OPTIMAL PERSONALIZED ADVERTISEMENT FOR VIRTUAL REALITY ENVIRONMENTS

by
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ENVIRONMENTS

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Abstract

Advertisements can be everywhere, even in the virtual environments such as social networks, digital games and applications of smart phones. The virtual places have changed the advertisement world rapidly in recent years. Advertisements are called personalized when they are in those places different from conventional ones. Innovative companies have started to notice opportunities of advertising in the virtual places. The owner of the virtual environments can display different advertisements to their users based on the specifications demanded by their advertisers, which is a significant advantage of advertising in virtual places over conventional ones. Though personalized advertisement has ensured substantial advantages to the companies, it has also brought some problems to the owners of the places. Assigning advertisements to proper users in accordance with the contract between companies and the owner of the places is a noteworthy problem for the owners to earn the maximum income from advertisements.

This study applies three different approaches to assign advertisements to the proper users. The first approach is a direct application of value iteration based dynamic programming to assign advertisements to the users. It is the main skeleton of assigning system. Second one is a finite difference approach which is constructed on the first approach with notable changes. Third one covers four different heuristics for the assignment transaction. The performance of these approaches are also compared in this study. The most effective one was selected for each different case and also for all situations. Thus the model may be suggested to owners of virtual places to maximize their incomes from advertisements.

SANAL GERÇEKLİK ORTAMLARI İÇİN OPTİMAL KİŞİSEL REKLAMCILIK

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Özet

Sosyal ağlar, dijital oyunlar ve akıllı telefon uygulamaları gibi pek çok sanal ortam da dahil her yerde karşımıza çıkar reklamlar. Sanal ortamlar, son zamanlarda reklam dünyasını oldukça hızlı değiştirmişlerdir. Sanal ortamlardaki kişisel reklamlar geleneksel yapıdakinden faklıdır. İnovatif şirketler bu ortamlardaki reklamların oluşturduğu fırsatları fark etmeye başlamışlardır. Sanal ortamların sahipleri, reklam verenlerin önceden belirledikleri kriterler doğrultusunda, internet sitelerinde farklı kullanıcılara farklı reklamlar gösterebilirler. Bu sanal ortamdaki reklamların en önemli avantajıdır ve onları geleneksel yapıdakilerden ayırır. Kişisel reklamcılık, reklam veren şirketlere pek çok avantaj sağlamasının yanı sıra, internet sitesi sahiplerine de bazı sorunlar çıkarabilir. Reklam veren şirketlerle yaptığı kontrat doğrultusunda, doğru kişiye doğru reklamı göstererek yüksek gelir elde edebilmek, sanal ortam sahiplerinin çözüme kavuşturması gereken önemli konulardan biridir.

Bu tez çalışması, uygun kullanıcıya uygun reklamı atayabilmek için üç farklı yaklaşım sunar. İlk yaklaşım iterasyon tabanlı dinamik programlamadır. Bu yaklaşım diğer önerilecek yöntemlerin de temelini oluşturur. İkincisi sonlu fark yaklaşımıdır, bu model iterasyon tabanlı dinamik programlama yaklaşımına dayanmaktadır; ancak belirgin farklılıları söz konusudur. Üçüncü yaklaşım ise dört farklı sezgisel yöntemden oluşmaktadır. Bu çalışma kapsamında ortaya konulmuş yaklaşımların performansları da kıyaslanır. Her durumda en etkili olanı belirlenir. Bu yaklaşım gelir maksimizasyonu sağlayabilmek için sanal ortam sahiplerine önerilebilir.

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Chapter 1

Introduction

Advertising is the best way to communicate with the customers. Advertising informs the customers about the products or services available in the market. Various different media types are utilized such as television, outdoor, radio, newspaper, magazine and digital platforms (mobile and desktop) in order to communicate with different faction of people.

As a business, advertising is among the most sustainable industries considering the growth performance of the last 25 years. With the advance of Internet technologies and the growing number of people and time they spend online, the Internet quickly became a major outlet for advertisement. Reports published by ZenithOptimedia say that, the Internet advertising continues to grow. It grew by 16.9% in 2014 and according to their forecast it will have an average of 15% annual growth between 2014 and 2017[1]. Figure 1.1 demonstrates the growth in the years between 2014 and 2017 for different advertisement channel. It might be easily detected that advertising via digital channel (mobile and desktop) has the highest growing rate.
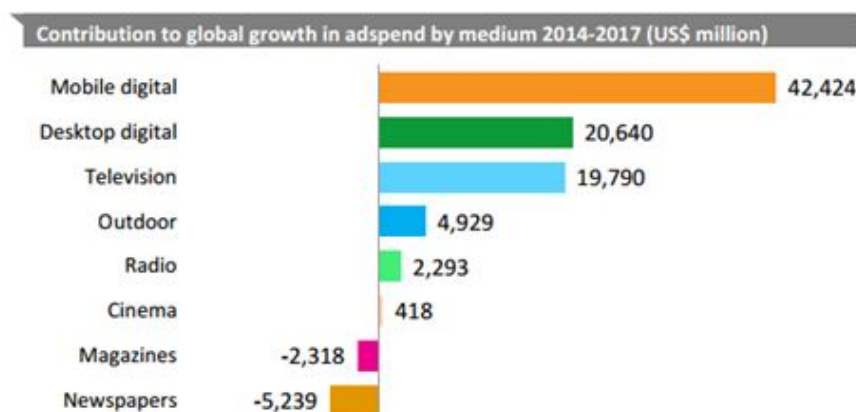

Figure 1.1: Growth Forecast of ZenithOptimedia on the Advertising channel [1]

Besides the increasing number of Internet users, other factors also stimulated the growth of Internet advertising. For example, Internet advertising offers to the advertis-

1

ers a different form of relationship with their customers, when we compare with other conventional advertising methods. Internet advertising has the advantage of a closed loop control system which means it receives the reaction from the customer and adopts itself accordingly. This difference yields a significant impact on advertising. By this way, it becomes possible and feasible (fast and cheap) to communicate directly with the customers that are targeted by the advertisers, i.e., paves the way for personalized advertisement.

Internet advertising has started a new era in advertising industry by offering a method other than mass advertising in a variety of ways. Improved advertising formats are making Internet display more interactive and attention grabbing. Companies mostly prefer video ads across desktop computers, tablets and television screens. According to forecast of ZenithOptimedia online video grow at 20% year for the rest of their forecast period. Meanwhile social media has holed the opportunities offered by the transition to mobile, and it is growing at 25% a year. The growth may be seen on the graph above[1].

Additionally, Internet enables the advertisers to collect large amount of data about the users. When information about users which is deep down inside this data is understood and effectively used, personalized advertisement becomes more effective. Advertisers may get the information regarding to the user in two different ways; either stored from online activities of users or obtained directly by the users (e.g., registration forms, online questionnaires, etc.). Personal information (demographics, interests, etc.) shared directly by the user and the online activities (such as the users frequency of visiting website, the duration that s/he spends on each page, the ads that the user click through, the time of the day the user logins, other users that s/he contacts with, etc.,) reveals invaluable information to the publishers (i.e., the owner of the media/platform that advertisement is made).

The current trend of increase in the number of users in social web sites and platforms provides more opportunities in Internet advertising. For instance, the virtual reality socialization platforms are similar to "Facebook" that users can share their information like interests, hobbies, relationship status, photos etc. with others as well as with the system providers. Therefore, personalized advertisement systems can utilize the data provided by the users. The virtual reality environments are mostly real life simulations. Users walk around in real world places modeled with 3D design technology. The virtual worlds simulate various places such as streets, buildings, squares, café, etc. in which users can interact with their virtual or real friends. Advertisements that are placed at certain locations within the virtual environments are among the major source of revenue for the company. While users are online at different places, e.g. when walking on the street occasionally a public bus with an advertisement on the sides might pass, the bus stop, the building across the street or the window of a café, etc. an advertisement might be displayed.

Figure 1.2 depicts the personalized advertisement and revenue generation setting of the virtual publishers. Generally speaking, the company that maintains such platforms
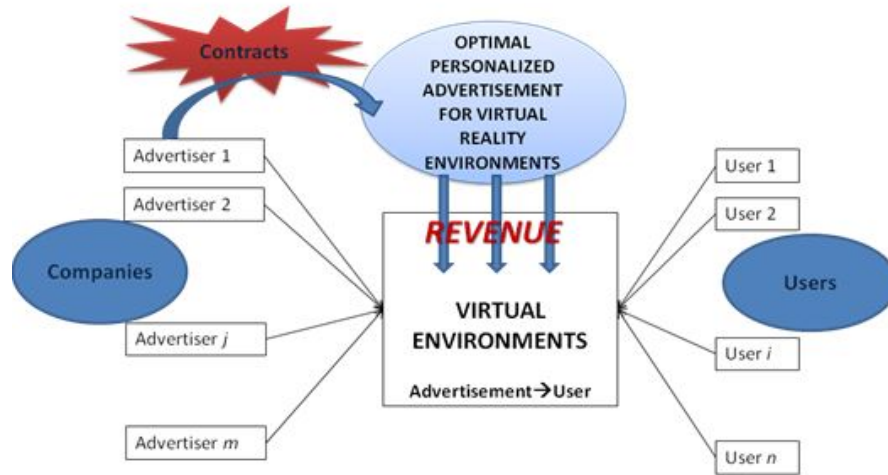
Figure 1.2: The Personalized Advertisement and Revenue Generation Setting

receives advertising revenue from two different sources, namely the display revenue and the click revenue. The display revenue is realized when a user exposed is to an advertisement. Therefore, the event is called exposure. Note that, in web browsing a user is categorized as "exposed" whenever the user visits the web page. On the other hand, exposure is defined differently in the 3D virtual environments. In such platforms, exposures are realized whenever the user is in the vicinity of a place of an advertisement location. The description of vicinity is based on the virtual distance (i.e., magnitude of the advertisement on the screen in terms of the total square pixel). There is no industrial standard and it should be negotiated between the owner of the virtual environment and advertisers, through a contract. It is, considered that depending on the contract terms between the publishers and advertisers time might be another issue. That is to state an exposure might be considered to be realized whenever the advertisement occupies a certain total square pixel of the screen for a certain amount of time. On the other hand, the click revenue is realized whenever the user clicks an advertisement, this event is called click-through.

This thesis motivates from a real life problem that a software company faces. The company develops a virtual based 3D socialization platform similar to the well known Second Life. The company requires a Personalized Advertisement System (PAS) [Figure 1.3]. Personalized advertising problem has two phases. Matching problem is the first phase of PAS in which the compatibility of candidate users with set of specifications that are established by the advertisers in terms of different kinds of featured are identified. Assignment phase is the second phase, where a specific ad is assigned to an advertisement place whenever the user is in the vicinity of an advertisement location[2].

In this study we focus on the assignment phase only and assume that the matching phase is already conducted. One of the main concerns about exposure of the advertisements in the assignment phase, is maximum and minimum display number per each per-
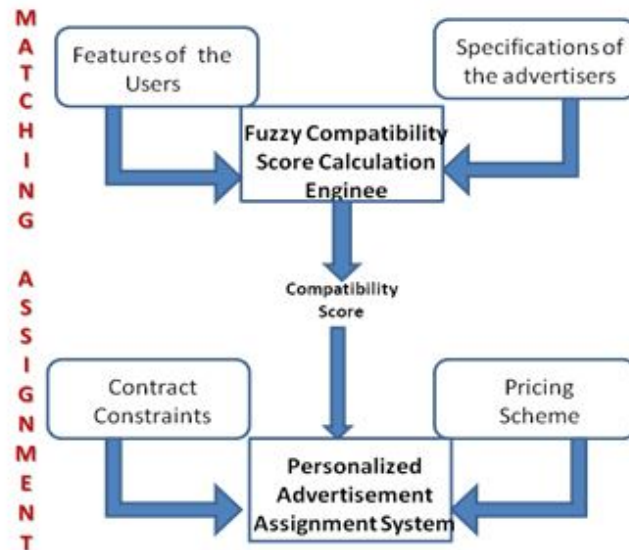
Figure 1.3: Personalized Advertisement System [2]

son. Oftenly displayed same advertisement that keeps popping up at every corner in a virtual web site can be quite annoying for the virtual place users which is undesirable for both the publisher and the advertiser. Furthermore, extensive repetition of the advertisement has no impact on viewers after a certain point (i.e. advertisements start to wear out) and advertisers are wasting their money if the message is already received by a particular user. Therefore, both the annoyance and the law of diminishing returns offer that setting a maximum display number per user is a widely acceptable and desirable constraint in practice.

On the other hand, advertisement may be inadequate without repetition so aim of the advertisement is not be accomplished. At the first couple of times, viewers can feel unfamiliar with ad and they ignore receiving the message, whereas some repetition would rise the viewer's ability to remember the advertisement in future. Thus a minimum display number is another desirable constraint for the advertisers. Note that, whenever a minimum display number is reached to the specific viewer, the publisher deserves for a payment.

Naturally, there is a maximum payment for every advertiser, which is called maximum budget constraint. The contracts would determine the maximum budget constraint which will specify the level that the advertiser would not pay anymore when the total revenue of the publisher from the exposures and the clicks would exceed that amount. Furthermore, the advertisers want to guarantee a set of number of displays so that they ensure to reach a critical mass that starts a word of mouth effect. This constraint is denoted as the minimum payment and the advertiser makes payment only if the advertisement is displayed more than a certain number of viewers (i.e., more than a threshold level).

It is always possible to set less quantities for all of the above identified constraints in application, which means that zero for the minimum display number and minimum

payment, and extremely large number for maximum display number and the maximum payment based on the contract between the publisher and the advertiser.

In this thesis, six different algorithms are developed and their performances are evaluated with an experimental analysis. The first approach is an application of value iteration based dynamic programming. Secondly, a finite difference approach is developed. The remaining four algorithms are heuristic based approaches.

Even though the setting of the problem is motivated from virtual environments, the algorithms that are developed in the study are also applicable for other platforms such as social media, web page and smart phone application advertising. Nowadays, applications are getting more and more popular in our smart phones and we may easily notice that almost all free applications show advertisement. That is to say, even though the study was inspired from the problem on a web site similar to Second Life, it may be extended for mobile and social media advertising.

Next, a review of relevant literature will be provided in Chapter 2. Chapter 3 is where the notation and the problem statement will be introduced. The iterative dynamic programming as well as the difference based methods will also be presented in Chapter 3. Later, in Chapter 4, the four heuristics as well as the results of the experimental analysis will be provided. We will finalize the thesis with some concluding remarks and future research suggestions in Chapter 5.

Chapter 2

Literature Review

Internet has changed the paradigm of advertising and introduced wide range of new opportunities. The advertisers did not have a practical and economical alternative other than the conventional mass advertising platforms (TV's, radios, magazines, boards, etc.) prior to the Internet revolution. Note that, conventional methods were not able to publish different advertisement for different users due to the physical constraints. Now, the advertisers are able to reach their target customers with lower cost and more efficiently by the Internet. That is to say, an essential change of paradigm is realization of personalized advertisement which is provided by the Internet (World Wide Web).

Advertisement is a significant source of revenue for online publishers and supports availability of free contents for the users. If users are presented with advertisements that match with their interest when they are visiting web sites, then advertisements are more likely to be responded. Thus web advertisements that are personalized are more likely to generate higher revenues. Furthermore, many of the Internet users want personalized content on the web sites, hence, personalized advertisements are not perceived as annoyances but rather even as a reason to visit a certain web site.

Personalized advertisement enables the display of an appropriate content to the appropriate user at the appropriate time. In conventional advertisement industry, it was possible to conduct personalized advertisement to a degree as well. Advertisers could target a certain group of people, by utilizing a platform that was mostly limited to the target group. However, web personalization can be based on individual behaviors, instead of considering a geographic location, or demographic characteristics (such as gender, age, marital status, etc.) in order to target a group. That is to say instead of group of users, single web user can be assigned appropriate advertisement through personalized web advertisement [3].

