EFFICIENT AND INCENTIVE COMPATIBLE COMMUNITY MEDIATION

by
HANDE NUR ÇELEBİ

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Approved by:

Dr. Öğr. Üyesi Selçuk Özyurt ....
(Thesis Supervisor)

Dr. Öğr. Üyesi Emin Karagözoğlu ....

Prof. Dr. Özgür Kıbrıs ....

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ABSTRACT

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HANDE NUR ÇELEBİ

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Thesis Supervisor: Asst. Prof. SELÇUK ÖZYURT

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Mediation is a highly popular alternative dispute resolution system and a growing industry. I analyze mediation as a mechanism design problem and construct an individually rational, efficient and incentive compatible mediation rule for a community when preferences over alternatives are common knowledge and ranking of the outside option is not. Different than Kesten and Özyurt (2019), this paper offers a mechanism when the dispute occurs among a community which has more than two parties. If there exist a single issue to dispute, we cannot construct the intended mediation rule. However, if there are more than one issue to dispute, under some assumptions we can find a possibility result.

ÖZET

BİR TOPLUMDA VERİMLİ VE MANİPÜLASYONA KAPALI ARABULUCULUK

HANDE NUR ÇELEBİ

EKONOMİ YÜKSEK LİSANS TEZİ, HAZİRAN 2019

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Anahtar Kelimeler: Alternatif uyuşmazlık çözümleri, arabuluculuk, mekanizma tasarımı, verimli, manipülasyona kapalı

Arabuluculuk sistemi alternatif uyuşmazlık çözüm sistemleri arasında oldukça popüler ve büyüyen bir endüstri haline gelmiştir. Bu makale, arabuluculuk sistemini bir mekanizma tasarımı problemi olarak analiz ediyor ve bir topluluk için bireysel rasyonel, verimli ve manipülasyona kapalı arabuluculuk kuralı inşa ediyor. Tarafların arabuluculuğa konu olan anlaşmazlık üzerindeki tercihleri herkes tarafından bilinirken, masadan kalkma haklarını nasıl kullanacakları her taraf için özel bilgidir. Kesten ve Özyurt'un (2019) makalesinden farklı olarak bu makale, uyuşmazlık ikiden fazla taraf arasında olduğunda dahi istenen sonuca ulaşılacak bir mekanizma arıyor. Sonuçta, eğer taraflar sadece bir konu üzerinde anlaşmazlığa düştüyse, istenen arabuluculuk kuralı bulunamıyor. Eğer anlaşmazlığa düşülen konu sayısı birden fazlaysa, bazı varsayımlar altında istenen sonuca ulaşılıyor.

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To Ceren Damar

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1. INTRODUCTION

The need of more efficient and more flexible ways in legal systems helps to find an alternative: mediation. Mediation is a rising topic in legal systems in all over the world. In 1996, the Alternative Dispute Resolution (ADR) Act is adopted by the U.S. Congress and this act ensures that the federal courts have right to send a case to the ADR agencies and even in some cases, the ADR process is mandatory [1]. Also, the courts can apply the ADR process by themselves. Since then, many of the departments in the U.S. government such as U.S. Department of Labor and Department of the Air Force has its own ADR programs. In 2017, 75% of the cases which were proceeded ADR voluntarily and 55% of the cases which were proceeded ADR mandatorily were resolved and approximately $15,500,000 and 14,000 days of attorney/staff time was saved in the U.S. [2] In Turkey, the proceeding is newer. “The Code of Mediation in Law Conflicts” is passed from the parliament in 2012. [3] In this code, the mediator and the mediation process were defined. In 2018, the mediation also became mandatory for some cases in Turkey.

Mediation can be explained as a way of ADR negotiating to resolve a dispute between two or more parties under the supervision of an independent third party. The independent third party, the mediator, should have certain conditions such as necessary training which are specified by the laws. Mediation is different than litigation and it has many advantages when it is compared to the traditional resolution methods. First, the parties are not obliged to agree with the outcome of the mediation. There is always an outside option such as litigation or simply, not resolving. Second, the mediator pursues the needs and requests of the parties and tries to find an optimal solution. Third, since mediation takes less time than a usual court process, it is cost-effective. Fourth, mediation process is strictly confidential, there are no publicly available hearing. And, mediation is a

[1] Administrative Dispute Resolution Act of 1996.

[2] The United States Department of Justice.

[3] https://www.mevzuat.gov.tr/MevzuatMetin/1.5.6325.pdf

flexible system in which the parties have the right to negotiate issues that are not directly related to the case.

It might be thought that the mediation process is usually between two parties such as between spouses who are going to divorce or the employer and the employee. However, there are many other cases which have multiple parties and need to be resolved by mediation, for instance, businesses who have contributed to the same project or a neighbourhood who have affected by the same change. Kesten and Özyurt (2019) paper, which is mentioned in detail in the literature section, analyzes the two-people mediation problem with an ordinal approach. This paper tries to extend their findings to a public dispute. This paper analyzes the multi-party mediation problems with an ordinal approach which is new in the literature and characterizes a strategy-proof, efficient and individually rational mediation rule to solve them. The ordinal approach gives a flexibility to dispose of specified utility functions.

The model consists of multiple parties who are in dispute and a mediator who runs the negotiations. In the issue subject to negotiation,there are finitely many alternatives to be the outcome which are relevant to the issue. The preferences of parties/negotiators over alternatives are commonly known. However, not all alternatives are acceptable for a negotiator. Mediation has always an outside option. Thus, if a mediator is not satisfied with the outcome, she can pursue alternative dispute resolution ways. The ranking of the outside option, in other words, the set of a negotiator's acceptable alternatives is each negotiator's private information. The mediator's aim is to construct a rule which guarantees strategy-proofness, efficiency and individual rationality.

I use clones to analyze the problem at first. I define the clones as negotiators who has exactly same preferences over alternatives and there is one different negotiator who has exactly opposite preferences over alternatives. We can see this problem as a employer-employees dispute. Employees are similar to each other and have similar preferences over alternatives in an issue like the settlement of the wages. Employer is the different one and has exactly opposite preferences over alternatives in the same issue. If there exists a single issue, issue X in this environment I cannot find a strategy-proof, efficient and individually rational mediation rule. Even if I adopt more issues in which the ranking of the outside option is private information, the result does not change. To guarantee the individual rationality, mediation rule f cannot give an outcome with an $x \in X$ which is not acceptable for at least one negotiator. Since preferences over issue $X \setminus \{o_x\}$ is the same for all clones, the least accepting clone is binding for the outcome where o_x is the outside option in issue X. Then the mediator solves the problem as if there exist two negotiators: the first one is the least accepting clone and the second one is the different negotiator.

Then, I adopt the second issue, issue Y. Negotiators rank the outside option as the worst alternative in issue Y and it is publicly known. Monetary issues might be an example for issue Y, i.e., if the issue is splitting an amount of money in a community, not splitting and going to the costly litigation is the worst option for all negotiators. Issue X may be the dispute about the location of the office, if negotiators are employees with similar preferences and employer with exactly opposite preferences. In this environment, the mediation rule offers a bundle (x, y) from the set of all available outcome bundles XxY to the negotiators. The negotiators give information about their preference about outside option in issue X to the mediator, and their preference about the outside option in issue Y is known by the mediator. Mediator tries to prevent the utility increase by lying. Again, the least accepting clone is binding for the outcome and the problem can be solved as if there are only two negotiators.

I use logrolling condition to handle with the violation of strategy-proofness of an outcome when efficiency and individual rationality is guaranteed. This condition leads a negotiator to substitute his loss from less preferred $x \in X$ with taking more preferred $y \in Y$ where the outcome bundle is (x, y). The mediation rule constructs a precedence order from the set of bundles which satisfies logrolling condition and specifies the outcome from this order when there are mutually acceptable alternatives.

