Analysis of Collusion and Competition in Electricity Markets Using an Agent-Based Approach

by

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Elektrik Piyasalarındaki Kartelleşme ve Rekabetin Ajan Temelli Bir Yaklaşımla İncelenmesi

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Özet

Serbestleştirilmiş elektrik piyasaları üreticiler arasındaki rekabet sayesinde kullanıcılara daha uygun fiyattan elektrik ulaştırabilmek için oluşturulmuştur. Her ne kadar bu pazar modelinin bu amaca hizmet etmesi beklense de, en eski serbest elektrik piyasaları bile rekabeti tehdit eden unsurlara maruz kalmaktadır. Elektrik pazarını idare etmekle sorumlu olan bağımsız sistem operatörü, kullanıcılara mümkün olan en düşük fiyattan elektrik sağlamayı hedeflerken rekabet eksikliği fiyatların yükselmesine neden olabilir. Pazardaki rekabet seviyesini etkilemesi beklenen ve birarada etkilerini göz önünde bulundurduğumuz üç faktör bağımsız sistem operatörünün stratejik seçimlerinden biri olan pazar-kapatma mekanizması, elektrik üretici firmaların stratejik fiyat verme politikaları ve iletim ağının özellikleri olarak belirlenmiştir.

Çalışmamızda bağımsız sistem operatörü ve üreticileri biraraya getiren pazar-kapatma mekanizmasını oyun kuramı yaklaşımıyla ele alarak hem matematiksel modelleme yaklaşımı

hem de ajan-temelli bir simülasyon modeli kullanıyoruz. Serbest elektrik pazarlarını inceleyen yazın güç üretici firmaların davranışlarını iletim ağını ve pazar katılımcıları üzerindeki etkilerini göz önünde bulundurmadan incelemişlerdir. Zira, iletim ağının modellere dahil edilmesi oyun-kuramı temelli yaklaşımları içinden çıkılmaz bir hale getirebilir. Her iki modelleme yaklaşımımızda da iletim ağını ve etkilerini göz önüne almakta iken, çalışmamızı gün-öncesi piyasasının incelenmesi ile sınırlandırmayı tercih ediyoruz.

Oyun kuramı temelli anlayış üreticilerin kartelleşmeye yönelik davranışlar içine girmelerini neden olacak koşulların anlaşılmasında kullanılıyor. Bu koşullar, bağımsız sistem operatörü ve üreticilerinin birbirleriyle çelişen amaç fonksiyonlarını barındıran iki seviyeli bir optimizasyon problemi içerisine yerleştiriliyor. Çok amaçlı iki seviyeli problemi çözmek için bir algoritma geliştiriyoruz, ve yeterli şartlar oluştuğunda üreticilerin optimal davranışlarının kartelleşmeye yönelik olduğunu gösterebiliyoruz.

Güç üretici firmaların stratejik davranışları ajan temelli bir simülasyon modeli kullanılarak farklı market kapatma mekanizmaları altında inceleniyor. Hem fiyatlandırma kurallarının hem de güç paylaşım politikalarının güç üretici firmaların davranışlarını etkileyebildikleri gözlemlenmektedir. Teklif-kadar-öde fiyatlandırma politikasının rasgele paylaşım politikasıyla birlikte kullanılması eş fiyatlandırma politikasının eşit paylaştırma politikasıyla birlikte kullanılmasına göre (fiyatların daha düşük olmasını sağlayarak) kamu yararına olduğu ortaya çıkmıştır. Geniş bir aralıkta kullanılan öğrenme modeli parametrelerinin farklı vakalar üzerinde kurgulandığı kapsamlı bir simülasyon deney sonuçları sunulmaktadır.

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Abstract

As a result of liberalization, deregulated electricity markets were formed to provide affordable electricity for consumers through promoting competition. Although the new market is expected to serve this purpose, even the earliest deregulated electricity markets are prone to threats that may disrupt the competition. While the independent system operator, responsible for administering the electricity markets, aims to provide the consumer with the lowest possible electricity price, lack of competition may increase prices. We consider the effect of three major factors hand-in-hand on that may affect the level of competition in the market: the independent system operator's market-clearing mechanism as a strategic choice, strategic bidding behavior of generation companies and the transmission network.

We use both a mathematical modeling approach and an agent-based simulation model with a game-theoretic understanding of the market clearance mechanism involving the independent system operator and the power generation companies (as the players). The literature on deregulated electricity markets mostly focus on analyzing the behavior of power generating companies without considering the transmission network and its impact on the players' behavior since including transmission network makes game-theoretic approaches intractable. While we consider the transmission grid in both modeling approaches, we confine the boundary of our analysis to the day-ahead market.

The game-theoretic understanding assists in characterizing a set of sufficient conditions for the generators to engage in a collusive behavior. These conditions are embedded into a bi-level optimization problem where the objectives of the independent systems operator are conflicting with those of the generators. We develop an algorithm to solve the multi-objective bi-level problem and we show that the generators' optimal behavior are collusive when sufficient conditions exist.

We investigate the strategic behavior of power generation companies under different market-clearing mechanisms by an agent-based simulation model. We observe that both pricing rules and rationing policies can alter the behavior of generation companies. We find that pay-as-bid pricing rule together with random dispatch policy improves social welfare more than uniform pricing with equal dispatch policy. Finally, we investigate the effects of risk attitude and capacity withholding. We present a complete set of results of simulation experiments using various cases for a wide range of learning model parameters.

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Chapter 1

Introduction

1.1 Electricity Market

Unlike other commodities, electricity can not be stored easily; therefore, it should be instantly consumed when it is generated. This unique feature brings technical and economical complexity to the trading mechanism. Electricity industry (almost) everywhere started with vertically integrated monopolies that were either state-owned or privately-owned. Poor performance of these regulated monopolies as a result of high operating costs and construction cost leads to high retail prices; the low performance together with development of more efficient generation technologies stimulate changes that would reduce electricity costs and retail prices ([54, 55]).

England and Wales's electricity market was among the first in the history that become deregulated in early 1990s after Chile restructured its electricity market in 1982. Shortly after that, Norway adopted a pool scheme forming the Nord Pool, which afterwards comprise Sweden, Finland and Denmark. This liberalization process has accelerated so rapidly that today most of the produced electricity is traded in deregulated markets.

The primal goal of deregulated electricity markets is to attain affordable electricity prices through a competitive market which leads to maximum social welfare. However, designing a deregulated market with perfect competition is extremely challenging. Level of competition and price volatility are affected by several factors including demand elasticity, market share of generation companies and their strategic behavior. It is also known that market power[1] and strategic behavior of suppliers may cause the market to divert from a perfectly competitive market. Besides, as studied by Harvey and Hogan [49] and Johnsen et al. [53], lack of transmission capacity (congestion) in the network and the grid structure also interferes with competition while affecting the strategic market behavior of the generators.

In fact, research has shown that some electricity markets are far from being competitive and act more like oligopolies. In David and Wen [25] the reasons for electricity markets to retain an oligopoly rather than a perfect competition are reported as

- limited number of generators,
- transmission constraints and congestion that isolates certain consumers from some generators,
- transmission losses that discourage consumers from distant producers, and
- entry barriers for new competitors, e.g. capital investment.

The oligopolistic nature of electricity market along with repetitive interaction of participants facilitates collusion. Collusion is an agreement between two or more parties to limit open competition. Explicit collusion in electricity market is forbidden but sometimes, it exists even in the absence of explicit agreement; then, it is called tacit collusion. To attain a competitive market, collusion among competitors should be mitigated. In general, it is not an easy task for the regulator to identify collusive behavior. For example, tacit collusion, as a major cause in increasing consumer prices and decreasing level of competition, is hard to recognize because there is no known agreement between suppliers.

By means of simulation-based models, research has shown that repetitive bargaining may result in tacit collusion [1, 12, 21, 99]. In real-life electricity markets, Guan et al. [46], Sweeting [98], and Fabra and Toro [35] have shown that generation companies might

[1] An ability which enables GenCos to maintain prices profitably above competitive levels for a significant period of time [81].

be engaged in tacit collusion in decentralized electricity markets of California, England and Wales, and Spain, respectively. The last incident was reported in the UK again when the big six energy suppliers which used to produce about 70% of electricity were accused of preventing effective competition. The parliament was about to freeze the price of electricity to avoid tacit collusion exactly after 15 years of opening the deregulated market [75].

Nowadays, power delivery can be regarded as consisting of all services, including generation, transmission, distribution and trading. Generation and trading layers consider electricity as a commodity, whereas transmission and distribution layers focus on providing electricity services.

1.1.1 Generation Companies

Power Generation Companies (GenCos) are the building blocks of any electricity market. GenCos produce electricity and sell the produced electricity into the market. The strategic behavior of GenCos is major determinant in identifying their market position. There are several factors which can be considered as an influential factor on GenCos' strategic behavior.

- One major factor is the type of technology they utilize. For example, a wind power generation company has lower production cost but it experiences higher risk due to stochastic nature of wind and higher setup cost per MW of capacity than a coal plant.
- The size of the company also matters; smaller producers are more sensitive towards risk than large ones because risk will have more intense effect on their business.

Cost of production and production capacity characterize GenCos bidding strategies. A GenCo determines its bids by looking into its production cost. In general, bids are higher than production cost (except for some rare case that government gives subsidy to a public plant to keep the price of electricity low for the end consumers). In the literature, production cost is mostly deterministic and usually considered as either quadratic or linear function of the amount of generated electricity. Generation capacity is also important since GenCos may hold their available production capacity to keep the electricity price high artificially and exercise their market power. In this study, the production cost is considered as a deterministic linear function of produced electricity.

1.1.2 Transmission Network

Although generation industry is deregulated in today's electricity markets, transmission is still left regulated [57]. GenCos' competition level and transmission grid structure are interrelated. Transmission bottlenecks can result in locational market power and provide opportunity for generation companies to exercise market power [68]. Thus, the regulator should design the transmission system such that it eliminates market power potentials and aids competition [73, 115].

In spite of abundant transmission capacity even in developed countries such as Germany, the responsible regulatory authority expects major congestion in the electricity transmission grid due to structural changes on the production side [105]. Therefore, considering transmission network is becoming indispensable part of studies.

In order to depict different players in the electricity market and their interrelations, we resort to a network representation where nodes represent the players and the arcs between nodes correspond to transmission lines. This network representation may also be considered as an abstract model of the transmission grid. In this network, a node with supply of electricity represents a GenCo while a node with no supply but demand for electricity represents a demand center which may also be referred to as a Load-Serving Entity (LSE[2]) transmitting or distributing the electrical power to end-users (residential or commercial) or to other LSEs.

The transmission line could afford to transmit only up to a certain level of electricity due to some physical restrictions such as a thermal constraint. The power network is said to be "congested" if a fully loaded transmission line cannot accommodate requests or ex-

[2]A Load-Serving Entity (LSE) is an electric utility, transmitting utility or Federal power marketing agency that has an obligation under Federal, State, or local law or under long-term contracts to provide electrical power to end-use (residential or commercial) consumers or to other LSEs with end-use consumers.

pectations for further use. Network congestion is managed either by increasing domestic supply in congested nodes or by allowing different prices at different nodes when congestion is experienced. The latter approach allows for an efficient management of congestion, since higher price at a node will decrease demand and resolve congestion.

1.1.3 Trading Mechanism

Electricity can be traded either via contracts or through centralized trading. With bilateral contracts, two participants (one seller and one buyer) negotiate and agree on the terms of contracts. Conversely, in centralized trading all offers from different participants (all sellers and buyers) are collected at one place and the regulator decides on the terms of contracts.

In deregulated electricity markets, Independent System Operator (ISO) plays the regulator's role to ensure the independent operational control of the transmission grid and assist competition among all the individual market participants. The ISO should be unbiased and independent to operate the market efficiently. The ISO is also responsible of managing congestion in the transmission grid.

Pool-based markets are considered in the class of centralized trading. A pool-based market is more effective than a market based on bilateral contracts because tracking agreements between pairs of traders is not necessary and this gives incentives to small consumers to have an active part in the electricity market. In pool-based market, the supply side bids are active while the demand side is approximated based on historical information. Pool markets are widely used in the United States. Unlike pool-based market model, exchange model utilizes both active supply and active demand; therefore, buyers and sellers can place bids in the market simultaneously. Exchange model is more common in Europe.

Day-ahead market is an example of centralized markets. There are two main approaches to day-ahead market design: USA pool models and European exchange models. Day-ahead market has special importance among all trading floors since it clears the market for the next day and the price of electricity in day-ahead market is used as a reference in other trading floors such as real-time and forward markets [28]. The focus of this study is solely on the day-ahead pool-based electricity market.

The ISO runs the day-ahead market hourly to determine which GenCo should produce how much electricity and the price. In the day-ahead market's auction, GenCos compete for the next day supply of an inelastic load demand. Each GenCo bids the minimum acceptable unit price of electricity for itself. Based on the predetermined market-clearing mechanism including a pricing rule, the ISO specifies the unit price of electricity (market clearance price) and each GenCo's assigned power.

1.1.4 Market-Clearing Mechanism

The most common pricing rules in electricity market literature are uniform and pay-as-bid pricing (see Cramton [22]). With uniform pricing, all GenCos with winning bids are paid the market-clearing price, whereas with pay-as-bid pricing, each GenCo is paid at its own bid.

These two pricing rules, in their original forms, fail to consider the physical limitations of the transmission lines. Based on uniform pricing, a more realistic rule, AC Optimal Power Flow (OPF) [89], has been developed that takes into account transmission line constraints. Researchers often consider a simplified version of AC-OPF, known as DC-OPF which is more tractable and common [19, 96]. In DC-OPF, each node in the grid may have a different price due to physical constraints of transmission lines, and GenCos in the same node are paid the same price.

An alternative to nodal pricing is zonal pricing, in which nodes are grouped into zones bounded by potential constraint interfaces and each zone has same price. This method encourages generators to be located within high-priced zones and focuses on relieving flow constraints in the congested interfaces between zones. In such a market, the boundaries must be updated from time to time to accommodate the generation and transmission expansion [83].

Even though nodal pricing is the efficient way to account for transmission constraints, most electricity markets still apply a uniform pricing rule in Europe as congestion is less of a problem [105]. However, the ISO has to take into account all network restrictions. If the obtained market solution is infeasible, re-dispatching becomes inevitable in order to avoid uniform pricing errors [43]. Therefore, taking uniform and pay-as-bid pricing rules into account is critical not only for academics but also for practitioners. In this section, we explain each pricing rule in details.

1.1.4.1 Without Transmission Network

In the day-ahead market, the ISO clears the bids sequentially for each hour of the next day through an auction mechanism. For any hour, each GenCo (GenCo-i) participates in the auction with its maximum capacity (P_i^{max}) and submits a minimum acceptable price (b_i) from a discrete set of available bids ($b_i \in B_i$) to the ISO. Based on the predetermined pricing rule, the ISO determines electricity price (λ) and the power to be dispatched by each GenCo ($P_i \leq P_i^{max}$). We now discuss different market-clearing mechanisms with respect to pricing rules and power dispatch (rationing) policies.

The combination of price and production quantity submitted to the ISO by a GenCo is referred to as the energy block of that GenCo. As illustrated in Figure 1.1, the ISO sorts received blocks in an increasing order of their prices ($b_{(i)} \leq b_{(i+1)}$), and accepts production offers starting from the cheapest block ($b_{(1)} \times P_{(1)}^{max}$) until demand is completely satisfied [80]. This procedure is known as the merit order.

Under *uniform* pricing, the ISO determines electricity price (λ) as that of the last accepted block offer. Winning GenCos, whose price bids were less than or equal to the price accepted by the ISO, are paid at λ for a unit of electricity. Since they are gaining no less than what they have asked in their bids, these GenCos accept the cleared price. Turkey is one of many countries in the world that employs *uniform* pricing rule [27].

Figure 1.1 depicts energy block offers accepted under uniform pricing by the merit order procedure. Blue shaded energy blocks are those of the winning GenCos. The partially shaded block determines the price while only part of the capacity of the corresponding GenCo is accepted by the ISO.

In this example, clearing price is determined as $b_{(3)}$ because $D \geq P_{(1)}^{max} + P_{(2)}^{max}$ and $D \leq P_{(1)}^{max} + P_{(2)}^{max} + P_{(3)}^{max}$. Thus, all winning GenCos are paid $\lambda = b_{(3)}$. As a result, payoff for GenCo-1 is $r_1 = P_1^{max}(\lambda - c_1)$, for GenCo-2 $r_2 = P_2^{max}(\lambda - c_2)$, and for GenCo-3 $r_3 = (D - P_{(1)}^{max} - P_{(2)}^{max})(\lambda - c_3)$. Here, c_i stands for the generation cost of

GenCo-i.

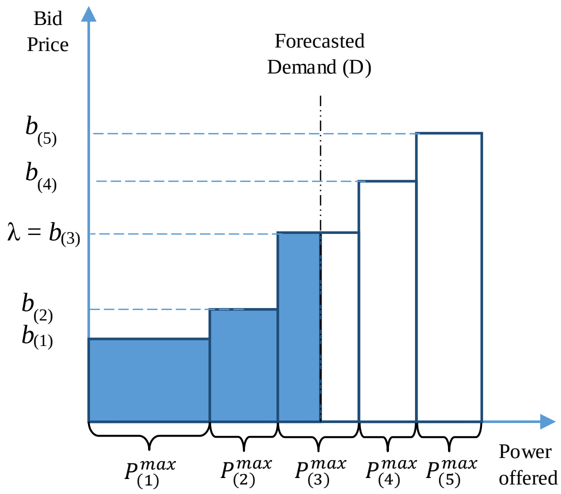


Figure 1.1: Supply curve from GenCos' bids

Different from *uniform* pricing, in *pay-as-bid* pricing rule, each GenCo is paid exactly at its own bid, but not more. *Pay-as-bid* pricing rule is used in Iran's electricity market[10, 41]. According to the example in Figure 1.1, the GenCo with the least expensive block will receive a price of $b_{(1)}$ instead of λ for $P_{(1)}^{max}$ unit of electricity. The next block will receive $b_{(2)}$ for $P_{(2)}^{max}$. Note that the last accepted block will receive λ for the accepted capacity under both uniform and pay-as-bid pricing rules.

An issue arises when more than one GenCo bids the price λ: How to assign the remaining demand? As Figure 1.2 illustrates with two such GenCos (G_1 and G_2), two rationing policies can be used:

1. *Random rationing*: All remaining demand is assigned to one of these GenCos, chosen randomly. This is illustrated in the left plot of Figure 1.2, where GenCo G_2 was chosen.

2. *Equal rationing*: Remaining demand is shared equally between these GenCos. This is illustrated in the right plot of Figure 1.2.

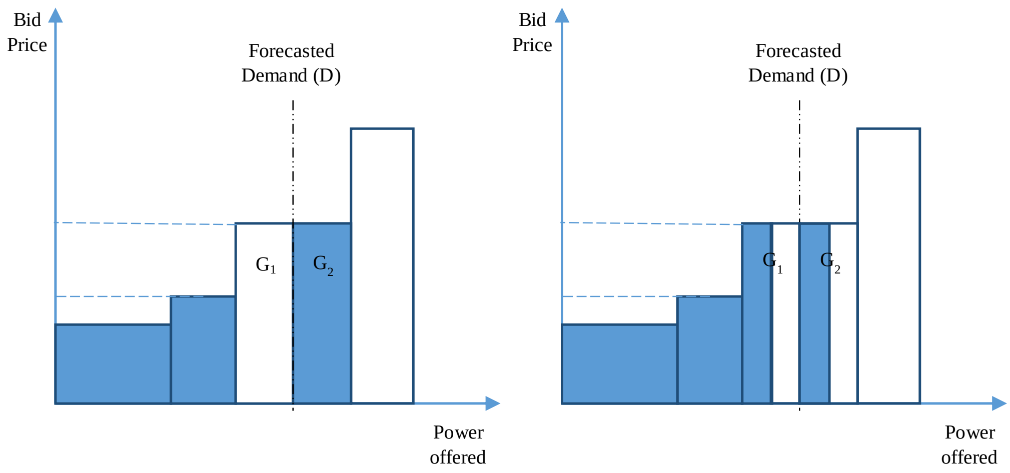


Figure 1.2: Random rationing policy (left) versus Equal rationing policy (right)

1.1.4.2 With Transmission Network

As we mentioned earlier, uniform and pay-as-bid pricing do not take the transmission network structure into consideration. Even though uniform pricing is the most common method to set prices in electricity markets, it may lead to infeasible solutions due to network constraints [105]. Therefore in a constrained network, a viable pricing method should also provide some economic signal to reflect the charge due to the physical constraints. This is what is done in the "Nodal Pricing" approach. Nodal pricing and Locational Marginal Pricing (LMP) can be used interchangeably.

The ISO handles physical constraints by considering congestion cost in calculating price of electricity at different locations. Nodal price at node-i (λ_i) corresponds to the minimum cost of fulfilling the demand for one additional unit of power (MWh) at that particular location. Transmission grid congestion is managed by the inclusion of congestion components, marginal generation cost, and cost of marginal losses in λs. Hence without loss of generality, we can say that nodal pricing is an extension of uniform pricing when we have more than one node.

In order to implement nodal pricing, an ISO may employ two methods: AC-OPF and DC-OPF. In practice, AC-OPF problems are typically approximated by more tractable

DC-OPF problems that focus exclusively on real power constraints in the linearized form.

DC-OPF problem is solved by ISO in order to maximize customers' welfare (minimizing cost of in demand electricity) and determines each GenCo's production, voltage angle[3], and λ_i on each node. For the sake of numerical stability, it is customary to consider all parameters and data in per unit (pu) terms so that parameters and decision variables become dimensionless. The solution is then converted back into International System (SI) units. Sun et al. [96] show how conversion in both ways are possible. In this respect, a node is selected as the reference so that the voltage angles of the other nodes would be calculated based on voltage angle of the reference node which is zero ($\theta_{\text{reference}} = 0$)[4].

The DC-OPF problem formulation is given as follows:

$$\text{Minimize}_{P_i,\theta_i} \quad z = \sum_i b_i P_i \tag{1.1}$$

$$\text{subject to} \quad P_i - D_i = \sum_{ij \in BR} y_{ij}\left(\theta_i - \theta_j\right) \qquad (\lambda_i) \qquad \forall i \tag{1.2}$$

$$P_i \leq P_i^{max} \qquad (\phi_i^{high}) \qquad \forall i \tag{1.3}$$

$$P_i \geq 0 \qquad (\phi_i^{low}) \qquad \forall i \tag{1.4}$$

$$\left|y_{ij}\left(\theta_i - \theta_j\right)\right| \leq F_{ij}^{max} \qquad \forall ij \in BR \tag{1.5}$$

Here, P_i denotes the power injected by GenCo-i (in pu), P_i^{max} the maximum capacity of GenCo-i (in pu), b_i the bid that GenCo-i submits to the ISO, D_i demand at node i (in pu), θ_i the voltage angle at node i (in pu), BR set of all available distinct transmission lines, y_{ij} the negative of the susceptance of the line (1 / reactance of the line) connecting node i to node j (in pu), and F_{ij}^{max} the maximum flow allowed in the transmission line connecting node i to node j (in pu).

Eq.(1.1) is the ISO's objective function which minimizes the cost of produced electricity. Eq.(1.2) is the flow balance constraint which ensures that extra power in each node will flow into the transmission lines to the connected nodes. Eq.(1.3) controls the maxi-

[3] Phase angle between two voltages which exist at the ends of a transmission line

[4] Without loss of generality, hereafter, we assume that base voltage (V_0) is 10 kVs and base apparent power (S_0) is 100 MVAs, therefore, base impedance is 1(ohms)

mum capacity of each GenCo and Eq.(1.5) limits the maximum allowed flow in each line of the transmission grid. The variable provided in the parentheses next to each equation denotes the dual variable of the corresponding constraint. The resulting λ_i yields the price of one unit of electricity at node i. In the presented formulation, the ISO does not consider any security criteria [5].

In this thesis, a pool-based day-ahead market is considered when ISO exercises DC-OPF, uniform, and pay-as-bid pricing rules to study each pricing rule's impact on the strategic bidding behavior of GenCos, as well as market performance.

1.2 Modeling of Electricity Markets

In the literature, three different market modeling trends are followed based on different research questions: optimization models, equilibrium models and simulation models. Optimization models can either focus on profit maximization of single GenCo or welfare maximization, while equilibrium models represent the overall market behavior taking into consideration competition among all participants. Perfect competition relates to price taking behavior whereas imperfect competition requires some kind of strategic company behavior ranging from classic Bertrand and Cournot competition to mathematically more demanding models such as Supply Function Equilibria (SFE) and Conjectural Models (CV). Simulation models are alternative to equilibrium models when the problem under consideration is too complex to be addressed within a formal equilibrium framework.

One advantage of *analytical methods* in equilibrium models is the straightforward derivation of equilibrium point under the conditions that each market participant tries to maximize its own profit. On the other hand, the expected outcomes of these models are not implementable and not necessarily observed in practice due to strict simplification assumptions in the analytical models [24]. One of these simplifying assumptions is associated with the length and the characterization of the considered time period for modeling. Most current approaches, e.g., Cournot modeling of players ([86], [59]), conjectural vari-

[5] Security criteria ensure that in the presence of problem to one or a collection of generators, other GenCos can collectively support system's demand.

ation ([87], [29]) and Equilibrium Problem with Equilibrium Constraints (EPEC) [88] are limited to observing the system during a fixed length and a very short time horizon. These models implicitly assume that behavior of GenCos do not change over time. In addition, analytical methods mostly do not consider the physical limitations of transmission lines ([86, 87, 88, 93]); Ruiz et al. [88] admit that representing the network with such limitations makes their proposed analytical approach intractable.

On the other hand, *Simulation models* are capable of capturing the dynamic behavior of the GenCos and other participants in the real-life market. If a GenCo changes its bidding strategy, the other GenCos may detect the new behavior after several failures, and will eventually adapt with the new environment and conditions. Many studies have investigated agent-based modeling of electricity markets ([94, 13, 105]). Recently, Li and Shi [66] claim that Agent-based modeling and simulation is a viable approach which provide realistic insights for the complex interactions among various market players.

In the following subsections, we describe each modeling approach briefly and enumerate their advantages and disadvantages.

1.2.1 Single Firm Optimization Models

With optimization models for a single firm, research mostly concentrates on the maximization of profit. However, the electricity price which is necessary to calculate the profit can be considered as either fixed and determined by the regulator from outside or a function of the demand.

1.2.1.1 Exogenous Price

The price clearing process is assumed to be independent of the GenCo's decisions.

- **Deterministic price:**
  Gross and Finlay [45]: The best offer of each generation unit consists of bidding its marginal cost.

- **Stochastic price:**

1. Rajaraman et al. [84] describe and solve the self-commitment problem of a generation firm in the presence of exogenous price uncertainty. The scheduling problem for each generating unit can be treated independently. The problem is solved using backward Dynamic Programming.
2. Fleten et al. ([37], [38]) address the medium-term risk management problem of GenCos that participate in the Nord Pool. They propose a stochastic programming model coordinating physical generation resources and hedging through the forward market. They model risk aversion by means of penalizing risk.
3. Unger [101] improves Fleten et al. [38] by explicitly measuring conditional value at risk (CVarR).
4. Pereira et al. [82] used benders decomposition to break the resulting large-scale problem into financial master problem and an operational sub-problem and both are solved by using Linear Programming (LP).

1.2.1.2 Price-demand Function

In the microeconomic theory, Varian [103] in 1992 shows that the behavior of a profit seeker firm when a given demand curve and supply curve of the rest of competitors taken into consideration is described by leader-in-price model.

- **Deterministic:**

  1. García et al. [40], addressed short-term schedule of thermal units in order to supply the electricity demand when linear residual-demand function is assumed. When market revenue is a quadratic function of firm's total output, they proposed a piecewise linearization procedure which enabled them to use Mixed Integer Linear Programming (MILP).
  2. Baíllo et al. [6] develop a MILP-based model focusing on the problem of one firm with significant hydro-resources. Their model supports non-concave market revenue functions.

- **Stochastic:**

    1. Anderson and Philpott [2] do not formulate the problem of optimal production but rather the problem of constructing the optimal offer curve of a generation firm. This approach constitutes an interesting starting point for the development of new models that convert the offer curve into a profitable risk hedging mechanism against short-term uncertainties in the marketplace.
    2. Baillo [5] advances the Anderson and Philpott [2] approach by incorporating a detailed modeling of the generating system which implies that offer curves of different hours are not independent.