Online advertising is becoming an increasing large fraction of the total advertising market as people spend more time on the Internet. Some of the most outstanding technology companies, including Google and Facebook, rely primarily on advertising through

the Internet to earn income. Broadly, online advertising is different because the technology that lies behind online advertising decreasing the cost of targeting. Targeting is the key difference between online and offline advertising [4].

Online advertising can be grouped into three categories: search based advertising, classified advertising, and display advertising. Firstly, search based advertising is the advertising that appears along with the algorithmic results on search engines such as Google or Bing. Because each search is a statement of goal, advertisers can show their ads in front of people at the exact moment that the latter are looking for something. On both Google and Bing, search based advertising is priced using a specialized auction mechanism, and each search query is priced using a separate auction. Advertisers pay whenever somebody clicks on their advertisement (called cost per click or CPC). However, even in its simplest form, search based advertising enables advertisers to target customers on the basis of specific keyword sequence.

Auction mechanisms are one of the biggest areas of research in online advertising. This area has been considerably interested by several famous economists. The reason of that can be claimed as economists probably have had such interest in auctions for online advertising is the well-established literature on auctions. It is driven by the need to price a large number of keywords in an effective manner by aiming consumers and a want to price which makes difference between advertisers fundamentally aiming advertisers. Both are the roles of the one-to-one communication between identifiable computers facilitated by the Internet. The first corporation to implement auctions that priced keyword-specific advertising in search engines was goto.com. After that, changed as Overture, and finally purchased by Yahoo!, goto.com implemented a straight auction in which advertisers would bid per click, and the advertiser with the highest bid would appear first in the search results. The properties of mentioned auctions were investigated by Varian [17]. and Edelman et al. [18]. Studies on auctions have been extended to incorporate reserve prices [19], clickweights [20], and the incorporation of consumer choices into the model [21] and [22]. Agarwal et al. [23] investigate differences between auctions in which bidders pay per click such as each time their ad is clicked by a user or pay per action as a given example, each time a user buys something from their website. Katona and Sarvary study the interaction between paid search results (the advertisements) and the algorithmic (organic) search results [24]. Many of these mentioned models are heterogeneous bidders and the requirement to the part of search engines in order to reach price differentiation between advertisers, in other saying, to aim prices to advertisers.

A couple of empirical papers have investigated these models, again mostly showing the roles of heterogeneous consumers and advertisers, and the targeting of advertising prices. Particularly, Yao and Mela show that developing the ability of advertisers to target searchers would increase search engine income [25]. A structural model of search engine advertising based on heterogeneous advertiser quality was used by Athey and Nekipelov

to indicate that bidders may have incentives to decrease their demand for advertising [26]. In an experimental study of display advertising prices across websites, Wu shows differences in the generalized second-price auction mechanism that is used by Google's display advertising network to the list value mechanism used by China's Taobao. He illustrates that the decision of mechanism is driven by the requirement of, and benefits of, price differentiation [27].

Secondly, classified advertising reveals on websites which do not render other media content or algorithmic search. Craigslist is the biggest of these websites and has been credited with the rejection of offline classified ads in local newspapers across the US [28]. Online jobs sites and online dating sites also given two examples for this category.

Lastly, display advertising is the core income provider for online media that are not search engines. It involves plain text and simple banner ads, media-rich ads, video ads, and the typical ads that are illustrated on social media websites in well-known social media websites like Facebook. According to website, display advertising is priced with different kind of systems. Some are priced by specialized auctions like search based advertising; some are priced depending on negotiated purchases like network television; and some have a stable price and can be purchased either online or via a sales force. Typically, companies pay per each display, and prices are communicated as cost per thousand impressions (or CPM). Rather than per impression some display advertising is priced per click, and a handful of companies have tried with a hybrid auction that allows advertisers to select whether to pay rely on cost per view or cost per every click [29]. Display advertising proposes different kinds of opportunities to target advertising. Online display advertising was mainly targeted and priced based on user demographics, in a sense similar to television in the decade between 1990 and 2000.

Internet describes the users through collection of data by two ways; either obtained directly by the users via questions and registration forms or provided indirectly from the web log of users. Indirect information stored from online activities of users is considered to be more reliable. Especially, advertisers would place their ads on websites with the proper demographic audience: make up ads on women.com and beer ads on espn.com. By the time of progress, demographic targeting has become more complicated. Advertisers can target specific demographic groups (married women living in Turkey aged 3034), based on information that users provide online. For instance, if a user has provided detailed demographic data elsewhere (say to Google or Microsoft because of their email account), then that data might be used for targeting (for example in ads managed by Google or Microsoft). Another form of targeting is called contextual targeting. In contextual targeting, ads match the context of the website. Search engine advertising is a form of contextual targeting in which advertisers match their ads with the content of the search. Contextual targeting is also common in display advertising: cars are advertised on cars.com, diapers are advertised on babycenter.com, and electronic gadgets are advertised

on techcrunch.com. Eventually, advertisers can use data based on past online behavior to target ads. This is called "behavioral targeting" in the industry. Typically, it involves the use of prior click stream data to decide whether a particular customer is a good match for an advertisement. Frequency of visiting, time spends in each page, the advertisement clicked by users, web pages that user is interested most, location of users are all invaluable information. Getting behavioral data of user may also be described as web usage mining and the obtaining data directly by questions may be called as web content mining [3]. Figure 2.1 demonstrates personalized advertising steps. Systems that are designed for
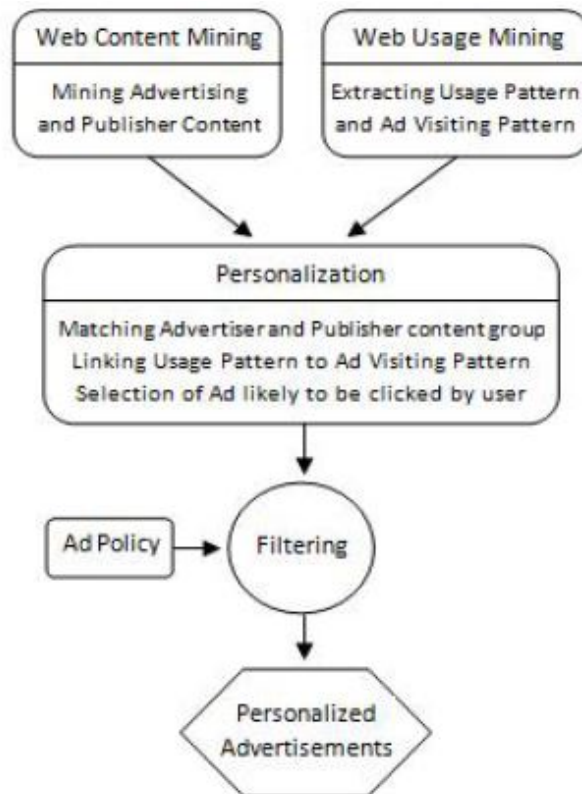
Figure 2.1: Advertising Personalization [3]

advertising the proper products the proper user are known as Recommendation Systems (RS). In the literature these systems are classified into three groups, namely the content based systems, the collaborative filtering systems, and hybrid approaches. Content based approaches are designed for advertising a product that is similar to the products that users have shown interest before. Collaborative filtering methods utilize preferences, demographics of the user and advertise a product which is similar to interests of user. Hybrid approaches has a system between content and collaborative [5].

Personalized advertisement systems and the recommendation systems, they both strive to advertise the most proper advertisements to the users. However, they are different in the sense that, a typical PAS tries to satisfy the requirements of the publishers and the advertisers at the same time as opposed to the RSs in which the overall aim is to satisfy

the desire of the publishers. Contracts between the advertisers and the publishers restrict the PASs freedom to always publish the most appropriate advertisement for the viewers to maximize the overall profit of the publisher. For instance, RS would choose an advertisement if the viewer has a high possibility to purchase the product, so that the publisher of the advertisement (i.e., the web site owner) could earn a certain amount of commission. However, the PAS should prefer to display an advertisement if it yields more profit, e.g., the corresponding advertiser pays more than the others. This means, due to the contract between the publisher and the advertisers, the shape of personalized advertisement systems were forced to modify by the objective function and the constraints. Therefore, RS are not applicable solutions for personalized advertisement problem [2].
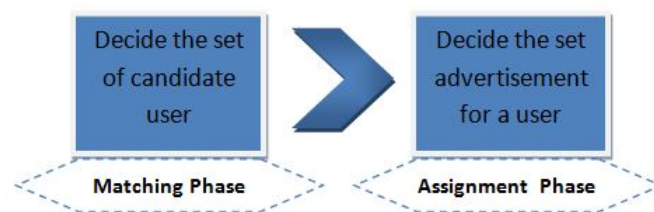
Figure 2.2: Simple Schema of Personalized Advertisement Problem

As stated in the introduction, there are two basic problems in PAS, i.e. matching the users to the advertisers and assigning (scheduling) the advertisements to the users on real time. Two different phases are represented by Figure 2.2. Most of the current literature regarding the PAS focus on the matching phase of the problem. A study proposes a web ad selector system based on fuzzy logic that matches the user preferences with the content of the advertisements [6]. Their structure does not include the scheduling constraints such as committed display budget, and the maximum or minimum display requirements for the advertisements. Though advertisers do not specify the requirements, writers decide what kind of users would view what kind of advertisements. Therefore the scheduling phase is far from dealing with the needs of the publishers in many real life applications. Afterward, a study based on fuzzy rules were generated from historical data collected from user' online activities is proposed [7]. But the assigning problem is not addressed in this study either.

A framework for personalized web advertising, both the matching and the scheduling problems are presented by Kazienko [8]. However, the details of the algorithms are not provided in their paper and mostly the framework was established in the paper. Zhou et al. also presents the structure of such a framework without any specific details [9]. The authors leave the scheduling part as future research topic in their paper. Using fuzzy intelligent agents, Yager studies a targeted e-commerce methodology [10]. It is comprehensively discussed that the fuzzy reasoning algorithm deals with the matching problem, while the scheduling problem is summarized and is depended on the bids that will be

offered by the innovator of advertisers. The bid level determination is not completely explained in the mentioned paper. A study is presented by Kilic who applies fuzzy reasoning for the matching phase and contributes a simple scoring index for the assignment phase [11]. The scoring index is limited and mainly constructed to consider merely the maximum budget and maximum display constraints.

A variety of assignment (scheduling) methods in target advertising are not involved in matching phase, as a given example, ADWIZ system is the one of them. The system sorts users based on the keywords they posted at the website and using linear programming approach assigns the advertisements [12]. The minimum number of advertisement exposures and the coverage constraint are two limitations that ensure the certain portion of the users with specific characteristics are subject to a particular advertisement. The second constraint aims the hard matching problem to a certain degree. The success of ADWIZ system is illustrated by Tomlin that based on mainly on the accurate assessment of the model parameters such as click rates[13]. In order to overcome accuracy problems, selection algorithm must be robust for ads which is asserted by the author. Furthermore, Tomlin offers a nonlinear programming model of the problem and also reconstructs the objective that it contains both exposure and click revenues. Nakamura and Abe, further improve the ADWIZ model and interpret it as a better applicable model [14]. Adler et al. described the advertisement placement problem in three attributes as Ad Geometry, Display Frequency and Time Interval ,and developed a heuristic for this problem [15]. After that, on a mathematical formulation and a heuristic solution are proposed for the problem introduced by Adler et al. [16]. The problem is divided into smaller sub problems in their method and they developed a heuristic that utilizes the consequences of these sub problems.

Advertisements are everywhere, even in the digital world of gaming. Beforehand, video and computer game players have been stereotypically thought as teenage boys. However, nothing could be further from the truth as, although these types of gamer exist, in 2013 it was observed that within the US, 39% of gamer were actually aged 36 and over. The average age of those playing games was 31 and 48% were female [30]. Part of the revenue made in the gaming industry comes from advertising associations with games and according to the Interactive Advertising Bureau, this can be called as "Game Advertising" [35] and [36]. After that, game advertising is defined as "The association of marketing communications messages with video and computer games to target consumers through Advergames, Around-Game Advertising or In-Game Advertising activities". Advergame is "A digital game specifically designed for the primary purpose of advertising and promotion of an organizations product, service or brand played via the Internet or on a compatible medium via a games disc or digital download". An alternative form which may be better suited to mainstream games is that of In-Game Advertising. It is defined as "The integration of non-fictional products and brands within the playing environment of

video and computer games through simulated real life marketing communications mechanisms". This aspect of Game Advertising is succeeded through placing products directly into the games via Product Placement or communicating with gamer through Marketing Displays. Lastly, Around-Game Advertising which is defined as 'Advertising and promotion linked to video and computer games through non-intrusive around game displays or licensing of game branding with associated third-party products [37]. Advertising through games ranks highly when compared to traditional methods. Figure 2.3 represents clearly the rank of advertisement methods. Although video game advertising lies at the bottom in terms of actual income it has the second highest growth percentage rate.

GLOBAL ADVERTISING BY CATEGORY (US $ MILLIONS)

CATEGORY	2008	2009	2010	2011	2012	2013e	2014e	2015e	2016e	2017e
Digital	61,438	63,761	75,676	90,189	106,772	125,223	144,718	165,992	188,307	212,163
Television	157,727	146,523	163,856	169,117	179,954	186,493	203,937	212,376	232,293	244,094
Audio	33,363	28,963	30,882	31,387	32,029	32,869	33,896	34,871	36,117	37,211
Cinema	1,854	1,828	2,047	2,073	2,175	2,252	2,350	2,481	2,621	2,759
Out-of-Home	31,630	27,438	28,816	29,592	30,440	31,466	32,882	34,435	36,187	37,967
Consumer Magazines	33,562	27,198	27,920	27,973	27,226	26,642	26,370	26,348	26,605	26,991
Newspapers	107,369	88,840	89,534	88,722	86,425	84,786	83,947	83,667	83,926	84,419
Video Games	1,374	1,602	1,864	2,147	2,442	2,745	3,036	3,344	3,643	3,939
TOTAL	428,317	386,153	420,595	441,200	467,463	492,476	531,136	563,514	609,699	649,543

GLOBAL ADVERTISING GROWTH BY CATEGORY (PERCENT)

CATEGORY	2008	2009	2010	2011	2012	2013e	2014e	2015e	2016e	2017e
Digital	18.7	3.8	18.7	19.2	18.4	17.3	15.6	14.7	13.4	12.7
Television	1.1	-7.1	11.8	3.2	6.4	3.6	9.4	4.1	9.4	5.1
Audio	-4.5	-13.2	6.6	1.6	2.0	2.6	3.1	2.9	3.6	3.0
Cinema	-0.5	-1.4	12.0	1.3	4.9	3.5	4.4	5.6	5.6	5.3
Out-of-Home	1.1	-13.3	5.0	2.7	2.9	3.4	4.5	4.7	5.1	4.9
Consumer Magazines	-2.0	-19.0	2.7	0.2	-2.7	-2.1	-1.0	-0.1	1.0	1.5
Newspapers	-7.9	-17.3	0.8	-0.9	-2.6	-1.9	-1.0	-0.3	0.3	0.6
Video Games	30.9	16.6	16.4	15.2	13.7	12.4	10.6	10.1	8.9	8.1
TOTAL	-0.3	-10.1	8.6	4.5	5.5	5.0	7.5	5.7	7.9	6.2

e = estimated

Figure 2.3: Global Advertising Revenue and Revenue Growth 2008-2017 [37]

Not only video games but also smartphone and tablet games have become significant platforms for targeted advertisement. Blake Commagere who is currently the Founder and CEO of MediaSpike (http://www.mediaspike.com) −a platform for product placement in social and mobile games− says companies started out a few years ago working to bring sponsored content to smartphone and tablet-based games. Now the company is considering how billboards, videos, and other types of product placement can fit into the computer-generated worlds viewed on devices like the Gear VR, as well as on headsets that don't yet have a firm release date. Dallas-based Airvirtise certainly hopes advertisers will be keen to strive to reach people inside virtual realities. It is working on virtual 3-D models that are integrated with real-world locations, which it realizes from longitude, latitude, and elevationthink a giant Angry Birds game in a park or a life-size virtual car you can walk around. Through smartphone apps and eventually through the lenses of augmented-reality eyewear these things would initially be distinguished [31].