Then, I extend my findings to the negotiators who do not restrict their preferences, i.e., there are no clones in this setting. The dispute does not mean exactly opposite preferences over issue X any more. The dispute occurs when at least one negotiator's top alternative is different the others. Except this, the model is the same with the clone model. If there exists a single issue, the I cannot find a strategy-proof, efficient and individually rational mediation rule f, simply because if the mediation rule satisfies efficiency and individual rationality, the rule cannot guarantee strategy-proofness when there exist more than one mutually acceptable alternatives.

In the last section, I adopt one more issue, issue Y as I did in the clone model. The preferences of negotiators about the outside option in issue Y is publicly known whereas the ranking of o_x is private information of each negotiator. I use a kind of logrolling condition, partial logrolling, which is used for substitution when it is needed to guarantee strategy-proofness while efficiency and individual rationality is satisfied. The mediation rule constructs a precedence order over partial logrolling bundles by considering the join semi lattice over alternatives in issue X and the outcome is specified according to this order and the mutually acceptable alternatives.

2. LITERATURE

Kesten and Özyurt (2019) is the main paper for this section. My thesis is an extension of this paper. Two negotiators with exactly opposite preferences over alternatives in an issue X are in dispute. The preferences are public information, but ranking of the outside option is private for each negotiator. If there is a single issue to dispute, the mediation rule f cannot give a strategy-proof, individually rational and efficient outcome. If they add one more issue and the outside option is inferior for all negotiators in that issue, there is a possibility result. They use logrolling bundles, and I adopt this condition from their paper. They characterize a precedence order on logrolling bundles. Efficiency, individual rationality and strategy-proofness are satisfied when the mediation rule gives the outcome by using the precedence order over mutually acceptable bundles. They proved that even if the preferences are not strictly opposite of each other, we can eliminate inefficient alternatives and find strictly opposite preferences for two negotiators. I inspired by their result when I use strictly opposite preferences in my first two model. They adopt an ordinal approach which is different than bargaining literature in general. And, I extend their model which is two-party mediation problem to a community mediation problem.

Myerson and Satterthwaite (1983) find that in a bargaining game, it is impossible to find an ex-post efficient allocation mechanism when individual rationality and strategy-proofness are satisfied. Mediation can be considered as a bargaining game with outside option. This paper differs from Myerson and Satterthwaite (1983) in two points: first, the parties' preferences are unknown to each other while preferences are public information in this paper. Second, their construction has cardinal approach, while this paper has an ordinal approach for the preferences. Bester and Warneryd (2006) states that if the outside option is distributed probabilistically and it is private information of the agents, the peaceful settlement is impossible even if it is ex-post efficient. Compte and Jehiel (2009) use the opposite preferences and the outside option as private information with cardinal approach in utilities.

In matchings literature, the ordinal approach is more common than in bargaining literature. Kesten and Özyurt (2019) paper defines some special rules in the outcome family, and one of them is negotiator-optimal rules. My model also allows me to define such rule and this rule resembles the proposing-optimal deferred acceptance algorithm from Gale and Shapley (1962). Single-issue mediation models in this paper and Kesten and Özyurt (2019) have common characteristics with voting rules in some papers such as Moulin (1980) and Ching (1997) Also, a special member of intended outcomes in my last model, the join semi lattice can be constructed by using Borda count.

3. SINGLE-ISSUE MEDIATION WITH CLONES

There exist n negotiators who are in dispute over a single issue and m available alternatives to the issue. Negotiators could have the outside option, o, as an outcome when they do not agree upon an alternative. Thus, the set $M = \{x_1, ...x_m, o\}$ is the set of all possible outcomes of the dispute.

$n-1$ of the negotiators are clones of each other. Therefore, their preferences over alternatives are the same. It is common knowledge that the preferences of clones and the dissimilar negotiator over alternatives are diametrically opposed, i.e., clones strictly prefer x_k to x_{k+1} and the dissimilar negotiator strictly prefers x_{k+1} to x_k. However, their ranking of the outside option is private information for each of them. Thus, each negotiator has m types, denoted as $\theta_i^{x_j}$, where i denotes the negotiator and x_j denotes the least acceptable alternative for negotiator i. For convenience, I assumed that each negotiator accepts at least one alternative.

We seek individually rational, efficient and strategy-proof mediation rules in this environment. Before the construction, we need to consider that the least accepting negotiator among clones is binding for us. Since we seek individual rationality, the mediation rule cannot give an alternative which is unacceptable for at least one negotiator. To satisfy this, the mediation rule should always give an alternative from the set of available solutions for the least accepting negotiator, i.e., $M_{\theta_c^{x_j}} = \{x_1, ...x_j, o\}$ where c denotes the least accepting negotiator belongs clones and x_j is the least acceptable alternative for that binding clone. Then, we can shrunk the mediation process into two-people single-issue mediation, $\theta_c^{x_j}$ and $\theta_d^{x_j}$ where c represent the least accepting clone and d represents the dissimilar negotiator.

To construct the mediation rule, first we consider individual rationality and efficiency and check for strategy-proofness later. Start with fixing $\theta_c^{x_1}$. We need to consider each $\theta_d^{x_j}$ where $j \in \{1, ..., m\}$. Since the least accepting clone only accepts x_1, if x_1 is mutually acceptable for both of them, then the mediation rule gives x_1 and otherwise it gives o. At step 2, fix $\theta_c^{x_2}$. If the set $\{x_1, x_2\}$ is mutually acceptable, then the mediation rule gives x_1 or x_2. If the only mutually acceptable alternative is x_2, then it gives x_2. Otherwise, it gives o. Continue this way by fixing each $\theta_c^{x_j}$ and finding individually rational and efficient alternatives for each case to reach the following matrix:

	$\theta_d^{x_1}$	...	$\theta_d^{x_m}$
$\theta_c^{x_1}$	x_1	...	o_x
...	...	...	...
$\theta_c^{x_m}$	$x_1, ..., x_m$	...	x_m

For strategy-proofness, fix one of the negotiators' type and check if the other negotiator can be better-off by misrepresent his preferences. For instance, if the mediation rule, μ, gives x_1 where negotiators are $\theta_c^{x_2}$ and $\theta_d^{x_1}$, i.e., $\mu(\theta_c^{x_2}, \theta_d^{x_1}) = x_1$, then $\theta_d^{x_1}$ wants to deviate to $\theta_d^{x_2}$ by not being truthful and can get x_2 which is more preferable for $\theta_d^{x_1}$. On the other hand, if $\mu(\theta_c^{x_2}, \theta_d^{x_1}) = x_2$, then $\theta_c^{x_2}$ can state his preferences as $\theta_c^{x_1}$ and get x_1 wihch is more desirable than x_2 for him.

This kind of contradiction with strategy-proofness can be found in all profiles, i.e., $(\theta_c^{x_j}, \theta_d^{x_k})$, that the individually rational and efficient mediation rule gives more than one alternative as outcome. When we fix one of the negotiators' type, the other negotiator always wants to deviate to the main diagonal by lying. Thus, we could not construct an individually rational, efficient and strategy-proof mediation rule in single-issue mediation with clones.

4. MULTI-ISSUE MEDIATION WITH CLONES

There exist n negotiators and two different issues, issue X and issue Y to dispute. There are m possible outcomes for each issue. There are outside options in each issue, o_x and o_y. If the settlement is not done in one of the issues, the outcome will be the outside option of this issue. Then, $X = \{x_1, ..., x_m, o_x\}$ and $Y = \{y_1, ..., y_m, o_y\}$ are the sets of all possible outcomes for each issue and $X \backslash \{o_x\}$ and $Y \backslash \{o_y\}$ are the sets of alternatives that are available.

