2(P_2 - c_{good_2}) + t_2 - c_{lot_2}]\\ & + \lambda_5 P_1 + \lambda_6 P_2 + \lambda_7 t_1 + \lambda_8 t_2\end{aligned} \tag{5.13}$$

where $\lambda_1, \lambda_2, \lambda_3, \lambda_4, \lambda_5, \lambda_6, \lambda_7$, and λ_8 are Lagrangian multipliers.

Taking the first-order conditions with respect to P_1, P_2, t_1, and t_2,

$$\begin{aligned}\mathcal{L}_{P_1} = {} & \rho_1[1 - F(\tilde{v}_1)] - f(\tilde{v}_1)\tilde{v}_{1,P_1}[\rho_1(P_1 - c_{good_1}) + t_1 - c_{lot_1}]\\ & + \lambda_1 \rho_1 u_{P_1}(w + v_1 - P_1 - t_1)(\tilde{v}_{1,P_1} - 1) + \lambda_3 \rho_1 + \lambda_5.\end{aligned} \tag{5.14}$$

$$\begin{aligned}\mathcal{L}_{P_2} = {} & \rho_2[1 - F(\tilde{v}_2)] - f(\tilde{v}_2)\tilde{v}_{2,P_2}[\rho_2(P_2 - c_{good_2}) + t_2 - c_{lot_2}]\\ & + \lambda_2 \rho_2 u_{P_2}(w + v_2 - P_2 - t_2)(\tilde{v}_{2,P_2} - 1) + \lambda_4 \rho_2 + \lambda_6.\end{aligned} \tag{5.15}$$

$$\begin{aligned}\mathcal{L}_{t_1} = {} & 1 - F(\tilde{v}_1) - f(\tilde{v}_1)\tilde{v}_{1,t_1}[\rho_1(P_1 - c_{good_1}) + t_1 - c_{lot_1}]\\ & + \lambda_1[\rho_1 u_{t_1}(w + v_1 - P_1 - t_1)(\tilde{v}_{1,t_1} - 1) - (1 - \rho_1)u_{t_1}(w - t_1)] + \lambda_3 + \lambda_7.\end{aligned} \tag{5.16}$$

$$\begin{aligned}\mathcal{L}_{t_2} = {} & 1 - F(\tilde{v}_2) - f(\tilde{v}_2)\tilde{v}_{2,t_2}[\rho_2(P_2 - c_{good_2}) + t_2 - c_{lot_2}]\\ & + \lambda_2[\rho_2 u_{t_2}(w + v_2 - P_2 - t_2)(\tilde{v}_{2,t_2} - 1) - (1 - \rho_2)u_{t_2}(w - t_2)] + \lambda_4 + \lambda_8.\end{aligned} \tag{5.17}$$

Equating the first-order condition in (5.14) to zero and using $\tilde{v}_{1,P_1} = 1$,

$$\rho_1[1 - F(\tilde{v}_1)] + \lambda_3 \rho_1 + \lambda_5 = f(\tilde{v}_1)[\rho_1(P_1 - c_{good_1}) + t_1 - c_{lot_1}]. \tag{5.18}$$

Notice that the expression $\rho_1(P_1 - c_{good_1}) + t_1 - c_{lot_1}$ cannot be zero, since $\rho_1[1 - F(\tilde{v}_1)]$ is greater than zero. Then, it must be that $\lambda_3 = 0$, since by complementary slackness condition, $\lambda_3[\rho_1(P_1 - c_{good_1}) + t_1 - c_{lot_1}] = 0$. Therefore, from (5.18),

$$1 - F(\tilde{v}_1) = \frac{f(\tilde{v}_1)[\rho_1(P_1 - c_{good_1}) + t_1 - c_{lot_1}] - \lambda_5}{\rho_1}. \tag{5.19}$$

Since $\tilde{v}_{1,t_1} > 1/\rho_1$,

$$\begin{aligned} 1 - F(\tilde{v}_1) &= \frac{f(\tilde{v}_1)[\rho_1(P_1 - c_{good_1}) + t_1 - c_{lot_1}] - \lambda_5}{\rho_1} \\ &< f(\tilde{v}_1)\tilde{v}_{1,t_1}[\rho_1(P_1 - c_{good_1}) + t_1 - c_{lot_1}] - \tilde{v}_{1,t_1}\lambda_5. \end{aligned} \tag{5.20}$$

Since $\tilde{v}_{1,t_1}\lambda_5 \geq 0$,

$$1 - F(\tilde{v}_1) < f(\tilde{v}_1)\tilde{v}_{1,t_1}[\rho_1(P_1 - c_{good_1}) + t_1 - c_{lot_1}]. \tag{5.21}$$

Equating the first-order condition in (5.16) to zero,

$$\begin{aligned} 1 - F(\tilde{v}_1) = {} & f(\tilde{v}_1)\tilde{v}_{1,t_1}[\rho_1(P_1 - c_{good_1}) + t_1 - c_{lot_1}] \\ & - \lambda_1[\rho_1 u_{t_1}(w + v_1 - P_1 - t_1)(\tilde{v}_{1,t_1} - 1) - (1 - \rho_1)u_{t_1}(w - t_1)] \\ & - \lambda_7. \end{aligned} \tag{5.22}$$

Notice that by (5.4), (5.22) becomes

$$1 - F(\tilde{v}_1) = f(\tilde{v}_1)\tilde{v}_{1,t_1}[\rho_1(P_1 - c_{good_1}) + t_1 - c_{lot_1}] - \lambda_7. \tag{5.23}$$

Then, by (5.21), $\lambda_7 > 0$. Hence, by complementary slackness condition, $t_1^* = 0$. Then, it must be that $P_1 > 0$, since the expression $\rho_1(P_1 - c_{good_1}) + t_1 - c_{lot_1}$ cannot be negative. Therefore, by complementary slackness condition, $\lambda_5 = 0$ and in equilibrium,

$$P_1^* = \frac{1 - F(\tilde{v}_1)}{f(\tilde{v}_1)} + \frac{c_{lot_1}}{\rho_1} + c_{good_1}, \quad \text{and} \quad t_1^* = 0. \tag{5.24}$$

To find the equilibrium price of the good and the parking fee for Type 2, similar steps are followed:

Equating the first-order condition in (5.15) to zero and using $\tilde{v}_{2,P_2} = 1$,

$$\rho_2[1 - F(\tilde{v}_2)] + \lambda_4\rho_2 + \lambda_6 = f(\tilde{v}_2)[\rho_2(P_2 - c_{good_2}) + t_2 - c_{lot_2}]. \tag{5.25}$$

Notice that the expression $\rho_2(P_2 - c_{good_2}) + t_2 - c_{lot_2}$ cannot be zero, since $\rho_2[1 - F(\tilde{v}_2)]$ is greater than zero. Then, it must be that $\lambda_4 = 0$, since by complementary slackness condition, $\lambda_4[\rho_2(P_2 - c_{good_2}) + t_2 - c_{lot_2}] = 0$. Therefore, from (5.25),

$$1 - F(\tilde{v}_2) = \frac{f(\tilde{v}_2)[\rho_2(P_2 - c_{good_2}) + t_2 - c_{lot_2}] - \lambda_6}{\rho_2}. \tag{5.26}$$

Since $\tilde{v}_{2,t_2} > 1/\rho_2$,

$$\begin{aligned} 1 - F(\tilde{v}_2) &= \frac{f(\tilde{v}_2)[\rho_2(P_2 - c_{good_2}) + t_2 - c_{lot_2}] - \lambda_6}{\rho_2} \\ &< f(\tilde{v}_2)\tilde{v}_{2,t_2}[\rho_2(P_2 - c_{good_2}) + t_2 - c_{lot_2}] - \tilde{v}_{2,t_2}\lambda_6. \end{aligned} \tag{5.27}$$

Since $\tilde{v}_{2,t_2}\lambda_6 \geq 0$,

$$1 - F(\tilde{v}_2) < f(\tilde{v}_2)\tilde{v}_{2,t_2}[\rho_2(P_2 - c_{good_2}) + t_2 - c_{lot_2}]. \tag{5.28}$$

Equating the first-order condition in (5.17) to zero,

$$\begin{aligned} 1 - F(\tilde{v}_2) = {} & f(\tilde{v}_2)\tilde{v}_{2,t_2}[\rho_2(P_2 - c_{good_2}) + t_2 - c_{lot_2}] \\ & - \lambda_2[\rho_2 u_{t_2}(w + v_2 - P_2 - t_2)(\tilde{v}_{2,t_2} - 1) - (1 - \rho_2)u_{t_2}(w - t_2)] \\ & - \lambda_8. \end{aligned} \tag{5.29}$$

Notice that by (5.8), (5.29) becomes

$$1 - F(\tilde{v}_2) = f(\tilde{v}_2)\tilde{v}_{2,t_2}[\rho_2(P_2 - c_{good_2}) + t_2 - c_{lot_2}] - \lambda_8. \tag{5.30}$$

Then, by (5.28), $\lambda_8 > 0$. Hence, by complementary slackness condition, $t_2^* = 0$. Then, it must be that $P_2 > 0$, since the expression $\rho_2(P_2 - c_{good_2}) + t_2 - c_{lot_2}$ cannot be negative. Therefore, by complementary slackness condition, $\lambda_6 = 0$ and in equilibrium,

$$P_2^* = \frac{1 - F(\tilde{v}_2)}{f(\tilde{v}_2)} + \frac{c_{lot_2}}{\rho_2} + c_{good_2}, \quad \text{and} \quad t_2^* = 0. \tag{5.31}$$

Therefore, under horizontal product differentiation where the types of customers have different preferences, parking is free for both Type 1 and Type 2 and the cost of the parking lot is embedded in the prices of Good 1 and Good 2. In this model, the types are observable to the mall since customers reveal their true preferences. The equilibrium prices of the goods are not affected by the probability of finding the other good.

