MATHEMATICS IN DIFFICULT TIMES

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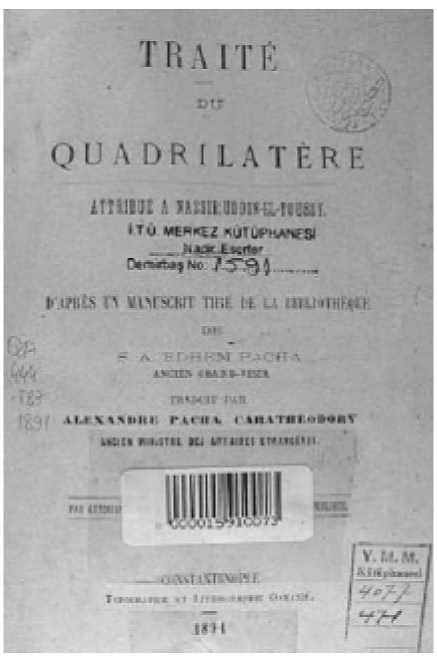
When I was in high school, my father used to tell me once in a while about his own years as a student, at the Universities of Göttingen and Munich, and his experiences during the early period of the Nazi regime in Germany. It was from him that I first heard the name of Constantin Carathéodory. While studying mathematics in my university, I frequently encountered Carathéodory's name in my textbooks and classes. Years later, I was able to find the book written by Maria Georgiadou [2] that described in detail Carathéodory's life and contributions to mathematics, and provided extensive information about the Carathéodory family, the political turmoils of the time, and the German academic world. Constantin Carathéodory had lived through and worked on mathematics during difficult times. Benefitting from Georgiadou's book, I gave a presentation on the "Carathéodory family" at the 2006 International Conference on the Teaching of Mathematics (ICTM). I also learned from this book that Alexander Carathéodory Pasha had translated a geometry book of Nasir al-Din al-Tusi from Arabic to French. I was able to find a copy of this book originally printed in 1891 [1], and, by also drawing on Salih Zeki's book named "Asar-ı Bakiyye" [5], learned quite a lot about Nasir al-Din al-Tusi, who lived approximately seven hundred years before Constantin Carathéodory. Nasir al-Din al-Tusi had also lived through difficult times.


TRAITÉ

DU

QUADRILATÈRE

D'APRÈS UN MANUSCRIT TIRÉ DE LA BIBLIOTHÈQUE

DE

EDHEM PACHA

TRADUIT PAR

ALEXANDRE PACHA CARATHEODORY

CONSTANTINOPLE

1891

[1]

Nasir al-Din al-Tusi, who was interested in many fields of knowledge such as *kalam*, *fiqh*, philosophy, logic, mathematics and chemistry, had written nearly 150 books during his life marked by sectarian rivalries, wars and devastation. So I often began to describe both Carathéodory and Nasir al-Din in talks I gave at different places and times. In the presentation entitled "Mathematics in Difficult

Times," which I gave during the National Mathematics Symposium held at Erciyes University in 2010, I once again spoke about Carathéodory and Nasir al-Din al-Tusi.

With his contributions to the calculus of variations, the theory of functions, measure theory, and the theories of geometry, optics and thermodynamics, Constantin Carathéodory (1873-1950) is undoubtedly one of the most important mathematicians of the 20th century. His name is mentioned in the mathematics literature with concepts and theories such as the Carathéodory measure, the Carathéodory theorem, the Carathéodory-Toeplitz theorem, and the Carathéodory metrics. In his lengthy article entitled "Investigations into the Foundations of Thermodynamics" published in 1909, Carathéodory successfully undertook – while still at the beginning of his career as a mathematician – the ambitious task of axiomatically formulating the basic laws of thermodynamics without using the concept of heat, and by only using known concepts of mechanics. Carathéodory always adopted different approaches in his books on the areas of variational calculus, real functions and theory of functions, and was advisor to 18 doctorate students during his years at the Universities of Munich, Göttingen and Berlin. These students included Paul Finsler, Georg Aumann, Ernst Peschl, Demetrious Kappos and Ahmet Nazım. A. Nazım Terzioğlu (1912-1976), one of the founders of the Turkish Mathematical Society, was also my father…