$n - 1$ of the negotiators have the same preferences over available alternatives in both issues, i.e., they are clones of each other. The remaining negotiator has opposite preferences. Same with the first model, it is common knowledge that the preferences of clones and the dissimilar negotiator over alternatives are diametrically opposed, i.e., clones strictly prefer x_k to x_{k+1} and the dissimilar negotiator strictly prefers x_{k+1} to x_k in X and clones strictly prefer y_k to y_{k+1} and the dissimilar negotiator strictly prefers y_{k+1} to y_k in Y. The ranking of o_x in X is private information of each negotiator while o_y is the inferior outcome for all negotiators and it is publicly known. Formally, $y_k \ \theta_i^Y \ o_y$ for all i and all $k \leq m$. The preference ordering of each negotiator over alternatives in issue Y is unique and publicly known and the preference orderings over alternatives in X can be one of the m different orderings. Thus, each negotiator has m types, denoted as $\theta_i^{x_j}$, where i denotes the negotiator and x_j denotes the least acceptable alternative for negotiator i. Θ_i denote the set of all possible types for negotiator i and $\Theta = \Theta_1 x ... x \Theta_n$is the set of all type profiles. For convenience, I assumed that each negotiator accepts at least one alternative.

$\Re$ is the set of all complete and transitive binary relations over the bundles $(x_k, y_k) \in X \times Y$, $R \in \Re$ and for two bundles $b, b' \in X \times Y$, $b \ R_i \ b'$ means b is at least as good as b' for negotiator i and P is a part of R and $b \ P_i \ b'$ means b is strictly better than b' for negotiator i. Λ is an extension map which assigns a non-empty set $\Lambda(\theta_i) \subseteq \Re$ of admissible orderings over bundles to all negotiators and types.

If an alternative is ranked higher than the outside option o_x for negotiator i, this alternative is acceptable for i. $A(\theta_i) = \{x \in X | x\theta_i o_x\}$ is the set of all acceptable alternatives in X for each negotiator i and type $\theta_i \in \Theta_i$. For each type profile $\theta = (\theta_1, ..., \theta_n) \in \Theta_i$, $A(\theta) = \{x \in X | x\theta_i o_x \forall i \in I\}$ is the set of mutually acceptable alternatives in X.

Definition 1 *The extension map Λ is consistent if the following holds for all i, $\theta_i \in \Theta_i$ and all $R_i \in \Lambda(\theta_i)$:*

(i) Monotonicity: For any $x, x' \in X$ and $y, y' \in Y$ with $(x,y) \neq (x',y')$, $(x,y)P_i(x',y')$ whenever $[x\theta_i x' or x = x']$ and $[y\theta_i y' or y = y']$.

(ii) Deal Breakers: For any $y, y' \in Y \setminus \{o_y\}$, $(x,y)R_i(x',y')$ whenever $x \in A(\theta_i) \cup \{o_x\}$, $x' \notin A(\theta_i)$ and $x \neq x'$.

(iii) Logrolling: For any i, there exist a one-to-one mapping $t_i : X \to Y$ such that for all $\theta_i \in \Theta_i$, $R_i \in \Lambda(\theta_i)$ and all $x, x' \in A(\theta_i)$ with $x \succ_i x'$,

$$(x', t_i(x'))R_i(x, t_i(x))$$

.

Monotonicity is a sensible assumption, since one always want to take a bundle that has more preferred alternative of at least one issue. Deal breakers assumption can be interpreted as one never accepts a bundle that has an unacceptable alternative for her in X no matter which $y_k \in Y$ is given in that bundle. Logrolling is the most important assumption that helps to guarantee strategy-proofness. If a negotiator gets a lower ranked but acceptable $x \in X$, she can substitute the loss with getting a higher ranked $y \in Y$. Negotiators can trade between x and y. The formal definitions below are the conditions that I seek in the mediation rule.

Definition 2 *The mediation rule f is strategy-proof if for all i and all $\theta_i \in \Theta_i$, $f(\theta_i, \theta_{-i})$ R_i $f(\theta_i', \theta_{-i})$ for all $R_i \in \Lambda(\theta_i)$, $\theta_i' \in \Theta_i$ and all $\theta_{-i} \in \Theta_{-i}$.*

Definition 3 *The mediation rule f is individually rational if for all i and all $(\theta_i, \theta_{-i}) \in \Theta_i$, $f(\theta_i, \theta_{-i})$ R_i (o_x, o_y) for all $R_i \in \Lambda(\theta_i)$.*

Definition 4 *The mediation rule f is efficient if there exists no $(\theta_i, \theta_{-i}) \in \Theta_i$ and $(x', y') \in X \times Y$ such that (x', y') R_i $f(\theta_i, \theta_{-i})$ for all $R_i \in \Lambda(\theta_i)$ and all $i \in I$, and for at least one $i \in I$, (x', y') P_i $f(\theta_i, \theta_{-i})$ for some $R_i \in \Lambda(\theta_i)$.*

The aim is to characterize a mediation rule in which to be better off by lying is not possible for negotiators, i.e., to announce the ranking of the outside option in issue X truthfully is the dominant strategy equilibrium. As in the first model, the least accepting clone is binding for the mediation outcome. The mediation rule cannot give an alternative which is not acceptable for at least one person. Moreover, the least accepting clone is always among the non-accepting negotiators if an alternative is not in the set of mutually acceptable alternatives. Thus, one can solve the mediation problem as if there exist two negotiators who are the least accepting clone and the dissimilar negotiator.

To illustrate, a simple example which has three negotiators and two alternatives for each issue is considered. Assume that negotiator 1 and negotiator 2 are clones of each other and negotiator 3 is the dissimilar one. Their rankings over alternatives are as follows:

- $\theta_1, \theta_2 : x_1 \ \theta_{1,2} \ x_2$ and $y_1 \ \theta_{1,2} \ y_2 \ \theta_{1,2} \ o_y$
- $\theta_3 : x_2 \ \theta_3 \ x_1$ and $y_2 \ \theta_3 \ y_1 \ \theta_3 \ o_y$

o_x does not exist in the ranking, because it is public information and it can be ranked right after the top alternative for a negotiator or as the worst option. Each negotiator has two different types according to the ranking of the outside option. For instance, the types of negotiator 1 are θ_1^{x1} and θ_1^{x2} and the preferences orderings for types are $x_1 \ \theta_1^{x1} \ o_x \ \theta_1^{x1} \ x_2$ and $x_1 \ \theta_1^{x2} \ x_2 \ \theta_1^{x2} \ o_x$ respectively. Suppose that negotiator 1 is the least accepting clone. The mediation rule f will give an outcome

	θ_3^{x2}	θ_3^{x1}
θ_1^{x1}	(o_x, y)	(x_1, y_2)
θ_1^{x2}	(x_2, y_1)	(x_1, y_2) or (x_2, y_1)

When all alternatives in X are acceptable for everyone, the logrolling condition states that if $x_1 \ \theta_1^{x2} \ x_2$, then $(x_2, y_1) \ R_i \ (x_1, y_2)$ should be satisfied and if $x_2 \ \theta_2^{x1} \ x_1$, then $(x_1, y_2) \ R_i \ (x_2, y_1)$ should be satisfied. The mediator can give any bundle from the set of logrolling bundles, $\{(x_1, y_2), (x_2, y_1)\}$ for $f_{2,1}$ and it will be strategy-proof, efficient and individually rational.

