

**OPTIMAL CAPITAL INCOME TAXATION UNDER CAPITAL-SKILL  
COMPLEMENTARITY**

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COMPLEMENTARITY

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Economics, Master of Arts Thesis, July 2017

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## **Abstract**

This paper analyses the impact of capital-skill complementarity on optimal capital income taxation problem for a redistributive government. We compare an infinite horizon heterogeneous agents incomplete market model in which the technology exhibits capital-skill complementarity to a model in which we keep all the properties of the former but eliminate capital-skill complementarity. We find that under capital-skill complementarity the optimal tax rate on capital income is 0.71, whereas it is 0.24 for the no complementarity case. The former is significantly higher due to extra redistributive channel that results from the relation between capital accumulation and skill premium.

**Keywords:** Capital-skill complementarity, capital income taxation, skill premium, incomplete markets, redistribution.

# SERMAYE-EĐİTİM TAMAMLAYICILIĐI ALTINDA OPTIMAL SERMAYE GELİRİ VERGİLENDİRMESİ

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## Özet

Bu çalışma sermaye-eđitim tamamlayıcılıđının optimal sermaye geliri vergilendirmesi problemindeki etkisini incelemektedir. Bunun için teknolojinin sermaye-eđitim tamamlayıcılıđı özelliđini taşıdıđı sonsuz zamanlı heterojen bireylerden oluşan bir eksik market modeli ile sermaye-eđitim tamamlayıcılıđının olmadığı ama diđer özelliklerin korunduđu bir modeli karşılaştırdık. Sonuçlarımıza göre, sermaye birikimi ve eđitim primi arasındaki ilişkidен kaynaklanan dolaylı yeniden bölüşüm kanalı nedeniyle, sermaye-eđitim tamamlayıcılıđı altında optimal sermaye geliri vergi oranı sermaye-eđitim tamamlayıcılıđının olmadığı modelden önemli ölçüde fazladır. Sermaye-eđitim tamamlayıcılıđı altında optimal vergi oranı 0.71 olurken, diđer modelde 0.24 olmuştur.

**Anahtar Kelimeler:**Sermaye-eđitim tamamlayıcılıđı, sermaye geliri vergilendirmesi, eđitim primi, eksik marketler, yeniden bölüşüm

*To my grandma*

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# 1 Introduction

In this study, we elaborate on optimal capital income taxation in the presence of capital-skill complementarity. We build an infinite horizon incomplete market model in which the production function exhibits capital-skill complementarity and agents are ex-ante different in their skill levels. As we summarize in the literature review part, taxation of capital income is a debatable topic as different paradigms assert different policies. The literature on representative-agent framework argues that optimal tax rate on capital income is zero in the long run. However, when market incompleteness is introduced and heterogeneity among households is allowed, optimal long-run capital taxes might be positive. We analyze optimal capital income taxation under incomplete markets by adding capital-skill complementarity assumption. The notion of capital-skill complementarity is formalized by Griliches (1969). It simply means that, capital is more complementary with skilled labor than unskilled labor. The main implication of capital-skill complementarity is that when the stock of capital grows, it tends to increase the relative marginal product of skilled labor. As a consequence, the ratio of marginal product of skilled labor to marginal product of unskilled labor, the skill premium, increases as capital stock grows. Under capital-skill complementarity, taxing capital decreases capital accumulation which then decreases the skill premium. Thus, taxing capital has an additional redistributive benefit which is the main point of our model. We aim to quantitatively assess the effects of this extra force while determining the optimal tax rate on capital income.

We construct two infinite horizon incomplete market models in which agents are ex-ante different in their skill levels. There is also ex-post heterogeneity since they are subject of uninsurable idiosyncratic skill group specific productivity shocks at every period, that is, at a given period of time they have different histories in terms of shocks. In the first model, technology exhibits capital-skill complementarity. To better understand the extra forces that results from capital-skill complementarity, we proceed with standard Cobb-Douglas production function in our second model. We calibrate our models to the US economy, and compute the tax rate on capital income which maximizes a utilitarian social welfare function with equal weights for each agent for both models.

Our results are in line with the intuition that we provide at the beginning. In the presence of capital-skill complementarity, optimal capital income tax rate is significantly higher than the optimal tax rate under no complementarity case, and the resulting allocation has more equal distribution as higher tax rate indirectly redistributes from the skilled agents to the unskilled ones. Higher tax rate decreases capital accumulation, and due to capital-skill complementarity, it affects the underlying distribution of labor income so that the skill premium declines. We find that optimal capital income tax rate is 0.7 under capital-skill complementarity. The corresponding skill premium is 1.74 which is lower than the skill premium under the status quo tax rate. At the optimal policy, the ratio of average consumption level of skilled type to average consumption level of unskilled type is declined to 1.31 from 1.44. Hence the corresponding allocation is better in terms of equality relative to the status quo allocation. To the contrary, under the no complementarity case optimal policy has more unequal distribution relative to the status quo policy. We find that optimal capital income tax rate is 0.24 in that case. The skill premium does not change as it is not depend on aggregates when we eliminate capital-skill complementarity. Also, the ratio of average consumption level of skilled type to average consumption level of unskilled type almost the same with the status quo.

## 2 Literature Review

This paper is related to two strands of the literature. First, the paper is related to the literature on taxation of capital income which is a controversial issue in the macroeconomics literature. In representative-agent paradigm, Chamley (1986) and Judd (1985) have shown that the optimal capital income tax rate is zero in the long-run. Lucas (1990) has shown that eliminating capital income tax can bring substantial welfare gains under complete-market infinite horizon setting. However, in incomplete market paradigm the optimal tax rate for capital income may be strictly positive in the steady state. Aiyagari (1995) has shown that optimal long-run capital income taxes might be positive when there is uninsured labor income risk due to incomplete markets. He point out that optimal steady state capital income tax is between 25% and 45% depending on parameters.

Imrohoroglu (1998) has shown that in an overlapping generation model with borrowing constraints and uninsurable idiosyncratic earnings risk, optimal tax rate on capital income is 15% at the steady state. Conesa, Kitao, and Krueger (2009) analyze optimal steady state capital income taxes in an Aiyagari economy. They point out that optimal

capital income tax rate is positive at 36%. Domeij and Heathcote (2004) compare a representative agent economy with an economy in which agents are exposed to uninsurable idiosyncratic labor income risks. They find that under representative-agent economy, eliminating capital income tax brings large welfare gain. However, when market incompleteness is introduced, most households have welfare losses under zero capital income tax. They find that when markets are incomplete, decreasing capital taxes increases productive efficiency (decrease consumption in the short run and increase it in the long run, and the latter dominates the former) but decreases redistribution (because it is the rich that hold most of the capital). The optimal rate that comes out of this trade off in their analysis is 39.7% when transitional dynamics are taken into account. Under pure steady-state comparison, they find that optimal tax rate on capital income is 17.6%.

