MULTI-DIMENSIONAL MODELLING OF CHATTER STABILITY IN PARALLEL TURNING OPERATION

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ABSTRACT

Recently, use of parallel turning operations has been increasing due to the potential they offer for improved material removal rate using multiple cutting tools. However, chatter vibrations resulting from process instability may limit the productivity in these operations. In order to determine favourable conditions for increased stability, dynamics of parallel turning operations must be modelled. Herein dynamics and stability of parallel turning operations including both cutter and workpiece flexibility is studied. Frequency and time domain models for multi-dimensional parallel turning have been developed where effects of process parameters on chatter behaviour are investigated.

Keywords: Parallel turning operation, Chatter stability, Cutting process modelling.

1. INTRODUCTION

Parallel machining operation makes the cutting process possible using multiple cutters, simultaneously. Extra provided cutting edges, in contrast with single cutter operation, enhances the Material Removal Rate (MRR); hence, the productivity of the process. One of the most fatal threats to a productive and precise cutting process in simultaneous machining is regenerative chatter phenomenon. An ongoing practical impediment on parallel turning operation, where chatter stability plays a major role in, is machining of slender and flexible workpieces for various industries such as aerospace and energy. In this process, not only the dynamic interaction between the tools but also the dynamic effect of the workpiece is an important factor in stability analysis; therefore, selecting appropriate process parameters in order to ensure a chatter-free condition would be possible by having a comprehensive insight over the dynamics of that system.

Tobias and Fishwick [1], and Tlusty and Polacek [2], almost half a century ago were the first researchers who introduced ‘regenerative chatter’ for a simple but practical orthogonal cutting process in turning. Oscillating tool leaves waves on workpiece surface due to vibrations which affects the chip thickness. In the next revolution, if the depth of cut is big enough, the oscillating wave’s amplitude and corresponding cutting force may increase. Successive increases in dynamic chip thickness and force called ‘regenerative chatter’. Even though, the number of publications on the one dimensional approach to chatter analysis is high, there are few authors who investigated a general model for stability analysis of a turning operation. Rao [3] developed an analysis taking into account the nose radius and side edge cutting angle effects in order to improve stability predictions. Later, Ozlu and Budak [4] presented an analytical multi-dimensional model to predict the stability limits in turning and boring processes. Their model considers all important parameter of the process geometry as well as the effects of tool and workpiece in different directions. Nevertheless, the stability of parallel turning operation has not been investigated extensively. Lazoglu [5] proposed a time-domain approach to stability of parallel turning system with two tools which are clamped on different turrets and cut different surfaces. The model includes the effect of both the tools and the workpiece. They demonstrated that for relatively flexible workpiece system, the stability limit in a conventional turning operation is higher than the one in parallel process. Later, Ozturk and Budak [6] proposed a frequency-domain solution to stability of orthogonal parallel turning operation. In their model, the cutters were coupled through the shared surface cutting different depth of cuts. They showed that stability limit could increase according to the dynamic interaction between the tools which creating an absorber effect. Brecher et al. [7] investigated the parallel turning operation considering various dynamic coupling through the machine structure. They showed that the radial angle between the tools has huge effects on stable depth of cut in the process. Ozturk et al. [8] for the first time show the effects of natural frequency ratio of the tools in chatter problem. By adding or removing mass, and changing the tool holder's length the system has been tuned for enhanced productivity. The results have been shown that dynamically identical tools give the worst stability limit.

In this paper, a multi-dimensional stability model for parallel turning operation is presented. For the first time, effect of side edge cutting angle of the tools has been considered in parallel turning operation. Consequently, tool and workpiece dynamic compliance have to be considered in order to have a more accurate model in predicting the stability limits for parallel turning operations. Frequency and time domain stability models for multi-dimensional parallel turning have been developed where effects of process parameters on chatter behavior are investigated.

2. STABILITY MODEL FOR PARALLEL TURNING OPERATION

Stability formulation has been presented for two tools which are clamped separately on different turrets but cutting a shared surface on the workpiece with a same depth of cut. The schematic configuration of the system has been demonstrated in Figure. 1.

