Searching for a Bargain: Power of Strategic Commitment*

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March 13, 2014

Abstract

This paper shows that in a multilateral bargaining setting where the sellers compete á la Bertrand, a range of prices that includes the monopoly price and 0 are compatible with equilibrium, even in the limit where the reputational concerns and frictions vanish. In particular, the incentive of committing to a specific demand, the opportunity of building reputation about inflexibility, and the anxiety of preserving their reputation can tilt players' bargaining power in such a way that being deemed as a tough bargainer is bad for the competing players, and thus, price undercutting is not optimal for the sellers.

Negotiators often use various bargaining tactics, manipulate the adversaries' beliefs and build false reputations to improve their bargaining positions and shares (Schelling 1960; Arrow et al. 1995). A growing literature on bargaining and reputation focuses particularly on a specific tactic—standing firm and not backing down from the initial offer—and analyzes its impacts on bilateral negotiations (Myerson 1991; Abreu and Gul 2000; Kambe 1999; Compte and Jehiel 2002; Atakan and Ekmekci 2010). This paper, on

*This research was supported by the Marie Curie International Reintegration Grant (# 256486) within the European Community Framework Programme. I would like to thank David Pearce, Wolfgang Pesendorfer, Larry Samuelson, Mehmet Ekmekçi, Ennio Stacchetti, Kalyan Chatterjee, Vijay Krishna, Ariel Rubinstein, Alessandro Lizzeri, Tomasz Sadzik, and two anonymous referees for helpful comments and suggestions. I also thank seminar participants at Caltech, Penn State, Carnegie Mellon (Tepper Business School), New York University, Maastricht, LUISS Guido Carli, Koc, Sabanci, Bogazici, Bilkent, TOBB, METU, and Central European University. All the remaining errors are my own.

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the other hand, highlights a new avenue through which reputations can tilt bargaining power when bargaining takes place in a multilateral setting in which a buyer cannot refrain from searching for a bargain.

I construct a simple market setup where the long side—the sellers—has *virtually* no market power. There are three defining features of the model: First, a single buyer negotiates with two sellers over the sale of one item. Second, the sellers make initial posted-price offers in the Bertrand fashion. The buyer can accept one of these costlessly or try to bargain for a lower price. Third, each player believes that its opponents might have some kind of commitment forcing them to insist on their initial offers. That is, the players can be obstinate with small probabilities, which affects the rational players' negotiating tactics and provides incentives to build a reputation on their resoluteness.

Obstinate (or commitment) types take an extremely simple form. Parallel to Myerson (1991) and Abreu and Gul (2000), a commitment player always demands a particular share and accepts an offer if and only if it weakly exceeds that share. An obstinate seller, for example, always offers his original posted price and never accepts an offer below that price. Similarly, an obstinate buyer always offers a particular amount and will never agree to pay more. Therefore, the reputation of a player is the posterior probability (attached to this player) of being the obstinate type. For analytical clarity, I construct the model with negligibly small frictions: the initial priors of each player being obstinate is small but positive, and the search cost that the rational buyer incurs at each time he switches his bargaining partner is very small but positive. Then I take the limit as these frictions converge to 0.

The analysis of the model shows that even in the limit where the frictions vanish, a range of prices including the monopoly price and 0 are compatible with equilibrium.[1] This conclusion is true because being deemed as a commitment type is bad for the competing players. This finding contrasts the standard conclusions of the bargaining and reputation literature, where the player who is believed to be a commitment type is immediately conceded by his rational opponent.

Undercutting in this framework involves mimicking a less-greedy commitment type than one's opponent. The seller's incentive to undercut his rival is eliminated not because undercutting reveals rationality or reduces the seller's reputation. In fact, if a seller undercuts, then the buyer fully believes that this seller is a commitment type. Undercutting is unattractive precisely because the buyer believes that the undercutting seller is obstinate and that a better deal is possible by bargaining with the undercutting seller's rival. In particular, the buyer bargains with the seller's rival, uses the more advantageous term offered by the undercutting seller as a threat point against the rival, and arrives

[1]This conclusion is true regardless of the players' time preferences.

at an agreement with a rational rival at the buyer's most preferred terms. Thus, the seller who undercuts does not steal the buyer from his rival and hence does not gain from undercutting.