1.2.2 Multiple-Firm Models (Strategic Interaction)

Several equilibrium models have been proposed to investigate markets that exhibit oligopolistic behavior, such as an emerging deregulated electricity market, which vary in terms of competition and market assumptions. Some of the most popular models include the Cournot, Bertrand, and Supply Function competition, while other approaches, such as the Stackelberg competition and the conjectural variations method, have also been used for electricity market analysis.

1.2.2.1 Cournot Competition

In Cournot competition, GenCos compete in quantity strategies and price will be specified through inverse price-demand function.

Pros:

- It allows a realistic modelling of electricity markets with a low level of computational complexity.
- It is well established in the microeconomics literature to analyze electricity markets; Cournot models are often encountered in the technical literature as they adequately represent producer behavior in real-world markets.

Cons:

- Each GenCo assumes that their production can affect the market clearing price; but it is not reflected on other players' production.
- GenCos have common knowledge about each other's cost function.
- The model is highly sensitive to the demand elasticity. Under the Cournot approach, GenCos' strategies are expressed in terms of quantities, but not in terms of offer curves. Hence, equilibrium prices are determined only by the demand function being therefore highly sensitive to demand representation and usually higher to those observed in reality.

1.2.2.2 Bertrand Competition

In the Bertrand model, prices are considered as strategic decision variables, instead of quantities. Bertrand model assumes that all GenCos can produce as much output as required to meet demand which is not applicable in every electricity market. In addition, a supplier cannot bid a lower price with the aim of increasing its output, since the marginal cost of generation is increasing as the output increases.

1.2.2.3 Supply function equilibrium (SFE)

Supply-curve bidding allows a GenCo to adapt better to changing conditions, such as electricity market. At the equilibrium point of the supply function game each player determines its optimum supply-curve bid that maximizes its profit based on how the other players will adjust their output to changes in market prices, anticipating their strategies. Klemperer and Meyer [60] have shown that price and quantity in any SFE are bounded by the Cournot and Bertrand outcomes.

Some studies show that there is a unique symmetric linear SFE for a market with linear demand and identical marginal cost curves if the range of demand is unbounded. If demand is bounded, as in real electricity markets where the load demand cannot be infinite, a continuum of equilibria exists, ranging from the highest prices of Cournot solutions to perfectly-competitive prices.

Pros:

- SFE models provide a realistic representation of electricity markets through gaming in both price and quantity.
- The SFE prices are not very sensitive to their demand-dependency as in Cournot competition, and the price predictions are more reliable.
- In contrast to the Cournot models, the SFE model offers the possibility of developing insights into the bidding behavior.

Cons:

- They are less advantageous in terms of computational complexity; SFE models turn out to be a set of differential equations (while Cournot models are a set of algebraic equations).
- In the presence of multiple SFE solutions, it is not clear which one is more qualified to represent GenCos' strategic behavior.
- Except for very simple versions of the SFE model, existence and uniqueness of a solution are very difficult to prove.
- Closed-form expressions of a solution are rarely obtained.
- Transmission constraints are only considered in extremely simplified versions of the SFE.

1.2.2.4 Conjectural Variation (CV)

The conjectural variation approach is used to estimate the strategic behavior of market players maximizing their profits while taking into account the reactions of their rivals with different levels of competition.

It can be shown that traditional market structures and game theoretical bidding strategies, such as perfect competition, Cournot, Stackelberg and monopoly, are special cases in the CV strategies and different Nash equilibria might be obtained corresponding to different CVs held by individual GenCo.

Pros:

- The CV models, like SFE models, overcome the demand elasticity issue in the Cournot equilibrium.
- The CV parameters allow representing different levels of competition in the market.

Cons:

- There are some arguments against CV models concerning the consistency of the conjectures and the possibility of multiple equilibria.
- Knowing all conjectural variation parameters of all rivals are essential which makes this method very hard to utilize in real electricity markets due to the number of players.

Ruiz et al. [88] find it intractable to represent the transmission network in the CV models and ask for a numerical approach instead of an analytical one if the network structure needs to be described.

1.2.2.5 Stackelberg and Multi-leader-follower Games

The Stackelberg model has been proposed in 1934 to investigate non-cooperative games. In Stackelberg game, a leader dominating the market acts strategically while followers observe leader's choice and act accordingly. An extension of the Stackelberg game is the multi-leader-follower game, in which two or more leaders act strategically and compete with each other. In the case of the electricity pool market, the ISO is considered to be a follower while the GenCos are represented by multiple leaders.

Microeconomics suggests that the Stackelberg equilibrium may fit better than other oligopolistic models with the long-term investment-decision problem due to its sequential decision-making process.

The Stackelberg model of Ventosa et al. [106] is developed with a Mathematical Programming Equilibrium Constraints (MPEC) formulation due to the fact that there is only one leader firm. In contrast, the Stackelberg-based model of Murphy and Murphy and Smeers [76] is an Equilibrium Problem with Equilibrium Constraints (EPEC) because multiple leaders may exist. Chen et al. [18] investigate the ability of the largest producer in an electricity market to manipulate both the electricity and emission allowances markets to its advantage. They model their problem as a Stackelberg games and solve it by MPEC. Leyffer and Munson [65] describe practical approaches for solving EPECs and apply these techniques to several medium-sized multi-leader-follower game models.

1.2.3 Simulation Models

Simulation models are alternative to equilibrium models when the problem under consideration is too complex to be addressed within a formal equilibrium framework. Simulation models typically represent each agent's strategic decision dynamics by a set of sequential rules that can range from scheduling generation units to constructing offer curves that include a reaction to previous offers submitted by competitors. The major advantage of a simulation approach lies in the flexibility it provides to implement almost any kind of strategic behavior.

1.2.3.1 Equilibrium Models

In many cases, simulation models are closely related to one of the families of equilibrium models. For example, when in a simulation model GenCos are assumed to take their decisions in the form of quantities, the Cournot equilibrium model may be used to support the adequacy of this approach. In this class of simulation models, market players follow certain rules without learning, and the resulting equilibrium under new set of conditions is observed. Song et al. [93] introduce a simulation model when GenCos try to find the CV parameters by following certain formula and bidding optimally by using CV method. In each iteration, GenCos try to minimize their perceptual errors about other rivals' levels of competition.

1.2.3.2 Agent-based models

In the agent-based models, modeling intelligence is necessary to some extent as the agents should be able to make decisions and act autonomous. In the real-world, GenCos need to learn in order to survive in the unknown market. Learning is helping GenCos to acquire and reinforce useful information to exhibit better performance in the future. Learning in the electricity market is important since the act of bidding and participating in the market is repetitive.

In current economic theory, the problem of learning is short-circuited by the imposition of a rational expectations assumption [100]. To the greatest extent, Game-based models suffer from absence of (dynamic) learning modelling for GenCos which can lead to inaccuracy in general conclusions [108]. However, Agent-based modelling provides enough flexibility to study effect of learning on strategic behavior of GenCos.

In the literature of electricity markets, different methodologies have been used to simulate learning; but researchers have put more emphasis on the reinforcement learning, particularly Q-learning. Q-learning is a specific type of reinforcement learning which can be used to find an optimal action-selection policy for a finite Markov decision process. An acceptable performance in accuracy together with ease of implementation and convergence, makes Q-learning a plausible choice for researchers.

Krause et al. ([62, 63]) investigate strategic behavior of GenCos using a generic Q-learning framework. They compare the resulting equilibrium with Nash equilibrium and conclude that existence of several Nash equilibria can effect cognitive ability of GenCos. Krause and Andersson [61] analyze various congestion management mechanisms using agent-based modeling when GenCos learn based on Q-learning mechanism. Concurrently, researchers try to enhance performance of Q-learning by employing different algorithms which produces variants of learning mechanisms. For instance, Bakirtzis and Tellidou ([7], [99]) and Wang [108] combine generic Q-learning with Simulated Annealing to tune exploration and exploitation rates. In their method, exploration rate decreases from a high value at beginning to a minimum as time progresses; contrarily, exploitation increases to a maximum value at the end of simulation run. Using simulated annealing in Q-Learning is a remedy for slow convergence issue.

Roth-Erev learning (Erev and Roth ([85], [32])) is a simpler version of reinforcement learning when n player are playing j pure strategies. Roth-Erev has just one state for each player unlike Q-learning that can have multiple state/action. By using one state for each player, they can avoid curse of dimensionality and infer faster from collected data. Veit et al. [105] use Roth-Erev method to examine effect of different congestion management mechanisms on German electricity market. Li and Shi [66] employ Roth-Erev learning method in an agent-based simulation method to investigate the effect of forecasting and wind penetration level on wind GenCos' net earnings.

1.3 Contributions

Some of the shortcomings in the literature have led us to identify open research questions that investigate the strategic behavior of GenCos. Considering its effect on public welfare, we narrow down our focus on identifying and investigating the existence of collusion. The research area has merely been involved particularly with this issue. In this endeavor, we are motivated with the following research questions:

- Under what conditions may collusive states exist?
- Does any pricing strategy effect competition in electricity markets?
- How can rationing policies assist ISOs in promoting competition?
- Which pricing strategy is more competition friendly and which one is more open to abuse of market power?
- What are the effects of risk aversion on GenCos' bid prices, profits and learning behavior?

In an attempt to answer such questions, we aim to shed light on corruptive market structures and conditions promoting collusion in order to help the system operators (ISOs) find defects and recognize collusion effectively.

In particular, we strive to contribute to the literature by achieving the following tasks:

- Develop a flexible agent-based simulation model to characterize the evolution of the dynamic electricity market under transmission grid constraints.
- Present a large-scale numerical analysis with a wide range of parameter combinations.
- Find patterns in the strategic bidding behavior of GenCos.
- Compare behavior of GenCos under different market-clearing mechanisms.
- Study the effects of risk aversion on GenCos' bid prices, profits and learning behavior by using a mean-variance approach.
- Analyze collusive strategy by considering the physical network structure and identify the properties of such states.
- Analyze if the ISO can prevent collusion and how.

1.4 Thesis Outline

Figure 1.3 summarizes introduction section. The scope of this dissertation is limited to green nodes. Contributions are organized inside pink round-rectangles. A vector-based version of Figure 1.3 is available at `http://APresenter.com/detail.faces?id=1275532714` that allows zooming and panning inside mind-map diagram to inspect details.

The thesis is organized as follows:

- In Chapter 2, we use a game-theoretic model to represent the market clearance mechanism involving the independent system operator and the generation companies in order to characterize the sufficient conditions that make it possible for the generators to engage in collusive behavior. We embed these conditions into a bi-level optimization problem where the objectives of the independent systems operator are conflicting with those of the generators. We develop an algorithm for the

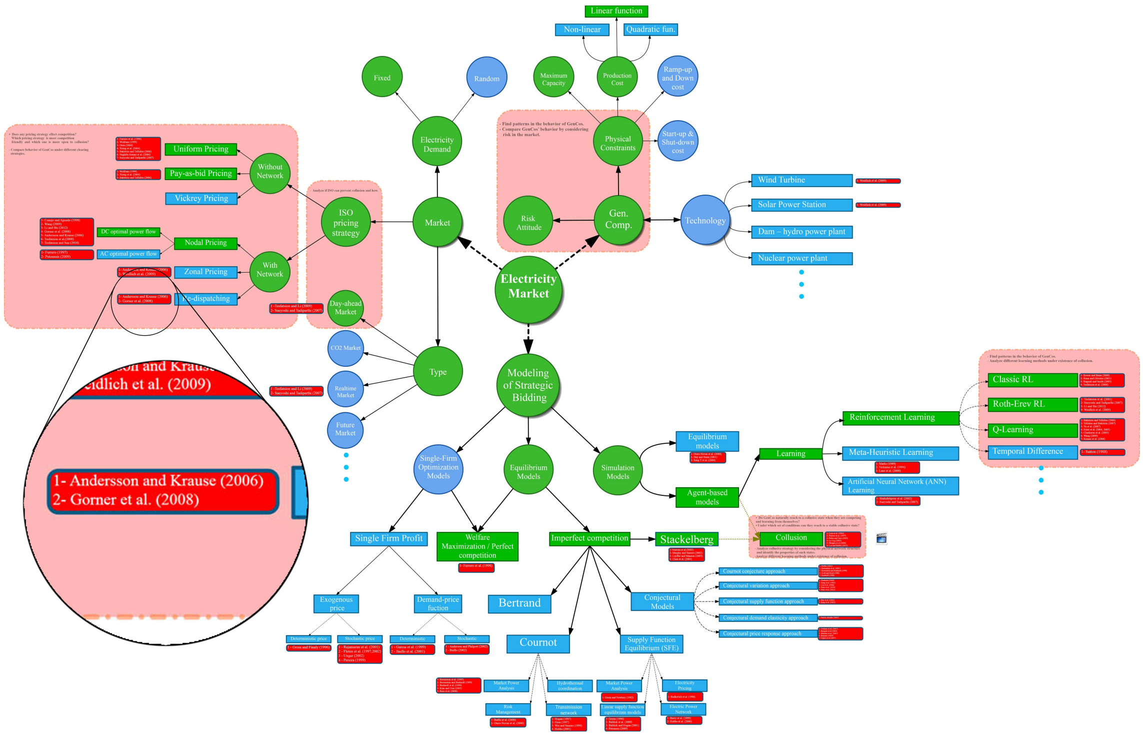


Figure 1.3: Overview of the thesis

bi-level problem to show that the optimal behavior of generators are collusive when sufficient conditions exist. We present numerical examples to illustrate our findings.

- In Chapter 3, we investigate the strategic behavior of power generation companies under different market-clearing mechanisms by an agent-based simulation model which integrates a game-theoretical understanding of the auction mechanism in the electricity market and generation companies' learning mechanism. We present results of our simulation experiments using various cases representing different market settings. Convergence to Nash equilibria or to similar states in terms of payoff are studied under different parameter settings. Furthermore, we investigate level of competition under different pricing rules and rationing policies of the regulator. Finally, effect of capacity withholding, risk attitude on strategic behavior of GenCos will be examined.

- Chapter 4 summarizes and concludes this research work. Also, some suggestions for further research are provided.

- The Appendixes at the end present some mathematical analysis about boundary conditions of presented learning algorithm. Moreover, the detailed information of each case study is presented at the end of Appendixes.

1.5 Publications and Conference Papers

1. Danial Esmaeili Aliabadi, Murat Kaya, and Güvenç Şahin (2017), **An Agent-based Simulation of Power Generation Company Behavior in Electricity Markets under Different Market-Clearing Mechanisms**, Energy Policy, 100, 191-205.

2. Danial Esmaeili Aliabadi, Murat Kaya, and Güvenç Şahin (2016), **Determining Collusion Opportunities in Deregulated Electricity Markets**, Electric Power Systems Research, 141, 432-441.

3. Danial Esmaeili Aliabadi, Murat Kaya, and Güvenç Şahin, **An Investigation on Behavior of Power Generation Companies in the Electricity Market under Different Market-Clearing Mechanisms**, 28th European Conference on Operational Research, Poznan, 3-7 July, 2016.

4. Danial Esmaeili Aliabadi, Güvenç Şahin, and Murat Kaya, **Determining Collusion Opportunities in Deregulated Electricity Markets**, International Conference on Operations Research (OR2015), Vienna, 1-5 September, 2015.

Chapter 2

Determining Collusion Opportunities

The primal goal of deregulated electricity markets is to attain affordable electricity prices through a competitive market which leads to maximum social welfare. However, designing a fully competitive market is extremely challenging. To achieve this goal, any kind of collusion (explicit or tacit) among competitors should be eliminated. Naturally, the law already forbids explicit collusion [17], but recognizing tacit collusion is not an easy task for the ISO.

The ISO may develop prohibitive policies when they have the potential to identify collusion in the market. In this respect, the extent and magnitude of collusion in the market shall be used as a measure to understand if the market is sufficiently collusion-free. On the other hand, knowing the existence of a possibly collusive market can be advantageous for the GenCos with the likelihood of higher marginal profit.

We use a game-theoretic approach to understand the market clearing process in an electricity market where GenCos first bid prices to the ISO and the ISO clears the market based on received bids. The clearing process is modeled as a multi-leader-follower problem as the decision of the follower (the ISO) is a function of the multiple leaders' (GenCos) bids. The market-clearing process is modeled as a bi-level optimization problem integrated with transmission network constraints such as the capacity of transmission lines. We develop an optimization-based method to identify the existence of collusion in a deregulated market. We also propose a heuristic approach to detect collusion in a market where sufficient conditions exist.

Although preventive mechanisms were developed for decades to attain a more conspiracy-free electricity market, the progress was not enough due to the oligopolistic nature which facilitates collusion. Lean et al. [64] measure how effective antitrust conduct remedies are in improving the performance against collusion in an oligopolistic industry such as the electricity market.

Strategic behavior of GenCos in an oligopoly market can be modeled based on a variety of assumptions; well-known models in the literature include Cournot, Bertrand, Supply Function Equilibrium, Conjectural Variation, and Stackelberg. Dixon et al. [31] conduct an experimental study to identify more beneficial strategic behaviors for players in GenCos' game (collusive, Cournot, and Stackelberg). Bernheim and Whinston [9] study the effect of multi-market contact in the framework of repeated competition; but they neglect the effect of network constraints on GenCos' strategic behaviors.

Liu and Hobbs [69] introduce a framework for modeling tacit collusion in which GenCos collectively maximize a Nash bargaining objective function in their study on a competitive pool-based electricity market operated by the ISO. To the best of our knowledge, this is the first study that considers network congestion in modeling tacit collusion; they propose MPEC and EPEC models. Although EPEC models are versatile, they are widely known for two issues. First, an equilibrium may not exist; second, it is hard to compute. Therefore, some heuristic algorithms are proposed to solve these models. Besides, the numerical solution of EPECs is a novel area with only few numerical studies [65]. We present an alternative formulation which is relatively easier to work with and can be handled with linear programming through some assumptions.

2.1 The ISO's Decision Problem

We consider a strategic bidding problem on a generic trading floor which can be exemplified by the day-ahead in a deregulated electricity market with transmission network constraints. As described in Section 1.1.2, the electricity grid consists of several nodes connected to each other by transmission lines. Each node operates independent from the others, representing a GenCo, a LSE, or a combination of both. For simplicity and without loss of generality, we suppose each node to have one GenCo and one demand center.

The ISO undertakes the daily operation of the transmission grid under a two-level settlement system using nodal pricing (DC-OPF implementation). Network congestion is managed by the inclusion of congestion components in λs.

We consider the following assumptions:

1) All GenCos are major players with power to affect other GenCos' strategic behaviors,

2) Price of electricity is capped by the ISO similar to [11, 99, 7, 68], and

3) Production cost and capacity of each GenCo are known to the problem solver.

The problem (1.1)-(1.5) is a linear optimization problem as we assume a DC representation of the transmission network. We adopt the matrix representation of the DC-OPF problem in Sun et al. [96] considering node 1 as the reference (Hence, $\theta_1 = 0$). Then, the matrix form of the formulation becomes

$$\begin{aligned} \text{Minimize} \quad & A^T x && (2.1)\\ \text{subject to} \quad & C_{eq}^T x = b_{eq} && (2.2)\\ & C_{iq}^T x \geq b_{iq} && (2.3) \end{aligned}$$

where

$$\begin{aligned} A^T &= \begin{bmatrix} b_1 & \dots & b_n & 0 & \dots & 0 \end{bmatrix}_{1\times(2n-1)} && (2.4)\\ x &= \begin{bmatrix} P_1 & \dots & P_n & \theta_2 & \dots & \theta_n \end{bmatrix}^T_{(2n-1)\times 1} \\ C_{eq}^T &= \begin{bmatrix} I & Y_r^T \end{bmatrix}_{n\times(2n-1)} \\ b_{eq} &= \begin{bmatrix} D_1 & \dots & D_n \end{bmatrix}^T_{n\times 1} \end{aligned}$$

$$
Y_r = \begin{bmatrix} y_{21} & -\sum_{k\neq 2} y_{k2} & y_{23} & \dots & y_{2n} \\ y_{31} & y_{32} & -\sum_{k\neq 3} y_{k3} & \dots & y_{3n} \\ \vdots & \vdots & \vdots & \ddots & \vdots \\ y_{n1} & y_{n2} & y_{n3} & \dots & -\sum_{k\neq n} y_{kn} \end{bmatrix}_{(n-1)\times n}
$$

$$
C_{iq}^T = \begin{bmatrix} O_t & -\mathbb{D}\mathbb{A}_r \\ -O_t & \mathbb{D}\mathbb{A}_r \\ I_p & O_p \\ -I_p & -O_p \end{bmatrix}_{(2m+2n)\times(2n-1)}
$$

$$
b_{iq} = \begin{bmatrix} F^{max} & F^{max} & O_m & P^{max} \end{bmatrix}^T_{(2m+2n)\times 1}
$$

$$
F^{max} = \begin{bmatrix} -F^{max}_{BI_1} & -F^{max}_{BI_2} & \dots & -F^{max}_{BI_m} \end{bmatrix}^T_{m\times 1}
$$

$$
P^{max} = \begin{bmatrix} -P^{max}_1 & -P^{max}_2 & \dots & -P^{max}_n \end{bmatrix}^T_{n\times 1}
$$

$$
O_m = \begin{bmatrix} 0 & \dots & 0 \end{bmatrix}^T_{n\times 1}
$$

with O_t denoting a $m \times n$ zero matrix, O_p denoting an $n \times (n-1)$ zero matrix, and I_p a $n \times n$ identity matrix. To define $\mathbb{D}\mathbb{A}_r$ in C_{iq}^T, we define BI as the list of all distinct transmission lines ($kl \in BR$) constituting the network, lexicographically sorted from lower to higher numbered nodes such that BI_m denotes the mth transmission line in BI. $\mathbb{A}_r$ with entries 1 and -1 is defined as

$$
\mathbb{A}_r = \begin{bmatrix} \mathbb{J}(2, BI_1) & \dots & \mathbb{J}(n, BI_1) \\ \mathbb{J}(2, BI_2) & \dots & \mathbb{J}(n, BI_2) \\ \vdots & \ddots & \vdots \\ \mathbb{J}(2, BI_m) & \dots & \mathbb{J}(n, BI_m) \end{bmatrix}_{m\times(n-1)}
$$

where

$$\mathbb{J}(i, BI_k) = \begin{cases} +1, & \text{if } BI_k \text{ takes the form} \\ & ij \in BR \text{ for some node } j > i \\ -1, & \text{if } BI_k \text{ takes the form} \\ & ji \in BR \text{ for some node } i > j \\ 0, & \text{otherwise} \end{cases}$$

for all nodes $i = 1, \ldots, n$ and all branches $k = 1, \ldots, m$. $\mathbb{D}$ is a $m \times m$ diagonal matrix whose diagonal entries correspond to y_{kl} values of all transmission lines $kl \in BR$.

By solving the DC-OPF problem, the ISO determines the production amount (dispatched power P_i) for each GenCo and the corresponding nodal price (λ_i) at each node $(i = 1, \ldots, n)$. λs can be obtained either as the shadow prices of constraint set (1.2) or by solving the dual of problem (2.1)-(2.3):

$$\text{Maximize} \quad w = b_{eq}^T L + b_{iq}^T T \tag{2.5}$$

$$\text{subject to} \quad C_{eq} L + C_{iq} T \leq A \tag{2.6}$$

$$T_{P^{max}} \leq 0 \tag{2.7}$$

where

$$L = \begin{bmatrix} \lambda_1 & \lambda_2 & \ldots & \lambda_n \end{bmatrix}_{n \times 1}^T, \tag{2.8}$$

$$T = \begin{bmatrix} N_1 & N_2 & \ldots & N_{2m+2n} \end{bmatrix}_{(2m+2n) \times 1}^T, \tag{2.9}$$

$$T_{P^{max}} = \{N_i | 2m + n \leq i \leq 2m + 2n\}.$$

GenCo-i calculates its profit (r_i) as

$$r_i = P_i(\lambda_i - c_i) \tag{2.10}$$

where c_i is the pu-adjusted production cost (\$/h) of electricity by GenCo-i, and $(\lambda_i - c_i)$ is the profit from producing one unit of power at GenCo-i.

2.2 A Game-Theoretic Understanding of Collusion

The electricity market has a hierarchical structure with GenCos bidding to the ISO and the ISO clearing the market by solving the DC-OPF problem. For instance, in a real day-ahead market, the ISO periodically clears the market every hour (or every half an hour). In each period, the ISO determines assigned power of each GenCo and electricity price of each node; then, GenCos calculate their profit based on the locational marginal prices, assigned power and their generation cost according to Eq. (2.10).

As a result, the market clearance process in each period can be modeled as a non-cooperative single-stage game G with finite number of players ($\mathcal{F} = \{\text{GenCo-1}, \ldots, \text{GenCo-}n\}$), strategy (action) space of $\mathfrak{B} = (B_1 \times \ldots \times B_n)$ where B_i denotes the feasible strategy space corresponding to possible bids of GenCo-i, and vector of all GenCos payoffs; that is, $r = (r_1, \ldots, r_n)$ where r_i denotes payoff function of GenCo-i. Thus, the normal-form representation of G is denoted by the triplet $(\mathcal{F}, \mathfrak{B}, r)$.

In each iteration, collection of submitted bids $(b_1 \in B_1, \ldots, b_n \in B_n)$ defines the "state" of the game as we assume all other parameters required to solve the ISO's decision making problem, DC-OPF, are known.

In our context, a bidding strategy $\mathbb{N} = (b_1^{\mathbb{N}}, \ldots, b_n^{\mathbb{N}})$ is called a Nash equilibrium if any GenCo-i cannot make a better payoff than the payoff of the Nash equilibrium $(r_i^{\mathbb{N}})$ by choosing another bid $(b_i \in B_i)$ as long as the other GenCos are not changing their bids, i.e.

$$(r_1^{\mathbb{N}}, \ldots, r_i^{\mathbb{N}}, \ldots, r_n^{N}) \geq (r_1^{\mathbb{N}}, \ldots, r_i, \ldots, r_n^{\mathbb{N}}), \qquad i \in \{1, \ldots, n\}. \tag{2.11}$$

The clearance process is repeated indefinitely many times. It is acceptable to model the interaction in the electricity market as an infinite game. As a result we define $G(\infty, \delta)$ as infinitely repeated simultaneous-move stage game G with discount factor of δ[1].

Using this modelling approach, in our first and foremost result, we shall make use of Folk Theorem [39] that proves the possibility of collusion under a set of given conditions under an infinitely repeated game.

Let $\mathbb{N}_j \in \mathfrak{B}$ constitute the jth Nash equilibrium where $\mathbb{N}_j = (b_1^{\mathbb{N}_j}, \ldots, b_n^{\mathbb{N}_j})$ and $b_i^{\mathbb{N}_j} \in B_i, \forall\, i$. Suppose $\mathfrak{N} = \{\mathbb{N}_1, \ldots, \mathbb{N}_m\}$ is the set of all Nash equilibria and $r_i^{\mathbb{N}_j}$ denotes the payoff of GenCo-i at $\mathbb{N}_j$. Also define $r_i^* = max\{r_i^{\mathbb{N}_1}, \ldots, r_i^{\mathbb{N}_m}\}$ as the maximum payoff of GenCo-i under any Nash equilibrium.

Theorem 1 (Folk Theorem) *For a finite static game of complete information G, let $(r_1^{\mathbb{N}}, \ldots, r_n^{\mathbb{N}})$ denote the payoff profile for a Nash equilibrium, and $(r_1, \ldots, r_n)$ denote any other feasible payoff profile. If $r_i > r_i^{\mathbb{N}}$ for every player i and δ is sufficiently close to one, there exists a subgame-perfect Nash equilibrium of the infinitely repeated game $G(\infty, \delta)$ that achieves $(r_1, \ldots, r_n)$ as the average payoff profile.*

In our problem context, Folk Theorem implies that GenCos prefer to collude when value of losses in the long run is expected to be higher than the gains in the short term when one of the GenCos abandon collusion.

Under the existence of multiple collusive states satisfying Folk Theorem, one may ask which collusive state is more attractive for GenCos to play. There are several answers to this question; however, one attractive alternative can be a state where the minimum payoff is maximized.