Slavik and Yazıcı (2014) and Slavik and Yazıcı (2015) are the first papers which studied capital taxation under capital-skill complementarity. These papers are the most relevant ones for our study. Slavik and Yazıcı (2014) have studied the optimality of differential capital taxation using a model with structure capital and equipment capital in the presence of equipment-skill complementarity. They show that it is optimal to tax equipments at a higher rate than structures since depressing accumulation of equipment capital lowers the skill premium hence provides indirect redistribution from the skilled to the unskilled agents. Also, Slavik and Yazıcı (2015) have analyzed the consequences of the uniform capital tax reform using an incomplete markets model with two types of capital and equipment-skill complementarity. They find that the reform not only increases productive efficiency by reallocating capital from low to high return capital but also improves equality by decreasing the skill premium. Different from Slavik and Yazıcı (2014), we analyze the optimal tax rate on overall capital under capital-skill complementarity in an incomplete market environment.

Second, since we have concerned with the role of capital-skill complementarity on capital income taxation problem, the paper is related to the literature on capital-skill complementarity and one of the its main consequences; the increase in skill premium after 1970s onward. Many studies have agreed on that latent skill-biased technical change is the main factor responsible for this increase. Bound and Johnson (1992) have concluded that much of the variation in the skill premium is attributed to a residual trend component that is often called skill-biased technical change. Acemoğlu (2002) has concluded that technical change favors more skilled workers, and intensifies inequality. Buera, Kaboski, and Rogerson (2015) have concluded that due to skill-biased structural change the demand for high-skill labor has increased and that increase accounts for between 25 and

30% of the overall increase of the skill premium over the period 1977 to 2005 in advanced economies.

Krusell, Ohanian, Rios-Rull, and Violante (2000) have developed a mechanism for understanding skill-biased technical change in terms of observable variables. They have introduced a four factor aggregate production function that explicitly distinguishes between equipment capital, capital structures, skilled labor, and unskilled labor and that allows for different elasticities of substitution between unskilled labor and capital equipment and between skilled labor and capital equipment. They have estimated the parameters that govern these elasticities. Their estimates imply that as stock of equipment capital increases, marginal product of skilled labor also increases, but marginal product of unskilled labor decreases. They also indicate that the stock of equipment capital has been growing at about twice the rate of either capital structures or consumption over much of the post-war period. Even though the relative supply of skilled labor has increased during this period, due to the increase in the stock of equipment capital, demand for skilled labor supply has also increased so that skilled wage has shown upward trend. Slavik and Yazıcı (2016) have studied the effect of the rise in wage risk on the skill premium by building an incomplete market model which exhibits equipment-skill complementarity. They find that a significant fraction of the rise in the skill premium over the years 1967 and 2010 is due to the rise in the wage risk that workers face.

## 3 Models

### 3.1 First Model: Capital-Skill Complementarity

We construct an infinite horizon incomplete market model in which agents are ex-ante different in their skill levels. There is also ex-post heterogeneity since agents receive uninsurable idiosyncratic skill group specific productivity shocks at every period. The production function uses skilled and unskilled labor, structure capital and equipment capital under constant returns to scale technology. The key property of the production function is equipment-skill complementarity.

**Households:** The economy is populated by infinitely lived households who are either skilled or unskilled. The skill types are permanent. The proportion of the skilled type in the population is  $\pi_s$ , and the proportion of the unskilled type is  $\pi_u$ , where  $\pi_s + \pi_u = 1$ . Each

household is indexed by  $i \in (S, U)$ . Households are endowed with one unit of time at every period which is allocated for work and leisure. Skilled workers and unskilled workers are not perfect substitutes, hence they work in different sectors. A worker of type  $i$  earns  $w_i$  for each unit of effective labor she supplies.

Households are subject of skill group specific idiosyncratic uninsurable productivity shocks at every period. Within each skill group, households draw their productivity shocks independent from one another according to a skill group specific stochastic process  $z_i$  where  $i \in (S, U)$ . The type-specific productivity shocks follow Markov chain with states  $Z_i = (z_{i_1}, \dots, z_{i_J})$  and transitions  $\pi_i(Z'|Z)$ . When an agent with shock  $z$  allocates  $l$  units of time to leisure, and works  $(1-l)$  units of time, her effective labor supply is  $(1-l)z$ .

There is uncertainty at the individual level, but there is no uncertainty at the aggregate level. Households derive utility from consumption and leisure. Preferences are described by:

$$E_i \left[ \sum_{t=0}^{\infty} \beta^t U(c_{it}, l_{it}) \right]$$

where  $\beta$  is the discount factor,  $c_{it}$  is the consumption level of type  $i$  at period  $t$ ,  $l_{it}$  is the leisure level of type  $i$  at period  $t$ , and expectation  $E_i$  is taken with respect to  $z_i$ .  $U(\cdot)$  is strictly increasing and concave in consumption and leisure.

Households can save through investing on capital or investing on risk free asset.

**Firm:** There is a representative firm which operates under constant returns to scale production function. It employs skilled and unskilled labor, and rents to types of capital, capital structures and equipment capital. Output equals to:

$$Y = F(K_s, K_e, N_s, N_u)$$

where  $K_s$ = capital structures,  $K_e$ = equipment capital,  $N_s$ = effective skilled labor,  $N_u$ = effective unskilled labor. The relative prices of  $K_s$  and  $K_e$  in terms of the output are 1 and  $q$ , respectively. In the quantitative analysis, we set  $q=1$ . One period depreciation rate for capital structures and equipment capital are  $\delta_s$  and  $\delta_e$ , respectively.

We use the production function that is introduced by Krusell, Ohanian, Rios-Rull, and Violante (2000) which exhibits equipment-skill complementarity. That is, the elasticity of substitution between equipment capital and unskilled labor is higher than that between equipment capital and skilled labor. On the other hand, the elasticity of substitution between capital structures and skilled labor is same as that between capital structures and

unskilled labor. Under equipment-skill complementarity, the growth of stock of equipment capital increases relative marginal product of skilled labor which implies that skill premium, hence wage inequality, increases as stock of equipment capital increases. The firm solves the following maximization problem at every period:

$$\max_{N_s, N_u, K_s, K_e} F(K_s, K_e, N_s, N_u) - r_s K_s - r_e K_e - w_s N_s - w_u N_u$$

where  $r_s$ = rental rate on capital structures at period t,  $r_e$ = rental rate on equipment capital at period t,  $w_s$ = wage rate for skilled workers at period t,  $w_u$ = wage rate for unskilled workers at period t.