The formalization I propose in this article has three major benefits: First, the model facilitates the investigation of the roles of strategic commitment and reputation that are elements missing in existing formal models of search and multilateral bargaining. Second, the model's predictions and the equilibrium dynamics are robust in many aspects. Third, given the sellers' initial offers, the equilibrium strategies in the multilateral bargaining game are essentially unique. This finding differs from the standard conclusion in noncooperative bargaining games that informational asymmetries give rise to multiplicities.[2] This makes the model a fruitful ground to answer further questions regarding the impacts of reputation on market outcomes and market microstructure.

Overview of the Results and of the Literature

Shelling (1960) points out the potential benefits of commitment in strategic and dynamic environments and asserts that one way to model the possibility of commitment is to explicitly include it as an action players can take. Crawford (1982), Muthoo (1996), and Ellingson and Miettinen (2008) follow this approach and show that commitment can be rationalized in equilibrium if revoking it is costly. However, I adopt an approach following Kreps and Wilson (1982) and Milgrom and Roberts (1982), where commitments are modeled as behavioral types that exist in the society, which rational players can mimic if they prefer to do so. Abreu and Sethi (2003) support the existence of commitment types from an evolutionary perspective and show that if players incur a cost of rationality, even if it is very small, the absence of such behavioral types is not compatible with the evolutionary stability in bargaining environments.

This paper is directly related to the reputation and bargaining literature initiated by Myerson (1991). Myerson investigates the impacts of one-sided reputation building on bilateral negotiations. Abreu and Gul (2000), Kambe (1999), and Compte and Jehiel (2002) consider two-sided versions of it. Compte and Jehiel (2002) consider a discrete-time bilateral bargaining problem in an Abreu-Gul setting and explore the role of exogenous outside options. They show that if both agents' outside options dominate yielding to the commitment type, then there is no point in building a reputation for inflexibility, and the unique equilibrium is again the Rubinstein (1982) outcome. The work of Atakan and Ekmekci (2010) is the most closely related to this paper as they study a market environment with multiple players. However, their main focus is substantially different.

[2]See, for example, Osborne and Rubinstein (1990).

They show—in a market with large numbers of buyers and sellers—that the existence of commitment types and endogenous outside options provide enough incentive for the rational players to create a false reputation on obstinacy. On the other hand, in this paper, I aim to answer how reputational concerns affect the market participants' pricing and search decisions.

This paper is also related (though indirectly) to the literature initiated first by Shaked and Sutton (1984) and Rubinstein and Wolinsky (1985) and later followed by Gale (1986a/b), Bester (1988, 1989), Binmore and Herrero (1988), Rubinstein and Wolinsky (1990), and Satterthwaite and Shneyerov (2007). This paper adds to this literature by showing that when players have reputational concerns, frictionless competitive markets need not be Walrasian.

An important finding of bargaining models in search markets is that an outside option plays a limited or no role when the continuation of negotiation is at least as valuable as that of the outside option. The current model, however, makes this prediction invalid by showing that the availability of an endogenous outside option substantially affects the outcome in the bargaining between a buyer and a seller if reputational concerns are present.

In the model, the rational buyer can costlessly learn and accept the sellers' posted prices. Therefore, price search is indeed costless. However, searching for a bargain price is assumed to be costly, for analytical convenience, as the buyer suffers a very small but positive switching cost each time he changes his bargaining partner. Regardless of his initial reputation, the rational buyer believes that he can achieve a lower price by haggling with the sellers, and the low cost of searching for a deal makes haggling more attractive than accepting a seller's posted price. In fact, the rational buyer strictly prefers to visit sellers if his initial reputation is high (i.e., the buyer is *strong*) and is indifferent between visiting stores and the immediate acceptance of the lowest price if the rational buyer is *weak* (i.e., the buyer's initial reputation is low enough).

Equilibrium analysis shows that sellers have no bargaining power when they fail to coordinate on their initial offers or when the buyer's initial reputation is sufficiently high (i.e., the buyer is strong). When sellers post different prices, the rational buyer can bargain with the seller whose posted price is higher (say seller 2) and uses the more advantages terms offered by seller 1 as a threat point against seller 2 and arrives at an agreement with the rational seller 2 at the buyer's most preferred terms. On the other hand, if the buyer's initial reputation is sufficiently high so that his expected payoff of visiting the other seller is no less than his continuation payoff with his current partner, then the rational buyer can give a "take it or leave it" ultimatum to the first seller he visits. In equilibrium, the rational sellers anticipate this, so they immediately accept the buyer's most preferred terms whenever he visits their stores first.