Theorem 1 *The mediation rule f is individually rational, efficient and strategy-proof if and only if the following hold:*

(i) If $j < k$, then $f_{j,k} = (o_x, \ y)$

(ii) If $j = k$, then $f_{j,k} = (x_j, y_{m+1-j})$

(iii) If $j > k$, then $f_{j,k} \in \{f_{j-1,k}, f_{j,k+1}\} \subset B$ and there exists a complete, transitive and anti-symmetric precedence order $\triangleright$ on $\boldsymbol{B}$ such that

$$f_{j,k} = \begin{cases} f_{j-1,k} & iff_{j-1,k} \triangleright f_{j,k+1} \\ f_{j,k+1} & otherwise \end{cases}$$

where the least accepting clone is $\theta_c^{x_j}$ *and the dissimilar negotiator is* $\theta_d^{x_k}$.

5. SINGLE-ISSUE MEDIATION WITHOUT RESTRICTION

Now, we change our first construction and abandon from any restriction upon any person's preferences. There are n negotiators and m alternatives over an issue x. Each negotiator has strict preferences over alternatives. We define the dispute different than the case with the clones. Dispute occurs when the first best alternative of at least one negotiator is different than the others'. Thus, the dispute does not restrict any ordering except the negotiators' first best. We can come up with two questions: First, what if only rankings over outside option, o_x, is private information? Second, what if rankings over all preferences are private information? Actually, the first question's answer implies the second's.

If rankings over o_x are the only private information, we could not find any strategy-proof, individually rational, and efficient mediation rule. If we try to construct such mediation rule f, we can start with assuring individual rationality and efficiency. For individual rationality, the mediation rule f shoud not give any alternative worse than o_x. For efficiency, f should give an alternative which is mutually acceptable for everyone. If there exist only one mutually acceptable alternative, f gives this. If there are more than one mutually acceptable alternative and some of them are inefficient, f picks one among the efficient alternatives. If there are more than one mutually acceptable alternative and any of them is not inefficient, f picks one of them to give. If there does not any mutually acceptable alternative, then f should give o_x.

Now, we try to find a strategy-proof rule among individually rational and efficient rules. If there is not any mutually acceptable alternative, f should give o_x and no one will be better off by lying. Because, if someone accepts more than his original type and leads to a mutually acceptable alternative, the rule f gives that one. However, it is not mutually acceptable alternative for him in truth, so he would not deviate. If there is only one mutually acceptable alternative, f should give this alternative. Then, if a negotiator deviates to a type that accepts more, f gives either the same alternative or an alternative that this negotiator ranks below the first alternative. If a negotiator deviates to a less accepting type, then f gives o_x, because there is not a mutually acceptable alternative any more. However, the first alternative is ranked above o_x for the negotiator. Thus, no one deviates to a better alternative by lying. If there are more than one mutually acceptable alternative, f should choose one of them. Let's say $\{x_i, x_k\}$ are mutually acceptable alternatives and wlog they are efficient. Since preferences over alternatives are strict and there is a dispute, at least one negotiator prefers x_i which is not the given one, more than x_k, the given one. Thus, the negotiator can deviate to a type that does not accept x_k and f should give x_i. Then, he will be better off by lying. If the rule f gives x_i, the same situation occurs again. Thus strategy-proofness is violated in all cases which has more than one acceptable and efficient alternatives. We can illustrate that with an example:

Suppose there are 3 people and 3 different alternatives in issue X. The ranking of each negotiator over alternatives is public information, thus, the rankings are as follows:

- $\theta_1 : x_1 \; \theta_1 \; x_2 \; \theta_1 \; x_3$
- $\theta_2 : x_2 \; \theta_2 \; x_3 \; \theta_2 \; x_1$
- $\theta_3 : x_3 \; \theta_3 \; x_1 \; \theta_3 \; x_2$

Since the ranking of the outside option, o is private information of each negotiator, each has three different types according to their acceptable alternatives. In this environment, individually rational and efficient alternatives, i.e., mutually acceptable alternatives at each type profile as follows:

$[\theta_3^{x_3}]$

	$\theta_2^{x_2}$	$\theta_2^{x_3}$	$\theta_2^{x_1}$
$\theta_1^{x_1}$	o	o	o
$\theta_1^{x_2}$	o	o	o
$\theta_1^{x_3}$	o	o	x_3

$[\theta_3^{x_1}]$

	$\theta_2^{x_2}$	$\theta_2^{x_3}$	$\theta_2^{x_1}$
$\theta_1^{x_1}$	o	o	x_1
$\theta_1^{x_2}$	o	o	x_1
$\theta_1^{x_3}$	o	x_3	x_1, x_3

$[\theta_3^{x_2}]$

	$\theta_2^{x_2}$	$\theta_2^{x_3}$	$\theta_2^{x_1}$
$\theta_1^{x_1}$	o	o	x_1
$\theta_1^{x_2}$	x_2	x_2	x_1, x_2
$\theta_1^{x_3}$	x_2	x_2, x_3	x_1, x_2, x_3

where the row negotiator is θ_1, column negotiator is θ_2 and table negotiator is θ_3 and superscript shows the least acceptable alternative for each type of each negotiator. From individually rational and efficient alternatives, we try to find a strategy-proof rule. For contradiction, suppose the mediation rule gives x_1 at the type profile $(\theta_1^{x_3}, \theta_2^{x_1}, \theta_3^{x_1})$. In this case, $\theta_2^{x_1})$ could be better off by revealing her type as $\theta_2^{x_2})$ and getting x_3. Thus, giving x_1 is not a strategy-proof rule. Now suppose the mediation rule gives x_3 at the type profile $(\theta_1^{x_3}, \theta_2^{x_1}, \theta_3^{x_1})$. In this case, $(\theta_1^{x_3}$ could be better off by revealing her type as $(\theta_1^{x_2}$ and getting x_1. Thus, giving x_3 is not a strategy-proof rule. At the end, any mutually acceptable alternative in the type profile $(\theta_1^{x_3}, \theta_2^{x_1}, \theta_3^{x_1})$ cannot give a strategy proof rule.

6. MULTI-ISSUE MEDIATION WITHOUT RESTRICTION

The model is the same with multi-issue mediation model with clones up to the assumption of restricted preferences. This model applies no restriction upon the negotiators' preference orderings. In other words, having diametrically opposed preferences is not obligatory in this setting. Thus, the dispute cannot be defined as diametrically opposed preferences. Now, the dispute occurs when at least one negotiator's top alternative is different than others. Preference ordering of each negotiator over available alternatives and o_y, the outside option in issue Y are common knowledge and the order of o_x, the outside option in issue X is private information of each negotiator as in the previous model. The notation is adopted from the previous model.

For simplicity, I use three alternatives for each issue and three negotiators to dispute. To be in a dispute, at least one of the negotiator's top alternative in one of the issues is different than others. Denote T_i as the top two alternatives for each negotiator i independent from their types. Formally,

$$T_i = \{x \in X | \exists x' \in X s.t. x \theta_i x' and x \neq x'\}.$$

θ^* is a join semi lattice that compares all $x, x' \in X$ such that $\{(x, x')\} \in \bigcup_{i=1}^{3} T_i$. θ^* should make pairwise comparison of top two alternatives of each negotiator. $A(\theta_i)$ is defined as the set of all acceptable alternatives for all i and all θ_i. Now, a restricted acceptable alternatives set is introduced. $A^R(\theta_i)$ consists of pairs of acceptable x's that negotiator i ranks reversely compared to the join semi lattice, θ^*. Formally,

$$A^R(\theta_i) = \{(x, x') \in X^2 | x \theta_i x' and x' \theta^* x \forall x, x' \in A(\theta_i) and x \neq x'\}$$

And, the set Θ_i^R is the set of all types for each negotiator i who contradicts with the join semi lattice θ^* at some point. Formally,

$$\Theta_i^R = \{\theta_i \in \Theta_i | A^R(\theta_i) \neq \emptyset\}$$

The new logrolling condition is constructed in the light of the sets that are defined above and the join semi lattice θ^*:

Definition 5 *The extension map* Λ *satisfies partial logrolling if there exist a one-to-one mapping* $t: X \to Y$ *such that* $\forall \theta_i \in \Theta_i^R$, $\forall R_i \in \Lambda(\theta_i)$ *and all* $(x,x') \in A^R(\theta_i)$ *with* $x\theta_i x'$ *we have*

$$(x', t(x'))\ R_i\ (x, t(x)).$$

The partial logrolling condition enables us to compare and trade the bundles in which the x's are ordered differently in θ^* and θ_i for all i.

