

Trajectory Control of a Quadrotor Using a Control Allocation Approach

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Abstract—A quadrotor is an underactuated unmanned aerial vehicle with four inputs to control the dynamics. Trajectory control of a quadrotor is a challenging task and usually tackled in a hierarchical framework where desired/reference attitude angles are analytically determined from the desired command signals, i.e. virtual controls, that control the positional dynamics of the quadrotor and the desired yaw angle is set to some constant value. Although this method is relatively straightforward, it may produce large and nonsmooth reference angles which must be saturated and low-pass filtered. In this work, we show that the determination of desired attitude angles from virtual controls can be viewed as a control allocation problem and it can be solved numerically using nonlinear optimization where certain magnitude and rate constraints can be imposed on the desired attitude angles and the yaw angle need not be constant. Simulation results for both analytical and numerical methods have been presented and compared. Results for constrained optimization show that the flight performance is quite satisfactory.

I. INTRODUCTION

Nowadays Unmanned Aerial Vehicles (UAV) are utilized in many civilian and military applications. Quadrotor is one of the most used kinds of UAV because of its ability to complete difficult tasks due to its vertical take off and landing (VTOL) capability. Quadrotor is a highly nonlinear system, so trajectory tracking is a challenging task. In recent years, there have been a number of papers dealing with various problems inherent to the exploitation of quadrotors dynamics. Trajectory control of quadrotors has gained great interest in the unmanned air vehicle community. Dynamic modeling issues were addressed in [1] and [2]. Bouabdallah [1] used the linear model and compared the results for PID controller with LQ controller. The results showed that hovering control was satisfactory for PID controller but for stability in the presence of disturbances, results were not so efficient. Different nonlinear control techniques such as sliding mode control, backstepping control and feedback linearization were used [3]–[6]. The sliding mode approach provided average results to stabilize the attitude while the structural changes affected the control quality because of the high-frequency disturbances. Comparison of feedback linearization and adaptive sliding mode to estimate uncertainty such as ground effects and noisy conditions was presented in [7]. Madani [8] used the technique of dividing

quadrotor dynamics into many linearly connected subsystems and proposed a full-state backstepping and sliding mode control technique based on the Lyapunov stability theory for quadrotor to track the desired trajectories. Adaptive fuzzy backstepping technique was used [9], in which a control law for the model was generated by fuzzy system using backstepping approach. Robust trajectory tracking was presented which used combination of integral backstepping and PID controller to stabilize the dynamics [10]. Global trajectory tracking control [11] was proposed without linear velocity measurements but efficiency in terms of position errors was not so good. Flatness-based control of a quadrotor via feedback linearization was performed in [12]. H_∞ and model predictive control were used to solve the problem of trajectory control [13]. Design and development of a tilt-wing quadrotor and its robust control using a hierarchical structure were considered in [14]–[16]. It consists of two parts: upper level control for positional dynamics which generates virtual controls and lower level control which provides tracking of desired attitude angles computed from virtual controls. Desired or reference angles were obtained through analytical formulas. More recently adaptive nonlinear hierarchical control of a tilt-wing quadrotor was developed in [17] and [18]. Chan also applied hierarchical control where motor speed control and attitude control were utilized [19]. Optimized trajectory planning algorithms had been applied using nonlinear optimization and results were shown for linear path where desired angles were obtained through analytical method [20].

In this work, a control allocation type technique is formulated on the nonlinear dynamics of the quadrotor to obtain optimum values for the desired attitude angles from command signals (virtual controls) developed for positional dynamics. Different from an existing study [21] where control allocation approach is applied to distribute the total control effort among real actuators optimally, here the command signals are utilized to get the values for desired attitude angles. With the help of hierarchical control, controller design is divided into two parts: position control and attitude control. Positional dynamics of the quadrotor is considered as the underdetermined part as it contains more unknown variables than the equations. In control allocation, nonlinear constraint optimization is used to obtain required actuator inputs according to command signals by solving an underdetermined system [22]. In our approach, high level controller is designed to obtain the desired command signals from the positional dynamics. Nonlinear constrained optimization is used to get desired attitude angles $(\phi_r, \theta_r, \psi_r)$ according to the command signals. Sequential quadratic programming (SQP) algorithm is used in nonlinear constraint optimiza-

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tion. Low level controllers are implemented to ensure that the attitude angles are adjusted according to desired trajectory. Analytical method for the calculation of reference angles is also applied and results are shown for the same trajectories. Simulation results for both analytical and numerical methods have been presented for comparison to show the efficiency of the proposed technique.