Moreover, in the literature there are some studies with methodologies close to our proposed methodology [9], [13], and [36]. One of the studies models planning of guaran-

teed display Internet advertising by an ad network, which acts as an intermediary between website publishers and advertisers. Advertisers purchase an advertising campaign from the ad network, which is the number of ads to be displayed, and a set of customer types, which describes who to show the campaign's ads to. This problem can be thought as a stochastic transportation problem with each customer type as a source with random supply and each advertising campaign as a sink with known demand. Each time a user loads a website enrolled in with the ad network, a decision must be made as to which advertisement to display. This paper focuses on the high level planning stage at the beginning of each optimization time period, determining what proportion of ads for each viewer type to allocate to each applicable campaign [32].

Another study has a fundamental approach to setting the frequency capping policy using Markov decision processes (MDP) which decides the best frequency capping policy for each user; it also focuses on the long term benefits of having such a visitor around. In nutshell, this study addresses two primary weaknesses of currently used practices: current approaches optimize for short term gains and largely ignore maximizing long term profits. First they study out different market segmentation strategies combined with feature selection algorithms by the way of constructing homogenous groups of consumers. Secondly, they extend their study to an online advertising problem setting which decides impact frequency capping rules for online marketing segments using Markov decision processes (MDP). Thus they may optimize a global marketing objective such as click through rate. A population-based search paradigm called genetic algorithms was adapted to discover good customer segmentations and corresponding influential frequency capping rules for individuals assigned to those segments [33].

Last but not least a novel idea was proposed in the allocation and serving of online advertising by Hojjat et al. [34]. They explore that by using predetermined fixed length streams of ads (which they call patterns) to serve advertising, they could incorporate a variety of interesting features into the ad allocation optimization problem. Particularly, under reach and frequency requirements their formulation optimizes to representativeness as well as user level diversity and pacing of ads. They reveal how the problem can be solved in an efficient way with a column generation scheme in which only a small set of best patterns are hold in the optimization problem. Their proposed algorithm has a promising run time and memory usage, with parallelization of the pattern generation process by numerical tests [34].

Note that, neither of these papers is directly related with our problem, however,their focus is to select the most efficient advertisements for users. At the moment the 3D platform on hand only displays one advertisement in a single advertising location at each exposure, i.e. mixed advertising is not allowed. The literature on the personalized advertisement systems is still growing. In the last decades, the application areas became more visible. Even though some research has been conducted which are closely related

(particularly for the matching phase), the field is still developing and the literature lacks efficient solution approaches that can be applied to a real life case, such as the problem on hand.

Chapter 3

Problem Statement and Solution Methods

Let $(\Omega, \mathcal{H}, \mathbb{P})$ be a probability space hosting n-many independent $\{0, 1\}$-valued Markov processes $X_1, \ldots, X_n$ representing the online/offline statuses of the members of a virtual environment. For $i \leq n$, $X_i(t) = 0$ if the ith user is offline at time t, and $X_i(t) = 1$ if s/he is online. If the current state of user i is $\{0\}$, then the time to next transition is an exponentially distributed random variable with parameter $\mu_{i,0}$, and at the next transition, the user will be online with probability one. If, on the other hand, the current state is $\{1\}$, then the time to next transition is exponentially distributed with parameter $\mu_{i,1}$. At the next transition the user may switch to another virtual location (i.e., re-enter the state $\{1\}$) independently with probability β_i, or go offline (jump to state $\{0\}$) with probability $1 - \beta_i$. In the sequel we let $T_1, T_2, \ldots$ be the transition times observed in the virtual reality environment and $Y_1, Y_2, \ldots$ denote the indices of the customers making transitions at those times. That is, $Y_k = \sum_{i \leq n} i \, 1_{\{X_i(T_k) \neq X_i(T_k-)\}}$ for $k \geq 1$.

Every time a user makes a transition into state $\{1\}$, the site manager can display an ad of one of the m-many advertisers willing to pay for such ads. At some terminal time T (for example, end of a day), the manager of the virtual environment collects some terminal reward from the advertisers depending on the number of adds displayed. In order to quantify this payment, we define the cumulative ad exposure matrix process $A \equiv \{A_t\}_{t \geq 0}$. For each t, $A(t)$ is an $n \times m$ matrix whose (i, j)th entry $A_{i,j}(t)$ records the number of times the user i is exposed to brand j until (and including) time t. In terms of the total exposure $A(T)$ at time T, the reward collected by these advertisers are given by $F(A(T))$, where F is a given function defined on a bounded set $\mathcal{N} \subset \mathbb{N}^{n \times m}$ shaped/defined according to the agreement with the advertisers. Here we assume that the domain of F is bounded to avoid overexposure to ads, which is a natural restriction placed by advertisers. The requirements/requests by the advertisers are also taken into account in defining the function

F precisely. For example, advertisers could provide some incentives if the number of times their ads are displayed/clicked exceed those of their competitors etc. Such requests can easily be reflected on the function F.

Given the function F, the site manager would like to decide dynamically which advertiser's ad should be displayed to the users when they make transitions into state $\{1\}$ (not displaying any ad is also an option) in order to maximize the expected revenue obtained at time T. Let us introduce $\{0, 1, \ldots, m\}$-valued random variables $D_1, D_2, \ldots$ to denote the display decisions made by the site manager at times $T_1, T_2, \ldots$ respectively. At the kth transition time, for $k \geq 1$, $D_k = j$ if jth advertiser's ad is selected, and $D_k = 0$ if no ad is displayed. By default, we display no add if a user goes off-line, or if a new add would cause the exposure matrix process to exit the domain $\mathcal{N}$. Clearly, each decision should be based on the information available by then, and looking into the future is not allowed. Below, we call such policies *admisssible*.

In this setup, the objective of the site manager is to compute the maximum expected revenue

$$\sup_{\mathcal{D} = (D_1, D_2, \ldots)} \mathbb{E}\left[F(A(T))\right], \tag{3.1}$$

and if exists, find an admissible policy $\mathcal{D} = (D_1, D_2, \ldots)$ attaining the supremum above. Clearly, the exposure matrix A depend on the policy $\mathcal{D}$. Here, we omit this dependence for notational convenience only.

3.1   The Dynamic Programming Operator

The formulation above implies that the problem is Markovian in terms of the cumulative exposure matrix, the online/offline state process $X(t) = (X_1(t), \ldots, X_n(t))$, and the remaining time t to the end of horizon T. Hence, for every (a, x, t) in

$$\Delta := \mathcal{N} \times \{0, 1\}^n \times [0, T], \tag{3.2}$$

we define the value function

$$V(a, x, t) := \sup_{\mathcal{D}} \mathbb{E}^{(a,x)}\left[F(A(t))\right], \tag{3.3}$$

where $\mathbb{E}^{(a,x)}$ denotes the expectation operator associated with the probability measure $\mathbb{P}^{(a,x)}$, under which $A(0) = a$, $X(0) = x$ with probability one. In plain words, $V(a, x, t)$ gives the maximum expected revenue that we can obtain given that there is t-time units to the end of the time horizon, the exposure matrix process starts from $A(0) = a$, and the

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initial state of the Markov process X is x.

On the set Δ, let us also introduce the sequence of functions $(V_k)_{k \in \mathbb{N}}$ as

$$V_k(a, x, t) := \sup_{\mathcal{D}} \mathbb{E}^{(a,x)} \left[F(A(t \wedge T_k)) \right]. \tag{3.4}$$

Each V_k represents the optimal expected revenue that the site manager can receive when s/he is allowed to make a decision at the first k transitions only. For all the other transitions (if any before t) no ad is displayed. Clearly, we have $F = V_0 \le V_1 \le V_2 \ldots \le V$. Lemma 1 below shows that V_k's converge to V uniformly.

Corollary 1. It follows immediately from the definitions of the functions V and V_k's that the mappings $t \mapsto V_k(\cdot, \cdot, t)$, $k \ge 1$, and $t \mapsto V(\cdot, \cdot, t)$ are non-decreasing.

Lemma 1. *For $k > 1$, we have*

$$V(a, x, t) - V_k(a, x, t) \le \frac{2\|F\| \, T \, \bar{\mu}}{k - 1} \qquad \text{for all } (a, x, t) \in \Delta, \tag{3.5}$$

where $\|F\| := \sup_{a \in \mathcal{N}} F(a)$ and $\bar{\mu} = \sum_{i=1}^{n} \mu_{i,0} \vee \mu_{i,1}$.

Proof. For an admissible policy $\mathcal{D}$ and $k > 1$, we have

$$\mathbb{E}^{(a,x)} \left[F(A(t)) \right] = \mathbb{E}^{(a,x)} \left[F(A(t \wedge T_k)) + 1_{\{T_k \le t\}} \left(F(A(t)) - F(A(T_k)) \right) \right]$$

$$\le \mathbb{E}^{(a,x)} \left[F(A(t \wedge T_k)) \right] + 2\|F\| \, \mathbb{P}^{(\cdot,\cdot)}\{T_k \le t\} \le V_k(a, x, t) + 2\|F\| \, \mathbb{P}^{(\cdot,\cdot)}\{T_k \le t\}.$$

It is possible to prove that $\mathbb{P}^{(\cdot,\cdot)}\{T_k \le t\} \le t \frac{\bar{\mu}}{k-1}$, hence we have

$$\mathbb{E}^{(a,x)} \left[F(A(t)) \right] \le V_k(a, x, t) + \frac{2\|F\| \, T \, \bar{\mu}}{k - 1}.$$

Since this is true for any admissible policy, the inequality (3.5) holds. $\qquad \square$

By the dynamic programming principle, we expect that the function V satisfies the equation $V = \mathcal{J}[V]$ where the operator $\mathcal{J}$ is defined as

$$\mathcal{J}[f](a, x, t) := \sup_{D_1} \mathbb{E}^{(a,x)} \Big[1_{\{t < T_1\}} F(a)$$

$$+ 1_{\{T_1 \le t\}} \Big(1_{\{X_{Y_1}(T_1)=1\}} \cdot \mathcal{S}_{Y_1, D_1}[f](a, X(T_1), t - T_1) + 1_{\{X_{Y_1}(T_1)=0\}} \cdot f(a, X(T_1), t - T_1) \Big) \Big]$$

$$\tag{3.6}$$

for a Borel function f on Δ. Here, the operator $\mathcal{S}_{i,j}$ is defined as

$$\mathcal{S}_{i,j}[f]\big(a, x, t\big) := \begin{cases} f(a + I_{i,j}, x, t) & j \in \{1, \ldots, m\}, \\ f(a, x, t) & j = 0, \end{cases} \tag{3.7}$$

for $1 \le i \le n$ and $0 \le j \le m$, in which $I_{i,j}$ denotes the $n \times m$ matrix whose (i, j)th entry is one, all others are zero. Here, we adopt the convention that $f(a + I_{i,j}, \cdot, \cdot) = -\infty$ whenever $a \in \mathcal{N}$ and $a + I_{i,j} \notin \mathcal{N}$. Clearly, the supremum above is attained if we take

$$D_1 = \begin{cases} \underset{0 \le j \le m}{\arg\max} \, \mathcal{S}_{Y_1,j}[f]\big(a, X(T_1), t - T_1\big) & \text{on } \{T_1 \le t\} \cap \{X_{Y_1}(T_1) = 1\}, \\ \\ 0 & \text{elsewhere,} \end{cases}$$

and with this choice of D_1, the operator $\mathcal{J}$ becomes

$$\mathcal{J}[f](a, x, t) = \mathbb{E}^{(a,x)}\Big[1_{\{t < T_1\}} \, F(a)$$
$$+ 1_{\{T_1 \le t\}} \Big(1_{\{X_{Y_1}(T_1)=1\}} \cdot \mathcal{S}_{Y_1}^*[f]\big(a, X(T_1), t - T_1\big) + 1_{\{X_{Y_1}(T_1)=0\}} \cdot f\big(a, X(T_1), t - T_1\big) \Big) \Big] \tag{3.8}$$

where

$$\mathcal{S}_i^*[f](a, x, t) := \max_{0 \le j \le m} \mathcal{S}_{i,j}[f](a, x, t), \qquad \text{for } i \le n.$$

The dynamic programming equation hints that the policy of advertising for the company j for which $\mathcal{S}_{i,j}[V]$ is maximized should be optimal when the ith user makes a transition to state $\{1\}$.

Let us introduce

$$r_i(x) = (1 - x_i) \, \mu_{i,0} + x_i \, \mu_{i,1} \quad \text{for } i \le n, \quad \text{and} \quad r(x) = \sum_{i \le n} r_i(x). \tag{3.9}$$

Then, in terms of the distribution

$$G^{(x)}(t) := \mathbb{P}^{(\cdot, x)}\big(T_1 \le t\big) = 1 - e^{-t \, r(x)}, \qquad t \ge 0, \tag{3.10}$$

of the first event time T_1 with the density

$$g^{(x)}(t) = \frac{\partial \, G^{(x)}(t)}{\partial t} = r(x) \, e^{-t \, r(x)}, \qquad t \ge 0,$$

we can rewrite the dynamic programming programming operator in (3.8) explicitly as the sum of

$$\mathbb{E}^{(a,x)}\Big[1_{\{t<T_1\}} F(a)\Big] = F(a) \cdot \big[1 - G^{(x)}(t)\big] \tag{3.11}$$

and

$$\mathbb{E}^{(a,x)}\Big[1_{\{T_1 \le t\}}\Big(1_{\{X_{Y_1}(T_1)=1\}} \mathcal{S}_{Y_1}^*[f]\big(a, X(T_1), t - T_1\big) + 1_{\{X_{Y_1}(T_1)=0\}} f\big(a, X(T_1), t - T_1\big)\Big)\Big]$$
$$= \int_0^t g^{(x)}(u) \sum_{i \le n} \frac{r_i(x)}{r(x)}\Big\{(1 - x_i) \cdot \mathcal{S}_i^*[f]\big(a, x + \bar{e}_i, t - u\big)$$
$$+ x_i\big[\beta_i \cdot \mathcal{S}_i^*[f]\big(a, x, t - u\big) + (1 - \beta_i) \cdot f\big(a, x - \bar{e}_i, t - u\big)\big]\Big\}du, \tag{3.12}$$

where $\bar{e}_i$ is a row vector of all zeros except that the i'th entry is one. Clearly, given a function f, $\mathcal{J}[f]$ can be computed by evaluating the expressions in (3.11-3.12) numerically. Numerical results will be in Chapter 4.

