How Do Local Interaction Patterns Affect the Global Behavior of a Community: Schelling Models Revisited

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HOW DO LOCAL INTERACTION PATTERNS AFFECT THE GLOBAL BEHAVIOR OF A COMMUNITY: SCHELLING MODEL REVISITED

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Bireylerin Yerel Etkileşimleri Toplumun Küresel Davranışlarını Nasıl Etkiler?

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Özet

Bu tezde, Schelling modellerini analiz ediyoruz ve bu modeller için eklentiler öneriyoruz. Schelling modellerinde, Bernoulli dağılımı kullanılarak kare örgüye iki ajan tipi (X ve Y) yerleştirilir. Her ajanın tanımlanmış bir mahallesi vardır ve eğer bir ajanın mahallesinde aynı tip komşularının oranı, ajanın eşiğinden daha küçük ise, o ajan mutsuzdur. Mutsuz ajanlar tiplerini değiştirir ve her ajan mutlu olduğunda, model bir dengeye ulaşır. Ajanların minimum enerji seviyesine sahip oldukları denge durumuna temel durum denir ve biz bu durumu konsensüs olarak adlandırıyoruz. Kare örgülerde, topluluklar belirli koşullar için bir konsensüse (tüm ajanların ayn tip olduu) ulaamazlar. Kare örgülerdeki bağlantıları yeniden düzenleyerek ajanların bağlantı sayılarını değiştirmenin toplulukların ortalama en kısa yol uzunluğunu ve kümelenme katsayısını azaltarak bir konsensüse ulaşmasına yardımcı olabileceğini gördük. Ayrıca, bu tezde şüpheli ajanları temsil eden yeni bir ajan tipi (XY) ve iki farklı model (DC and PC modelleri) yaratıyoruz. DC modellerinde, ajanlar komşularında şüphe yaratabilirler ve topluluklar her bir ajanın şüphenin yararına sahip olduğu bir konsensüse ulaşabilirler. Öte yandan, PC modellerinde ajanlar komşularına şüphenin yararını veremez ve sonuç olarak, şüpheli ajanlar ağdan kaybolur. Son olarak, bireylerin kendi çevrelerinde çeşitlilik arayışında bulundukları yeni bir yaklaşım sunuyoruz ve labirent desenlerinin, ajanları uzak çevrelerinde çeşitlilik arayışına girdiğinde ortaya çıktını gösteriyoruz.

How Do Local Interaction Patterns Affect the Global Behavior of a Community: Schelling Model Revisited

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Keywords: Schelling models, networks, random links, ground state, consensus, the benefit of the doubt, diversity seeking, maze patterns

Abstract

In this thesis, we analyze Schelling models and propose extensions for these models. In Schelling models, two agent types (X and Y) are placed to a regular square lattice using Bernoulli distribution. Agents have their defined neighborhoods and if the percentage of the same type neighbors of an agent is smaller than its threshold, the agent is unhappy. Unhappy agents change their types and when every agent in the network is happy, the model reaches an equilibrium. Equilibrium state where agents have the minimum energy level is called ground state and we name ground state, consensus. In the square lattice, communities cannot reach a consensus (where all agents are the same type) for specific conditions. We found that changing node degrees by rewiring the links in the square lattice can help communities to reach a consensus by decreasing average shortest path length and clustering coefficient Moreover, we introduce a new agent type (XY) which represents doubtful agents and two different models (DC and PC models). In DC models, agents can give the benefit of doubt to their neighbors and communities can reach a consensus where every agent has the benefit of the doubt. On the other hand in PC models, agents cannot give the benefit of the doubt and consequently, doubtful agents disappear from the network. Lastly, we present a new approach to diversity seeking behavior where individuals seek diversity in their vicinities. We show that maze patterns emerge when agents seek diversity in their distant vicinities.

Contents

List of Figures

List of Tables

Chapter 1

Introduction

People have different genders, nationalities, religions, social and political views. As Easley and Kleinberg [8] stated, individuals tend to be friends with people who have similar characteristics with themselves and this behavior is called homophily. These differences and homophily led to separation between people through the whole history. Homophily principle forces people to link with the people from same race, ethnicity, etc. and this cause people with the same attributes to cluster. Also, Kossinets and Watts [16] claim that similar pairs more likely to form links with each other.

Nobel Prize Winner Thomas Schelling's works are one of the first studies in the literature to analyze homophily behavior and segregation on the modern cities [22] [23]. Schelling analyzed the racial segregation in Chicago and stated how individual incentives affect global outcomes [24]. He claimed that even though the individuals do not attempt to live segregated with people from different races, to satisfy the desire's of everyone, society eventually became segregated. Schelling models can be used to understand not only the racial segregation but any distinctions which are twofold, exhaustive and recognizable [23].

Schelling presented two different models. The first one is an agent based spatial model which comprises a basis for this thesis. The second model is a non-spatial "Bounded Neighborhood model" where all agents belong to the same neighborhood. Bounded Neighborhood model is not covered in this thesis. In Schelling's spatial models there are two agent types (denoted by X and Y) randomly placed to a network. Every agent has its defined neighborhood and utility function which defines whether the agent is happy or unhappy. Unhappy agents move to empty nodes on the network in each time step, and simulation ends when all agents satisfy their utility functions. In 2001, Young [32] introduced "Kawasaki dynamics" into Schelling models. In Young's model, there are no empty nodes in the network and two unhappy agents from different types change their places in order to satisfy their utility functions. Another approach that Young contributed to the literature is the noise. Noise means that agents can act against their benefits with a small probability. These models with a noise are called non-zero temperature models. In this thesis, zero temperature models without a noise are analyzed. Afterwards, Barmpalias put forward "Glauber dynamics" in which agents do not change their places pairwise instead of this unhappy agents change their types [3][4]. Glauber dynamics employed in the Schelling models suggest that unhappy agents move out of the boundaries of the network and agent who can be happy in that defined neighborhood (which is an opposite type agent) moved to the place of the unhappy agent.

Schelling models are discrete time agent based models, where agents act according to the rules in their defined neighborhood at each time step (denoted as iterations). Schelling models are also one of the most famous models of self-organizing behavior. Across the science, there are models similar to the Schelling models. These models are Ising models [27], Cellular Automata [31], Hopfield networks [14], Spin Glass models [1] and cascading phenomena models [15]. In the Ising models where phase transitions are analyzed, two agent types are randomly distributed to a lattice denoted by "up-spins" and "down-spins" move according to their utility functions. Stauffer and Solomon elaborately analyzed the similarities between Schelling models and Ising models [28]. Ising Models will be mentioned in the following chapters. Another model which will be mentioned in the following chapters is the Cellular Automata. Hegselmann [12] mentioned the resemblances of the Cellular Automata and Schelling's model.

Even though it has been almost 50 years since Schelling first proposed his models, mathematical proofs and underlying factors in the segregation are still not entirely analyzed and understood. First analytical explanations made by Young [32] who used stochastic processes to analyze Schelling models in 1 dimension (ring lattice). Afterwards, Zhang developed Young's work to 2-dimensional networks (square lattice) by using the stochastic evolutionary game theory techniques and analyzed the long-term dynamics of the models [33]. Zhang continued his analytical works and showed that segregation emerges even in integrationist societies [34]. Furthermore, in 2011 he published another paper in which the tipping points of Schelling's model are analyzed [35]. However, all these analytical studies are applied to the non-zero temperature (perturbed) models. First proofs of zero temperature (unperturbed) Schelling's Model are introduced by Brandt, Immorlica, Kamath and Kleinberg [6] in 2012. Then Barmpalias, Elwes and Lewis-Pye extended this proofs on 1-dimensional Schelling's model [4] [3], and presented the first rigorous analysis of 2-dimensional models [3].

1.1 Our Contribution

Our first and most important contribution to the literature is analyzing Schelling models with Glauber dynamics in different network topologies. Fagiolo, Valente and Vriend [9] studied Schelling's spatial model for different network structures. We extend their research by removing empty nodes from the network and employing Glauber dynamics. Also, underlying mechanisms of segregation in the small world and random networks are explained. These analyses are conducted for both Ising models and Schelling models for different unhappy agent type changing dynamics. In addition to these, sociological meaning of ground state [27] (in the thesis denoted as consensus state) used in the physics model is analyzed, and we provide an understanding on how societies reach a consensus equilibrium state. Moreover, we based our models on mutable characteristics, and we were able to study the diffusion of social norms in the societies. Our second contribution to the literature is introducing the individuals who give their neighbors the benefit of doubt. These individuals analyzed using two different models which we propose, Doubtful Community models (DC models) and Persistent Community models (PC models). These two models are investigated under different network topologies. Lastly, we contribute to the literature by showing that maze patterns emerge when individuals seek diversity in their vicinity.

Chapter 2

Model

2.1 Model Properties

In the models analyzed in this chapter $n\ x\ n$ square lattice is employed, where n denotes the length of an edge of a square. Since square lattice is the most suitable network type to represent spatial (physical) settlements of neighborhoods and cities, they are commonly used in the literature starting from Schelling's model [22] until recent studies. Also, in the physics models mentioned in the previous chapter, square lattice represents the 2D surface. For the square lattices employed in this thesis n is chosen large enough to ensure that neighborhoods are small enough to not affect the whole network, and small enough to decrease computational complexity.

Square lattices consist of $n\ x\ n$ nodes and they are completely ordered. This means that all the nodes in the square lattice have the same degree k. Node degree shows how many edges that node has. In order to have completely ordered square lattice, periodic boundary conditions are used in the models. Periodic boundary conditions provide a connection between the nodes located at the edges of the square lattice. For instance, periodic boundary conditions provide connection between the nodes in the right edge and the nodes in the left edge of the square lattice. Demonstration of the periodic boundary conditions can be seen in the Appendix A. These conditions ensure that the nodes in the boundaries of the network have the same degree with the other nodes.

Node degrees (number of neighbors) determined by Moore neighborhood. In the literature especially in the Ising models, Von Neumann neighborhood is frequently used. The difference between these two neighborhood definitions can be seen in Appendix B. Moore neighborhood simply creates $w\ x\ w$ square, where w is the neighborhood range and denotes the neighborhood of the node in the center of this square. Equation 2.1 shows how the number of neighbors is calculated based on

Moore Neighborhood. Therefore when w is equal to 1, the square that defines the neighborhood consists of 9 nodes. and if the node in the center, which neighborhood definition is based on, is subtracted each node has 8 neighbors. Consequently, when w is 2 agents have 24 neighbors and so on.

$$k = (2w + 1)^2 - 1 \tag{2.1}$$

For each node in the network either X or Y are assigned using Bernoulli distribution with parameter 0.5. X and Y represent two different stands on an issue. For instance, X denotes people who recycle while Y denotes people who do not recycle. In the thesis X's and Y's are assigned to nodes called agents. For a single replication of the model, initial percentages of X's and Y's are not necessarily equal but their excepted value for multiple replications is 50%. Because there are no differences between the agent types.

After creating the network and assigning the initial agent distribution, the algorithm starts. The algorithm has two steps. In the first step, the algorithm checks all agents randomly whether they are happy or unhappy. Then unhappy agents are placed to a list. In the second step, unhappy agents are randomly chosen one by one from the list, and if the chosen agent is still unhappy it switches its type. For instance, if the chosen agent is X and it is still unhappy, the agent changes its type to Y. One iteration of the algorithm contains these two steps. The algorithm stops when there are no unhappy agents to place to the list in the first step. This state of a network is called equilibrium state. The question "why algorithm does not stop in the first iteration, if all the unhappy agents in the network change their type?" may arise. It is because while an agent changes its type and becomes happy, this change can make other agents unhappy.

Definition of happiness is as follows; an agent is happy if the percentage of same type agents in its neighborhood is equal or greater than its threshold (t), and an agent is unhappy if the percentage of same type agents in its neighborhood is smaller than its threshold. For an agent X, same type agents are the X agents and for an agent Y, same type agents are the Y agents. Schelling models with different thresholds are elaborately analyzed in the literature [20]. In this thesis, thresholds are 0.5. This means that an agent is unhappy if the number of same type agents is smaller than the number of different type agents in its neighborhood. In case of equality in the numbers of different agent types in the neighborhood, agents use themselves as tie-breakers and they become happy. For instance, if an X agent has 4 X and 4 Y neighbors then the X agent is happy. However, in the literature there are models using different rule in case of equality. For example in the Ising models [27], agents stay as same type with a 0.5 probability and change their types with a 0.5 probability when there is equality in the numbers of different agent types.

In this thesis, Glauber dynamics are adopted. As mentioned above, in the Glauber dynamics unhappy agents are considered as moved out of the network boundaries (community) and an agent who can be happy in the neighborhood moved to that node [3]. Even though the algorithmic approach used is the same as the literature, here different social approach used. In this thesis, unhappy agents do not considered as moved out of the network instead they considered as change their types. Since the models in this thesis based on mutable characteristics, unlike Schelling's model which is based on unmutable characteristics such as race, it is assumed that unhappy agents change their views on an issue and become different type agents.