The prices of Good 1 and Good 2 are unique. For each good, the price of the good depends negatively on the probability of finding the good. As customers become less likely to buy the good, they pay more for that good. Hence, the mall aims to attract the customers with a higher probability of buying the good by offering them a lower price while making the customers with a lower probability of buying the good pay more for the good.

Therefore, the following proposition is established:

Proposition 5. Under horizontal product differentiation where the types of customers have different preferences, the prices of Good 1 and Good 2 are unique and cover the costs of the parking lots. Moreover, as customers become less (more) likely to buy the good, they pay more (less) for the good.

6. PRICE DISCRIMINATION MODELS WITH N TYPES OF CUSTOMERS

This chapter studies the generalization of the models constructed in Chapter 4 for more than two types of customers. We assume that there are n types of customers: Type 1, Type 2,..., and Type N. Type i has a valuation $v_i \in [0, \bar{v}_i]$ for the good. The mall sells the good at a price P_i and provides parking lot to Type i customers at a fee t_i.

6.1 First-Degree Price Discrimination

In this model, there is one type of good. Suppose that the mall can perfectly differentiate the customers and their willingness to pay for the good. Therefore, the mall can set different prices for the good and the parking fee for each different type of customer.

Type i goes to the mall if

$$\rho u(w+v_i-P_i-t_i)+(1-\rho)u(w-t_i) \geq u(w+r). \tag{6.1}$$

The marginal customer of Type i who is indifferent between going to the mall or not has a valuation $\tilde{v}_i(P_i,t_i)$,

$$\tilde{v}_i(P_i,t_i) \equiv u^{-1}\left(\frac{u(w+r)-(1-\rho)u(w-t_i)}{\rho}\right)-w+P_i+t_i, \quad \forall i=1,...,N. \tag{6.2}$$

Therefore, for all $i=1,...,N$,

$$\tilde{v}_{i,P_i}=1, \tag{6.3}$$

and

$$\tilde{v}_{i,t_i} = 1 + \frac{(1-\rho)}{\rho}\frac{u_{t_i}(w-t_i)}{u_{t_i}(w+\tilde{v}_i - P_i - t_i)} \geq \frac{1}{\rho}. \tag{6.4}$$

Notice that if $\tilde{v}_{i,t_i} = 1/\rho$, then $\tilde{v}_i = P_i$. But then, from the equation (6.2), it must be that in equilibrium, $t_i^* = -r$. Since this is not feasible, it must be that $\tilde{v}_{i,t_i} > 1/\rho$.

The mall's profit is $\Pi(P_1, P_2, ..., P_N, t_1, t_2, ..., t_N)$, which is the summation of the profits it receives from each type of customer,

$$\Pi(P_1, P_2, ..., P_N, t_1, t_2, ..., t_N) = \sum_{i=1}^{N}(1 - F(\tilde{v}_i))[\rho(P_i - c_{good}) + t_i - c_{lot}]. \tag{6.5}$$

The mall maximizes its profit subject to individual rationality constraints of each Type i,

$$(IR_i): \quad \rho u(w + v_i - P_i - t_i) + (1-\rho)u(w - t_i) \geq u(w + r), \quad \forall i = 1, ..., N, \tag{6.6}$$

and the non-negativity constraints,

$$\rho(P_i - c_{good}) + t_i - c_{lot} \geq 0, \quad \text{and} \quad P_i, t_i \geq 0, \quad \forall i = 1, ..., N. \tag{6.7}$$

The Lagrangian function is

$$\begin{aligned}\mathcal{L}(P_1, ..., P_N, t_1, ..., t_N) = &\sum_{i=1}^{N}[1 - F(\tilde{v}_i)][\rho(P_i - c_{good}) + t_i - c_{lot}] \\ &+ \sum_{i=1}^{N}\lambda_i[\rho u(w + v_i - P_i - t_i) + (1-\rho)u(w - t_i) - u(w + r)] \\ &+ \sum_{i=1}^{N}\mu_i[\rho(P_i - c_{good}) + t_i - c_{lot}] + \sum_{i=1}^{N}\delta_i P_i + \sum_{i=1}^{N}\gamma_i t_i\end{aligned} \tag{6.8}$$

where $\lambda_i, \mu_i, \delta_i$, and γ_i, for all $i = 1, ..., N$ are Lagrangian multipliers.

Taking the first-order conditions with respect to $P_1, P_2 ..., P_N$, and $t_1, t_2, ..., t_N$,

$$\begin{aligned}\mathcal{L}_{P_i} = &\rho[1 - F(\tilde{v}_i)] - f(\tilde{v}_i)\tilde{v}_{i,P_i}[\rho(P_i - c_{good}) + t_i - c_{lot}] \\ &+ \lambda_i \rho u_{P_i}(w + v_i - P_i - t_i)(\tilde{v}_{i,P_i} - 1) + \mu_i\rho + \delta_i, \quad \forall i = 1, ..., N.\end{aligned} \tag{6.9}$$

$$\begin{aligned}\mathcal{L}_{t_i} = &1 - F(\tilde{v}_i) - f(\tilde{v}_i)\tilde{v}_{i,t_i}[\rho(P_i - c_{good}) + t_i - c_{lot}] \\ &+ \lambda_i[\rho u_{t_i}(w + v_i - P_i - t_i)(\tilde{v}_{i,t_i} - 1) - (1-\rho)u_{t_i}(w - t_i)] \\ &+ \mu_i + \gamma_i, \quad \forall i = 1, ..., N.\end{aligned} \tag{6.10}$$

Equating (6.9) to zero for all $i = 1, ..., N$ and using $\tilde{v}_{i,P_i} = 1$,

$$1 - F(\tilde{v}_i) = \frac{f(\tilde{v}_i)[\rho(P_i - c_{good}) + t_i - c_{lot}] - \mu_i \rho - \delta_i}{\rho}, \quad \forall i = 1, ..., N. \tag{6.11}$$

Notice that the expression $\rho(P_i - c_{good}) + t_i - c_{lot}$ cannot be zero, since $1 - F(\tilde{v}_i)$ is strictly greater than zero. Then, by complementary slackness condition, it must be that $\mu_i = 0$, for all $i = 1, ..., N$. Therefore, (6.11) becomes

$$1 - F(\tilde{v}_i) = \frac{f(\tilde{v}_i)[\rho(P_i - c_{good}) + t_i - c_{lot}] - \delta_i}{\rho}, \quad \forall i = 1, ..., N. \tag{6.12}$$

Since $\tilde{v}_{i,t_i} > 1/\rho$,

$$\begin{aligned} 1 - F(\tilde{v}_i) &= \frac{f(\tilde{v}_i)[\rho(P_i - c_{good}) + t_i - c_{lot}] - \delta_i}{\rho} \\ &< f(\tilde{v}_i)\tilde{v}_{i,t_i}[\rho(P_i - c_{good}) + t_i - c_{lot}] - \delta_i \tilde{v}_{i,t_i}, \quad \forall i = 1, ..., N. \end{aligned} \tag{6.13}$$

Since $\delta_i \tilde{v}_{i,t_i} \geq 0$,

$$1 - F(\tilde{v}_i) < f(\tilde{v}_i)\tilde{v}_{i,t_i}[\rho(P_i - c_{good}) + t_i - c_{lot}], \quad \forall i = 1, ..., N. \tag{6.14}$$

Equating the first-order condition in (6.10) to zero and using (6.4),

$$1 - F(\tilde{v}_i) = f(\tilde{v}_i)\tilde{v}_{i,t_i}[\rho(P_i - c_{good}) + t_i - c_{lot}] - \gamma_i, \quad \forall i = 1, ..., N. \tag{6.15}$$

Then, by (6.14), it must be that $\gamma_i > 0$, for all $i = 1, ..., N$. Then, by complementary slackness condition, $t_i^* = 0$, for all $i = 1, ..., N$. Then, it must be that $P_i > 0$, since the expression $\rho(P_i - c_{good}) + t_i - c_{lot}$ cannot be negative. Then, by complementary slackness condition, $\delta_i = 0$, for all $i = 1, ..., N$ and in equilibrium,

$$P_i^* = \frac{1 - F(\tilde{v}_i)}{f(\tilde{v}_i)} + \frac{c_{lot}}{\rho} + c_{good}, \quad \text{and} \quad t_i^* = 0, \quad \forall i = 1, ..., N. \tag{6.16}$$

Therefore, we have the following proposition:

Proposition 6. Proposition 1 holds when there are n types of customers.

