Resource Portfolio Problem under Relaxed Resource Dedication Policy in a Multi-Project Environment

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1 Introduction

The characterization of the way resources are used by individual projects in the multi-project environment is called resource management policy in this study. The common resource usage approach in multi-project scheduling literature allows the sharing of resources without any restrictions or costs among projects. An extension is proposed Krüger and Scholl (2009) and Krüger and Scholl (2010) which allows for resource sharing with sequence dependent transfer times. Another resource management policy called resource dedication policy is proposed by Besikci et al. (2011) and further investigated in Besikci et al. (2012), where resources cannot be shared among the projects and must be dedicated. According to the characteristics of resources and projects, resource dedication policy can be extended to relaxed resource dedication policy where the renewable resources dedicated to a particular project can be transferred after that project's finish time to other projects that are yet to start.

In some multi-project environments, the resource availability values can be considered as another set of decisions, which can be thought as a higher decision level, resulting in general resource capacities. This problem is defined as the resource portfolio problem and can be modeled in different forms based on the particular resource management policy. Here, we will deal with the resource portfolio problem under relaxed resource dedication (RPP-RRD) policy.

2 Resource Portfolio Problem under Relaxed Resource Dedication Policy

The mathematical formulation for the resource portfolio problem under relaxed resource dedication policy, model RPP-RRD is given below.

Sets:

- V set of projects, $v \in V$
- J_v set of activities of project v, $j \in J_v$
- P_v set of all precedence relationships of project v
- M_{vj} set of modes for activity j of project v, $m \in M_{vj}$
- K set of renewable resources, $k \in K$
- I set of nonrenewable resources, $i \in I$
- T set of time periods, $t \in T$

Parameters:

- E_{vj} Earliest finish time of activity j of project v
- L_{vj} Latest finish time of activity j of project v
- d_{vjm} Duration of activity j, operating on mode m
- r_{vjkm} Renewable resource k usage of activity j of project v, operating on mode m
- w_{vjim} Nonrenewable resource i usage of activity j of project v, operating on mode m
- dd_v Assigned due date for project v
- c_v Relative weight of project v
- cr_k Unit cost of renewable resource k
- cw_i Unit cost of nonrenewable resource i
- tb Total resource budget
- Ω A big number

Decision Variables:

$$x_{vjmt} = \begin{cases} 1 \text{ if activity } j \text{, operating on mode } m \text{, in project } v \text{ is finished at period } t \\ 0 \text{ otherwise} \end{cases}$$

BR_{vk} = Amount of renewable resource k dedicated to project v
BW_{vi} = Amount of nonrenewable resource i dedicated to project v
TC_v = Weighted tardiness cost of project v
R_k = Total amount of required renewable resource k
W_i = Total amount of required nonrenewable resource i
f_v = Release time of project v
$S_{vv'k}$ = Amount of renewable resource k given to project v' from project v

$$y_{vv'} = \begin{cases} 1 \text{ if project } v' \text{ is released after project} v \text{ is finished} \\ 0 \text{ otherwise} \end{cases}$$

Mathematical Model RPP-RRD

$$min.\ z = \sum_{v \in V} TC_v \tag{1}$$

Subject to

$$\sum_{m \in M_{vj}} \sum_{t=E_{vj}}^{L_{jv}} x_{vjmt} = 1 \qquad \forall\, j \in N_v \text{ and } \forall\, v \in V \tag{2}$$

$$\sum_{m \in M_{vj}} \sum_{t=E_{vb}}^{L_{vb}} (t - d_{vbm}) x_{vbmt} \geq \sum_{m \in M_{vj}} \sum_{t=E_{va}}^{L_{va}} t x_{vamt} \qquad \forall\, (a,b) \in P \text{ and } \forall\, v \in V \tag{3}$$

$$\sum_{j \in N_v} \sum_{m \in M_{vj}} \sum_{q=t}^{t+d_{vjm}-1} r_{vjkm} x_{vjmq} \leq BR_{vk} + \sum_{v' \in V} SR_{v'vk} \qquad \forall\, k \in K\ \forall\, t \in T\ \forall\, v \in V \tag{4}$$