As a result, when reputational concerns are present, if the buyer's outside option is high enough—which is the case when the sellers post different prices or when the buyer's initial reputation is sufficiently high—then the buyer's bargaining power becomes substantially strengthened, and the sellers accept any positive share the buyer offers. This conclusion is in contrast with the standard bargaining models without obstinate types. In those models, a seller can always offer the buyer's continuation value and prevent the buyer from leaving him empty-handed. However, this is never the case when commitment types are present. When players have reputational concerns, offering something different than his posted price would reveal a seller's rationality, which yields surplus no more than what the seller would achieve by accepting the buyer's offer (see Myerson 1991; Compte and Jehiel 2002).

However, when the buyer is weak, then the rational buyer's desire to make a better deal turns into a trap. This trap drags the rational buyer into a situation where he may get much less than what he would achieve if he would have committed himself to accept the lowest posted price. The problem is that the rational buyer cannot commit himself to accept one of the posted prices immediately because searching for a bargain is equally attractive to the buyer when he is weak. For this reason, the rational sellers do not have to compete with each other over their posted prices, making positive prices consistent with equilibrium.

In particular, when the buyer is weak, positive prices are consistent with equilibrium because (1) reputation has a *lock-in* effect (analogous to Klemperer, 1987) for the buyer, which provides leverage to the sellers, and (2) price undercutting is not optimal for the sellers. When the buyer is weak and the sellers post the same price, conceding to the first seller is at least as good for the rational buyer as visiting the second seller. The rational buyer can credibly terminate the negotiation with the first seller and visit the second seller only if the buyer maintains a sufficiently high posterior probability of him being an obstinate type while negotiating with the first seller. However, this is possible if the rational buyer plays a mixed strategy in which he accepts the seller's price with a positive probability before the buyer leaves the first seller. Because the rational buyer plays a mixed strategy, the rational sellers receive ex-ante positive expected surplus in equilibrium.

We reach the conclusion that price undercutting is not optimal for the sellers because of two reasons: First, if a seller price undercuts, then the buyer fully believes that this seller is a commitment type. Second, as I argued previously, posting different prices will improve the rational buyer's bargaining power remarkably. As a result, being perceived as an obstinate seller reduces the chance that his offer will be accepted by the buyer because the rational buyer prefers to visit the undercutting seller's rival—who is likely to be rational—first, and this restrains a rational seller from underbidding his competitor. This observation contrasts with the predictions of the bilateral bargaining models of Kambe (1999), Abreu and Gul (2000), and Compte and Jehiel (2002). In their models, being perceived as an obstinate type is immediately followed by a concession from the rational opponent. High search cost clearly makes this trap go away as the rational buyer knows that high cost decreases the attractiveness of searching for a deal.

The current model presumes that the buyer's moves throughout the haggling process are observable to the sellers. Therefore, the buyer can use his reputation that is built in one store against the other seller. This might be a strong assumption for large markets, where the buyers are usually anonymous. For this reason, in Section 3, I relax this condition and suppose that the buyer's arrival time to stores, initial offers, and the time he spends in each store are not publicly observable. The simple extension of the model shows that anonymity increases the sellers' market power even further. Nevertheless, to be deemed as a tough bargainer is still bad for the competing players, and so price undercutting is not optimal.

Finally, the model's predictions are robust in many aspects. For instance, in Section 2 (Theorem 3), I check if the impacts of reputation decrease in "larger" markets, where the number of sellers is greater than two, and show that a range of prices, including the monopoly price and 0 are still consistent with equilibrium. In addition, Section 3 shows that the premises on the obstinate buyer's store selection have no significant effect. That is, even if the obstinate buyer is committed to immediately leave a seller's store once his offer is not accepted, then the lock-in effect of the reputation will still be in play, making price undercutting suboptimal and positive prices consistent with equilibrium. Finally, I show that reputational concerns of the players overwhelm their behaviors so that equilibrium has a war of attrition structure. As a result, the equilibrium of the haggling process is "independent" of the exogenously assumed bargaining protocols.[3]

[3]Likewise, Chatterjee and Samuelson (1987); Samuelson (1992); Caruana, Eirav, and Quint (2007); and Caruana and Einav (2008) show that credible commitment to certain promises, threats, or actions would wash out technical specifications of the bargaining procedures.

1 The Competitive-Bargaining Game in Continuous Time

Here, I define the *competitive-bargaining game* in continuous time. Section 2 presents the main results. Sections 3 offers some extensions of the model and provides some robustness results.