Definition 2 (The Most Collusive State) *A state is called the Most Collusive State, if its minimum payoff over all GenCos is greater or equal than the minimum payoff for all GenCos in all other collusive states.*

We characterize the existence of collusion by identifying the profitability of a collusive state over all Nash equilibria. Considering the infinitely repeated game, when a

[1]The discount factor can be defined as $\delta = (1 - p)/(1 + \iota)$, where $\iota \in [0; 1]$ is an interest rate and $p \in [0; 1]$ represents a probability that the repeated game will end in the next time period.

GenCo unilaterally deviates from a collusive state for the higher short term profit, others learn about the deviation at the next iteration and play a specific Nash equilibrium strategy where all players are punished with lower payoffs. Yet, the effect of punishment is not the same on all GenCos; therefore, distinguishing between low and high punishment effects seems necessary for characterizing collusion. For this purpose, we define two types of collusive states: weak and strong. Weak collusion is more general than strong collusion since the minimum determined payoff of each GenCo is lower. On the other hand, weak collusion does not correspond to an equilibrium state while strong collusion can be identified as a stable equilibrium state because GenCos are punished severely if they deviate from a strong collusive state.

In this respect, we first formally define both versions of the collusive states along with the sufficiency conditions for their existence. Then, we develop a bi-level mathematical model representing the decision making process of one iteration of the game when a weak collusive state exists.

Definition 3 (Strong Collusive Equilibrium (SCE)) *A state $SCE \in \mathfrak{B}$ but $SCE \notin \mathfrak{N}$ with*

$$r_i^{SCE} > r_i^* \geq 0, \ \forall i \tag{2.12}$$

constitutes a Strong Collusive Equilibrium where r_i^{SCE} is the payoff of the GenCo-i in SCE.

As it is apparent from Definition 3, the payoffs of all GenCos under a *SCE* state need to be positive. So, any state with a payoff of zero for even one GenCo can not be a *SCE* for sure.

Proposition 4 (SCE necessary condition) *Under SCE, the nodal price of electricity at node-i is $b_i - \phi_i^{low} - \phi_i^{high}$ which is bigger than generation cost at node-i.*

Proof. From definition of SCE, we know that $r_i^{SCE} > r_i^* \geq 0 \Rightarrow r_i^{SCE} = P_i(\lambda_i - c_i) > 0$. Therefore, all GenCos should have positive power assignments ($P_i > 0$). When all the assigned powers are positive, the corresponding dual constraint $\lambda_i + \phi_i^{low} + \phi_i^{high} = b_i$.

Also to attain a positive r_i^{SCE}, λ_i should be greater than c_i for all GenCos. By combining previous sentences, we have $\lambda_i = b_i - \phi_i^{low} - \phi_i^{high} > c_i$ which is the necessary condition for SCE. □

This equilibrium is denoted as strong because all GenCos benefit from this type of collusion and hence they have no incentive to deviate. If a GenCo deviates, the game will move to a specific Nash equilibrium which is inferior with respect to payoffs for all GenCos.

Some interesting properties of *SCE* states can be identified when the number of strategies for each GenCo is finite.

Corollary 5 *In the neighborhood* [2] *of a* SCE *state, no Nash equilibrium exists.*

Proof. Suppose $\mathbb{N}_j \in \mathfrak{N}$ is a Nash equilibrium in the neighborhood of a SCE state. From definition of *SCE*, $r_i^{SCE} > r_i^* \geq r_i^{\mathbb{N}_j}$ for all GenCos. However, this contradicts the definition of a static Nash equilibrium; therefore, $\mathbb{N}_j$ cannot constitute a Nash equilibrium. □

From Corollary 5, one can easily claim that neighborhoods of Nash states can be excluded from the search for a *SCE* state. Thus, the search space can be narrowed down to a smaller subspace.

Corollary 6 *There exists at least one state $H \in \mathfrak{B}$ in the neighborhood of a* SCE *state such that at least one GenCo makes higher profit under H than it does in the* SCE.

Proof. Suppose there is no such state H and *SCE* offers better payoffs for all GenCos than all of its neighborhoods. Then, no GenCo has incentive to deviate from *SCE*. This implies that *SCE* is a Nash equilibrium which contradicts the definition of *SCE*. Thus, there is always at least one state H in the neighborhood of *SCE* which provides better payoffs to at least one GenCo, i.e. $r_i^H > r_i^{SCE} > r_i^*$, $\exists i$. □

Table 2.1 shows an example illustrating the existence of a *SCE* in an electricity market with two GenCos. Here, state $(b_1 = 20, b_2 = 30)$ is the only pure Nash equilibrium while

[2] We define state *A* as the neighborhood of state *B* if and only if *A* is different from *B* for only one GenCo's strategy.

state $(b_1 = 30, b_2 = 40)$ has better payoff for both GenCos; hence, it is a *SCE*. We observe that $(b_1 = 30, b_2 = 20)$ satisfies the definition of state H in Corollary 6.

Table 2.1: Payoff tables of two GenCos in an electricity market with collusion

$B_1 \backslash B_2$	20	30	40	50
20	(700, 0)	**(4000, 700)**	(6000, 0)	(6000, 0)
30	(350, <u>2500</u>)	(4000, 700)	**(6000, <u>1500</u>)**	(6000, 0)
40	(0, 6000)	(0, 6000)	(0, 6000)	(5000, 0)
50	(0, 6000)	(0, 6000)	(0, 6000)	(0, 6000)

We now, try to exploit the sufficiency conditions for existence of a *SCE*. For checking the existence of a *SCE*, the proposition requires the existence of at least one "best Nash equilibrium" as defined next.

Definition 7 (The Best Nash Equilibrium) *A Nash equilibrium $\mathbb{N}^* \in \mathfrak{N}$ is called the best Nash equilibrium when $r_i^{\mathbb{N}^*} = r_i^*, \ \forall i$.*

In existence of unique pure Nash equilibrium, the only Nash equilibrium is trivially the best Nash equilibrium.

Proposition 8 (SCE sufficiency condition) *Suppose $\mathbb{N}^* = (b_1^e, \ldots, b_n^e)$ is a best Nash equilibrium with a DC-OPF optimal power dispatch $x^e = (P_1^e, \ldots, P_n^e)$. If there exists at least two GenCos $(i$ and $j)$ who can increase the nodal prices $(\lambda_i^{'} > \lambda_i^e, \forall\, i)$ by increasing their bids $(b_i^{'} > b_i^e$ and $b_j^{'} > b_j^e)$, while x^e is still optimal, then $(b_1^e, \ldots, b_i^{'}, \ldots, b_j^{'}, \ldots, b_n^e)$ constitutes a SCE.*

Proof. When only one GenCo deviates from an initial Nash equilibrium $\mathbb{N}^*$ by increasing its bid, the new state is in the neighborhood of $\mathbb{N}^*$. From Corollary 5, such a state cannot constitute a *SCE*. Therefore, at least two GenCos need to collaborate in an attempt to increase all nodal prices.

Let σ denote a $2n \times 1$ column vector with at least two positive elements σ_i and σ_j for $i, j \in [1, \ldots, n]$. Increasing the coefficients of Eq. (2.4) from a Nash state (A^e) to

$A^e + \sigma$ such that x^e is still optimal and $\lambda_j > b_j^e$, $\forall j \in [1, \ldots, n]$ means $r_j' > r_j^e$ by Eq.(2.10). First, the new bids cannot constitute a Nash as the initial state $\mathbb{N}^*$ is a best Nash equilibrium itself. Second, since derived state is not in the neighborhood of $\mathbb{N}^*$, it constitutes a *SCE* by Definition 3. $\square$

Next, we expand the notion of a "collusive state" by defining a weaker condition than the one required for *SCE*. We will use this weaker condition later to characterize the solution space for optimization.

Definition 9 (Weak Collusive State (WCS)) *A state $WCS \in \mathfrak{B}$ but $WCS \notin \mathfrak{N}$ with*

$$r_i^{WCS} \geq r_i^*, \forall i \tag{2.13}$$

constitutes a Weak Collusive State where r_i^{WCS} is the payoff of GenCo-i in WCS.

A *WCS* implies that a GenCo that has no incentive to stay in collusion may deviate from the collusive state with an effort to increase its payoff in short term before the game goes to a specific Nash equilibrium. In response, other GenCos are inclined to move to a specific Nash equilibrium following a grim trigger strategy. For instance, consider a *WCS* where $r_{i'}^{WCS} = r_{i'}^* = 0$ for a particular GenCo-i'. Since the profit of GenCo-i' is zero, it has no incentive to stay in equilibrium, and it may exploit chances of better payoffs by deviating. When a *WCS* is abandoned by at least one such GenCo, all GenCos move to a particular Nash to prevent further losses. Certainly, these short term profits have no effect on long-term profit of GenCo-i' as $r_{i'}^* = 0$ under all Nash equilibria.

Proposition 10 (WCS sufficiency condition) *Suppose $\mathbb{N}^* = (b_1^e, \ldots, b_n^e)$ is a best Nash equilibrium with a DC-OPF optimal solution $x^e = (P_1^e, \ldots, P_n^e)$. If x^e is still optimal when $b_i^e = \lambda_i^e$ increases to b_i' for at least one GenCo-i with $P_i^e > 0$ then $(b_1', \ldots, b_i', \ldots, b_n')$ where $b_j' \geq b_j^e, \forall j \in \{1, \ldots, n\}$ and $\exists i\ b_i' > b_i^e$ constitutes a WCS.*

Proof. If x^e, DC-OPF optimal solution for $\mathbb{N}^*$, stays optimal when coefficients in A^e (Eq. (2.4)) increase from b_i^e to b_i' for GenCo-i with $b_i^e = \lambda_i^e$, then λ_i' (nodal price of GenCo-i after bidding b_i') has to increase to $b_i' > b_i^e$. Otherwise, x^e changes since GenCo-i will not be a given $P_i > 0$ any longer. Therefore, $r_i' - r_i^e = P_i^e(\lambda_i' - \lambda_i^e) > 0$ for GenCo-i by Eq.(2.10).

The rest of the nodal prices are either larger (because GenCo-i may have participated in providing electricity in those nodes) or equal for all winning bids. To prove the aforementioned statement, we examine the changes in GenCos' payoffs in three different cases: GenCos with losing bids ($P_i = 0$), GenCos with winning bids with an allocated power less than their production capacity ($0 < P_i < P_i^{max}$), and GenCos that could sell out their entire production capacity ($0 < P_i = P_i^{max}$).

Recall that the corresponding dual variables for $P_i \leq P_i^{max}$ and $P_i \geq 0$ are ϕ_i^{high} and ϕ_i^{low}, respectively.

- $P_i = 0$: The payoff trivially stays at zero ($r_i^{'} = r_i = 0$) under any scenario even if $\lambda_i^{'} < \lambda_i$.

- $0 < P_i < P_i^{max}$: In this case $\lambda_i = b_i$ implies that $\lambda_i + \phi_i^{low} + \phi_i^{high} = b_i$. As $0 < P_i < P_i^{max}$, $\phi_i^{low} = \phi_i^{high} = 0$. By increasing b_i to $b_i^{'}$ then $\lambda_i^{'} = b_i^{'} \geq b_i = \lambda_i$ since $0 < P_i < P_i^{max}$. $\lambda_i > b_i$ implies that $b_i^{'}$ will stay the same as b_i when $\lambda_i \neq b_i$. Also $\phi_i^{low} = \phi_i^{high} = 0$ since $0 < P_i < P_i^{max}$. Thus, $\lambda_i + \phi_i^{low} + \phi_i^{high} = b_i = b_i^{'} \Rightarrow \lambda_i = b_i = b_i^{'}$ and this contradicts the assumption.

- $P_i = P_i^{max}$: In this case $\lambda_i = b_i$ implies that b_i to $b_i^{'} \geq b_i$. We have $\lambda_i^{'} + \phi_i^{high} = b_i^{'}$ and since $\phi_i^{high} \leq 0$ then $\lambda_i^{'} = b_i^{'} - \phi_i^{high} \geq b_i = \lambda_i$. $\lambda_i > b_i$ implies that $\lambda_i + \phi_i^{high} = b_i = \lambda_i^{'} + \phi_i^{high'}$. When price of electricity in other nodes are higher $\lambda_j^{'} \geq \lambda_j$ then adding one unit of power to capacity of cheaper GenCo-i can increase the customers' welfare z^* more. Since $z^{*'} \geq z^*$ then $\phi_i^{high'} \leq \phi_i^{high} \Rightarrow \lambda_i^{'} \geq \lambda_i$.

Therefore, all GenCos' payoffs are (at least) as good as those under $\mathbb{N}^*$ ($r_j' \geq r_j^e \; \forall j$). Finally, because $(b_1^{'}, \ldots, b_i^{'}, \ldots, b_n^{'})$ provides a better payoff profile than (the best Nash) $\mathbb{N}^*$, it could not be a Nash equilibrium and it constitutes a *WCS* by Definition 9. $\square$

2.3 A Bi-level Mathematical Model for Collusion

Bi-level programming problems are hierarchical optimization problems combining decisions of at least two decision makers: the leader and the follower. They are commonly used to model hierarchical games (Weber and Overbye [112], Hu and Ralph [51]), in particular, for finding the equilibrium in Stackelberg games. Bi-level programming problems are special cases of multi-level programming problems.

A collusive state (*SCE* or *WCS*) of GenCos can be characterized with a mathematical model in the form of a multi-objective non-linear bi-level optimization problem where GenCos choose their bids at the leader level and the ISO makes decisions on nodal prices and assigned powers in the follower level. The strategic bidding problem with embedded collusion can be prescribed as follows:

$$\text{Maximize}_{\{b_1,\ldots,b_n\}} \ (r_1, r_2, \ldots, r_n) \tag{2.14}$$

$$\text{subject to} \quad r_i \geq r_i^* \qquad \forall i \tag{2.15}$$

$$\text{Minimize}_{\{P_1,\ldots,P_n,\theta_1,\ldots,\theta_n\}} \ A^T x \tag{2.16}$$

$$\text{subject to} \quad C_{eq}^T x = b_{eq} \qquad (L) \tag{2.17}$$

$$C_{iq}^T x \geq b_{iq} \qquad (T) \tag{2.18}$$

$$b_i \in B_i \qquad \forall i \tag{2.19}$$

where $r_i = P_i(\lambda_i - c_i)$. The objective function (2.14) maximizes the payoffs for all GenCos. Constraint set (2.15) characterizes a payoff profile under a collusive state where payoff of each GenCo is better than any payoff it can obtain under any Nash equilibrium. (2.16)-(2.18) represent the ISO's DC-OPF problem. The corresponding dual variables for constraint sets of (2.17) and (2.18) are shown in parenthesis which were previously defined in Eq. (2.8) and (2.9), respectively.

The resulting problem is multi-objective; but we can first simplify this function as maximize $\omega = min_i\{r_i\}$ based on Definition 2. In this respect ω can be used as a threshold value for the payoffs while representing the most collusive state through (2.14)-(2.19). This threshold guarantees that minimum received payoff of all GenCos is greater than ω.

Farahi and Ansari [36] present a similar approach to convert a multi-objective problem into a single objective. The formulation becomes:

$$\text{Maximize}_{\{b_1,\ldots,b_n\}} \;\; \omega \tag{2.20}$$
$$\text{subject to} \quad \omega \le r_i \qquad \forall i \tag{2.21}$$
$$r_i \ge r_i^* \qquad \forall i \tag{2.22}$$
$$\text{Minimize}_{\{P_1,\ldots,P_n,\theta_1,\ldots,\theta_n\}} \; A^T x \tag{2.23}$$
$$\text{subject to} \quad C_{eq}^T x = b_{eq} \tag{2.24}$$
$$C_{iq}^T x \ge b_{iq} \tag{2.25}$$
$$b_i \in B_i \qquad \forall i \tag{2.26}$$

Problem (2.20)-(2.26) is still a bi-level optimization problem and solving a bi-level problem can be challenging since the basic theory of bi-level optimization does not allow cooperation between the upper level and the lower level decision-makers (Candler and Norton [15]). A single level optimization problem can be derived by exploiting the following proposition together with the strong duality theorem.

Proposition 11 *Problem* (2.23)-(2.25) *always has a bounded feasible solution if and only if the feasible region is not empty.*

Proof. As P_i is always bounded by the capacity P_i^{max}, and b_i is a finite number ($c_i \le b_i \le b_i^{cap} < \infty$), $b_i P_i$ is finite for all GenCos. Although, θ_i are unbounded $\forall\, i$, their coefficients in (2.23) are 0. Hence, $\sum_i b_i P_i < \infty$. □

Remark 12 (strong duality theorem) *If the primal problem has finite optimal solution, then the dual also has an optimal solution with an objective value equal to that of a primal optimal solution [102].*

The problem in the follower level (DC-OPF) can be rewritten as a system of equations whose feasible solution would be an optimal solution to DC-OPF. In this respect, we resort to the equality of the optimal objective function values of a primal-dual pair. This is only attained when the feasibility conditions of primal problem (2.28)-(2.29) and the corresponding dual problem are satisfied simultaneously. Recently, Singh [92] use the same approach to solve a bi-level quadratic-linear programming problem. As a result, a feasible solution to the system of equations

$$A^T x = b_{eq}^T L + b_{iq}^T T \tag{2.27}$$

$$C_{eq}^T x = b_{eq} \tag{2.28}$$

$$C_{iq}^T x \geq b_{iq} \tag{2.29}$$

$$C_{eq} L + C_{iq} T \leq A \tag{2.30}$$

$$T_{P^{max}} \leq 0 \tag{2.31}$$

corresponds to the optimal solution of (2.23)-(2.25). Replacing the second-level problem with the above system of equations converts the bi-level programming problem into a single-level problem as follows:

$$\begin{aligned}
WCP\ :&\text{Maximize}_{\{b_1,\ldots,b_n\}} : \omega \\
\text{subject to} \quad & \omega \leq P_i\lambda_i - P_i c_i \qquad \forall i \\
& P_i\lambda_i - P_i c_i \geq r_i^N \quad \forall i \\
& A^T x = b_{eq}^T L + b_{iq}^T T \\
& C_{eq}^T x = b_{eq} \\
& C_{iq}^T x \geq b_{iq} \\
& C_{eq} L + C_{iq} T \leq A \\
& T_{P^{max}} \leq 0 \\
& b_i \in B_i \qquad\qquad\qquad \forall i
\end{aligned} \tag{2.32}$$

We also substitute constraint (2.32) with

$$c_i \leq b_i \leq b_i^{Cap} \qquad \forall i \tag{2.33}$$

in order to solve *WCP* with less computational effort. In (2.33), we suppose that each bid of GenCo-i is greater or equal than the production cost (c_i) of that GenCo and is capped by a specific price cap b_i^{Cap} which is exogenously determined.

In order to solve *WCP*, we should first tackle the nonlinearity in (2.27) since b_i and P_i both are decision variables. In this respect, we make use of the following important results from the theory of linear programming as in Vanderbei [102]:

Remark 13 *The feasible region of the linear program has at least one vertex and at most finite vertices if it is non empty.*

Remark 14 *If there exists an optimal solution to the linear program, it must be at least one of the vertex of the feasible region.*

Let $S = \{(L,T,A)|C_{eq}L + C_{iq}T - A \leq 0, T_{P^{max}} \leq 0\}$ denote the set of all vertices of the dual problem of (2.23)-(2.25) represented with the feasible solution space described by (2.30)-(2.31). Hence, the optimal solution to *WCP* is in S. We obtain all vertices of S using linear algebra as $S = \{S_1, S_2, \ldots, S_n\}$. For a given $S_v = (L_v, T_v, A_v)$ with $\lambda_i \in L_v \geq b_i \in A_v$, we obtain a system of linear equations:

$$A_v{}^T x = b_{eq}^T L_v + b_{iq}^T T_v \tag{2.34}$$

$$C_{eq}^T x = b_{eq} \tag{2.35}$$

$$C_{iq}^T x \geq b_{iq} \tag{2.36}$$

$$C_{eq}L_v + C_{iq}T_v \leq A_v \tag{2.37}$$

$$T_{P^{max}} \leq 0 \tag{2.38}$$

which can be solved for x. For a given x (including $P_i,\ \forall i$), the corresponding payoff profile can be calculated from (2.10). Accordingly, we may define $\overline{S} = \{(L_v, T_v, A_v) \in$

$S|r_i \geq r_i^* \forall i\}$ as the subset of these vertices that satisfies *WCS* condition. $\overline{S}_v \in \overline{S}$ and $\overline{S}_v \neq \emptyset$ implies a *WCS*.

For $\overline{S}_v \neq \emptyset$, the optimal solution is $\{A_v|\omega_{\overline{S}_v} \geq \omega_{\overline{S}_z}, \forall \overline{S}_z \in \overline{S}\}$ where $\omega_{\overline{S}_v} = min_i\{P_i\lambda_i - P_ic_i|\overline{S}_v\}$.

Proposition 15 *In the existence of a* SCE*, the optimal solution to* WCP *corresponds to a* SCE *state.*

Proof. Even though in *WCP* a weak collusion is modeled by forcing $r_i \geq r_i^*$ as a constraint, any *SCE* is feasible solution for *WCP* since $SCE \subset WCS$. Moreover, $\omega^{SCE} = min\{r_i^{SCE} > r_i^*|\forall\ i\} \geq \omega^{WCS} = min\{r_i^{WCS} \geq r_i^*|\forall\ i\}$; therefore, by increasing the threshold through maximizing objective function we obtain $\omega \rightarrow \omega^{SCE}$. $\square$

Last but not the least, finding all vertices of a polytope/polyhedron is very difficult, even impossible, for large-scale problems [58]. Next, we present a heuristic algorithm to solve the large-scale instances of *WCP* in a more efficient compact search space.

2.4 Proposed Algorithm

The algorithm relies on exploiting the sufficiency conditions for *WCS*s and *SCE*s provided in Proposition 8 and Proposition 10. In order to implement the algorithm for a given set of GenCos,

- all Nash equilibria should be known, and
- at least one Nash equilibrium should have the same payoff profile as r^*.

If there is no such equilibrium, the algorithm is not applicable.

All Nash equilibria can be computed with the algorithm proposed in Li et al. [68]. By determining all Nash equilibria, one may calculate r_i^* of each GenCo, and in consequence, r^* can be found. If there is a unique Nash equilibrium, the second condition is trivially satisfied; otherwise, we need to check all Nash equilibria to find one with the same payoff profile as r^*. If there is any, using Proposition 8 and Proposition 10, the algorithm finds the most collusive state as in Definition 2. Ruiz et al. [88] proposed a mixed-integer linear programming problem formulation to select meaningful equilibria; one may resort to their algorithm in order to find all Nash equilibria to initialize Algorithm 1.

The algorithm finds *WCS* and *SCE* in $\mathfrak{B}$ regardless of whether the bids (b_i) are continuous or discrete. To switch from the continuous case to the discreet case, a resolution parameter (Δ) is used in the algorithm. Δ can be interpreted as the difference between two consecutive bids or accepted accuracy of monetary unit. It is worth noting that the accuracy is increased as $\Delta \to 0$; therefore, Δ is zero for continuous bid prices.

The algorithm begins searching the solution space from the best Nash equilibrium (N^*). In line 8, we maximize the bids such that $b_i^{'} \leq b_i^{cap}$. It searches over all possible subsets of the GenCos (G_m) sorted in descending order based on their cardinality to find σ_{G_m} by which the minimum payoff of GenCos in the subset G_m can be improved. In the presence of such σ_{G_m} on subset G_m, we find at least one*WCS* based on Proposition 10. Based on Proposition 8, if the condition on line 9 holds for the subset G_1, the algorithm finds at least one collusive state as in Definition 3 since the subset G_1 includes all the GenCos. However, not finding any $\sigma_{G_1} > 0$ at the beginning does not mean the problem does not have any *SCE*; sometimes, the combined *WCS* may result in a *SCE*. Lines 22-28 find out whether the resulting strategy is a *WCS* or *SCE*.

2.5 Numerical Examples

In order to illustrate the algorithm, we present two examples: one with a unique Nash equilibrium and another one with multiple Nash equilibria. Recognizing the existence of collusive states in the examples, we also discuss possible actions for the ISO to cope with and avoid collusion in these markets.

2.5.1 Case 1: An electricity market with a unique Nash equilibrium

The transmission network has five nodes with six transmission lines. The properties of transmission lines are given in Table 4.3. The network structure along with generation

Algorithm 1 Finding SCE and WCS

$\sigma^{'} = \overrightarrow{0}$
Find all Nash equilibria $\mathfrak{N} = \{N_1, N_2, \ldots, N_m\}$
Find $N^* \in \mathfrak{N}$ with same payoff profile as r^*.
if N^* exists **then**
  Solve DC-OPF for N^* and find x^e
  for $m = 1$ to $m < 2^n$ **do**
    Consider mth subset of GenCos (G_m)
    maximize σ_{G_m} such that x^e stays optimal when $b_i^{'} = \{b_i + \sigma_{G_m} | \sigma_{G_m} \leq b_i^{cap} - b_i\}, i \in G_m$
    if $\sigma_{G_m} \geq \Delta$ **then**
      if $\Delta > 0$ **then**
        $\sigma_{G_m} = \lfloor \frac{\sigma_{G_m}}{\Delta} \rfloor \Delta$
      end if
      for $i = 1$ to n **do**
        if $i \in G_m$ **then**
          $b_i = b_i + \sigma_{G_m}$
        end if
      end for
      Calculate λ_i at each node $(i = 1, \ldots, n)$
      Calculate r_i and $\omega = min_i\{r_i\}$
    end if
  end for
  if $(r_i > r_i^*, \forall\, i)$ **then**
    SCE is found
  else
    if $(\exists\, i, r_i > r_i^*)$ **then**
      WCS is found
    end if
  end if
end if

Table 2.2: Transmission Line Properties in Case 1

Src (k)/ Dst (l)	y_{kl}	F_{kl}^{max}
$\{1/2, 1/3, 3/4, 4/5\}$	4	No Limit
2/4	4	150
2/5	4	100

Table 2.3: Parameters of GenCos in Case 1

ID	P_i^{max}	c_i	B_i
GenCo-1	300	20	$\{20, 30, 40, 50\}$
GenCo-2	300	20	$\{20, 30, 40, 50\}$
GenCo-5	250	30	$\{40\}$

capacities and demand load data are shown in Fig.2 and Table 4.4, respectively. Node 3 is the reference bus in this system. GenCo-1 and GenCo-2 can bid any value between their marginal cost of \$20$/MW$ and price cap of \$50/MW with increments of \$10/MW (hence $\Delta = 10$) whereas GenCo-5 can only bid \$40/MW. Knowing that GenCo-5 has no alternative bid to change the state of the game, we ignore GenCo-5 in our calculations.

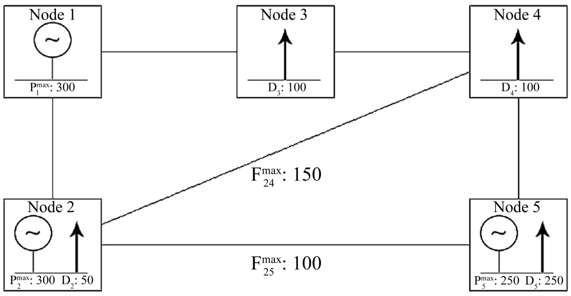


Figure 2.1: Transmission Network in Case 1

Table 2.4 shows the payoffs in the stage game for all possible bidding strategies. The execution of the algorithm on Case 1 can be explained step by step as follows:

1. $N^* = (b_1 = 20, b_2 = 30)$ is the only Nash equilibrium and it is trivially the best Nash equilibrium.

2. $x^e = (P_1^e = 300, P_2^e = 78.57)$ is the DC-OPF optimal solution for N^*.

3. From Table 2.5, we observe $\sigma_{G_1} = 10 = \Delta$ as the maximum permissible increase in the bid price for both GenCo-1 and GenCo-2; therefore, $(b_1 = 30, b_2 = 40)$ constitutes a *SCE*.

4. From $(b_1 = 30, b_2 = 40)$, the algorithm (from line 6 to 21) will try all the other subsets of GenCos to find another σ_{G_m} which preserves the optimality of x^e. The subset $\{GenCo - 1\}$ lets x^e to stay optimal when $\sigma_{G_m} = 9.\overline{9}$; however, $9.\overline{9} < \Delta$.