The organization of the paper is as follows: Section II outlines the dynamics of the quadrotor. Section III presents the control allocation and nonlinear optimization. Section IV presents the controller design. Section V and VI present the simulation results and the summary of the paper respectively.

II. QUADROTOR DYNAMICS

The quadrotor is a type of aerial robot which consists of four rotors separated in a cross structure. The crossed configuration shows robustness even the mechanically linked motors are heavier than the frame [23]. Reduction gears are used to connect propellers to the motors. The motion of the quadrotor depends upon the direction of the rotation of the propellers of rotors. Front and rear propellers rotate counter clockwise, while the left and the right ones turn clockwise. There is no need of the tail rotor because of opposite pairs directions configuration unlike the standard helicopter structure. Fig. 1 shows the model in a hovering state, where all the propellers have the same speed. The motion of a

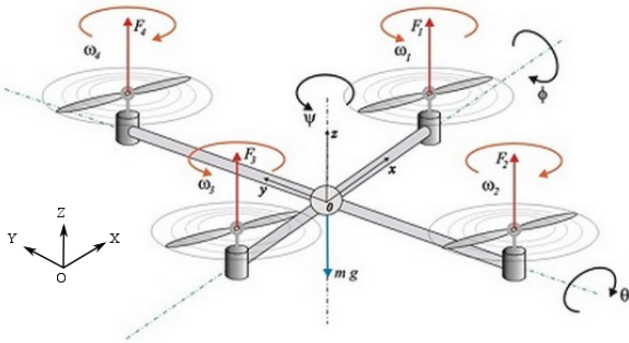


Fig. 1: Quadrotor Dynamics (Adapted from [24])

quadrotor is described by two frame of references, one of which is fixed and called inertial frame and the other one which is moving, called body frame. The vertical force w.r.t body-fixed frame is provided by thrust command (U_1) which is used to raise (or lower) the quadrotor by increasing and decreasing the speed of the propellers equally respectively. Similarly, increasing (or decreasing) the left propeller speed and decreasing (or increasing) the right one results into roll command (U_2), which makes the quadrotor to turn due to the torque with respect to the x axis. Pitch command (U_3) is very similar to the roll but in this case, increase (or decrease) in the rear propeller speed and decrease (or increase) in the front one leads to a torque with respect to the y axis which makes the quadrotor to turn. In order to enable the quadrotor to turn along z axis, torque is provided by the yaw command (U_4)

which is provided by increasing (or decreasing) the front-rear propellers speed and by decreasing (or increasing) the speed of the left-right couple propellers. Detailed description of the quadrotor dynamics can be found in [23]. The quadrotor positional dynamics expressed in world frame and attitude dynamics expressed in body frame are given as

$$\begin{aligned}\ddot{X} &= (\sin \psi \sin \phi + \cos \psi \sin \theta \cos \phi) \frac{U_1}{m} \\ \ddot{Y} &= (-\cos \psi \sin \phi + \sin \psi \sin \theta \cos \phi) \frac{U_1}{m} \\ \ddot{Z} &= -g + (\cos \theta \cos \phi) \frac{U_1}{m} \\ \dot{p} &= \frac{I_{yy} - I_{zz}}{I_{xx}} qr - \frac{J_{prop}}{I_{xx}} + \frac{U_2}{I_{xx}} \\ \dot{q} &= \frac{I_{zz} - I_{xx}}{I_{yy}} pr + \frac{J_{prop}}{I_{yy}} + \frac{U_3}{I_{yy}} \\ \dot{r} &= \frac{I_{xx} - I_{yy}}{I_{zz}} pq + \frac{U_4}{I_{zz}}\end{aligned}\quad (1)$$

where U_1, U_2, U_3 and U_4 are defined as

$$\begin{aligned}U_1 &= b(\omega_1^2 + \omega_2^2 + \omega_3^2 + \omega_4^2) \\ U_2 &= lb(-\omega_2^2 + \omega_4^2) \\ U_3 &= lb(-\omega_1^2 + \omega_3^2) \\ U_4 &= d(-\omega_1^2 + \omega_2^2 - \omega_3^2 + \omega_4^2)\end{aligned}\quad (2)$$

where $\omega_1, \omega_2, \omega_3$ and ω_4 are the speed of front, right, rear and left propellers; l, b and d are the length of the rotor arm, thrust factor and drag factor, respectively. First three equations represent the positional dynamics and last three equations describe the orientation of the quadrotor in (1).