The question "why the models in this thesis are based on mutable characteristics" needs further explanation. In this thesis, how social norms evolve and how communities reach a consensus are analyzed. Therefore, models are based on mutable characteristics which help to understand how individuals change their opinions and reach a consensus. Consensus term presented in this thesis is the sociological equivalent of the ground state term used in the physic models. Ground state is analyzed in the Ising models and it represents the minimum energy level in which all spins are in the same direction (up or down) and totally homogenous. Consensus represents the network where all agents belong to the same agent type, and consequently community in which all people have the same view on a certain issue. View of the majority of the community on a certain issue is the social norm. Therefore to reach a consensus, social norm must be adopted by every member of the community not only by the members of some local neighborhoods. Since the thresholds in this thesis are 0.5, if an agent is unhappy than the other agent type is in the majority in the neighborhood and their views represents the social norm of the neighborhood [19]. Suppose an unhappy Y agent and Y denotes people who do not recycle. X agents who are in the majority in the neighborhood represent people who do recycle, and consequently social norm in this community is recycling. This norm put a social pressure on the unhappy Y agent, and to avoid social rejection and to become member of a community the Y agent change its type [5] [7].

Another point that needs further explanation in the algorithm is how agents in the unhappy list change their types. Here, agents in the unhappy list change their types one-by-one (asynchronously). The reason for asynchronous change is in the society if the previous changes made a person happy, there is no need for that person to change its type. In the literature, there is also synchronous change which means all agents in the unhappy list change their types at the same time. Differences between asynchronous and synchronous change and how these differences may lead to distinct consequences can be understood from three network examples presented in the below figures. In the examples network structures and initial conditions are the same. Each agent has three neighbors. Periodic boundary (dotted lines) conditions are employed and each node denoted with a number above them. The array shows the agents in the unhappy list. Figure 2.1 shows synchronous change dynamics. In the initial network (top left) agents 2, 3, 6 and 7 are unhappy. They all change their types at the same time and in the resulting network unhappy agents are the same as the initial network (top right). Same agents change their types one more time and network reaches initial placement (bottom). This process repeats itself and model never reaches the equilibrium state where all agents satisfy their utility functions.

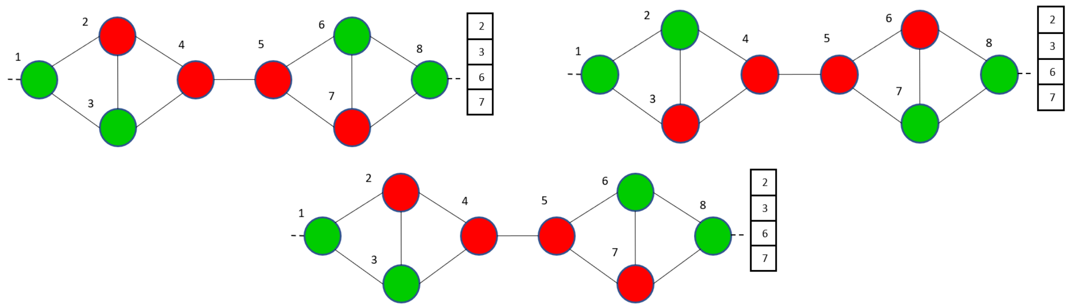

Figure 2.1: Network consists of 8 nodes and each node has 3 neighbors. Periodic boundary conditions and synchronous change are employed. Figure on the top left is the inital network, figure on the top right is the network after the first iteration and the bottom figure is the network after the second iteration. Array shows the unhappy list.

Figure 2.2 shows the asynchronous change dynamics. Since the initial network (top right) is same as the above example initial unhappy list is also same. From unhappy list agent 2 is randomly chosen to change its type. After agent 2 changes its type there are 3 agents left in the unhappy list and agent 6 is randomly chosen (top left). Agents left in the unhappy list (3, 7) become happy due to previous changes (middle left). In this network second iteration started and another unhappy list is created (middle right). Agent 4 is randomly chosen from the unhappy list changes its type (bottom right). Afterwards, agent 8 changes its type and model reaches the equilibrium state (bottom left). From the examples presented in Figure 2.1 and 2.2, it is clear that synchronous and asynchronous change can lead to different outcomes even with the same initial conditions.

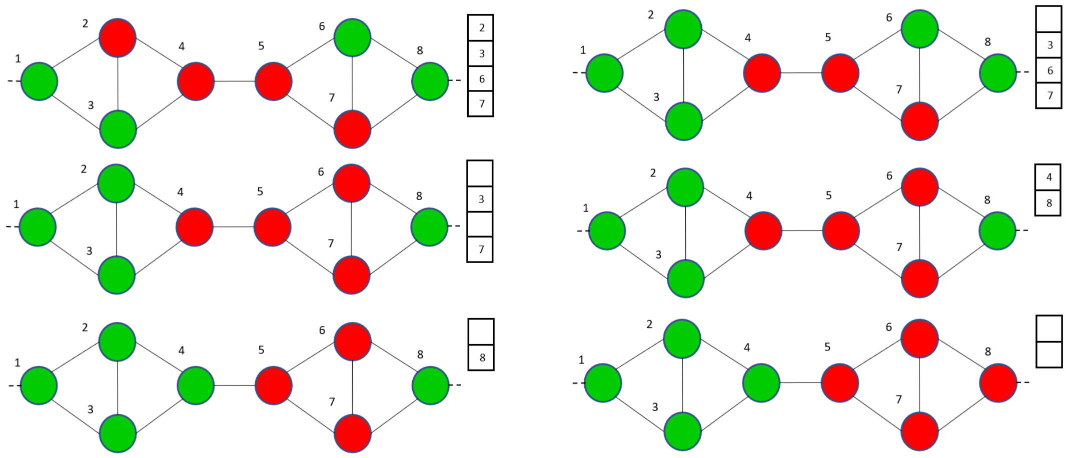

Figure 2.2: Network consists of 8 nodes and each node has 3 neighbors. Periodic boundary conditions and asynchronous change are employed. Figure on the top left is the inital network and figure on the top right is the network after agent 2 changes its type. Figure on the bottom right is the equilibrium network. Array shows the unhappy list.

Moreover, the order of agents chosen from the unhappy list can also lead networks to different outcomes. In Figure 2.3 first agent chosen from the list is same as Figure 2.2, but suppose the second agent chosen from the list is 7 instead of 6. In this example model reaches consensus in the equilibrium. This shows that even in the synchronous change, different outcomes can be reached because of random choosing mechanism.

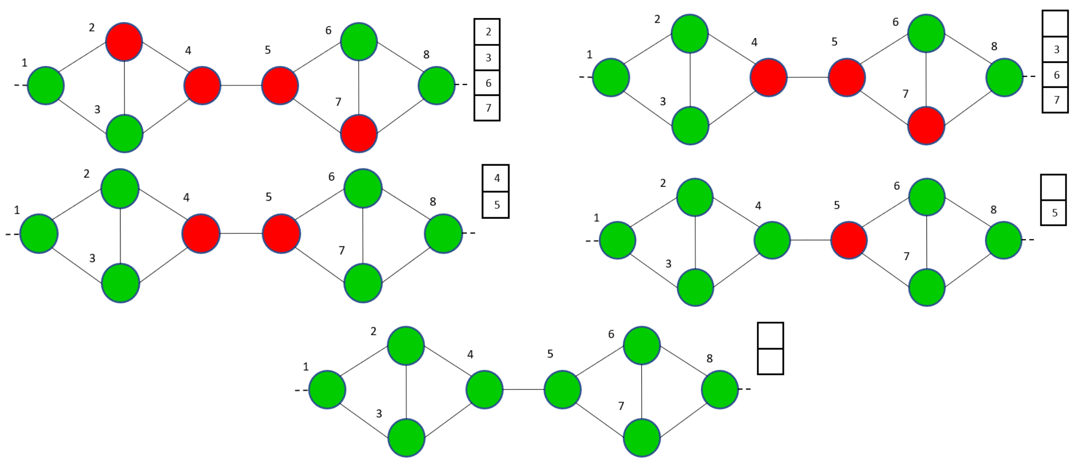

Figure 2.3: Network consists of 8 nodes and each node has 3 neighbors. Periodic boundary conditions and asynchronous change are employed. Figure on the top left is the inital network and figure on the top right is the network after agent 2 changes its type. Figure on the bottom is the equilibrium network. Array shows the unhappy list.

2.2 Analysis of Schelling Models on Square Lattices

In order to interpret the results from the equilibrium state, it is crucial to understand initial networks. Initially, agents are distributed over the network with Bernoulli distribution with parameter 0.5. Hence in each node probability of having X or Y agents are equal. Figure 2.4 shows the initial distribution of same type neighbors. As it can be seen from the figure number of same type neighbors follows a normal distribution. Same type neighbors are X agents in the neighborhood for an agent X, and Y agents in the neighborhood for an agent Y. Percentages of initially clustered neighbors, where the vast majority of agents are the same type, are less than one percent (where x axis is 0,1,7 and 8). Although the percentage of initially clustered neighborhoods are low, they are still extremely important. To understand the importance of initial clusters suppose a network which is divided into two parts with a straight line. One part consists of only X agents and the other part consists of only Y agents. In this initial network all agents satisfy their utility functions, therefore the initial network is the equilibrium state. Even though the borders are not straight lines and there are some agents on the borders who are unhappy, the only change will occur at the borders and the clustered structure will be preserved. This shows that in the square lattice initial clusters are preserved over the time.

Initial neighborhoods mostly consist of both agent types. 28% of all neighborhoods in the network have equality in the number of different agent types. These results confirm that initially, model has an integrated network.

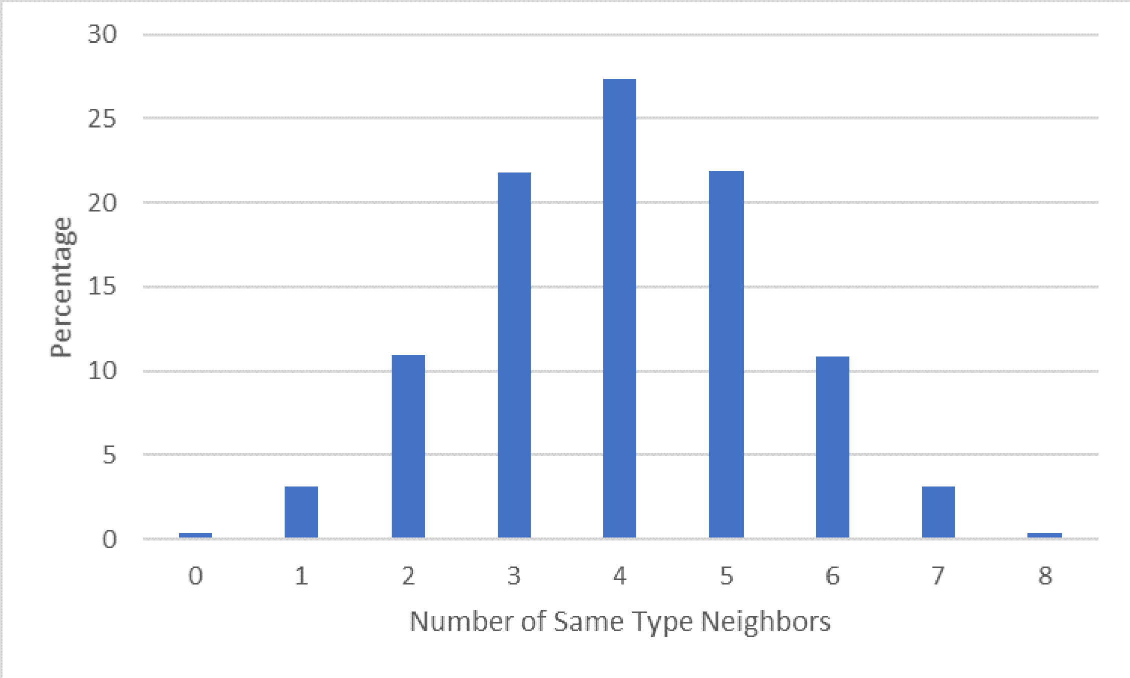

Figure 2.4: Same type neighbor distribution of agents in the initial network. Same type neighbors are X agents in the neighbors for an agent X and Y agents in the neighbors for an agent Y. Network is 120x120 square lattice. Periodic boundary conditions and Moore neighborhood are employed. w=1. Distributions obtained from 500 runs.

Same type neighbor percentages can also be calculated using an analytical approach and this confirms the accuracy of our models. Since the probability of assigning X and Y agents to each node is 0.5, the only distinction between having a different number of same type neighbors is in how many ways that number of same type neighbors can be placed to the neighborhood. For instance, a neighborhood which has 1 X agent and 7 Y agents can be placed in 8 different ways while a neighborhood which has zero X agents and 8 Y agents can only be placed in 1 way. Hence to calculate the probability of having s same type neighbors, dividing how many ways s neighbors can be located in the neighborhood with in total how many different ways neighborhood can be located is enough. X and Y agents can be placed to a neighborhood consists of 8 neighbors in 2^8 different ways. Suppose s is 1, and 1 same type neighbor can be placed in 8 different ways, dividing this two numbers will give 0,03 which gives the probability of having 1 same type neighbor. If these probabilities are multiplied with 100, the initial percentage of same type neighbors can be calculated. Using these formulation initial unhappy agent, who have less than 4 same type neighbors, the percentage is 36.