5. The algorithm terminates where $(b_1 = 30, b_2 = 40)$ is discovered as a *SCE* state.

Table 2.4: Payoff profile of GenCo-1 and GenCo-2

$B_1 \backslash B_2$	20	30	40	50
20	(857.14, 0)	**(3428.57, 785.71)**	(6000, 0)	(6000, 0)
30	(416.67, <u>2500</u>)	(3428.57, 785.71)	**(6000, <u>1571.43</u>)**	(6000, 0)
40	(0, 6000)	(0, 6000)	(0, 6000)	(5000, 0)
50	(0, 6000)	(0, 6000)	(0, 6000)	(0, 7500)

Table 2.5: Dispatched power of GenCo-1 and GenCo-2

$B_1 \backslash B_2$	20	30	40	50
20	(300, 78.57)	$x^e = (300, 78.57)$	(300, 0)	(300, 0)
30	(41.67, 300)	(300, 78.57)	**(300, 78.57)**	(300, 0)
40	(0, 300)	(0, 300)	(0, 300)	(250, 0)
50	(0, 300)	(0, 300)	(0, 300)	(0, 250)

GenCos may inevitably engage into a collusion at a *SCE* state. Yet, the ISO may prevent this collusion through some actions that change the market configuration as suggested bellow:

1. Establishment of a new subsidized GenCo: Establishing a new generator at node 4 with a maximum capacity of 65MW ($P_4^{max} = 65$) will cause collusion to break. The difference in total payoffs in each hour is $\sum_i (r_i^{SCE} - r_i^*) = 1778.58$ which is 20.24% of the total payoffs in *SCE*. Thus, by buying electricity from a new GenCo

Table 2.6: Parameters of GenCos in Case 2

ID	P_i^{max}	c_i	B_i
GenCo-1	139	20	$\{20, 25, 30, 35, 40, 45, 50\}$
GenCo-2	527	20	$\{20, 25, 30, 35, 40, 45, 50\}$
GenCo-5	560	30	$\{30, 35, 40, 45, 50\}$

at node 4 with $P_4^{max} \geq 65$MW under \$27/MW the ISO would improve customers' welfare as well as breaking collusion of GenCo-1, GenCo-2 and GenCo-5.

2. Modifying the transmission network configuration: Relieving congestion can break collusion, however it does not necessarily increase customer's welfare. For instance, relieving congestion of all transmission lines breaks the collusion; but, the total payoffs of all GenCos ($\sum_i r_i^*$) will be \$12000/h. On the other hand, by decreasing the capacity of transmission line from node 2 to node 4 to 55 (pu), the ISO can isolate the GenCos with market power and save about \$602 in each hour in addition to breaking collusion.

3. Introducing a price cap at some nodes: Collusion may be prevented easily by limiting the price of electricity at some nodes. Table 2.4 shows that an upper bound on GenCo-2 at \$30/MW shrinks the payoff table and excludes the collusive state. Note, however that this solution may not always be viable due to regulations.

2.5.2 Case 2: An electricity market with multiple Nash equilibria

We illustrate the algorithm on another case in which there exist multiple Nash equilibria and two best Nash equilibria as in Definition 7. The transmission network has five nodes with six transmission lines. The network structure along with generation capacities and demand load data are given in Fig.2.2 and Table 2.6, respectively. Node 3 is assumed as the reference bus. All GenCos can bid any value between their marginal cost and a price cap of \$50/MW with increments of \$5/MW (hence $\Delta = 5$).

According to Table 2.7 where the resulting payoff profiles of all possible bidding scenarios are seen, four pure Nash equilibrium (shown in bold characters) exists. Among

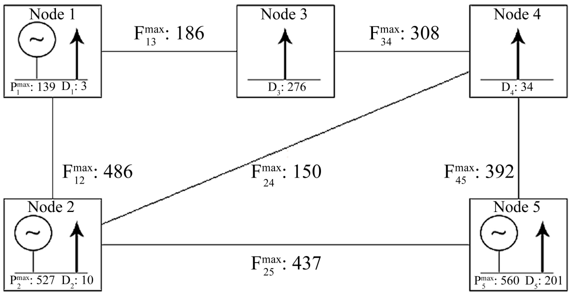


Figure 2.2: transmission network of case 2

these, $N_1^* = (b_1 = 25, b_2 = 35, b_5 = 35)$ and $N_2^* = (b_1 = 30, b_2 = 35, b_5 = 35)$ have the payoff profile of r^*, so they are the best Nash equilibria. The execution of the algorithm on this example can be explained step by step as follows:

1. $N_1^* = (b_1 = 25, b_2 = 35, b_5 = 35)$ is one of the best Nash equilibria and $x^e = (P_1^e = 139, P_2^e = 374, P_5^e = 11)$ is the DC-OPF optimal solution of N_1^*.

2. The maximum permissible increase for all GenCos is $\sigma_{G_1} = 3\Delta = 15$. Therefore, $(b_1 = 40, b_2 = 50, b_5 = 50)$ constitutes a *SCE*.

3. From $(b_1 = 40, b_2 = 50, b_5 = 50)$, the algorithm will try all the other combinations of GenCos to find another σ_{G_m} which satisfies the optimality condition of x^e. The subset $\{GenCo - 1\}$ lets x^e to stay optimal when $\sigma_{G_m} = 5$; therefore, it leads to $(b_1 = 45, b_2 = 50, b_5 = 50)$.

4. The algorithm terminates where *SCE* = $(b_1 = 45, b_2 = 50, b_5 = 50)$ is discovered.

Similar to Case 1, we may list alternative courses of actions for the ISO to avoid the collusion between GenCos as follows:

1. Establishments of a new subsidized GenCo: Establishing two generators at node 3 and node 4 with maximum capacity of 262MW $(P_3^{max} = P_4^{max} = 262)$ will break collusion. The difference of total payments in each hour is $\sum_i (r_i^{SCE} - r_i^*) = 15610$. Thus, the ISO could be confident that buying at least 262MW electricity from each

Table 2.7: Profit profile of (r_1, r_2, r_5)

$b_5 = 30$	20	25	30	35	40	45	50
20	(0, 0, 0)	(0, 1943.33, 0)	**(1390, 3740, 0)**	(1390, 0, 0)	(1390, 0, 0)	(1390, 0, 0)	(1390, 0, 0)
25	(667.5, 0, 0)	(667.5, 1952.5, 0)	**(1390, 3740, 0)**	(1390, 0, 0)	(1390, 0, 0)	(1390, 0, 0)	(1390, 0, 0)
30	(0, 0, 0)	(0, 2175, 0)	(0, 4350, 0)	(0, 0, 0)	(0, 0, 0)	(1390, 0, 0)	(1390, 0, 0)
35	(0, 0, 0)	(0, 2175, 0)	(0, 4350, 0)	(0, 0, 0)	(0, 0, 0)	(0, 0, 0)	(0, 0, 0)
40	(0, 0, 0)	(0, 2175, 0)	(0, 4350, 0)	(0, 0, 0)	(0, 0, 0)	(0, 0, 0)	(0, 0, 0)
45	(0, 0, 0)	(0, 2175, 0)	(0, 4350, 0)	(0, 0, 0)	(0, 0, 0)	(0, 0, 0)	(0, 0, 0)
50	(0, 0, 0)	(0, 2175, 0)	(0, 4350, 0)	(0, 0, 0)	(0, 0, 0)	(0, 0, 0)	(0, 0, 0)
$b_5 = 35$	20	25	30	35	40	45	50
20	(0, 0, 0)	(0, 1943.33, 0)	(0, 3886.67, 0)	(2085, 0, 1925)	(2085, 0, 1925)	(2085, 0, 1925)	(2085, 0, 1925)
25	(667.5, 0, 0)	(667.5, 1952.5, 0)	(676.67, 3886.67, 0)	**(2085, 5610, 55)**	(2085, 0, 1925)	(2085, 0, 1925)	(2085, 0, 1925)
30	(1335, 0, 0)	(1335, 1952.5, 0)	(1335, 3905, 0)	**(2085, 5610, 55)**	(2085, 0, 1925)	(2085, 0, 1925)	(2085, 0, 1925)
35	(0, 0, 445)	(0, 2175, 445)	(0, 4350, 445)	(0, 6525, 445)	(0, 0, 2620)	(0, 0, 2620)	(0, 0, 2620)
40	(0, 0, 445)	(0, 2175, 445)	(0, 4350, 445)	(0, 6525, 445)	(0, 0, 2620)	(0, 0, 2620)	(0, 0, 2620)
45	(0, 0, 445)	(0, 2175, 445)	(0, 4350, 445)	(0, 6525, 445)	(0, 0, 2620)	(0, 0, 2620)	(0, 0, 2620)
50	(0, 0, 445)	(0, 2175, 445)	(0, 4350, 445)	(0, 6525, 445)	(0, 0, 2620)	(0, 0, 2620)	(0, 0, 2620)
$b_5 = 40$	20	25	30	35	40	45	50
20	(0, 0, 0)	(0, 1943.33, 0)	(0, 3886.67, 0)	(0, 5610, 110)	(2780, 0, 3850)	(2780, 0, 3850)	(2780, 0, 3850)
25	(667.5, 0, 0)	(667.5, 1952.5, 0)	(676.67, 3886.67, 0)	(676.67, 5830, 0)	(2780, 7480, 110)	(2780, 0, 3850)	(2780, 0, 3850)
30	(1335, 0, 0)	(1335, 1952.5, 0)	(1335, 3905, 0)	(1353.33, 5830, 0)	(2780, 7480, 110)	(2780, 0, 3850)	(2780, 0, 3850)
35	(0, 0, 890)	(2002.5, 1952.5, 0)	(2002.5, 3905, 0)	(2002.5, 5857.5, 0)	(2780, 7480, 110)	(2780, 0, 3850)	(2780, 0, 3850)
40	(0, 0, 890)	(0, 2175, 890)	(0, 4350,890)	(0, 6525, 890)	(0, 8700, 890)	(0, 0, 5240)	(0, 0, 5240)
45	(0, 0, 890)	(0, 2175, 890)	(0, 4350,890)	(0, 6525, 890)	(0, 8700, 890)	(0, 0, 5240)	(0, 0, 5240)
50	(0, 0, 890)	(0, 2175, 890)	(0, 4350,890)	(0, 6525, 890)	(0, 8700, 890)	(0, 0, 5240)	(0, 0, 5240)
$b_5 = 45$	20	25	30	35	40	45	50
20	(0, 0, 0)	(0, 1943.33, 0)	(0, 3886.67, 0)	(0, 5830, 0)	(695, 7480, 165)	(3475, 0, 5775)	(3475, 0, 5775)
25	(667.5, 0, 0)	(676.67, 1943.33, 0)	(676.67, 3886.67, 0)	(676.67, 5830, 0)	(695, 7480, 165)	(3475, 0, 5775)	(3475, 0, 5775)
30	(1335, 0, 0)	(1335, 1952.5, 0)	(1335, 3905, 0)	(1353.33, 5830, 0)	(1353.33, 7773.33, 0)	(3475, 9350, 165)	(3475, 0, 5775)
35	(2002.5, 0, 0)	(2002.5, 1952.5, 0)	(2002.5, 3905, 0)	(2002.5, 5857.5, 0)	(2030, 7773.33, 0)	(3475, 9350, 165)	(3475, 0, 5775)
40	(0, 0, 1335)	(0, 2175, 1335)	(2670, 3905, 0)	(2670, 5857.5, 0)	(2670, 7810, 0)	(3475, 9350, 165)	(3475, 0, 5775)
45	(0, 0, 1335)	(0, 2175, 1335)	(0, 4350, 1335)	(0, 6525, 1335)	(0, 8700, 1335)	(0, 10875, 1335)	(0, 0, 7860)
50	(0, 0, 1335)	(0, 2175, 1335)	(0, 4350, 1335)	(0, 6525, 1335)	(0, 8700, 1335)	(0, 10875, 1335)	(0, 0, 7860)
$b_5 = 50$	20	25	30	35	40	45	50
20	(0, 0, 0)	(0, 1943.33, 0)	(0, 3886.67, 0)	(0, 5830, 0)	(0, 7773.33, 0)	(1390, 9350, 220)	(4170, 0, 7700)
25	(667.5, 0, 0)	(676.67, 1943.33, 0)	(676.67, 3886.67, 0)	(676.67, 5830, 0)	(676.67, 7773.33, 0)	(1390, 9350, 220)	(4170, 0, 7700)
30	(1335, 0, 0)	(1335, 1952.5, 0)	(1353.33, 3886.67, 0)	(1353.33, 5830, 0)	(1353.33, 7773.33, 0)	(1390, 9350, 220)	(4170, 0, 7700)
35	(2002.5, 0, 0)	(2002.5, 1952.5, 0)	(2002.5, 3905, 0)	(2002.5, 5857.5, 0)	(2030, 7773.33, 0)	(2030, 9716.67, 0)	(4170, 11220, 220)
40	(0, 0, 1780)	(2670, 1952.5, 0)	(2670, 3905, 0)	(2670, 5857.5, 0)	(2670, 7810, 0)	(2706.67, 9716.67, 0)	(4170, 11220, 220)
45	(0, 0, 1780)	(0, 2175, 1780)	(0, 4350, 1780)	(3337.5, 5857.5, 0)	(3337.5, 7810, 0)	(3337.5, 9762.5, 0)	<u>(4170, 11220, 220)</u>
50	(0, 0, 1780)	(0, 2175, 1780)	(0, 4350, 1780)	(0, 6525, 1780)	(0, 8700, 1780)	(0, 10875, 1780)	(0, 13050, 1780)

new GenCo at node 3 and 4 under \$29.79/MW improves customers' welfare as well as breaking collusion.

2. Investment on the transmission network to relieve congestion: Adding a transmission line between node 3 and node 5 with $F_{35}^{max} = 50$ disrupts the collusive equilibrium. The new market will not have any pure Nash equilibrium. Hence, collusion does not exist according to Definition 3 and Definition 9.

3. Introducing a price cap at some nodes: Setting $b_5^{cap} = 35$ breaks the collusion; however, forcing GenCo-5 to bids under price cap may cause him to leave this node. In the long-term, node 5 may suffer from lack of domestic generator and thus, may become highly dependent on power import. Consequently, satisfying demand of node 5 will be harder when congestion occurs.

2.6 Remarks

We study the existence and identification of collusion among GenCos in a deregulated (oligopolistic) electricity market when transmission network constraints are under consideration within the market clearance mechanism of the ISO. Firstly, we examine characteristics of collusion based on market parameters and strategic behaviors of GenCos. Strategic behavior of GenCos is modeled within an infinite horizon game. Then, we develop a bi-level mathematical programming problem to model the market clearance mechanism of the ISO where the behavior of GenCos and network constraints are considered. The problem has multiple non-linear objective functions where GenCos compete at the leader level and the optimal power flow is determined at the follower level. Using linear programming theory and the methods in multi-objective optimization, the problem is simplified to a constrained optimization problem with a linear objective function. An optimization-based approach is proposed to solve the problem. In our computational study, we present case studies which indicate how collusion can be detrimental for the end consumers disrupting the competition. Based on the cases, we also discuss alternative courses of action for the ISO to cope with and avoid collusion.

Chapter 3

Strategic Bidding Behavior of Power Generation Companies

Focus of this chapter is on the day-ahead market in where GenCos compete for the next day supply of an inelastic load demand. In the day-ahead market's auction, each GenCo bids the minimum acceptable unit price of electricity for itself. Based on the predetermined market-clearing mechanism including a pricing rule, the ISO specifies the unit price of electricity (market clearance price) and each GenCo's assigned power. Our particular focus in this chapter is the effect of various factors such as pricing rules on the strategic bidding behavior of GenCos.

First, we analyze the effect of different pricing rules using an agent-based simulation model. We simulate a repetitive auction process where GenCos employ reinforcement learning to learn from the history of their own actions to bid a more profitable price. We compare the outcomes of the simulation model with the Nash equilibrium of the single-stage game between the GenCos. Then, we extend the bid to price and available capacity. Finally, we consider risk sensitivity factor in bidding process of GenCos when ISO considers transmission lines' constraints. By analyzing the results of our simulation experiments, we aim to answer the following questions:

- Market-cleaning mechanisms:

- How does different market-clearing mechanisms affect the strategic bidding behavior of GenCos in the presence of learning?
  - How should GenCos exploit available information (that is, realized profit at each iteration) to increase earnings in the future?
  - Which pricing rule is more competition-friendly?

- Capacity withholding:
  - Is there any benefits for GenCos to not offer their full generation capacity to the ISO?
  - As the GenCos repetitively bidding, can they find and maintain a strategy with capacity withholding?
  - What can the ISO do to hinder GenCos to keep their generation capacity?

- Risk-sensitivity:
  - How can risk affect GenCos behavior and payoffs?
  - How can a GenCo compete with others when GenCos do not share any information?
  - In sharp competition with other powerful GenCos, how should a GenCo behave?

3.1 Related Work

The related literature of current chapter can be divided in to three categories: learning and game-theory, applications of agent-based simulation, and analysis of pricing rules.

3.1.1 Learning and Game-theory

Due to repetitive nature of auctions in the electricity markets, GenCos are expected to learn by gathering new information in each repetition of the auction and improve their performance over time. In this respect, analyzing GenCos' behavior without a learning mechanism would lead to inaccurate results. Even in the early years of game-theory, researchers have been interested in learning models. Exhibiting convergence to Nash equilibrium in the presence of learning has attracted a lot of attention from the game-theory modelers as well as energy-economics community.

Aumann [3] claims that Nash equilibrium concept is one of the most applied concepts in economics; yet, it is not crystal clear under what condition players might be expected to play a Nash equilibrium. Mailath [71] discusses various justifications that have been advanced for equilibrium analysis and points out learning as the least problematic justification. Also, Mailath notes that convergence to Nash equilibria is a necessary condition in the evolutionary dynamics for any reasonable model of social learning when the number of players is large enough. Kalai and Lehrer [56] show that under some simplifying assumptions, rational learning leads to Nash equilibrium.

Hart and Mas-Colell [47] propose "reinforcement" models in which all players can lead to an equilibrium of the stage game. Their learning procedure, unlike the "regret-matching" procedure [48], does not need to observe all past payoffs, and players do not need to know their own payoff function.

Wang and Sandholm [109] state that even agents with non-conflicting interests may not be able to learn an optimal coordination policy in the presence of multiple Nash equilibria. As a solution, these authors propose a new learning mechanism based on reinforcement learning that converges to an optimal Nash equilibrium with probability one in any team Markov game.

3.1.2 Agent-based Simulation of Electricity Markets

Although analytical models can be employed to study learning mechanisms, the expected outcomes of these models are not necessarily observed in practice due to strict simplifying assumptions [26]. A widely accepted alternative tool is Agent-based Modeling and Simulation; it can provide better understanding of real-life markets especially when analytical models show poor tractability in investigating complicated problems. Li and Shi [66] claim that agent-based modeling and simulation is a viable approach which provide realistic insights for the complex interactions among various market players.

In the previous studies with agent-based approaches Reinforcement Learning has been adopted to simulate the intelligence of the players in the electricity markets ([108], [62], [66]) as it is expected that agents should be agile and be able to respond to new opportunities and threats. Reinforcement Learning includes the temporal difference algorithm proposed by Sutton [97] and the Q-learning of Watkins ([110], [111]). These algorithms have been extensively used in many other applications such as industrial control, time sequence prediction [52], and robot soccer competition [90]. Erev and Roth [32] have shown that models based on reinforcement learning outperform the equilibrium predictions in certain games.

Existence of multiple Nash equilibria can disrupt GenCos' learning process in such a way that the long-run equilibrium is not necessarily achieved. Krause et al. [62] study a day-ahead market where GenCos learn by reinforcement learning. It turns out that these authors' simulation does not converge in the existence of multiple Nash equilibria. The GenCos' strategies pendulate between those Nash equilibria. The oscillation between different Nash equilibria in the reinforcement learning process can be overcome by making better use of collected information. To this end, Wang [108] used the SA-Q-learning algorithm with Metropolis criterion.

Naghibi-Sistani et al. [77] apply Q-learning for agents' bidding in a pool-based power market with uniform pricing. They show that a participant with reinforcement learning capability could ultimately learn the optimal policy and could adapt himself to unknown parameters in the environment. The authors also find that under reinforcement learning, bids can converge and stay in the Nash equilibrium for a two-participant case. Nevertheless, these authors have not studied pricing rules other than uniform pricing and their impact on convergence.

3.1.3 Pricing Rules

Selecting a pricing rule is a vital decision for the ISO as it is likely to affect GenCos' strategic bidding behavior. By using agent-based simulation, Yu et al. [115] show that flaw in the regulations can be captured and exploited by GenCos even without having to know others' historical bidding data. Thus, researchers investigate the characteristics of pricing rules to improve the functionality of underlying markets.

Uniform pricing is believed to be a rational choice for the ISO since it provides sufficient incentives to bidding GenCos to reveal their true cost [79] while it is also accused for the high price volatility [74]. On the other hand, pay-as-bid pricing results in a flat supply function which is a remedy for price volatility. However, pay-as-bid pricing also assists powerful players that have information about market-clearing price in obtaining higher profits [113].

Xiong et al. [114] compare uniform and pay-as-bid pricing rules using agent-based simulation and show that pay-as-bid results in lower market prices and price volatility. They also claim that demand side response has less effect on market prices with pay-as-bid policy. Bakirtzis and Tellidou [7] show that high price levels are due to exercised market power with both uniform and pay-as-bid policies. Azadeh et al. [4] study three different pricing rules (uniform, pay-as-bid, and Vickrey) by using Principal Component Analysis (PCA). They conclude pay-as-bid pricing rule with one permissible step to be the best pricing rule.

Sugianto and Liao [95] use agent-based modeling approach to investigate the impact of different auction pricing rules on the market performance. They conclude that the pay-as-bid pricing rule can complicate the way bidders learn and react to each other's strategy. Also, their results suggest that Vickrey pricing provides a balance between managing the total cost and its stability in the presence of unequal GenCo market shares.

In addition to the pricing rule, we also discuss another aspect of the market-clearing mechanism, the "rationing policy". The rationing policy determines the allocation of the remaining demand at the market cleaning price when multiple GenCos' (the marginal GenCos) bids coincide at that price. This possibility arises due to the discrete nature of bid price and quantity. Rationing rule is especially important when the bid prices are likely to accumulate at certain values. Holmberg [50] and the references therein discuss rationing rules to break ties between multiple bids at the market clearing price in general multi-unit auctions. In auctions where all bids are cleared simultaneously, standard practice is pro-rata rationing where the same percentage of bid is accepted for each marginal bidder.

In continuous trading, priority can be given to marginal bids that arrive early. Madlener and Kaufmann [70] describe the rationing rules employed in European power exchanges. For instance, in case of a supply surplus at the market clearing price, APX and OMEL exchanges distribute the demanded quantity in proportion to the bid quantities. Borzen, EEX and EXAA exchanges, on the other hand, prioritize according to the size of bid or time of submission. Different from these, we propose a rationing policy where the priority ordering of marginal GenCos is randomly determined (random rationing), and another policy where the remaining demand is equally distributed to marginal GenCos (equal rationing). We are not aware of any other work that models rationing policies in electricity markets.

3.2 Simulation Process

In our agent based simulation model, agents represent the GenCos that are expected to satisfy demand on the transmission grid. GenCos submit bids sequentially for each hour of the next 24 hours to the ISO. The bidding process is synchronic for all GenCos, and each iteration in the simulation corresponds to the auction of an hour in the day-ahead-market. The simulation is run for a finite number of iterations (max_t). At the end of each iteration/bid, each GenCo-i calculates its payoff r_i.

In the simulation model, we assume that

- The demand is inelastic and constant, i.e., it does not change from one hour to the next.
- GenCos participate only in the day-ahead market (but not in the futures or real-time markets).
- No line or generation outage is experienced.
- GenCos do not change their technology that would alter P_i^{max} and c_i.
- Capacity withholding is not allowed; each GenCo offers its maximum capacity (we will relax this assumption later).

- GenCos do not share information with each other; they are not aware of others' production costs, available bids and submitted bids.

In essence, the bidding process of GenCos is a decision-making problem with incomplete information as each GenCo is only aware of its own cost and bids. Each GenCo determines what to bid through a Q-learning mechanism (to be explained) based on historical payoff information from its own bids in the previous iterations. Thus, the profit at an iteration affects the GenCo's subsequent bid decisions.

We model GenCos' learning mechanism by reinforcement learning. We improve the standard Q-learning mechanism by making the two following parameters time-dependent:

- Recency rate $(\alpha_{it} \in [0,1])$ determines the weight given by GenCo-i to the most recent observed outcome (profit).

- Exploration parameter $(\epsilon_{it} \in [0,1])$ measures the tendency of GenCo-i at iteration t to explore, i.e., to use a randomly selected bid rather than using its best identified bid.

Recall that GenCo-i has a set of bids (B_i) to choose from. For each bid, the Q-value in the learning algorithm denoted by Q_{ij} corresponds to the average realized profit of GenCo-i when jth bid from B_i is used in the previous iterations. Initially, all Q-values are zero. At the end of each iteration t, based on the observed payoff r_i, the Q-value of the submitted bid is updated as follows

$$Q_{ij} = (1 - \alpha_{it})Q_{ij} + \alpha_{it}(r_i). \tag{3.1}$$

A high α-value represents a GenCo that is primarily concerned about the most recent outcomes it experienced, and is less affected by the earlier ones. In our modified Q-learning algorithm, α_{it} starts from a high value (α_{i0}) at the beginning and diminishes linearly over iterations to a lower value of $(\frac{\alpha_{i0}}{10})$. In this respect, we use a linear decreasing function of time for α_{it} as

$$\alpha_{it} = \left(1 - \frac{t}{max_t}\right)(\alpha_{i0}) + \left(\frac{t}{max_t}\right)\left(\frac{\alpha_{i0}}{10}\right). \tag{3.2}$$

We use a descending recency rate because GenCo is assumed to become less sensitive to individual recent observations over time due to gained experience.

We refer to the bid price that maximizes the Q_{ij} value as b_i^* corresponding to GenCo-i's "best identified bid". In the proposed Q-learning algorithm, at iteration t, GenCo-i either selects b_i^* (with prob $1 - \epsilon_{it}$) or explores by choosing a random bid from B_i (with with probability ϵ_{it}). Therefore, with lower ϵ, the GenCo explores less and sticks to its best identified bid more often. We assume that GenCos explore more in initial iterations using random bids, but they are more likely to use their best identified bid in latter iterations. To represent such behavior, the exploration parameter (ϵ_{it}) decreases linearly from the base value of ϵ_{i0} (when $t = 0$) to almost zero at iteration number $\lceil \frac{max_t \epsilon_{i0}}{8(1-\epsilon_{i0})} \rceil$. Afterwards, exploration is minimum and exploitation is maximum.

Our Q-learning algorithm is presented in Algorithm 2. In line 2, Q-values are set to zero for initialization. In lines (7 - 11), GenCo determines its price bid to the ISO. This decision is governed by the Q-learning parameters. Following the market clearance by the ISO in line 12, GenCo-i will update the Q-value of the selected bid in line 13.

Algorithm 2 The simulation model with the proposed Q-learning algorithm for each GenCo-i.

1: $t \leftarrow 1$

2: $Q_{ij} \leftarrow 0 \quad \forall j$

3: **repeat**

4:     R $\leftarrow$ Random Number $\in [0, 1]$

5:     $\epsilon_{it} \leftarrow max\{0.001, 1 - (1 - \epsilon_{i0})\left(1 + \frac{8t}{max_t}\right)\}$

6:     $\alpha_{it} = (1 - \frac{t}{max_t})(\alpha_{i0}) + (\alpha_{i0}/10)(\frac{t}{max_t})$

7:     **if** $R > \epsilon_{it}$ **then**

8:         $b_i \leftarrow$ Select Best bid (b_i^*)

9:     **else**

10:         $b_i \leftarrow$ Select a bid randomly from B_i

11:     **end if**

12:     $r_i \leftarrow$ CLEARMARKET($\{b_i : \forall i\}$)

13:     $Q_{ij} \leftarrow (1 - \alpha_{it})Q_{ij} + \alpha_{it}(r_i)$

14:     $t \leftarrow t + 1$

15: **until** $(t < max_t)$

The increasing exploitation of the best identified bid cancels out the effect of GenCos' random choices at early iterations. Therefore, the Q-value of the best identified bid b_i^* for each GenCo-i converges to the profit (r_i) of GenCo-i at equilibrium state in the long-run.