The relationship between angular velocities of quadrotor in body frame (η) and the Euler rates in world frame (Ω) is given by

$$\eta = E(\Theta)\Omega \quad (3)$$

where $E(\Theta)$ is the velocity transformation matrix defined as

$$E(\Theta) = \begin{bmatrix} 1 & \sin\phi\tan\theta & \cos\phi\tan\theta \\ 0 & \cos\phi & -\sin\phi \\ 0 & \frac{\sin\phi}{\cos\theta} & \frac{\cos\phi}{\cos\theta} \end{bmatrix}$$

where $\Theta = [\phi, \theta, \psi]^T$, $\Omega = [\dot{\phi}, \dot{\theta}, \dot{\psi}]^T$ and $\eta = [p, q, r]^T$.

III. CONTROL ALLOCATION AND NONLINEAR OPTIMIZATION

A. Control Allocation

Motion control is used to control the motion of the mechanical systems. All mechanical systems require control forces which are produced by the end effectors. Actuators are the devices which are used to control these forces. Sometimes there are more actuators than required. In that case system becomes underdetermined due to more number of unknown variables than the number of equations. Control allocation approach is used for such underdetermined systems. The purpose of the control allocation is to distribute the desired

control effort to all redundant actuators. In order to provide optimal solution in the presence of coupling among the actuators due to underdetermined nature, optimization techniques are used for this purpose. Control allocation is a hierarchical type algorithm which consists of the following three parts [25].

High level controller is used to produce virtual command inputs.

Optimization is used to distribute the total virtual command among the actuators through linear and nonlinear optimization depending upon the cost function to be minimized and constraints.

Low level controller is used to produce required force at each end effector with the help of actuators.

Control allocation algorithm is advantageous due to the following reasons.

- 1) It is used to solve problems like actuator position and rate saturation.
- 2) It is used to produce fault tolerant system in case of failure of any actuators.
- 3) It is used to produce size and cost effective mechanical design by choosing the optimal set of the actuators rather than the small number of the actuators.

In this work positional dynamics of the quadrotor is exploited to apply the control allocation approach. If we look at the positional dynamics of the quadrotor in (1), it consists of three equations and four unknown variables $(\phi, \theta, \psi, U_1)$, so it can be considered as underdetermined system. In [22] control allocation approach had been used to solve underdetermined system where nonlinear optimization problem had been formulated. As underdetermined part of the quadrotor consists of nonlinear equations, nonlinear optimization is required to get the optimal solution. The purpose of the control allocation is to generate command inputs that must be produced jointly by all actuators which in our case are ϕ, θ, ψ and U_1 . Optimization problem along with the nonlinear and linear constraints is formulated as

$$J(\zeta) = \frac{1}{2}(s^T s) \quad (4)$$

where $J(\zeta)$ is the cost function to be minimized and s is a slack variable. In our problem ζ is given by:

$$\zeta = [\phi, \theta, \psi, U_1] \quad (5)$$

Cost function is minimized subject to the following nonlinear and linear constraints

$$s = \tau - B(\zeta) \quad (6)$$

$$\zeta_{min} \leq \zeta \leq \zeta_{max} \quad (7)$$

$$\Delta\zeta \leq C \quad (8)$$

where τ is the desired command inputs that is provided by the high level controller of the hierarchical control. τ and $B(\zeta)$ are given as

$$\tau = [\ddot{X} \ \ddot{Y} \ \ddot{Z}]^T \quad (9)$$

$$B(\zeta) = \begin{bmatrix} (\sin \psi \sin \phi + \cos \psi \sin \theta \cos \phi) \frac{U_1}{m} \\ (-\cos \psi \sin \phi + \sin \psi \sin \theta \cos \phi) \frac{U_1}{m} \\ -g + (\cos \theta \cos \phi) \frac{U_1}{m} \end{bmatrix} \quad (10)$$

ζ_{min} and the ζ_{max} are the constrained range for ζ . Actuator rate constraint $\Delta\zeta$ is included in the formulation by limiting the change in the control inputs ζ from the last sampling instant to some constant C .

B. Nonlinear Optimization

Optimization deals with finding the feasible solution for n variables to minimize or maximize any function. Nonlinear programming is a mathematical tool, used to minimize the cost function subject to linear and nonlinear constraints. Feasible regions show the set of optimized variables which lie in the range of constraints. In unconstrained optimization feasible solutions converge to some finite values. In constrained optimization, nonlinear problem is converted into many subspaces and then different algorithms are applied to obtain optimum solution according to constraints.