In almost every description of his personal background, Constantin Carathéodory is described as a German mathematician of Greek descent. However, he is actually a member of the Carathéodory, one of prominent Ottoman Greek (*Rum*) families of Fener. His father, Stephanos Carathéodory (1836-1907), had entered the Ministry of Foreign Affairs of the Ottoman Empire after completing his doctorate studies at the University of Berlin. In 1866, he served as a counsellor at the Ottoman embassy in Russia, and became the head clerk of the Ottoman embassy in Berlin in 1871. Constantin was born in Germany in 1873. Due to a special assignment given by the Sultan, Stephanos Carathéodory spent years 1874 and 1875 in Istanbul. Stephanos was so successful in this assignment – on which we have very little information – that he was then appointed as the Ottoman Empire's ambassador to Brussels. After serving at this post for twenty five long years, he retired from the Ministry of Foreign Affairs, and remained in Brussels for the rest of his life.

For centuries, many members of the famous Ottoman Greek families, who were referred to as the "Fener Greeks" or "Fener Beyzade," served in important positions in the Ottoman Empire. Some of these families traced their roots to the Byzantine Empire, while others were families of Albanian, Wallachian or Moldovan origin who had become Greek Christians. Families from Fener constantly looked out after one another, ensured that their children received good education, and knew how to increase their political influence within the Empire through carefully arranged marriages. Starting from the mid-17th century, the Fener Greeks often served as diplomats and translators in the Ottoman Empire, and were in close contact with the Europeans in Istanbul. They had gained the Ottoman State's trust to such an extent that all *Beys* of Wallachia and Moldova during the 18th century were selected from members of these families. The influence of these distinguished and wealthy families could be felt within Patriarchate as well, which is located in the Fener district in İstanbul. Since a branch of every family was engaged in trade/commerce, Fener Greeks eventually became established in cities outside of Istanbul, such as Alexandria, Bucharest, Constanta, Livorno and Marseilles

The Fener Greeks closely followed the new currents and schools of thought that emerged following the French Revolution, and began sending their children to renowned universities in European starting from the early 19^{th} century. Many of these children became important physicians, lawyers, engineers and administrators. To provide an example, let us briefly look at the background of two members from the Carathéodory family.

Constantin Carathéodory's namesake Constantin Antonious Carathéodory (1802-1879) studied medicine in Pisa and Paris, and completed his residency education in London. From 1830 until his death, he worked as professor at the *Tıbbiye-yi Şahane* School of Medicine in Istanbul. In addition, he also served as Mahmut II's, and later as Abdülmecid's, personal physicians.

Constantin Carathéodory's great uncle and father-in-law Alexander Pasha Carathéodory (1833-1906) studied law and mathematics in Paris before entering the Ministry of Foreign Affairs of the Ottoman Empire. He served in important posts such as Ambassador to Rome, Governor of Crete, the Minister of Foreign Affairs, and Abdülhamit's chief translator. At the Berlin Congress in 1878, Alexander Carathéodory was the Ottoman Empire's fully authorized representative.