3.2   A finite Difference Approach to Compute the Value Function

For every $(a, x, t) \in \Delta$, the policy $\widetilde{D}^{(\infty)}$ attains the supremum in (3.3) and we have

$$V(a, x, t) = \mathbb{E}^{(a,x)}\Big[F\big(\widetilde{A}^{(\infty)}(t)\big)\Big].$$

For small $h < t$, let $\widetilde{\mathcal{F}}_h^{(\infty)}$ be the information available at time t. The conditioning on $\widetilde{\mathcal{F}}_h^{(\infty)}$, we can write

$$V(a, x, t) = \mathbb{E}^{(a,x)}\Big[\mathbb{E}^{(a,x)}\big[F\big(\widetilde{A}^{(\infty)}(t)\big)|\mathcal{F}_h\big]\Big] = \mathbb{E}^{(a,x)}\Big[V\big(\widetilde{A}^{(\infty)}(h), X(h), t - h\big)\Big], \tag{3.13}$$

thanks to the structure of the optimal policy and the Markov property. Recall that for small h we have

$$\mathbb{P}^{(a,x)}(T_1 > h) = 1 - r(x)\,h + o(h) \quad \text{and} \quad \mathbb{P}^{(a,x)}(T_1 \le h, Y_1 = i) = r_i(x)\,h + o(h)$$

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for $i \le n$. Using these probabilities in (3.13) together with the structure of the optimal policy, we obtain

$$V(a, x, t) = \Big[1 - r(x)h + o(h)\Big] V(a, x, t - h) + \sum_{i=1}^{n} \big[(1 - x_i(0))\,\mu_{i,0} h + o(h)\big]\, \mathcal{S}_i^*[V]\big(a, x + \bar{e}_i, t - h\big)$$

$$+ \sum_{i=1}^{n} [x_i(0)\,\mu_{i,1} h + o(h)]\,\Big[\beta_i \cdot \mathcal{S}_i^*[V]\big(a, x, t - h\big) + (1 - \beta_i) \cdot V\big(a, x - \bar{e}_i, t - h\big)\Big].$$

After arranging the terms, this gives

$$\frac{V(a, x, t) - V(a, x, t - h)}{h} = -\Big[r(x) + \frac{o(h)}{h}\Big] V(a, x, t - h)$$

$$+ \sum_{i=1}^{n} \Big[(1 - x_i(0))\,\mu_{i,0} + \frac{o(h)}{h}\Big]\, \mathcal{S}_i^*[V]\big(a, x + \bar{e}_i, t - h\big)$$

$$+ \sum_{i=1}^{n} \Big[x_i(0)\,\mu_{i,1} + \frac{o(h)}{h}\Big]\,\Big[\beta_i \cdot \mathcal{S}_i^*[V]\big(a, x, t - h\big) + (1 - \beta_i) \cdot V\big(a, x - \bar{e}_i, t - h\big)\Big],$$

and letting $h \searrow 0$, we obtain the left partial derivative of V with respect to t as

$$D^- V(a, x, t) = -r(x)V(a, x, t) + \sum_{i=1}^{n}(1 - x_i(0))\,\mu_{i,0}\,\mathcal{S}_i^*[V]\big(a, x + \bar{e}_i, t\big)$$

$$+ \sum_{i=1}^{n}[x_i(0)\,\mu_{i,1}]\,\Big[\beta_i \cdot \mathcal{S}_i^*[V]\big(a, x, t\big) + (1 - \beta_i) \cdot V\big(a, x - \bar{e}_i, t\big)\Big] =: \mathcal{L}[V](a, x, t).$$

$$(3.14)$$

Repeating the arguments similarly between $t + h$ and t, we observe that the right partial derivative $D^+ V(a, x, t)$ coincide with $\mathcal{L}[V](a, x, t)$ above. Hence, we conclude that V is differentiable with respect to t. This observation can be used to devise a finite-difference approach to compute the value function numerically. Numerical studies for the finite difference approach algorithm also take a place in Chapter 4. In particular, when h is small, we can use the approximation

$$V(a, x, t + h) \approx V(a, x, t) + h \cdot \mathcal{L}[V](a, x, t) \tag{3.15}$$

with the initial condition $V(a, x, 0) = F(a)$, for all $(a, x, t) \in \Delta$.

Chapter 4

Experimental Analysis and Results

In order to test the performance of the proposed algorithms, namely the value iterations based dynamic programming algorithm and the finite difference approach, various experimental analyses were conducted. On top of these two algorithms, four heuristics, which are referred to as A, B, C and Random, were also developed and included to the experimental analysis.

Briefly speaking, the heuristics are myopic greedy heuristics and only the degree of shortsightedness of each one varies from the others. That is to say, other than the random heuristic (as the name implies, that one just randomly assigns an advertisement to the active user), the developed heuristics checks the immediate return and assigns the highest paying advertisement to the active user while considering a subset of the constraints that were imposed by the contract between the advertiser and the publisher. That is to say, the subset of the constraints that are considered by a particular heuristic is what sets it apart from the other two heuristics.

The first heuristic, namely the Heuristic A, considers the maximum advertisement display number, minimum advertisement display number, maximum budget and minimum budget constraints. The heuristic checks all of the constraints for each one of the advertisement, rules out the advertisements that are not going to generate any revenue in case displayed due to the constraints and assigns the advertisements that yields highest immediate return for the active user among the remaining ones. On the other hand, Heuristic B, takes into account only the maximum advertisement display and the maximum budget constraints during the candidate advertisements selection process. Likewise, the third heuristic, namely Heuristic C, neglects all of the constraints and dispatches the advertisement that yields the highest immediate return for the active user. Finally, the fourth heuristic is the Random Heuristic which dispatches one of the advertisements randomly (with equal probability).

Recall that the theoretical background and the most significant equations that were used in the value iteration based dynamic programming algorithm and the finite differ-

ence approach were presented in Chapter 3. The pseudo codes of the algorithms are also presented in order to clarify the details of the algorithms and improve the understanding of the readers in Appendix A. Furthermore the pseudo codes of the heuristics are also available in Appendix A.

Various experimental analysis are developed in order to understand the effects of the parameters associated with the algorithms ($Structural\ Analysis$) and compare the performances of the algorithms ($Performance\ Analysis$) under different experimental conditions, i.e., sensitivity of the performance of the algorithms with respect to the problem parameters. The general framework of the experimental analysis is depicted in Figure 4.1. The first set of analysis provided us insights regarding to the execution of the algorithms as well as the parameter tuning phase. The latter set of experiments revealed the conditions under which the algorithms performed better than the others.
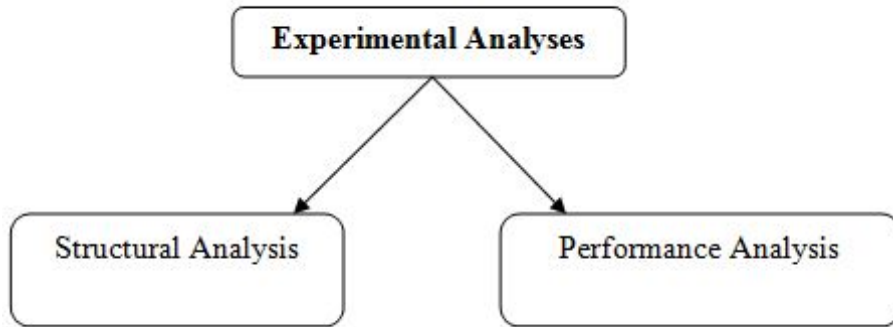

Figure 4.1: Classification of Experimental Analyses

In order to conduct the experimental analysis, replications of sample realizations ($i.e., sample\ paths$) were randomly generated. Note that, in the following results, the performances of the heuristics are solely the results based on the sample path realizations. On the other hand, for the value iteration based dynamic programming algorithm and the finite difference approach, we also have theoretical results ($i.e., expected\ revenue$) along with the revenue realizations based on the sample paths. The latter realized revenues are obtained basically by using the partial theoretical result from the algorithm and assigning the advertisement that has the highest $expected\ revenue$ for a particular active user at a particular time and state.

Next, the results of the Structural Analysis and Performance Analysis are provided in two consecutive sections, namely Section 4.1 and Section 4.2.

4.1 Structural Analysis

First set of analyses were based on the parameters which influences the performance of the algorithms. Note that, these analyses did not provided insights regarding to the executions of the algorithms, but also provided us a set of reasonable parameters for further analysis. As we will discuss later in more detail, one of the significant issues regarding to the developed algorithms, in particular for the dynamic programming based algorithms, is the high computational requirements. Therefore, these analyses allowed us to determine the parameter values that can be used in order to conduct runs in reasonable amount of time. Finally, one another reason for these analyses was for the purpose of the verification and validation of the algorithms. For various parameters, we have also tested extreme values and checked if the expected results were obtained.

Algorithms			
Critical Parameters	**Dynamic Programming**	**Finite Difference Approach**	**Heuristics**
Number of sample paths	√	√	√
Iteration number	√		
Resolution	√		
h value		√	
β probability	√	√	√

Table 4.1: First Group of Analysis (Structural Analysis)

Table 4.1 tabulates the parameters that were analyzed in this stage. These are namely the *sample path* (for all of the algorithms), *iteration number* and *resolution* (for the value iteration based dynamic programming algorithm), $h-value$ (for the finite difference algorithm) and $\beta\ probability$ (for all of the algorithms).

The *number of sample paths* denotes the number of random replications that were used in the experiments. Note that, the results of the analysis that are based on low numbers of replications are dubious due to the randomness. Therefore, sufficient numbers of replications are required where the variation due to randomness is swept away yet it is also computationally feasible. Note that, the sample path parameter is applicable for all of the algorithms.

The *iteration number* (k) and the *resolution* are the two significant parameters that directly influence the performance of the value iteration based dynamic programming algorithm both in terms of the accuracy of the calculated expected revenues and the computational requirements. Note that, even though, a theoretical result is provided regarding to the appropriate selection of the *iteration number* (k) earlier, for the sake of decreasing the computational efforts, we conducted analysis in order to determine reasonably good values for this parameter. On the other hand the *resolution* is the parameter that is required to approximate the integration available in the recursion equation [Equation 3.12]. Note that higher the *resolution*, better the approximation but higher the computational time required. Therefore, an appropriate value for *resolution parameter* is also

required.

The $h-value$ is the counter part of the $resolution$ for the finite difference algorithm. It is used in the approximation presented in Equation 3.15. Similar to the $resolution$, higher the $h-value$, better the approximation but higher the computational time required.

Finally, recall that the user may switch to another virtual location while she was already online. The $\beta\ probability$ is the probability of re-entering the state 1 in the next transition while the current state of the user is 1. Unlike the former four parameters this parameter is associated with the $problem$ rather than the $algorithms$, that is to say it should be estimated from the historical data and used in the algorithms. We have conducted various analyses with respect to the $\beta\ probability$ for validation and verification of the purposes.

Note that, various other parameters that are associated with the $problem$ also exist, such as the parameters that are due to the contract between the advertiser and the publisher, namely the $maximum\ budget$, $minimum\ budget$, $maximum\ advertisement$ $display\ number$ and $minimum\ advertisement\ display\ number$. Some bounds associated with some of these parameters are present in the literature based on empirical researches. For example, according to Novak and Hoffman [38] less than three exposures are ineffective for the user to perceive the message given. On the other hand, Novak and Hoffman [38] also state that exposures after 7, 8 or 10 have almost no effect and it can also be annoying for users.

Therefore, generally speaking, the minimum advertisement display number ($when$ $applicable$) was set to be equal to three, and the maximum advertisement display number was set to be equal to eight during the analysis. In some analysis due to computational reasons, the maximum advertisement display number was set to be a value lower than eight, in case higher value of this parameter would not change the conclusion of the particular analysis (that is to say, value of the parameter is somewhat irrelevant for that particular experiment). Furthermore, in various experiments minimum advertisement display number was set to the trivial value ($i.e.$, $zero$) or rearranged according to the maximum advertisement display number which was used in the analysis (e.g., if the maximum advertisement display number was set to be equal to three, the minimum advertisement display number was set to be equal to two).

Generally speaking, for most of the analysis, for the maximum budget and minimum budget levels, the trivial values were used, i.e., $infinite$ and $zero$ respectively. Only in some of the analysis these parameters were set to other values in order to create various scenarios (such as, tight budget, etc.).

Another problem specific parameter used in the analysis is the $event\ rates$, which is the parameter that is used to specify the time to next transition of the users. Note that, in our analysis we selected various event rates from 1 to 30. The value or the set of values of the event rates used in a particular experiment is presented in Table 4.2 along with the

corresponding values of the other variables. Due to the immense number of parameters associated with the algorithms and problem scenarios a full factorial experimental analysis was computationally prohibitive. Therefore only the conditions that were appropriate for a particular objective in an analysis was considered. These conditions are presented in Table 4.2. The last column (namely, Considering Bounds column) in Table 4.2 specifies which contract based parameters are utilized in the analysis, that is to say, those that are not listed in a particular experiment are set to the trivial values. Furthermore, three different levels are assumed in the analysis for the *maximum advertisement display number*, namely high level, middle level and low level, which are set to be equal to 8, 5 and 3 respectively. The corresponding level(s) used in a particular experiment for the *maximum advertisement display number* are also tabulated in Table 4.2.

Critical Parameters of Algorithms			
Analysis	Advertisement Display Number	Event Rate	Considering Bounds
Sample paths	middle level	30	max. display, min. display
Iteration number	min. and middle level	10, 20	max. display
Resolution	min level	1, 5, 10, 20	max. display, min. display
h value	middle and high level	10, 30	max. display, min. display max. budget, min. budget
β probability	min level	10	max. display, min. display

Table 4.2: Test Conditions (First Group of Analysis)

In order to determine the appropriate number of sample paths (i.e., *replications*) that should be utilized in the analysis for a random problem scenario (where event rate is set to be equal to 30, maximum advertisement display number is set to be middle level and all of the contracting constraints are considered as presented in Table 4.2 (Test Conditions) for 1000 random replications are generated randomly and the results of the algorithms are observed. Figure 4.2 depicts the average revenue realizations and Figure 4.3 illustrates the variance of the average revenue realizations for the algorithms obtained from different number of replications (i.e., 1 to 1000). Note that, this analysis was conducted for all of the algorithms with the same set of replications. As we can observe from the figure, the results converge after 100 sample paths to a particular level and the change after this value is subtle. Hence we decided to use 100 as number of sample paths (i.e., *replications*) during the rest of the analyses. Figures 4.2 and 4.3 represent a case without no constraints, the case which is minimum advertisement display was considered may be seen in the appendix.

Iteration number (k) is one of the fundamental factors that effects the results of the value iteration based dynamic programming algorithm. Recall that this value represents the kth transition time and the value iteration based dynamic programming algorithm calculates the expected revenues for all of the state-space combinations assuming that the previous k−1 decisions are optimal. Therefore, this value should be chosen very carefully; it should be large enough so that it should cover the event realizations and generate
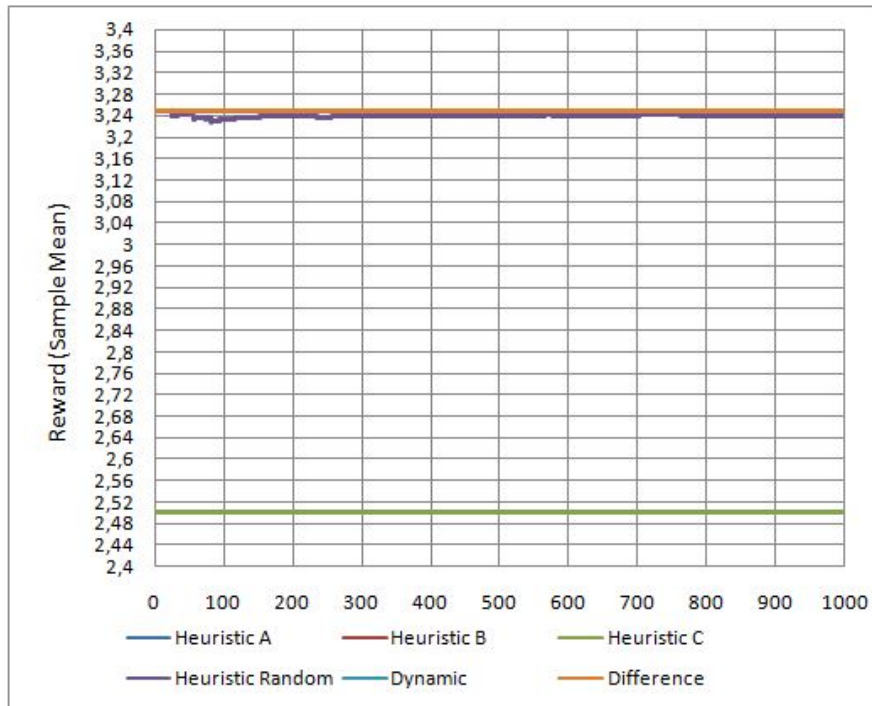
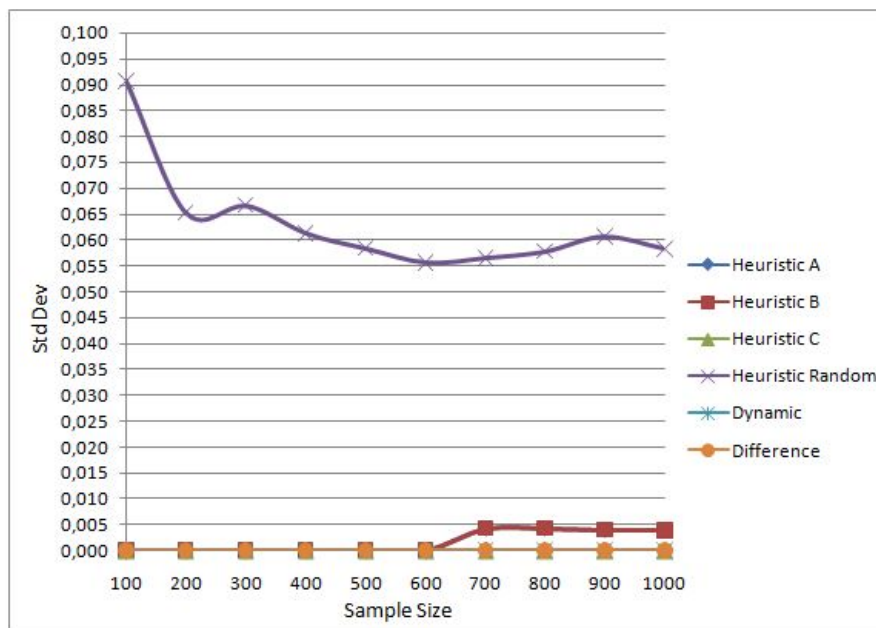Figure 4.2: Mean of Sample (Sample Path Analysis)

Figure 4.3: Sample Standard Deviation (Sample Path Analysis)

a decision, at the same time small enough to overcome the problems associated with the computational requirements of the experimental analysis. Even though a theoretical value depending on an epsilon parameter, supremum revenue, time horizon, and total event rate for each case, in the experimental analyses an efficient value was determined which would not influence the performance of the algorithm for the most part. Recall that the equation

for iteration number calculation is presented earlier as follows:

$$k \geq \frac{\epsilon + (2\|F\| \, T \, \bar{\mu})}{\epsilon} \qquad (4.1)$$

In these analyses only the maximum advertisement display number was considered as variable (two values are assumed as 3 and 5) and for the other contracting constraints (i.e., the budget constraints and the minimum advertisement display number) were assumed to be their trivial values. Two different event rate values are considered as 10 and 20. Note that, higher the event rate, one expects that there is a need for higher iteration number (since more decisions are expected to be made) in order to converge to the true expected revenue.