3.3 Nash Equilibrium

We assume that GenCos start bidding with no information about the potential profit of each bid in their set of bids, and the possible bids of other GenCos. Throughout the iterations GenCos collect information on the outcomes of their own bids but not those of competitors. With the simulation analysis, we expect to understand if the market reaches a Nash equilibrium. In this respect, each iteration of the simulation corresponds to a stage of the multi-stage game.

The market clearance process for an hour in the day-ahead market can be modeled as a non-cooperative single-stage game G with finite number of players, $\mathcal{F} = \{$GenCo-1, ..., GenCo-$n\}$, an action space of $\mathfrak{B} = (B_1 \times \ldots \times B_n)$ and a vector of payoffs $r = (r_1, \ldots, r_n)$. Thus, the normal-form representation of G is denoted by the triplet $(\mathcal{F}, \mathfrak{B}, r)$. In each iteration, collection of submitted bids $(b_1 \in B_1, \ldots, b_n \in B_n)$ defines the "state" of the game.

When random rationing policy is used (under both uniform and pay-as-bid pricing), the same set of bids from GenCos can lead to different power dispatches, leading to different profit vectors. This is because of the ISO's random choice among the GenCos that submit the same bid at the market-clearing price. Therefore, in random rationing policy, we calculate average payoff of each GenCo with respect to probability of each realized profit for a given state. The vector of average payoffs will be used to identify Nash equilibria.

In our context, a bidding strategy $N = (b_1^N, \ldots, b_n^N)$ is called a Nash equilibrium if any GenCo-i cannot make a better payoff than the payoff of the Nash equilibrium (r_i^N) by choosing another bid as long as the other GenCos are not changing their bids, i.e.

$$(r_1^N, \ldots, r_i^N, \ldots, r_n^N) \geq (r_1^N, \ldots, r_i, \ldots, r_n^N), \qquad i \in \{1, \ldots, n\}. \tag{3.3}$$

We also define $S = (b_1^S, \ldots, b_n^S)$ as a *semi-Nash* state if

$$(r_1^S, \ldots, r_n^S) = (r_1^N, \ldots, r_n^N) \qquad\qquad \exists j \text{ such that } b_j^S \neq b_j^N.$$

A semi-Nash state is defined with respect to a particular Nash equilibrium. If one of the GenCos (say, GenCo-i) in a Nash equilibrium can change its bid without affecting the payoff of any GenCo including itself, we refer to the resulting state as a semi-Nash. A semi-Nash state is not a Nash equilibrium because at least one of the GenCos (other than GenCo-i) can increase its payoff by deviating from this state. During our experimental simulations, however, such a GenCo may or may not realize this profitable deviation opportunity. Hence, the simulation may end up converging to a semi-Nash state just as it may converge to a Nash equilibrium.

3.4 Computational Experiments

We conduct simulation experiments on four case studies. In each case study, GenCos are subject to challenges due to a variety of environmental settings. The underlying characteristics of the case studies can be summarized as follows:

- Case 1: A public GenCo and a private GenCo compete in a limited competition market where the public GenCo always bids its generation cost.
- Case 2: Two private GenCos and a public GenCo participate in a competitive market where only private GenCos are learning agents.
- Case 3: The public GenCo in Case 2 is replaced with a GenCo that can learn.
- Case 4: Three active GenCos compete to satisfy demand of a single node. As the demand can be satisfied to a great extent by any one of three GenCos, competition between GenCos is tight.

The details of the case studies are presented in Appendix A. Table 3.1 reports the number of Nash equilibria and semi-Nash states under each pricing rule.

Table 3.1: Number of Nash equilibria and semi-Nash states in case studies

Case Study	Active GenCos	Uniform Pricing		Pay-as-bid Pricing		DC-OPF Pricing	
		Nash	semi-Nash	Nash	semi-Nash	Nash	semi-Nash
Case 1	1	1	0	1	0	1	0
Case 2	2	3	1	3	1	1	0
Case 3	3	6	4	3	2	2	3
Case 4	3	3	10	3	7	3	15

In what follows, we first use case 1 to illustrate how our Q-learning algorithm operates. Next, using cases 2, 3 and 4 we study whether our simulations converge to theoretical Nash equilibria and/or semi-Nash states. Finally, we discuss which pricing rule is more competition-friendly.

To describe the behavior of the only learning agent (private GenCo) in Case 1, we limit the public GenCo to offer one price; therefore, the described Q-learning algorithm does not apply to this GenCo.

Figure 3.1 shows the evolution of expected profits (Q-values) for the private GenCo for each bid option throughout the iterations. Each bid option is shown with a different line style. The results clearly indicate one outcome: The private GenCo gradually learns to bid higher prices to the ISO as it discovers along iterations that the public company cannot fulfill the demand. Eventually, the private GenCo reaches the maximum Q-value of 2700 by bidding 40. In fact, this bid, along with the fixed bid of the public GenCo correspond to the unique Nash equilibrium of the stage game. The bid also happens to be the optimal one for the private GenCo. We observed this result to hold true independent of the pricing rule and rationing policy in Case 1.

3.4.1 Convergence to Nash Equilibria and Semi-Nash States

We first intend to compare the state converged at the end of simulation experiments against game-theoretic expectations. Each simulation run is terminated after 2000 iterations ($max_t = 2000$), and it is replicated for 30 times. The initial values of the Q-learning parameters (α_{i0} and ϵ_{i0}) are same for all GenCos as we suppose that they are subject to

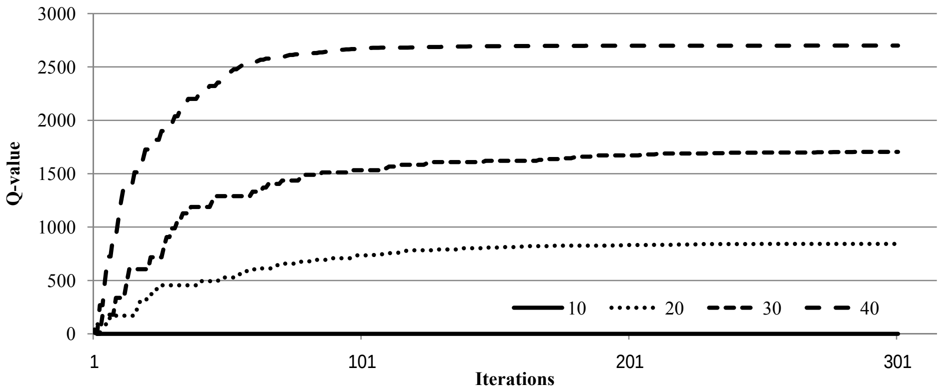


Figure 3.1: Evolution of Q-values of the private GenCo over iterations

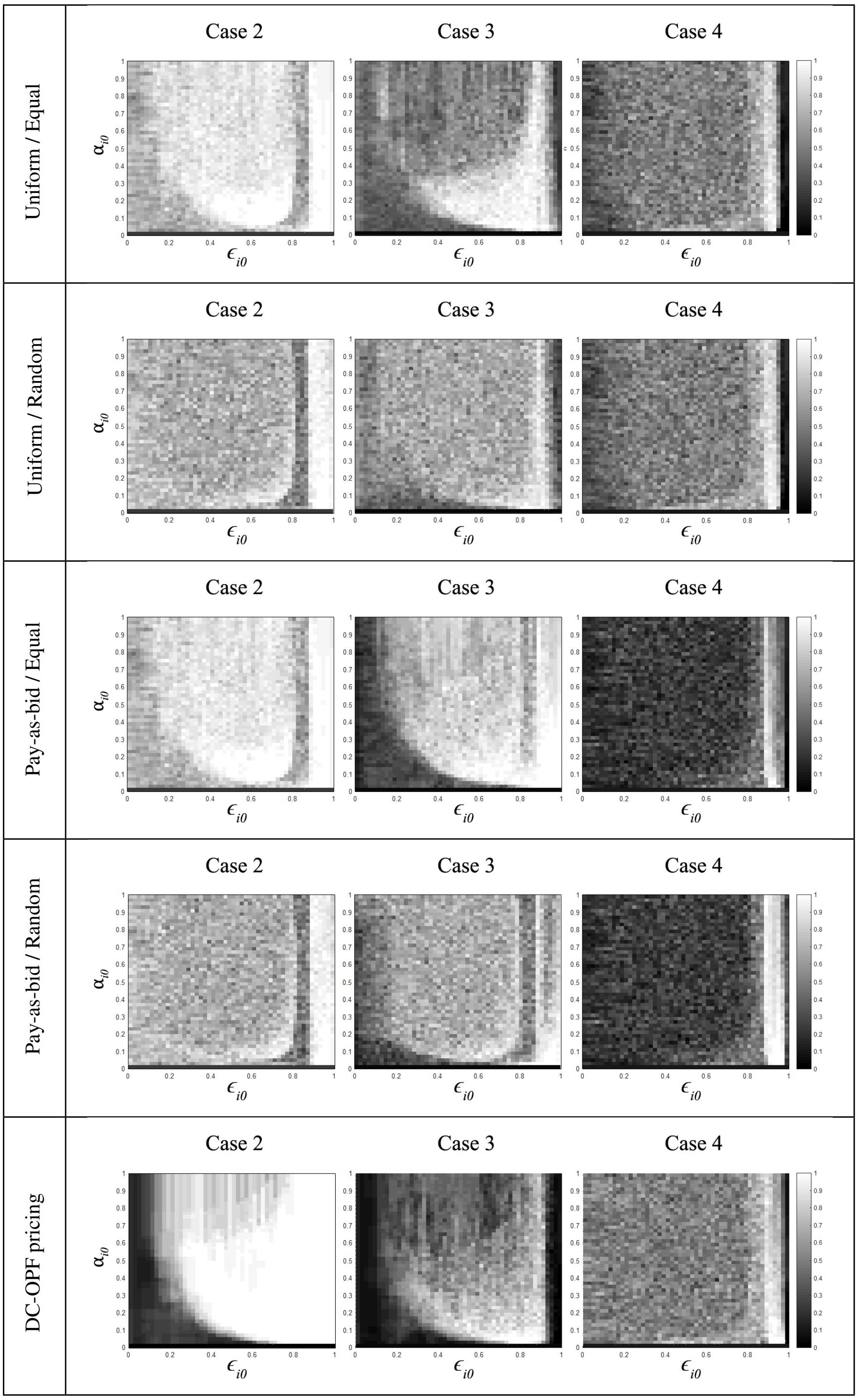
similar learning mechanisms.

We are interested in whether the simulation will converge to a Nash equilibrium, a semi-Nash state or neither. We define “convergence frequency to a state” as the fraction of time the state is reached at the end of the simulation over 30 replications. We examine the convergence frequency under different values of α_{i0} and ϵ_{i0} ranging from 0 to 1.00 with increments of 0.02 for each (corresponding to 2601 parameter combinations, or settings). Table 3.2 displays convergence frequency to Nash equilibria under different pricing rules and rationing policies for each case in our experimental study. The lighter a point on charts, the higher the Nash convergence frequency of the corresponding setting.

Overall, we observe that the frequency of convergence to Nash equilibria decreases as the complexity of the case increases (from Case 2 to Case 4). This is because the number of states is substantially higher in the more complex cases. In all case studies, GenCos converge to Nash equilibria more frequently when $\epsilon_{i0} \in [0.7, 0.9]$, and they fail to converge when $\epsilon_{i0} < 0.1$; convergence frequencies tend to decline as one moves left in any of the charts. In fact, in Appendix B, we show very low exploration to disrupt learning. Furthermore, we observe a high-convergence zone around $\epsilon_{i0} \approx 0.9$ for any α_{i0}; this is due to the shape of the linear decay function of ϵ_{it}.

An important difference between the equal and random rationing policies is seen in the figures: Under equal rationing, the regions of high and low convergence are clearly separated from each other whereas we do not observe such separation under random ra-

Table 3.2: Convergence frequency to Nash equilibria



tioning. Recall from the definition of rationing policies that the random rationing policy of the ISO introduces another stochastic event to the decision-making process of GenCos when the ISO chooses the winner arbitrarily among GenCos with the same bid. Imposing more randomness to the system disrupts the learning process of GenCos, by blurring the link between successful bids and high profits. Thus, the ISO can use a random rationing policy in cases where it needs to interfere with GenCos' learning. This can be the case, for instance, when collusion opportunities [33] are present for GenCos.

The figures also allow comparing the effectiveness of changing the pricing rule and changing the rationing policy in convergence to Nash equilibria. We observe the result of the comparison to be case-dependent: While the rationing policy change is more influential in Case 2, pricing rule is more important in Case 4. In Case 3, on the other hand, both factors seem to be influential.

Under DC-OPF pricing, settings with a high convergence frequency create a distinctive wedge shape extending from $\alpha_{i0} \approx 0.04$ and $\epsilon_{i0} \approx 0.9$ to $\alpha_{i0} \approx 1$ and $\epsilon_{i0} \approx 0.9$ while it is curved around $\alpha_{i0} \approx 0.1$ and $\epsilon_{i0} \approx 0.4$. This observation suggests that GenCos with low tendency to explore while giving sufficient importance to the last observed outcome are more likely to converge to Nash equilibria especially in Case 2 and Case 3.

Table 3.3 summarizes the observations from Table 3.2 by presenting the average (over different parameter settings) observed convergence frequencies to Nash equilibrium under different pricing rules and rationing policies. Convergence frequency is high for the simple case (Case 1) and relatively low for the more complex ones (say, Case 4). Uniform pricing causes higher convergence frequency to Nash equilibrium than pay-as-bid for most cases (except Case 3, under equal rationing). For Case 1, we observe no impact of either the pricing rule or the rationing policy on the behavior of the private GenCo.

Differences are clearer when one also includes convergence to semi-Nash states. Table 3.4 presents the convergence frequency to semi-Nash states and to either Nash or semi-Nash states (in parentheses). That latter frequency is observed to be higher under uniform than under pay-as-bid pricing rule and higher under random than under equal rationing policy. Overall, we shall claim that simulations with learning agents do converge to Nash equilibria, or a state that has identical payoffs with a particular Nash equilibrium (a semi-

Table 3.3: Average convergence frequency to Nash equilibria

Case Study	Uniform Equal	Uniform Random	Pay-as-bid Equal	Pay-as-bid Random	DC-OPF
Case1	0.8242	0.8242	0.8242	0.8242	0.8242
Case2	0.8396	0.7096	0.8396	0.7049	0.7865
Case3	0.5776	0.6230	0.6673	0.6133	0.4443
Case4	0.4848	0.4891	0.2906	0.3269	0.5246

Nash state) for the majority of instances.

Table 3.4: Convergence frequency to semi-Nash* states

Case Study	Uniform Equal rat.	Uniform Random rat.	Pay-as-bid Equal rat.	Pay-as-bid Random rat.	DC-OPF
Case 2	0.1081 (0.9477)	0.2683 (0.9779)	0.1081 (0.9477)	0.2722 (0.9771)	N/A (0.7865)
Case 3	0.1864 (0.7640)	0.2748 (0.8978)	0.0845 (0.7518)	0.2517 (0.8650)	0.2253 (0.6696)
Case 4	0.4369 (0.9217)	0.4367 (0.9258)	0.3314 (0.6220)	0.3430 (0.6699)	0.4418 (0.9664)

* Values inside parentheses indicate the summation of convergence frequencies to Nash equilibria and semi-Nash states.

3.4.2 Competition Analysis

We are interested in the relation between the bids and the cost of GenCo-i. As it is associated with the level of competition in the market. In a highly competitive market, GenCos are likely to cut their bids, leading to a smaller difference between the offered bid and the generation cost [79]. In general, uniform pricing is known to provide more incentive to bidders (GenCos in our case) to bid a closer price to their cost than pay-as-bid pricing does.

To study this relation, we define $\Delta_i^k = b_i^{(k)*} - C_i$ for GenCo-i in replication k. For a given parameter setting (ϵ_{i0} and α_{i0}), we calculate the average Δ_i^k over all N GenCos and 30 replications as $\Delta = \frac{\sum_k \sum_i \Delta_i^k}{30 \times N}$.

Figure 3.2 presents the difference in Δ between the two pricing rules and the two rationing policies for Case 3 (as an example). For instance, Figure 3.2(A) shows the difference in Δ between equal and random rationing, under uniform pricing. All indicated

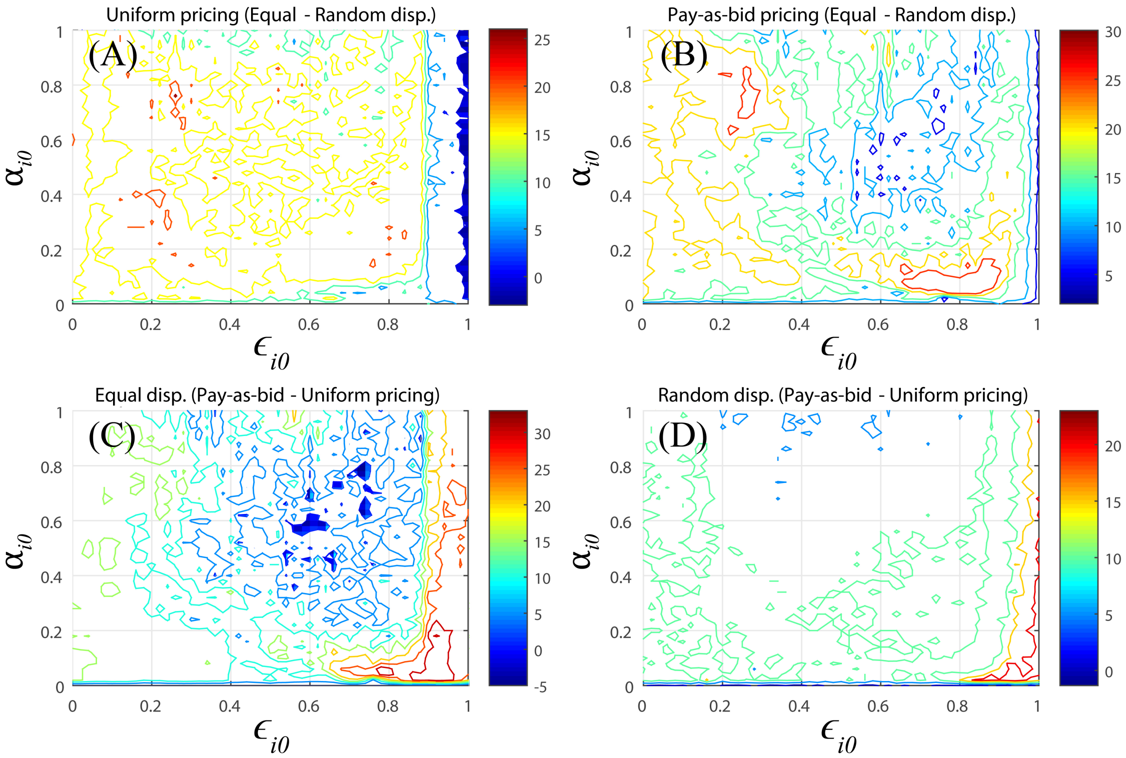


Figure 3.2: Difference in Δ between the two pricing rules and the two rationing policies for Case 3

difference values are found to be positive with the only exceptions marked with darker shades. These exceptions, where the Δ difference is negative, are found in the rightmost side of Figure 3.2(A), and in a few islands in the middle of Figure 3.2(C). Thus, uniform pricing is found to be more successful in making GenCos bid closer bids to their costs for almost all parameter settings. Likewise, equal rationing is observed to be more successful than random rationing. These observations are summarized in Table 3.5 which provides the average Δ values ($\overline{\Delta}$) over all parameter combinations. Uniform pricing and random rationing together lead to the lowest $\overline{\Delta}$ values. Hence, an ISO can use this combination to stimulate higher competition among GenCos in the market.

Table 3.5: $\overline{\Delta}$ comparison

Case Study	Uniform Equal rat.	Pay-as-bid Equal rat.	Uniform Random rat.	Pay-as-bid Random rat.	DC-OPF
Case 1	**27.510**	**27.510**	**27.510**	**27.510**	**27.510**
Case 2	21.244	21.244	**10.896**	10.915	20.592
Case 3	26.247	37.176	**12.781**	22.107	40.680
Case 4	19.209	27.696	**19.110**	26.969	19.753

3.4.3 Profit Analysis

We observe uniform pricing to achieve lower bid prices than pay-as-bid. However, this difference does not necessarily lead to lower GenCo profits under uniform pricing because of the difference in payment mechanisms. To acknowledge the difference, we compare GenCos' total profits under different market clearing mechanisms. This analysis is important because higher GenCos profits indicates that the market fails to provide affordable electricity for consumers. For a given parameter setting $(\alpha_{i0}, \epsilon_{i0})$, we first calculate the average profit of each GenCo-i over 2000 iterations and 30 replications as $\overline{r_i} = \frac{\sum_{k=1}^{30} \sum_{t=1}^{max_t} r_{it}^k}{30 \times max_t}$. Next, we calculate GenCo-i's average profit over all parameter settings. These are reported for all GenCos over Case 2, Case 3, and Case 4 in Table 3.6, Table 3.7, and Table 3.8, respectively. We observe clearly that switching from uniform to pay-as-bid pricing rule decreases GenCos' total profits, regardless of the rationing policy. For Case 2, the effect is more tangible because GenCo-2 has only one bid price which is equal to its generation cost.

Table 3.6: GenCos' profits under different market-clearing mechanisms

	Case Study 2				
	Uniform Equal rat.	Uniform Random rat.	Pay-as-bid Equal rat.	Pay-as-bid Random rat.	DC-OPF
GenCo-1	1944.07	1966.18	1944.07	1965.28	1007.61
GenCo-2	4647.61	3542.70	0.00	0.00	1060.85
GenCo-5	571.73	172.90	571.73	172.04	2264.00
Total	7163.40	5681.78	2515.79	2137.32	4332.47

Table 3.7: GenCos' profits under different market-clearing mechanisms

	Case Study 3				
	Uniform Equal rat.	Uniform Random rat.	Pay-as-bid Equal rat.	Pay-as-bid Random rat.	DC-OPF
GenCo-1	3393.48	2822.74	2709.80	2504.06	5085.20
GenCo-2	3434.35	2829.77	2756.82	2517.76	1844.68
GenCo-5	755.65	275.91	893.53	332.77	2233.11
Total	7583.48	5928.43	6360.15	5354.60	9162.99

Table 3.8: GenCos' profits under different market-clearing mechanisms

	Case Study 4				
	Uniform Equal rat.	Uniform Random rat.	Pay-as-bid Equal rat.	Pay-as-bid Random rat.	DC-OPF
GenCo-2	5273.27	5318.39	5505.16	5422.45	4962.80
GenCo-3	14783.93	14796.97	12757.13	12454.59	15900.05
GenCo-4	439.73	421.12	336.70	335.89	375.96
Total	20496.93	20536.48	18598.98	18212.93	21238.81

We shall also investigate the profit difference between the market-clearance mechanisms statistically using 2601 parameter settings $(\alpha_{i0}, \epsilon_{i0})$ as the samples (see Figure 3.3). For each case study and each rationing policy, we test whether the difference of the median profits between uniform pricing and pay-as-bid pricing is zero or positive while the difference is calculated by subtracting the profit under pay-as-bid pricing from that under uniform pricing.

Figure 3.4 shows the histograms for the three case studies. Kolmogorov-Smirnov normality test (Figure 3.5) confirms that none of them are normally distributed; hence, we use a nonparametric Sample Sign test that doesn't assume normality or symmetric data.

We observe almost all differences to be positive, indicating higher GenCo profits under uniform pricing for almost all parameter settings. In fact, the median profit difference is found to be statistically higher than zero (*Nonparametric Sign Test* with $p-$values around 0.0000). In addition, in Case 2 and Case 3, the profit difference between the pricing rules under equal rationing is observed to be more pronounced from the one under random

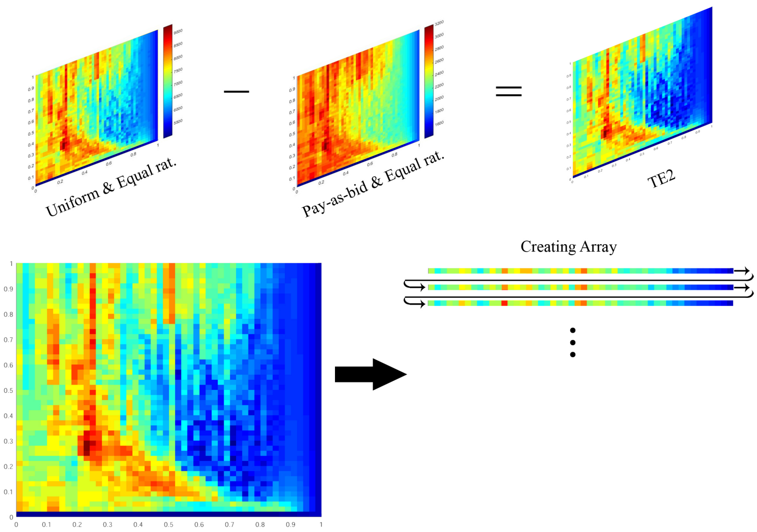


Figure 3.3: The resulting profit matrices are reshaped to vectors

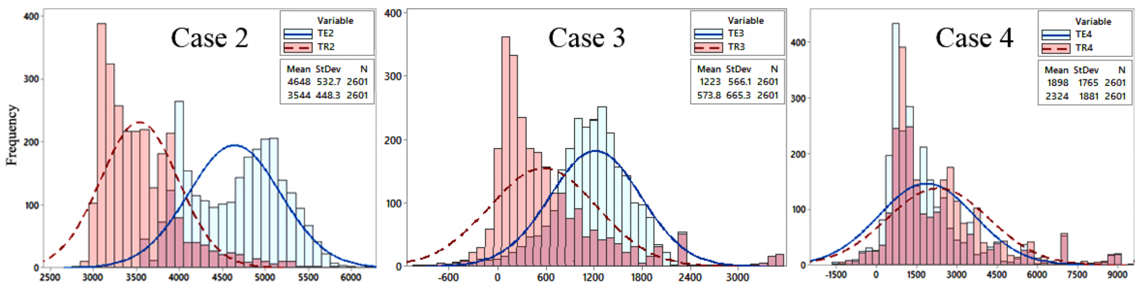


Figure 3.4: Histogram plot of profit differences between uniform and pay-as-bid pricing where TXY denotes the difference vector under rationing policy X (E represents Equal and R represents Random) in Case Y

rationing. Finally we observe the profit distribution among GenCos to be quite different under the DC-OPF rule compared to those under the other clearance mechanisms. This result highlights the role of locational marginal pricing in DC-OPF, which takes the transmission grid structure and constraints into accounts in determining local electricity prices.

Figure 3.6 demonstrates GenCos' total profit under each pricing rule and rationing policy for each initial setting. This detailed figure confirms that GenCos learning param-

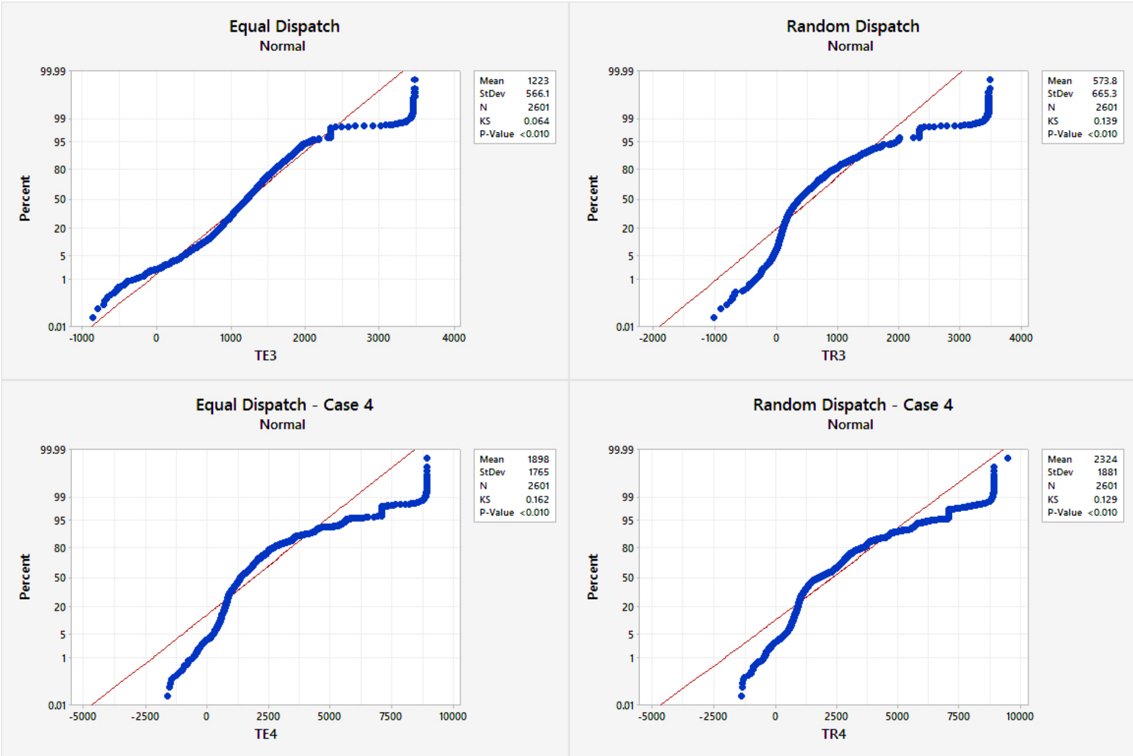


Figure 3.5: Kolmogorov-Smirnov normality test for Case 3 (up) and Case 4 (down)

eters are influential to GenCos' behaviors and achieved payoffs. Also, by focusing on Nash and semi-Nash states in Figure 3.7, we observe that most of Nash equilibria have low total profit.