6.2 Second-Degree Price Discrimination

In this section, there are n types of goods: Good 1, Good 2,..., Good N. Good i has a marginal cost $c_{good_i} > 0$ and the probability of finding it is $\rho_i \in (0,1)$. The marginal cost of the parking lot for Good i buyers is $c_{lot_i} > 0$. The mall knows that there are different types of customers but cannot identify the type of a given customer.

6.2.1 Types are differing in probability of buying the good

Type i customer goes to the mall if and only if

$$\rho_i u(w+v_i-P_i-t_i)+(1-\rho_i)u(w-t_i) \geq u(w+r), \quad \forall i=1,...,N. \tag{6.17}$$

Type i customer who is indifferent between going to the mall or not has a valuation $\tilde{v}_i(P_i,t_i)$,

$$\tilde{v}_i(P_i,t_i) \equiv u^{-1}\left(\frac{u(w+r)-(1-\rho_i)u(w-t_i)}{\rho_i}\right)-w+P_i+t_i, \quad \forall i=1,...,N. \tag{6.18}$$

Therefore, for all $i=1,...,N$,[1]

$$\tilde{v}_{i,P_i}=1, \tag{6.19}$$

and

$$\tilde{v}_{i,t_i}=1+\frac{(1-\rho_i)}{\rho_i}\frac{u_{t_i}(w-t_i)}{u_{t_i}(w+\tilde{v}_i-P_i-t_i)} \geq \frac{1}{\rho_i}. \tag{6.20}$$

Notice that if $\tilde{v}_{i,t_i}=1/\rho_i$, then $\tilde{v}_i=P_i$. But then, from the equation (6.18), it must be that in equilibrium, $t_i^*=-r$. Since this is not feasible, it must be that $\tilde{v}_{i,t_i}>1/\rho_i$.

The mall's profit is $\Pi(P_1,...,P_N,t_1,...,t_N)$, which is the summation of the profits it receives from each type of customer,

$$\Pi(P_1,P_2,...,P_N,t_1,t_2,...,t_N)=\sum_{i=1}^{N}(1-F(\tilde{v}_i))[\rho_i(P_i-c_{good_i})+t_i-c_{lot_i}]. \tag{6.21}$$

The mall maximizes its profit, subject to individual rationality constraints of each

[1] Note that $\tilde{v}_{i,P_{-i}}=0$ and $\tilde{v}_{i,t_{-i}}=0, \forall i \in \{1,...,N\}, \forall -i \in \{1,...,i-1,i+1,...,N\}$.

type i,

$$(IR_i): \quad \rho_i u(w+v_i-P_i-t_i)+(1-\rho_i)u(w-t_i) \geq u(w+r), \quad \forall i=1,...,N, \tag{6.22}$$

and the incentive compatibility constraints of each type,

$$\begin{aligned}(IC_i): \quad & \rho_i u(w+v_i-P_i-t_i)+(1-\rho_i)u(w-t_i) \\ & \geq \rho_{-i}u(w+v_{-i}-P_{-i}-t_{-i})+(1-\rho_{-i})u(w-t_{-i}), \\ & \forall i \in \{1,...,N\}, \quad \forall -i \in \{1,...,i-1,i+1,...,N\},\end{aligned} \tag{6.23}$$

and the non-negativity constraints,

$$\rho_i(P_i-c_{good_i})+t_i-c_{lot_i} \geq 0, \quad P_i,t_i \geq 0, \quad \forall i=1,...,N. \tag{6.24}$$

The Lagrangian function is

$$\begin{aligned}\mathcal{L}(P_1,...,P_N,t_1,...,t_N) = & \sum_{i=1}^{N}(1-F(\tilde{v}_i))[\rho_i(P_i-c_{good_i})+t_i-c_{lot_i}] \\ & +\sum_{i=1}^{N}\lambda_i[\rho_i u(w+v_i-P_i-t_i)+(1-\rho_i)u(w-t_i)-u(w+r)] \\ & +\sum_{i\neq 1}^{N}\mu_{1i}[\rho_1 u(w+v_1-P_1-t_1)+(1-\rho_1)u(w-t_1) \\ & -\rho_i u(w+v_i-P_i-t_i)-(1-\rho_i)u(w-t_i)] \\ & +\sum_{i\neq 2}^{N}\mu_{2i}[\rho_2 u(w+v_2-P_2-t_2)+(1-\rho_2)u(w-t_2) \\ & -\rho_i u(w+v_i-P_i-t_i)-(1-\rho_i)u(w-t_i)] \\ & \vdots \\ & +\sum_{i=1}^{N-1}\mu_{Ni}[\rho_N u(w+v_N-P_N-t_N)+(1-\rho_N)u(w-t_N) \\ & -\rho_i u(w+v_i-P_i-t_i)-(1-\rho_i)u(w-t_i)] \\ & +\sum_{i=1}^{N}\tau_i[\rho_i(P_i-c_{good_i})+t_i-c_{lot_i}]+\sum_{i=1}^{N}\delta_i P_i+\sum_{i=1}^{N}\gamma_i t_i\end{aligned} \tag{6.25}$$

where $\lambda_i,\mu_{1i},...,\mu_{Ni},\tau_i,\delta_i,$ and γ_i, for all $i=1,...,N$ are Lagrangian multipliers.

Taking the first-order conditions with respect to $P_1,...,P_N$, and $t_1,...,t_N$,

$$\begin{aligned}\mathcal{L}_{P_1} &= \rho_1[1-F(\tilde{v}_1)] - f(\tilde{v}_1)\tilde{v}_{1,P_1}[\rho_1(P_1-c_{good_1})+t_1-c_{lot_1}] \\ &+ \left(\lambda_1+\sum_{i\neq 1}^{N}\mu_{1i}-\sum_{i\neq 1}^{N}\mu_{i1}\right)\rho_1 u_{P_1}(w+v_1-P_1-t_1)(\tilde{v}_{1,P_1}-1) \\ &+\tau_1\rho_1+\delta_1.\end{aligned} \tag{6.26}$$

$$\vdots$$

$$\begin{aligned}\mathcal{L}_{P_N} &= \rho_N[1-F(\tilde{v}_N)] - f(\tilde{v}_N)\tilde{v}_{N,P_N}[\rho_N(P_N-c_{good_N})+t_N-c_{lot_N}] \\ &+ \left(\lambda_N+\sum_{i=1}^{N-1}\mu_{Ni}-\sum_{i=1}^{N-1}\mu_{iN}\right)\rho_N u_{P_N}(w+v_N-P_N-t_N)(\tilde{v}_{N,P_N}-1) \\ &+\tau_N\rho_N+\delta_N.\end{aligned} \tag{6.27}$$

$$\begin{aligned}\mathcal{L}_{t_1} &= 1-F(\tilde{v}_1) - f(\tilde{v}_1)\tilde{v}_{1,t_1}[\rho_1(P_1-c_{good_1})+t_1-c_{lot_1}] \\ &+ \left(\lambda_1+\sum_{i\neq 1}^{N}\mu_{1i}-\sum_{i\neq 1}^{N}\mu_{i1}\right)[\rho_1 u_{t_1}(w+v_1-P_1-t_1)(\tilde{v}_{1,t_1}-1)-(1-\rho_1)u_{t_1}(w-t_1)] \\ &+\tau_1+\gamma_1.\end{aligned} \tag{6.28}$$

$$\vdots$$

$$\begin{aligned}\mathcal{L}_{t_N} &= 1-F(\tilde{v}_N) - f(\tilde{v}_N)\tilde{v}_{N,t_N}[\rho_N(P_N-c_{good_N})+t_N-c_{lot_N}] \\ &+ \left(\lambda_N+\sum_{i=1}^{N-1}\mu_{Ni}-\sum_{i=1}^{N-1}\mu_{iN}\right)[\rho_N u_{t_N}(w+v_N-P_N-t_N)(\tilde{v}_{N,t_N}-1) \\ &-(1-\rho_N)u_{t_N}(w-t_N)]+\tau_N+\gamma_N.\end{aligned} \tag{6.29}$$

Equating $\mathcal{L}_{P_i}$ to zero and $\tilde{v}_{i,P_i}=1$,

$$1-F(\tilde{v}_i) = \frac{f(\tilde{v}_i)[\rho_i(P_i-c_{good_i})+t_i-c_{lot_i}]-\tau_i\rho_i-\delta_i}{\rho_i}, \quad \forall i=1,...,N. \tag{6.30}$$