Constantin Carathéodory's mother, Despina Petrocochino-Carathéodory, was also a Fener Greek. Shortly after Stephanos Carathéodory began his assignment as ambassador in Brussels, Despina Carathéodory died of pneumonia at the age of 28. At the time of his mother's death, Constantin was only six-years-old, while his sister was four-years-old. After this event, Constantin's maternal grandmother began living with them in Brussels. The children learned Greek at home and French and school; the family also hired a German maid, such that the children could learn German. At the age of 17, Constantin was enrolled as a foreign student in the "Ecole Militaire de Belgique," a military engineering school. This school, modeled after the famous "Ecole Polytechnique," one of France's leading schools, taught technical subjects through a strong and comprehensive emphasis on mathematical and scientific knowledge. Constantin graduated from this school in 1895 as an artillery and engineering officer. During the summer of 1895, he visited Alexander Pasha, the General Governor of Crete. For a two-year period, he assisted his cousin working on the highways plans for Lesbos and Samos islands. Constantin had also learned English while at school. Between 1898 and 1900, he worked as an engineer at a dam construction in Egypt conducted by the British, and wrote an article about the pyramids. But in the end, Constantin Carathéodory decided to leave engineering behind, and to start studying mathematics to become a mathematician. His friends and family were astounded by this decision. For them, leaving engineering – an occupation that held great promise – to restart school at the age of 27 was absurd. Maybe after all those years he would spend studying, Constantin was to become just an ordinary mathematics teacher. The only person who supported Constantin's wish and eagerness to become a mathematician was Alexander Pasha.

Constantin was eventually able to convince his friends and family about his decision, and began studying mathematics at the University of Berlin. Two years later, he transferred to the University of Göttingen. Within a very brief period of time, he acquired an entirely new circle of friends. His instructors included very famous mathematicians such as Hilbert, Klein, Frobenius, Schwarz and Zermelo. Constantin Carathéodory received his doctorate degree in 1904 at the University of Göttingen after submitting his thesis entitled "On the Discontinuous Solutions in the Calculus of Variations." His doctorate advisor was H. Minkowski. Constantin was now 31, and at the threshold of an entirely new career.

Constantin Carathéodory first began working at the University of Göttingen. Although he did not have a salary at the time, which did not matter so much, since he was still the member of a wealthy family. During this time, he published numerous researches. His father died in 1907. Constantin spent the summer of 1908 at Kuruçeşme, Istanbul, where he worked on the laws of thermodynamics. In 1909, he was married at Kuruçeşme with Euphrosyne, the daughter of Alexander Pasha. He then began teaching at the Technical Universities of Hannover and Breslau, where he was now earning a salary. During those years, the University of Göttingen was one of the world's leading centers in mathematics and physics. After the retirement of Felix Klein, one of the famous mathematics professors at the University of Göttingen, Constantin Carathéodory was appointed as professor in his stead. Following this outstanding achievement, he was elected to the editorial board of the "Mathematische Annalen" journal. In 1918, he published the book entitled "Vorlesungen über reelle Funktionen." Rising rapidly in his career as a mathematician – which he had started after deciding to leave engineering and reenter school to study mathematics – one would expect that Constantin Carathéodory would be satisfied with his new life. His large social circle included mathematicians and physicists, as well as artists, philosophers and writers. However, his life would be carried towards an entirely different direction following his meeting with E. Venizelos in Paris on September 1919.

Constantin Carathéodory had first met with Venizelos at the island of Crete in 1895. Their first encounter coincided with a meeting between Venizelos and Alexander Pasha, the General Governor of Crete, where the latter was trying to convince Venizelos to take part in the planned local elections at the island. After Venizelos insisted on boycotting the elections, Alexander Pasha harshly reprimanded him. Young Constantin, who was visiting his grand uncle Alexander Pasha at the time, witnessed this exchange with Venizelos, and was impressed by him. Venizelos was an ardent support of the "Megali Idea," which means the "Great Idea." In the new Greek State, which had recently gained its independence from the Ottoman Empire, politics were largely shaped around the notion of "Megali Idea." The main goal of the "Megali Idea" was to revive the Byzantine Empire that had vanished centuries ago, and to bring together all Greeks under the flag of this new state. This ideal had deeply influenced young intellectuals from Fener, including Constantin Carathéodory, while both Constantin's father and Alexander Pasha viewed the "Megali Idea" as a romantic dream detached from reality.