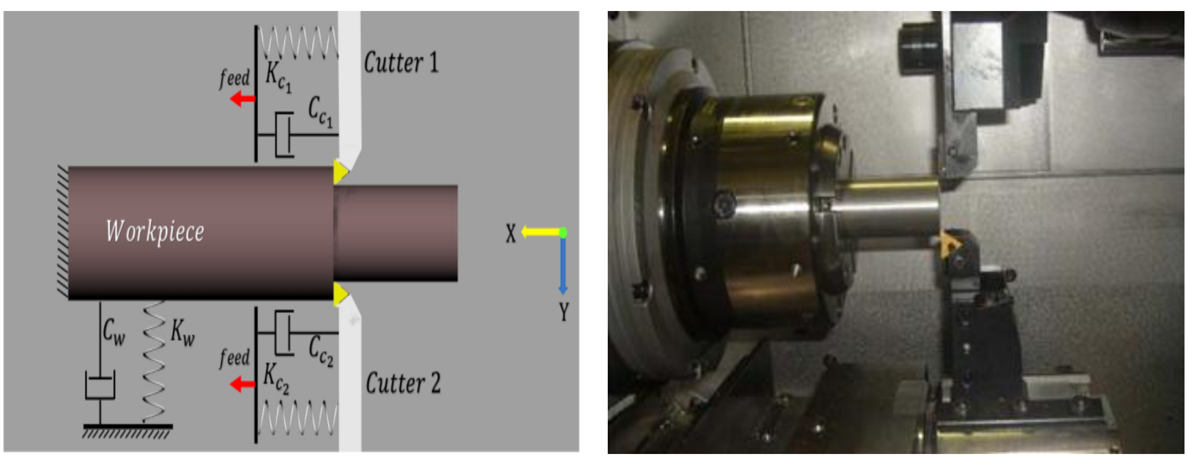


Figure. 1. Parallel turning process

As it can be seen from Figure. 1. both cutters are cutting the same depth of material. In order to make a cut, one of the tools must be ahead of the others. But it should be noted that the offset must be less than half period of a revolution in order to have both of the tools remove material simultaneously. The tools are clamped on different turrets; hence, they are not dynamically coupled. However, both cutters are cutting a shared surface of the workpiece, and thus waviness on the surface due to vibrations of one of tools causes variation of the chip thickness on the other tool. Therefore, they are dynamically dependent. Since presence of side edge cutting angle affects the modulated chip thickness in both feed and radial directions, the dynamic effect of system in both directions must be included. In fact, the tools are assumed to be flexible in the feed direction whereas the workpiece is assumed to be flexible in the radial direction only.

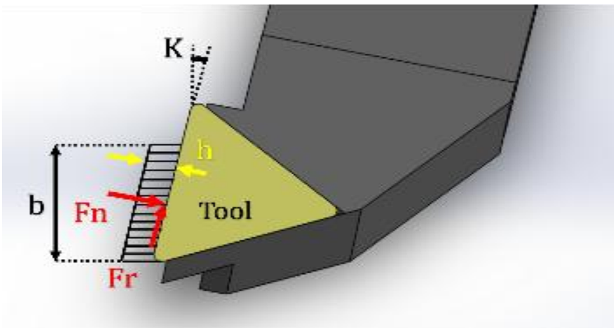


Figure. 2. Forces and modulated chip thickness on a tool with side edge cutting angle

In order to develop a stability model, the modulated chip thickness and forces parallel and perpendicular to the cutting edges of the tools must be projected in the global coordinates of the lathe. It is worth noticing that tangential force acting on the rake face of the tools does not affect the dynamic modulated chip thickness.

$$h_1(t)=\frac{h_0}{2}\cos K_1+\left[x_{c1}(t)-x_{c2}\left(t-\frac{\tau}{2}\right)\right]\cos K_1+\left[-y_w(t)+y_w\left(t-\frac{\tau}{2}\right)\right]\sin K_1 \quad (1)$$

$$h_2(t)=\frac{h_0}{2}\cos K_2+\left[x_{c2}(t)-x_{c1}\left(t-\frac{\tau}{2}\right)\right]\cos K_2+\left[y_w(t)-y_w\left(t-\frac{\tau}{2}\right)\right]\sin K_2$$