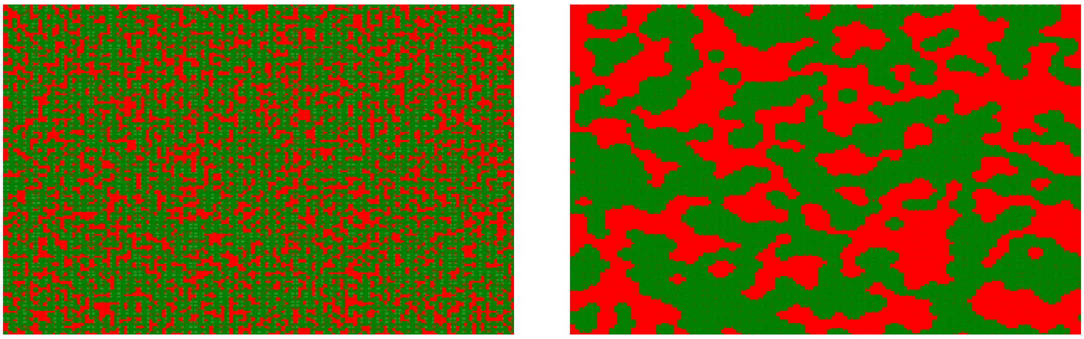

Figure 2.5: Initial (left) and equilibrium (right) networks for 120x120 square lattice with periodic boundary conditions and Moore Neighborhood. Red colors are X agents and green colors are Y agents. w=1.

After analyzing and understanding initial network distributions, the equilibrium state can be focused on. In the equilibrium state, X's and Y's expected to be segregated from the previous studies [22]. But what does it mean to be segregated? As it can be seen from Figure 2.5 and above paragraphs in the initial networks X's and Y's are not clustered also, green and red colors are randomly distributed over the network. On the other hand in Figure 2.5 it is clear that X's and Y's are clustered. This clustered structure of different agent types what is called segregation. Previously, segregation analyzed using different indexes. For instance Mixity, which is the ratio of heterogenous links (the link between an X agent and a Y agent) to total links, is employed by Banos [17], and Pancs and Vriend [21]. Freeman's Segregation Index (FSI) [10], number of separatists [23], Neighborhood Distribution Function (NDF) [11] and Conditional Neighborhood Distribution Function (CNDF) [20] are also used as measures of segregation.

Here rather than these indexes mentioned above, distribution of the neighbors is focused on explaining segregation. While the initial same type neighbors (Figure 2.1) follows a normal distribution since there are also unhappy agents who have more other type neighbors than same type neighbors, in the equilibrium state a different distribution occurs. In the equilibrium same type neighbor distribution which can be seen from Figure 2.6, distribution is skewed to the right since each agent has at least 4 same type neighbors as themselves. This shows the segregation clearly because more than 50% of agents only have neighbors in their type. These agents are the ones in the middle of clusters, and the remaining agents are the ones in the boundaries of clusters.

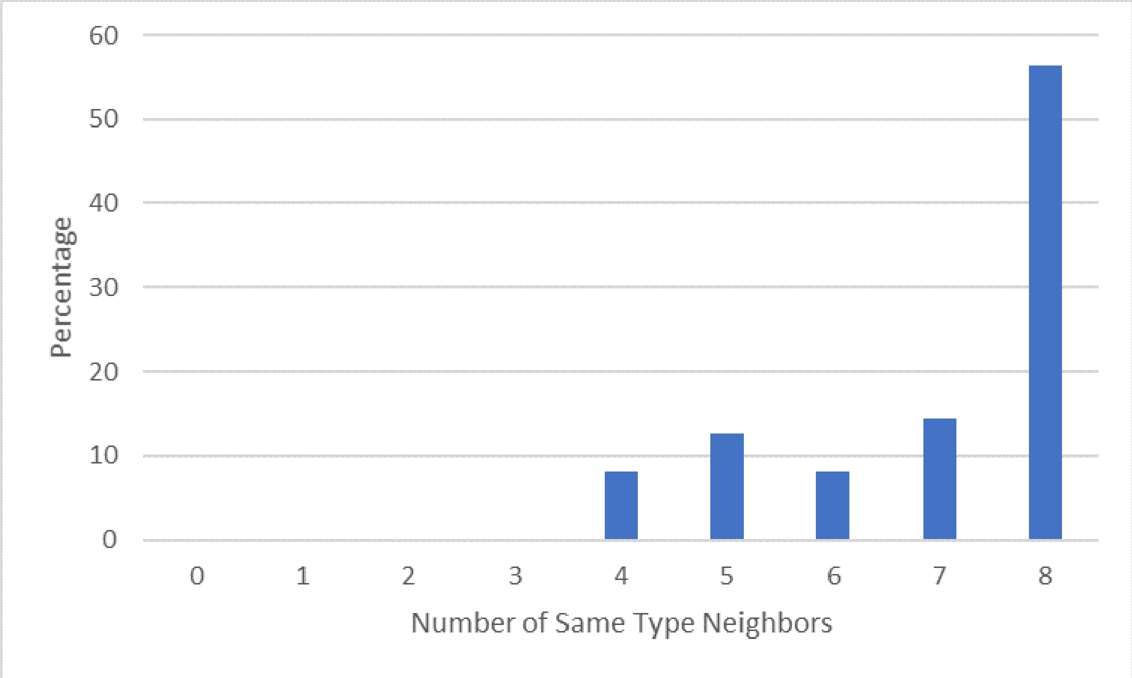

Figure 2.6: Same type neighbor distribution of agents in the equilibrium network. Same type neighbors are X agents in the neighbors for an agent X and Y agents in the neighbors for an agent Y. Network is 120x120 square lattice. Periodic boundary conditions and Moore neighborhood are employed. w=1. Distributions obtained from 500 runs.

Furthermore, in order to explain segregation better distribution of neighbors of an agent's neighbors is analyzed. Neighbors of an agent's neighbors are the agents whose shortest path lengths are 2 from the agent. Figure 2.8 shows the distribution of neighbors of an agent's neighbors both for initial and equilibrium networks. It can be seen that these distributions follow a similar pattern with the same type neighbor distribution: Initially normally distributed and in the equilibrium network skewed to the right with a huge increase in the neighborhoods consist of only same type agents. These distributions are crucial to understanding the segregation. Because neighbors of an agent's neighbors do not affect the agents' utility functions but segregation still can be seen in their distribution. This shows that people need to cluster to satisfy everyone's utilities in the network.

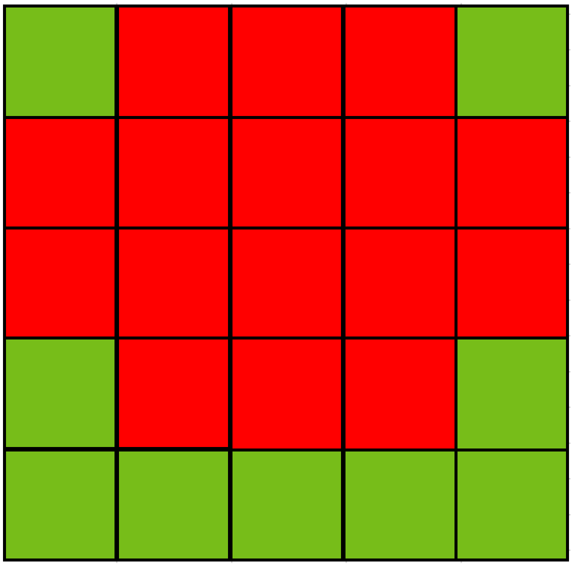

Figure 2.7: 5x5 part of a network. Moore neighborhood is employed. Green colors represent Y agents and red colors represent X agents.

The smallest possible cluster can be observed in the equilibrium state is demonstrated in Figure 2.7. In this cluster, agents have at least 4 same type neighbors and at least 5 same type agents in the neighbors of their neighbors. Since it is the smallest possible cluster, this example explains the findings in Figure 2.8.

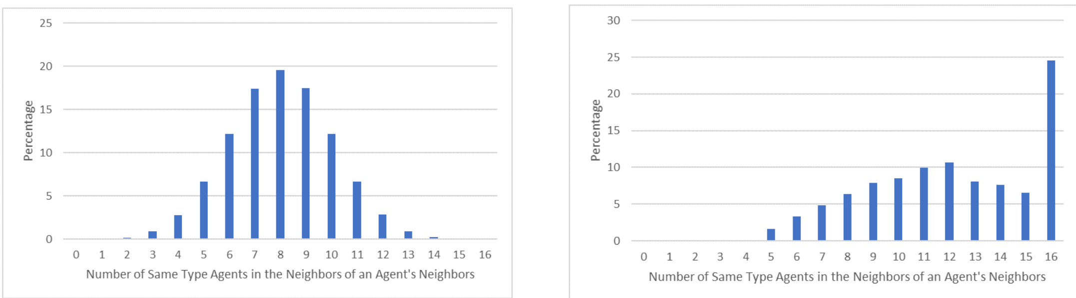

Figure 2.8: Initial (left) and Equilibrium (right) network distributions of neighbors of an agent's neighbors. 120x120 square lattice with periodic boundary conditions and Moore neighborhood is employed. w=1. Distributions obtained from 500 runs.

What if neighbors of an agent's neighbors are included into the agent's neigborhood? This means increasing w to 2, and therefore degree of each node is increased to the 24. Figure 2.9 shows the initial and equilibrium networks when w is equal to 2. Initial network follows the same pattern with w=1 since changing w does not affect initial conditions. Although in the equilibrium network, it can be seen that clusters are enlarged. The reason for that is when w is 2, agents are concerned with a larger region to satisfy their utility functions, and consequently clusters become larger.

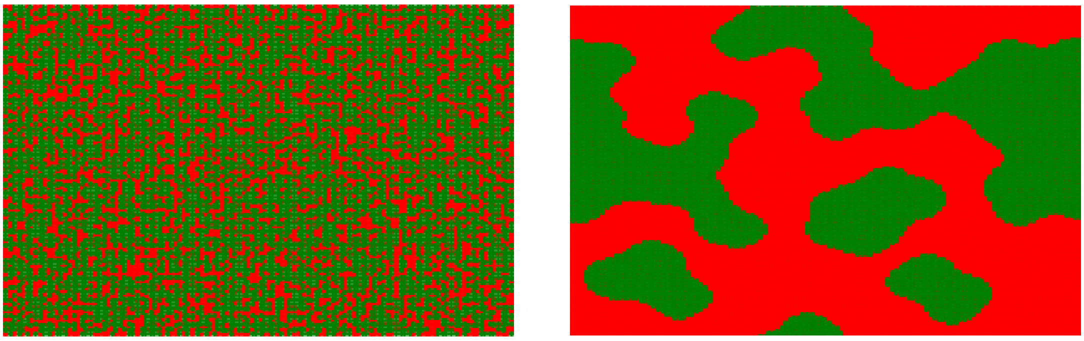

Figure 2.9: Initial (left) and Equilibrium (right) networks for 120x120 square lattice with periodic boundary conditions and Moore Neighborhood. Red colors are X agents and green colors are Y agents. w=2.

It is observed that an increase in the w, enlarges the clusters. These results are consistent with the previous studies that showed as the radius increases segregation increases [18]. What if neighborhood range is increased even more? Suppose a network consists of only one neighborhood and the number of X and Y agents is equal to e. In this network, all the agents are neighbors with every other agent in the network. Initially, every agent is unhappy because the number of different type agents will be one more in the neighbors. Since agents do not consider themselves in the neighborhood, unless there is a tie, number of same type agents will be e-1 and number of other type agents will be e in the neighborhood. Unhappy list consists of all agents in the network. The first agent randomly chosen from the list will change its type and afterwards, that agent type will always be in the minority. Consequently, type of the first agent randomly chosen from unhappy list will disappear, and network reaches the consensus where all the agents are the same type.

Chapter 3

How Do Random Links Help a Community to Reach a Consensus?

As mentioned at the end of the last chapter, if w is increased enough, one agent type can take the dominance of the whole network and other agent type disappears from the network. These model runs where only one agent type lefts in the equilibrium network called ground state in the physics literature [27] [13] and consensus in this thesis and also in social physics [26]. To remind one more time, ground state represents the minimum energy level in which all spins are in the same direction (up or down) and totally homogenous. Consensus represents the network where all agents are the same type meaning all people in the community have the same views about an issue. Since all agents are the same type, every agent in the network is happy and consequently when consensus arise model reaches equilibrium.

What are the other factors can lead community to a consensus? First of all the most intuitive answer will be the initial conditions. If the initial percentage of one agent type is increased large enough, that agent type can take the dominance of the whole network. For instance, if one agent type is initially distributed as the majority in every neighborhood in the network, then other agent type will disappear from the network. Secondly, in the literature how changes in the thresholds (utility functions of agents) can lead a community to a consensus is analyzed [2]. Another way to reach a consensus is changing the node degrees by rewiring edges in square lattice randomly as discussed in this chapter.