Recall that the *resolution* parameter that is used in the approximation of the integration in the recursion equation of the algorithm also influences the performance of the algorithm. Therefore, two resolution levels are considered in the analysis, which are chosen to be 30 and 60 after some initial trials. Note that, higher the resolution rate, the chance of observing an event is higher (the events occurs only in these discrete points), hence more decisions are to be made and as a result higher iteration number is required in order to converge to the true expected revenue.

The results of these analyses are depicted in Figure 4.4 and Figure 4.5 for resolution numbers 60 and 30 respectively. The results are parallel to our initial expectations hence somewhat the verification of the code was attained. Note that it may be observed from the figures that after 30 iteration number there is almost no alteration and after 20 iteration number, the results change less than one percent for the particular parameter settings. As a result rather than the higher theoretical values for the iteration number, a much lower *practical* iteration number was chosen for the rest of the analyses which reduced the computational time required for the rest of the analyses immensely and do not change the conclusions much.

Figure 4.6 and 4.7 shows changes of results coming from dynamic programming model up to value of resolution. These graphics show theoretical values for the dynamic programming models. Event rate "μ" is taken as four different sets they are 1,5,10 and 20 for each user, model was run for maximum 3 advertisement display. Studies are designed with and without constraints. Both type of analysis represent that after 30 resolution value, the results change less than one percent for the particular parameter settings. After studies on resolution and iteration number we decided to specified value of them for the following experiments to 20 for iteration and 30 for the resolution.

Finite difference approach has a critical parameter "h" is a small time interval. As mentioned in the problem statement part, h should be small and close to zero to get the optimal result from the model. Therefore we have studied on the value of "h" and tried six
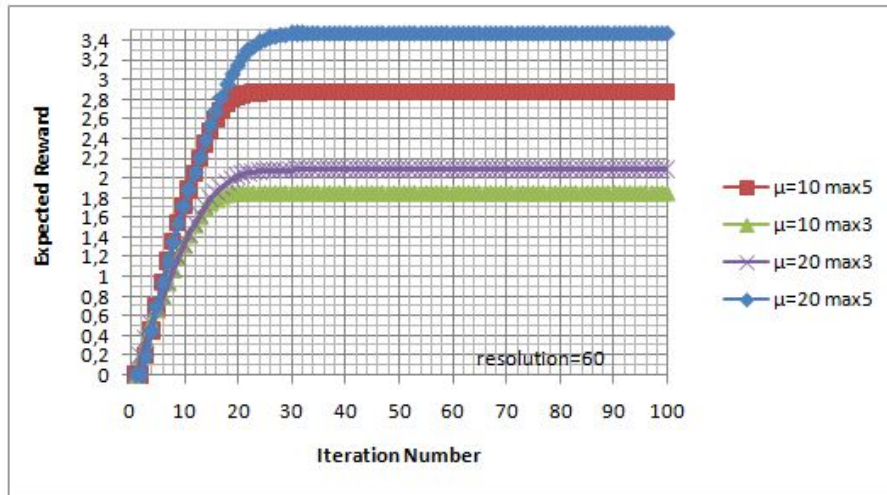
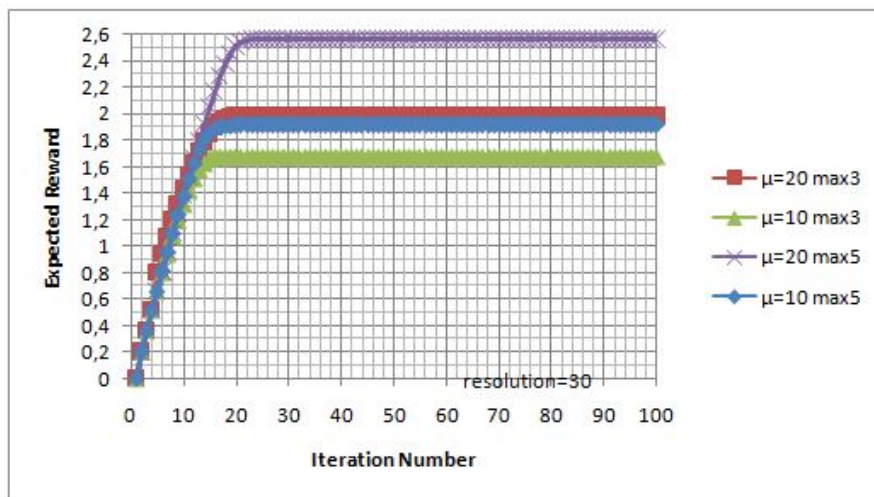Figure 4.4: Iteration Number Analysis for Resolution 60

Figure 4.5: Iteration Number Analysis for Resolution 30

different h values for different cases. The case has low event rate ($\mu = 10$) and more constraint (maximum 5 and minimum 3 advertisement display, minimum 1.5 and maximum 2 budget) could give acceptable result for the high "h = 0.05" value. When we increase the maximum advertisement display level or add maximum advertisement display constraint we detected that "h" value should have been selected as smaller than the other situations. For the following analyses we selected the "h" value as 0.005. Figure 4.8 demonstrates results for h value analysis.

In a time horizon for the whole models we assumed that users can change their status from online to online, it is possible while they are changing their stage, page or environment, and they stay to be online. Independently with probability of we have described this event in our formulations. Also the user can go offline (jump to state 0) with probability 1-β according to our description. An examination on the probability are on the Figures 4.9 and 4.10 with and without minimum advertisement display constraints.
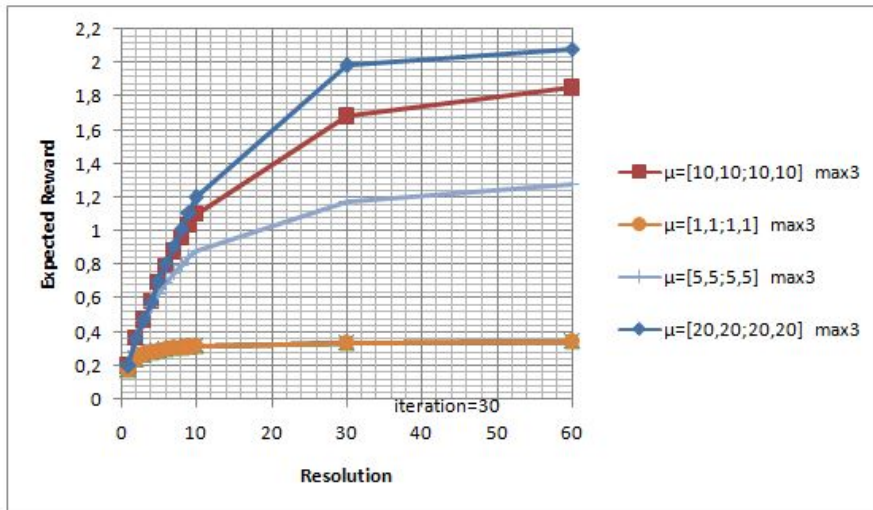
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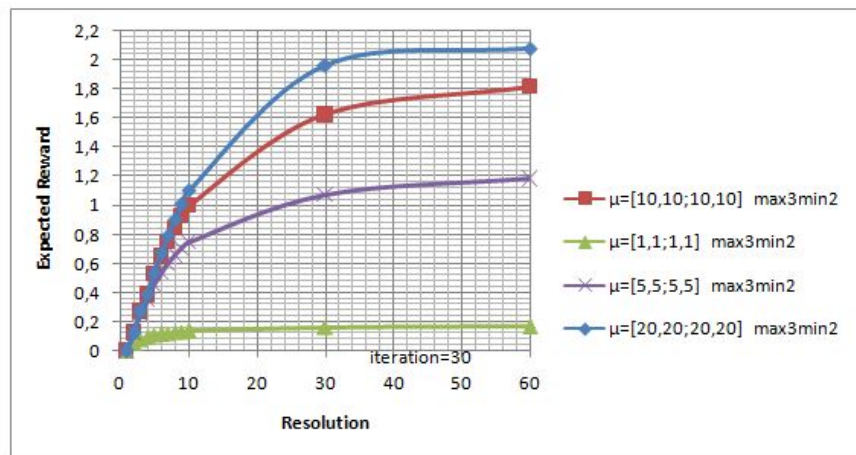Figure 4.6: Resolution Analysis without Constraint

Figure 4.7: Resolution Analysis with Minimum Display Constraint

Results of heuristics, theoretical and experimental finite difference and dynamic programming approach are on the graphic. Three different set of β ([0.7, 0.9], [1, 1], [0.1, 0.25]) have been applied with and without constraint cases. When we have increased the , results have also increased as we predicted. The highest value for β is 1 we have detected the highest revenue among same conditions with constraint and without constraint. After these examination we decided to fix the value for β to [0.5, 0.7] for the following studies.
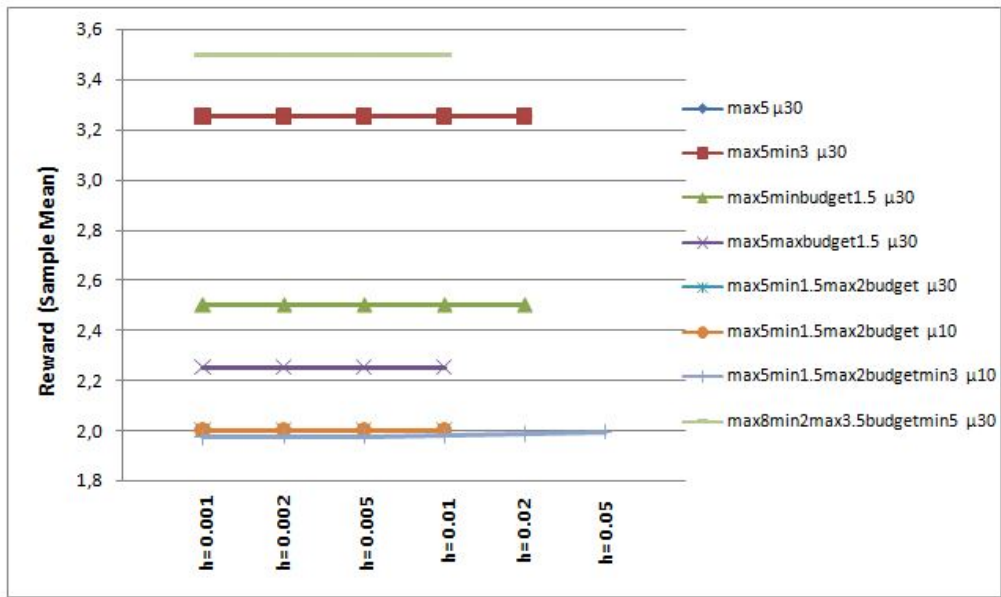
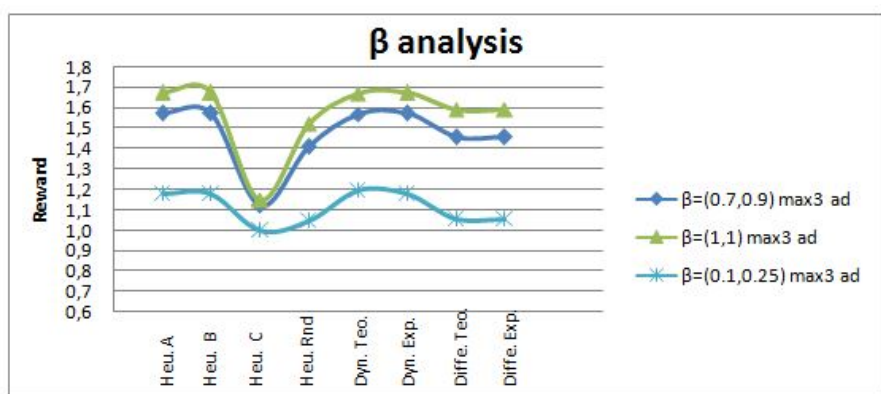Figure 4.8: h Value Analysis for Finite Difference Approach
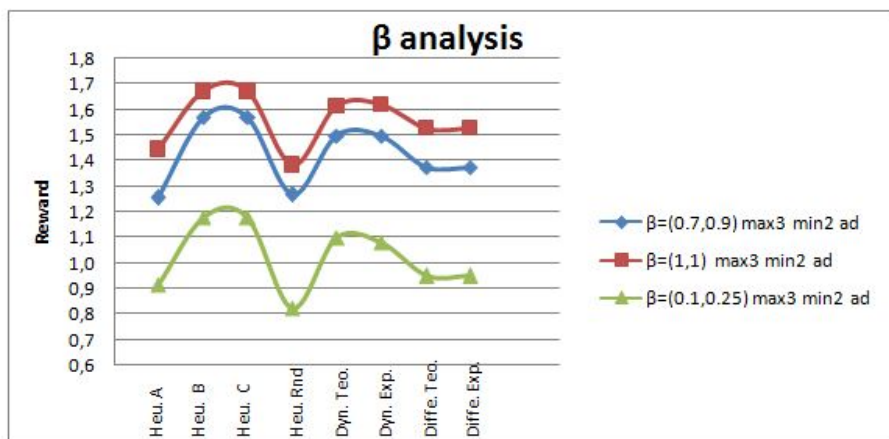

Figure 4.9: β Analysis without Constraint

Figure 4.10: β Analysis with Minimum Display Constraint

4.2   Performance Analysis

Analysis was gathered into two groups, the first which I explained above is to examine the structure of the algorithm and select the most appropriate values for the critical parameters. The second group of the analysis is to measure the performance of proposed algorithms and compare them. Test conditions for the second group of analysis are on Table 4.3. After the analyses the most effective model may be suggested to the owner of the web page to maximize its revenue from the advertisement display. Firstly, computational times of all algorithms are below.