Dynamic chip thickness on a tool is affected by the displacement of the tool at the present time and the displacement of the other tool at a half rotation period (τ /2) before. The feed per revolution (h_0) is shared equally between the tools the static chip thickness. Since the static chip thicknesses on the tools do not affect the regeneration mechanism, they can be eliminated from the stability formulation. In Equation 1, t represents the time, x_{c1}, x_{c2}, y_w are the dynamic displacement of the first and the second cutter and workpiece in feed and radial directions, respectively. Furthermore, K_1 and K_2 are the side edge cutting angles for the first and the second tools. After determining the chip thickness for each tool, we can calculate the acting forces parallel and perpendicular to the cutting edge as follows:

$$\begin{Bmatrix} F_f^{c1} \\ F_r^{c1} \end{Bmatrix} = \frac{b}{\cos K_1} \begin{bmatrix} K_f^{c1} \\ K_r^{c1} \end{bmatrix} \times \begin{bmatrix} \cos K_1 & \sin K_1 \end{bmatrix} \begin{Bmatrix} +x_{c1}(t) - x_{c2}\left(t - \frac{\tau}{2}\right) \\ -y_w(t) + y_w\left(t - \frac{\tau}{2}\right) \end{Bmatrix} \tag{2}$$

$$\begin{Bmatrix} F_f^{c2} \\ F_r^{c2} \end{Bmatrix} = \frac{b}{\cos K_2} \begin{bmatrix} K_f^{c2} \\ K_r^{c2} \end{bmatrix} \times \begin{bmatrix} \cos K_2 & \sin K_2 \end{bmatrix} \begin{Bmatrix} x_{c2}(t) - x_{c1}\left(t - \frac{\tau}{2}\right) \\ y_w(t) - y_w\left(t - \frac{\tau}{2}\right) \end{Bmatrix}$$

where b is the depth of cut and K_f^{c1}, K_r^{c1}, K_f^{c2} and K_r^{c2} are cutting coefficients in feed and radial direction for first and second tool correspondingly. Since edge forces do not take part in regenerative process they are not considered in the dynamic cutting force formulations. By transforming the local coordinate (on the tool side edge) to the global coordinate (on the lathe), the cutting forces can be written as follows:

$$\begin{Bmatrix} F_x^{c1} \\ F_y^{c1} \end{Bmatrix} = \begin{bmatrix} -\cos K_1 & \sin K_1 \\ -\sin K_1 & -\cos K_1 \end{bmatrix} \begin{Bmatrix} F_f^{c1} \\ F_r^{c1} \end{Bmatrix} \tag{3}$$

$$\begin{Bmatrix} F_x^{c1} \\ F_y^{c1} \end{Bmatrix} = \begin{bmatrix} -\cos K_2 & \sin K_2 \\ \sin K_2 & \cos K_2 \end{bmatrix} \begin{Bmatrix} F_f^{c2} \\ F_r^{c2} \end{Bmatrix}$$

Dynamic displacement can be expressed in terms of dynamic properties of the system and the dynamic forces as follows:

$$\begin{Bmatrix} x_p \\ y_p \end{Bmatrix} = \begin{bmatrix} G_{xx}^p & 0 \\ 0 & G_{yy}^p \end{bmatrix} \begin{Bmatrix} F_x^p \\ F_y^p \end{Bmatrix} \quad , \quad p = c1, c2, w \tag{4}$$

The geometry of tools and workpieces in most of the turning operations is symmetrical and beamlike structures; thus, owing to relatively stiff behavior of either the tool or the workpiece in certain direction, the cross transfer functions could be neglected. In order to determine the marginally stable depth of cut, the roots of the characteristic equation of the system must have zero real part, and imaginary part of the equation contains the vibrating chatter frequency (ω_c). It is the same procedure that Budak and Altintas [9] have used. As a consequence, the dynamic displacements and the cutting forces when the system is critically stable can be expressed as:

$$\{x,y\}_p e^{i\omega_c t} = \left[G_{x,y}\right]_p \{F_{x,y}\}_p e^{i\omega_c t} \qquad p = c1, c2, w \tag{5}$$