Definition 6 *The pair of the extension map and the function* (Λ, t) *is partially consistent if the followings hold for all* $i \in I$, $\theta_i \in \Theta_i$ *and all* $R_i \in \Lambda(\theta_i)$*:*

(i) Monotonicity

(ii) Deal breakers

(iii) Partial Logrolling

and, $B^p(t) = \{(x, t(x)) \in X \times Y | (\Lambda, t)$ *is partially consistent* $\}$.

Monotonicity and deal breakers are defined as exactly the same with the Multi-issue mediation with clones part. $B^p(t)$ denotes the set of partially logrolling bundles when (Λ, t) is partially consistent.

Theorem 2 *The mediation rule f is efficient, strategy-proof and individually rational if and only if the following hold:*

(i) $A(\theta) = \emptyset$*, then* $f = (o_x, y)$ *for some* $y \in Y$

(ii) $A(\theta) = \{x\}$*, then* $f = (x, t(x)) \in B^p(t)$

(iii) $|A(\theta)| > 1$*, then* $\exists$ *a complete, transitive and antisymmetric precedence order* $\rhd$ *on* $B^p(t)$ *that satisfies* $x\theta^* x \Rightarrow (x, t(x)) \rhd (x', t(x'))$ *such that*

$$f = \max_{A(\theta,t)} \rhd$$

where $A(\theta, t) = \{(x, t(x)) \in B^p(t) | x \in A(\theta)\} \subseteq B^p(t)$.

If there does not exist a mutually acceptable alternative in issue X, then the mediation rule should give the bundle that contains the outside option, o_x. The rule cannot give a bundle which contains an alternative $x \neq o_x$, because this alternative x is unacceptable for at least one negotiator for sure and thus, it contradicts with individual rationality. If there exist only one mutually acceptable alternative in issue X, then the mediation rule should give the partial logrolling bundle that contains that $x \in X$. It cannot give a different $x' \in X$, because x' is unacceptable for at least one negotiator for sure and thus, it contradicts with efficiency and individual rationality. If there exist more than one mutually acceptable alternatives, the mediation rule should choose which x to give in the outcome bundle according to the partial logrolling condition. When the rule guarantees efficiency and individual rationality, it needs partial logrolling to satisfy the strategy-proofness.

7. CONCLUSION

This paper seeks a mechanism that gives efficient, individually rational and incentive compatible mediation rule when there exist more than two parties in dispute. It differs from Kesten and Özyurt's (2019) paper, because their paper analyzes the mediation problem with only two parties. In this sense, this paper can be seen as an extension of Kesten and Özyurt (2019).

Along the paper, the ranking of the outside option for each negotiator is private information for that negotiator. The paper contains four different parts. First, it analyzes a single-issue mediation problem with clones. In this case, there exist a single issue in dispute and all people but one in the community have exactly same preferences over the alternatives in the issue and and one have exactly opposite preferences over the same alternatives. In such environment, the mediator cannot find the intended mediation rule. Second, I add a second issue to dispute and I still have the clone assumption. Now, the paper analyzes a multi-issue mediation problem with clones. I know the ranking of the outside option of all negotiators in the new issue. The outside option is the worst one for all. In that case, the mediator can find the intended mediation rule if the preferences satisfy monotonicity, deal breakers and logrolling assumptions. Since the least accepting clone is binding for the outcome, the problem can be shrunk into two-party mediation problem and these first two results of the paper can be derived from Kesten and Özyurt (2019).

Third, I drop the clone assumption, then negotiators do not have a restriction upon their preferences. Now the paper analyzes a single-issue mediation problem with no restriction. Since there cannot be exactly opposite preferences over alternatives for disputing parts, the dispute occurs unless all negotiators' top alternatives are the same. In such an environment, the mediator cannot find the intended mediation rule. In other words, even if the dispute is minimal, there does not exist an efficient, individually rational and incentive compatible mediation rule. Forth and last, the paper analyzes three-party multi-issue mediation problem with no restriction. The multi-party analysis is left for publication. Again, there exist a second issue and the outside option of the second issue is worst outcome for that issue for all negotiators. In that case, the mediator can find the intended mediation rule if the preferences satisfy monotonicity, deal-breakers and partial logrolling assumptions.

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APPENDIX A

Proof of Theorem 1

Proof. *Proof of "if" part:* According to the outcomes on the right hand side assumptions of the theorem, the mediation rule f always gives more preferred bundles than (o_x, o_y). Thus, f is individually rational.

For efficiency, suppose that both of the least accepting clone and the dissimilar negotiator accept all alternatives in X. Then, by assumption *(iii)*, $\mu(\theta_c^{x_m}, \theta_d^{x_1}) \in \mathbf{B}$, say b. Suppose that the mediation rule f gives a different bundle, a, from the set of logrolling bundles, $\mathbf{B}$. In such case, one of the negotiators will certainly be worse off. The reason is that preferences of the negotiators over logrolling bundles are diametrically opposed, i.e., if $a\ R_c\ b$, then $b\ R_d\ a$ for all bundles in $\mathbf{B}$. Suppose now that f gives (o_x, y) where $y \in Y \setminus \{o_y\}$. In that case, each of the negotiators will be worse off by deal-breakers assumption. Suppose that f gives bundle c which is neither a logrolling bundle nor consists of the outside option. The consistency assumption puts no restriction on comparing such a bundle with a logrolling bundle. Thus, there exists a negotiator $i \in \{c, d\}$ and a consistent preference ordering R_i such that $b\ P_i\ c$. Then, the negotiator i will be worse off. We can apply this to all types that accept less alternatives in X.

Moreover, If there does not exist mutually acceptable alternatives in$X$, f gives (o_x, y). Suppose f gives some other bundle b, then one of the negotiators will be worse off by deal-breakers assumption. Thus, f is efficient.

To find the mediation rule f is strategy-proof, without loss of generality, we can consider the least accepting clone's deviations. There exist three cases: $j < k$, $j = k$, and $j > k$. Suppose that the least accepting clone's type is $\theta_c^{x_j}$ with $j < k$. Since there is not any mutually acceptable alternative in X, f gives (o_x, y). If $\theta_c^{x_j}$ deviates to some $x_{j'}$ with $j' = k$, then he gets $(x_{j'}, y_{m+1-j'})$. However, $\theta_c^{x_j}$ will be worse off, by deal-breakers assumption, because $x_{j'}$ is an unacceptable alternative for him. If $\theta_c^{x_j}$ deviates to some $x_{j'}$ with $j' > k$, then f gives some $x_c \in X$ where $c \in \{k, .., j'\}$ by adjacency. However, all $x_{j'}$s are unacceptable for $\theta_c^{x_j}$ and he will be worse off by deal-breakers assumption. Thus,

$\theta_c^{x_j}$ with $j < k$ has no profitable deviation from $\mu(\theta_c^{x_j}, \theta_d^{x_k})$. For the second case, assume that the least accepting clone's type is $\theta_c^{x_j}$ with $j = k$. In that case, f gives (x_j, y_{m+1-j}) and there exist only one mutually acceptable alternative, x_j, in X. If $\theta_c^{x_j}$ deviates to some $x_{j'}$ with $j' < k$, then f gives (o_x, y) and this bundle is clearly less preferred than (x_j, y_{m+1-j}) for $\theta_c^{x_j}$, by deal-breakers assumption. If $\theta_c^{x_j}$ deviates to some $x_{j'}$ with $j' > k$, f gives some $x_c \in X$ where $c \in \{k, .., j'\}$ by adjacency. Then, $\theta_c^{x_j}$ gets either the same bundle (x_j, y_{m+1-j}) or a bundle with an unacceptable alternative for him, i.e., he is either indifferent or worse off in that case. Thus, $\theta_c^{x_j}$ with $j = k$ has no profitable deviation from $\mu(\theta_c^{x_j}, \theta_d^{x_k})$.