On May 15, 1919, the Greek Army carried by an armada consisting of British, French, American and Greek warships landed in Izmir, and gained control of most of the Aegean Region by the autumn of 1919 despite resistance in certain locations. A Greek High Commissioner Office was soon established in Izmir. The perception that the Megali Idea was becoming reality stirred great enthusiasm also among some Anatolian Greeks. It was during this period that Venizelos met with Constantin Carathéodory in Paris, and proposed him to become the rector of a new Greek University that would be established in Izmir – an offer which promptly Constantin accepted.

Carathéodory thus departed from Göttingen University, where he had been previously appointed as professor in Klein's place, and traveled to Izmir. In the meantime, he was also appointed as geometry professor at the University of Athens. As the rector of the new university, he was to work directly under the Greek High Commissioner Office. A mansion was soon rented at Buca for the Carathéodory family. The law concerning the foundation of a Greek university in Izmir – named "Ionia University" – was approved by the Greek Parliament on July 14, 1920. The university was planned to consist of four faculties and two independent institutes. The names considered for these faculties were the Agricultural and Natural Sciences Faculty, the Social and Economic Sciences Faculty, the Commerce and Public Works Faculty, and the Oriental Languages Faculty. The Hygiene Institute and the High Islamic Institute were also to be part of Ionia University, independently of these four faculties. Greek was selected as the language of education at the University, although plans were also made to provide certain courses in other languages under special circumstances. The High Commissioner Office allocated the area of the Jewish cemetery in Izmir for use as the university campus. During 1920, Carathéodory met twice with Venizelos onboard a Greek warship stationed at the Gulf Izmir (since it was considered dangerous for Venizelos to set foot in Izmir). In the Greek elections held towards the end of the same year, Venizelos' party unexpectedly lost by a wide margin. Following this defeat, Venizelos declared that he was leaving politics, and left for Paris in self-imposed exile. Soon after, the exiled king Constantine returned to Greece. The newly formed Greek government quickly dismissed many officers and officials known to be "Venizelists." Constantin Carathéodory was also affected by this purge, losing his professorship at the University of Athens. However, he retained his office as the rector of Ionia University. Besides, the new Greek government was pledging it would do a better job pursuing the Megali Idea project than Venizelos.

In 1921, while Carathéodory was holding meetings in various European countries concerning the Ionia University, the Greek Army in Anatolia suffered defeats at the Battles of Inönü, and was decisively routed after making a last push that advanced as far as Sakarya. Cathéodory undoubtedly followed these events closely; however, he was deeply attached to the Megali Idea, and confident that things would eventually work out, which is why he had returned from Europe to Izmir. As the remnants of the Greek Army began abandoning Anatolia in a state of complete disorder after the Turkish Great Offensive, Constantin Carathéodory escaped from Izmir to Greece onboard a warship a day before the Turkish cavalry entered the city. The Ionia University thus vanished before it had even opened its doors, and without a single student. The adventures of those pursuing Megali Idea day dream ended with what they themselves would call the "Asia Minor Disaster."

One cannot help but wonder how the Megali Idea could be so influential as to prompt Constantin Carathéodory – a world-renown mathematics professor from a distinguished and wealthy family who knew the world and spoke many languages – to abandon his brilliant career at the University of Göttingen and travel to an Izmir under occupation? Megali Idea was certainly one of the romantic nationalist movements of the 19th century; and just as any other romantic nationalist movement, the ideal of establishing a Greater Greece of two continents and five seas created its own myths, heroes and epics. Starting from the 1840s, its discourse and notions began to deeply influence Greeks all around the world. Every romantic nationalist movement has an internally consistent logic that requires individuals to turn a blind-eye to certain realities, and to accept certain notions unquestioningly. So if you firmly believe in the superiority of the Greek nation, race and culture over other nations and their cultures, then the Megali Idea might begin to sound reasonable! Most nationalist movements eventually transform into introverted ideologies detached from reality; however, as an ideology, the Megali Idea actually received intellectual support from nations other than Greece as well.