Afterward, the presented formulations are substituted into Equation. 3, and then in Equation2, the cutting forces at the limit of stability after some algebraic manipulation can be written as follows:

$$\begin{Bmatrix} F_x^{c1} \\ F_y^{c1} \\ F_x^{c2} \\ F_y^{c2} \end{Bmatrix} e^{i\omega_c t} = b[A]\left[\bar{G}\right] \begin{Bmatrix} F_x^{c1} \\ F_y^{c1} \\ F_x^{c2} \\ F_y^{c2} \end{Bmatrix} e^{i\omega_c t} \tag{7}$$

where:

$$\bar{G} = \begin{bmatrix} G_{xx}^{c1} & 0 & -G_{xx}^{c2} e^{-i\omega_c \frac{\tau}{2}} & 0 \\ 0 & G_{yy}^{w}\left(1 - e^{-i\omega_c \frac{\tau}{2}}\right) & 0 & -G_{yy}^{w}\left(1 - e^{-i\omega_c \frac{\tau}{2}}\right) \\ -G_{xx}^{c1} e^{-i\omega_c \frac{\tau}{2}} & 0 & G_{xx}^{c2} & 0 \\ 0 & -G_{yy}^{w}\left(1 - e^{-i\omega_c \frac{\tau}{2}}\right) & 0 & G_{yy}^{w}\left(1 - e^{-i\omega_c \frac{\tau}{2}}\right) \end{bmatrix} \tag{8}$$

$$A = \begin{bmatrix} -\cos k_1 & \sin k_1 & 0 & 0 \\ -\sin k_1 & -\cos k_1 & 0 & 0 \\ 0 & 0 & -\cos k_2 & \sin k_2 \\ 0 & 0 & \sin k_2 & \cos k_2 \end{bmatrix} \begin{bmatrix} K_f^{c1} & 0 \\ K_r^{c1} & 0 \\ 0 & K_f^{c2} \\ 0 & K_r^{c2} \end{bmatrix} \begin{bmatrix} 1 & \tan k_1 & 0 & 0 \\ 0 & 0 & 1 & \tan k_2 \end{bmatrix}$$

After some arrangements, Equation. 7 could be presented as follows:

$$\left[I - bA\bar{G}\right]\{F\} = 0 \tag{9}$$

where *I* is an 4×4 identity matrix. Equation. 9, has non-trivial solution, if and only if the determinant of $\left[I - bA\bar{G}\right]$ matrix is zero.

Generally, all chatter problems result in a similar equation to solve. However, according to complexity of the problem, researchers have been using various approaches to determine the solution. Analytical relations could be derived between various parameters in some of chatter problems thus the solution can be obtained fully analytically [4-9-10]. However, there is no fully analytical solution for the stability of parallel turning operations. Ozturk [6-8] has derived a mathematical equation for the determinant of the eigenvalue problem and separate the real and the imaginary part of the determinant. Later, by setting the real part to zero and substituting in the imaginary part, one equation left with two unknowns. In contrast, in current study the intricate equation of the determinant cannot be derived analytically. This introduces serious computational efforts and complexities to the solution. The outcome is a complex equation that has both real and imaginary parts which must be set to zero. There are three unknowns in the problem, chatter frequency (ω_c), period of rotation (τ) and the depth of cut of the tools (*b*) which must be found. It should be noted that each spindle speed (*n*) corresponds to a rotational period (τ) by n=60/τ. Firstly, a reasonable range and an incremental step for each known parameter must be considered, in order to discretize the space. For instance, chatter frequency is usually close to the natural frequency of the dominant modes of the system. Also, desirable working speed of the spindle could be specified. Each spindle speed, natural frequency and depth of cut represents a point in the discretized space of the parameters. In order to find the solution, each of these points (sets of *b*, ω_c and *n*) should be inserted in the equation and compared to their adjacent point. If there is a sign change in both the real and the imaginary parts of the equation of these two adjacent points, a part of the solution lay between these two points. Hence; the set of *b*, ω_c and *n* correspond to the points must be bracketed. As the final step, bisection method [11] must be used between these two points in order to interpolate the exact solutions. Evidently, the finer is the incremental step for each parameter, the more accurate results could be reached.