Before the last case, we need to specify some characteristics of the mediation rule f. We can construct a region called value region for a logrolling bundle in any mediation rule.

Definition 7 *Given a mediation rule f,* ***the value region of a bundle*** $a \in B$, $V(a)$, *consists of bundles in the intersection of all columns lower than a and all rows higher than a.*

Lemma 1 *For two bundles on the main diagonal, a and b, the adjacent rule that is described in Theorem 1 safisfies:*

(i) a does not exist outside of $V(a)$

(ii) a and b do not exist in $V(a) \cap V(b)$ together

(iii) If a is above b on the same column or a is left of b on the same row, then a is above b on the main diagonal.

Proof.

(i) According to the construction of the rule f, any bundle on the main diagonal continues to be seen only left of itself or below itself. Since all logrolling bundles are unique in the main diagonal, it could not exist outside $V(a)$.

(ii) let $a = f_{k,k}$ and $b = f_{r,r}$ with $k < r$ on the main diagonal. We can construct the proof of condition (ii) in three cases. First, suppose a $(f_{r_a,s})$ and b $(f_{r_b,s})$ are on the same column in $V(b) \cap V(a)$ and a is above b. Then, $r \leq r_a < r_b \leq m$. By adjacency, if we move along row r from $b = f_{r,r}$ to $f_{r,s}$, the left bundles are ranked either the same or higher. Then, at column s, if we move from $f_{r,s}$ to $a = f_{r_a,s}$, the below bundles are ranked either the same or higher, again by adjacency. Thus $a \rhd b$ must hold by transitivity. It contradicts with the fact that $b \rhd a$ which must hold because b is below a at the column s.

Second, suppose $a = f_{s,c_a}$ and $b = f_{s,c_b}$ are on the same row in $V(b) \cap V(a)$ where a is the left of b, i.e., $1 \leq c_a < c_b \leq k$. By using the same argumentation with the first case of (ii), if we move along column k from a to $f_{s,k}$, $f_{s,k}$ is ranked either the same or higher than a, by adjacency. Then we move from $f_{s,k}$ to $b = f_{s,c_b}$ along row s, b is ranked either the same or higher than $f_{s,k}$, by adjacency. Thus, $b \rhd a$ must hold by transitivity. This contradicts with the fact that $a \rhd b$ must hold since a is the left of b.

Lastly, suppose a and b are neither on the same column nor row. Without loss of generality, let $a = f_{r_a,c_a}$ and $b = f_{r_b,c_b}$ where $r \leq r_a < r_b \leq m$ and $1 \leq c_a < c_b \leq k$. In such case, if we move from $b = f_{r,r}$ to f_{r,c_a} along row r, i.e., if we move from right to left, f_{r,c_a} is ranked either the same or higher than $b = f_{r,r}$, by adjacency. Thus, $a \rhd b$ holds. On the other hand, if we move from $a = f_{k,k}$ to $f_{r_b,k}$ along column k, $f_{r_b,k}$ is ranked either the same or higher than $a = f_{k,k}$. Then, continue to move to the left, to $b = f_{r_b,c_b}$, $b = f_{r_b,c_b}$ is ranked either the same or higher than $f_{r_b,k}$. Thus, $b \rhd a$ holds. Then, we have a contradiction.

(iii) This condition follows from condition (ii). If a and b are on the same column and a is above b, we need to show that a is above b on the main diagonal. Suppose that a is below b on the main diagonal for a contradiction. Let $b = f_{k,k}$ and $a = f_{r,r}$ where $k < r$. Then a and b cannot be in $V(b) \cap V(a)$ together, by (ii). Moreover, a cannot be in $V(b) \setminus V(a)$, by (i). Thus, there is not any possibility for them to be in the same column, contradicts with the assumption at the beginning. The same reasoning is valid for the case that a and b are on the same row.

■

Finally, suppose that the least accepting clone's type is $\theta_c^{x_j}$ with $j > k$ and let c $\in$ **B** the bundle when $\theta_c^{x_j}$ reports his type truthfully. If $\theta_c^{x_j}$ deviates to some j' with j' $<$ k, then he will be worse off by getting (o_x, y) (deal-breakers assumption). If textit$\theta_c^{x_j}$ deviates to j' with $j > j' \geq k$, he gets some bundle, say a. Then, a is above c because of $j > j'$. Then, a appears above c on the main diagonal, by condition (iii) of Lemma 1. By logrolling, $c \succeq_c a$. Thus, deviating from bundle c to bundle a is not profitable for $\theta_c^{x_j}$. If $\theta_c^{x_j}$ deviates to some j' with $j' > j$ to get a bundle called b. b appears below c on the main diagonal, because of condition (iii) of Lemma 1. By conditions (i) and (ii), bundle c must be in region I, while bundle b must be in region II. Then, no matter where bundle c is in region I, bundle b has an unacceptable alternative of x for θ_c. Then $\theta_c^{x_j}$ will not deviate from c to b. Hence, f is strategy-proof.

Proof of "only if" part: First of all, we need to prove that if the mediation rule f is individually rational, strategy-proof and efficient, then the least accepting clone is binding for us. Since f is individually rational, it never gives a bundle worse than (o_x, o_y). Thus, f is individually rational for all other types of clones. Since f is efficient for the least accepting clone, if there exists an improvement for any other type of clone, then θ_d will be definitely worse off. Thus, f cannot provide any pareto improvement for any of them. Finally, f is strategy-proof for the least accepting clone. Without loss of generality, we can pick one of the clones that has a different type. Then we should consider three cases: If he deviates to a more accepting type, the outcome does not change, because f gives a bundle which has a mutually acceptable option in X, to preserve efficiency. If he deviates to a less accepting type but more than the least, the outcome does not change by the same reason. If he deviates to a type which accepts less than the least, since the rule is strategy-proof for for the least accepting clone at the beginning, he cannot be better off by lying. Thus, the least accepting clone is binding. Then, we can start to prove the conditions:

(i) Since f is individually rational and efficient, and since there does not a mutually acceptable alternative for the least accepting clone $\theta_c^{x_j}$ and $\theta_d^{x_k}$ where $j < k$, f should give o_x for the issue X. Because, any outcome in$X$ other than o_x will be unacceptable for one of the negotiators and contradicts with the deal-breakers assumption. Since f is strategy-proof, it should give the same y in issue Y. Without loss of generality, suppose f gives $y' \neq y$ at $\mu(\theta_c^{x_j}, \theta_d^{x_k})$ where $j < k$ for a contradiction. Then, one of the negotiators should strictly prefer y' more than y, because of the diametrically opposed preferences. If $\theta_c^{x_j}$ prefers y' more than y, then $\theta_d^{x_k}$ prefers y more than y'. The latter one can deviate to $\theta_d^{x_{k'}}$ where $j < k'$ to get (o_x, y) and will be better off. On the other hand, if $\theta_d^{x_k}$ prefers y' more than y, $\theta_c^{x_j}$ can deviate to $\theta_c^{x_{j'}}$ where $j' < k$ to get (o_x, y) by the same reasoning. Thus, the mediation rule f, should give (o_x, y) at every type bundles, i.e., $(\theta_c^{x_j}, \theta_d^{x_k})$ with $j < k$.