**Asset Market:** There is a single risk-free asset which has one period maturity. Households can save by using this asset, but they are not allowed to borrow.  $A=[0, \infty]$  represents the possible asset levels that households can hold.

**Government:** The government imposes linear taxes on capital income and labor income at every period. Capital structures and equipment capital are taxed at the same rate,  $\tau_k$ . Similarly, there is a single labor income tax rate  $\tau_n$  for skilled and unskilled agents. Tax revenue is used to finance government expenditures and the remaining part is redistributed equally to the households through lump-sum transfers. Here, lump-sum transfers play an important role. Under linear taxation, low income taxpayers are treated unequally relative to high income taxpayers. Lump-sum transfers partially fix for this unequal treatment as everybody receives the same amount of transfer but it has a relatively greater impact on low income taxpayers' disposable income. Finally, the government expenditures equal to a certain fraction of output at every period.

### Stationary Recursive Competitive Equilibrium:

**Definiton:** Let  $a \in A=[0, \infty]$  and  $z_i \in Z_i$  for  $i \in (S, U)$ . Given borrowing limit, prices  $w_u, w_s, r_s, r_e$ , and  $R$ , a *stationary recursive competitive equilibrium* consists of two value functions  $V_u(a, z_u), V_s(a, z_s)$ , policy functions  $c_u(a, z_u), c_s(a, z_s), l_u(a, z_u), l_s(a, z_s), a'_u(a, z_u), a'_s(a, z_s)$ , the firm's decision rules  $K_s, K_e, N_s, N_u$ , the government's policies  $\tau_n, \tau_k, G$  and two stationary distributions  $\lambda_u(a, z_u), \lambda_s(a, z_s)$  such that

1. Given prices and government policies, the value function  $V_i(a, z_i)$ , and the policy functions  $c_i(a, z_i), l_i(a, z_i), a'_i(a, z_i)$  solve type i's problem for all  $i \in (S, U)$ :

$$V_i(a, z_i) = \max_{c_i, l_i, a'_i} (U(c_i, l_i) + \beta E_i[V_i(a', z'_i)])$$

s.t.  $c_i + a'_i = \tau_n w_i z_i (1 - l_i) + R a_i + l_s$  where  $l_s$  is the lump-sum government transfer and  $R = 1 + (r_s - \delta_s)(1 - \tau_k) = \frac{q + (r_e - q\delta_e)(1 - \tau_k)}{q}$  is the after-tax net return on asset.

2. Given prices, the allocation  $(K_e, K_s, N_s, N_u)$  solves the firm's problem:

$$\max_{N_s, N_u, K_s, K_e} F(K_s, K_e, N_s, N_u) - r_s K_s - r_e K_e - w_s N_s - w_u N_u$$

3. For each  $i \in (S, U)$ ,  $\lambda_i(a, z_i)$  satisfies the following:

$$\lambda'_i(z, a) = \lambda_i(z, a)$$

$$\text{where } \lambda_i(z', a') = \sum_{z \in Z_i} \pi_i(z' | z) \int_{a: a'_i(z, a) \leq a'} d\lambda_i(z, a) \quad \forall (z', a')$$

4. All markets clear:

$$F(K_e, K_s, N_s, N_u) + (1 - \delta_e)K_e + (1 - \delta_s)K_s = C + G + LS + K_e + K_s$$

where LS: total lump-sum transfers, G: government expenditures, and C: total consumption

$$C = \pi_u \int_A \int_{Z_u} c_u(a, z_u) d\lambda_u(a, z_u) + \pi_s \int_A \int_{Z_s} c_s(a, z_s) d\lambda_s(a, z_s)$$

$$K_s + K_e = \pi_u \int_A \int_{Z_u} a'_u(a, z_u) d\lambda_u(a, z_u) + \pi_s \int_A \int_{Z_s} a'_s(a, z_s) d\lambda_s(a, z_s)$$

$$N_u = \pi_u \int_A \int_{Z_u} (1 - l_u(a, z_u)) d\lambda_u(a, z_u)$$

$$N_s = \pi_s \int_A \int_{Z_s} (1 - l_s(a, z_s)) d\lambda_s(a, z_s)$$

5. The government runs a balanced budget:

$$G + LS = \tau_n (w_u N_u + w_s N_s) + \tau_k (r_s - \delta_s) K_s + \tau_k (r_e - \delta_e) K_e$$



### 3.2 Second Model: No Complementarity

Since the main goal of this paper is to understand the effects of capital-skill complementarity in optimal capital income taxation problem, in our second economy we eliminate capital-skill complementarity from the production function and preserve all other properties of our first model. We do not distinguish between equipment capital and structure capital, we have one type of capital which depreciates every period at rate  $\delta$ . We use the following Cobb-Douglas production function:

$$Y = F(K, N_s, N_u) = TK^\theta(\mu N_s + N_u)^{1-\theta}$$

where T: total factor productivity,  $\theta$ = share of capital, and  $\frac{MPN_s}{MPN_u} = \mu$

Under this production function, the ratio of marginal product of skilled labor to marginal product of unskilled labor, hence the skill premium, is constant and equals to  $\mu$ . That is, skill premium does not depend on the aggregates. The changes in the aggregate capital level do not affect skill premium, therefore here capital income taxation has no impact on wage inequality.

## Stationary Recursive Competitive Equilibrium (Second Model) :

**Definiton:** Let  $a \in A=[0,\infty]$  and  $z_i \in Z_i$  for  $i \in (S,U)$ . Given borrowing limit, prices  $w_u$ ,  $w_s$ ,  $r$ , and  $R$ , a *stationary recursive competitive equilibrium*, henceforth SRCE, consists of two value functions  $V_u(a, z_u)$ ,  $V_s(a, z_s)$ , policy functions  $c_u(a, z_u)$ ,  $c_s(a, z_s)$ ,  $l_u(a, z_u)$ ,  $l_s(a, z_s)$ ,  $a'_u(a, z_u)$ ,  $a'_s(a, z_s)$ , the firm's decision rules  $K$ ,  $N_s$ ,  $N_u$ , the government's policies  $\tau_n$ ,  $\tau_k$ ,  $G$  and two stationary distributions  $\lambda_u(a, z_u)$ ,  $\lambda_s(a, z_s)$  such that

1. Given prices and government policies, the value function  $V_i(a, z_i)$ , and the policy functions  $c_i(a, z_i)$ ,  $l_i(a, z_i)$ ,  $a'_i(a, z_i)$  solve type  $i$ 's problem for all  $i \in (S,U)$ :

$$V_i(a, z_i) = \max_{c_i, l_i, a'_i} (U(c_i, l_i) + \beta E_i[V_i(a', z'_i)])$$

s.t.  $c_i + a'_i = \tau_n w_i z_i (1 - l_i) + R a_i + l_s$  where  $l_s$  is the lump-sum government transfer and  $R = 1 + (r - \delta)(1 - \tau_k)$  is the after-tax net return on asset.