However, the converged state, apart from being Nash state or not, is not the only influential factor on total profit; for instance, one may notice that convergence to Nash or semi-Nash results in a different average total profit over 2000 iterations with respect to initial settings $(\alpha_{i0}, \epsilon_{i0})$, especially in Case 2 under DC-OPF pricing rule. There is only one Nash equilibrium, and no any semi-Nash state; however, total average profit is higher when $\epsilon_{i0} \geq 0.9$.

For this peculiar case, GenCo-2 benefits from high exploration rate of other GenCos, and therefore, GenCo-2 can keep the total profit high. Meanwhile, GenCos with market power cannot obtain a decent profit because of the same reason. Now, if we decrease

Table 3.9: Mean test results for 2601 different values for initial settings

	Sample size	Below	Equal	Above	P-value	Median
TE_3	2601	53	0	2548	0.0000	1206
TR_3	2601	189	1	2411	0.0000	371.9
TE_4	2601	93	0	2508	0.0000	1341
TR_4	2601	76	0	2525	0.0000	1766

ϵ_{i0} to a value around 0.6, GenCo-2 loses this opportunity and ends up with no profit that decreases the total profit. But, other GenCos will obtain better expected profits. The difference between high and low exploration rates has been depicted in Figure 3.8. In Figure 3.8, the vertical axes is the expected payoff and horizontal axes shows iterations. Bids evolve during time and the expected payoff corresponding to each bid is drawn with different color.

3.5 Effect of Capacity Withholding

One of the general assumptions in the simulation model speculate that GenCos bid for their entire production capacity. However, it is possible to modify the simulated bidding process in such a way GenCos may withhold their production capacities to some extent. We presume zero salvage value for the idle capacity; there is no future market or real-time market that GenCos can participate with the remaining capacity.

We aim to examine whether GenCos would rather lose their selling opportunity than participating in the market with full capacity. If some GenCos withhold their capacity, prices may increase which may harm consumers. Capacity withholding also may reduce the revenue of the GenCo if the price does not increase significantly. Therefore, we aim to answer the following questions:

1. Are there any benefits for GenCos to not offer their full generation capacity?
2. As the GenCos repetitively bidding, can they find and maintain a strategy with capacity withholding?

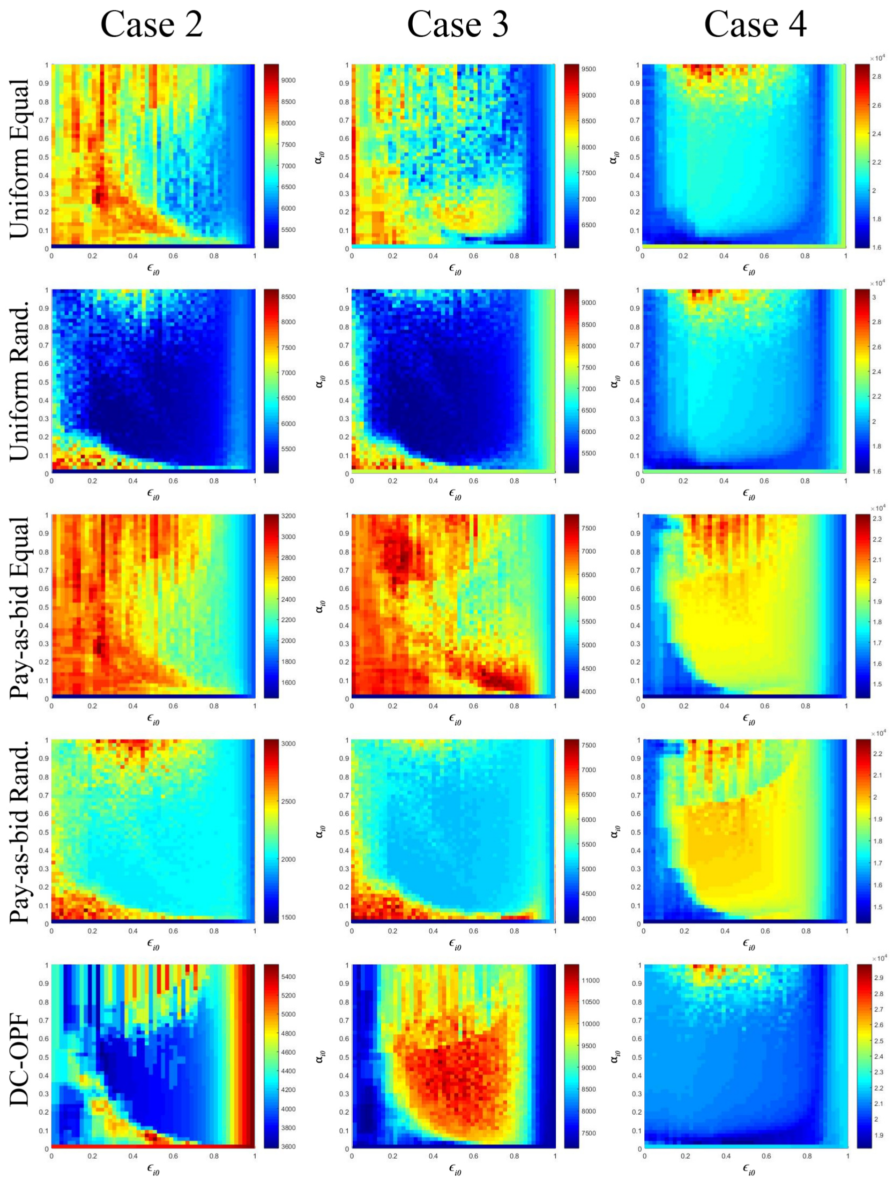


Figure 3.6: Total profit profile under each setting of Case 2, Case 3, and Case 4

3. What can the ISO do to hinder GenCos to keep their generation capacity?

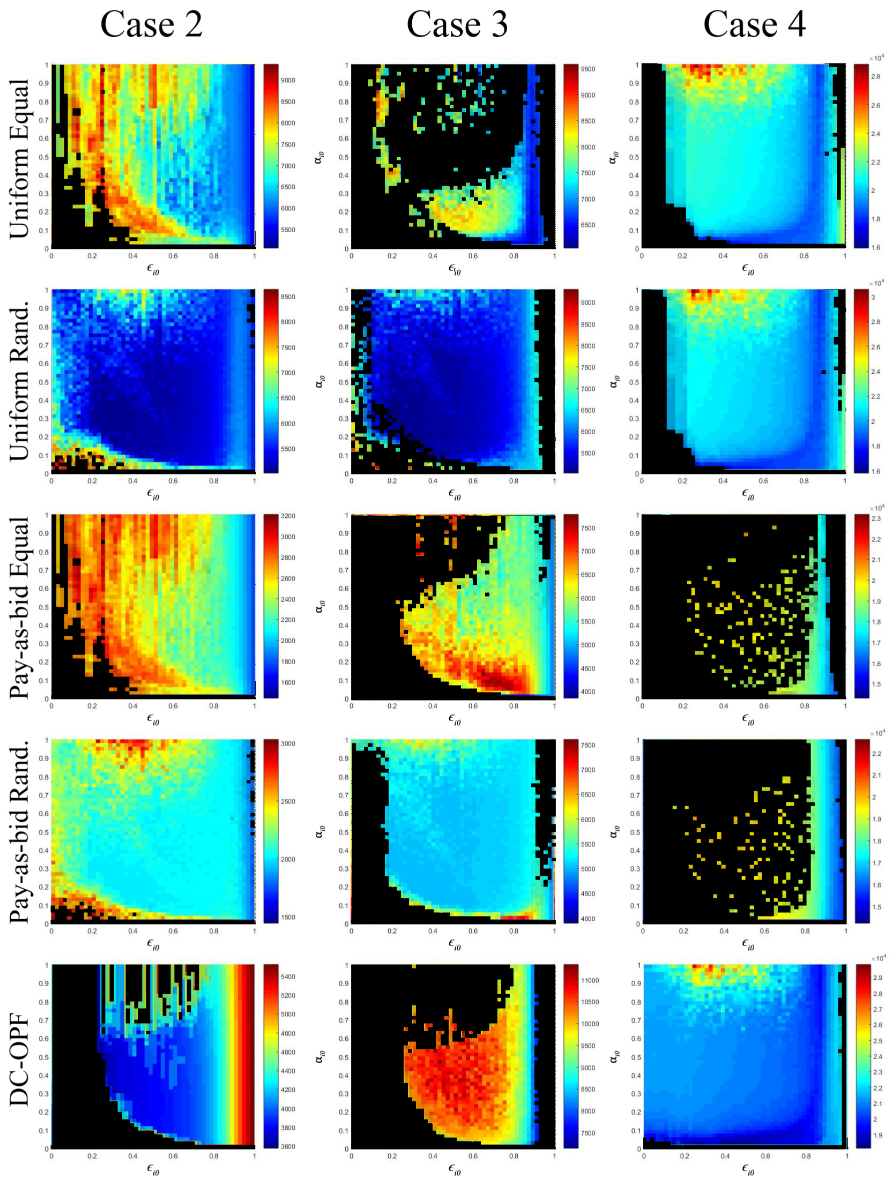


Figure 3.7: Covering settings that converge to a non-Nash or non-semi-Nash states

3.5.1 Simulation's Configuration

In order to keep the simulation setting manageable while relaxing the full capacity bid assumption, we allow each GenCo to bid 80% of generation capacity as an alternative. As

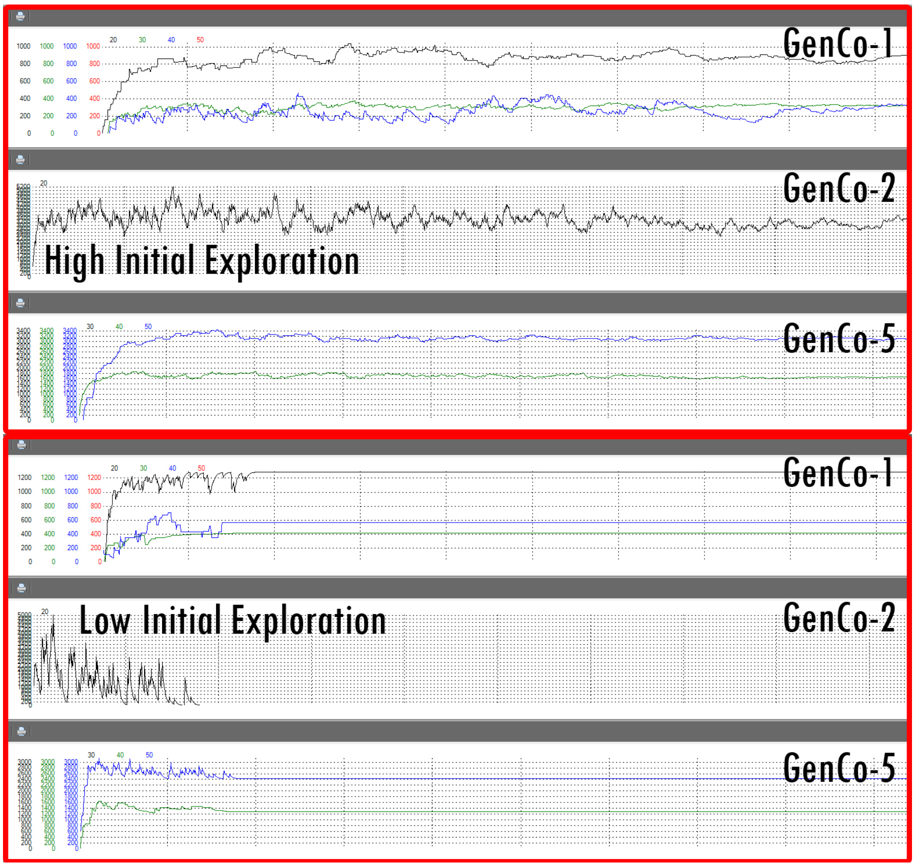


Figure 3.8: Effect of low and high exploration rates on Q-values of different GenCos in Case 2 with DC-OPF pricing

a result, each bid now becomes a price-capacity pair; therefore, the notation of bid can be extend to $G_i = (b_i, P_i)$ to include both GenCo-i's bid price b_i and offered capacity P_i. For the sake of simplicity, multi-market interaction is ignored; therefore, we are still focusing on a day-ahead market and unsold capacity will be unused. ISO clears the market with uniform pricing rule.

We also define υ as a measure to evaluate robustness of decisions when GenCos withhold their capacity. To this end, first, we define N_s as the number of time strategy s is observed. We sort strategies in decreasing order of their frequency (N_s values). υ is defined as $\sum_{i=0}^{30} \left(iN_{\{i\}}\right)$ where $N_{\{0\}}$ is the number of times that we observe the most visited strategy in 30 runs. The highest value of υ is observed when all strategies are visited equally and in this case $\upsilon^{max} = \sum_{i=0}^{30}(i) = \frac{30\times31}{2} = 465$ and the lowest value (υ^{min}) is 0.

Figure 3.9 illustrates two scenarios when two unique strategies are observed with different ratios at the end of 30 replications. Labels in horizontal axis are the weights of corresponding bars. The robustness of decisions in the left plot is higher since 2/3 of times simulation converged to a particular state.

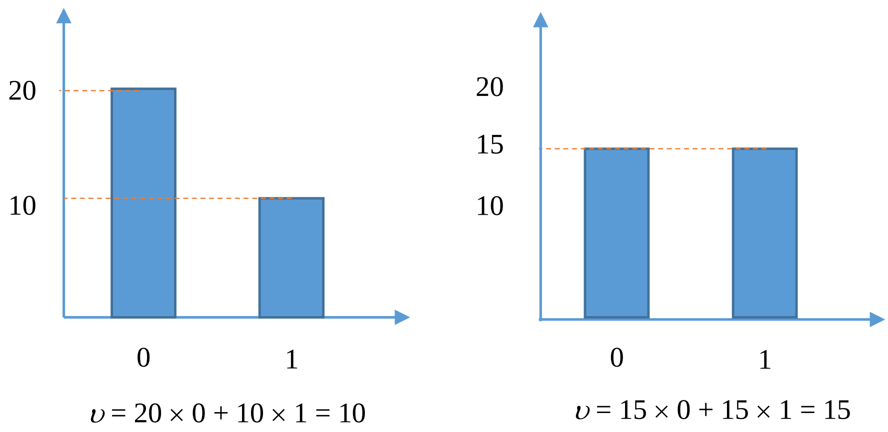


Figure 3.9: Variability of converged strategy in two different scenarios

3.5.2 Modified Case Study 1 with Capacity Withholding

As a case study, we select an extension of Case 1 where public GenCo (GenCo-1) can offer either $100MWh$ or $80MWh$ for its only bid (\$20 per MWh). However, private GenCo (GenCo-3) can submit various prices with $100MWh$ and $80MWh$. The aggregate demand is concentrated in Node 2 ($100MWh$). The list of possible offers (\$ per MWh) for both companies are given in Table 4.2 along with their production capacities and production costs are available Table 3.10.

The public GenCo is better of with 80% of the capacity while the private GenCo can increase the cleared-price by bidding higher values. Indeed, the expected behavior is observed in the simulation. By running the simulation 30 times when $(\alpha_{i0} = 0.1, \epsilon_{i0} = 0.9)$,

Table 3.10: GenCos' Parameters in the modified Case Study 1

ID	P_i^{max}	c_i	B_i
1	$\{80, 100\}$	10	$\{20\}$
3	$\{80, 100\}$	10	$\{20, 50, 90, 120\}$

we obtained $G_1 = (\$20, 80MWh)$ and $G_3 = (\$120, 80MWh)$ as the converged bids at the end of simulation for all the replications.

We also observe more variability on Q-values since a fixed offer (bid, power) will have more options against (at lease two-fold) that even some of which might be infeasible to fulfill the demand. This issue indicates that simulation time span (max_t) has to be extended to alleviate convergence problem.

In Figure 3.10, we repeat the same experiment under various $(\alpha_{i0}, \epsilon_{i0})$. Settings correspond to dark blue regions withhold capacity at its maximum permissible level (80%). Interestingly, GenCos' decisions in these dark blue regions show more robustness as well.

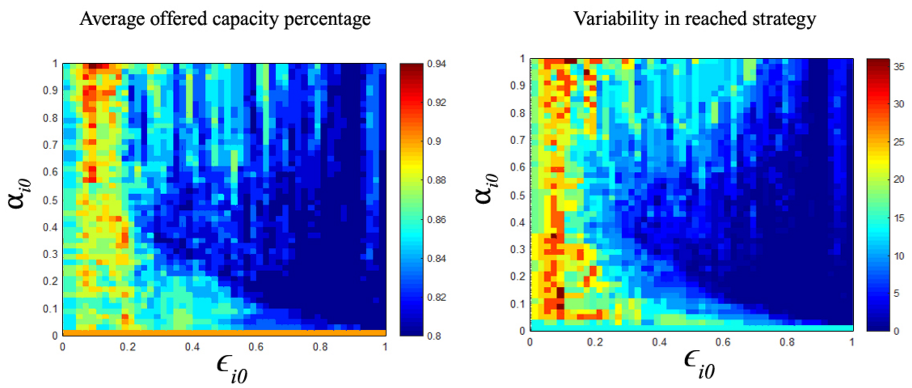


Figure 3.10: Average offered capacity percentage (left) and variability in reached strategy (right)

If we change the ISO pricing strategy to pay-as-bid, the public GenCo has no incentive to share the demand. Therefore, it will offer its total generation capacity to the ISO. On the other hand, private GenCo will lose the market if it offers any price above \$20 per MWh; so, the private GenCo will go for full capacity exactly with \$20 per MWh.

Uniform pricing strategy will lead to same results as DC-OPF. This behavior is ex-

Table 3.11: GenCos' parameters in Case 2 with Capacity withholding

ID	P_i^{max}	c_i	B_i
1	$\{240, 300\}$	20	$\{20, 30, 40, 50\}$
2	$\{240, 300\}$	20	$\{20\}$
5	$\{200, 250\}$	30	$\{30, 40, 50\}$

pected since in the absence of congestion, DC-OPF gives similar prices as uniform pricing at all nodes [116].

3.5.3 Case Study 2 with Capacity withholding

We now have three GenCos: GenCo-1 and GenCo-2 with the lowest generation cost of \$20 per MWh and GenCo-5 with \$30 per MWh.

After running the simulation for 30 replications, GenCos converge to the following strategies at the end of simulation:

1. GenCo-1 ($b_1 = 30, P_1 = 240$): Because generation cost imposes GenCo-1 to offer a price above 20, it turns out to be exactly 30. By offering \$30, GenCo-1 guarantees its winning as production cost of GenCo-5 is \$30. However, if it offers all its generation capacity, GenCo-5 will not have any chance to win; therefore, the price of electricity will be \$30. By presenting $240MWh$ instead of $300MWh$, GenCo-1 together with GenCo-2 keep $20MWh$ extra demand for GenCo-5 to increase the market price of electricity for its benefit.

2. GenCo-2 ($b_2 = 20, P_2 = 240$): GenCo-2 should offer \$20 since it has no other choice. However, GenCo-2 shall not offer $300MWh$ since GenCo-5 does not have any chance to increase the market-cleared price. It needs both GenCo-1 and GenCo-2 keep some of their generation capacity.

3. GenCo-5 ($b_5 = 50, P_5 = 200$): GenCo-5 shall not offer \$30 since its generation cost is 30. Bidding above \$30, which is above GenCo-1s bid, causes the remaining demand ($(D = 500) - (P_1 + P_2 = 480) = 20MWh$) to be allocated to GenCo-5.

Since the assigned power to GenCo-5 is fixed, it is better to bid as high price as possible to increase the marginal profit per MWh.

In Figure 3.11, we repeat this experiment with various $(\alpha_{i0}, \epsilon_{i0})$. GenCos withhold capacity around 80.20% when $(\alpha_{i0}, \epsilon_{i0})$ is chosen from dark regions. Similar to Figure 3.10, GenCos' decisions in dark regions are more robustness.

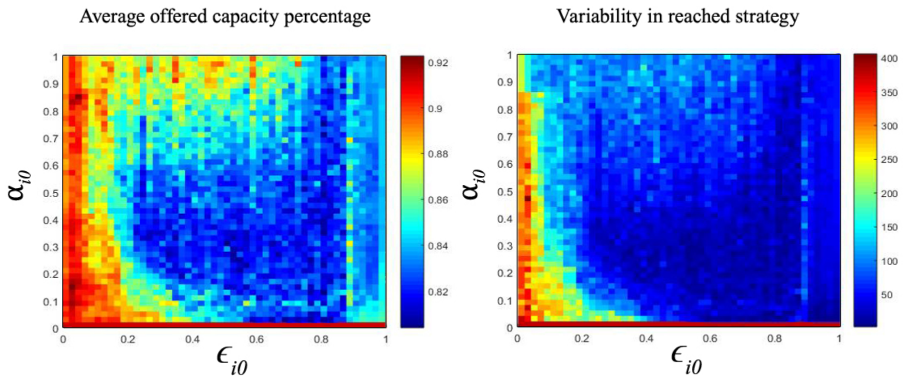


Figure 3.11: Average offered capacity percentage (left) and variability in reached strategy (right)

The results exhibit the key role of market-cleared price in GenCos' coordination. Market-cleared price carries necessary information to GenCos which can assist them in adjusting themselves with others. Although unused capacity is assumed to be lost, the results confirm the profitability of capacity withholding. This means participating in other consecutive markets (e.g., future market) to sell unused capacity may motivate GenCos to withhold capacity even more.

3.6 Effect of Risk

Electricity markets are oligopolies and electricity demand is often considered inelastic in the short term with respect to price. In addition, transmission line constraints and the relative locations of electricity demand and supply can provide market power to individual GenCos. Due to all these reasons, GenCos can bid strategically above their marginal costs and obtain positive profit. This possibility, and the importance of the power sector for the economy has triggered a wave of research into GenCos' strategic bidding behavior [See, for example, 24, 67].

Although convergence to a Nash equilibrium is an important theoretical question, it has been somewhat over-emphasized in academic literature [91]. Electricity market literature has a shortage of works that focus on the level and variability of GenCos' profits, which motivates studies on the effect of risk aversion. Literature that addresses GenCo behavior assumes risk-neutral decision makers whose objective is to maximize expected profit only. In reality, however, GenCos are exposed to increased levels of risk due to fluctuations in hourly prices and dispatched power quantities. Thus, they may act risk-averse in bidding. Dahlgren et al. [23] provide an early review of risk assessment methods in energy trading. We study the effect of GenCo's risk aversion on bidding behavior and profits. To this end, we adopt a model where risk is captured through the variance of past realized profits.

A number of researchers have formulated stochastic programming models to develop bidding strategies under the supply and price risks that GenCos face. For example, Ni et al. [78] consider the bidding risk of a producer that owns hydro, thermal and pumped-storage units. The authors illustrate how profit variability can be reduced through the risk management algorithm they introduce. Morales et al. [72] develop a stochastic programming model to addresses the supply and price uncertainties in the day-ahead, adjustment and balancing markets that a wind producer faces. Cabero et al. [14] modeled the market risk management problem in an oligopolistic market where network structure is ignored. They solve resulted equilibrium problem with Bender decomposition.

Conejo et al. [20] address the self-scheduling problem of a power producer. The authors consider the trade-off between maximizing profit and minimizing risk by taking into account the variance of the market-clearing price in the day-ahead market. Dicorato et al. [30] study a similar problem with a convex optimization model where risk aversion in captured through a constraint on the conditional value-at-risk (CVaR) of daily profit.

The aforementioned papers assume price-taking GenCos operating under perfect competition. Ventosa et al. [107]'s survey cites Batlle et al. [8] as the only work that addresses risk management of firms under imperfect competition. Batlle et al. [8] develop a Monte Carlo simulation model to capture hydro production and demand risks in electricity markets under Cournot competition. The authors use risk measures such as value-at-risk (VaR) and profit-at-risk (PaR). Gountis and Bakirtzis [44], similar to our study, consider a competitive market with network constraints. The authors solve a bi-level optimization problem in which competing GenCos submit linear supply functions at the first stage, and the ISO determines the dispatch in the second stage. Each GenCo holds a probabilistic view of competitor behavior and system load. Expected profits are characterized with Monte Carlo simulations, whereas the optimal bidding strategy is found through Genetic Algorithms. The authors illustrate how risk aversion affects GenCos' bidding strategy. Caruso et al. [16] consider both a pool and a bilateral-contract structure. Expected profits are determined through a Monte Carlo simulation, and risk is quantified through VaR and CVaR.

Another stream of risk-related papers are those that address the generation portfolio selection problem of a single GenCo. For instance, Fleten et al. [38] present a stochastic programming model for an integrated portfolio selection and scheduling problem for a risk-averse hydro producer participating in the Nord Pool. Gielis [42] discuss the effect of risk aversion on power plant investment decisions using agent-based simulation and Monte Carlo approaches on the EMLab-Generation model. The author compares results under CVaR, and two utility approaches: constant absolute risk aversion (CARA) and constant relative risk aversion. Vehviläinen and Keppo [104] develop a Monte-Carlo simulation to optimize a power portfolio composed of physical and financial assets.

The major contributions of our work can be summarized as follows.

- Building on Krause et al. [62], we develop a flexible agent-based simulation model to characterize the evolution of the dynamic electricity market under transmission grid constraints. In particular, we extend the learning model of Krause et al. [62] and Krause et al. [63] by considering time-dependent learning model parameters, similar to Esmaeili Aliabadi et al. [34]

- Using a mean-variance approach, we study the effects of risk aversion on GenCos' bid prices, profits and learning behavior.

- Different from most studies in literature, we present a large-scale numerical analysis with a wide range of parameter combinations.

3.7 Model with Risk-Averse GenCos

In this model, GenCos are allowed to be *risk-averse*, where each GenCo maximizes its expected utility. This risk-averse model encompasses the risk-neutral model of Section 3.2 as a special case. The utility of bid price alternative b_{ij} to GenCo-i is increasing in its Q-value Q_{ij}, and decreasing in the standard deviation of past realized profits from that alternative (which are recorded in the set H_{ij}). Accordingly, GenCo-i's best identified bid price is determined as

$$b_i^* = \underset{b_{ij}}{Max} \left\{Q_{ij}^r\right\} \tag{3.4}$$

$$\text{where} \qquad Q_{ij}^r = (1-\beta_i)Q_{ij} - \beta_i \sqrt{\frac{\sum_{r_{ij} \in H_{ij}} (r_{ij} - Q_{ij})^2}{|H_{ij}| - 1}}. \tag{3.5}$$

Parameter $\beta \in \{0, 1\}$ denotes the *risk aversion level* of the GenCo where $\beta = 0$ corresponds to the risk-neutral case and higher β values correspond to more risk-aversion. Note that risk aversion does not affect how the Q-value associated with each bid price alternative is updated; the update is still based on profits as given in Equation (3.1).