Analysis	Advertisement Display Level	Event Rate	Considering Bounds
Computational time	min. level	5, 10, 30	max. display, min. display max. budget,min. budget
Heuristics comparison	middle and high level	5, 30	max. display
All Models comparison	middle level	5, 30	max. display, min. display
Heuristics failing condition	high level	5	max. display, min. display max. budget, min. budget

Table 4.3: Test Conditions (Second Group of Analysis)

Hereby I will present computational times for all models and the effecting parameters of their running time. They may be seen on the Figure 4.11. It is observed that heuristics only need seconds to calculate the results while other algorithms need minutes and for some cases hours. When iteration times and resolution increase, dynamic programming algorithm needs more time to calculate the optimal revenue. Both of them have increasing effect on the run time, resolution's effect is more. When it increases the algorithm needs much more time than iteration number increases to compute the result. On the side of finite difference approach model "h value" influence its running time. Event rate "μ" has almost no effect on the computational time.
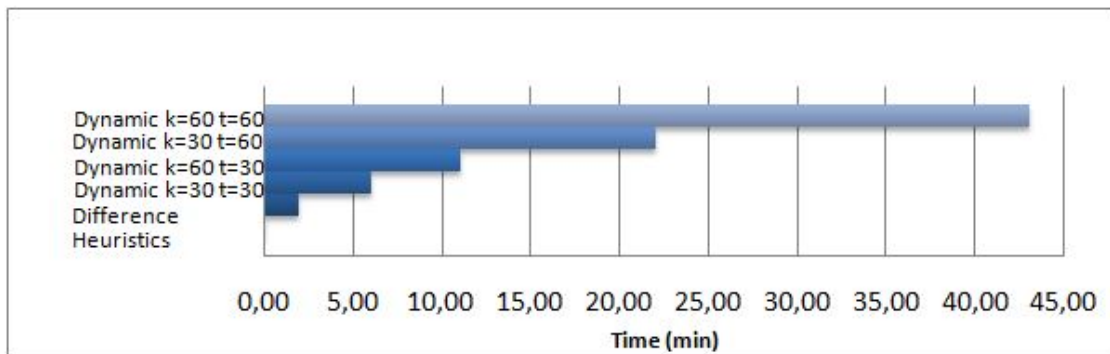

Figure 4.11: Computational Time

After the analysis of computational time, rewards those are generated by heuristics were compared. Maximum advertisement display was selected in two different levels as 5 and 8 on the analyses below. Both analyses designed in the variety of constraint and event

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rate cases those are maximum budget, minimum budget and minimum display constraints with event rate of 5 and 30. In all cases virtual environments acquire the best result when they apply Heuristic B. Figures 4.12 and 4.13 represent revenues generated by heuristics. Although heuristic B has only maximum display and maximum budget constraints to assign the advertisement to the user of virtual environments, we detected the best result. The heuristic do not stick on the minimum display condition for assignment so it can display ad to higher incomes bringer.
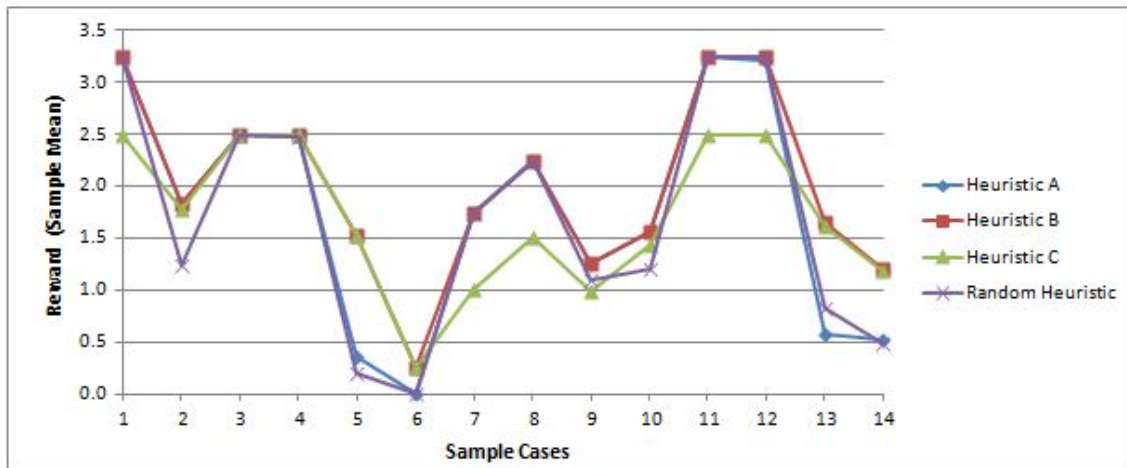
Figure 4.12: Heuristics Results Comparison Maximum 5 Advertisement Display
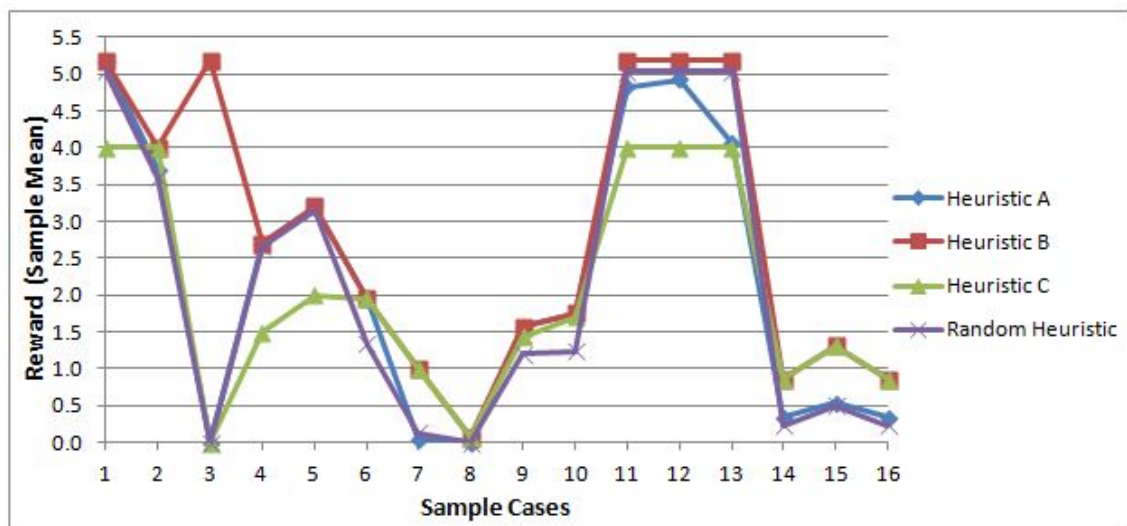
Figure 4.13: Heuristics Results Comparison Maximum 8 Advertisement Display

To measure performance of all proposed algorithms and to compare them in a reasonable time, maximum advertisement display number selected as middle one which is five. In some specific conditions majority of algorithms have to be fail according to our prediction. In a set of experiment we specified minimum budget constraints for two different event rates one is low (5) and the other is high (30). It is detected that on the condition of

Figure 4.14: Comparison of Algorithms with min. Budget Constraint

low event rate and minimum budget constraints finite difference approach provided better result than other heuristic and the dynamic algorithm. Results on the different conditions at Table 4.4 and on Figure 4.14.
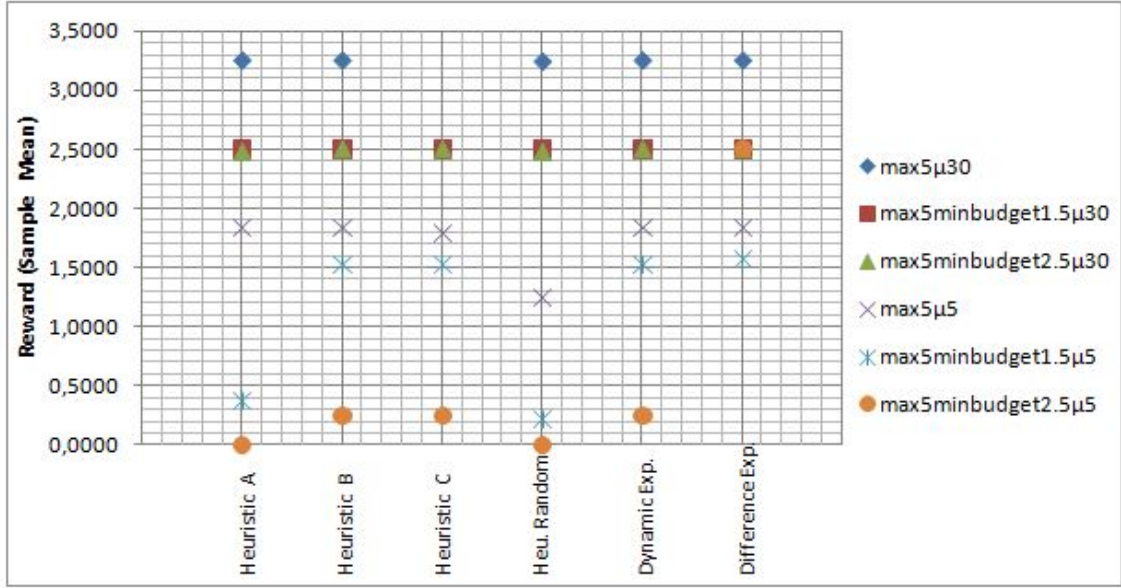
	A	B	C	Random	Dynamic	Difference
max5μ30	**3.2500**	**3.2500**	2.5000	3.2410	**3.2500**	**3.2500**
max5minbudget1.5μ30	**2.5000**	**2.5000**	**2.5000**	2.4980	**2.5000**	**2.5000**
max5minbudget2.5μ30	2.4750	**2.5000**	**2.5000**	2.4750	**2.5000**	**2.5000**
max5μ5	**1.8260**	**1.8260**	1.7830	1.2395	**1.8260**	**1.8260**
max5minbudget1.5μ5	0.3590	1.5200	1.5200	0.2070	1.5200	**1.5600**
max5minbudget1.5μ5	0.0000	0.2500	0.2500	0.0000	0.2500	**2.5000**

Table 4.4: Comparison of Algorithms with min. Budget Constraint

In another set of experiment we defined minimum ad display constraints for two different event rates one is low (5) and the other is high (30). It is detected that on the condition of low event rate and minimum ad display constraints dynamic programming model afforded better than other heuristic and the finite difference approach. Results on the different conditions at Table 4.5 and on Figure 4.15.

	A	B	C	Random	Dynamic	Difference
max5μ30	**3.2500**	**3.2500**	2.5000	3.2410	**3.2500**	**3.2500**
max5minbudget1.3μ30	3.2420	**3.2500**	2.5000	3.2410	**3.2500**	**3.2500**
max5minbudget2.4μ30	3.2180	**3.2500**	2.5000	2.4750	**3.2500**	**3.2500**
max5μ5	**1.8260**	**1.8260**	1.7830	1.2395	**1.8260**	**1.8260**
max5minbudget1.3μ5	0.5675	1.6600	1.6170	0.8235	**1.6620**	1.6605
max5minbudget2.4μ5	0.5145	1.2120	1.1820	0.4880	**1.2275**	1.2145

Table 4.5: Comparison of Algorithms with min. Advertisement Display Constraint
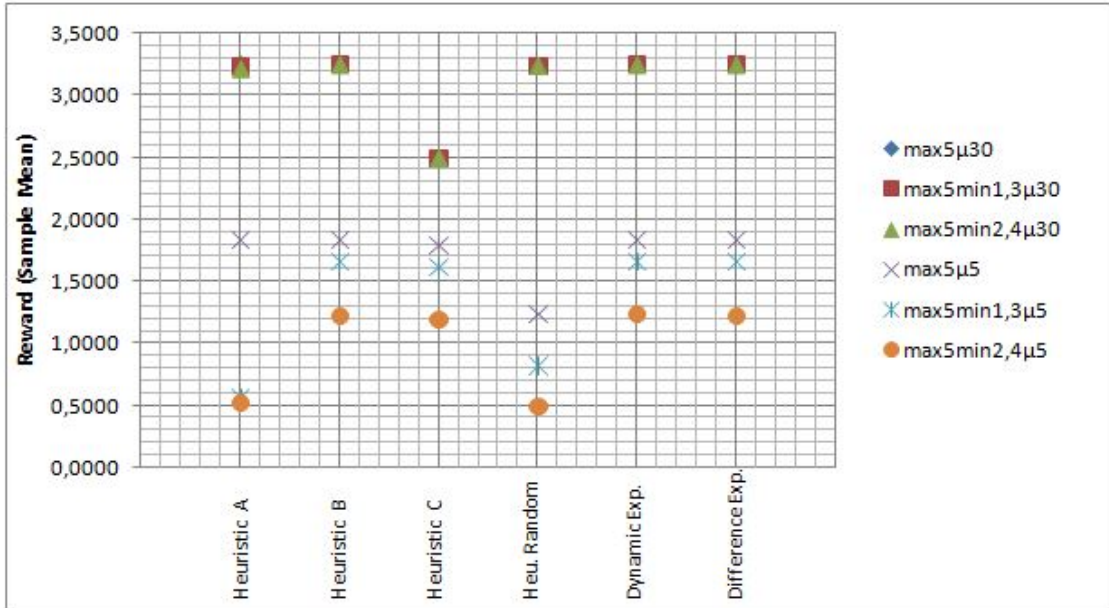
Figure 4.15: Comparison of Algorithms with min. Advertisement Display Constraint
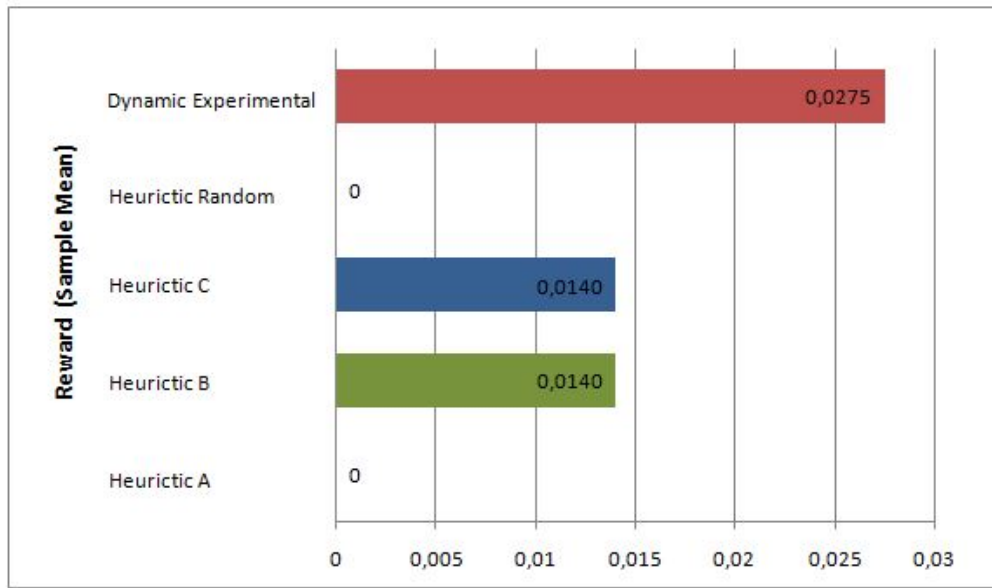

Figure 4.16: Heuristics Fail Situation

Heuristics are more straightforward when we compare with the dynamic programming or the finite different approach. They can generate result in a short time. Although they only need seconds to calculate the optimal revenue, till now we have observed that their results close to the dynamic and the finite difference approach models results.

According to nature of the algorithms it is expected that heuristics cannot compute optimal solution in some conditions. On the case of high level advertisement display, high level of minimum advertisement display constraint and low event rate heuristics failed. It is observed that results of dynamic 49% higher than heuristic B and C, also heuristic A

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and random could not compute any result. Results of them may be seen on the Figure 4.16.

These results provide that our defense is true for the heuristics. They can generate a solution in a short time and their structure can be more simple but they are non-functional for the situations which are I explained above. Thus the fundamental algorithm that is the dynamic one is more effective and trustable in spite of its complexity and high computational time.