Sequential quadratic programming (*SQP*) is a nonlinear optimization tool which is one of the most effective iterative methods. Advantage of using *SQP* is to find good initial point for feasible solution. Active set method is used in *SQP*. It converts nonlinear problem into the subspace based on Lagrange approximation. Lagrange function is given by:

$$L(\zeta, \lambda) = J(\zeta) + \sum_{i=1}^m \lambda_i \cdot g_i(\zeta) \quad (11)$$

where $g_i(\zeta)$ is a gradient and λ_i is a Lagrange multiplier. Detailed description of *SQP* is given in [26], briefly it consists of following three parts:

Hessian Matrix part uses positive definite quasi Newton optimization method for Lagrange function using Broyede-Fletcher-Golfarb-Shanon(*BFGS*) [26] method at every instant k .

$$H_{k+1} = H_k + \frac{q_k q_k^T}{q_k^T l_k} - \frac{H_k l_k l_k^T H_k}{l_k^T H_k l_k} \quad (12)$$

where

$$l_k = \zeta_{k+1} - \zeta_k \quad (13)$$

$$q_k = \nabla J(\zeta_{k+1}) + \sum_{i=1}^m \lambda_i \cdot g_i(\zeta) - (\nabla J(\zeta_k) + \sum_{i=1}^m \lambda_i \cdot g_i(\zeta))$$

This equation is used to neutralize the gradients between cost function and active constraints, so for balancing, the magnitudes of the Lagrange multipliers are necessary. Positive definiteness of the Hessian matrix is ensured providing $q_k^T l_k$ is positive at each iteration. When $q_k^T l_k$ is not positive, q_k is modified on element by element basis so that q_k is positive [26].

Quadratic Programming Solution part consists of two steps: first step gives the feasible point and the second step produces iterative process of feasible points to converge

within the constraints. During this part of nonlinear programming, problem is converted to subproblems and then QP is used to solve, which is formulated as

$$\begin{aligned} \min & \frac{1}{2} d^T H_k d + \nabla J(\zeta_k)^T d \\ \nabla g_i(\zeta_k)^T d + g_i(\zeta_k) &= 0, i = 1, 2, \dots, m_e \\ \nabla g_i(\zeta_k)^T d + g_i(\zeta_k) &\leq 0, i = m_e + 1, \dots, m \end{aligned} \quad (14)$$

The following QP form is used at every iteration.

$$\min q(d) = \frac{1}{2} d^T H d + c^T d \quad (15)$$

subject to following equality and inequality constraints.

$$\begin{aligned} A_i d &= b_i, i = 1, 2, \dots, m_e \\ A_i d &\leq b_i, i = m_e + 1, \dots, m \end{aligned} \quad (16)$$

The solutions of quadratic programming will give d_k which is feasible region search direction. Active constraints are updated at every iteration to form a basis for new search direction d_k .

Optimal Line Search part of the programming is used to produce new iteration using vector d_k .

$$\zeta_{k+1} = \zeta_k + \alpha d_k \quad (17)$$

where α is the step length parameter, which ensures that ζ_{k+1} remains in the feasible region.

IV. CONTROLLER DESIGN

Hierarchical control is used as control approach for controller design. PID controllers are used as high level controller to provide virtual command inputs which are used to get reference angles $(\phi_r, \theta_r, \psi_r)$ with the help of nonlinear optimization. Acquired reference angles are used to produce desired trajectory with the help of the low level controller as shown in closed loop control structure in Fig. 2.

A. Position Control

From the positional dynamics of quadrotor in (1)

$$\begin{aligned} \mu_X &= \ddot{X} = (\sin \psi \sin \phi + \cos \psi \sin \theta \cos \phi) \frac{U_1}{m} \\ \mu_Y &= \ddot{Y} = (-\cos \psi \sin \phi + \sin \psi \sin \theta \cos \phi) \frac{U_1}{m} \\ \mu_Z &= \ddot{Z} = -g + (\cos \theta \cos \phi) \frac{U_1}{m} \end{aligned} \quad (18)$$

Errors are defined as

$$\begin{aligned} e_X &= X_r - X \\ e_Y &= Y_r - Y \\ e_Z &= Z_r - Z \end{aligned} \quad (19)$$

Error dynamics can be formulated as

$$\begin{aligned} \dot{e}_X &= \dot{X}_r - \dot{X} \Rightarrow \ddot{e}_X = \ddot{X}_r - \ddot{X} = \ddot{X}_r - \mu_X \\ \dot{e}_Y &= \dot{Y}_r - \dot{Y} \Rightarrow \ddot{e}_Y = \ddot{Y}_r - \ddot{Y} = \ddot{Y}_r - \mu_Y \\ \dot{e}_Z &= \dot{Z}_r - \dot{Z} \Rightarrow \ddot{e}_Z = \ddot{Z}_r - \ddot{Z} = \ddot{Z}_r - \mu_Z \end{aligned} \quad (20)$$