2. Given prices, the allocation  $(K, N_s, N_u)$  solves the firm's problem:

$$\max_{N_s, N_u, K} F(K, N_s, N_u) - rK - w_s N_s - w_u N_u$$

3. For each  $i \in (S,U)$ ,  $\lambda_i(a, z_i)$  satisfies the following:

$$\lambda'_i(z, a) = \lambda_i(z, a)$$

where  $\lambda_i(z', a') = \sum_{z \in Z_i} \pi_i(z'|z) \int_{a: a'_i(z, a) \leq a'} d\lambda_i(z, a) \forall (z', a')$

4. All markets clear:

$$F(K, N_s, N_u) + (1 - \delta)K + = C + G + LS + K$$

where LS:total lump-sum transfers, G: government expenditures, and C:total consumption

$$C = \pi_u \int_A \int_{Z_u} c_u(a, z_u) d\lambda_u(a, z_u) + \pi_s \int_A \int_{Z_s} c_s(a, z_s) d\lambda_s(a, z_s)$$

$$K = \pi_u \int_A \int_{Z_u} a'_u(a, z_u) d\lambda_u(a, z_u) + \pi_s \int_A \int_{Z_s} a'_s(a, z_s) d\lambda_s(a, z_s)$$

$$N_u = \pi_u \int_A \int_{Z_u} (1 - l_u(a, z_u)) d\lambda_u(a, z_u)$$

$$N_s = \pi_s \int_A \int_{Z_s} (1 - l_s(a, z_s)) d\lambda_s(a, z_s)$$

5. The government runs a balanced budget:

$$G + LS = \tau_n(w_u N_u + w_s N_s) + \tau_k(r - \delta)K$$

### 3.3 The Optimal Tax Problem

The optimal tax problem is to find the marginal tax rate  $\tau_k$  on capital income that produces a competitive equilibrium such that the utilitarian social welfare function which treats all agents equally is maximized in the steady state.

$$\max_{\tau_k} W = \pi_s \int_a \int_{Z_s} V_s(a, z_s) d\lambda_s(a, z_s) + \pi_u \int_a \int_{Z_u} V_u(a, z_u) d\lambda_s(a, z_u)$$

s.t. for every possible  $\tau_k$  along with other policy parameters allocation is given by the corresponding stationary recursive competitive equilibrium

## 4 Parameterization and Calibration

We fix a number of parameters to values from the data or from the literature, table 1 summarizes these parameters. We pick the parameters so that they match to the parameters of US economy in 2000s. The remaining parameters are calibrated to match specific targets of US economy for the relevant years. Table 2 summarizes our calibration procedure. The model period corresponds to one year, hence all parameter values are reported in yearly basis.

First of all, we assume that the utility function has the Balanced Growth Path compatible form, that is,

$$U(c, l) = \frac{[c^\phi l^{(1-\phi)}]^\frac{1-\sigma}{\phi} - 1}{\frac{1-\sigma}{\phi}}$$

$\sigma$  is the relative risk aversion parameter,  $\phi$  together with  $\sigma$  controls for the average labor supply and Frisch elasticity of labor. We set  $\sigma=2$ , and calibrate  $\phi$  to match the aggregate labor supply in US economy. Our target for the aggregate labor supply is 1/3, the value that is commonly used in the macro literature.

The productivity shocks follow the following AR(1) process:

$$\log z_{t+1} = \rho_i \log z_t + \epsilon_{i,t} \quad \forall i \in (S, U)$$

We use the values from Krueger and Ludwig (2015) for parameterization of the processes for productivity shocks. Following Kopecky and Suen (2010), we use the Rouwenhorst method to approximate these processes by finite number Markov chains. The proportion of the skilled population is  $\pi_s = 0.4376$  which is consistent with the 2011 US Census data. Skilled agents are those who have college education or above, and rest of the agents are unskilled ones.

In our first economy, we use the following production function that is introduced by Krusell, Ohanian, Rios-Rull, and Violante (2000):

$$Y = F(K_s, K_e, N_s, N_u) = K_s^\alpha (\nu [\omega K_e^\rho + (1 - \omega) N_s^\rho]^\frac{\eta}{\rho} + (1 - \nu) N_u^\eta)^\frac{1-\alpha}{\eta}$$

In this function,  $\rho$  stands for the degree of complementarity between equipment capital and skilled labor, and  $\eta$  stands for the degree of complementarity between equipment capital and unskilled labor. The elasticity of substitution between equipment capital and unskilled labor is  $\frac{1}{1-\eta}$ , this is also the elasticity of substitution between skilled and unskilled labor. The elasticity of substitution between capital equipment and skilled labor is  $\frac{1}{1-\rho}$ . Equipment-skill complementarity implies that  $\rho < \eta$ .  $\omega$  and  $\nu$  control the income share of equipment capital, skilled and unskilled labor in output. We use Krusell, Ohanian, Rios-Rull, and Violante (2000)'s estimates for  $\alpha$ ,  $\eta$ ,  $\rho$ . Since they do not have estimates for  $\omega$  and  $\nu$ , we calibrate these parameters. We target to match labor share of income and skill premium to calibrate  $\omega$  and  $\nu$ , respectively. We take the labor share as 2/3 which is the average labor share during the period 1980-2010 as reported in the National Income

and Product Accounts (NIPA) data. According to Slavik and Yazıcı (2016), the skill premium,  $w_s/w_u$ , equals to 1.9 in 2010, our benchmark specifications are in line with this finding.

In our second economy, we eliminate equipment-skill complementarity, and use the following production function:

$$Y = F(K, N_s, N_u) = TK^\theta(\mu N_s + N_u)^{1-\theta}$$

In the above production function, we set capital's share of output,  $\theta$ , to 0.36 as is commonly used in the macro literature.  $\mu$  stands for the skill premium, we include it to capture the wage inequality between skilled and unskilled agents under the Cobb-Douglas specification, hence we set  $\mu=1.9$ . We include  $T$  as total factor productivity. Our choice of  $T$  guarantees that under the benchmark specifications, two economies have the same level of output.

In our benchmark specifications, we take labor income tax rate and capital incomes tax rate as 0.28 and 0.36, respectively. These values are the average tax rates in the US economy during the period 1995-2007 which calculated by Trabandt and Uhlig (2011). We set the government expenditures-to-output ratio to 16% which is close to the average ratio over the years 1980 to 2012 according to NIPA.