$$\sum_{j \in N_v} \sum_{m \in M_{vj}} \sum_{t=E_{vj}}^{L_{vj}} w_{vjim} x_{vjmt} \leq BW_{vi} \qquad \forall\, i \in I \text{ and } \forall\, v \in V \tag{5}$$

$$BR_{vk} + \sum_{v' \in V} SR_{v'vk} \geq \sum_{v' \in V} SR_{vv'k} \qquad \forall\, k \in K \text{ and } \forall\, v \in V \tag{6}$$

$$\sum_{v \in V} BR_{vk} \leq R_k \qquad \forall\, k \in K \tag{7}$$

$$\sum_{v \in V} BW_{vi} \leq W_i \qquad \forall\, i \in I \tag{8}$$

$$\sum_{i \in I} cw_i W_i + \sum_{v \in V} cr_k R_k \leq tb \tag{9}$$

$$f_{v'} - f_v - \sum_{t=E_{vN}}^{L_{vN}} \sum_{m \in M_{vN}} t x_{vNmt} \leq \Omega(y_{vv'}) \qquad \forall\, v, v' \in V \tag{10}$$

$$f_v + \sum_{t=E_{vN}}^{L_{vN}} \sum_{m \in M_{vN}} t x_{vNmt} - f_{v'} \leq \Omega(1 - y_{vv'}) \qquad \forall\, v, v' \in V \tag{11}$$

$$SR_{vv'k} \leq \Omega(y_{vv'}) \qquad \forall\, v, v' \in V \text{ and } \forall\, k \in K \tag{12}$$

$$TC_v \geq C_v(f_v + \sum_{t=E_{vN}}^{L_{vN}} \sum_{m \in M_{vN}} x_{vNmt} - dd_v) \qquad \forall\, v \in V \tag{13}$$

$$x_{vjmt} \in \{0,1\} \qquad \forall\, j \in J, \forall\, t \in T, \forall\, m \in M_{vj} \text{ and } \forall\, v \in V \tag{14}$$

$$BR_{vk},\ BW_{vi},\ R_k,\ W_i,\ TC_v,\ f_v \in Z^+ \qquad \forall\, v \in V, \forall\, k \in K \text{ and } \forall\, i \in I \tag{15}$$

$$y_{vv'},\ SR_{vv'k} \in \{0,1\} \qquad \forall\, v \in V \text{ and } \forall\, k \in K \tag{16}$$

The objective function (1) is the minimization of the total weighted tardiness cost for all projects. Constraint sets (2) and (3) are for activity finish and precedence relations. Constraint set (4) limits the renewable resources employed for each project with the dedicated renewable resources and the transferred renewable resources from the other projects. Constraint set (5) calculates the nonrenewable resource dedication values for each project. In constraint set (6), the total resource that can be transferred by a project is limited with the total resource dedicated to this project and the total resource it gained from transfers. Constraint sets (7) and (8) calculate the total renewable and nonrenewable resource requirements, respectively. Constraint (9) limits the sum of the total renewable and nonrenewable resource costs with the general resource budget.

Constraint sets (10) and (11) set decision variable $y_{vv'}$ to 1, if project v is finished before project v' is released, and to 0 otherwise. Thus, the $SR_{vv'k}$ values will only have positive values, if project v is finished before project v' is released with constraint set (12). And finally, constraint set (13) calculates the weighted tardiness value for each project.

3 A Modified Branch and Cut Procedure for Resource Portfolio Problem under Relaxed Resource Dedication Policy

The given mathematical model RPP-RRD is a complex scheduling problem such that it is very difficult even to reflect all the aspects of the problem into a heuristic approach. *ILOG CPLEX* employs a branch and cut (B&C) procedure for solving mixed integer programs (MIP) (ILOG CPLEX 11.0 Users Manual 2007). To solve the model RPP-RRD the B&C procedure of *ILOG CPLEX* will be modified with different incumbent solution approaches, branching strategies and cuts.