Interest in Ancient Greek culture, which initially began during the Renaissance, gradually increased in Northern Europe and America until it transformed into an exaggerated admiration by the early 19th century. During the 19th century, Homer's epics were translated again and again, over twenty times, into English. Ancient Greek culture was the source of inspiration of romantic poets such as Byron, Keats, Schiller and Shelley. Nearly all monumental buildings constructed in countries such as the United

Kingdom, Germany and France during the 19th century carried the lines and traces of Ancient Greek temples. The same can be said for the governmental buildings and monuments in Washington, the capital of the United States. It had become custom in the West to seek and find all that is good, aesthetically beautiful and morally right in the ancient Greek civilization – a period considered as a golden age of sorts. But in fa**c**t, that civilization had long since become history; however this admiration in Europe and America had become a strong external supporting the Megali Idea, blurring the lines between reality and dreams, and between the past and the present. When years later a Turkish student, my father, asked his mentor and teacher about his adventure and experiences in Izmir, a painful smile appeared on Constantin Carathéodory's face, who said that he was prepared to speak and discuss any subject with his young student, except that particular one.

In 1922, Carathéodory began working at the University of Athens. Representatives of the new Greek Government had deemed it appropriate for the famous professor from the University of Göttingen to teach Calculus to chemistry students. But not only were his students not interested in his classes, but they also acted disrespectfully towards him as their instructor. For some reason, the students saw him as a foreigner. When one day, several of his students mocked him by saying ”noch einmal!” (”one more time”) in German while he teaching in Greek, Carathéodory shouted "I am Greek! Greek!" and bolted out the amphitheater. Meanwhile, the Treaty of Lausanne sealed the Megali Idea's fate. In 1924, Carathéodory accepted a job offer by Munich University and returned to Germany. He tried to forget his unpleasant experiences by fully focusing on mathematics. He began publishing many articles and books. Following an invitation of the American Mathematical Society (AMS) in 1928, he performed a successful tour of conferences in the United States and Canada that lasted for nine months. He even declined a professorship offer by Stanford University, and returned to Munich.

After Venizelos came back to power in Greece in 1928, he first focused of normalizing Greece's relations with its neighbors. İnönü and Venizelos had met during the Lausanne negotiations. Following reciprocal visits by both prime ministers, a friendship agreement was signed between Turkey and Greece. In those years, higher education reform was a top priority on both countries' agenda. Turkey had assigned a Swiss professor, A. Malche, for university reform in Turkey, while Greece had assigned Constantin Carathéodory for this task, who had asked for a temporary leave from the University of Munich. However, when Venizelos lost the elections in 1932, the new Greek government promptly dismissed Carathéodory, who then returned to the University of Munich.

Becoming Germany's chancellor in 1933, Hitler began to gradually implement the Nazi party's racist and totalitarian ideology. Professors who were considered non-Aryan by the Nazis were quickly sacked, while students were incited to identify "harmful" books in their university libraries, and to ceremoniously burn them at the city center at night. Although Carathéodory was considered Aryan, this was not enough: Like all Aryan professors who did not wish to lose their jobs and posts, he was forced to swear allegiance to Hitler.

Many of the academicians who were forced to leave German universities at the time were Carathéodory's close acquaintances. Among them, Albert Einstein stayed at the United States after traveling there to give conferences. Emmy Noether and Hermann Weyl also migrated to the United States. After receiving an offer from Istanbul University, Richard Courant asked for Carathéodory's opinion about this new post; however, after remaining in Istanbul for a few weeks, he chose to migrate to the United States instead. On the other hand, Von Mises and the philosopher H. Reichenbach joined the academic staff of Istanbul University. Bochner found a job at the United States, while physicist Max Born found a new post in the United Kingdom. Otto Neugebauer refused to swear allegiance to Hitler, and left for the United States, where he initiated the publication of the Mathematical Reviews. To understand the scope of the devastation caused by the Nazi regime on the German academic world, it suffices to say that until 1935, 18 mathematics professors were forced to leave the University of Göttingen alone.