Finally, we calibrate discount factor  $\beta$  so that the capital-to-output ratio equals to 3 which is the average of 1970-2011 according to NIPA and Fixed Asset Tables data.

Table 1: Benchmark Parameters

| Parameters  | Symbol            | Value  | Source      |
|---|-------------------|--------|-------------|
| Relative risk aversion parameter  | $\sigma$          | 2      |             |
| Technology (First Model)  |                   |        |             |
| Structure capital depreciation rate   | $\delta_s$        | 0.056  | GHK         |
| Equipment capital depreciation rate   | $\delta_e$        | 0.124  | GHK         |
| Measure of elasticity of substitution between equipment capital and unskilled labor | $\eta$            | 0.401  | KORV        |
| Measure of elasticity of substitution between equipment capital and skilled labor   | $\rho$            | -0.495 | KORV        |
| Share of structure capital in output  | $\alpha$          | 0.117  | KORV        |
| Technology (Second Model)   |                   |        |             |
| Capital's share of output   | $\theta$          | 0.36   |             |
| Coefficient of the skilled labor in the production function                         | $\mu$             | 1.9    |             |
| Depreciation rate of capital  | $\delta$          | 0.07   | KL          |
| Fraction of skilled agents  | $\pi_s$           | 0.4376 | U.S. Census |
| Productivity Shocks   |                   |        |             |
| Productivity persistence of skilled agents  | $\rho_s$          | 0.9690 | KL          |
| Productivity volatility of skilled agents   | $Var(\epsilon_s)$ | 0.0100 | KL          |
| Productivity persistence of unskilled agents  | $\rho_u$          | 0.9280 | KL          |
| Productivity volatility of unskilled agents   | $Var(\epsilon_u)$ | 0.0192 | KL          |
| Government policies   |                   |        |             |
| Labor income tax rate   | $\tau_n$          | 0.28   | TU          |
| Capital income tax rate   | $\tau_k$          | 0.36   | TU          |
| Government expenditures   | $G/Y$             | 0.16   | NIPA        |

This table shows the benchmark parameters that we take from the literature or the data.

NIPA: National Income and Product Accounts, KL: Krueger and Ludwig (2015), TU: Trabandt and Uhlig (2011), GHK: Greenwood, Hercowitz, and Krusell(1997), KORV: Krusell, Ohanian, Rios-Rull and Violante (2000)

Table 2: Internally Calibrated Parameters

| Parameters                | Symbol       | Value  | Target                                    | Data | Source   |
|---------------------------|--------------|--------|---|------|----------|
| First Model               |              |        |   |      |          |
| Production parameter      | $\omega$     | 0.4305 | labor share                               | 2/3  | NIPA     |
| Production parameter      | $\nu$        | 0.6982 | skill premium in 2010                     | 1.9  | SY       |
| Disutility of labor       | $\phi$       | 0.3892 | labor supply                              | 1/3  |          |
| Discount factor           | $\beta$      | 0.9750 | capital-to-output ratio                   | 3    | NIPA,FAT |
| Second Model              |              |        |   |      |          |
| Disutility of labor       | $\phi^{nc}$  | 0.4227 | labor supply                              | 1/3  |          |
| Discount factor           | $\beta^{nc}$ | 0.9577 | capital-to-output ratio                   | 3    | NIPA,FAT |
| Total factor productivity | $T$          | 0.39   | benchmark output level in the first model |      |          |

This table shows our calibration strategy.

SY: Slavik and Yazıcı (2016), NIPA: National Income and Product Accounts, FAT: Fixed Asset Tables

## 5 Results

### 5.1 Optimal Policy vs. Benchmark Policy Under Equipment-Skill Complementarity

We find that optimal capital income tax rate under equipment-skill complementarity is 0.71; that is, the utilitarian social welfare function which puts equal weights to all agents is maximized when  $\tau_k = 0.71$ . Table 3 compares the steady states under benchmark economy and under optimal policy.

There are remarkable differences between two steady states. Output decreases by 12.42% as all of the inputs decline under the new steady state. Switching to the optimal policy distorts the total capital stock by 26.81%. The most drastic decline is observed in the stock of structure capital, it decreases by 33.06%, whereas stock of equipment capital decreases by 19.16%. One of the key properties of our production function is that

the complementarity between structure capital and unskilled labor and the complementarity between structure capital and skilled labor are the same. Hence, the decline in the stock of structure capital does not affect the skill premium. On the other hand, due to equipment-skill complementarity the decline in stock of equipment capital lowers the relative marginal product of skilled labor. As a result, the skill premium is decreased to 1.75 from 1.9. While both types of capital stock decline, their pre-tax returns increase significantly.

Under the optimal policy, both types supply less labor to the market relative to the benchmark economy. Skilled labor supply is 3.85% lower than its benchmark counterpart, while the unskilled labor supply is lower than 7.03% relative to its benchmark value. To understand the reasons behind these changes, we should remember that we are working with balanced growth path compatible preferences hence we have constant intertemporal elasticity of substitution which equals to  $\sigma$ . In our models, since  $\sigma$  equals to two, income effect dominates substitution effect when there is a change in prices. Here, agents face with three different income effects. The decline in wages induces agents to work more, on the other hand, the increase in net interest rate and the increase in lump-sum transfers affect labor supply decision negatively. As lump-sum transfers and net interest rate show more significant changes relative to the decline in wages, net effect on labor supply is negative for both types. Here, it is noteworthy to underline that the decline in unskilled labor supply is more rapid than the decline in the skilled labor supply, as a result, even though unskilled agents work more than skilled agents in both steady states, the difference between unskilled labor supply and skilled labor supply is decreased by 21.26% in the new steady state. This means that the inequality in terms of hours worked is less under the optimal tax policy. What are the reasons behind the relatively more severe decline in unskilled labor supply? First notice that the wage of skilled type shows a more rapid change relative to the wage of unskilled type. The former declines by 12.29% whereas the latter declines by 4.86%, that is, the income effect due to change in wages is higher for skilled agents. In other words, upward pressure on labor supply is higher for the skilled type. On the other hand, the downward pressure on labor supply decisions that comes from the increase in net interest rate is higher for the unskilled type. This is because, average unskilled capital income increases more than average skilled capital income relative to their benchmark values. The former increases by 61.61%, and the latter increases by 49.68%, so that the cumulative effect on labor supply is stronger for the unskilled type.