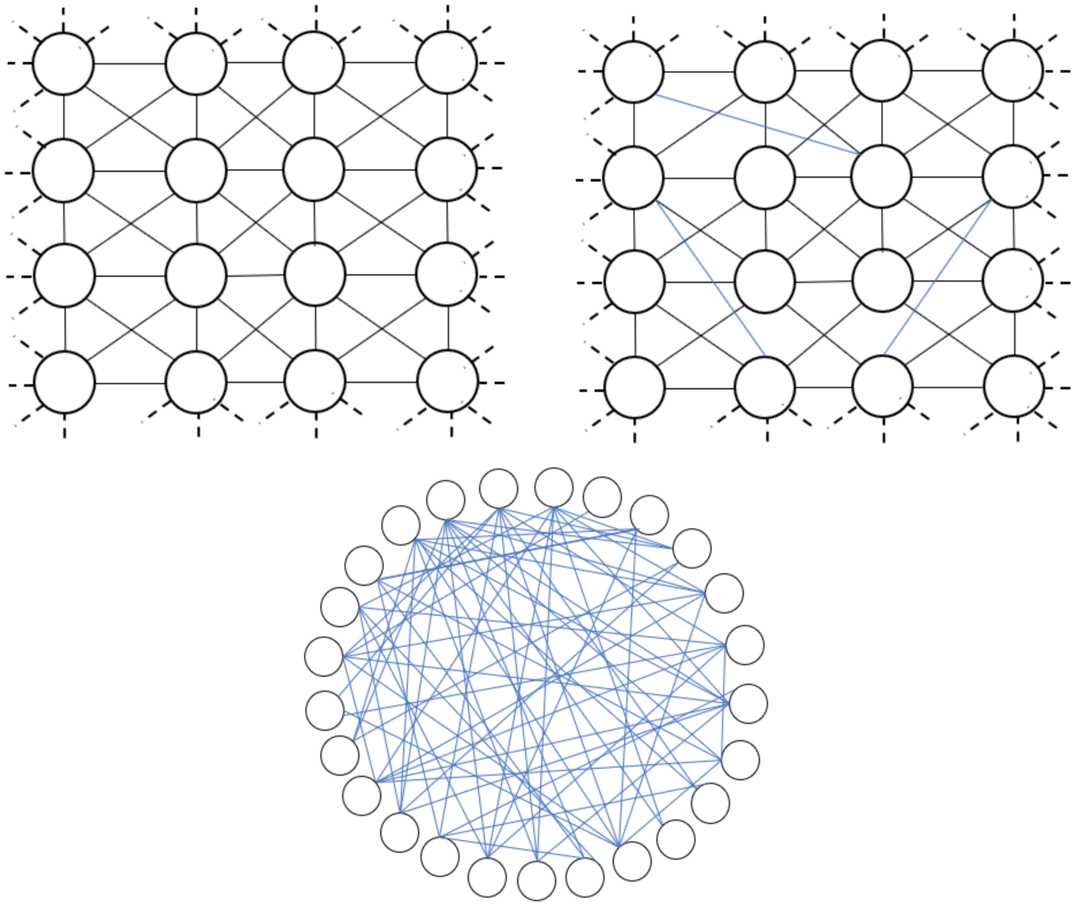

Figure 3.1: Figure on the left displays regular square lattice ($p = 0$) with periodic boundary conditions and Moore neighborhood (w=1) where the degrees of all nodes are equal. Dotted lines represent periodic boundaries. Figure on the right shows the small-world network where some links (blue) from square lattice are broken and rewired to nodes that are randomly chosen. Number of links is preserved. Clustering coefficient is still high as regular square lattice but the average shortest path length is decreased. Figure on the bottom displays random network ($p = 1$) where all edges are rewired.

In order to change node degrees, Watts-Strogatz randomization process [30] is followed by only one difference. Watts and Strogatz proposed randomization process for the ring lattices while in this thesis process adopted to square lattices. Illustration of randomization process can be seen from Figure 3.1. The randomization process follows: For each node in the square lattice, edges that node has chosen one-by-one. For each edge, a random number is generated and if this number is smaller than rewiring probability (p), the edge is broken from the agent's neighbor and wired to another node which is not in the agent's neighborhood. Generated random number must be bigger than or equal to zero and smaller than one. Therefore when rewiring probability is zero no edge will be rewired and when rewiring probability is one all edges will be rewired. For rewiring probability values between 0 and 1, each edge has p probability to rewire.

Square lattices ($p = 1$) display high clustering coefficients (c) and high average shortest path lengths (l). Clustering coefficient is the ratio of how many links exist between agent's neighbors and how many possible links can be established between agent's neighbors. In regular square lattice when w is one each agent has 8 neighbors, and these 8 neighbors have 12 links between them. 8 nodes can be linked in total 28 ways, therefore clustering coefficient of the regular square lattice, when w is 1, is 0.428. The average shortest path length of regular square lattices employed in this thesis, when w is, is 40.

If rewiring probability is equal to one, then all edges will be rewired and network will be a random network. In random networks, there are no local clusters like regular lattices which leads to smaller clustering coefficient. Clustering coefficient of random networks is approximately equal to k/N [30] (N denotes number of nodes) which is 0.0005 in this thesis since k is 8 and N is 14400. Simulation results confirm these conditions. Average shortest path lengths are significantly small in random networks compared to regular lattices since each edge is randomly assigned and these random edges create shortcuts between different parts of the network. The average shortest path length of a random network is approximately equal to $ln(n)/ln(k)$ [30] which is 4.6 for the models employed in this thesis. Both the clustering coefficient and the average shortest path length values obtained from simulations are consistent with the theoretical values, and this shows the accuracy of the randomization process employed in this thesis.

Networks where clustering coefficient is as high as square lattice and average shortest path length is as small as random networks, are called small-world networks [30]. As it can be seen from Figure 3.2 these conditions correspond rewiring probability values around 0.01. These networks have highly clustered local knits because random links are not so many to disorder initial network structure of square lattice. On the other hand, these networks have small average shortest path lengths because random link number is enough to bound different parts of the network and create shortcuts.

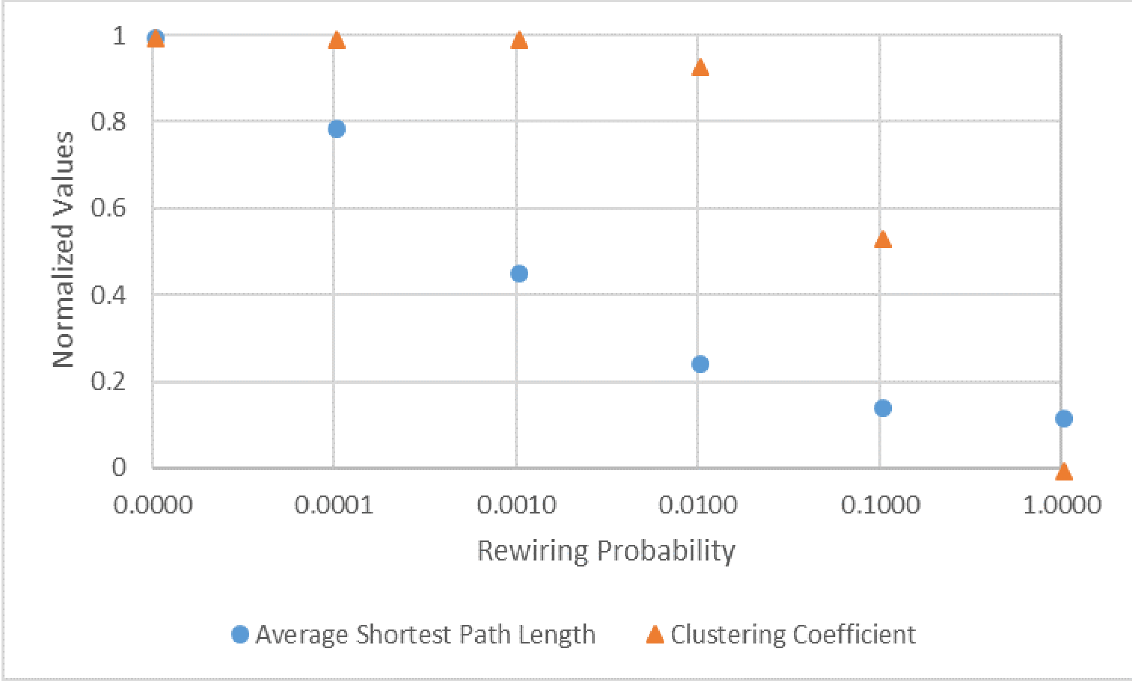

Figure 3.2: Clustering coefficient and average shortest path length values for different rewiring probabilities. Initial network is 120x120 regular square lattice with periodic boundary conditions and Moore Neighborhood, w=1.

Figure 3.2 shows how clustering coefficients and average shortest path lengths normalized with their values for square lattice, change with rewiring probability. Average shortest path length decreases even with a few number of random links, while clustering coefficient decreases significantly when 10% of the edges are rewired.

Table 3.1: Initial unhappy agent percentages for different rewiring probabilities

Rewiring Probability	Mean	Std	Miniimum	Maximum
0.0	36.2	0.4	35.2	37.2
0.0001	36.3	0.4	35.3	37.0
0.001	36.5	0.4	35.4	37.6
0.01	38.1	0.4	37.2	39.2
0.1	42.8	0.6	41.5	44.1
0.3	43.0	0.5	41.8	44.5
0.5	42.9	0.5	41.4	44.2
0.7	42.9	0.5	41.4	44.3
1.0	43.0	0.6	41.4	44.9

Initial unhappy agent percentages increase as the rewiring probability increases as can be seen in Table 3.1. Change in the node degrees increases the probability of being unhappy initially.

After explaining the randomization process and how random links affect clustering coefficient and average shortest path length, the question "why changing node degrees can lead community to consensus" can be answered clearly. As mentioned in Chapter 2, initially there are neighborhoods where X's and Y's are in the majority. In the first iterations, these initial X and Y clusters are shaped and become larger. After initial clusters are formed they remain until the equilibrium. Later iterations cause changes only in the boundaries of initially formatted different color clusters, as a consequence consensus can not be observed. Changing node degrees with random links can prevent initial clusters to preserve themselves until equilibrium state, therefore helps communities to reach a consensus. Because random links decrease average shortest path lengths and this leads to each node in the network to become connected in few steps. Furthermore, random links decrease clustering coefficient which helps initial clusters to affect from other agents in the networks.

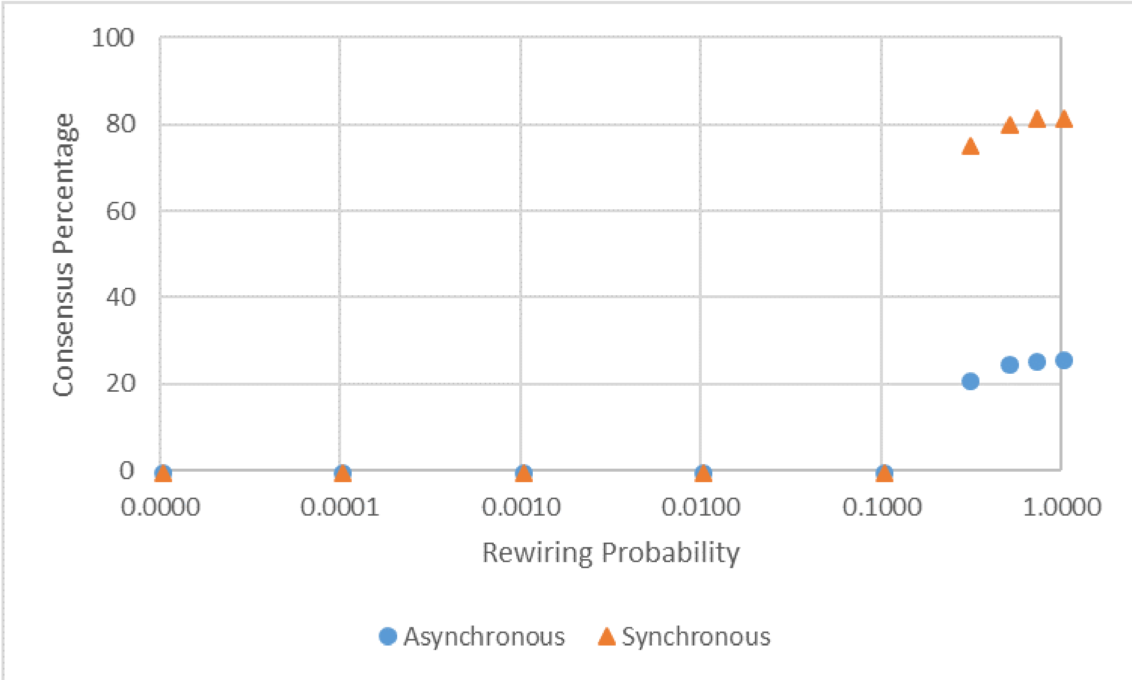

Figure 3.3: Percentage of communities that reach a consensus for synchronous and asycnhronous change. Initial network is 120x120 regular square lattice with periodic boundary conditions and Moore neighborhood. w=1. All networks are connected.

Figure 3.3 shows the percentage of communities that reach a consensus for different dynamics. Here two different dynamics are analyzed. The only difference between the two dynamics is how agents in the unhappy list change their types.