Carathéodory could very well have left Germany to work in another country. He was a famous mathematician, and could teach in several languages. However, he still chose to remain in Germany. We do not his exact reasons for this. Maybe just like his friend Courant, he assumed that the Nazi regime was a temporary episode of collective madness that would soon end by itself. He never became a member to the Nazi party. During these difficult times, he endeavored to reduce as much as he could the regime's destructive effect on the academic world, while also paying attention to not endangering himself or his family.

After Germany annexed all of Austria and part of Czechoslovakia, the invasion and partition of Poland by Germany and the Soviet Union led to beginning of World War 2. Germany's ally Italy soon declared war on Greece. The Germans swiftly annexed Belgium, France and Yugoslavia, and also attacked Greece to aid their Italian allies, whose campaign was running into difficulties. During those dark days, Carathéodory lead efforts for the release of the Serbian mathematician Saltykow arrested by the Germans, and was eventually successful. He also tried to save the famous Polish mathematician J. Schauder, who was accused of listening the BBC radio, but could not prevent his execution by a Gestapo firing squad. During World War 2, the Nazi party's racist policies reached unbearable proportions. All non-Aryans, and especially the Jews, were now being taken to concentration camps. Felix Hausdorff, one of the pioneers of modern topology, had long since lost his job because he was Jew, but had nevertheless decided to stay in Germany like Carathéodory. However, after learning that he was to be sent to a concentration camp, he committed suicide together with his wife.

When occupying forces forcefully confiscated agricultural products in Greece, many people living in larger cities began to die of starvation. By obtaining permission from both the occupying German forces and the British who were blockading Greece from the sea, Turkey sent food aid with ships from Istanbul to Piraeus starting from autumn of 1941, which continued until the end of the war. Living in Munich, Carathéodory's family certainly did not have any difficulties in obtaining food; however, Constantin Carathéodory always complained about not being able to find real coffee. My father regularly sent Red Crescent food aid packages from Istanbul to his professor, and added as much coffee as he could to these packages. Munich was bombed many times by the allied air forces. Carathéodory's house was not hit, although the bombings wrecked his wife's nerves. Constantin Carathéodory lost his wife in 1947, and he passed away on February 2, 1950.

Alexander Pasha translated into French a handwritten geometry book he found in the library of Edhem Pasha, one of the Ottoman grand viziers of 19th century, and published this translation together with its original Arabic text in 1891 [1]. The author of this book is believed to be Nasir al-Din al-Tusi (1201-1274), one of the important scholars of the 13th century. Although Alexander Pasha translated the work's title as "*Traite du Quadrilatere*," a more accurate translation of the Arabic title would have been "*A theorem that renders the theorem of secants unnecessary*." [5] In spherical geometry, the theorem we today know as the Menelaus theorem is referred to by Tusi as the theorem of secants. If we designate the corner angles of a triangle on a unit sphere as α, β and γ, and if we designate the lengths of the arcs which connect these corners as a, b and c (Figure); the theorem which, in Tusi's own words, renders the theorem of secants unnecessary is:

$$(\sin \alpha / \sin a) = (\sin \beta / \sin b) = (\sin \gamma / \sin c)$$

which is in fact the "sine theorem." If you wish to determine, for example, the value of sin a by using this theorem, you need to also know the values of sin α, sin β and sin b. On the other hand, to solve the same problem with Menelaus' theorem, you must know five values instead of three. This is why Tusi chose to name his book "*A theorem that renders the theorem of secants unnecessary*." Describing that this important theorem in spherical geometry was first proved by Abu al-Wafa' Buzjani (940- 998), Nasir al-Din al-Tusi provides a different proof for the sine theorem, along with many interesting examples of its application.