Another important change between the two steady states is observed in the decomposition of capital ownership and average consumption levels. The amount of capital



that is held by unskilled type is 45.71% under the optimal policy, which was 43.81% in the benchmark economy. In both steady states, the skilled type consumes more than the unskilled type, but under the optimal policy the difference between their consumption levels is lowered by 30.73%. Thus, the economy ends up with less inequality in terms of capital ownership, consumption levels, and hours worked. The improvements in terms of equality have two sources. First, under capital-skill complementarity taxing capital has an additional redistributive benefit; it indirectly redistributes from the skilled to the unskilled as a consequence of the decline in the skill premium. Second redistribution channel is the lump-sum transfers, the amount of lump-sum transfers increase by 82.85% relative to its benchmark value. Lump-sum transfers can be considered as a tool to partially correct inequality that results from linear income taxation. All agents receive the same amount of transfer, but its effect on the disposable income of the poor is higher than its effect on the rich.

Under the optimal tax policy, total welfare in the society is 1.97% higher than the welfare level of the benchmark economy. Switching to the optimal policy decreases total welfare of the skilled group by 6.64%, whereas it improves the total welfare of the unskilled group by 5.85%. While skilled agents would prefer to stay in benchmark policy, the unskilled type would benefit from high capital income tax rate.

As we indicated above, the degree of equality between skilled and unskilled type has increased under the optimal policy. Although the economy benefits from the improvements in terms of equality, we may say that moving to the optimal policy distorts the economy in terms of productive efficiency.

Table 3: The Comparison of Two Steady States Under Capital-Skill Complementarity

| Variable    | Benchmark | Optimal |
|-------------|-----------|---------|
| $\tau_k$    | 0.36      | 0.71    |
| $\tilde{r}$ | 0.0214    | 0.0453  |
| $w_s$       | 0.5513    | 0.4835  |
| $w_u$       | 0.2901    | 0.2760  |
| $w_s/w_u$   | 1.9       | 1.7522  |
| ls          | 0.0105    | 0.0192  |
| Y           | 0.2117    | 0.1854  |
| K           | 0.6350    | 0.4647  |
| K/Y         | 3         | 2.51    |
| $K_e$       | 0.3150    | 0.2506  |
| $K_s$       | 0.32      | 0.2142  |
| $N_s$       | 0.1557    | 0.1497  |
| $N_u$       | 0.1905    | 0.1771  |
| $c_s$       | 0.0713    | 0.0639  |
| $c_u$       | 0.0495    | 0.0488  |
| $c_s/c_u$   | 1.44      | 1.31    |
| W           | -277.73   | -272.25 |
| $W_s$       | -86.195   | -91.91  |
| $W_u$       | -191.53   | -180.33 |

where  $\tilde{r}$ : return on asset,  $w_s$ : wage of skilled type,  $w_u$ : wage of unskilled type, ls: lump-sum transfer, Y: aggregate output, K: aggregate capital,  $K_e$ : aggregate equipment capital,  $K_s$ : aggregate structure capital,  $N_s$ : aggregate skilled labor supply,  $N_u$ : aggregate unskilled labor supply,  $c_s$ : average consumption level of the skilled type,  $c_u$ : average consumption level of the unskilled type, W: total welfare,  $W_s$ : total welfare of skilled type,  $W_u$ : total welfare of unskilled type

## 5.2 Optimal Policy vs. Benchmark Policy Under Cobb-Douglas Economy

In our second model we eliminate capital-skill complementarity and use Cobb-Douglas production function. We find that optimal capital income tax rate is 24% which is lower than its benchmark value. Table 4 compares the steady states under benchmark economy and under optimal policy.

Unlike the first model, in Cobb-Douglas case optimal capital income tax rate is lower than its benchmark value, hence in the optimal steady state the capital stock is 12.45% higher than its benchmark level. As capital stock increases, the net return on asset is lowered by 15.20% in the new steady state. The increase in capital stock affects marginal product of both types positively. Here, since we do not have capital-skill complementarity,

the change in the capital stock affects both wage rates almost at the same rate so that the skill premium does not change. Remember that under our Cobb-Douglas specification the skill premium does not depend on aggregates, it is constant.

The increase in wages and the decline in the net interest rate have opposite income effects on labor supply decisions. Since the change in interest rate is more rapid relative to the change in wages, the decline in  $\tilde{r}$  dominates the increase in wages so that both types supply more labor to the market.

The average return on assets decreases by 5.13% for the skilled type, and decreases by 4.04% for the unskilled type. Even though income effect that comes from the change in interest rate higher for the unskilled type, the change in unskilled labor supply is more substantial. This may be due to 25.49% decline in the lump-sum transfers as they play a more important role in unskilled agents' disposable income.

As all inputs increase, output increases by 5.30%. However, the new allocation is relatively worse than the initial one in terms of equality. In both steady states, the unskilled type works more but under the optimal policy inequality in terms of hours worked is widened by 10.38%. In the same manner, the inequality in terms of consumption level is increased by 4.86%. Under the optimal policy, average welfare level of unskilled type is declined by 0.42%, while 1.67% increase is observed in the average welfare of the skilled type. Unlike our first model, here optimal policy favors skilled type.

Table 4: The Comparison of Two Steady States in Cobb-Douglas Case

| Variable    | Benchmark | Optimal   |
|-------------|-----------|-----------|
| $\tau_k$    | 0.36      | 0.24      |
| $\tilde{r}$ | 0.05      | 0.0424    |
| $w_s$       | 0.5231    | 0.5428    |
| $w_u$       | 0.2753    | 0.2857    |
| $w_s/w_u$   | 1.9       | 1.9       |
| ls          | 0.0153    | 0.0114    |
| Y           | 0.2093    | 0.2204    |
| K           | 0.6279    | 0.7061    |
| K/Y         | 3         | 3.20      |
| $N_s$       | 0.1578    | 0.16      |
| $N_u$       | 0.1867    | 0.1912    |
| $c_s$       | 0.0772    | 0.06797   |
| $c_u$       | 0.0546    | 0.056     |
| $c_s/c_u$   | 1.41      | 1.42      |
| W           | -151.9542 | -151.5911 |
| $W_s$       | -48.0847  | -47.2831  |
| $W_u$       | -103.8701 | -104.308  |

where  $\tilde{r}$ : return on asset,  $w_s$ : wage of skilled type,  $w_u$ : wage of unskilled type, ls: lump-sum transfer, Y: aggregate output, K: aggregate capital,  $N_s$ : aggregate skilled labor supply,  $N_u$ : aggregate unskilled labor supply,  $c_s$ : average consumption level of the skilled type,  $c_u$ : average consumption level of the unskilled type, W: total welfare,  $W_s$ : total welfare of skilled type,  $W_u$ : total welfare of unskilled type

### 5.3 Comparison of the Final Steady States

When we ignore capital-skill complementarity and proceed with standard Cobb-Douglas production function, we are removing indirect redistributive effect of capital income taxation. This effect comes from the reverse relationship between capital income tax rate and wage inequality under capital-skill complementarity. However, under Cobb-Douglas specification, capital income taxation has no impact on wage distribution. In the first model, the economy ends up with a lower skill premium, and lower inequality in terms of hours worked and consumption levels under the new steady state, on the other hand, the skill premium stays constant and inequality in terms of hours worked and consumption levels increased when the second model moves to the optimal policy.