Then, it must be that $\rho_i(P_i-c_{good_i})+t_i-c_{lot_i}>0$. Then, by complementary slackness, $\tau_i=0$, for all $i=1,...,N$. Then, (6.30) becomes

$$1-F(\tilde{v}_i) = \frac{f(\tilde{v}_i)[\rho_i(P_i-c_{good_i})+t_i-c_{lot_i}]-\delta_i}{\rho_i}, \quad \forall i=1,...,N. \tag{6.31}$$

Since $\tilde{v}_{i,t_i} > 1/\rho_i$, for all $i = 1, ..., N.$,

$$\begin{aligned} 1 - F(\tilde{v}_i) &= \frac{f(\tilde{v}_i)[\rho_i(P_i - c_{good_i}) + t_i - c_{lot_i}] - \delta_i}{\rho_i} \\ &< f(\tilde{v}_i)\tilde{v}_{i,t_i}[\rho_i(P_i - c_{good_i}) + t_i - c_{lot_i}] - \delta_i \tilde{v}_{i,t_i}. \end{aligned} \tag{6.32}$$

Since $\delta_i \tilde{v}_{i,t_i} \geq 0$,

$$1 - F(\tilde{v}_i) < f(\tilde{v}_i)\tilde{v}_{i,t_i}[\rho_i(P_i - c_{good_i}) + t_i - c_{lot_i}]. \tag{6.33}$$

Equating $\mathcal{L}_{t_i}$ to zero and using (6.20),

$$1 - F(\tilde{v}_i) = f(\tilde{v}_i)\tilde{v}_{i,t_i}[\rho_i(P_i - c_{good_i}) + t_i - c_{lot_i}] - \gamma_i. \tag{6.34}$$

By (6.33), $\gamma_i > 0$, for all $i = 1, ..., N$. By complementary slackness, $t_i^* = 0$, for all $i = 1, ..., N$. Then, since $\rho_i(P_i - c_{good_i}) + t_i - c_{lot_i} > 0$, for all $i = 1, ..., N$, it must be that $P_i > 0$, for all $i = 1, ..., N$. By complementary slackness, $\delta_i = 0$, for all $i = 1, ..., N$. Therefore, by (6.31), in equilibrium,

$$P_i^* = \frac{1 - F(\tilde{v}_i)}{f(\tilde{v}_i)} + \frac{c_{lot_i}}{\rho_i} + c_{good_i}, \quad \text{and} \quad t_i^* = 0, \quad \forall i = 1, ..., N. \tag{6.35}$$

Therefore, we have the following proposition:

Proposition 7. Proposition 2 holds when there are n types of customers and n types of goods.

6.2.2 Types are differing in their attitudes towards risk

Suppose that customers have different utility functions and degree of risk-aversion. Let Type i customers have a utility function u_i. Further, without loss of generality, suppose that from Type 1 to Type N, the degree of risk-aversion is decreasing. That is, Type 1 is the most risk-averse type, Type 2 is less risk-averse than Type 1 but more risk-averse than Type 3 and so on. Type N is the least risk-averse type. Let $h_2, h_3, ...h_N$ be increasing concave functions such that $u_1 = h_2 \circ u_2$, $u_2 = h_3 \circ u_3$, and so on. Therefore, $u_1 = h_2 \circ h_3 \circ h_4 \circ ... \circ h_N \circ u_N$.[2]

[2]Note that the composition of increasing concave functions is also concave.

Type i goes to the mall if

$$\rho_i u_i(w+v_i-P_i-t_i)+(1-\rho_i)u_i(w-t_i)\geq u_i(w+r). \tag{6.36}$$

The marginal customer who is indifferent between going to the mall or not has a valuation $\tilde{v}_i(P_i,t_i)$,

$$\tilde{v}_i(P_i,t_i)\equiv u_i^{-1}\left(\frac{u_i(w+r)-(1-\rho_i)u_i(w-t_i)}{\rho_i}\right)-w+P_i+t_i. \tag{6.37}$$

Therefore, for all $i=1,...,N$,[3]

$$\tilde{v}_{i,P_i}=1, \tag{6.38}$$

and

$$\tilde{v}_{i,t_i}=1+\frac{(1-\rho_i)}{\rho_i}\frac{u_{i,t_i}(w-t_i)}{u_{i,t_i}(w+\tilde{v}_i-P_i-t_i)}\geq\frac{1}{\rho_i}. \tag{6.39}$$

Notice that if $\tilde{v}_{i,t_i}=1/\rho_i$, then $\tilde{v}_i=P_i$. But then, from the equation (6.37), it must be that in equilibrium, $t_i^*=-r$. Since this is not feasible, it must be that $\tilde{v}_{i,t_i}>1/\rho_i$.

Then, the mall maximizes its profit, $\Pi(P_1,P_2,...,P_N,t_1,t_2,...,t_N)$,which is the summation of the profits it receives from each type of customer,

$$\Pi(P_1,P_2,...,P_N,t_1,t_2,...,t_N)=\sum_{i=1}^{N}(1-F(\tilde{v}_i))[\rho_i(P_i-c_{good_i})+t_i-c_{lot_i}], \tag{6.40}$$

subject to individual rationality constraints of each type i,

$$(IR_i):\quad \rho_i u_i(w+v_i-P_i-t_i)+(1-\rho_i)u_i(w-t_i)\geq u_i(w+r),\quad \forall i=1,...,N, \tag{6.41}$$

and the incentive compatibility constraints of each type,

$$\begin{aligned}(IC_i):\quad &\rho_i u_i(w+v_i-P_i-t_i)+(1-\rho_i)u_i(w-t_i)\\ &\geq\rho_{-i}u_i(w+v_{-i}-P_{-i}-t_{-i})+(1-\rho_{-i})u_i(w-t_{-i}),\\ &\forall i\in\{1,...,N\},\quad \forall -i\in\{1,...,i-1,i+1,...,N\},\end{aligned} \tag{6.42}$$

and the non-negativity constraints,

$$\rho_i(P_i-c_{good_i})+t_i-c_{lot_i}\geq 0,\quad P_i,t_i\geq 0,\quad \forall i=1,...,N. \tag{6.43}$$

[3]Note that $\tilde{v}_{i,P_{-i}}=0$ and $\tilde{v}_{i,t_{-i}}=0,\forall i=1,...,N,\forall -i\in\{1,...,i-1,i+1,...,N\}$.

The Lagrangian function is

$$
\begin{aligned}
\mathcal{L}(P_1,...,P_N,t_1,...,t_N) = & \sum_{i=1}^{N}(1-F(\tilde{v}_i))[\rho_i(P_i-c_{good_i})+t_i-c_{lot_i}] \\
& + \sum_{i=1}^{N}\lambda_i[\rho_i u_i(w+v_i-P_i-t_i)+(1-\rho_i)u_i(w-t_i)-u_i(w+r)] \\
& + \sum_{i\neq 1}^{N}\mu_{1i}[\rho_1 u_1(w+v_1-P_1-t_1)+(1-\rho_1)u_1(w-t_1) \\
& - \rho_i u_1(w+v_i-P_i-t_i)-(1-\rho_i)u_1(w-t_i)] \\
& + \sum_{i\neq 2}^{N}\mu_{2i}[\rho_2 u_2(w+v_2-P_2-t_2)+(1-\rho_2)u_2(w-t_2) \\
& - \rho_i u_2(w+v_i-P_i-t_i)-(1-\rho_i)u_2(w-t_i)] \\
& \vdots \\
& + \sum_{i=1}^{N-1}\mu_{Ni}[\rho_N u_N(w+v_N-P_N-t_N)+(1-\rho_N)u_N(w-t_N) \\
& - \rho_i u_N(w+v_i-P_i-t_i)-(1-\rho_i)u_N(w-t_i)] \\
& + \sum_{i=1}^{N}\tau_i[\rho_i(P_i-c_{good_i})+t_i-c_{lot_i}]+\sum_{i=1}^{N}\delta_i P_i+\sum_{i=1}^{N}\gamma_i t_i
\end{aligned}
\tag{6.44}
$$

where $\lambda_i,\mu_{1i},...,\mu_{Ni},\tau_i,\delta_i,$ and γ_i, for all $i=1,...,N$ are Lagrangian multipliers.