(ii) Since f is individually rational and efficient, there exist a unique mutually acceptable alternative of issue X where $j = k$ which is x_j and f should give that. For y, suppose f gives some $y' \neq y_{m+1-j}$ at $(\theta_c^{x_k}, \theta_d^{x_k})$ for a contradiction. The bundle (x_k, y') is no longer in $\mathbf{B}$. Since there is no restriction on how to rank bundles which are not in $\mathbf{B}$, we can always find a bundle which is ranked higher than (x_k, y'). Thus, to preserve strategy-proofness, $f_{k+1,k}$ should also be (x_k, y'). By the same arguments above, $f_{k+1,k+1}$ should also be (x_k, y'). However, x_k is not mutually acceptable for $f_{k+1,k+1}$. Hence, it contradicts with the rule that is individually rational and efficient. This proof holds for any $k \in \{1, ..., m\}$.

(iii) We want to prove that if there is an individually rational, strategy-proof and efficient mediation rule f, then the rule is adjacent, i.e., $f_{j,k} \in \{f_{j-1,k}, f_{j,k+1}\} \subset \mathbf{B}$ where

$j < k$, and there exist a precendence order on the elements of $\mathbf{B}$ which is complete, transitive and strict. First we have a lemma to prove:

Lemma 2 *Suppose $f_{j,k} \in \{f_{j-1,k}, f_{j,k+1}\} \subset \boldsymbol{B}$ holds where $j < k$. Let a and b are bundles in $\boldsymbol{B}$. If a is above b on the main diagonal, then a is above b on all diagonals that a and b are both on.*

Proof. Suppose a is above b on the main diagonal and a and b appear on the diagonals up to diagonal t According to the adjacent rule, a and b can move to the left or to the down. First, suppose a moves to the left on the main diagonal. Then, regardless of how b moves, a is still above b on the second diagonal. Second, suppose a moves to the down. Then, there are two cases: 1) a and b are on the same row, and 2) a is above b. If the first case occurs, then b should drop down to continue existing. Thus, a is above b again. If the second case occurs, then it is obvious. We can do this up to diagonal t, which is the last diagonal than a and b both appear. ■

Now we can do the condition (iii)'s proof step by step:

1. Adjacent rule: We will show that individually rational, efficient, and strategy proof mediation rule f gives $f_{j,k} \in \{f_{j-1,k}, f_{j,k+1}\} \subset \mathbf{B}$ where $j > k$. For induction, we should first prove that this statement holds for $j = k+1$. Because we know that the elements at the main diagonal are all belong to $\mathbf{B}$. Suppose $f_{j,k} \notin \{f_{j-1,k}, f_{j,k+1}\} \subset \mathbf{B}$ where $j = k+1$ for contradiction. Since the rule f is individually rational and efficient, it should give some $x \in \{x_k, x_{k+1}\}$. Suppose f gives some y. If $f_{j,k} = (x_k, y)$, $y \succ_c \widehat{y}_k$ should be satisfied for strategy-proofness of f, because otherwise θ_c^{k+1} deviates to θ_c^k and will be better off by getting $f_{j-1,k} = (x_k, \widehat{y}_k)$. Suppose $y \succ_c \widehat{y}_k$. With this, deviation will be reversed, i.e., θ_c^k deviates to θ_c^{k+1} to get (x_k, y). This contradicts with strategy-proofness of f. If $f_{j,k} = (x_{k+1}, y)$, $y \succ_d \widehat{y}_{k+1}$ should be satisfied for strategy-proofness of f, since otherwise θ_d^k deviates to θ_d^{k+1} to get $(x_{k+1}, \widehat{y}_{k+1})$. In this case, however, θ_d^{k+1} gets $(x_{k+1}, \widehat{y}_{k+1})$ and will deviate to θ_d^k to get (x_{k+1}, y). This contradicts with strategy-proofness of f. Since we could not find such y, $f_{j,k} \in \{f_{j-1,k}, f_{j,k+1}\} \subset \mathbf{B}$ must hold for $j = k+1$.
By induction, suppose $f_{j,k} \in \{f_{j-1,k}, f_{j,k+1}\} \subset \mathbf{B}$ holds up to $j = k+b$. We should show that it holds for $j = k+b+1$. Suppose for a contradiction that $f_{j,k} \notin \{f_{j-1,k}, f_{j,k+1}\} \subset \mathbf{B}$ where $j = k+b+1$. First consider $f_{j,k} \in \mathbf{B}$ and WLOG, consider θ_c. There exist three cases: first, suppose $f_{j,k+1} \succsim_c f_{j-1,k} \succsim_c f_{j,k}$. We can find a consistent preference relation which satisfies $f_{j-1,k} \succ_c f_{j,k}$. Then θ_c^j will be better of by stating himself as θ_c^{j-1}, contradicting with strategy-proofness of f. Second, suppose $f_{j,k} \succsim_c f_{j,k+1} \succsim_c f_{j-1,k}$. Because of the same reason above, there exist a consistent preference relation which satisfies $f_{j,k} \succ_c f_{j-1,k}$ so θ_c deviates from

$f_{j-1,k}$ to $f_{j,k}$ to be better off, contradicting with strategy-proofness of f. Third, suppose $f_{j,k+1} \succsim_c f_{j,k} \succsim_c f_{j-1,k}$. Because of the same reason above, we can find a preference relation such that $f_{j,k} \succ_c f_{j-1,k}$, so θ_c wants to deviate from $f_{j-1,k}$ to $f_{j,k}$. Thus, $f_{j,k}$ cannot be in $\mathbf{B}$. Consider $f_{j,k} \notin \mathbf{B}$.

2. Precedence Order: Now I know that the desired mediation rule f should give $f_{j,k} \in \{f_{j-1,k}, f_{j,k+1}\} \subset \mathbf{B}\ \forall j > k$. For a well-defined mediation rule the mediator needs an order on these pairs of $\{f_{j-1,k}, f_{j,k+1}\}$. First I will show that the order is asymmetric. Suppose that $a \triangleright b$ and $b \triangleright a$ hold together for a contradiction. Suppose t is the smallest diagonal which a and b are adjacent to each other at and without loss of generality, $f_{j-1,k-1} = a$ and $f_{j,k} = b$, i.e., a is above b. This means $b\ R_c\ a$ when they are both acceptable for θ_c. Suppose that $a \triangleright b$ at t. By assumption, there exists some $t' > t$ which a and b are adjacent to each other at, $f_{p-1,r-1} = a$ and $f_{p,r} = b$, and $b \triangleright a$ holds, i.e., $f_{p,r-1} = b$. By lemma 2 and adjacent rule principle, $p > j$. a and b are both acceptable for $\theta_c^{x_{p-1}}$. Thus, $\theta_c^{x_{p-1}}$ can deviate to $\theta_c^{x_p}$ and get b instead of a and would be better off by lying. I contradicts with the strategy-proofness of the rule.

Now I will show the transitivity of the precedence order $\triangleright$. Suppose $a \triangleright b$ and $b \triangleright c$ hold alongside with $c \triangleright a$ for a contradiction. WLOG, assume that t is the first diagonal which a and b are adjacent to each other at and b is above a, $f_{j-1,k-1} = b$ and $f_{j,k} = a$.

Suppose that b and c are adjacent to each other and b is above c at some $t' > t$. If c exists in t', it should exist and below b at t. Since a is adjacent to and below b at t, c should be below a at t. By assumption of $a \triangleright b$, b can only move to the left and by $c \triangleright a$, c can move to the left and below. Since a is in between b and c at t and b and c cannot move towards each other, b and c cannot be adjacent in some t'.