Table 5 compares the steady states. In model 2, social welfare function is maximized at a lower rate than the current tax rate, hence capital stock, aggregate labor supply and

aggregate output increases. Yet, the optimal policy favors the skilled type and most of the population cannot take advantage from the 5.30% expansion in output. The only redistribution channel is lump-sum transfers but they declined by 25.49%. As a result, the difference between the average welfare levels of skilled and unskilled type is deepened by 2.22%.

In contrast, when we allow for capital-skill complementarity the economy improves in terms of equality but the output declines by 12.43%. In this case, there are two redistribution channels. The direct redistribution comes from lump-sum transfers, and they exhibit a rapid increase relative to the benchmark case. Second, due to the 7.77% decline in the skill premium, there is an indirect redistribution from the skilled type to the unskilled type. So that, the difference between the average welfare levels of skilled and unskilled type is diminished by 2.22%. Also, inequality in consumption levels and hours worked declines substantially.

Table 5: The Comparison of Optimal Policies

| Variable    | Benchmark<br>(Model 1) | Optimal<br>(Model 1) | Benchmark<br>(Model 2) | Optimal<br>(Model 2) |
|-------------|------------------------|----------------------|------------------------|----------------------|
| $\tau_k$    | 0.36                   | 0.71                 | 0.36                   | 0.24                 |
| $\tilde{r}$ | 0.021                  | 0.045                | 0.05                   | 0.042                |
| $w_s/w_u$   | 1.9                    | 1.75                 | 1.9                    | 1.9                  |
| $ls$        | 0.0101                 | 0.0192               | 0.0153                 | 0.0114               |
| $Y$         | 0.21                   | 0.19                 | 0.21                   | 0.22                 |
| $K$         | 0.64                   | 0.47                 | 0.63                   | 0.71                 |
| $K/Y$       | 3                      | 2.51                 | 3                      | 3.20                 |
| $N_s$       | 0.1557                 | 0.1497               | 0.1578                 | 0.16                 |
| $N_u$       | 0.1905                 | 0.1771               | 0.1867                 | 0.1912               |
| $\Delta N$  | 0.0348                 | 0.0274               | 0.029                  | 0.03                 |
| $c_s$       | 0.0713                 | 0.0639               | 0.0772                 | 0.06797              |
| $c_u$       | 0.0495                 | 0.0488               | 0.0546                 | 0.056                |
| $c_s/c_u$   | 1.44                   | 1.31                 | 1.41                   | 1.42                 |
| $\Delta c$  | 0.0218                 | 0.0151               | 0.0226                 | 0.0237               |
| $W$         | -277.73                | -272.25              | -151.95                | -151.59              |
| $W_s$       | -86.2                  | -91.9                | -48.085                | -47.28               |
| $W_u$       | -191.53                | -180.334             | -103.87                | -104.31              |
| $\Delta W$  | 105.34                 | 88.42                | 55.79                  | 57.02                |

$\tau_k$ : tax rate on capital,  $\tilde{r}$ : net return on asset,  $w_i$ : wage rate for type  $i$ ,  $ls$ : lump-sum transfer,  $Y$ : output,  $K$ : agg. capital,  $N_i$ : effective labor supply of type  $i$ ,  $c_i$ : average consumption level of type  $i$ ,  $W$ : total welfare,  $W_i$ : welfare of type  $i$  for  $i \in \{S, U\}$

## 6 Results for Different Social Welfare Function Specifications

Up until now, we use a utilitarian social welfare function which puts equal weights to all agents. As an extension, we use the following social welfare functions which treat skill groups differently.

$$W^s = 2\pi_s \int_a \int_{Z_s} V_s(a, z_s) d\lambda_s(a, z_s) + \pi_u \int_a \int_{Z_u} V_u(a, z_u) d\lambda_s(a, z_u)$$

$$W^u = \pi_s \int_a \int_{Z_s} V_s(a, z_s) d\lambda_s(a, z_s) + 2\pi_u \int_a \int_{Z_u} V_u(a, z_u) d\lambda_s(a, z_u)$$

$W^s$  puts more weights on aggregate welfare level of the skilled type, whereas  $W^u$  favors the unskilled type. Intuitively, as the weight on unskilled type increases, the motivation for redistribution through lump-sum transfers increases so that optimal capital income tax rate increases. Reversely, when the weight on skilled type increases, the return on redistribution in terms of total welfare declines, hence the economy ends up with a lower capital income tax rate. This intuition applies to both models. Apart from that, under capital-skill complementarity due to the presence of indirect redistribution channel, maximizing  $W^u$  requires higher tax rate on capital income. Our findings are in line with this intuition. Table 6 shows the results for this analysis. Under capital-skill complementarity,  $W^s$  is maximized when  $\tau_k=0.58$ , and  $W^u$  reaches its maximum level at  $\tau_k=0.79$ . When we eliminate capital-skill complementarity, optimal capital income tax rates for  $W^s$ , and  $W^u$  are 0.12, and 0.31, respectively.

Table 6: Results for Different Social Welfare Functions

| Variable    | $W_c^s$ | $W_c$  | $W_c^u$ |
|-------------|---------|--------|---------|
| $\tau_k^*$  | 0.58    | 0.71   | 0.79    |
| $\tilde{r}$ | 0.0320  | 0.0453 | 0.0609  |
| $w_s/w_u$   | 1.83    | 1.75   | 1.68    |
| ls          | 0.0151  | 0.0192 | 0.0225  |
| Y           | 0.1988  | 0.1854 | 0.1724  |
| K           | 0.5474  | 0.4647 | 0.3925  |
| K/Y         | 2.75    | 2.51   | 2.28    |
| $N_s$       | 0.1528  | 0.1497 | 0.1466  |
| $N_u$       | 0.1837  | 0.1771 | 0.1713  |
| $c_s$       | 0.0678  | 0.0639 | 0.06    |
| $c_u$       | 0.0493  | 0.0488 | 0.048   |
| $c_s/c_u$   | 1.38    | 1.31   | 1.25    |

$W_c$ : allocation under social welfare function with equal weights

$W_c^s$ : allocation under social welfare function that favors skilled type

$W_c^u$ : allocation under social welfare function that favors unskilled type

## 7 Optimal Taxation of Labor Income and Capital Income

Up until now, we take tax rate on labor income as given and maximize total welfare by choosing the tax rate on capital income. In this section, we solve the optimal tax problem by choosing the tax rate on labor income and capital income at the same time. We find that total welfare is maximized when labor income tax rate is 39% and capital income tax rate is 53%. Under the corresponding allocation, the skill premium is 1.754 and the ratio of average consumption level of skilled type to average consumption level of unskilled type is 1.29. Hence, in terms of equality choosing the both tax rates at the same time produces almost the same result when we keep labor income tax rate at its status quo level 28% and maximize welfare at capital income tax rate 71%. Under both policies, the output level, wage inequality and consumption inequality is almost the same but when we choose both tax rates the economy ends up with higher aggregate capital and lower labor supply.