3. TIME DOMAIN RESPONSE

Created forces during cutting process will cause vibrating the tools and workpiece. Subsequently, the modulated dynamic chip thickness of the tools will be influenced by the dynamic properties of the system. Since the cutting forces depend on dynamic chip thickness, modulated chip thickness in current revolution of cut will be affecting the cutting forces in the next revolution. Intuitively, the governing delayed differential equation of the system could be modeled (see Figure 3.) in MATLAB/Simulink [12]. In contrast with frequency domain approach, the static chip thickness (feed rate) and the edge forces must be included in the simulations. Even though considering static chip thickness and edge forces in the time domain model will not move the stability boundaries, it leads to obtain accurate force and displacement values. It is worth noticing that, step size must be selected small enough in order to have enough simulation points in each chatter wave. Runga-Kutta method [11] used to solve the DDE.

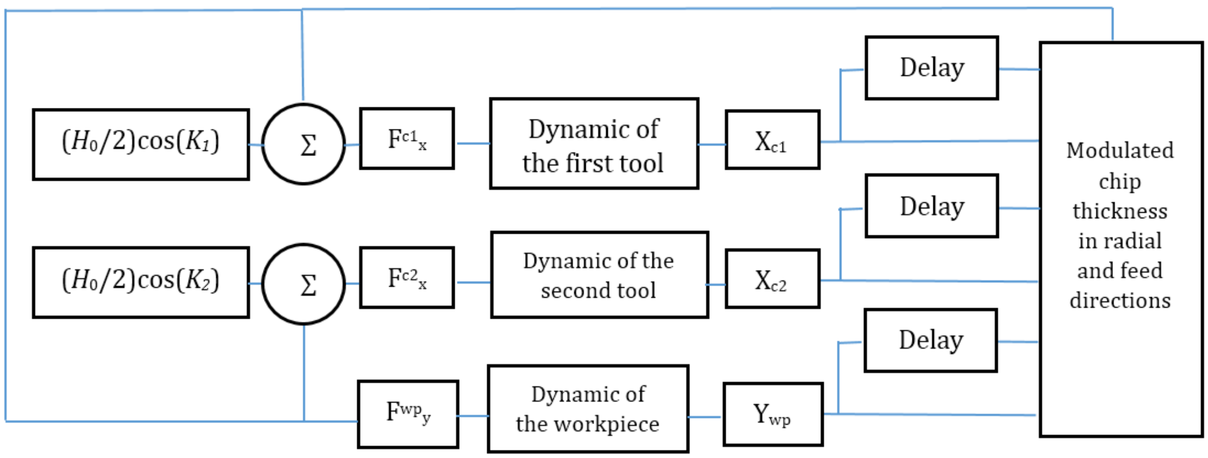


Figure. 3., schematic chatter mechanism in parallel turning operation

4. RESULTS AND DISCUSSIONS

The results for chatter stability of parallel turning operation has been presented in this section. Firstly, results of the presented approach are compared with available results in the literature for verification. Later, a parametric investigation has been considered to illustrate the effects of tools and workpiece dynamics on the response of the system.