3.8 Simulation Study with Risk-Averse GenCos

To address risk aversion, two modifications are made in the simulation algorithm. First, the best identified bid price alternative is now determined based on the risk-modified Q^r-values as shown in Equation (3.4). Second, the initial η iterations of the simulation are defined as the *risk ignorance periods*. During these iterations, the variance component of Equation (3.4) is ignored in GenCo-i's best bid price determination as there are only a few observations in initial iterations to support meaningful variance calculations. Therefore, we set $\eta = max_t/2$.

We study the effects of risk aversion on GenCos' bid prices and profits using a new case study, Case 4. All reported results are averages over 30 random runs. The number of iterations in each run is 2000. The initial Q-learning parameters are $\epsilon_{i0} = 0.85$ and $\alpha_{i0} = 0.15$ for all GenCos. Note that this structure provides GenCo-3 advantage due to zero generation cost, whereas GenCo-4 is at an unfavorable position with a relatively high generation cost.

3.8.1 Identical Risk Aversion Level

For this analysis, we assume all three GenCos to have the same risk aversion level. We initially focus on the picture at the end of the simulation, that is, at iteration number 2000. Figure 3.12(a) provides the best identified bid price at the end of the simulation for each GenCo, averaged over 30 runs. The horizontal axis depicts the identical risk aversion level β. Figure 3.12(b) presents the corresponding profit values.

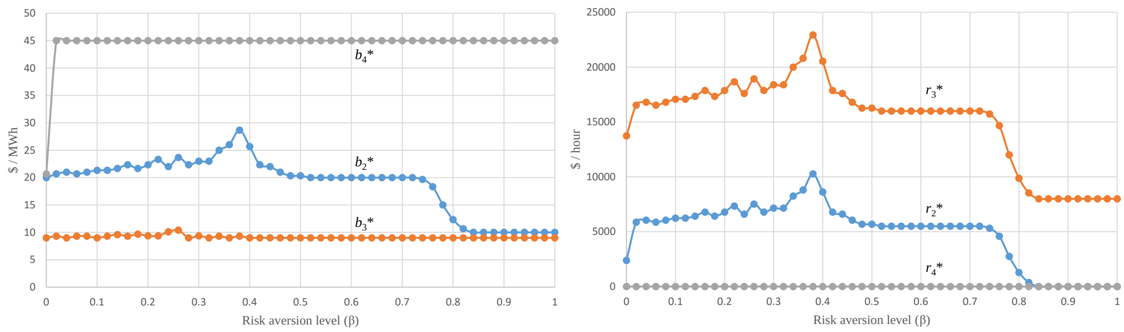


Figure 3.12: End-of-simulation results (a) The best identified bid prices (b) Profits

From Figure 3.12(a), we observe the best bid prices of GenCo-3 and GenCo-4 to be quite stable for different risk aversion levels. GenCo-3 mostly bids 9. It ventures into bidding 18 only for relatively small β values. GenCo-4 bids its maximum price of 45 for any β value except zero. GenCo-2, on the other hand, responds to different levels of risk aversion. In fact, the profit results observed in Figure 3.12(a) are driven by the changes in GenCo-2's bid price b_2. As β increases from 0.00 to around 0.38, b_2 increases. When risk-neutral, GenCo-2 usually bids 20, but as it becomes risk-averse, it tries higher bid prices such as 30 or 40 more frequently. These higher bid prices lead to higher profits not only for GenCo-2 itself, but also for its rival GenCo-3 as well. In fact, both GenCos' individual profits, and also the total profit of all GenCos are maximized at $\beta = 0.38$. Thus, we observe that some level of risk aversion in the market can benefit all GenCos.

After reaching a maximum at β values around 0.38, GenCo-2's average bid decreases for higher risk aversion levels. In fact, for $\beta \in [0.74, 0.82]$, GenCo-2 becomes excessively concerned about the variability in profits and bids its marginal cost 10 more frequently. For even higher β values, GenCo-2 only bids 10, resulting in zero profits. Such low bids by GenCo-2 causes a significant reduction in the profit of rival GenCo-3 as well. For sufficiently high β values, both GenCo-2 and GenCo-3 submit their marginal generation costs to minimize the variability in their profits. GenCo-4, meanwhile, is observed to obtain zero profit at the end of the simulation independent of its risk aversion level.

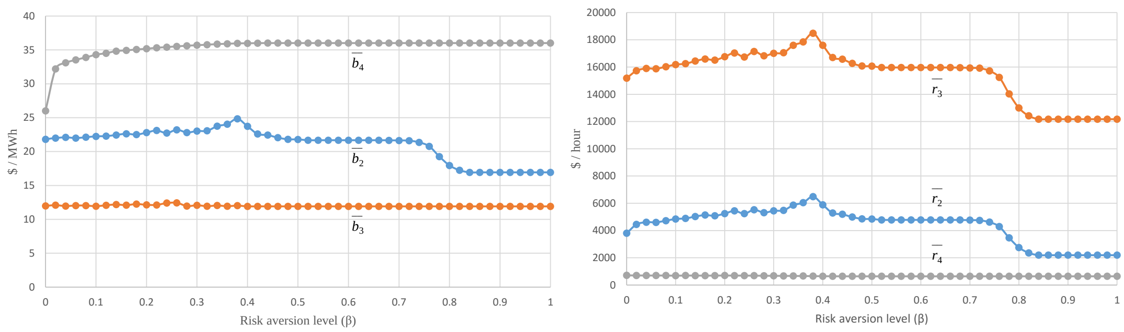


Figure 3.13: Simulation averages (a) The best identified bid prices (b) Profits

We have discussed the end-of-simulation results when each GenCo-i bids its best identified price b_i^* as of iteration 2000. While these converged results are of interest, they do not necessarily represent what has happened throughout the 2000 iterations of the simulation, especially in the initial iterations where the most of learning takes place. Figures 3.13(a) and (b) provide the average bid prices and profit values over all 2000 iterations of the simulation, again averaged over 30 runs. A comparison between Figure 3.12 and Figure 3.13 illustrate the effects of GenCo learning and strategic interaction over time.

The similarities in shapes indicate strong convergence in bid prices. The differences in bid prices point to changes in GenCo bidding behavior over time. In particular, the effect of risk aversion on GenCo-2's bids become sharper over iterations. GenCo-3 bids higher prices than 9, and GenCo-4 bids lower prices than 45 throughout the iterations. Accordingly, GenCos' profits converge to more extreme levels at the end of the simulation. For $\beta < 0.74$, the competing GenCos, GenCo-2 and GenCo-3, achieve higher profits at the end of the simulation than in the initial iterations. For higher β values however, the extreme risk aversion of GenCo-2 causes a reduction in both GenCos' profits. Meanwhile, as expected, GenCo-4's profits converge to zero over iterations. All these observations underscore the importance of risk aversion on GenCo bidding behavior and resulting profit levels in an environment shaped by dynamic learning and competition.

Figure 3.14 presents the corresponding DC-OPF optimal solution value, $\Sigma b_i P_i$, and total payment to GenCos, $\Sigma(LMP)_i P_i$ as a function of the identical risk aversion level. Comparing the end-of-simulation and simulation-average results, we make the following two observations:

- **DC-OPF optimal value:** Under almost all risk aversion levels, the end-of-simulation DC-OPF objective function value is lower than the simulation-average value. The ISO's auction mechanism seems to be successful in driving GenCos' bid prices down throughout the simulation. Note that the difference becomes larger for $\beta > 0.74$.

- **Total payment to GenCos:** For $\beta < 0.45$, the total payment to GenCos (hence, their total profit) is higher at the end of the simulation than the average payment during simulation. As long as the GenCos are not very risk-averse, they collectively learn to obtain better profits over time. For $\beta > 0.45$, however, the observation is reversed; risk-averse behavior of GenCo-2 causes a reduction in total GenCo profits. This reduction is especially acute for $\beta > 0.74$, where GenCo profits are adversely affected by extreme price competition.

This analysis sheds light onto the effect of risk aversion level on GenCos' bids and profits. Overall, while some level of risk aversion can be beneficial to GenCos' total profits, high levels of risk aversion is observed to degrade profits due to extreme price

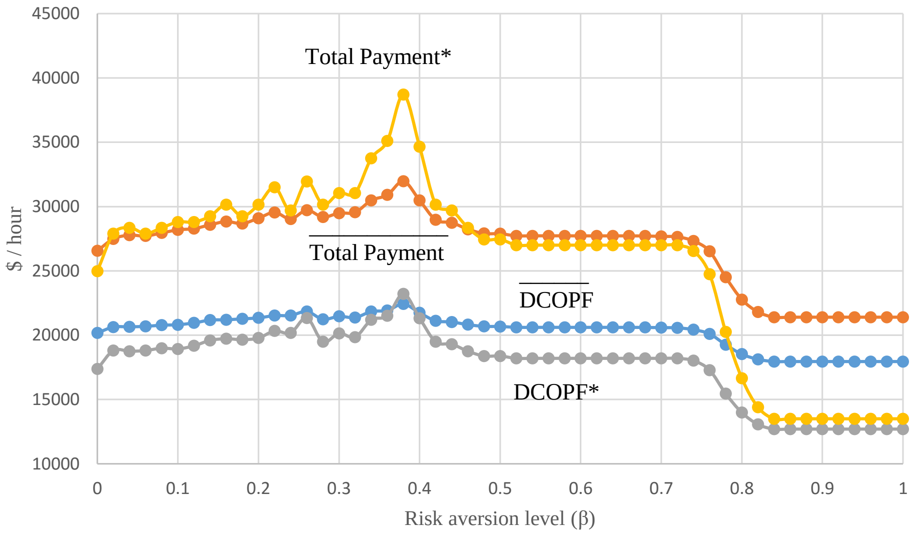


Figure 3.14: DC-OPF objective value and total payments to GenCos (Presenting both end-of-simulation and simulation-average results)

competition. Another important observation shows that the ISO is optimizing wrong criterion. We suggest ISO to optimize (minimize) $\sum_i (LMP_i \times P_i)$ instead of $\sum_i (b_i \times P_i)$ or change its payment strategy from paying nodal price of LMP_i to bid price b_i per MWh to winning bids.

3.8.2 Differing Risk Aversion Levels

We analyze the effects of changes in the risk aversion levels of individual GenCos, focusing initially on GenCo-2. Figure 3.15 presents the average profits and average bid prices of each GenCo at a separate column, as a function of β_2 (in the y axis) and the other two β values (in the x axis, assumed equal to each other) over the whole simulation run. If β_2 increases, while keeping β_3 and β_4 constant, we observe the profit of GenCo-2 to decrease. This is expected as this GenCo now bids lower prices. Interestingly, GenCo-3's profit also decreases due to increased competition. GenCo-4's average profit, too, is reduced for most instances. The only exception with GenCo-2 occurs for very high β_2 values where GenCo-2 bids its minimum price 10 most of the time. In this case, GenCo-4 has a chance to make some profit only if β_4 is relatively low (the northwest corner of the graph). Figure 3.16 presents the end-of-simulation version of the same analysis.

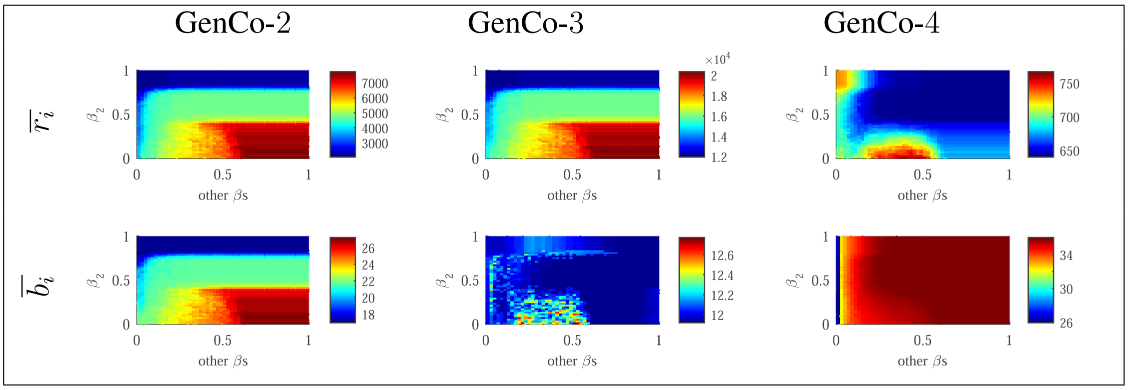


Figure 3.15: Bid prices and profits as a function of β_2 vs. $\beta_3 = \beta_4$, simulation average

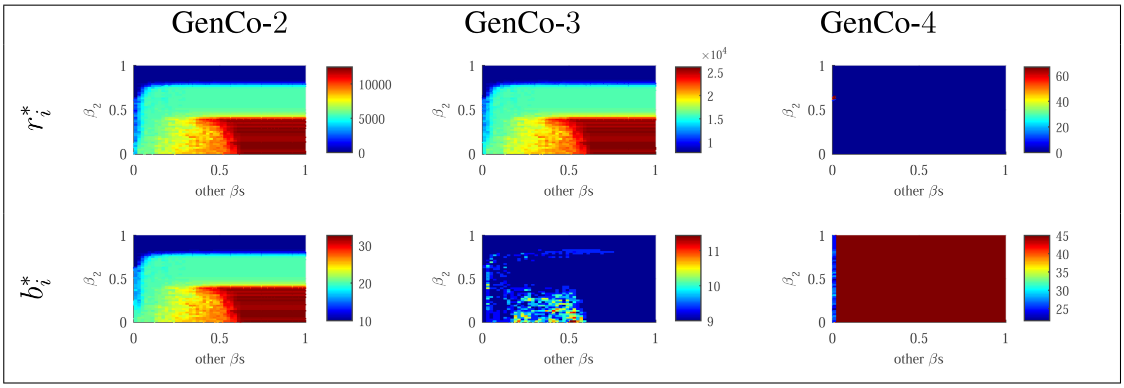


Figure 3.16: Bid prices and profits as a function of β_2 vs. $\beta_3 = \beta_4$ at the end of simulation

Next, we investigate the effects of a simultaneous increase in β_3 and β_4, while keeping β_2 fixed, for example, at zero. When the competitor GenCos become more risk-averse, they might be expected to reduce their bid prices, leading to a decrease in GenCo-2's profit. Our simulation, however, yields the opposite outcome. As β_3 and β_4 increase, we observe GenCo-2 to increase its bid price, leading to an increase in its profit. The key to understanding this counterintuitive result is GenCo-4's behavior. GenCo-4 simply bids its highest price alternative of 45. For this price, this GenCo is assigned no dispatch and receives zero profit. If GenCo-4 bids one of the lower prices, there is a slight chance that it will be assigned some dispatch and some profit. When this happens, however, the variability in profit also increases which is not desirable for a risk-averse GenCo.

Figure 3.17 illustrates the average bid price and profit level of each GenCo-i (in a column) over the simulation as a function of its own β_i (in the y axis) and the other GenCos' β values (in the x axis). The leftmost column is the same as that of Figure 3.15. The middle column, for example, illustrates GenCo-3's average bid price and profit as a function of β_3, and ($\beta_2 = \beta_4$). We observe that GenCo-3's profit depends mostly on the risk aversion level of the other GenCos, particularly that of GenCo-2, than its own β_3. For instance, GenCo-3 bids high prices only if its risk aversion level is low. Otherwise, this GenCo sticks to the advantageous bid price of 9. Given this b_3, GenCo-3's profit level becomes a function of b_2, which decreases if β_2 increases. Note the emergence of $\beta = 0.38$ as a critical value again in this graph. GenCo-3 profits, in particular, are maximized when the other β values are around $\beta_2 = \beta_4 = 0.38$. GenCo-4 makes a much lower profit compared to GenCos 2 and 3. GenCo-4's profit is maximized when β_4 is at intermediate values, while β_2 and β_3 are relatively low; that is, when the other two GenCos' risk aversion level is low and consequently they do not engage in intense price competition.

Figure 3.18 presents the corresponding graphs at the end of simulation. Compared to the simulation-average values, we observe GenCo-2's and GenCo-3's profits to be higher. GenCo-4's end-of-simulation profit, on the other hand, has converged to zero independent of its own risk aversion level.

3.8.3 Learning Dynamics

To understand the effects of learning dynamics, next, we drill further down into the details of the learning model. This discussion illustrates how the learning, strategic interaction and risk aversion components of our model interact with each other. Figure 3.19 presents how the three GenCo's Q^r-values, hence the best identified bid prices, change over iterations for a given risk profile $(\beta_2, \beta_3, \beta_4)$. The figure on left presents the case of the risk profile $(0, 0, 0)$, corresponding to the bottom left corner of the relevant graph in Fig-

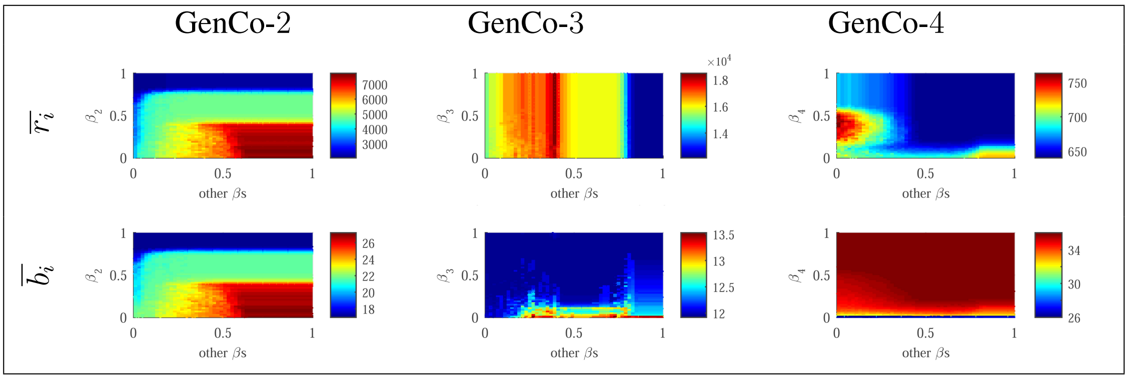


Figure 3.17: Bid prices and profits as a function of risk aversion levels, simulation average

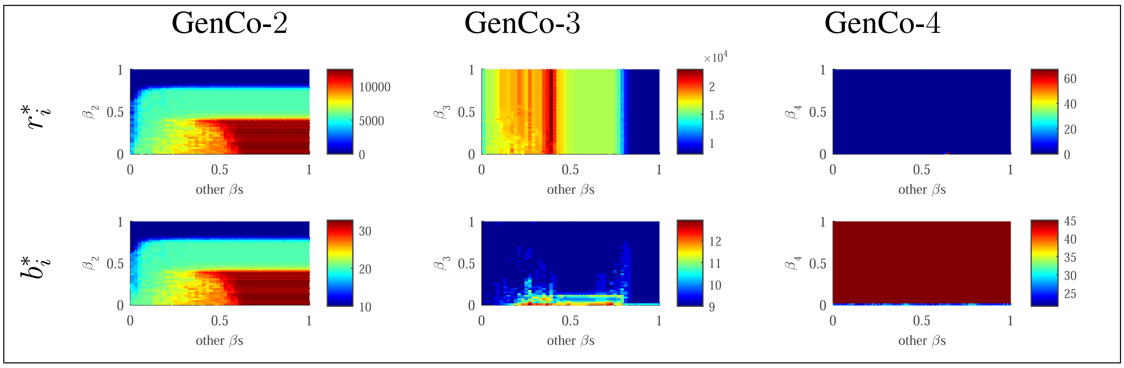


Figure 3.18: Bid prices and profits as a function of risk aversion levels at the end of simulation

ure 3.16. The figure on right presents the case of the risk profile $(0, 1, 1)$. Recall that our model ignores the effect of risk during the so-called risk ignorance periods; the risk model kicks in after iteration 1000. In fact, iteration 1000 is indicated with a dashed vertical line at each graph.

When all GenCos are risk-neutral (Figure 3.19(a)), we observe the best identified bid prices of GenCo-2 and GenCo-3 to take some time to converge, due possibly to the tight competition between these two GenCos. The best identified bid price emerges as 20 for GenCo-2 and 9 for GenCo-3. Once this equilibrium between GenCos 2 and 3 is reached, GenCo-4's bids become irrelevant as it is driven out of the market.

When GenCo-2 is risk-neutral but GenCos 3 and 4 are extremely risk-averse (Figure 3.19(b)), the best identified bid price for GenCo-2 changes from 20 to 30 once risk aversion kicks in. For GenCo-3, 9 arises as the best identified bid price. Recall that bidding 9 brings in a decent profit to GenCo-3 while not having the profit variability disadvantage of the higher bid prices.

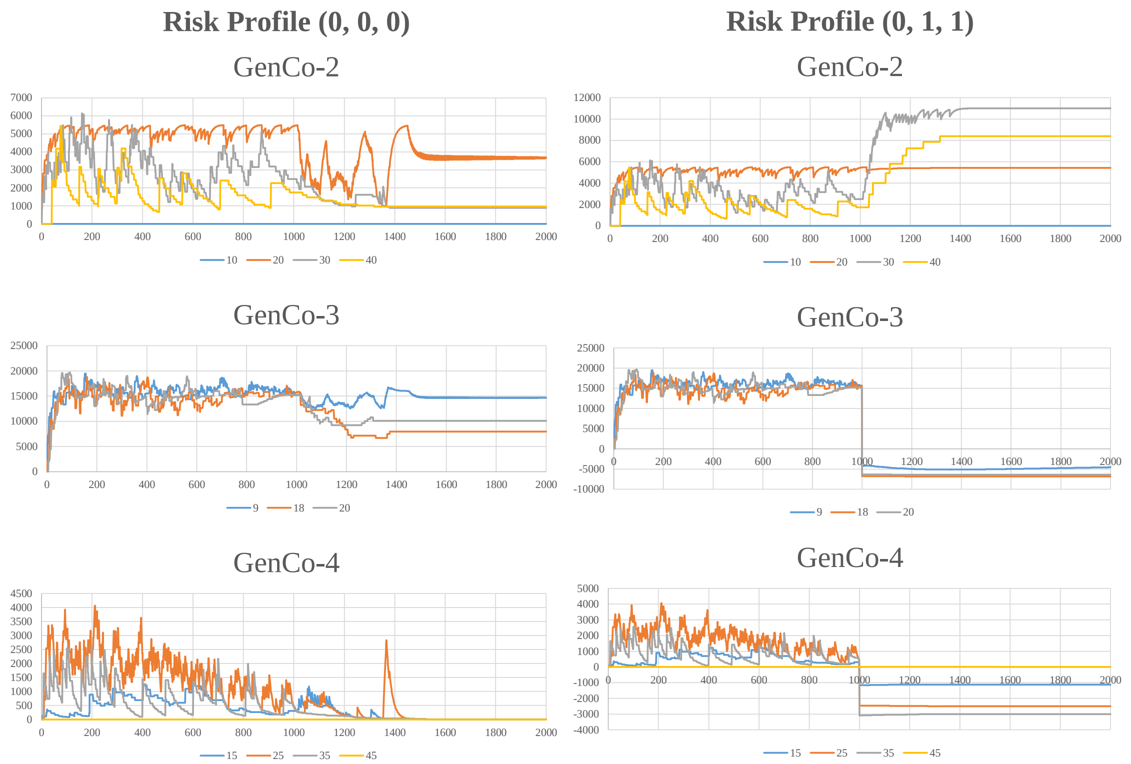


Figure 3.19: Q^r-value evolution graphs (a) Risk profile $(0, 0, 0)$, (b) Risk profile $(0, 1, 1)$

3.9 Effects of the Q-Learning Parameters

We study the effects of the Q-learning parameters ϵ and α on GenCos' profits. Because these two parameters are defined as time-decaying, we analyze the effect of their initial values ϵ_0 and α_0. A comprehensive simulation study is conducted using the network structure of Case 4, which has three learning GenCos. We report the results focusing on one of the GenCos (the *GenCo-i*), while referring to the other two GenCos as the *rivals*. For all simulation runs, $max_t = 2000$ and $\eta = 1000$ values are used.

GenCo-i is assumed to be risk neutral. For this GenCo, results are reported under 21 x 21 = 441 parameter combinations of $(\epsilon_{i0}, \alpha_{i0})$, where each of the two parameters range between 0 and 1, with an increment size of 0.05.

For a given $(\epsilon_{i0}, \alpha_{i0})$ combination of GenCo-i, we speak of different *scenarios* characterizing the parameters of the two rival GenCos (GenCo-j where $j \neq i$). Each of the two rival GenCos' parameters are chosen from the following sets: $(\alpha_{-j0} \in \{0, 0.2, 0.8\}, \beta_{-j0} \in \{0, 0.4, 0.8\}, \epsilon_{-j0} \in \{0.2, 0.4, 0.8\})$. Thus, $3^3 \times 3^3$ = 729 scenarios are considered. In each scenario, the same stream of random numbers are used, and the results are averaged over 10 simulation runs. GenCo-i is assumed to have no information about the parameters of its two competitors; hence, it believes all scenarios to be equally likely. Consequently, for each $(\epsilon_{i0}, \alpha_{i0})$ combination of GenCo-i, the average Q-value $(\overline{Q_i})$ and the average cumulative profit $(\overline{CP_i})$ over all 729 scenarios are reported. All in all, this comprehensive simulation study required the DC-OPF problem to be solved 19,289,340,000 times (3 GenCo-i $\times$ 441 combinations $\times$ 729 scenarios $\times$ 10 runs $\times$ 2000 iterations). The study took around 2000 hours on an Intel Core i7 @ 3.2GHz computer with 24GB RAM.

Figure 3.20 presents the results where each column corresponds to a GenCo-i. Graphs in the first row illustrate GenCo-i's expected profit, that is, the Q-value of the best identified bid at the end of the simulation; whereas those in the second row illustrate the cumulative profit (CP) throughout the simulation. First, α_{i0} is observed not to have a major impact on profit results unless its value is very low. That is, the profits are robust to the initial value of recency rate alpha as long as some updating of Q-values occur. The exploration parameter ϵ_{i0}, on the other hand, is seen to have a significant impact on profits. The direction of this impact, however, is ambiguous. For GenCo-2, high exploration levels lead to better profits. For GenCo-3, this is true for the end-of-simulation profit; yet, the cumulative profit first increases then decreases with the exploration parameter. For GenCo-4, profit is uniformly increasing in the exploration parameter. Recall that GenCo-4 is at a disadvantageous position compared to the other GenCos. As suggested in Figure 3.20, this GenCo can maximize its expected profit by acting as randomly as possible (corresponding to $\epsilon_{40} = 1$), thereby disrupting the learning of the other two GenCos.

We compare the end-of-simulation expected profit (the first row graph) and the cumulative profit over the simulation (the second row graph) for each GenCo-i. For GenCos 2 and 4, these two graphs exhibit overall parallel results. That is, $(\epsilon_{i0}, \alpha_{i0})$ combinations that yield the highest expected profit at the end of the simulation also happen to provide the highest cumulative profit throughout the simulation. For GenCo-3, however, we observe significant differences. This GenCo can identify better profit opportunities at the end of the simulation by exploring excessively, however, this comes at a cost of achieving a lower cumulative profit. Overall, the only parameter that has a major profit impact at the end of the simulation turn out to be the exploration parameter of GenCo-2: High values of this parameter is observed to significantly increase the expected profit of GenCo-2.