Taking the first-order conditions with respect to $P_1,...,P_N$, and $t_1,...,t_N$,

$$
\begin{aligned}
\mathcal{L}_{P_1} = & \rho_1(1-F(\tilde{v}_1))-f(\tilde{v}_1)\tilde{v}_{1,P_1}(\rho_1(P_1-c_{good_1})+t_1-c_{lot_1}) \\
& + \lambda_1\rho_1 u_{1,P_1}(w+v_1-P_1-t_1)(\tilde{v}_{1,P_1}-1) \\
& + \left(\sum_{i\neq 1}^{N}\mu_{1i}\right)\rho_1 u_{1,P_1}(w+v_1-P_1-t_1)(\tilde{v}_{1,P_1}-1) \\
& - \left(\sum_{i\neq 1}^{N}\mu_{i1}u_{i,P_1}(w+v_1-P_1-t_1)\right)\rho_1(\tilde{v}_{1,P_1}-1)+\tau_1\rho_1+\delta_1.
\end{aligned}
\tag{6.45}
$$

$$\vdots$$

$$\begin{aligned}
\mathcal{L}_{P_N} &= \rho_N(1-F(\tilde{v}_N)) - f(\tilde{v}_N)\tilde{v}_{N,P_N}(\rho_N(P_N - c_{good_N}) + t_N - c_{lot_N}) \\
&\quad + \lambda_N \rho_N u_{N,P_N}(w + v_N - P_N - t_N)(\tilde{v}_{N,P_N} - 1) \\
&\quad + \left(\sum_{i=1}^{N-1} \mu_{Ni}\right) \rho_N u_{N,P_N}(w + v_N - P_N - t_N)(\tilde{v}_{N,P_N} - 1) \\
&\quad - \left(\sum_{i=1}^{N-1} \mu_{iN} u_{i,P_N}(w + v_N - P_N - t_N)\right) \rho_N(\tilde{v}_{N,P_N} - 1) + \tau_N \rho_N + \delta_N.
\end{aligned} \tag{6.46}$$

$$\begin{aligned}
\mathcal{L}_{t_1} &= 1 - F(\tilde{v}_1) - f(\tilde{v}_1)\tilde{v}_{1,t_1}(\rho_1(P_1 - c_{good_1}) + t_1 - c_{lot_1}) \\
&\quad + \lambda_1[\rho_1 u_{1,t_1}(w + v_1 - P_1 - t_1)(\tilde{v}_{1,t_1} - 1) - (1-\rho_1) u_{1,t_1}(w - t_1)] \\
&\quad + \left(\sum_{i\neq 1}^{N} \mu_{1i}\right) [\rho_1 u_{1,t_1}(w + v_1 - P_1 - t_1)(\tilde{v}_{1,t_1} - 1) - (1-\rho_1) u_{1,t_1}(w - t_1)] \\
&\quad - \sum_{i\neq 1}^{N} \mu_{i1}[\rho_1 u_{i,t_1}(w + v_1 - P_1 - t_1)(\tilde{v}_{1,t_1} - 1) - (1-\rho_1) u_{i,t_1}(w - t_1)] \\
&\quad + \tau_1 + \gamma_1.
\end{aligned} \tag{6.47}$$

$$\vdots$$

$$\begin{aligned}
\mathcal{L}_{t_N} &= 1 - F(\tilde{v}_N) - f(\tilde{v}_N)\tilde{v}_{N,t_N}(\rho_N(P_N - c_{good_N}) + t_N - c_{lot_N}) \\
&\quad + \lambda_N[\rho_N u_{N,t_N}(w + v_N - P_N - t_N)(\tilde{v}_{N,t_N} - 1) - (1-\rho_N) u_{N,t_N}(w - t_N)] \\
&\quad + \left(\sum_{i=1}^{N-1} \mu_{Ni}\right) [\rho_N u_{N,t_N}(w + v_N - P_N - t_N)(\tilde{v}_{N,t_N} - 1) - (1-\rho_N) u_{N,t_N}(w - t_N)] \\
&\quad - \sum_{i=1}^{N-1} \mu_{iN}[\rho_N u_{i,t_N}(w + v_N - P_N - t_N)(\tilde{v}_{N,t_N} - 1) - (1-\rho_N) u_{i,t_N}(w - t_N)] \\
&\quad + \tau_N + \gamma_N.
\end{aligned} \tag{6.48}$$

Equating $\mathcal{L}_{P_i}$ to zero and $\tilde{v}_{i,P_i} = 1$,

$$\rho_i(1-F(\tilde{v}_i)) = f(\tilde{v}_i)(\rho_i(P_i - c_{good_i}) + t_i - c_{lot_i}) - \tau_i\rho_i - \delta_i, \quad \forall i = 1, ..., N. \tag{6.49}$$

Then, it must be that $\rho_i(P_i - c_{good_i}) + t_i - c_{lot_i} > 0$. Then, by complementary slackness, $\tau_i = 0$, for all $i = 1, ..., N$. Then, (6.49) becomes

$$1 - F(\tilde{v}_i) = \frac{f(\tilde{v}_i)(\rho_i(P_i - c_{good_i}) + t_i - c_{lot_i}) - \delta_i}{\rho_i}, \quad \forall i = 1, ..., N. \tag{6.50}$$

Since $\tilde{v}_{i,t_i} > 1/\rho_i$, for all $i = 1,...,N$,

$$\begin{aligned} 1 - F(\tilde{v}_i) &= \frac{f(\tilde{v}_i)[\rho_i(P_i - c_{good_i}) + t_i - c_{lot_i}] - \delta_i}{\rho_i} \\ &< f(\tilde{v}_i)\tilde{v}_{i,t_i}[\rho_i(P_i - c_{good_i}) + t_i - c_{lot_i}] - \delta_i\tilde{v}_{i,t_i}, \quad \forall i = 1,...,N. \end{aligned} \tag{6.51}$$

Since $\delta_i\tilde{v}_{i,t_i} \geq 0$,

$$1 - F(\tilde{v}_i) < f(\tilde{v}_i)\tilde{v}_{i,t_i}[\rho_i(P_i - c_{good_i}) + t_i - c_{lot_i}], \quad \forall i = 1,...,N. \tag{6.52}$$

We examine the cases for Type 1 and Type N separately. First, we find the equilibrium price for the most risk-averse type, Type 1:

Equating $\mathcal{L}_{t_1}$ to zero and using (6.39),

$$\begin{aligned} 1 - F(\tilde{v}_1) = {} & f(\tilde{v}_1)\tilde{v}_{1,t_1}(\rho_1(P_1 - c_{good_1}) + t_1 - c_{lot_1}) \\ & + \sum_{i \neq 1}^{N} \mu_{i1}[\rho_1 u_{i,t_1}(w + v_1 - P_1 - t_1)(\tilde{v}_{1,t_1} - 1) - (1 - \rho_1)u_{i,t_1}(w - t_1)] \\ & - \gamma_1. \end{aligned} \tag{6.53}$$

Note that by (6.39), for all $i = 2,...,N$,

$$\begin{aligned} & \rho_1 u_{i,t_1}(w + v_1 - P_1 - t_1)(\tilde{v}_{1,t_1} - 1) - (1 - \rho_1)u_{i,t_1}(w - t_1) \\ & = (1 - \rho_1)\left[u_{i,t_1}(w + v_1 - P_1 - t_1)\frac{u_{1,t_1}(w - t_1)}{u_{1,t_1}(w + \tilde{v}_1 - P_1 - t_1)} - u_{i,t_1}(w - t_1)\right]. \end{aligned} \tag{6.54}$$

Since $u_1 = h \circ u_i$ for some increasing concave function h,

$$\frac{u_{1,t_1}(w - t_1)}{u_{1,t_1}(w + \tilde{v}_1 - P_1 - t_1)} = \frac{h_{t_1}(u_i(w - t_1))u_{i,t_1}(w - t_1)}{h_{t_1}(u_i(w + \tilde{v}_1 - P_1 - t_1))u_{i,t_1}(w + \tilde{v}_1 - P_1 - t_1)}. \tag{6.55}$$

Since u_i is an increasing function and $\tilde{v}_1 > P_1$, $u_i(w + \tilde{v}_1 - P_1 - t_1) > u_i(w - t_1)$. Since h is a concave function, its derivative is a decreasing function. Therefore, $h_{t_1}(u_i(w - t_1)) > h_{t_1}(u_i(w + \tilde{v}_1 - P_1 - t_1))$. Since the derivative of the function u_i is positive,

$$\frac{h_{t_1}(u_i(w - t_1))u_{i,t_1}(w - t_1)}{h_{t_1}(u_i(w + \tilde{v}_1 - P_1 - t_1))u_{i,t_1}(w + \tilde{v}_1 - P_1 - t_1)} > \frac{u_{i,t_1}(w - t_1)}{u_{i,t_1}(w + \tilde{v}_1 - P_1 - t_1)}. \tag{6.56}$$

Therefore, (6.54) is strictly positive. Hence,

$$\sum_{i \neq 1}^{N} \mu_{i1}[\rho_1 u_{i,t_1}(w + v_1 - P_1 - t_1)(\tilde{v}_{1,t_1} - 1) - (1 - \rho_1)u_{i,t_1}(w - t_1)] \geq 0, \tag{6.57}$$

with equality if and only if $\mu_{i1}=0$, for all $i=2,...,N$. Hence, by (6.52), γ_1 is strictly positive. By complementary slackness, $t_1^*=0$. Then, since the expression $\rho_1(P_1-c_{good_1})+t_1-c_{lot_1}$ cannot be negative, $P_1>0$. Hence, by complementary slackness $\delta_1=0$. Then, by (6.50), in equilibrium,

$$P_1^*=\frac{1-F(\tilde{v}_1)}{f(\tilde{v}_1)}+\frac{c_{lot_1}}{\rho_1}+c_{good_1}, \quad \text{and} \quad t_1^*=0. \tag{6.58}$$