For the asynchronous change, consensus can be observed starting from rewiring probability 0.3. This leads to the conclusion of only small average shortest path length is not enough to observe consensus. If only small average shortest path length would be enough, consensus should have been observed for rewiring probability 0.1 where the average shortest path length is nearly same as random networks. The difference that reaches communities to a consensus for rewiring probability 0.3, instead of 0.1 is clustering coefficient. For rewiring probability 0.1 clustering coefficient is still very close to a square lattice. On the contrary, for rewiring probability 0.3, clustering coefficient is nearly half of the square lattice. Having these in mind to observe consensus by randomization, average shortest path length must be small in order to agents to reach every other node in the network easily, and clustering coefficient must be small to break highly clustered local knits.

Theoretically, Ising models with asynchronous change reach ground state with 0.66 probability and reach stripe state with 0.33 probability [27] in regular square lattices. The only difference between Ising models and the presented model, how agents behave when there is an equality in the numbers of different type agents in their neighbors. How can this difference lead communities to reach a consensus with 0.66 probability? It is because, as mentioned communities cannot reach a consensus because of initial clusters that remain until the equilibrium. In thesis dynamics, initial cluster formation is easier than Ising models since even with the equality in the number of different type neighbors, agents can satisfy their utility functions. Furthermore, deciding to change the agent type with a probability when there is equality in the numbers of different type agents in the neighborhood can also be seen as another way of randomization.

When synchronous change is employed, consensus is more frequently observed than asynchronous change. The reason for this is that in asynchronous change, clusters can emerge more easily than synchronous change. Even though there are random links, clusters can emerge among the same type agents due to the order of random agent choosing from the unhappy list as explained in previous chapter. However, in synchronous change as average shortest path length and clustering coefficient become smaller, the probability of having different type clusters decreases faster.

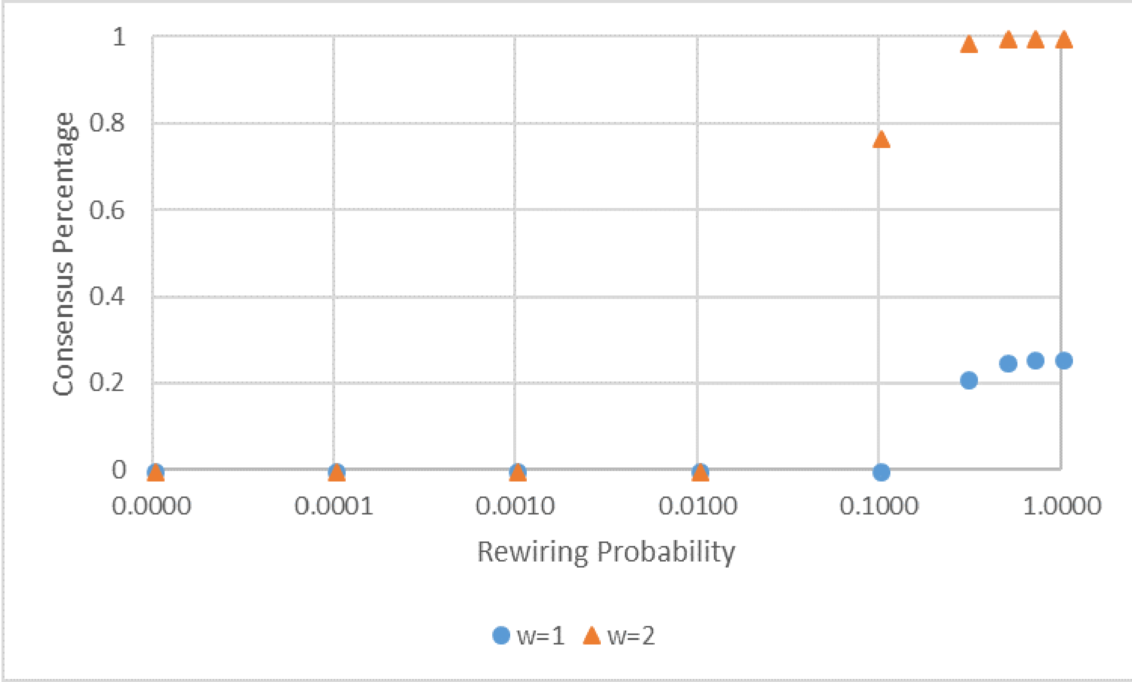

Figure 3.4: Percentage of communities that reach a consensus for w=1 and w=2. Initial network is 120x120 square lattice with periodic boundary conditions and asynchronous change.

Lastly, the effect of randomization on the networks with different initial node degrees is analyzed. For networks with higher initial node degrees (w=2) percentage of communities that reach a consensus is higher. Another difference between $w = 1$ and $w = 2$ is that when $w = 2$ communities can reach a consensus with smaller rewiring probabilities. This is because in models with higher node degrees, there are more random links for the same rewiring probability. Therefore, average shortest path length and clustering coefficient decrease faster for $w = 2$ than $w = 1$ as can be seen in Appendix D.

Chapter 4

The Effect of Individuals Who Give Their Neighbors the Benefit of the Doubt

The models previously mentioned in this thesis and in the literature consist of two agent types: X and Y. In the social sciences literature, two agent types represent two different sides of unmutable characteristics. For instance, in Schelling models, X and Y represent two different races in Chicago in order to understand racial segregation. Also, in the physics literature X and Y represent up and down spins. However, the models proposed in this thesis based on mutable characteristics. These mutable characteristics can be opinions, attitudes or beliefs about any social and political issue. These characteristics are strongly related to an individual's personal background, political ideology and religious beliefs, moreover they are developed over a long time [19]. Therefore, changing these characteristics and adapting different ones are compelling. Suppose an example in which issue is recycling and X represents people who recycle while Y represents people who do not recycle. Recycling behavior changes with personality and attitudes of environmental concerns [25]. It is not realistic to expect to one to change its environmental concerns and personality in a short time period and take the opposite stand. Therefore, a new agent type proposed in order to model, changes in characteristics more realistic.

A new agent type proposed is called XY. XY represents the people who have doubts about their opinions. While X and Y represent two opposite stands on an issue, XY represents the moderate stand. In recycling example, XY's are the people, regardless of whether they are recycling or not, who question their stands. In the models presented in this chapter, unhappy X and Y agents turn to XY instead of turning to each other. Unhappy agents are under the social pressure because the majority of the people in their neighborhood have different views from them. Since opposite stand on an issue is a social norm in that neighborhood, this social pressure forces agents to change their views [19]. Instead of turning to totally opposite side, at first agents turn to moderate stand where they can have the benefit of doubt and question their opinions and if they are still unhappy than they take the opposite stand.

The models presented in this chapter are same as the previous models until agents in the first unhappy list change their types. The first unhappy list consists of only X's and Y's. These unhappy X and Y agents turn to XY asynchronously. Since the initial percentage of unhappy agents for square lattices and for different rewiring probabilities are known (Table 3.1) and XY's initially created from the first unhappy list, these initial unhappy agent percentages are also upper bounds for initial XY percentage.

After XY's are introduced to the network, X agents are unhappy if the majority of their neighborhood is Y or XY. If the number of Y's and XY's are equal to each other and greater than the number of X's, X agents are also unhappy. Unhappy X agents turn to XY, regardless of their neighborhood structure. Y agents display a similar behavior as X agents. They are unhappy if the majority of their neighborhood is X or XY. In addition to that, they are unhappy if the number of X's and XY's are equal to each other and greater than the number of Y's. Similar to X agents, Y agents turn to XY if they are unhappy. X's and Y's turn to XY when XY's are in the majority because people who have doubt put a social pressure on others and make them question their opinions. Therefore this model is called Doubtful Community model (DC model) because doubtful agents can give their neighbors, who have a stand on an issue, the benefit of doubt.

XY agents are unhappy if the majority of their neighbors are X or Y. They turn to X if X's are the majority, and turn to Y if Y's are the majority. XY agents are happy when the number of X's and Y's are equal to each other and greater than the number of XY's in the neighborhood.

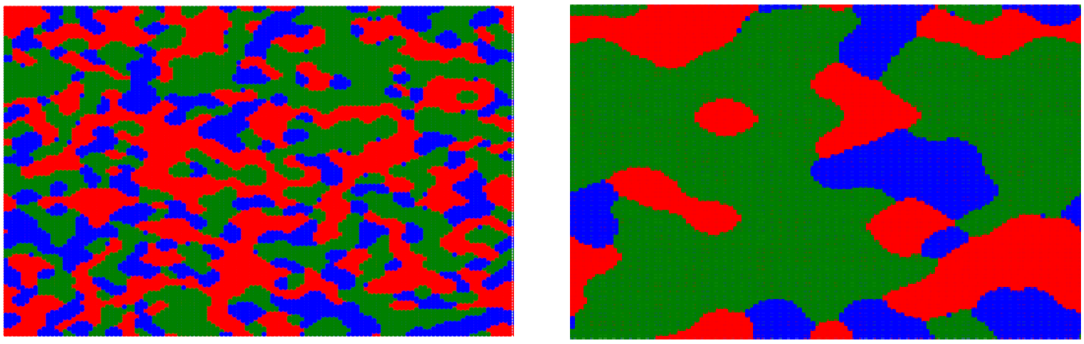

Figure 4.1: Equilibrium networks for DC model on 120x120 square lattice with periodic boundary conditions and Moore neighborhood. Figure on the left shows w=1 and figure on the right shows w=2. Red colors are X agents, green colors are Y agents and blue colors are XY agents.

Figure 4.1 shows the equilibrium networks for DC model. When w is 1 in the equilibrium state, the network is not dominated by XY's even though all unhappy X and Y agents turn to XY. Since the expected X and Y percentages are always equal and 23% of all agents are XY (Table 4.1), X and Y percentages in the equilibrium network are 38.5%. XY is always in the minority in the equilibrium network, even though initially they can be in the majority. To explain this, consider an unhappy X or Y agent in the initial network. Since this agent is unhappy, other agent type (X or Y) must be the majority in this agent's neighborhood, and this agent must change its type to XY. Even though this agent change its type to XY, unless other changes in the network did not cause a critical change in the neighborhood, still X's or Y's are the majority in the neighborhood. As it can be seen from Figure 4.2, initially the huge part of the XY's are minority in their neighborhood. Therefore XY agent will be unhappy in the next iteration and change its type to the agent type in majority. This is the main reason that prevents XY's to dominate the network. As the neighborhood size increases, clusters become larger as explained in Chapter 2 and DC model follows the same behavior.

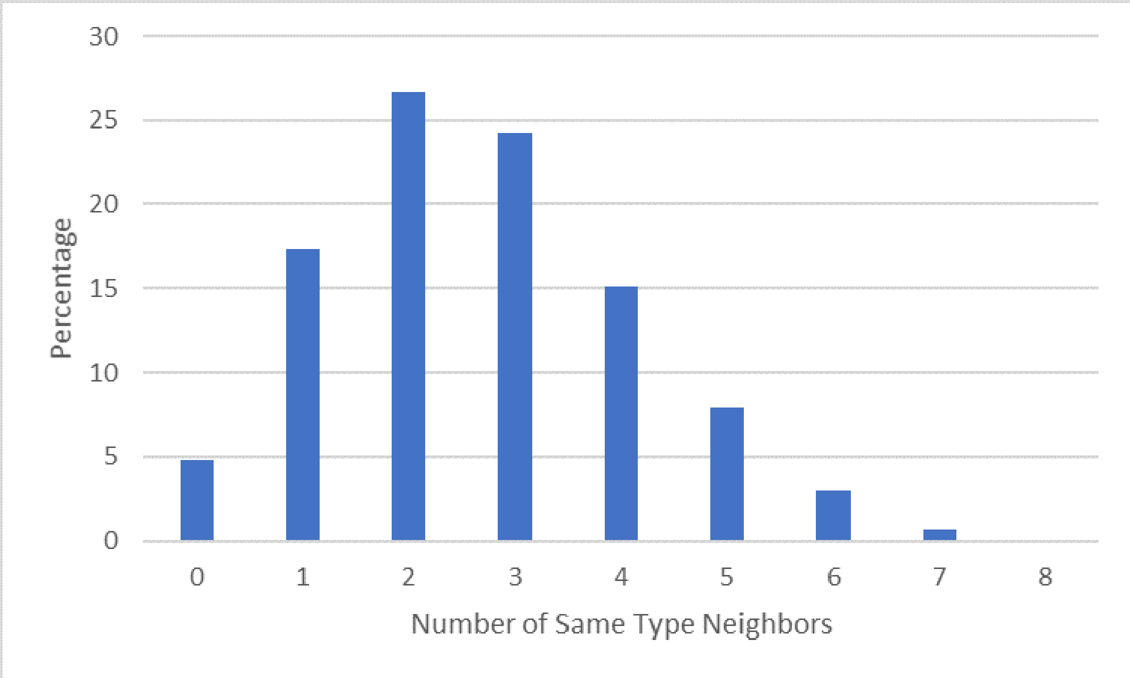

Figure 4.2: Same type neighbor distribution of XY agents in the initial network. Same type neighbors are XY agents in the neighbors for an agent XY. Network is 120x120 square lattice. Periodic boundary conditions and Moore neighborhood are employed. w=1. Distributions obtained from 500 runs.