4.1 Verification of the models

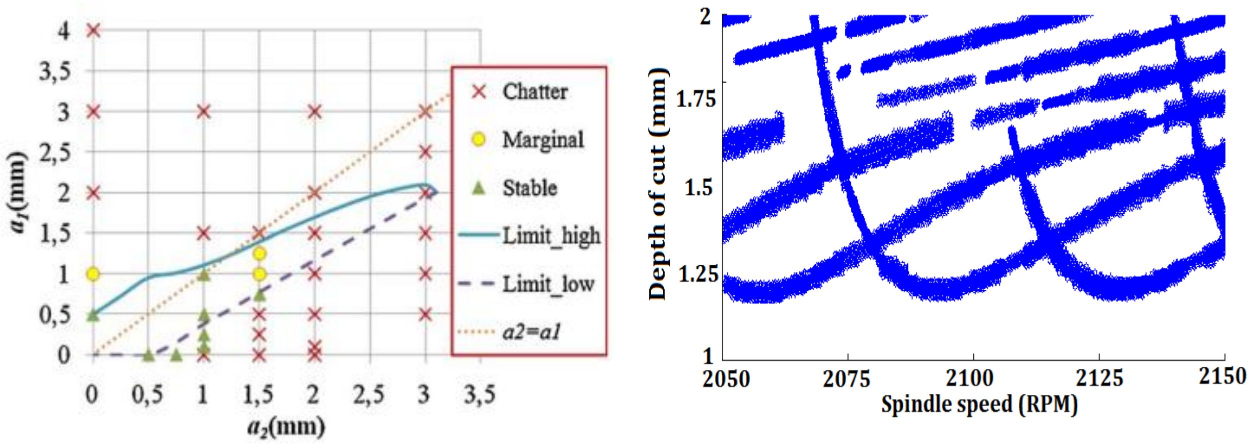
Stability limits determined by introduced model in the current study have been verified with other available case in the literature. The FRF data of the system has been tabulated in the Table 1. The workpiece material is a 32 mm diameter cylinder made out of 1050 steel. The edge and cutting force coefficients in feed and radial direction of the tools are mechanistically have been calibrated using linear edge force model as 86.5 N/mm, 1100 and 300 MPa, respectively [6].

Table 1, Modal data of the system

		Mode	Natural Frequency (Hz)	Stiffness (N/m)	Damping (%)
Tool 1 [6]	G^1_{xx}	1	1688.1	1.495×10^7	3.85
		2	2060.2	2.482×10^8	0.87
Tool 2 [6]	G^2_{xx}	1	1922.1	6.429×10^6	4.72
Workpiec*	G^w_{yy}	1	402	1.51×10^6	0.677

*The workpiece's dynamic has been measured for this study.

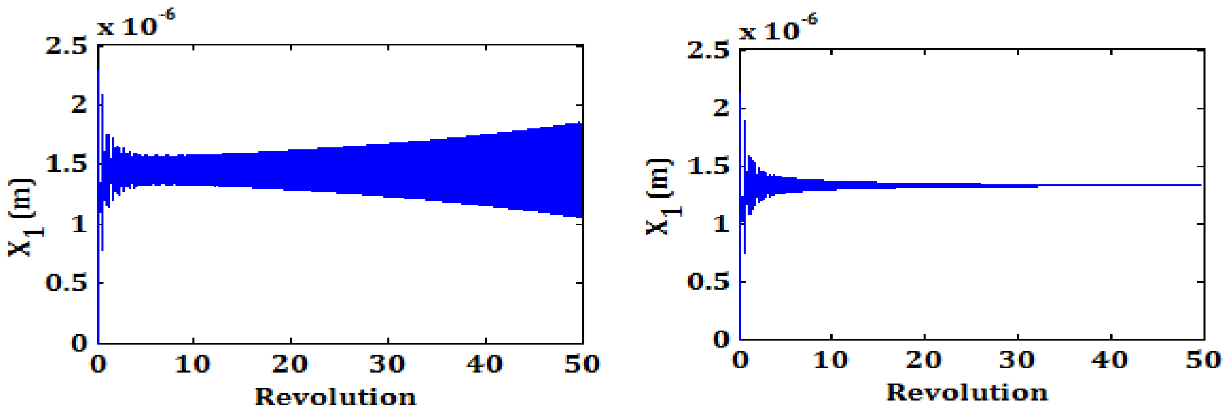
Ozturk et al [6] have assumed that the workpiece was relatively rigid in radial direction and tools were vibrating in the feed direction. By setting the side edge cutting angle to zero and choosing a high stiffness value for the workpiece in the radial direction, the current formulation can be simplified to that case. Furthermore, their results have been presented for two tools which cut different depth of cuts. In fact, the stability limits of one of the tools has been determined for a given depth of cut of the other tool. By plotting the stability limits, stability map for the system were obtained (see Figure 4-a) for 2100 RPM. Stable cutting is guaranteed within the stability boundaries. In addition to stability of the system for cutters with different depth of cuts, the line which shows the a_1=a_2 in Figure. 4-a, is demonstrating the stability limit when the tools have similar depth of cut. It can be observed that the line leaves the stable area around a_1=a_2=1.2 mm.