The Players: There are two sellers having an indivisible homogeneous good and a single buyer who wants to consume only one unit.[4] The valuation of the good is one for the buyer and 0 for the sellers. Both the buyer and the sellers have some small positive probability of being a "commitment" type. An obstinate (or commitment) type of player $n \in \{1, 2, b\}$, where b represents the buyer and 1 and 2 represent the sellers, is identified by a number $\alpha_n \in [0, 1]$. A type α_i of seller $i \in \{1, 2\}$ always demands α_i, accepts any price offer greater or equal to α_i, and rejects all smaller offers. On the other hand, a type α_b of the buyer always demands α_b, accepts any price offer smaller or equal to α_b, and rejects all greater offers. I use the terms "rational" or "obstinate" with the identity of a player (buyer or seller) whenever I want to differentiate the types of the player. Not mentioning these terms with the identity of a player should be understood that I mean both rational and obstinate types of that player.

I denote by $\mathcal{C} \subset [0, 1)$ with $0 \in \mathcal{C}$ the *finite* set of obstinate types for all three players and by $\pi(\alpha_n)$ the conditional probability that player n is obstinate of type α_n given that he is obstinate.[5] Thus, π is a probability distribution on $\mathcal{C}$ satisfying $\pi(\alpha) > 0$ for all $\alpha \in \mathcal{C}$. For simplicity, I assume that π is common for all three players. In case I need to emphasize different obstinate types of player n, I use α_n, α'_n, and so on. The initial probability that n is obstinate (i.e., player n's initial reputation) is denoted by z_n. I restrict my attention to the case where the sellers' initial reputations are the same (i.e., $z_i = z_s$ for $i = 1, 2$) and that z_b and z_s take sufficiently small values. Finally, I denote by r_b and r_s the rate of time preferences of the rational buyer and the sellers, respectively.

The Timing of the Game: The competitive-bargaining game between the sellers and the buyer is a two-stage, infinite-horizon, continuous-time game. The sellers make initial posted-price offers; the buyer can accept one of these costlessly (say over the phone) or visit one of the stores and try to bargain for a lower price. The buyer can negotiate

[4] At the end of Section 2, I consider the case where the number of sellers is some $N > 2$. In Section 2, I show that positive prices can be supported in equilibrium even though the buyer has monopsony power. In this respect, having more than one buyer can only strengthen the main findings of the paper.

[5] Having $1 \notin \mathcal{C}$ does not affect the analyses and the results of the paper but eliminates additional cases that produce nothing new.

only with the seller whom he is currently visiting. The buyer is free to walk out of one store and try another, but at a cost (delay) of switching, which is assumed to be very small. The reader may wish to picture this market as an environment where the sellers' stores are located at opposite ends of a town, so changing the bargaining partner is costly for the buyer because it takes time to move from one store to the other, and the buyer discounts time.

More formally, the first stage starts and ends at time 0, and the timing within the first stage is as follows: Initially, each seller simultaneously announces (posts) a demand (price) from the finite set $\mathcal{C}$, and it is observable to the buyer.[6] After observing the sellers' demands, the buyer has two options: He can accept one of the posted prices and finish the game. Or he can make a counteroffer that is observable to the sellers and visit one of the sellers to start the second stage (the bargaining phase).

Note that if seller i is rational and posts the price of $\alpha_i \in \mathcal{C}$ in stage 1, then this is his strategic choice. If he is the obstinate type, then he merely declares the demand corresponding to his type. Given the description of the obstinate players, if the buyer accepts α_i and finishes the game at time 0, then he is either rational and finishes the game strategically or is obstinate of type α_b such that $\alpha_b \geq \alpha_i$. Likewise, if the buyer makes a counteroffer $\alpha_b \in \mathcal{C}$, which is incompatible with the sellers' demands (i.e., $\alpha_b < \min\{\alpha_1, \alpha_2\}$), then this may be because the buyer is rational and strategically demands this price or because the buyer is the obstinate type α_b.[7]

Upon the beginning of the second stage (at time 0), the buyer and seller i, who is visited by the buyer first, immediately begin to play the following *concession game*: At any given time, a player either accepts his opponent's initial demand or waits for a concession. At the same time, the buyer decides whether to stay or leave store i. If the buyer leaves store i and goes to store $j \in \{1, 2\}$ with $j \neq i$, the buyer and seller j start playing the concession game upon the buyer's arrival at that store.[8] Assuming that the sellers are spatially separated, let δ denote the discount factor for the buyer that occurs due to the time $\Delta > 0$ required to travel from one store to the other. That is, $\delta = e^{-r_b\Delta}$. Note that $1 - \delta$ (the search friction) is the cost that the buyer incurs each time he switches his bargaining partner.[9] I assume that the search friction is very

[6]For analytical simplicity, I assume that the set of offers is common for all the players and is equal to the set of obstinate types $\mathcal{C}$. This restriction is dispensable and can be removed with no impact on equilibrium outcomes.