What if doubtful agents cannot give the benefit of doubt to their neighbors, and people persist on their stands? In this case, X and Y agents do not become unhappy if XY's are the majority in their neighborhood. X's and Y's do not feel social pressure as in DC model, therefore persist on their views and do not have the benefit of doubt. This version of XY models called Persistent Community model (PC model). The only difference between PC model and DC model is, how X and Y agents act when doubtful agents (XY) are in the majority in their neighborhood. Figure 4.3 shows the equilibrium networks for PC model for $w = 1$ and $w = 2$. In PC model, XY percentage in the equilibrium state is less than 1% and distinctly less than DC model. Because in the PC model even XY's are in the majority of the neighborhoods of X's and Y's, X's and Y's do not turn to XY. This prevents XY's to become clusters and eventually doubtful XY agents leave their doubts. Remaining XY agents in the equilibrium networks are the ones placed on the boundaries between X's and Y's where they have equal number of X and Y as neighbors.

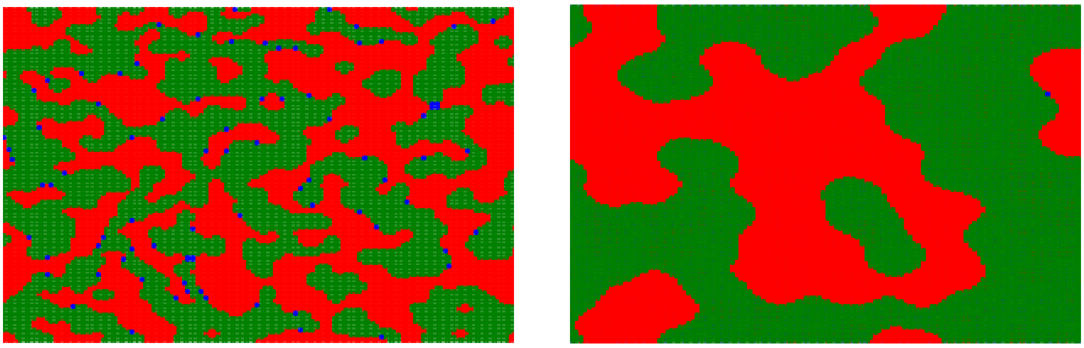

Figure 4.3: PC model on 120x120 square lattice with periodic boundary conditions and Moore neighborhood. Figure on the left shows w=1 and figure on the right shows w=2. Red colors are X agents, green colors are Y agents and blue colors are XY agents.

In PC model if w is increased to 2, X and Y clusters become larger while XY's nearly disapper from the network. It is because when w is 1 there are already few XY's remaining in the network, and this XY's mostly are the ones who have equality in the number of X's and Y's in their neighborhood. When w increases the probability of having equal number of X's and Y's decreases and this leads XY percentage to become less.

In regular lattice, both DC and PC models cannot reach a consensus. What if random links are introduced to these two models? Do doubtful agents increase the probability of reaching a consensus? Can a community where everyone is doubtful be observed? To answer these questions, the same randomization process in Chapter 3 employed for these two models. Since the same randomization process is employed, network structures, average shortest path lengths and clustering coefficients are the same as the models in Chapter 3.

Figure 4.4 shows the percentage of communities that reach a consensus. In both models, consensus can be observed starting from rewiring probability 0.3 when w is 1, and from rewiring probability 0.1 when w is 2 as the models in Chapter 3. This leads to the conclusion that reaching a consensus is not related to the number of agent type, instead it is related to the network structure. When w is 1 in both models, the percentage of communities that reach a consensus differs from zero starting from rewiring probability 0.3, yet there are huge differences between these two models.

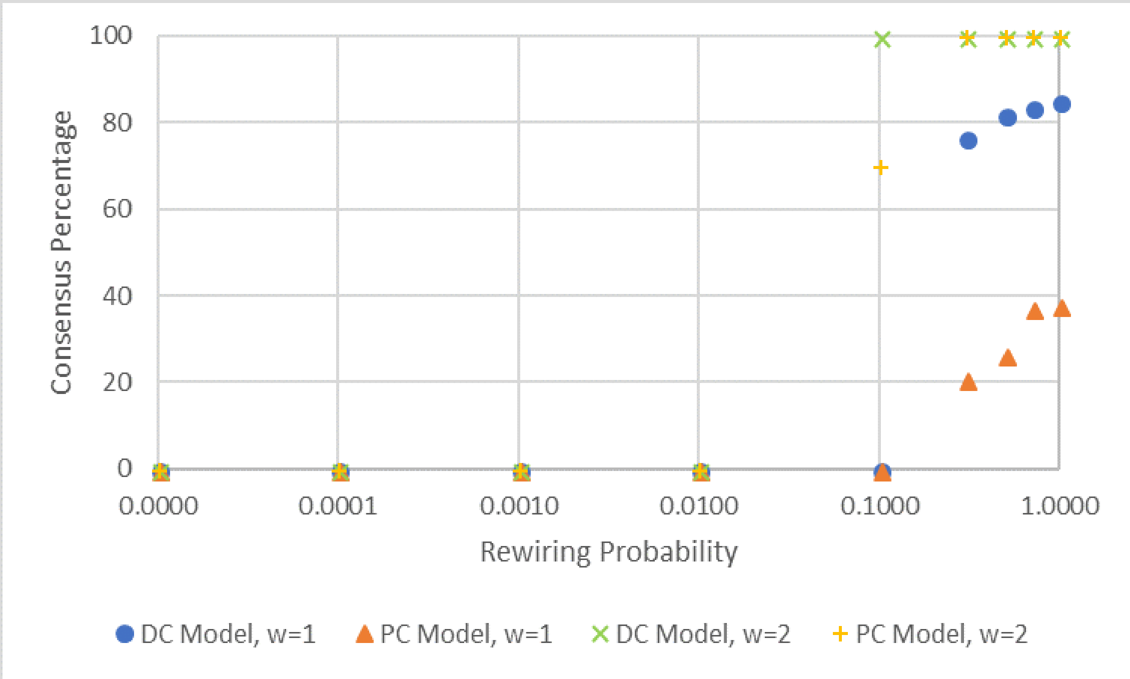

Figure 4.4: Percentage of communities that reach a consensus for DC and PC models and for different neighborhood ranges. Initial network is 120x120 regular square lattice with periodic boundary conditions. All networks are connected.

The most crucial difference between DC and PC models is, in consensus state, all agents are XY in DC model, while in PC model all agents are either X or Y. As the rewiring probability increases, the initial unhappy percentage also increases (Table 3.1) therefore initial XY percentage increases. This increase makes XY agents majority in each run starting from rewiring probability 0.3 as it can be seen from Table 4.1. In addition to these, the effect of being the majority in the initial network increases with the rewiring probability. In square lattice, even though one agent type is in the majority, other agent type can cluster in some parts of the network since the clustering coefficient is high. However, random links decrease the clustering coefficient and prevent these small clusters to emerge. Therefore, agents in the minority cannot cluster and disappear from the network. Consequently, in DC model consensus can be reached when every agent in the community is doubtful. Even though in PC model, XY's are also in the majority initially, since X and Y agents do not turn to XY when XY's are the majority, consensus with doubtful agents cannot be reached in this model.

Both models reach a consensus with a higher probability when w is increased to 2. DC model always reaches a consensus when the rewiring probability is equal or greater than 0.1 and PC model when the rewiring probability is equal or greater than 0.3. These patterns are consistent with the models analyzed in Chapter 3.

Equilibrium state XY percentages can help to understand DC and PC models with random links. For DC model (Table 4.1) XY percentages in the equilibrium state significantly increase from 0.01 to 0.1 and from 0.1 to 0.3. However, for rewiring probability 0.1 the maximum observed XY percentage is 49, while for rewiring probability 0.3 is 100 which shows the communities that reach a consensus. Standart deviations are higher for rewiring probabilities that consensus can be reached. Because reaching a consensus is highly related to how random links are placed.

Table 4.1: Equilibrium state XY percentages in DC model for different rewiring probabilities

Rewiring Probability	Mean	Std	Minimum	Maximum
0.0	23.0	1.82	19.1	27.3
0.0001	23.0	1.70	19.4	28.9
0.001	23.4	1.8	19.5	28.7
0.01	25.3	2.2	19.7	31.3
0.1	37.4	4.0	27.8	49.0
0.3	86.3	25.1	35.0	100.0
0.5	90.1	20.8	35.2	100.0
0.7	90.9	19.0	35.6	100.0
1.0	89.5	22.8	34.6	100.0

In PC model average percentage of XY in the equilibrium state is always less than 1. For rewiring probability values equal to or greater than 0.3, consensus can be observed and in consensus, there are no doubtful agents in the community. Standart deviation increases with rewiring probability because as mentioned above random links can lead to distinct consequences.

Table 4.2: Equilibrium state XY percentages in PC model for different rewiring probabilities

Rewiring Probability	Mean	Std	Minimum	Maximum
0.0	0.56	0.09	0.29	0.86
0.0001	0.55	0.09	0.35	0.79
0.001	0.56	0.65	0.41	0.74
0.01	0.50	0.07	0.35	0.64
0.1	0.39	0.06	0.25	0.53
0.3	0.50	0.27	0.00	0.87
0.5	0.56	0.36	0.00	1.04
0.7	0.51	0.39	0.00	1.09
1.0	0.48	0.37	0.00	0.97

XY agents are individuals who become doubtful about their opinions because of social norms in their neighborhood. They may want to remain their doubtfulness for longer time periods in order to see how social norms in their neighborhood will shape over time. To model this behavior, networks in each iteration denoted as N_t, where t is the number of iteration. For instance, the initial random network consists of only X's and Y's denoted as N_0 and the network where XY's are first created denoted as N_1. In the previous models, XY agents change their type if they are unhappy regardless of which agent type they were in the previous iterations. Here, XY agents do not change their type if they were X or Y in $N_{t\text{-}p}$ and p represents the persistence step which shows for how many iterations XY's preserve their doubtfulness.

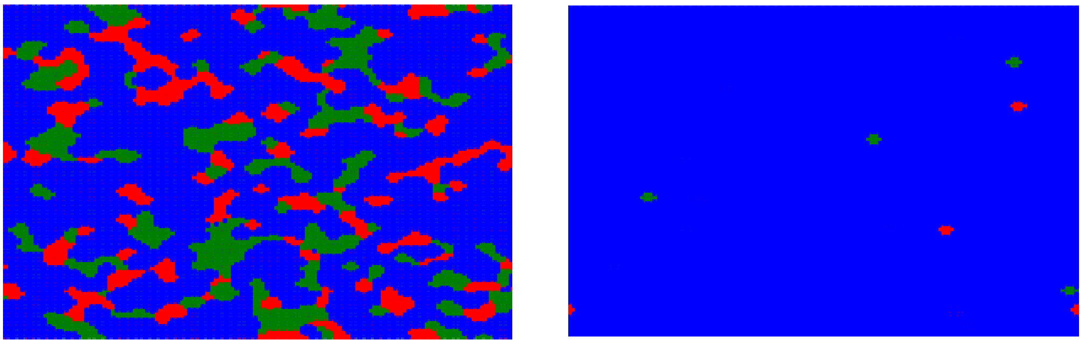

Figure 4.5: DC models with persistence step. Network is 120x120 square lattice with periodic boundary conditions and Moore neighborhood. w=1. Figure on the left shows the equilibrium network for persistence step 1, and figure on the right shows the equilibrium network when persistence step is equal to the number of iterations until equilibrium.

Figure 4.5 shows the equilibrium network for DC models with persistence step. Figure on the left is the equilibrium network when persistence step is 1 and figure on the right is the equilibrium network when XY agents remain their doubtfulness until the equilibrium. X and Y percentages decrease when persistence step is increased to 1 from 0, as it can be seen from the figure. The main reason for this, as it mentioned above color clusters predominantly formed in the first iterations. Since in N_0 all the agents are X and Y, in the first iteration XY's do not change their types and they gain an advantage to form clusters. When XY's remain doubtful until the equilibrium, there are few X and Y clusters left. These clusters form at the initial network and they remain until the equilibrium.

Communities can not reach a consensus in square lattice even if XY agents remain their doubtfulness until the equilibrium and do not change their types. Doubtful agents remain their doubtfulness but they can not give the benefit of doubt to the neighborhoods where X or Y agents are in the majority initially.

Chapter 5

Individuals That Seek Diversity in Their Vicinity and Emergence of Maze Patterns

In the previous chapters, agents' desire to have similar people in their vicinities are analyzed. Although in real life people may have different preferences about who they want to live within their vicinities and how important this desire can vary. For instance, people may desire similar people in their immediate vicinities but diversity in their distant vicinities. Furthermore, having similar people in immediate vicinities can be more important than having diversity in distant vicinities.

In order to model these preferences, neighbors are divided into categories. When w is equal to 1, the 8 neighbors that agents have denoted as the first degree neighbors. The additional 16 neighbors that agents have when w is increased to 2, denoted as the second degree neighbors. Generally, using this logic neighbors divided to categories as *i'th* degree neighbors where i is the value of w. In this thesis, only first and second degree neighbors are included in the models. First degree neighbors are considered as the immediate vicinity of a person and second degree neighbors are considered as the distant vicinity of a person.