Chapter 5

Concluding Remarks

This thesis inspired from a problem that a virtual social platform developer company faces during revenue generation process from the advertisements. The virtual environment allowed the developer company (i.e., publisher) to personalize the advertisements and maximize its profit by targeting the users that are compatible with the specifications set by the advertisers. Even though the motivation of the research was the problem that this virtual social platform developer company faced, similar problem is faced by many other digital media companies (web sites, social networks, mobile services, etc.) that mostly relies on advertisements revenues and have detailed personal information. This problem encompass two phases; namely, the matching phase where the compatibility with the users' profiles and the advertisers specifications are determined and the assignment phase where the best advertisement among the set of advertisements is assigned whenever a user becomes active (i.e., an opportunity to expose an advertisement realizes).

Based on the contracts between the advertisers and the publishers various constraints exists in the problem. These constraints are referred to as maximum payment, minimum payment, maximum advertisement display number and minimum advertisement display number. Furthermore, the advertisers pay different amounts for the display of their advertisements to the users which are based on the degree of compatibility that is determined after the matching phase. The constraints as well as the pricing scheme increases the complexity of the problem and discourages myopic approaches.

In this study, six different personalized advertisement assignment algorithms are developed and their performances are evaluated. First, a value iteration based dynamic programming algorithm is developed in order to solve the personalized advertisement assignment problem in a finite time horizon. Secondly, a finite difference algorithm was developed for the same problem. Finally, four different greedy heuristics with varying level of "consciousness" is developed and compared with the former two algorithms.

Two sets of analyses are conducted during the experimental analysis step. The first set was conducted in order to tune the essential parameters so that the rest of the analysis

could be conducted in feasible time and validate and verify the algorithms. The second set of experiments was conducted in order to measure their performance in terms of the revenues. First, numbers of sample paths (i.e., the replications) was determined where the trade of is between the computational time (lower the number of replication the better) and the error due to randomness (higher the number of replications the better). Second, a set of analysis was conducted in order to select appropriate values for the iteration number and resolution parameters of the value iteration based dynamic programming algorithm. Third, different h values were experienced to select a reliable value for the different finite approach model.

Proposed Models	Conditions			
Dynamic Programming	Loose conditions	Minimum budget	Minimum ad display	Tight conditions
Finite Difference Approach	Loose conditions	Minimum budget	-	-
Heuristics	Loose conditions	-	-	-

Table 5.1: Suggestion of Algorithms for Variety of Conditions

The most challenging part of our study is to compare all proposed models to solve the advertisement assignment problem in an efficient way. First of all, the execution times of the algorithms were compared. As expected that the value iteration based dynamic programming algorithm required the highest time to calculate the expected revenues, while the computational time required for heuristics were virtually negligible. Moreover, when heuristic models compared between each other, it is detected that heuristic B generates the best result on the average.

In order to determine the most reliable proposed algorithm under different conditions that are considered in the contracts, an experimental analysis was conducted. The results revealed that, for the basic conditions (no or loose constraints considering), the greedy heuristics performs as good as the more elaborate (and time consuming) value iteration based and finite difference method algorithms. Considering only one bound such as minimum display and for the minimum display constraint until three(minimum level) display number, which is described as minimum level for the experimental analyses, may be called as loose constraints. Regarding more than one bound and specifying high level for the each bound may also be call tight constraints.

Based on the analysis, none of the algorithms outperforms the rest and generates the highest performance under all circumstances. For example when minimum budget constraint was considered, the finite difference algorithm outperforms the others on the average but its results were not significantly different than the result of the value iteration based dynamic programming algorithms results. Furthermore, when a minimum advertisement display condition was considered this time the value iteration based dynamic programming model generated the expectedly best revenue. Last but not least, in order to determine the most reliable model in each conditions, all constraints were involved in

the high advertisement display level. As a result of examinations, value iteration based dynamic programming algorithm has been computed as best solutions. Revenues which are coming from value iteration dynamic programming, is approximately 49% percent higher than heuristics. Table 5.1 shows outcome of comparisons, it may be seen clearly which one may offer in what conditions.

Within the context of future studies, user of the web sites may click ads, so the click revenue should also be included in the algorithms. We have not encounter with any application of punishment if number of displayed ad is lower than minimum ad display constraint on the area of personalized advertisement. Punishment has large implementation on the logistics revenue management, therefore, it might be taken into account in the next researches. As a conclusion, this study introduces a new aspect to developing business of personalized advertisement. Different algorithms were generated and examined. Performances of all proposed models were measured and compared. A route was also described for future studies.

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Chapter A

Appendix: Algorithms

A.1 *Algorithm: Value Iteration Based Dynamic Programming*

Notation:

i: 1…n index for the users (NumberofUsers)

j: 1…m index for the advertisers (NumberofAdvertisers)

$X_i(t)$ *: State of the user i (CurrentStateofUserArr)*

$\mu_{i,0}$ *: event rate of the user I when its current state offline (TransitionRateMuforUsers)*

$\mu_{i,1}$*: event rate of the user I when its current state online (TransitionRateMuforUsers)*

β_i *: online to online transaction probability of the user I (TransitionProbBetaforUsers)*

$A_{i,j}(t)$ *: exposure matrix j ads to i users (ExposureMatrixArr)*

F(A(T)): reward function (TOTALREWARD)

T: time horizon (TimeHorizon)

t: remaining time (RemainingTime)

Maximum budget bound: MaximumBudgetConstraint

Minimum budget bound: MinimumBudgetConstraint

Maximum display bound: MaximumDisplayConstraint

Minimum display bound: MinimumDisplayConstraint

Payment for each exposure: ExposurePaymentMatrixArr

1. Define Input Variables

2. Read Input Data: from .txt files reading constraints, exposure payment matrix, transition rates and probability

3. Read sample paths that is coming from heuristics

4. Probability of exceeding remaining time calculation

For i = 1 To NumberofUsers

$$= e^{-t\left[\sum_{i=1}^{n}(1-x_i(0))\,\mu_{i,0}+x_i(0)\,\mu_{i,1}\right]} \tag{A.1.1}$$

5. Compute optimal expected revenue for the first iteration

6. Calculate optimal expected revenue for user i: advertisement assigning to get expectedly maximum reward

For i = 1 To NumberofUsers

For j = 1 To NumberofAdvertisers

If (TemporaryUser = i) Then increase ExposureMatrixArr( ) calculate new Address

from Address compute optimal expected revenue for user i

7. Determine the ad for user I for sample path

For i = 1 To NumberofUsers

For j = 1 To NumberofAdvertisers

If (TemporaryUser = i) i is coming through sample route Then increase ExposureMatrixArr( )

calculate new Address from Address compute optimal expected revenue for user i

8. Find reward value for Addresses: getting reward from described addresses

9. Compute optimal expected revenue for the next iteration

$$\sum_{i\leq n}\frac{(1-x_i(0))\,\mu_{i,0}+x_i(0)\,\mu_{i,1}}{\left[\sum_{k\leq n}(1-x_k(0))\,\mu_{k,0}+x_k(0)\,\mu_{k,1}\right]} \tag{A.1.2}$$

For i = 1 To RemainingTime calculate the probability that there will be an event between [time(x)] and [time(x)-1]

calculate ProbabilityofUserIMakesTheTransition = (TransitionRate / SumTransitionRateForAllUsers)

For i = 1 To NumberofUsers

ProbabilityofUserIMakesTheTransition = (TransitionRate / SumTransitionRateForAllUsers)

If Current state of user is offline Then calculation OptimalExpectedRevenueforUseri

$$(1-x_i(0)) \cdot \mathcal{S}_i^*[f]\big(a, x+\bar{e}_i, t-u\big) \tag{A.1.3}$$

state converges 0→1, Else If Current state of user is offline Then calculation OptimalEx-

pectedRevenueforUseri

$$x_i(0)\big[\beta_i \cdot \mathcal{S}_i^*[f]\big(a, x, t - u\big) \tag{A.1.4}$$

state converges 1→1 or

$$(1 - \beta_i) \cdot f\big(a, x - \bar{e}_i, t - u\big) \tag{A.1.5}$$

state converges 1→0

10. Calculate Reward: Constraint is Considered

For i = 1 To NumberofUsers

For j = 1 To NumberofAdvertisers

If ExposureMatrixArr(i, j) > MaximumDisplayConstraint(j) Then modify Exposure-Matrix

If ExposureMatrixArr(i, j) < MinimumDisplayConstraint(j) Then modify Exposure-Matrix

If TemporaryPayment(i) > MaximumBudgetConstraint(i) Then modify Temporary-Payment

ElseIf TemporaryPayment(i) < MinimumBudgetConstraint(i) Then modify Temporary-Payment

11. Calculate Address

12. Generate instances for first iteration: storing reward for current situation

13. Generate instances for next iteration: assigning reward to the addresses

14. Update arrays from first to the next iterations

15. Write results to excel cells

16. Calculate rewards for sample paths: computing address for the routes then getting rewards from address

A.2 *Algorithm: Finite Difference Approach*

Notation:

i: 1…n index for the users (NumberofUsers)

j: 1…m index for the advertisers (NumberofAdvertisers)

$X_i(t)$: State of the user i (CurrentStateofUserArr)

$\mu_{i,0}$: event rate of the user I when its current state offline (TransitionRateMuforUsers)

$\mu_{i,1}$: event rate of the user I when its current state online (TransitionRateMuforUsers)

β_i : online to online transaction probability of the user I (TransitionProbBetaforUsers)

$A_{i,j}(t)$: exposure matrix j ads to i users (ExposureMatrixArr)

F(A(T)): reward function (TOTALREWARD)

T: time horizon (TimeHorizon)

t: remaining time (RemainingTime)

Maximum budget bound: MaximumBudgetConstraint

Minimum budget bound: MinimumBudgetConstraint

Maximum display bound: MaximumDisplayConstraint

Minimum display bound: MinimumDisplayConstraint

Payment for each exposure: ExposurePaymentMatrixArr

1. Define Input Variables

2. Read Input Data: from .txt files reading constraints, exposure payment matrix, transition rates and probability

3. Calculate summation of all users' transition rate

For i = 1 To NumberofUsers

$$\sum_{i=1}^{n}(1 - x_i(0))\,\mu_{i,0} + x_i(0)\,\mu_{i,1} \tag{A.2.6}$$

4. Compute optimal expected revenue for the first iteration

5. Calculate optimal expected revenue for user i: advertisement assigning to get expectedly maximum reward

For i = 1 To NumberofUsers

For j = 1 To NumberofAdvertisers

If (TemporaryUser = i) Then increase ExposureMatrixArr( ) calculate new Address from Address compute optimal expected revenue for useri

6. Find reward value for Addresses: getting reward from described addresses

7. Calculatie of Second Term:

$$h \cdot D^+V(a, x, t) \tag{A.2.7}$$

For i = 1 To NumberofUser

If Current state of user is offline Then calculation OptimalExpectedRevenueforUseri

$$\sum_{i=1}^{n} (1 - x_i(0)) \, \mu_{i,0} \, \mathcal{S}_i^*[V]\big(a, x + \bar{e}_i, t\big) \qquad \text{(A.2.8)}$$

state converges 0→1, Else

If Current state of user is offline Then calculation OptimalExpectedRevenueforUseri

$$\sum_{i=1}^{n} [x_i(0) \, \mu_{i,1}] \, \big[\beta_i \cdot \mathcal{S}_i^*[V]\big(a, x, t\big) + (1 - \beta_i) \cdot V\big(a, x - \bar{e}_i, t\big)\big] \qquad \text{(A.2.9)}$$

state converges 1→1 or 1→0

8. Calculate Reward: Constraint is Considered For i = 1 To NumberofUsers

For j = 1 To NumberofAdvertisers

If ExposureMatrixArr(i, j) > MaximumDisplayConstraint(j) Then modify Exposure-Matrix

If ExposureMatrixArr(i, j) < MinimumDisplayConstraint(j) Then modify Exposure-Matrix

If TemporaryPayment(i) > MaximumBudgetConstraint(i) Then modify Temporary-Payment

ElseIf TemporaryPayment(i) < MinimumBudgetConstraint(i) Then modify Temporary-Payment

9. Calculation Address

10. Write results to excel cells

11. Calculate rewards for sample paths: computing address for the routes then getting rewards from address

12. Read sample paths that is coming from heuristics

13. Calculate rewards for sample paths: computing address for the routes then getting rewards from address.

A.3 *Algorithm: Heuristics*

Notation:

i: 1…n index for the users (NumberofUsers)

j: 1…m index for the advertisers (NumberofAdvertisers)

$X_i(t)$ *: State of the user i (CurrentStateofUserArr)*

$\mu_{i,0}$ *: event rate of the user I when its current state offline (TransitionRateMuforUsers)*

$\mu_{i,1}$ *: event rate of the user I when its current state online (TransitionRateMuforUsers)*

β_i *: online to online transaction probability of the user I (TransitionProbBetaforUsers)*

$A_{i,j}(t)$ *: exposure matrix j ads to i users (ExposureMatrixArr)*

F(A(T)): reward function (TOTALREWARD)

T: time horizon (TimeHorizon)

t: remaining time (RemainingTime)

Maximum budget bound: MaximumBudgetConstraint

Minimum budget bound: MinimumBudgetConstraint

Maximum display bound: MaximumDisplayConstraint

Minimum display bound: MinimumDisplayConstraint

Payment for each exposure: ExposurePaymentMatrixArr

Random Seed: RandomSeeds, NumberofRandomSeed

Problem Size: ProblemSizeArr(3)=(NumberofUsers, NumberofAdvertisers, TimeHorizon)

1. Define Input Variables

2. Read Input Data: from .txt files reading constraints, exposure payment matrix, transition rates and probability

3. Read Random Seeds: from .txt files reading random seeds

4. Calculate Reward

For i = 1 To NumberofUsers

For j = 1 To NumberofAdvertisers

If ExposureMatrixArr(i, j) > MaximumDisplayConstraint(j) Then modify Exposure-Matrix

If ExposureMatrixArr(i, j) < MinimumDisplayConstraint(j) Then modify Exposure-Matrix

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If TemporaryPayment(i) > MaximumBudgetConstraint(i) Then modify Temporary-Payment

ElseIf TemporaryPayment(i) < MinimumBudgetConstraint(i) Then modify Temporary-Payment

5. Generate Event For i = 1 To NumberofUsers, summarize transition rates for all users(TotalRateofTransition) Randomly generate event time:

If NextEventTime = 0 Then

Do While NextEventTime = 0

NextEventTime = (-Log(Rnd) / TotalRateofTransition)

Loop Else

NextEventTime = (-Log(Rnd) / TotalRateofTransition)

End If

Randomly change the state of the user:

If StateofUserArr = 0 Then

NextStateofUser = 1

Else

StateChangeRandomNumber = Rnd

If StateChangeRandomNumber > TransitionProbBetaforUsers(NextUser) Then

NextStateofUser = 0

Else

NextStateofUser = 1

End If

End If

6. Make Decision Myopic A

RandomNumberforCandidates = Rnd

ProbabilityofEachCandidate = 1 / (CountNumberofCandidates)

CumulativeProbabilityforSelection = 1 / (CountNumberofCandidates)

For k = 1 To CountNumberofCandidates If (CumulativeProbabilityforSelection ≥ RandomNumberforCandidates) And done Then

SelectedAdvertisement = CandidateAdvertisementArray(k) done = False Else

CumulativeProbabilityforSelection = CumulativeProbabilityforSelection + ProbabilityofEachCandidate

End If

Next k

DecisionMyopicArr(i) = SelectedAdvertisement

REWARD is computed by 4. Calculate Reward

7. Calculate Reward B

For i = 1 To NumberofUsers

For j = 1 To NumberofAdvertisers

If ExposureMatrixArr(i, j) > MaximumDisplayConstraint(j) Then modify Exposure-Matrix

If TemporaryPayment(i) > MaximumBudgetConstraint(i) Then modify Temporary-Payment

8. Make Decision Myopic B

RandomNumberforCandidates = Rnd

ProbabilityofEachCandidate = 1 / (CountNumberofCandidates)

CumulativeProbabilityforSelection = 1 / (CountNumberofCandidates)