(a) (b)

Figure. 4, Comparison of the predictions with the ones in [6].

As it can be seen from Figure. 4, the predicted stability limit provided by the current study in 2100 RPM is in good agreement with those which presented in [6]. The time domain response of the system also has been presented in Figure. 4, for depth of cuts slightly higher and lower than the absolute stability limit predicted by the frequency domain solution.



(a) Unstable *b*=1.25 (mm)

(b) Stable, *b*=1.15 (mm)

Figure. 3, time domain response of the system in 2100 RPM.

4.2 Parametric study

The main objective of this study is to investigate the effect of side edge cutting angle in parallel turning process. As aforementioned before, the effect of workpiece dynamic in radial direction on the stability limit are considered herein, for the first time. The influence of the side edge cutting angle of the tools in addition to dynamic stiffness ratio of the tools and workpiece on the absolute chatter stability limit have been studied. Same material properties as the *verification of the models* have been used. Since tools are mounted on different turrets, there is no strong dynamic interaction between them.

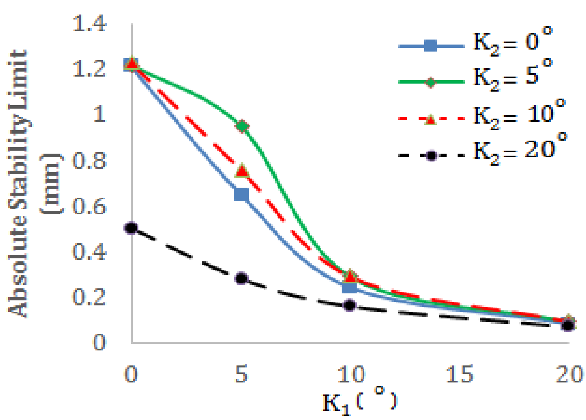


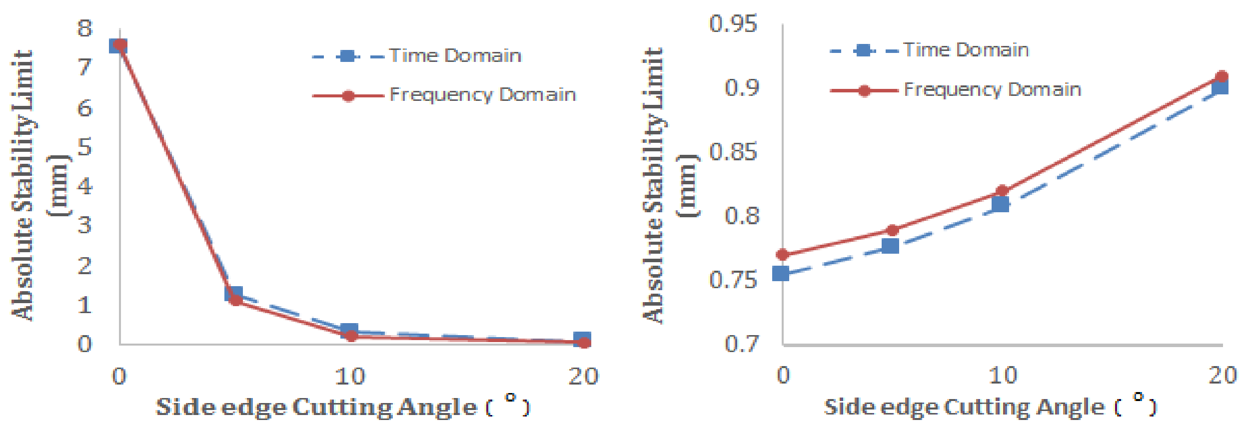
Figure. 5, Effect of side edge cutting angle of the tools on absolute stability limit of the process in 2100 RPM