3.1 Branching on Project Order Decision Variables

When decision variables $y_{vv'}$ have their values set, the remaining problem becomes a resource dedication problem with potential resource transfers defined by the decision variables $y_{vv'}$. Branching on decision variables $y_{vv'}$ can facilitate the branch and cut procedure since the formulation given above allows for a separation of the projects when $y_{vv'}$ and resource related decision variables are determined. For this, the branch callback function of *CPLEX* is used and branches are generated from integer infeasible variables from the linear relaxation solution on the node explicitly and feed to branch and cut procedure of *CPLEX*. Without this branching modification *CPLEX* was not able to find any feasible solutions for the test problems. The $y_{vv'}$ variable selection basically favors projects without any sequence relations and/or projects that are predecessor in their sequence relations and priorities projects with higher weights.

3.2 Upper Bound Heuristic

At each viable node, *CPLEX* attempts to generate a feasible solution close to the solution of the linear programming (LP) relaxation of the problem. If good (in some cases any) feasible solutions can be generated, *CPLEX* can use these solutions to facilitate the execution of the B&C algorithm. The proposed branching strategies basically prioritize branching on $y_{vv'}$ variables, this strategy results integer feasible values for these variables in the early stages of B&C procedure. This structure can be used to generate feasible solutions, since when $y_{vv'}$ variables are known, the remaining problem reduces RPP with possible resource transfers according to the values of $y_{vv'}$ decision variables. The solution approaches developed for Resource Dedication Problem (RDP) and RPP in Besikci et al. (2011) and Besikci et al. (2012) can be modified and used to generate feasible solutions at these stages of the B&C procedure.

The proposed feasible solution generation procedure basically determines general resource capacities, resource dedication values and resource transfers (according to values of $y_{vv'}$ variables) proportional to no-delay resource requirements of the projects and generates an initial solution (not necessarily feasible). Note that, when all resource related decision variables are set, the problem reduces to solving multi-mode resource constraint project scheduling problems (MRCPSP) for each project. An then, this initial solution is improved with CA for RD and CA for RP improvement heuristics which are based on preference concept for resources. These heuristics basically separate problem into resource dedication (BR_{vk} and BW_{vi}) and resource portfolio (R_k and W_i) parts and try to move the current resource state of the problem to a more preferable state. The preference calculation approaches couples the separated parts of the problem.

4 Experimental Results

To test the modifications in the branch and cut procedure of *CPLEX* (version 11.2) a set of multi-project problems are generated from PSPLIB (Kolish and Sprecher 1996) each of which has 6 projects with different number of activities (14, 22, 32). The problems are modified such that the total resource budget concept is applicable. A time limit of 240 minutes is set as the termination criterion. The computational results are given in the Table 1 below. *Modified CPLEX* is the branch and cut procedure with branching and heuristic solution modifications, LB is the lower bound found by the procedure and RT is the run time in minutes.

Table 1. Test results for selected problems

	Unmodified CPLEX				Modified CPLEX			
Problems	Status	Objective	LB	RT	Status	Objective	LB	RT
Problem1	Infeasible	NA	8.82358	240	Feasible	37	8.28921	240
Problem2	Infeasible	NA	13.94295	240	Feasible	40	13.96027	240
Problem3	Infeasible	NA	9.45726	240	Feasible	31	7.23539	240
Problem4	Infeasible	NA	12.91288	240	Feasible	56	8.83703	240
Problem5	Infeasible	NA	15.11832	240	Feasible	32	12.81082	240
Problem6	Infeasible	NA	14.67669	240	Feasible	41	14.01215	240
Problem7	Infeasible	NA	13.88472	240	Feasible	41	13.77914	240
Problem8	Infeasible	NA	11.16413	240	Feasible	33	9.19310	240
Problem9	Infeasible	NA	11.64536	240	Feasible	34	9.58541	240
Problem10	Infeasible	NA	14.67312	240	Feasible	45	14.05152	240

As it can be seen from the results the branching and heuristic solution modifications have greatly improved the results by finding feasible solutions. But the lower bound values of the modified branch and cut procedure is still not good enough to result in an optimal solution termination.

5 Conclusions and Further Research Topics

A new resource management policy and its mathematical formulation are proposed under resource portfolio problem. A modified branch and cut procedure for *ILOG CPLEX* is presented using some special branching strategies and a feasible solution heuristic. The results show that branching strategy and feasible solution heuristic are efficient, but the lower bound values need to be improved. Thus for further research, cut generation procedures will be investigated to improve the lower bound values.

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