Instead of looking neighbors as a whole dividing them into categories give a more realistic perspective about agents' preferences. First degree neighbors can be thought of as a family and close friends, while second degree neighbors can be thought of as people encountered in daily life such as co-workers. Therefore, the importance of different degree neighbors can differ.

Importance of different degree neighbors given by the value of α and preference of same type or different type agents in the different degrees given by the sign of α.

Value of α shows the relative importance of different degrees according to the first degree neighbors. When $| \alpha |$ is smaller than 1, the first degree neighbors are more important than the second degree neighbors. Equation 5.1 shows the condition for an agent to be happy where s_i denotes the same type agents in *i'th* degree neighbors.

$$s_1 - 4 + \alpha(s_2 - 8) \geq 0 \tag{5.1}$$

When individuals in a community do not seek diversity and give the same importance to their first and second degree neighbors ($\alpha = 1$), models become exactly same as the models analyzed in Chapter 3 for $w = 2$. In this case, Equation 5.1 becomes $s_1 + s_2 \geq 12$, which shows the total of same type first degree and second degree agents must be equal to or greater than half of the total neighbors. Inequalities for different α values can be seen from Figure 5.1. For the positive values of α if an agent's same type first degree and second degree neighbors are on the dotted line or above the dotted line agents are happy, otherwise they are unhappy. On the other hand, for the negative values of α if an agent's same type first degree and second degree neighbors are on the dotted line or below the dotted line agents are happy, otherwise they are unhappy. Agents are happy regardless of α value if they have 4 same type first degree neighbors and 8 same type second degree neighbors.

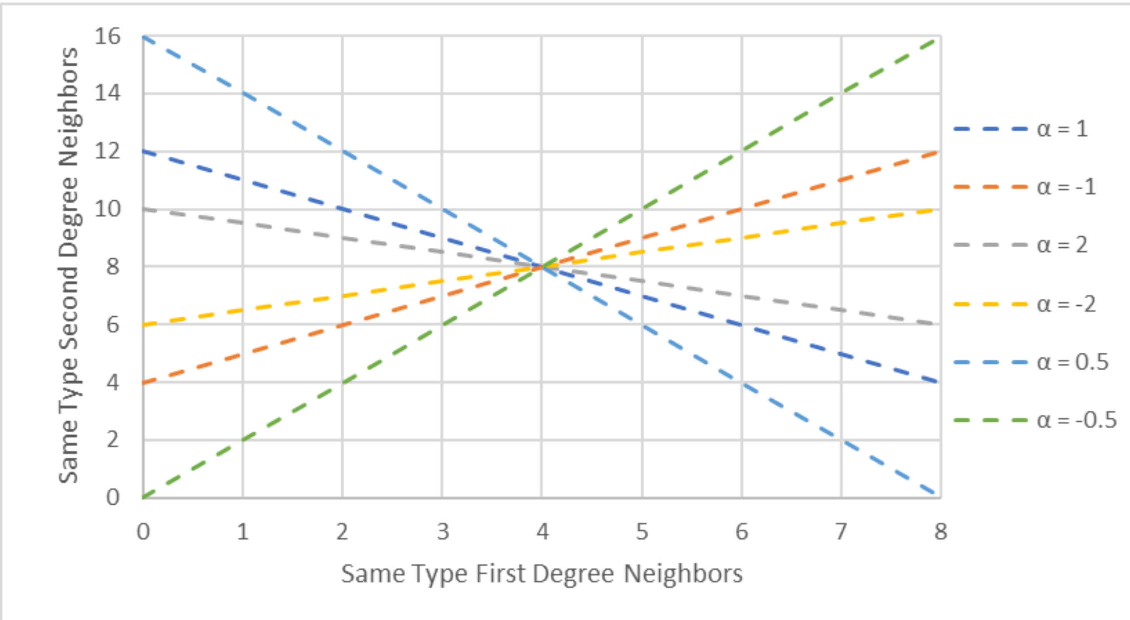

Figure 5.1: Inequalities show the numbers of same type first degree and second degree neighbors agents need to satisfy their utility functions for different α values.

It is useful to analyze these preferences in two parts: α is greater than zero and α is smaller than zero. When α is greater than zero, agents prefer the same type agents as themselves in their immediate and distant vicinities. As mentioned above $\alpha = 1$ means agents have no preference over the same type agents in their vicinities. For α values between 0 and 1, agents give more importance to having the same type agents in their immediate vicinities than their distant vicinities. On the other hand for α values greater than 1, having same type agents in distant vicinities is more important than having the same type agents in immediate vicinities. Figure 5.2 shows the equilibrium networks for the positive values of α.

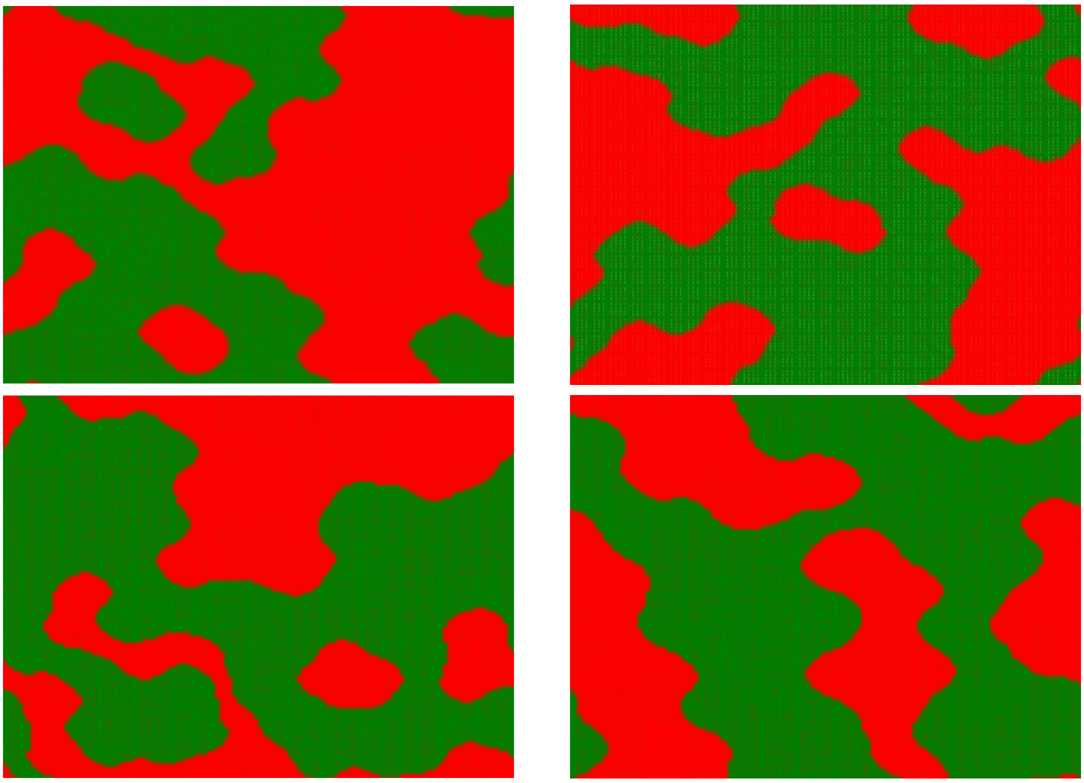

Figure 5.2: Equilibirum networks for the positive values of α. Networks are 120x120 square lattices with periodic boundary conditions and Moore neighborhood. Figure on the top left is for $\alpha = 0.125$, figure on the top right is for $\alpha = 0.5$, figure on the bottom left is for $\alpha = 1$ and figure on the bottom right is for $\alpha = 2$.

In order to satisfy each agent's utility function for $w = 1$, agent's second degree neighbors become segregated as mentioned in Chapter 2. Consequently, there are no substantial differences between the positive values of α.

In the case where α is smaller than zero, agents seek diversity in their second degree neighbors. Diversity seeking behavior of individuals excessively examined in the literature [20] [11]. In the literature diversity seeking behavior analyzed mostly by employing an upper bound. Agent thresholds (lower bound) are minimum same type neighbor percentage needed to satisfy agent's utility function and the upper bound is maximum same type neighbor percentage needed to satisfy agent's utility function. For instance, if lower bound is 0.5 (as in this thesis) when $w = 1$ agents must have at least 4 same type neighbors and if upper bound is 0.95 agents must have at most 7 same type neighbors in order to satisfy their utility functions.

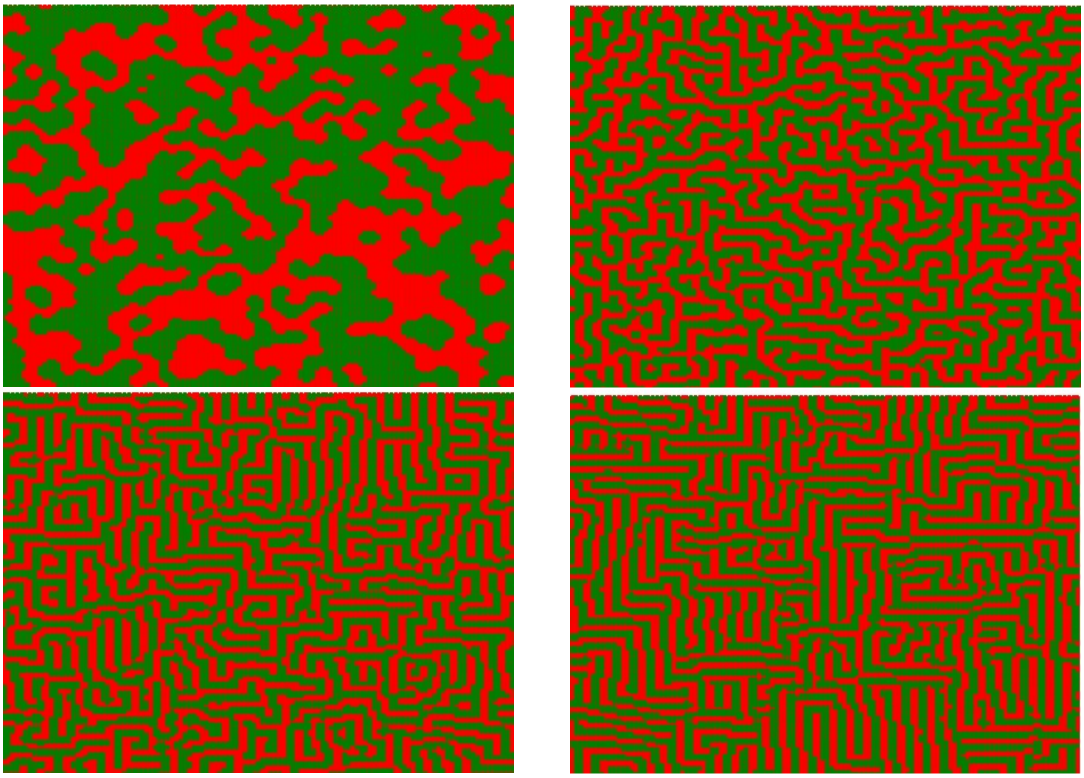

Figure 5.3: Equilibirum networks for the negative values of α. Networks are 120x120 square lattices with periodic boundary conditions and Moore neighborhood. Figure on the top left is for $\alpha = -0.125$, figure on the top right is for $\alpha = -0.5$, figure on the bottom left is for $\alpha = -1$ and figure on the bottom right is for $\alpha = -2$.

Here, a different approach to diversity seeking behavior is proposed. Agents seek diversity but specifically in their second degree neighbors. This behavior can be explained by various reasons. For instance, an agent wants to be comfortable in its immediate vicinity but at the same time agent can desire to be informed from different type agents or to have these contacts in its distant vicinity for the social advantages [29].

Equilibrium networks change to a great extent when agents seek diversity in their second degree neighbors. Maze-like patterns emerge when α is -0.5 and become apparent when α is -1. When α is -1, agents prefer similar people in their immediate vicinities and diversity in their second degree neighbors. Even though agents have 8 same type neighbors in their immediate vicinities, to satisfy their utility functions agents need different type agents in their distant vicinities. Therefore, similar agents cluster, however, these clusters are small enough for every agent in the cluster to have different agent types in their second degree neighbors. Inevitably maze patterns emerge.

Chapter 6

Conclusion and Future Work

We started our studies by analyzing the Schelling models with Glauber dynamics on the regular square lattice. We obtained consistent results with the literature. Schelling models (when threshold is 0.5) on 120x120 square lattice with Glauber dynamics ,periodic boundary conditions and Moore neighborhood ($w = 1$) where agents initially distributed using Bernoulli distribution with parameter 0.5 cannot reach a consensus (ground state). Knowing this, we investigated if random links can help communities to reach a consensus.

Changing node degrees by random links can help communities to reach a consensus. The main reason for this is, randomness decreases clustering coefficients and average shortest path lengths. In the regular square lattice, agents initially cluster with the same type agents on different parts of the network and these highly clustered knits remain until the equilibrium. As average shortest path length decreases, different parts of the network are become connected to each other and as clustering coefficient decreases, these highly clustered knits break. Therefore, communities can reach a consensus.