μ_X, μ_Y and μ_Z consist of feed forward and feedback terms, which are defined as

$$\begin{aligned} \mu_X &= \ddot{X}_r + K_{p,X} e_X + K_{d,X} \dot{e}_X + K_{i,X} \int e_X dt \\ \mu_Y &= \ddot{Y}_r + K_{p,Y} e_Y + K_{d,Y} \dot{e}_Y + K_{i,Y} \int e_Y dt \\ \mu_Z &= \ddot{Z}_r + K_{p,Z} e_Z + K_{d,Z} \dot{e}_Z + K_{i,Z} \int e_Z dt \end{aligned} \quad (21)$$

In [15] analytical method was used to calculate the desired attitude angles of the aerial vehicle from desired acceleration vector (τ) by assuming yaw angle (ψ) to be some fixed value (ψ^*) . Desired angles are calculate as

$$U_1 = \sqrt{\mu_X^2 + \mu_Y^2 + (\mu_Z + g)^2} \quad (22)$$

$$\theta_r = -\arcsin\left(\frac{\mu_X}{U_1}\right) \quad (23)$$

$$\phi_r = \arcsin\left(\frac{\mu_Y}{U_1 \cos \theta_r}\right) \quad (24)$$

B. Attitude Control

In order to develop controllers for attitude control, from (1) attitude dynamics can be linearized around hover conditions i.e. $\phi, \theta, \psi \approx 0$ and $\dot{\phi}, \dot{\theta}, \dot{\psi} \approx 0$. Angular accelerations in body and world frames will be approximately equal after linearization i.e. $\ddot{p} \approx \ddot{\phi}, \ddot{q} \approx \ddot{\theta}, \ddot{r} \approx \ddot{\psi}$ [15]. Resulting attitude dynamics is given by:

$$\ddot{\phi} = \frac{U_2}{I_{xx}}, \ddot{\theta} = \frac{U_3}{I_{yy}}, \ddot{\psi} = \frac{U_4}{I_{zz}} \quad (25)$$

Errors are defined as

$$e_\phi = \phi_r - \phi, e_\theta = \theta_r - \theta, e_\psi = \psi_r - \psi \quad (26)$$

Similarly error dynamics are defined as

$$\begin{aligned} \dot{e}_\phi &= \dot{\phi}_r - \dot{\phi} \Rightarrow \ddot{e}_\phi = \ddot{\phi}_r - \ddot{\phi} \\ \dot{e}_\theta &= \dot{\theta}_r - \dot{\theta} \Rightarrow \ddot{e}_\theta = \ddot{\theta}_r - \ddot{\theta} \\ \dot{e}_\psi &= \dot{\psi}_r - \dot{\psi} \Rightarrow \ddot{e}_\psi = \ddot{\psi}_r - \ddot{\psi} \end{aligned} \quad (27)$$

Attitude controllers are designed as

$$\begin{aligned} U_2 &= I_{xx} \left(\ddot{\phi}_r + K_{p,\phi} e_\phi + K_{d,\phi} \dot{e}_\phi + K_{i,\phi} \int e_\phi dt \right) \\ U_3 &= I_{yy} \left(\ddot{\theta}_r + K_{p,\theta} e_\theta + K_{d,\theta} \dot{e}_\theta + K_{i,\theta} \int e_\theta dt \right) \\ U_4 &= I_{zz} \left(\ddot{\psi}_r + K_{p,\psi} e_\psi + K_{d,\psi} \dot{e}_\psi + K_{i,\psi} \int e_\psi dt \right) \end{aligned} \quad (28)$$

V. SIMULATION RESULTS

A control allocation type approach has been implemented where reference angles are generated through numerical method using nonlinear optimization. Analytical method for reference angles calculation is also considered for comparison. For simulation, the following two scenarios are taken into account.

- 1) During the first scenario yaw angle (ψ) is fixed to a constant value (ψ^*) for both numerical and analytical methods.