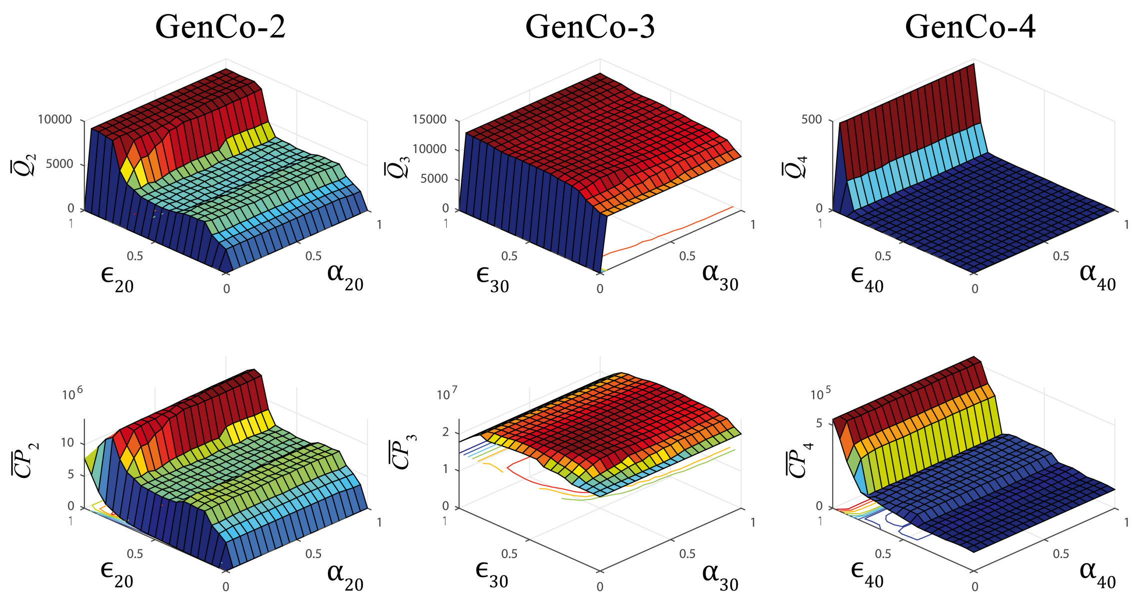


Figure 3.20: $\overline{Q_i}$ (first row) and $\overline{CP_i}$ (second row) for different $(\alpha_{i0}, \epsilon_{i0})$ combinations

3.10 Results

We study the effect of the ISO's market-clearing mechanism and risk aversion on the bidding behavior of GenCos in an electricity market. We compare the results under two well-known pricing rules, uniform and pay-as-bid pricing, as well as under equal and random rationing policies. Learning is modeled through a modified Q-learning algorithm with time-decaying parameters and risk aversion is captured as aversion to variability in profits. Given GenCos' bids, to determine locational marginal prices and GenCos' power dispatches, the ISO solves a DC-OPF problem that considers the physical network characteristics.

We implement the simulation model on four case studies representing different levels of market complexity. Results are reported under all possible combinations of the two key learning-model parameters.

Our simulation results indicate that under most parameter settings the market does converge to either a Nash equilibrium or a state that has identical payoffs with a particular Nash Equilibrium (a semi-Nash state, as we define it). The convergence frequency to Nash Equilibria is found to be lower for more complex cases. Another important result is about the level of competition in the market, which we measure through the difference between the best identified bid and production cost for each GenCo. Uniform pricing with random rationing policy is observed to be the most successful in making GenCos submit closer bids to their production costs, hence in promoting competition among GenCos. In particular, the random rationing policy is seen to be effective in disrupting GenCos' learning process, which can be instrumental if the ISO needs to prevent GenCos' learning towards, for instance, a collusive equilibrium.

Another major finding is that some level of risk aversion may indeed be beneficial for GenCos' total profits compared to the risk-neutral case. On the other hand, high levels of risk aversion is shown to intensify price competition and degrade profits. We illustrate how altering the risk aversion level of even one GenCo can trigger changes in the bidding behavior and profit levels of all GenCos through learning and market interaction among GenCos.

Our findings highlight the importance of what static game-theoretical models fail to capture: Dynamics of the interaction between competing GenCos that learn from experience. One should be cautious in using such models to investigate a dynamic markets such as the day-ahead electricity market. In addition, we illustrate the role of risk aversion in shaping GenCo behavior and market evolution. This aspect is overlooked in most electric market studies, game-theoretical or agent-based simulation alike.

The agent-based simulation model that we developed for this study is a detailed and versatile one. We plan to extend this model to address further questions on strategic interactions in electricity markets. One future research direction is the study of GenCos' collusive behavior (i.e., studying GenCos' bidding behavior in existence of *SCE* states by using agent-based simulation). Another can be on the effects of a second market (e.g., futures market). One may also use a more sophisticated measure of risk such as CVaR.

Chapter 4

Conclusion

In this dissertation, the bidding behavior of power generation companies (GenCos) is studied using an agent-based simulation model. GenCos are modeled as agents that bid prices repeatedly for each hour of the day-ahead market. Learning is modeled through a modified Q-learning algorithm with time-decaying parameters, and risk aversion is captured as aversion to variability in profits. Given GenCos' bids, to determine nodal prices and GenCos' power dispatches, the ISO solves a pricing problem that considers the physical network characteristics.

4.1 Original Contribution

The contribution of the present work is threefold:

- We study the existence and identification of collusion among GenCos in a deregulated (oligopolistic) electricity market when transmission network constraints are under consideration within the market clearance mechanism of the ISO. We examine characteristics of collusion based on market parameters and strategic behaviors of GenCos. Strategic behavior of GenCos is modeled within an infinite horizon game. We develop a bi-level mathematical programming problem to model the market clearance mechanism of the ISO where the behavior of GenCos and network constraints are considered. The problem has multiple non-linear objective functions where GenCos compete at the leader level and the optimal power flow is determined at the follower level. Using linear programming theory and the methods in multi-objective optimization, the problem is simplified to a constrained optimization problem with a linear objective function. An optimization-based approach is proposed to solve the problem. In our computational study, we present case studies which indicate how collusion can be detrimental for the end consumers disrupting the competition. Based on the cases, we also discuss alternative courses of action for the ISO to cope with and avoid collusion.

- In the next chapter, we study the effect of the ISO's market-clearing mechanism and risk aversion on the bidding behavior of GenCos in an electricity market. We compare the results under two well-known pricing rules, uniform and pay-as-bid pricing, as well as under equal and random rationing policies. We implement the simulation model on four case studies representing different levels of market complexity. Results are reported under all possible combinations of the two key learning-model parameters. Our simulation results indicate that under most parameter settings the market does converge to either a Nash equilibrium or a state that has identical payoffs with a particular Nash Equilibrium (a semi-Nash state, as we define it). The convergence frequency to Nash Equilibria is found to be lower for more complex cases. Another important result is about the level of competition in the market, which we measure through the difference between the best identified bid and production cost for each GenCo. Uniform pricing with random rationing policy is observed to be the most successful in making GenCos submit closer bids to their production costs, hence in promoting competition among GenCos. In particular, the random rationing policy is seen to be effective in disrupting GenCos' learning process, which can be instrumental if the ISO needs to prevent GenCos' learning towards, for instance, a collusive equilibrium.

  The next major finding is that some level of risk aversion may indeed be beneficial for GenCos' total profits compared to the risk-neutral case. On the other hand, high levels of risk aversion is shown to intensify price competition and degrade profits.

We illustrate how altering the risk aversion level of even one GenCo can trigger changes in the bidding behavior and profit levels of all GenCos through learning and market interaction among GenCos.

- The results when GenCos are capable of withholding capacity exhibit the key role of market-cleared price in GenCos' coordination. Market-cleared price carries necessary information to GenCos which can assist them in adjusting themselves with others. Although unused capacity is assumed to be lost, the results confirm the profitability of capacity withholding. This means participating in other consecutive markets (e.g., future market) to sell unused capacity may motivate GenCos to withhold capacity even more.

Our findings highlight the importance of what static game-theoretical models fail to capture: Dynamics of the interaction between competing GenCos that learn from experience. One should be cautious in using such models to investigate a dynamic markets such as the day-ahead electricity market. In addition, we illustrate the role of risk aversion in shaping GenCo behavior and market evolution. This aspect is overlooked in most electric market studies, game-theoretical or agent-based simulation alike.

Our results have important regulatory and managerial implications. Effects of market-clearing mechanism and risk aversion would be a particularly important concern for markets that have a number of relatively small GenCos at key grid positions. The ISOs may use simulation models such as ours for the purpose of customizing their market-clearing mechanisms based on the risk aversion levels (related to firm size and financial status etc.) and learning capabilities (related to human resources and access to information etc.) of the GenCos in their markets.

4.2 Future Work

In order to focus on the strategic level questions, as it is customary in related literature, we considered only a simplistic power system in this thesis. This work can definitely be extended to address more complex networks, or more operational-level problems. The goal is to strike a good balance between capturing practical system details and avoiding expositional complexity.

Although the agent-based simulation model that we developed for this thesis is a detailed and versatile one, we plan to extend this model to address further questions on strategic interactions in electricity markets. One future research direction is the study of GenCos' collusive behavior. Another can be on the effects of a second market. One may also use a more sophisticated measure of risk such as CVaR. Likewise, a different learning model can be used; or our Q-learning model can be extended to achieve better performance. For instance, a GenCo may improve its profit by "tracking and reacting to" Q-values that decrease. Alternatively, the power dispatch results in each period can be used in Q-value updates.

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Appendix

A Details of case studies

A.1 Case 1: Public GenCo versus Private GenCo

The simplified transmission grid is illustrated in Figure 4.1. Node 1 represents the public GenCo company and Node 3 represents the private one while Node 2 represents a load/demand center. The properties of transmission lines are shown in Table 4.1; the first column $Src(k)/Dst(l)$ shows the source and the destination nodes of transmission line; the second column depicts the value of y_{kl} of the line k to l and the last column shows the maximum flow on the line. The public GenCo benefits from subsidies to keep the price of energy as low as possible. As a result, it offers a bid price which is equal to its production cost (C_1). The list of possible offers ($ per MW) for both companies are given in Table 4.2 along with their production capacities and production costs.

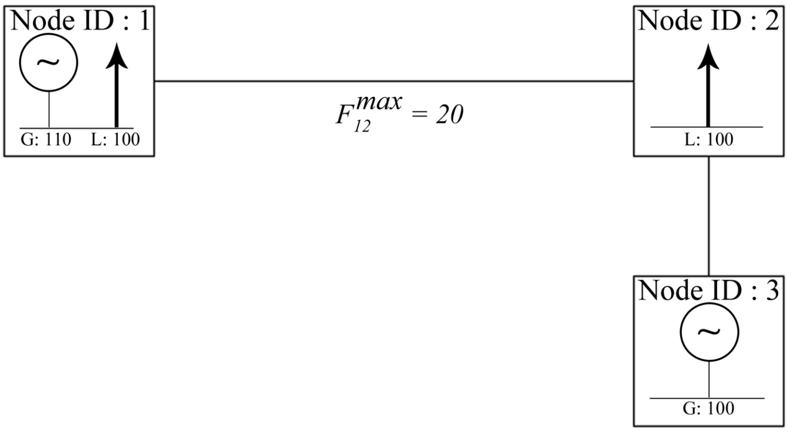

Figure 4.1: The transmission grid for Case 1

Table 4.1: Transmission line properties

Src (k)/ Dst (l)	y_{kl}	F_{kl}^{max}
1/2	4	20
2/3	4	No Limit

Table 4.2: Parameters of the GenCos

ID	P_i^{max}	C_i	B_i
1	110	10	$\{10\}$
3	100	10	$\{10, 20, 30, 40\}$

A.2 Case 2: Two GenCos as Learning Agents

In the second case study, we have a five-node transmission grid governing the power market. The properties of transmission lines are given in Table 4.3. The network structure along with generation capacities and demand load data are given in Fig.4.2 and Table 4.4, respectively. Node 3 is the reference bus in this system (i.e. the voltage angle of reference bus is zero in DC-OPF). Node 1 and Node 5 are the GenCos that benefit from learning.

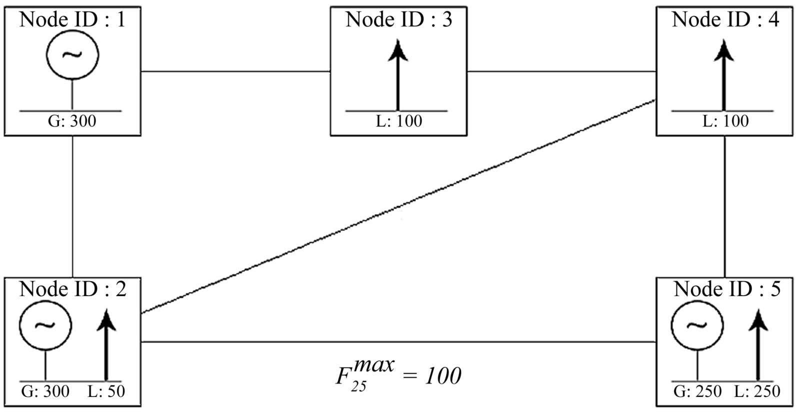

Figure 4.2: The network for Case 2 and Case 3

Table 4.3: Transmission line properties in Case 2

Src (k)/ Dst (l)	y_{kl}	F_{kl}^{max}
$\{1/2, 1/3, 2/4, 3/4, 4/5\}$	4	No Limit
2/5	4	100

Table 4.4: Parameters of GenCos in Case 2

ID	P_i^{max}	C_i	B_i
1	300	20	$\{20, 30, 40, 50\}$
2	300	20	$\{20\}$
5	250	30	$\{30, 40, 50\}$

A.3 Case 3: Three GenCos behaving as active agents

If we change the second case study's bid alternatives for b_2 from $\{20\}$ to $\{20, 30, 40, 50\}$ the system would have multiple Nash equilibria. Table 4.5 shows all possible outcomes under DC-OPF pricing: light-gray cells are the best responses of the GenCo-2 to the action of GenCo-1, bold-text cells are the best responses of the GenCo-1 to the action of the GenCo-2, and italic-text cells are the best responses of the GenCo-5 to the given action of GenCo-1 and GenCo-2. The intersection of all best responses are highlighted in dark-gray; they represent the Nash equilibria.

A.4 Case 4: Three active GenCos with a centralized demand node

In this case, we have created a small market with three GenCos; Fig.4.3 shows structure of undertaken market. The second GenCo benefits from wind power technology; this is why, production cost is negligible in Table 4.6. Thus, GenCo-2 can offer a lower price to the ISO (cost of not fulfilling promised demand is considered in the price of electricity).

B Boundary analysis of the proposed method

To keep tracking the evolution of Q-values over iterations, we modify the Q-value and payoff notations. $Q_{ij}^{(t)}$ stands for the Q-value of b_{ij} at iteration t. Also, the received payoff

Table 4.5: Profit of each policy $\{r_1, r_2, r_5\}$ - Rows: B_1, Columns: B_2 and separated tables: B_5

$b_5 = 30$	20	30	40	50
20	**(428.57, 0, 0)**	**(3000, 785.71, 0)**	**(3000, 0, 0)**	(3000, 0, 0)
30	(0, 3000, 0)	(0, 3000, 0)	(2500, 0, 0)	(2500, 0, 0)
40	(0, 3000, 0)	(0, 3000, 0)	*(0, 5000, 2500)*	***(5000, 0, 2500)***
50	(0, 3000, 0)	(0, 3000, 0)	(0, 5000, 2500)	*(0, 7500, 5000)*

$b_5 = 40$	20	30	40	50
20	**(857.14, 0, 1214.29)**	**(3428.57, 785.71, 1214.29)**	**(6000, 1571.43, 1214.29)**	**(6000, 0, 2000)**
30	(416.67, 2500, 1583.33)	**(3428.57, 785.71, 1214.29)**	**(6000, 1571.43, 1214.29)**	**(6000, 0, 2000)**
40	(0, 6000, 2000)	(0, 6000, 2000)	(0, 6000, 2000)	*(5000, 0, 2500)*
50	(0, 6000, 2000)	(0, 6000, 2000)	(0, 6000, 2000)	*(0, 7500, 5000)*

$b_5 = 50$	20	30	40	50
20	***(1285.71, 0, 2428.57)***	***(3857.14, 785.71, 2428.57)***	***(6428.57, 1571.43, 2428.57)***	***(9000, 0, 4000)***
30	*(416.67, 2000, 3166.67)*	***(3857.14, 785.71, 2428.57)***	***(6428.57, 1571.43, 2428.57)***	***(9000, 2357.14, 2428.57)***
40	*(833.33, 5500, 3166.67)*	*(833.33, 5500, 3166.67)*	**(6428.57, 1571.43, 2428.57)**	**(9000, 2357.14, 2428.57)**
50	*(0, 9000, 4000)*	*(0, 9000, 4000)*	*(0, 9000, 4000)*	(0, 9000, 4000)

Table 4.6: GenCos bidding sets and costs

ID	P_i^{max}	C_i	B_i
2	1200	10	$\{10, 20, 30, 40\}$
3	800	0	$\{9, 18, 20\}$
4	1000	15	$\{15, 25, 35, 45\}$

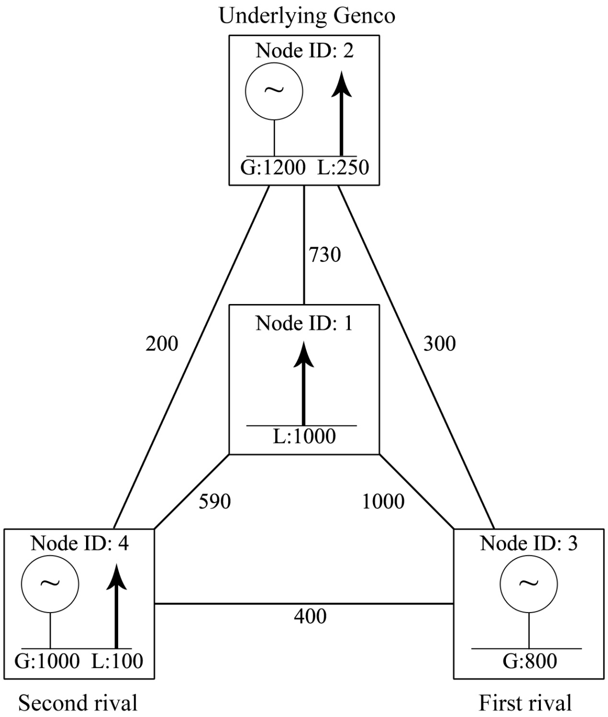

Figure 4.3: Structure of market

of GenCo-i at iteration t is $r_i^{(t)}$.

B.1 Conventional Q-learning

by using mathematical induction we know,

$$Q_{ij}^{(n)} = (1-\alpha_i)Q_{ij}^{(n-1)} + \alpha_i r_i^{(n)}; Q_{ij}^{(0)} = 0$$
$$\text{when } t = 1 \Rightarrow Q_{ij}^{(1)} = (1-\alpha_i)Q_{ij}^{(0)} + \alpha_i r_i^{(1)}$$
$$\text{when } t = 2 \Rightarrow Q_{ij}^{(2)} = (1-\alpha_i)Q_{ij}^{(1)} + \alpha_i r_i^{(2)} = \alpha_i r_i^{(2)} - (\alpha_i^2 - \alpha_i) r_i^{(1)}$$

The closed form is as follow.

$$Q_{ij}^{(n)} = (\alpha_i) \sum_{t=1}^{n} r_i^{(t)} (1-\alpha_i)^{(n-t)} \tag{B.1}$$

Boundary Condition 1.

When $\alpha_i = 1 \Rightarrow Q_{ij}^{(n)} = r_i^{(n)}$ from Eq.(B.1) by setting $\alpha_i = 1$ just for $t = n$ we get 1 from $(1-\alpha_i)^{n-t} = 0^0 = 1$.

Boundary Condition 2.

When $\epsilon_i = 0$. ϵ parameter determines how much exploration should be done. Therefore, if $\epsilon_i = 0$ then the GenCo only selects b_{ij} corresponds to $Q_{ij}^{(1)}$ (by assuming positive $r_i^{(1)}$), because all other $Q_{iJ}^{(1)} = Q_{iJ}^{(0)} = 0$ when $J \neq j$.

$$\begin{aligned} Q_{ij}^{(n)} &= r_i^{(1)} (1-\alpha_i)^n \left(\left(\frac{1}{1-\alpha_i} \right)^n - 1 \right) \\ &= r_i^{(1)} (1-(1-\alpha_i)^n) = r_i^{(1)} - (1-\alpha_i)^n r_i^{(1)} \end{aligned} \tag{B.2}$$

So, if we assume to conduct experiment up to infinity ($max_t = \infty$) then $\lim_{n\to\infty} Q_{ij}^{(n)} = r_i^{(1)}$. Therefore, GenCos cannot make more profit per iteration by increasing the number of iterations.

Boundary Condition 3.

Finally, if $\alpha_i = 0$ then $Q_{ij}^{(n)} = 0$ from Eq.(B.1).

B.2 Q-learning with variable learning rate

The closed form of Q-values in presence of variable learning rate is as follows.

$$Q_{ij}^{(n)} = \sum_{t=0}^{n} \alpha_{it} r_i^{(t)} \Pi_{\hat{t}=t+1}^{n} (1-\alpha_{i\hat{t}}) \tag{B.3}$$

Also, we have $\alpha_{it} = \alpha_{i0} - \frac{t}{n}(\alpha_{i0} - \frac{\alpha_{i0}}{10}) = \alpha_{i0} - \frac{9t\alpha_{i0}}{10n}$. Because in each iteration, $Q_{ij}^{(n)}$ is a convex combination of non-negative r_is, $Q_{ij}^{(n)}$ is non-negative.

Boundary Condition 4.

if $\epsilon = 0$ then from Eq.(B.3) we get following equation.

$$Q_{ij}^{(n)} = r_i^{(1)} \sum_{t=0}^{n} \alpha_{it} \Pi_{\hat{t}=t+1}^{n} (1 - \alpha_{i\hat{t}}) \tag{B.4}$$

because $r_i^{(n)} = r_i^{(1)}$. In this situation $Q_{ij}^{(n)}$ is not only affected by $r_i^{(1)}$ but is also a function of maximum number of iterations and the initial learning rate. We refer to $f(n) = \sum_{t=0}^{n} \alpha_{it} \Pi_{\hat{t}=t+1}^{n} (1 - \alpha_{i\hat{t}})$ as learning function when $\alpha_{it} = \alpha_{i0} - \frac{9t\alpha_{i0}}{10n}$. Fig.4.4 depicts the evolution of $f(n)$ over different n, considering different α_{i0}.

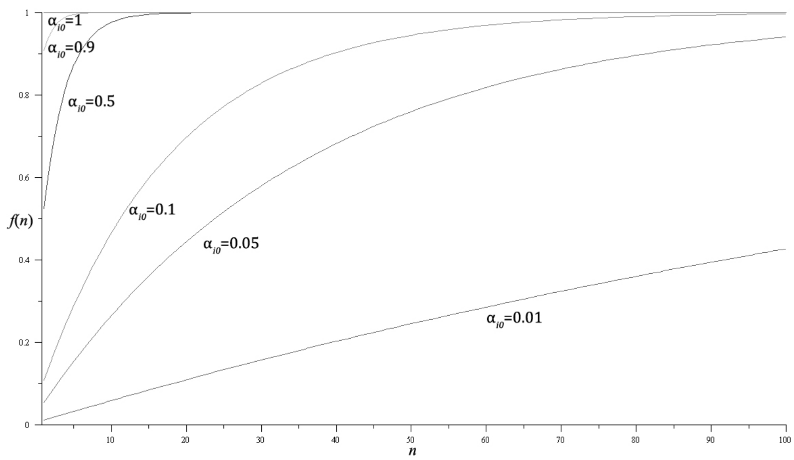

Figure 4.4: effect of $f(n)$ for different α_{i0} over different n

Proposition 16 (Bounded Learning Function) *$f(n)$ is an increasing bounded function that converges to a number between* $[0, 1]$.

Proof. we know that $f(n) = \sum_{t=0}^{n} \alpha_{i0} T_t \Pi_{\hat{t}=t+1}^{n} (1 - \alpha_{i0} T_{\hat{t}})$ when $T_t = 1 - \frac{9t}{10n} \in [1, 0.1]$ is a scale parameter and decreasing. Therefore, $f(n) \geq 0$ and $f(n) = 0$ when $\alpha_{i0} = 0$

which means no learning employed. However, we also need to find an upper bound for $f(n)$. The most dramatic increase in $f(n)$ happens when $\alpha_{i0} = 1$. Hence,

$$
\begin{aligned}
f(n)|_{\alpha_{i0}=1} &= \sum_{t=0}^{n} T_t \Pi_{\hat{t}=t+1}^{n} (1 - T_{\hat{t}}) \\
&= \left(\frac{9}{10n}\right)^{n+1} \Gamma(n+1) \sum_{t=0}^{n} \frac{(1 - \frac{9t}{10n})}{\left(\frac{9}{10n}\right)^{t+1} \Gamma(t+1)} \\
&= -\frac{9(n+1)\left(1 - \frac{9(n+1)}{10n}\right)\left(\frac{9}{10n}\right)^{n+1} \Gamma(n+1)}{(-n+9)(\frac{9}{10n})^{n+2}\Gamma(n+2)} = 1
\end{aligned}
$$

Thus, $f(n)$ is a bounded function $|f(n)| \leq 1$. Also, from definition of $f(n)$ we know, $f(n+1) - f(n) = \alpha_{in+1}(1 - f(n))$ and when $f(n) \leq 1$ therefore, $f(n+1) \geq f(n)$ hence $f(n)$ is an increasing sequence.

By using the Bolzano-Weistrass theorem, $f(n)$ converges to some point such as $f' \in [0, 1]$. □

By using Proposition.16, $Q_{ij}^{(n)} \leq r_i^{(1)}$. Also, $Q_{ij}^{(n)} = r_i^{(1)}$ when $\alpha_{i0} = 1$ for every $n > 0$ and $Q_{ij}^{(n)} = 0$ when $\alpha_{i0} = 0$.

Proposition 17 (Continuity of Q-value function) *for every* $\varepsilon > 0$ *there exists* $\delta_\alpha > 0$ *such that* $|\alpha_{i0} - 1| < \delta_\alpha$ *then* $\left|Q_{ij}^{(n)} - r_i^{(1)}\right| < \varepsilon$

Proof. By assuming n as real number, $f(n)$ is a continuous function, $Q_{ij}^{(n)} = r_i^{(1)} f(n)$ is also continuous and by definition of continuous function, we can find such an interval. Therefore, by increasing (n) a GenCo cannot make better profit than $r_i^{(1)}$. □

Thus, by checking our algorithm, one can comprehend by choosing $\epsilon_{i0} < \frac{8}{9}$ there exists $t \in [1, max_t]$ such that $\epsilon_{i\hat{t}} = 0$ when $\hat{t} \geq t$. Thus, conducting simulation for more iterations than t won't help GenCo-i to gain more profit per iteration than the best bid at time t (see Fig.4.5).

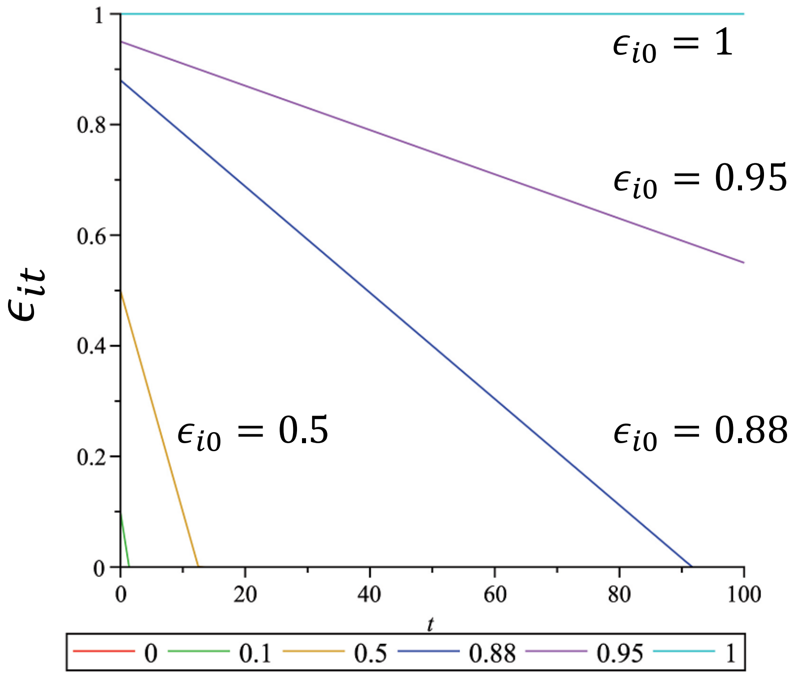

Figure 4.5: Linear decaying function with different ϵ_{i0} over time

Boundary Condition 5.

Contrary to **Boundary Condition 1**, $Q_{ij}^{(n)} \neq r_i^{(n)}$ when $\alpha_{i0} = 1$ because α-value changes during iterations. Hence, the historical payoff information from GenCo-i's bids in previous iterations are considered when α_{it} is a monotone decreasing sequence ($\alpha_{it} > \alpha_{i\hat{t}}$ for $t < \hat{t}$).