In addition to the models above, we introduced a new agent type called XY which represents doubtful agents who have the benefit of doubt. Two different models are created according to what X and Y agents do when doubtful agents are in the majority in their neighborhoods. First model is DC model where XY agents can give the benefit of doubt to X and Y agents if they are the majority. DC model can reach a consensus, where every agent in the network is doubtful, with random links. Second model is PC model where XY agents cannot give the benefit of doubt to X and Y agents. In PC model X and Y agents persist on their views and reject to become doubtful. These communities can also reach a consensus however, no doubtful agents remain in the equilibrium.

Afterwards, we asked the question "what happens if doubtful agents remain doubtful for longer time periods?". We showed that even if the doubtful agents remain doubtful until the equilibrium, communities cannot reach a consensus without having random links. These findings are consistent with our explanations on how do random links help communities to reach a consensus. Because even though one agent type never changes in the square lattice initially there are highly knitted clusters consist of one agent type which remain until the equilibrium.

Lastly, we introduced a new diversity seeking approach where agents seek diversity in their distant vicinities. We found that maze patterns emerge if agents give equal or more importance to having diversity in their vicinities than having similar people in their immediate vicinities.

In this thesis, our models have one dimensional issue space. Schelling focuses only to the race in his models, and we focus on social norms and issues such as recycling. For future studies, the dimension of issue space can be increased. For instance, focusing on race, demographic and socioeconomic attributes at the same time is a more realistic approach. Sturgis [29] shows that different age groups who belong to the same ethnicity may have different preferences about who they want to live with. Therefore, increasing the dimension of issue space can help to understand segregation in real life better.

Instead of increasing the dimension of issue space, more agent types can be introduced to the models. DC and PC models are the examples of models with more agent types. Suppose a line segment between -1 and 1 which represent one dimensional issue space. Since X and Y represent opposite stands on an issue, in this line -1 denotes X agents, 1 denotes Y agents and 0 denotes XY agents. Introducing agents who place themselves between these points to the models would be the more realistic approach.

Furthermore, random links can be added to the models in which individuals that seek diversity in their distant vicinities for further studies.

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Appendices

Appendix A

Periodic Boundary Conditions

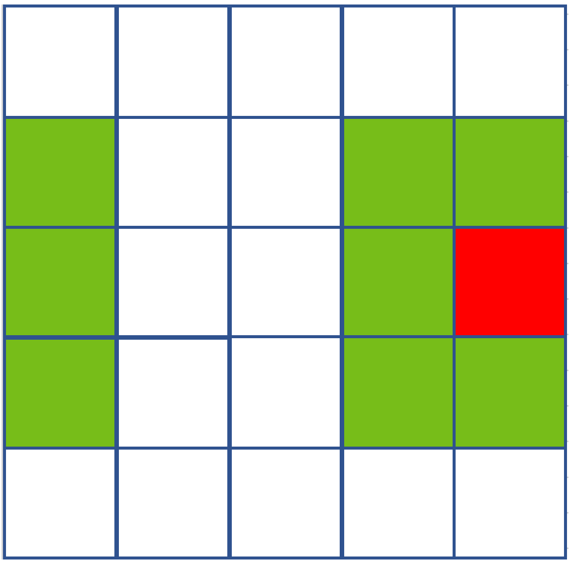

Figure A.1: Representation of periodic boundary conditions on a 5x5 square lattice. Green nodes are the neighbors of the red node.

Appendix B

Von Neumann and Moore Neighborhoods

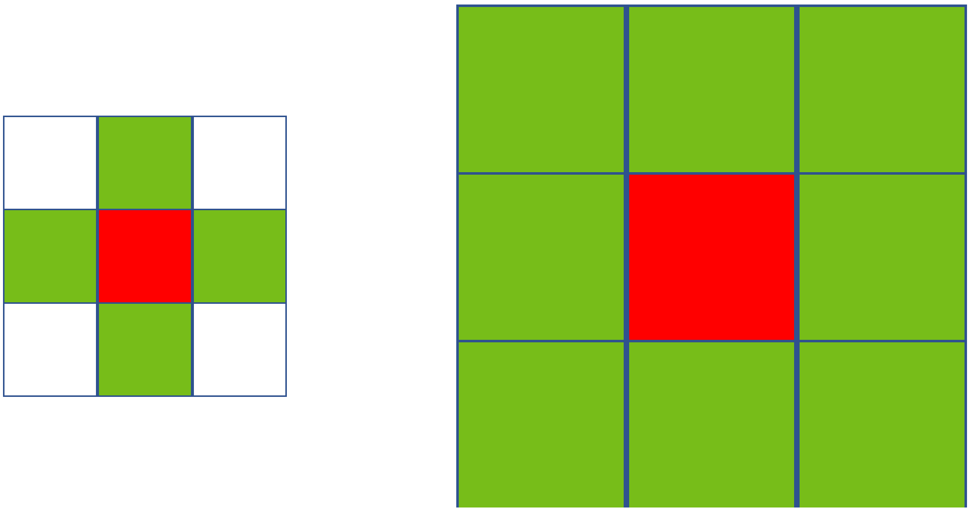

Figure B.1: Differences between these two neighborhoods are demonstrated in 3x3 part of a square lattice. Green nodes are the neighbors of the red nodes. Figure on the left demonstrates Von Neumann Neighborhood and figure on the right demonstrates Moore Neighborhood.

Appendix C

Initial and Equilibrium Network Examples

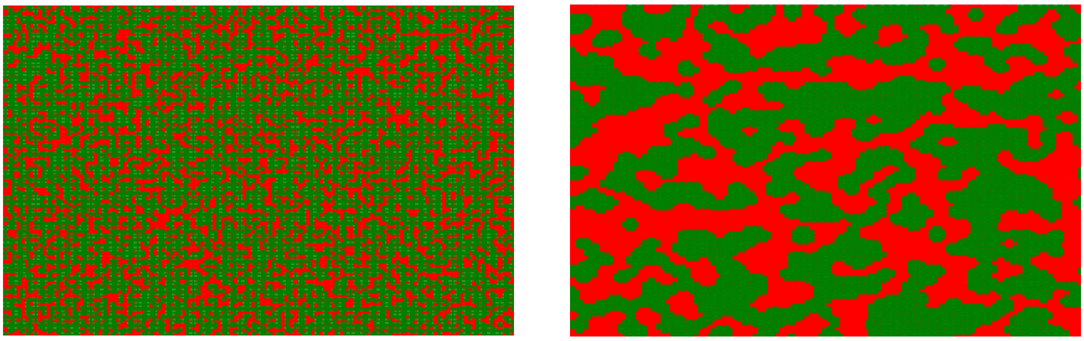

Figure C.1: First example of initial and equilibrium networks. Network is 120x120 square lattice with periodic boundary conditions and Moore neighborhood. w=1. Figure on the left is the initial network and figure on the right is the equilibrium network.

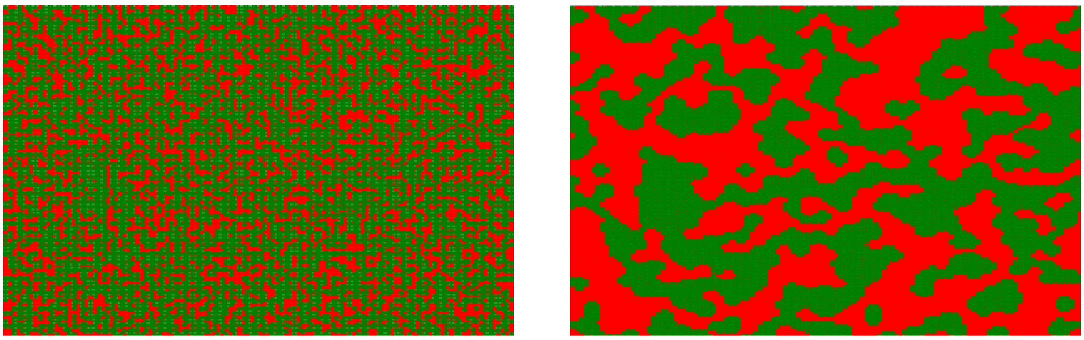

Figure C.2: Second example of initial and equilibrium networks. Network is 120x120 square lattice with periodic boundary conditions and Moore neighborhood. w=1. Figure on the left is the initial network and figure on the right is the equilibrium network.

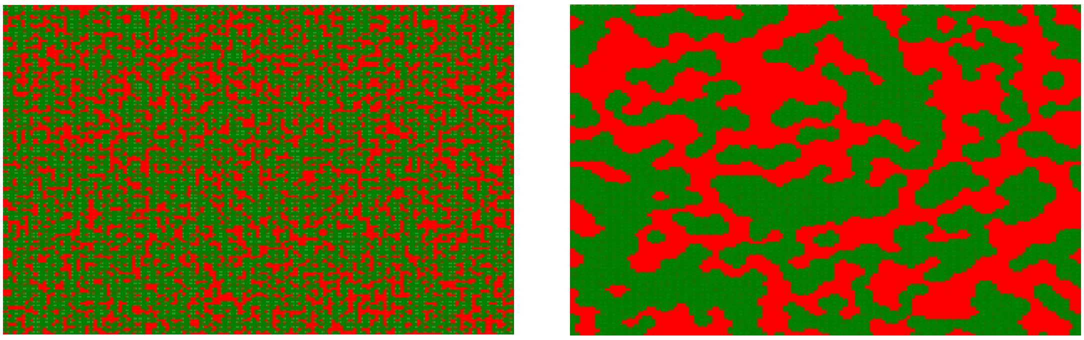

Figure C.3: Third example of initial and equilibrium networks. Network is 120x120 square lattice with periodic boundary conditions and Moore neighborhood. w=1. Figure on the left is the initial network and figure on the right is the equilibrium network.

Appendix D

Clustering Coefficients and Rewiring Probabilities for Different Rewiring Probabilities

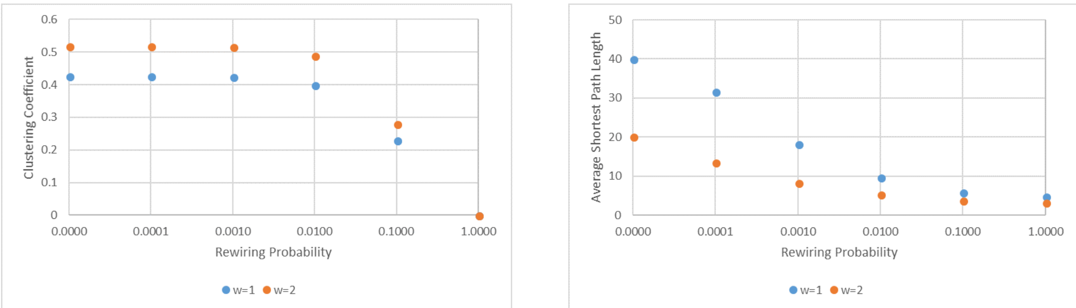

Figure D.1: Figure on the left shows clustering coefficient values and figure on the right shows average shortest path length for different rewiring probabilities. Initial network is 120x120 square lattice with periodic boundary conditions and Moore neighborhood.

Appendix E

Degree Distributions for Different Rewiring Probabilities

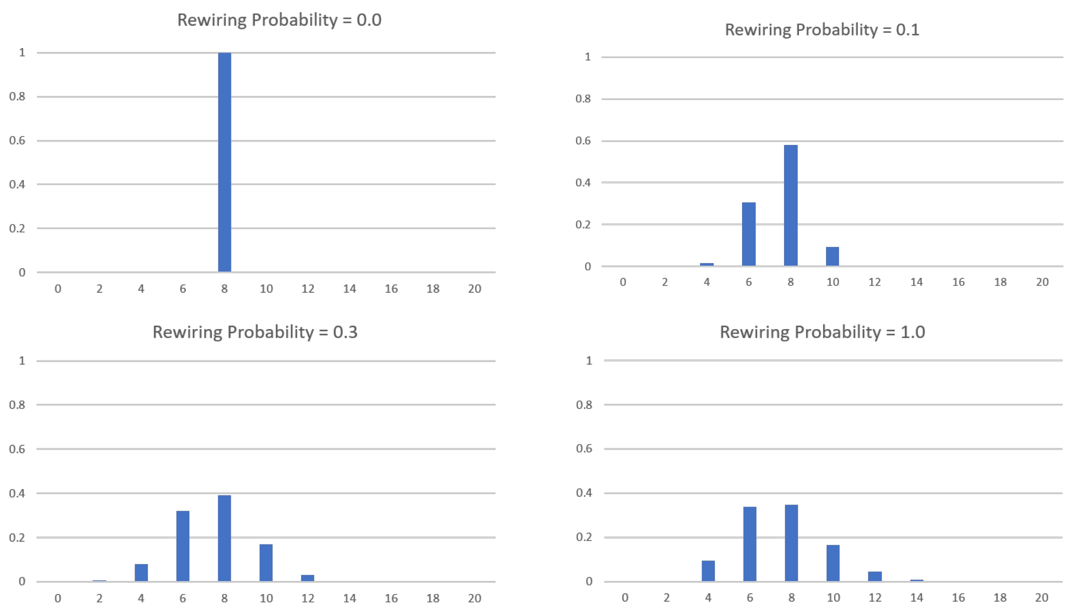

Figure E.1: Degree distributions for different rewiring probabilities. Initial network is 120x120 square lattice with periodic boundary conditions and Moore neighborhood. w=1.