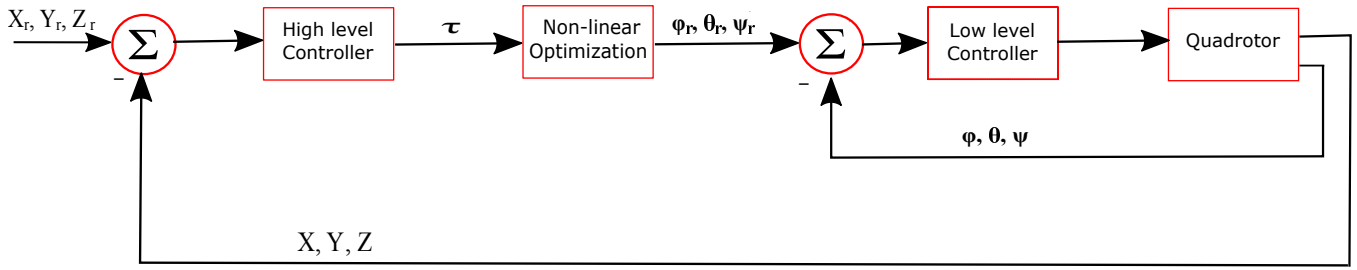


Fig. 2: Closed Loop Control System

2) During the second scenario, numerical method has been taken into account and yaw angle (ψ) is not fixed and constraints have been put on all reference angles. For nonlinear optimization, stopping criterion specifications are summarized in the TABLE I.

TABLE I: Nonlinear Optimization Specifications

Specifications	Range
<i>Max Iterations</i>	400
<i>Function Tolerance</i>	1e-06
<i>Constraint Tolerance</i>	1e-06
<i>Max Function Evaluations</i>	100
<i>First Order Optimality Measure</i>	1e-06

For trajectory it is assumed that quadrotor takesoff vertically from the spot along the z axis, then hovers at desired altitude, takes a straight flight along the x axis, moves in a circular loop and lands at the desired spot after taking some more straight flight. Trajectory has been shown in Figure 3.

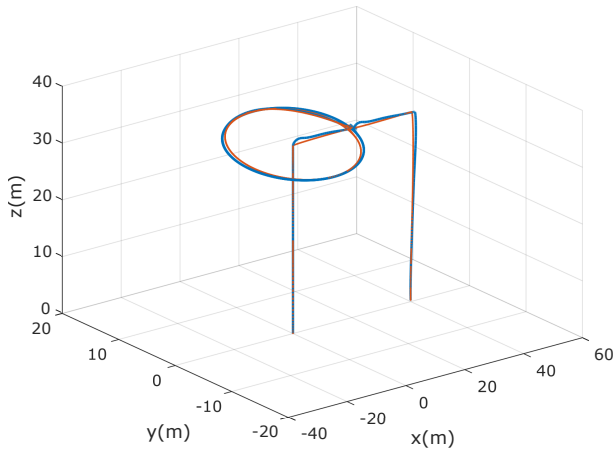


Fig. 3: Trajectory Tracking (Desired=red, Actual=blue--)

During the first scenario of the simulation yaw angle (ψ) is taken to be fixed at 3.5° for the calculation of reference angles through analytical and numerical method. Cartesian positions of the quadrotor during the trajectory for both methods have been shown in Figure 4 which shows that quadrotor successfully tracked the desired trajectory for both methods.

Position errors in Figure 5 show the efficiency of tracking for the numerical method. It has been observed that cartesian

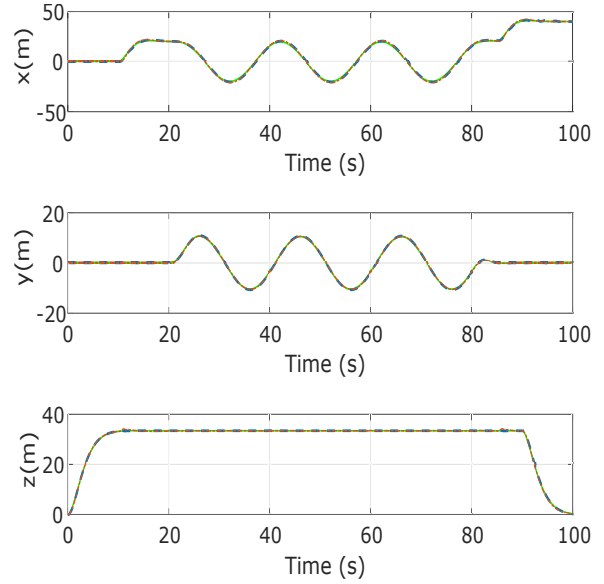


Fig. 4: Cartesian positions of the quadrotor vs Time (Desired=green, Analytical=blue-- , Numerical=red-.)