## 8 Sensitivity Analysis

### 8.1 Sensitivity Analysis for Risk Aversion

$\sigma$  stands for the relative risk aversion parameter. Intuitively, as  $\sigma$  increases, agents become more risk averse, so that they want more insurance. As their desire for insurance increases, taxation of capital becomes more favorable for all agents since it increases insurance through lump-sum transfers. Second,  $\sigma$  controls income effect and substitution effect when there is a change in prices. Higher  $\sigma$  stimulates these effects. Also,  $\sigma$  controls equality preference. As  $\sigma$  increases, the utility function becomes more concave. Increase in curvature leads to an increase in equality preference of individuals in the society and social planner. Given that the government aims redistribution, the motivation for capital taxation increases as curvature increases. Also, since the skilled type holds most of the capital, taxation of capital is a favorable tool for a redistributive government. Therefore, we conduct sensitivity analysis to see whether our results are sensitive to our risk aversion parameter choice. Our benchmark value for  $\sigma$  is 2, in addition to that we calibrate our models for  $\sigma = 1$  and  $\sigma = 3$ , and then solve for optimal capital income tax rate. Table 7 summarizes the results for sensitivity analysis.

Our results are in line with the above arguments. As  $\sigma$  increases optimal tax rate increases, the skill premium declines and equality in terms of consumption improves. When we calibrate our first model for  $\sigma=3$  and solve for optimal tax rate, we find that optimal capital income tax rate is 0.78 and the corresponding skill premium is 1.68. As we indicated above, when  $\sigma>1$  income effect dominates substitution effect. Therefore, when  $\sigma=3$  we expect higher income effect relative to  $\sigma=2$  case. Our findings are in line with this expectation. Skill labor supply is declined by 5.61% relative to its benchmark value. The corresponding decline was 3.85% when  $\sigma=2$ . Unskilled labor supply is declined by 9.8%, and it was 7.03% decline under  $\sigma=2$ . As a result, the difference between skill labor supply and unskilled labor supply narrowed by 27.93% relative to the benchmark difference (The decline in the difference was 21.26% when  $\sigma=2$ ). This means that, equality in terms of hours worked is also higher relative to  $\sigma=2$  case. Optimal tax rate is 0.55 and the corresponding skill premium is 1.84 for  $\sigma=1$ . As equality preference is declined, optimal tax rate is lower relative to other cases, and the economy ends up with higher inequality in terms wage distribution and consumption levels between skilled types. In that case, substitution effect and income effect cancels out each other, so the change in labor supply is smaller relative to previous cases.

Table 7: Sensitivity Analysis for  $\sigma$ 

| sigma        | $\tau_k^*$ | skill premium | $c_s/c_u$ |
|--------------|------------|---------------|-----------|
| Model 1      |            |               |           |
| $\sigma = 1$ | 0.55       | 1.84          | 1.37      |
| $\sigma = 2$ | 0.71       | 1.75          | 1.31      |
| $\sigma = 3$ | 0.78       | 1.68          | 1.26      |
| Model 2      |            |               |           |
| $\sigma = 1$ | -0.13      | 1.9           | 1.4       |
| $\sigma = 2$ | 0.24       | 1.9           | 1.41      |
| $\sigma = 3$ | 0.41       | 1.9           | 1.42      |

The positive relationship between  $\sigma$  and optimal tax rate is preserved under our second economy. However, since we remove the indirect redistribution channel, we do not have improvements in terms of equality in that case.

## 8.2 Sensitivity Analysis for Elasticity Parameters

We also conduct sensitivity analysis for elasticity parameter  $\rho$ . Remember that  $\rho$  controls the elasticity of substitution between equipment capital and skilled labor. Higher  $\rho$  implies higher elasticity of substitution hence less equipment skill complementarity. Intuitively, as equipment capital and skilled labor becomes less complement, the additional redistributive benefit that comes from equipment-skill complementarity declines so that optimal tax rate on capital income also declines. Our results are in line with this intuition, as we increase  $\rho$ , optimal tax rate decreases and the economy ends up with higher inequality. Table 8 summarizes the results for different  $\rho$  values.

First row shows our main result. When we increase  $\rho$  in the second and third columns, optimal tax rate decreases and consumption inequality and skill premium increase. In the

Table 8: Sensitivity Analysis for  $\rho$ 

| $\rho$               | $\tau_k^*$ | $\frac{w_s}{w_u}$ | $\frac{c_s}{c_u}$ |
|----------------------|------------|-------------------|-------------------|
| $\rho = -.495$       | 0.71       | 1.75              | 1.31              |
| $\rho = -.1$         | 0.69       | 1.79              | 1.34              |
| $\rho = .1$          | 0.67       | 1.82              | 1.36              |
| $\rho = \eta = .401$ | 0.64       | 1.87              | 1.40              |

last row, we set  $\rho$  equal to  $\eta$ , that is, we eliminate equipment-skill complementarity (technology exhibits equipment-skill complementarity when  $\rho < \eta$ ). In that case, optimal tax rate is lower than the previous cases but it still significantly higher than no complementarity case that implies the only difference between the production function that is introduced by Krusell et al(2000) and Cobb-Douglas production function is not that the former allows for different elasticities of substitutions between inputs.

## 9 Conclusion

In this paper, we aim to assess the role of capital-skill complementarity in optimal capital-income taxation problem by constructing an infinite horizon incomplete market model in which agents are either skilled or unskilled. Since capital income taxation decreases capital accumulation, it lowers the skill premium in the presence of capital-skill complementarity. We find that optimal capital income tax rate is 0.71, and switching to this policy from the current rate, 0.36, decreases skill premium by 7.77%. The decrease in the skill premium indirectly redistributes from the skilled to the unskilled, hence equality in terms of consumption, hours worked, and capital ownership improves. Whereas, the economy ends up with a lower output. On the other hand, when we eliminate capital-skill complementarity assumption from the model, the indirect redistribution channel through capital income taxation disappears, and the economy ends up with lower equality but higher output. As a further study, it would be a good idea to analyze the transitional dynamics of this model.

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