Now, we find the equilibrium price of the good and the parking fee for the other types of customers except Type 1 and Type N: Fixing any $i=2,...,N-1$, equating $\mathcal{L}_{t_i}$ to zero and using (6.39),

$$\begin{aligned}1-F(\tilde{v}_i)=&f(\tilde{v}_i)\tilde{v}_{i,t_i}(\rho_i(P_i-c_{good_i})+t_i-c_{lot_i})\\&+\sum_{\substack{j\neq i\\ j=1,...,N}}^{N}\mu_{ji}[\rho_i u_{j,t_i}(w+v_i-P_i-t_i)(\tilde{v}_{i,t_i}-1)-(1-\rho_i)u_{j,t_i}(w-t_i)]\\&-\gamma_i.\end{aligned} \tag{6.59}$$

Note that by (6.39), $\forall j\neq i$,

$$\begin{aligned}&\rho_i u_{j,t_i}(w+v_i-P_i-t_i)(\tilde{v}_{i,t_i}-1)-(1-\rho_i)u_{j,t_i}(w-t_i)\\&=(1-\rho_i)\left[u_{j,t_i}(w+v_i-P_i-t_i)\frac{u_{i,t_i}(w-t_i)}{u_{i,t_i}(w+\tilde{v}_i-P_i-t_i)}-u_{j,t_i}(w-t_i)\right]\end{aligned} \tag{6.60}$$

which is strictly positive for j's who are less risk-averse than i while strictly negative for j's who are more risk-averse than i. Hence, except for the most risk-averse (Type 1) and the least risk-averse (Type N) types, (6.60) could be (strictly) positive or negative. Then,

$$\sum_{\substack{j\neq i\\ j=1,...,N}}^{N}\mu_{ji}[\rho_i u_{j,t_i}(w+v_i-P_i-t_i)(\tilde{v}_{i,t_i}-1)-(1-\rho_i)u_{j,t_i}(w-t_i)] \tag{6.61}$$

is zero if and only if $\mu_{ji}=0, \forall j\neq i$.

For any case where the expression in (6.60) is zero or positive, it must be that γ_i is strictly positive due to (6.52) and (6.59). Then, by complementary slackness, $t_i=0$. Then, since the expression $\rho_i(P_i-c_{good_i})+t_i-c_{lot_i}$ cannot be negative, $P_i>0$. Hence, by complementary slackness, $\delta_i=0$. Then, by (6.50), in equilibrium,

$$P_i^*=\frac{1-F(\tilde{v}_i)}{f(\tilde{v}_i)}+\frac{c_{lot_i}}{\rho_i}+c_{good_i}, \quad \text{and} \quad t_i^*=0, \quad \forall i=2,...,N-1. \tag{6.62}$$

Note that (6.60) is zero if and only if $\mu_{ji} = 0, \forall j \neq i$, and one of the cases where it is positive is when $\mu_{ji} = 0$, for all j's that are more risk-averse than i.

For any case where (6.60) is strictly negative, γ_i could be zero or strictly positive. Hence, there are infinitely many optimal solutions of parking fee and the price of the good. These solutions include the cases where the mall charges zero parking fee as well as the cases where it charges a strictly positive parking fee. The equilibrium prices in (6.62) are also in this infinite set.

For Type N, equating $\mathcal{L}_{t_N}$ to zero and using (6.39),

$$\begin{aligned}1 - F(\tilde{v}_N) = f(\tilde{v}_N)\tilde{v}_{N,t_N}(\rho_N(P_N - c_{good_N}) + t_N - c_{lot_N})& \\ + \sum_{i=1}^{N-1} \mu_{iN}[\rho_N u_{i,t_N}(w + v_N - P_N - t_N)(\tilde{v}_{N,t_N} - 1) - (1-\rho_N)u_{i,t_N}(w - t_N)]& \\ - \gamma_N.&\end{aligned} \tag{6.63}$$

Note that by (6.39), for all $i = 1, ..., N-1$,

$$\begin{aligned}&\rho_N u_{i,t_N}(w + v_N - P_N - t_N)(\tilde{v}_{N,t_N} - 1) - (1-\rho_N)u_{i,t_N}(w - t_N) \\ &= (1-\rho_N)[u_{i,t_N}(w + v_N - P_N - t_N)\frac{u_{N,t_N}(w - t_N)}{u_{N,t_N}(w + \tilde{v}_N - P_N - t_N)} - u_{i,t_N}(w - t_N)].\end{aligned} \tag{6.64}$$

Since $u_i = g \circ u_N$, for some increasing concave function g,[4]

$$\frac{u_{i,t_N}(w - t_N)}{u_{i,t_N}(w + v_N - P_N - t_N)} = \frac{g_{t_N}(u_N(w - t_N))u_{N,t_N}(w - t_N)}{g_{t_N}(u_N(w + v_N - P_N - t_N))u_{N,t_N}(w + v_N - P_N - t_N)}. \tag{6.65}$$

Since $v_N > P_N$ and u_N is increasing, $u_N(w + v_N - P_N - t_N) > u_N(w - t_N)$. Since g_{t_N} is decreasing, $g_{t_N}(u_N(w + v_N - P_N - t_N)) < g_{t_N}(u_N(w - t_N))$. Hence,

$$\frac{u_{N,t_N}(w - t_N)}{u_{N,t_N}(w + \tilde{v}_N - P_N - t_N)} < \frac{u_{i,t_N}(w - t_N)}{u_{i,t_N}(w + v_N - P_N - t_N)}. \tag{6.66}$$

Therefore, (6.64) is strictly negative, and so

$$\sum_{i=1}^{N-1} \mu_{iN}[\rho_N u_{i,t_N}(w + v_N - P_N - t_N)(\tilde{v}_{N,t_N} - 1) - (1-\rho_N)u_{i,t_N}(w - t_N)] \leq 0, \tag{6.67}$$

[4] g_x stands for $\partial g / \partial x$.

with equality if and only if $\mu_{iN} = 0$, for all $i = 1, ..., N-1$. If $\mu_{iN} = 0$, for all $i = 1, ..., N-1$, then (6.67) holds with equality and γ_N is strictly positive due to (6.52) and (6.63). By complementary slackness, $t_N = 0$. Then, since the expression $\rho_N(P_N - c_{good_N}) + t_N - c_{lot_N}$ cannot be negative, $P_N > 0$. Hence, by complementary slackness, $\delta_N = 0$. Then, by (6.50), in equilibrium,

$$P_N^* = \frac{1 - F(\tilde{v}_N)}{f(\tilde{v}_N)} + \frac{c_{lot_N}}{\rho_N} + c_{good_N}, \quad \text{and} \quad t_N^* = 0. \tag{6.68}$$

However, if for any $i = 1, ..., N-1$, $\mu_{iN} > 0$, then (6.64) is strictly negative. Then, γ_i could be zero or strictly positive. Hence, there are infinitely many optimal solutions of parking fee and the price of the good. These solutions include the cases where the mall charges zero parking fee as well as the cases where it charges a strictly positive parking fee. The equilibrium prices in (6.68) are also in this infinite set.

Under second-degree price discrimination where the types of customers have different attitudes towards risk and probabilities of finding (or buying) the goods, parking is free for the type of customer who is the most risk-averse among all types of customers. Thus, the mall has an incentive to insure the most risk-averse customer for the risk of not finding the good. The cost of the parking lot of this type is embedded in the prices of the good they buy. Moreover, the price is unique and depends negatively on the probability of finding the good. As they become less likely to buy the good, they pay more for the good.

For every other type, while free parking is an optimal solution, it is not the unique solution. That is, the mall can set parking fee to be strictly positive or free. Hence, the mall is flexible in embedding the cost of parking lot reserved for other types of customers into the price of the goods. If the mall chooses to provide parking for free for the other types, then the optimal price it can set is unique. In that case, the price of the good depends negatively on the probability of finding the good. As the customer becomes less likely to buy the good, she pays more for that good. To make free parking optimal, one of the things the mall can do is to set the prices such that every type strictly prefers revealing their true types.

Therefore, the following proposition is established:

Proposition 8. Under second-degree price discrimination where customers differ in their degree of risk-aversion, the price of the good that the most risk-averse customer buys is unique and covers the costs of the parking lots. Moreover, as the most risk-averse customer becomes less (more) likely to buy the good, she pays more (less) for the good. On the other hand, these results may not hold for other types of customers.