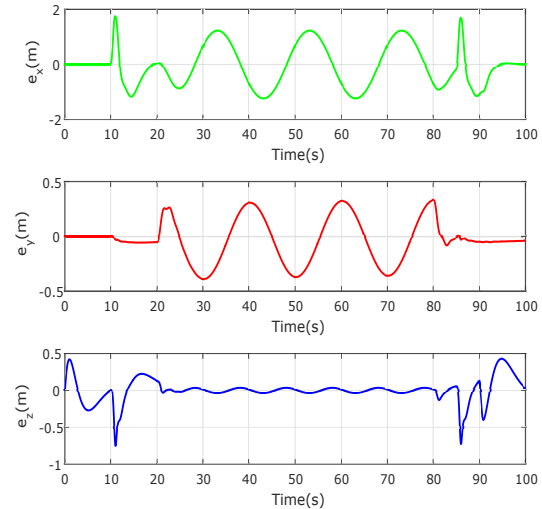


Fig. 5: Position Errors

positions of the quadrotor demonstrate similar behavior during the circular path but during the transition from hovering

TABLE II: Euler Angles and Total Thrust (U_1) Bound Values

Euler Angles	Range	Rate
ϕ	-14 to 14 (deg)	≤ 8.5 (deg/sample)
θ	-14 to 14 (deg)	≤ 8.5 (deg/sample)
ψ	-7 to 7 (deg)	≤ 5.5 (deg/sample)
U_1	0 to 20	≤ 10

state to straight flight, numerical method showed smoother results than analytical method of finding reference Euler angles which can be seen during 10 to 20 seconds and after 85 seconds in Figure 6.

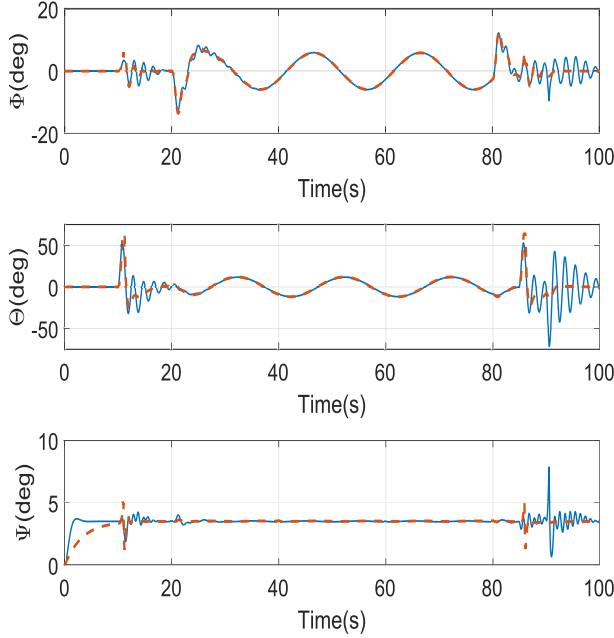


Fig. 6: Euler Angles (Analytical=solid, Numerical=dashed)

In the second scenario, yaw angle (ψ) is not taken to be fixed; instead constraints have been put on all reference angles which are given in TABLE II. Simulation results for trajectory curves have been shown for constrained optimization in Figure 7, which shows that quadrotor successfully tracked the trajectory. In Figure 8 comparison of Euler angles for constrained optimization in the first scenario when the yaw angle (ψ) is fixed and for constraint optimization for range of values for all reference angles including yaw angle (ψ), shows that Euler angles not only change in a smooth fashion but also remain within the bounds for the same trajectory. In Figure 9 control efforts for numerical method when yaw angle (ψ) is not fixed are plotted and it is clear that each one of them stays within physical bounds.

VI. CONCLUSION AND FUTURE WORK

In this paper, a control allocation like method using nonlinear constraint optimization is implemented on the nonlinear dynamics of the quadrotor, which consists of high level control and low level control. High level controller is used to get necessary desired generalized command signals. Nonlinear constraint optimization is used at every instant to get the

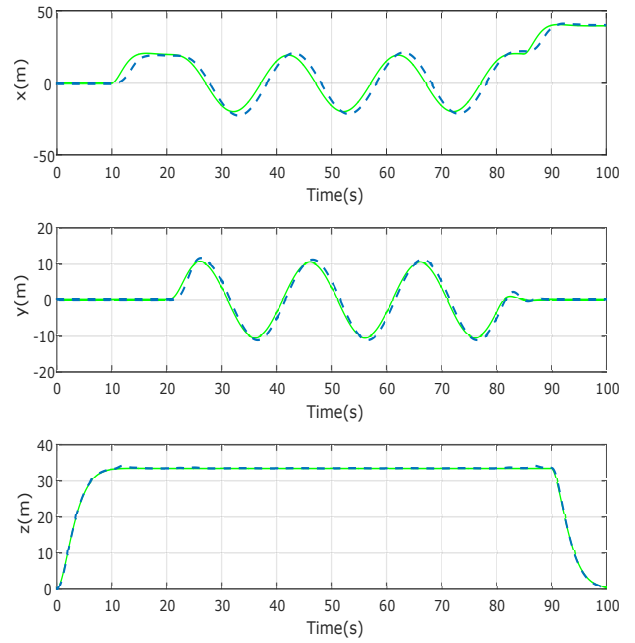


Fig. 7: Cartesian positions of the quadrotor vs Time for bounded angles optimization (Desired=solid, Actual=dashed)