As it can be seen from Figure. 5, side edge cutting angle has a salient influence on the absolute stability limit of the system. When both K_1 and K_2 are zero, the effect of system's dynamic in radial direction is not accounted for. Clearly, it represents a one dimensional (feed direction) chatter analysis. However, by changing K_1 and K_2 values, the addressed problem is no longer one dimensional. Therefore, a 2D (feed and radial directions) model is needed for an accurate solution. According to the aforementioned formulation, the stability limit not only is a function of spindle speed and the dynamics of the system, but also a function of tool's geometry. In fact, the cutters stiffness ratio, cutters and workpiece stiffness ratios, cutting force coefficients ratios in feed and radial direction will affect the stability limit of the system. For the provided experimental dataset, the first tool is slightly stiffer than the second tool whereas the second tool is slightly stiffer than the workpiece. When K_1 is zero, it means that the first tool is in its maximum stiffness condition in the radial direction. Since the second tool is already more flexible than the first one and slightly stiffer than the workpiece, increasing its side edge cutting angle won't change the absolute stability, up to certain degrees. In fact, by increasing the side edge cutting angle of the second tool, effect of workpiece is being added in the radial direction. As stated before, slight increase in side edge cutting angle of the second tool will not affect the absolute stability limit remarkably, since dynamic of the second tool remains dominant. However, when K_2 is increased to almost 20 degrees, the contribution of dynamics effects in radial direction is considerable in comparison with those of feed direction. In plainer words, contribution of flexible workpiece is big enough to be the dominant dynamic component of the system. As can be seen from figure 5, when K_2 is 20 degrees, the absolute stability limit decreased significantly. Furthermore, since the first tool is much stiffer than the workpiece, increasing K_1, will affect the stability limit extremely, in comparison to the second tool. As a detailed parametric study of the side edge cutting angles on the chatter stability limit, two different extreme cases have been investigated. Firstly, the tools are much stiffer than the workpiece, and secondly, the workpiece is much stiffer than the tools. The data has been used are shown in Table 3**.**

Table 3, The stiffness value ranges used in the simulation

case	K_{wp}/K_{tools}	K_{tools} (N/m)	K_{wp} (N/m)
1 – Flexible workpiece	0.01	1.51×10^8	1.51×10^6
2 – Flexible tools	10	1.51×10^7	1.51×10^8

The results of absolute stability limit for two different cases can be seen in Figure. 6. The absolute stability limits for both the cases have been determined using the frequency and time domain solutions where a good agreement can be observed. Clearly, the values of the side edge cutting angles will determine the contributions of the workpiece (radial direction) and the tool (feed direction) dynamics on the cutting stability of the systems. As a consequence, increasing the side edge cutting angles increases the dynamic effects in radial direction. In case 1, for a flexible workpiece, increasing side edge cutting angle lessens the total rigidity of the system. Therefore, stability limit will decrease. However, in case 2, for flexible tools, increasing side edge cutting angle increases the total rigidity of the system, and hence the stability limit.



(a) (b)

Figure. 6, Stability limit for (a) flexible workpiece, and (b) flexible tools in 2100 RPM

5. CONCLUSION

Frequency and time domain approaches have been presented in order to address chatter stability and dynamics of parallel turning operations including both tools and workpiece dynamics. The results obtained by both methods are in good agreement. Predictions have shown that dynamics of the workpiece influence the stability behaviour of the process immensely. Depending on the stiffness ratios in the process, the influence of the side edge cutting angle can be different. It is worth noting that in processes with flexible workpiece, it is necessary to include the effect side edge cutting angle, otherwise the results would be deviated from predictions, drastically. As one may note, provided formulation presents a better insight to understand the actual parallel turning operation. Nose radii inclusion will increase the accuracy of the predictions to a great extent.

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