With the encouragement of its father and uncle, Nasir al-Din al-Tusi began studying *kalam*, philosophy, astronomy and mathematics at an early age. He was member of a family with strong Shiite traditions. In 1214, he was forced to leave Tus, his place of birth, together with his family who had sensed the approaching Mongol threat. Nasir al-Din continued his education at Nishapur. With its famous libraries and madrasahs, Baghdad under the sovereignty of the Abbasid caliph was an important center of science and culture. Maybe with the hope of being invited to Baghdad, the young Nasir al-Din wrote an eulogy praising the caliph of his time. However, not only did this eulogy fail to fulfill his hope, but also earned him the ire of his teachers and friends who were strongly devoted to the Twelvers sect of Shiism. Certain sources state that he then took refuge at Alamut Castle, while others state that the young scholar was abducted and taken to Alamut Castle. The castle was under the control of Hassan Sabbah's successor, who was a member of the Nizari branch of the Seveners sect of Shiism. Although he was allowed to work at the castle, he was constantly kept under watch, leading a semi-captive life. Difficult times now lay ahead of Nasir al-Din. The Mongols eventually captured Alamut Castle, and the Ilkhanid ruler Hülagu Khan, who had heard of the young scholar's fame, took Nasir al-Din under his protection.

In 1258, Hülagu conquered Baghdad, and the Mongol army plundered the city for a whole week. Hundreds of thousands of people were killed in the city, and the Great Library, which was unique and unrivaled across the world, was demolished and its books were thrown into the Tigris river. For Nasir al-Din, who witnessed all of these horrible events, his dream of working one day in Baghdad had turned into a nightmare. However, he never gave up on pursuing research, and even convinced his Mongol patrons to build an observatory. He chose Maraga as location, which is located in present-day Azerbaijan. Through Nasir al-Din's efforts, the Maraga Observatory was rapidly built and developed, and soon became an important center with its astronomy and mathematics scholars from all around the world, and its rich library. The celestial atlas and star map formed through twelve years of observations and calculations at the observatory was named "Ziyci-Ilhani." This work remained a main source of reference in this area for the following two centuries. Nasir al-Din died in 1274, and was buried in the environs of Baghdad.

Over the past two centuries, most Western historians of science have set the beginning of mathematics in Ancient Greece, and claimed that the development of mathematics continued only during the Renaissance after being forgotten for nearly one thousand years. They made little or no mention of the contributions of the Egyptian, Mesopotamian, Chinese and Indian civilizations to mathematics. The generally held view in the Western science community during those years was that, although the Islamic World had made *some* contributions in the area of algebra, they generally had not added anything which was essentially new to mathematics. How had Nasir al-Din's book found its way into the library of Edhem Pasha, one of the Ottoman grand viziers? Did Edhem Pasha know the importance of this work from the 13th century? Or was it Alexander Pasha who had noticed, with his law and mathematics education in Paris, the importance of this handwritten work? I do not have the answers to these questions. However Alexander Pasha probably knew that Western historians of science attached little importance to the Islamic World and its achievements. Maybe it was the intention of undermining this assumption and preconception that lead Alexander Pasha to translate and publish, through great efforts and pains, Nasir al-Din's book on spherical geometry. And his efforts did indeed prove to be successful, since Paul Tannery, one of the important science historians of the 19th century, wrote an article describing Nasir al-Din's book translated by Alexander Pasha [4]. In his article, Tannery describes that it is a mistake to underestimate the contributions of Muslim scholars to mathematics, and that the field of spherical geometry, in particular, had appeared in the Islamic World, as clearly demonstrated by Alexander Pasha's book. Writing that Nasir al-Din knew nearly every modern trigonometry formulae in the 13th century, Tannery concluded his article by congratulating the distinguished translator who had allowed this important work to be discovered by the scientific community.

The tendency of Western historians of science to ignore or underestimate the contributions that non-Western civilizations have made to science throughout history has changed significantly in recent years. For example, looking at the list of names given by the International Astronomical Union (IAU) to the craters of the moon, we can see that the list nowadays includes names such as Al-Khwarizmi, Al-Biruni, Ulugh Bey and Nasir al-Din along with Archimedes, Apollonius, Copernicus and Newton. An important reference book edited by Victor J. Katz [3] is another important indicator of this change.

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