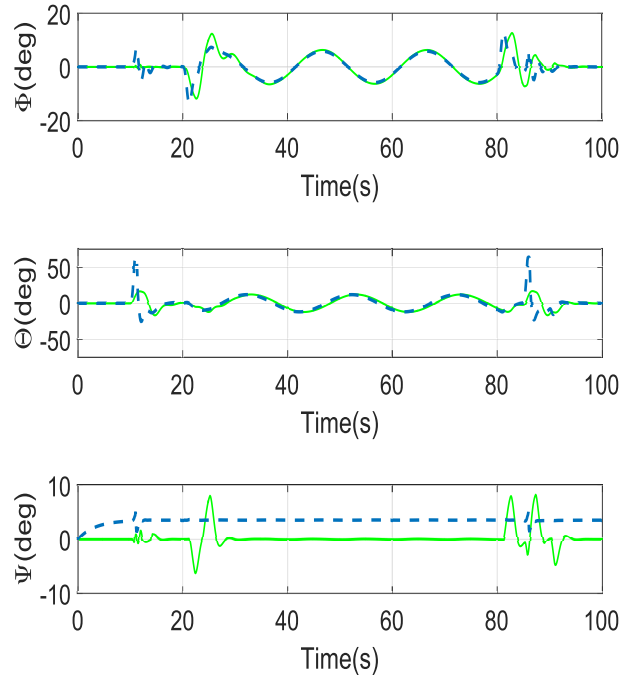


Fig. 8: Euler Angles (For fixed psi (ψ^*)=dashed, for bounded values of psi (ψ)=solid)

reference angles for the desired trajectory tracking according to generalized commands. Low level controllers are used for attitude control. Hierarchical control allows us to design separately the controllers for position and attitude for highly coupled nonlinear dynamics of quadrotor. For trajectory, it has been considered that vehicle moves in a circular loop after hovering at the desired altitude. For trajectory tracking, two scenarios have been taken into account based on yaw

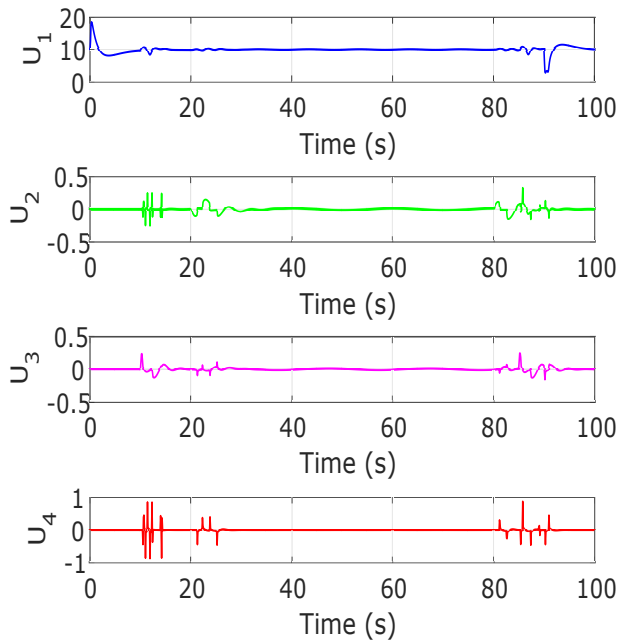


Fig. 9: Control Efforts

angle (ψ). Simulation results for both scenarios have been shown for both analytical and numerical methods of finding reference angles for comparison. From results it has been inferred that both methods provided successful trajectory tracking but numerical method showed smoother changes in Euler angles than the analytical method. Simulation results have also been presented for the second scenario when the yaw angle is not taken as fixed; instead bounded constraints have been considered for optimization. Results showed that quadrotor successfully tracked the trajectory with the desired attitude angles remaining within the bounds for the whole time period.

As future work, we will work on making the proposed technique real-time and then test it on a physical quadrotor.

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