For k = 1 To CountNumberofCandidates

If (CumulativeProbabilityforSelection ≥ RandomNumberforCandidates) And done Then

SelectedAdvertisement = CandidateAdvertisementArray(k) done = False Else

CumulativeProbabilityforSelection = CumulativeProbabilityforSelection + Probabil-ityofEachCandidate

End If

Next k

DecisionMyopicArr(i) = SelectedAdvertisement REWARD is computed by 7. Calcu-late Reward B

9. Make Decision Myopic C: same probability structure with the A and B is used, for REWARD calculation there is no constraint consider

10. Make Decision Myopic Random:

SelectionProbability = 1 / (NumberofAdvertisers)

For i = 1 To NumberofEvents

If UsersNewState(i) = 1 Then RandomProbability = Rnd

CumulativeProbability = SelectionProbability done = False

For j = 1 To NumberofAdvertisers

If ((CumulativeProbability ≥ RandomProbability) And Not (done)) Then done = True

SelectedAdvertisement = j Else

CumulativeProbability = CumulativeProbability + SelectionProbability

End If

Next j DecisionRandomArr(i) = SelectedAdvertisement Else there is no decision

End If

Next i
REWARD calculation, there is no constraint consider.
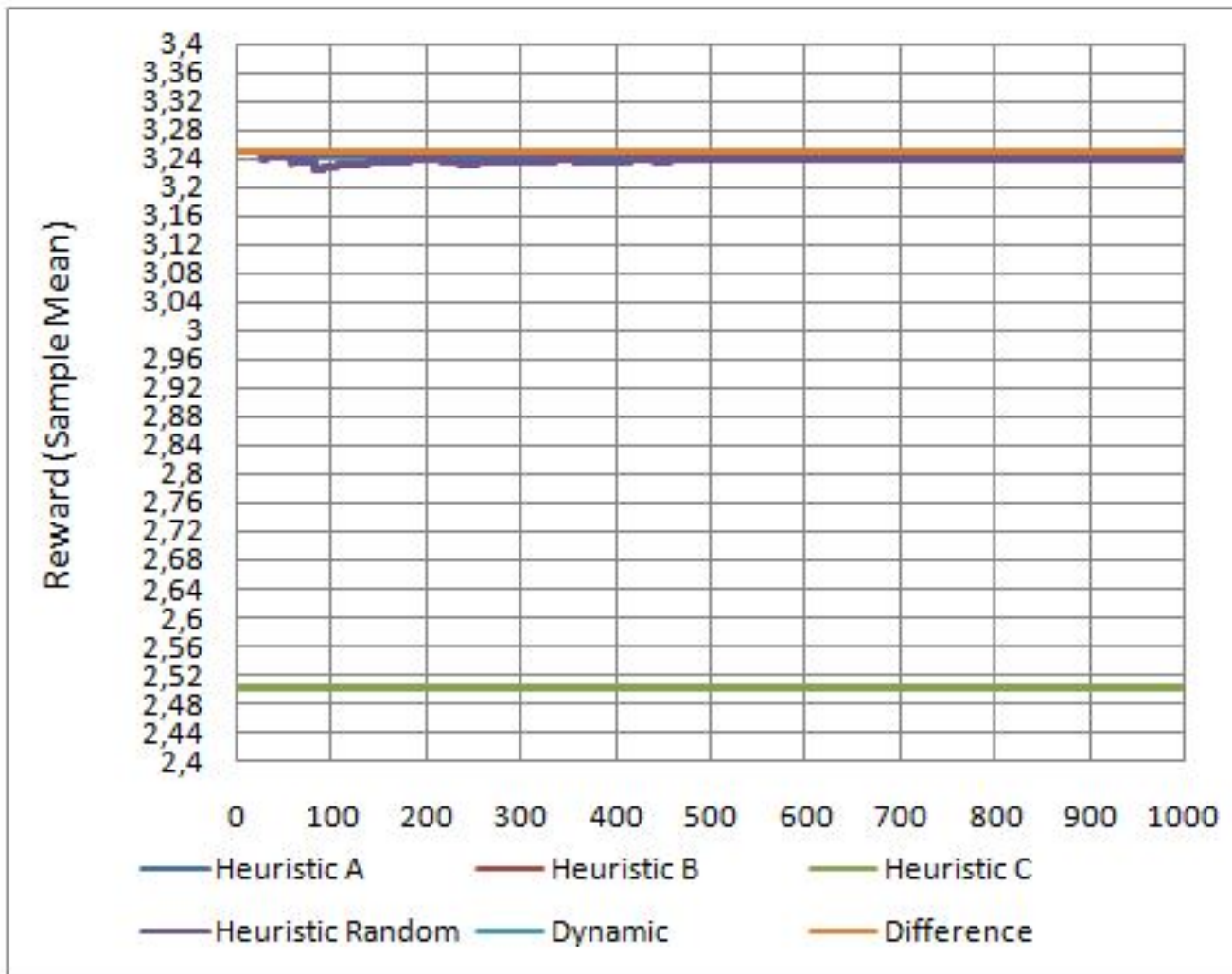
Chapter B

Appendix

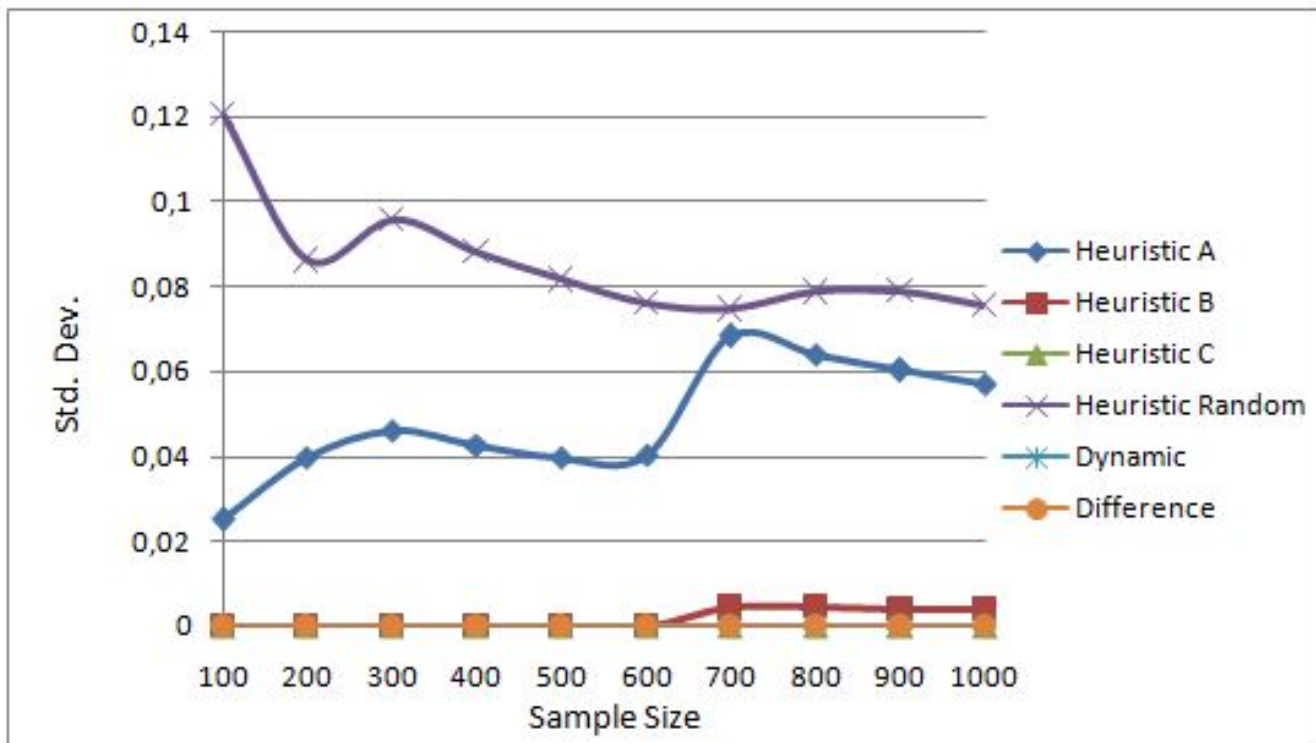**Figure B.1: Sample Path Analysis with Minimum Display Constraint**

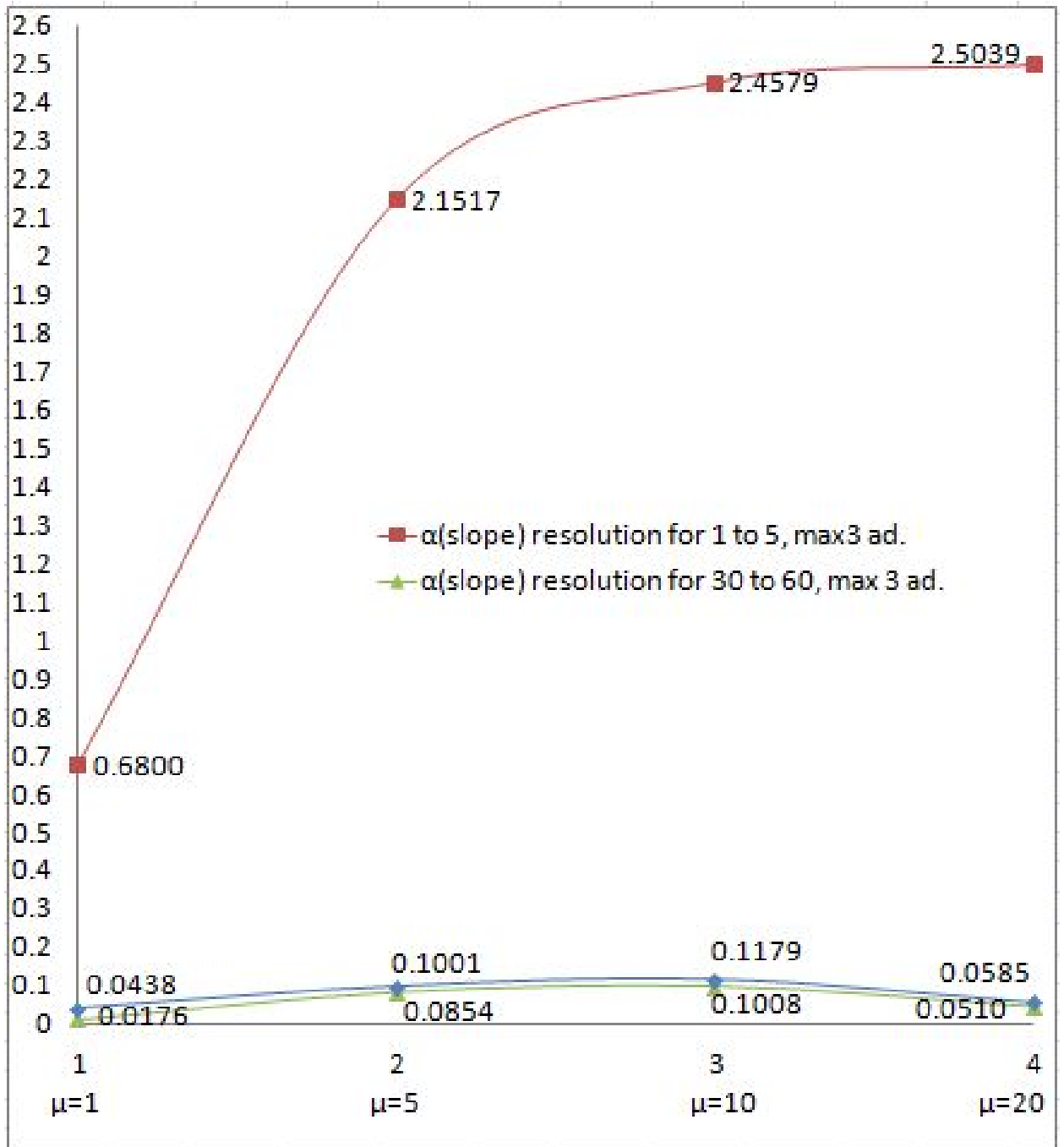**Figure B.2: Sample Standard Deviation for Sample Path Analysis with Minimum Display Constraint**
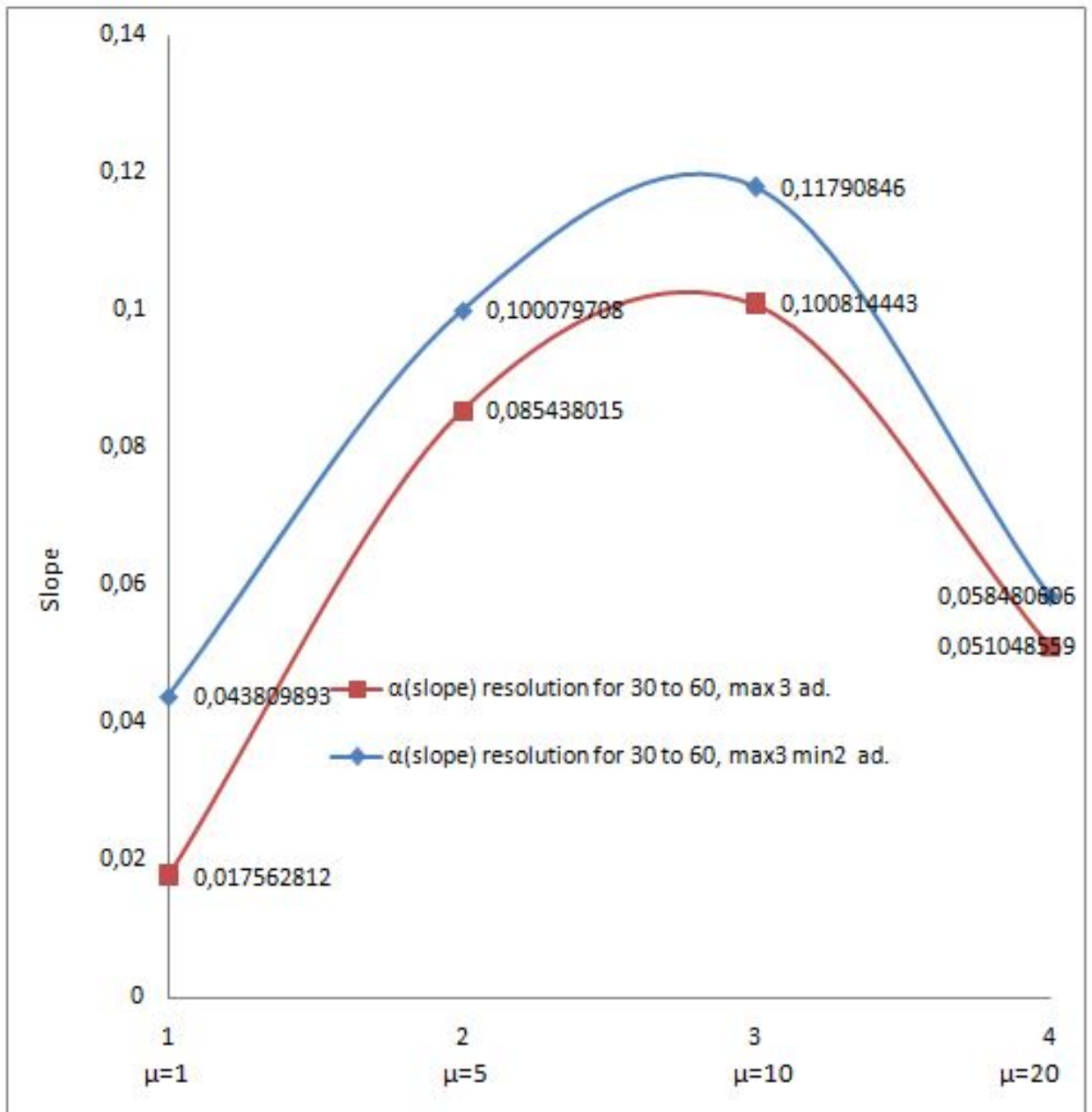
Figure B.3: Slope of Iteration Numbers Curves

Figure B.4: Slope of Iteration Numbers Curves