Suppose that b and c are adjacent to each other and b is below c at some $t' > t$, say $f_{p,r} = b$ and $f_{p-1,r-1} = c$. Suppose a is adjacent to c at $t'' > t'$. Otherwise is not possible, because b is in between c and a at t. c is surely above row $p-1$ because $b \triangleright c$. $p < j$ because of the value region of b and a is surely below row j. At t'', a and c cannot be adjacent.

Suppose that b and c are adjacent to each other and c is above b at some $t' < t$, say $f_{p,r} = b$ and $f_{p-1,r-1} = c$. If a is at t, then it should be at t' by lemma 2, and a should be below b at t'.By the assumption of $c \triangleright a$, c and a should be adjacent to each other at some t''. $t'' < t'$ is not possible, because b is in between c and a at t'. Then it must be the case that $t'' > t$. However, by $b \triangleright c$, c can only move to the left and cannot exist below row $p-1$ at t''. By $a \triangleright b$, a should exist below row p at t''.

Thus, c and a cannot be adjacent to each other. The last point which is b and c are adjacent to each other and c is below b at some $t' < t$ can be proved easily with the same arguments.

■

APPENDIX B

Proof of Theorem 2

Proof. *Proof of "if" part:* If there does not exist a mutually acceptable alternative, the rule should give o_x, because otherwise someone leaves the table. If there exist one or more mutually acceptable alternative, say x, it is sure that this alternative is better than o_x. Since o_y is inferior the mediator can pick an alternative $y \in Y$ and it is better than o_y for all negotiators. (o_x, y) R_i (o_x, o_y) $\forall i$ and $(x, t(x))$ R_i (o_x, o_y) $\forall i$ by monotonicity. Thus, the mediation rule characterized by the three points of the theorem 2 is individually rational.

For efficiency, take the case where each negotiator accepts all alternatives. Then, the mediation rule f should give $b \in A(\theta, t)$. Suppose for a contradiction, f gives $b' \in A(\theta, t)$ and b R_i b' for at least one negotiator i, because of partial logrolling. Thus b' R_i b cannot be true for all i. Suppose for a contradiction, f gives (o_x, y), the mutually acceptable alternative x in b is better than o_x for all i. y is random in (o_x, y) and all $y \in Y \backslash \{o_x\}$ is efficient, there exists a negotiator i, $y' \in b$ θ_i y. Thus, b R_i (o_x, y) for that i. Suppose for a contradiction that f gives $c = (x', y') \notin A(\theta, t)$. The mediation rule does not put a restriction on how b and c are compared. Thus, there exist a negotiator i and R_i such that b P_i c. Thus, that i will be worse off. This process can be applied when there exist a mutually acceptable alternative. Suppose there does not exists a mutually acceptable alternative and f gives $(x', y') \neq (o_x, y)$. There exists a negotiator i who does not accept x'. By deal breakers and monotonicity, that i will be worse off. Thus, the mediation rule f that is characterized by theorem 2 is efficient.

For strategy-proofness, we will check each scenario. WLOG, suppose that the outcome is (o_x, y) and negotiator i can manipulate the system. If negotiator i lies and reveals her type as a less accepting type than her true type, she will get (o_x, y) again. Thus, she does not manipulate. If she reveals her type as a more accepting type than her true type, she can get either (o_x, y) or $(x, t(x))$. In former case, she does not need to deviate. In latter, since she ranks o_x higher than x, by deal breakers, she will be worse off. Thus, again she does not manipulate.

WLOG, suppose that the outcome is $(x,t(x))$ and there exists only one mutually acceptable alternative and negotiator i can manipulate the system. If she lies and reveals her type as a less accepting type than her true type, she will get (o_x,y). Since x is the mutually acceptable alternative, we know that negotiator i ranks x higher than o_x. Thus, by deal breakers and monotonicity, $(x,t(x))\ R_i\ (o_x,y)$ and she will be worse off. Thus, she does not manipulate the system in that way. If she reveals her type as a more accepting type, she can get either $(x,t(x))$ or $(x',t(x')) \in A(\theta,t)$. In former case, she does not need to deviate. In latter, since $x' \notin A(\theta_i)$, by deal breakers, she will be worse off. Thus, she does not manipulate.

WLOG, suppose that the outcome is $(x,t(x))$ and there exist more than one mutually acceptable alternative and negotiator i can manipulate the system. If she lies and reveals her type as a less accepting type than her true type, then she can get (o_x,y), $(x,t(x))$ or $(x',t(x')) \in A(\theta,t)$. In the first case, she will be worse off by monotonicity. Thus she does not manipulate in this way. In the second case, she gets exactly same bundle, thus she does not need to deviate. In the third case, she will deviate if and only if $(x',t(x'))\ R_i\ (x,t(x))$. By partial logrolling, it is satisfied if and only if $x\ \theta_i\ x'$ and $x,x' \in A(\theta_i)$. Thus, any less accepting type of negotiator i than x does not accept x'. It is a contradiction. Thus she does not manipulate in that way.

If negotiator i lies and reveals her type as a more accepting type than her true type, she can get either $(x,t(x))$ or $(x',t(x')) \in A(\theta,t)$. In former case, she does not need to deviate. Thus, she does not manipulate the system in that way. In latter case, if $x' \notin A(\theta_i)$, then by deal breakers, she does not manipulate the system in that way. If $x' \in A(\theta_i)$, then $(x',t(x'))\ R_i\ (x,t(x))$ should be satisfied for a manipulation. By partial logrolling, $x\ \theta_i\ x'$ and $x'\ \theta^*\ x$ should be satisfied. However, if $x'\ \theta^*\ x$ is satisfied and $x' \in A(\theta_i)$, f should give $(x',t(x'))$, instead of $(x,t(x))$. It is a contradiction. Thus, she does not manipulate in that way. Thus, the mediation rule that is characterized by theorem 2 is strategy-proof.

Proof of "only if" part: By individual rationality and efficiency, if $A(\theta) = \emptyset$, the outcome bundle should be (o_x,y). Since there is no mutually acceptable alternative in X, o_x is a better outcome for at least one negotiator i for all $x \in X$. All $y \in Y$ are efficient for the outcome by construction. By strategy-proofness, the rule cannot give different y's at different type profiles with $A(\theta) = \emptyset$. Thus, for all θ with $A(\theta) = \emptyset$, the mediation rule f should give (o_x,y).

By efficiency, individual rationality and consistency of preferences, if $A(\theta)$ is singleton and is equal to x, then $f^x = x$. Suppose $\theta_i^{x_i}$ gets (x_i, y). $\theta_i^{x_{i+1}}$ gets either $(x_i, t(x_i))$ or $(x_{i+1}, t(x_{i+1}))$. In former case, by monotonicity, y should be equal to $t(x_i)$. In latter case, by strategy-proofness, y should be equal to $t(x_i)$. Thus, for all θ with singleton $A(\theta)$, the mediation rule f should give $(x, t(x))$.

By efficiency and individual rationality, if $|A(\theta)| > 1$, then, $f^x = x$ where $x \in A(\theta)$. WLOG, a negotiator can take the same bundle with one less accepting type of her, say (x, y) or she can take a bundle with her different acceptable alternative, say (x', y'). If she takes the former, there is no problem. If she gets the latter, then by strategy proofness, she should rank (x', y') over (x, y) while we know that she ranks x over x'. The mediator should gather all problematic types such as the more accepting type of the negotiator above in a set and problematic couples such as (x, x') in another set and construct a partial logrolling condition according to these substitution need of strategy-proofness. Then, the third part of the theorem will be satisfied.

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