6.3 Third-Degree Price Discrimination

In this model, every customer is interested in the same good. The mall can separate the customers into groups and charge each group differently. There are n types of customers. Type i customer has a probability $\rho_i \in (0,1)$ of buying the good. The marginal cost of the parking lot of Type i customer is c_{lot_i}.

Type i goes to the mall if

$$\rho_i u_i(w+v_i-P_i-t_i)+(1-\rho_i)u_i(w-t_i) \geq u_i(w+r). \tag{6.69}$$

The marginal customer who is indifferent between going to the mall or not has a valuation $\tilde{v}_i(P_i,t_i)$,

$$\tilde{v}_i(P_i,t_i) \equiv u_i^{-1}\left(\frac{u_i(w+r)-(1-\rho_i)u_i(w-t_i)}{\rho_i}\right)-w+P_i+t_i, \quad \forall i=1,...,N. \tag{6.70}$$

Therefore,[5]

$$\tilde{v}_{i,P_i}=1, \tag{6.71}$$

and

$$\tilde{v}_{i,t_i}=1+\frac{(1-\rho_i)}{\rho_i}\frac{u_{i,t_i}(w-t_i)}{u_{i,t_i}(w+\tilde{v}_i-P_i-t_i)} \geq \frac{1}{\rho_i}. \tag{6.72}$$

Notice that if $\tilde{v}_{i,t_i}=1/\rho_i$, then $\tilde{v}_i=P_i$. But then, from the equation (6.70), it must be that in equilibrium, $t_i^*=-r$. Since this is not feasible, it must be that $\tilde{v}_{i,t_i}>1/\rho_i$.

Then, the mall maximizes its profit, $\Pi(P_1,P_2,...,P_N,t_1,t_2,...,t_N)$, which is the summation of the profits it receives from each type of customer,

$$\Pi(P_1,P_2,...,P_N,t_1,t_2,...,t_N)=\sum_{i=1}^{N}(1-F(\tilde{v}_i))[\rho_i(P_i-c_{good})+t_i-c_{lot_i}], \tag{6.73}$$

subject to individual rationality constraints of each type i,

$$(IR_i): \quad \rho_i u_i(w+v_i-P_i-t_i)+(1-\rho_i)u_i(w-t_i) \geq u_i(w+r), \quad \forall i=1,...,N, \tag{6.74}$$

and the non-negativity constraints,

$$\rho_i(P_i-c_{good})+t_i-c_{lot_i} \geq 0, \quad \text{and} \quad P_i,t_i \geq 0, \quad \forall i=1,...,N. \tag{6.75}$$

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[5] Note that $\tilde{v}_{i,P_{-i}}=0$ and $\tilde{v}_{i,t_{-i}}=0, \forall i=1,...,N, \forall -i \in \{1,...,i-1,i+1,...,N\}$.

The Lagrangian function is

$$\begin{aligned}\mathcal{L}(P_1,...,P_N,t_1,...,t_N) = &\sum_{i=1}^{N}[1-F(\tilde{v}_i)][\rho_i(P_i-c_{good})+t_i-c_{lot_i}]\\ &+\sum_{i=1}^{N}\lambda_i[\rho_i u_i(w+v_i-P_i-t_i)+(1-\rho_i)u_i(w-t_i)-u_i(w+r)]\\ &+\sum_{i=1}^{N}\mu_i[\rho_i(P_i-c_{good})+t_i-c_{lot_i}]+\sum_{i=1}^{N}\delta_i P_i+\sum_{i=1}^{N}\gamma_i t_i\end{aligned} \tag{6.76}$$

where $\lambda_i, \mu_i, \delta_i,$ and γ_i, for all $i=1,...,N$ are Lagrangian multipliers.

Taking the first-order conditions with respect to $P_1, P_2..., P_N$, and $t_1, t_2, ..., t_N$,

$$\begin{aligned}\mathcal{L}_{P_i} = &\rho_i[1-F(\tilde{v}_i)] - f(\tilde{v}_i)\tilde{v}_{i,P_i}[\rho_i(P_i-c_{good})+t_i-c_{lot_i}]\\ &+\lambda_i\rho_i u_{i,P_i}(w+v_i-P_i-t_i)(\tilde{v}_{i,P_i}-1)+\mu_i\rho_i+\delta_i, \quad \forall i=1,...,N.\end{aligned} \tag{6.77}$$

$$\begin{aligned}\mathcal{L}_{t_i} = &1-F(\tilde{v}_i) - f(\tilde{v}_i)\tilde{v}_{i,t_i}[\rho_i(P_i-c_{good})+t_i-c_{lot_i}]\\ &+\lambda_i[\rho_i u_{i,t_i}(w+v_i-P_i-t_i)(\tilde{v}_{i,t_i}-1)-(1-\rho_i)u_{i,t_i}(w-t_i)]\\ &+\mu_i+\gamma_i, \quad \forall i=1,...,N.\end{aligned} \tag{6.78}$$

Equating (6.77) to zero for all $i=1,...,N$ and using $\tilde{v}_{i,P_i}=1$,

$$1-F(\tilde{v}_i) = \frac{f(\tilde{v}_i)[\rho_i(P_i-c_{good})+t_i-c_{lot_i}]-\mu_i\rho_i-\delta_i}{\rho_i}, \quad \forall i=1,...,N. \tag{6.79}$$

Notice that $\rho_i(P_i-c_{good})+t_i-c_{lot_i}$ cannot be zero, since $1-F(\tilde{v}_i)$ is strictly greater than zero. Therefore, by complementary slackness, $\mu_i=0$, for all $i=1,...,N$. Therefore, (6.79) becomes

$$1-F(\tilde{v}_i) = \frac{f(\tilde{v}_i)[\rho_i(P_i-c_{good})+t_i-c_{lot_i}]-\delta_i}{\rho_i}, \quad \forall i=1,...,N. \tag{6.80}$$

Since $\tilde{v}_{i,t_i} > 1/\rho_i$,

$$\begin{aligned}1-F(\tilde{v}_i) &= \frac{f(\tilde{v}_i)[\rho_i(P_i-c_{good})+t_i-c_{lot_i}]-\delta_i}{\rho_i}\\ &< f(\tilde{v}_i)\tilde{v}_{i,t_i}[\rho_i(P_i-c_{good})+t_i-c_{lot_i}]-\delta_i\tilde{v}_{i,t_i}, \quad \forall i=1,...,N.\end{aligned} \tag{6.81}$$

Since $\delta_i\tilde{v}_{i,t_i} \geq 0$,

$$1-F(\tilde{v}_i) < f(\tilde{v}_i)\tilde{v}_{i,t_i}[\rho_i(P_i-c_{good})+t_i-c_{lot_i}], \quad \forall i=1,...,N. \tag{6.82}$$

Equating the first-order condition in (6.78) to zero and using (6.72),

$$1-F(\tilde{v}_i)=f(\tilde{v}_i)\tilde{v}_{i,t_i}[\rho_i(P_i-c_{good})+t_i-c_{lot_i}]-\gamma_i, \quad \forall i=1,...,N. \tag{6.83}$$

But then, by (6.82), it must be that $\gamma_i>0$, for all $i=1,...,N$. By complementary slackness, $t_i^*=0$, for all $i=1,...,N$. Then, it must be that $P_i>0$, since the expression $\rho_i(P_i-c_{good})+t_i-c_{lot_i}$ cannot be negative. Therefore, by complementary slackness, $\delta_i=0$, for all $i=1,...,N$ and in equilibrium,

$$P_i^*=\frac{1-F(\tilde{v}_i)}{f(\tilde{v}_i)}+\frac{c_{lot_i}}{\rho_i}+c_{good}, \quad \text{and} \quad t_i^*=0, \quad \forall i=1,...,N. \tag{6.84}$$

Therefore, we have the following proposition:

Proposition 9. Proposition 4 holds when there are n types of customers.

7. HORIZONTAL PRODUCT DIFFERENTIATION MODEL WITH N TYPES OF GOODS

This chapter studies the generalization of the model constructed in Chapter 5 for more than two types of goods.

Suppose that there are n types of customers: Type 1, Type 2,..., Type N, and n types of goods: Good 1, Good 2,..., Good N in the mall. Further, suppose that each type of customer is interested in exactly one of the goods and they decide to buy a good based on their subjective preferences such as color and taste. Let Type i customers be interested in Good i. The mall decides on the price and the fee of the parking lot for each type separately.

Type i goes to the mall if

$$\rho_i u(w+v_i-P_i-t_i)+(1-\rho_i)u(w-t_i) \geq u(w+r). \tag{7.1}$$

The marginal customer of Type i who is indifferent between going to the mall or not has a valuation $\tilde{v}_i(P_i,t_i)$,

$$\tilde{v}_i(P_i,t_i) \equiv u^{-1}\left(\frac{u(w+r)-(1-\rho_i)u(w-t_i)}{\rho_i}\right)-w+P_i+t_i, \quad \forall i=1,...,N. \tag{7.2}$$

Therefore, for all $i=1,...,N$[1]

$$\tilde{v}_{i,P_i}=1, \tag{7.3}$$

and

$$\tilde{v}_{i,t_i}=1+\frac{(1-\rho_i)}{\rho_i}\frac{u_{t_i}(w-t_i)}{u_{t_i}(w+\tilde{v}_i-P_i-t_i)} \geq \frac{1}{\rho_i}. \tag{7.4}$$

Notice that if $\tilde{v}_{i,t_i}=1/\rho_i$, then $\tilde{v}_i=P_i$. But then, from the equation (7.2), it must be that in equilibrium, $t_i^*=-r$. Since this is not feasible, it must be that $\tilde{v}_{i,t_i}>1/\rho_i$.

Then, the mall maximizes its profit, $\Pi(P_1,P_2,...,P_N,t_1,t_2,...,t_N)$, which is the sum-

[1]Note that $\tilde{v}_{i,P_{-i}}=0$ and $\tilde{v}_{i,t_{-i}}=0, \forall i=1,...,N, \forall -i \in \{1,...,i-1,i+1,...,N\}$.

mation of the profits it receives from each type of customer,

$$\Pi(P_1,P_2,...,P_N,t_1,t_2,...,t_N) = \sum_{i=1}^{N}(1-F(\tilde{v}_i))[\rho_i(P_i-c_{good_i})+t_i-c_{lot_i}], \tag{7.5}$$

subject to individual rationality constraints of each type i,

$$(IR_i): \quad \rho_i u(w+v_i-P_i-t_i)+(1-\rho_i)\,u\,(w-t_i) \geq u(w+r), \quad \forall i=1,...,N, \tag{7.6}$$

and the non-negativity constraints,

$$\rho_i(P_i-c_{good_i})+t_i-c_{lot_i} \geq 0, \quad \text{and} \quad P_i,t_i \geq 0, \quad \forall i=1,...,N. \tag{7.7}$$

The Lagrangian function is

$$\begin{aligned}\mathcal{L}(P_1,...,P_N,t_1,...,t_N) = &\sum_{i=1}^{N}[1-F(\tilde{v}_i)][\rho_i(P_i-c_{good_i})+t_i-c_{lot_i}]\\ &+\sum_{i=1}^{N}\lambda_i[\rho_i u(w+v_i-P_i-t_i)+(1-\rho_i)u(w-t_i)-u(w+r)]\\ &+\sum_{i=1}^{N}\mu_i[\rho_i(P_i-c_{good_i})+t_i-c_{lot_i}]+\sum_{i=1}^{N}\delta_i P_i+\sum_{i=1}^{N}\gamma_i t_i\end{aligned} \tag{7.8}$$

where $\lambda_i,\mu_i,\delta_i,$ and γ_i, for all $i=1,...,N$ are Lagrangian multipliers.

Taking the first-order conditions with respect to $P_1,P_2...,P_N$, and $t_1,t_2,...,t_N$,

$$\begin{aligned}\mathcal{L}_{P_i} = &\rho_i[1-F(\tilde{v}_i)]-f(\tilde{v}_i)\tilde{v}_{i,P_i}[\rho_i(P_i-c_{good_i})+t_i-c_{lot_i}]\\ &+\lambda_i\rho_i u_{P_i}(w+v_i-P_i-t_i)(\tilde{v}_{i,P_i}-1)+\mu_i\rho_i+\delta_i, \quad \forall i=1,...,N.\end{aligned} \tag{7.9}$$

$$\begin{aligned}\mathcal{L}_{t_i} = &1-F(\tilde{v}_i)-f(\tilde{v}_i)\tilde{v}_{i,t_i}[\rho_i(P_i-c_{good_i})+t_i-c_{lot_i}]\\ &+\lambda_i[\rho_i u_{t_i}(w+v_i-P_i-t_i)(\tilde{v}_{i,t_i}-1)-(1-\rho_i)u_{t_i}(w-t_i)]\\ &+\mu_i+\gamma_i, \quad \forall i=1,...,N.\end{aligned} \tag{7.10}$$

Equating (7.9) to zero for all $i=1,...,N$ and using $\tilde{v}_{i,P_i}=1$,

$$1-F(\tilde{v}_i) = \frac{f(\tilde{v}_i)[\rho_i(P_i-c_{good_i})+t_i-c_{lot_i}]-\mu_i\rho_i-\delta_i}{\rho_i}, \quad \forall i=1,...,N. \tag{7.11}$$

Notice that the expression $\rho_i(P_i-c_{good_i})+t_i-c_{lot_i}$ cannot be zero, since $1-F(\tilde{v}_i)$ is strictly greater than zero. Therefore, by complementary slackness condition, $\mu_i=0$, for all $i = 1, ..., N$. Then, (7.11) becomes

$$1 - F(\tilde{v}_i) = \frac{f(\tilde{v}_i)[\rho_i(P_i - c_{good_i}) + t_i - c_{lot_i}] - \delta_i}{\rho_i}, \quad \forall i = 1, ..., N. \tag{7.12}$$

Since $\tilde{v}_{i,t_i} > 1/\rho_i$,

$$\begin{aligned} 1 - F(\tilde{v}_i) &= \frac{f(\tilde{v}_i)[\rho_i(P_i - c_{good_i}) + t_i - c_{lot_i}] - \delta_i}{\rho_i} \\ &< f(\tilde{v}_i)\tilde{v}_{i,t_i}[\rho_i(P_i - c_{good_i}) + t_i - c_{lot_i}] - \delta_i \tilde{v}_{i,t_i}, \quad \forall i = 1, ..., N. \end{aligned} \tag{7.13}$$

Since $\delta_i \tilde{v}_{i,t_i} \geq 0$,

$$1 - F(\tilde{v}_i) < f(\tilde{v}_i)\tilde{v}_{i,t_i}[\rho_i(P_i - c_{good_i}) + t_i - c_{lot_i}], \quad \forall i = 1, ..., N. \tag{7.14}$$

Equating the first-order condition in (7.10) to zero and using (7.4),

$$1 - F(\tilde{v}_i) = f(\tilde{v}_i)\tilde{v}_{i,t_i}[\rho_i(P_i - c_{good_i}) + t_i - c_{lot_i}] - \gamma_i, \quad \forall i = 1, ..., N. \tag{7.15}$$

Then, by (7.14), it must be that $\gamma_i > 0$, for all $i = 1, ..., N$. Therefore, by complementary slackness condition, $t_i^* = 0$, for all $i = 1, ..., N$. Since the expression $\rho_i(P_i - c_{good_i}) + t_i - c_{lot_i}$ cannot be negative, it must be that $P_i > 0$. Then, by complementary slackness condition, $\delta_i = 0$, for all $i = 1, ..., N$ and in equilibrium,

$$P_i^* = \frac{1 - F(\tilde{v}_i)}{f(\tilde{v}_i)} + \frac{c_{lot_i}}{\rho_i} + c_{good_i}, \quad \text{and} \quad t_i^* = 0, \quad \forall i = 1, ..., N. \tag{7.16}$$

Therefore, the following proposition is established:

Proposition 10. Proposition 5 holds when there are n types of customers and n types of goods.

8. CONCLUSION

In this thesis, we construct a model to analyze how a monopolist shopping mall determines the prices of the goods and parking fees when it has some degree of ability to identify the types of customers. Customers have valuations over the goods and when they visit the mall and examine the goods, they may decide to leave the mall empty-handed. Hence, they take a risk by going to the mall.

Under first and third-degree price discrimination and horizontal product differentiation, the mall is able to distinguish the types of the customers, either because customers reveal their true types themselves or the mall can make them. Then, the mall provides free parking for all and embeds the cost of parking into the prices of the goods. Hence, the mall uses parking as a loss leader. Moreover, there is always a negative relationship between the price of the good and the probability of buying the good. Hence, as customers become less likely to buy the good, they pay more for the good.

Under second-degree price discrimination, we analyze two cases: In the first one customers have different probabilities of buying the good and in the second one, they have different attitudes towards risk. In the initial case, the results are similar to the other types of price discrimination. However, in the latter one, the results are more complex in the sense that while the mall still provides free parking to the most risk-averse customer, it may charge a positive fee for other types of customers. Thus, while the mall has an incentive to insure the most risk-averse customer for the risk of not finding or buying the good, it does not necessarily have such an incentive for the others. Hence, the degree of risk-aversion is essential in this setup. In addition, even though the negative relationship between the price and the probability of buying the good still holds for the most risk-averse customer, it is not necessarily true for the other types of customers.

One future research topic is to analyze the pricing strategy of the shopping mall when there are many goods and customers decide how many and which goods to buy. Then, the mall may prefer mixed bundling. Another topic is to study our model when the shopping mall is not a monopoly. In that case, customers decide whether to stay at home or go to a mall, and if they decide to go, they choose a mall to visit.

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