[7]Therefore, if the buyer makes a counteroffer and demands α_b that is greater than or equal to the minimum of the posted prices, then the buyer is rational and strategically demanding this price.

[8]After leaving store i and traveling partway to store j, the buyer could, if he wished, turn back and enter store i again. However, the buyer will never behave that way in equilibrium.

[9]One may assume a switching cost for the buyer that is independent of the "travel time" Δ, but this change would not affect our results. However, incorporating the search friction in this manner simplifies

small (i.e., $1-\delta$ is very close to 0) and thus, the finite set $\mathcal{C}$ is coarse relative to the search friction.[10] More specifically, I assume that for all $\alpha, \alpha' \in \mathcal{C}$ with $\alpha > \alpha'$, we have $(1-\alpha) < \delta(1-\alpha')$. The idea behind this assumption is very simple: the friction should not prevent the rational buyer to walk away from a store if he knows that the other seller has posted a lower price.[11] Concession of the buyer or seller i while the buyer is in store i marks the completion of the game; if the agreement $\alpha \in \{\alpha_b, \alpha_i\}$ is reached at time t, then the payoffs to seller i, the buyer, and seller j are $\alpha e^{-r_s t}$, $(1-\alpha)e^{-r_b t}$, and 0, respectively. In case of simultaneous concessions, surplus is split equally.[12]

I denote the two-stage competitive-bargaining game in continuous time by G. The second stage of the competitive-bargaining game is modeled as a modified war of attrition game. Alternatively, for example, we could suppose that players can modify their offers (in the second stage) at times $\{1, 2, ...\}$ in alternating orders but can concede to an outstanding demand at any $t \in [0, \infty)$. Given the behaviors of the obstinate types, modifying his offer would reveal a player's rationality, and in the unique equilibrium of the continuation game, he should concede to the opponent's demand immediately. Hence, in equilibrium, rational players would never modify their demands. These arguments are formally investigated in Appendix B for appropriately chosen parameter values.

The Information Structure: There is no informational asymmetry regarding the players' valuations and time preferences. However, players have private information about their resoluteness. That is, each player knows its own type but does not know the opponents' true types.

In addition, I assume that all three players' initial offers, the buyer's timing, and store selection are observable to the public. In Section 3, I consider a case where the buyer's arrival to the market and moves in negotiating with a seller are unobservable to the public.

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the notation substantially.

[10]In some markets, search friction may shape the market participants' behavior significantly. However, there are many examples where search cost is negligible (e.g., Alibaba.com, eBay, Amazon, and similar e-commerce platforms).

[11]This inequality follows from the dynamics of the rational buyer's haggling activities. Suppose that the buyer is in store 1 and playing the concession game with seller 1 whose posted price is α. If the buyer concedes to seller 1, the buyer's instantaneous payoff will be $1-\alpha$. However, if the buyer (immediately) leaves seller 1 and goes directly to the second seller to accept his posted price α' (where $\alpha' < \alpha$), his discounted payoff will be $\delta(1-\alpha')$. Hence, the inequality $(1-\alpha) < \delta(1-\alpha')$ ensures that the rational buyer will not hesitate to walk away from a store to accept the other seller's lower price.

[12]This particular assumption is not crucial because simultaneous concession occurs with probability 0 in equilibrium.

More Details on Obstinate Types: The obstinate types are defined by the strategies they pursue, and so they are *strategy types*. Details of their strategies are important in determining the equilibrium behavior of the rational players. The critical assumption for our results is that an obstinate player never backs down from his initial offer during the concession games. The remaining details of the obstinate players' strategies have minor impact on the main results in Section 2, and I prove this by analyzing some possible alternatives in Section 3.

The remaining details of the strategies of the obstinate types are as follows: The obstinate buyer of any type (or demand) $\alpha_b \in \mathcal{C}$ understands the equilibrium and leaves his bargaining partner permanently when he is convinced that his partner will never concede. If the sellers' posted prices (α_1 and α_2) are the same or the obstinate buyer's type (α_b) is incompatible with these prices, then the obstinate buyer visits each seller with equal probabilities. Moreover, if a seller's posted price is compatible with the obstinate buyer's type α_b (i.e., $\min\{\alpha_1, \alpha_2\} \leq \alpha_b$), then he immediately accepts the lowest price and finishes the game at time 0. Finally, the obstinate buyer with demand α_b never visits a seller who is known to be the commitment type with demand $\alpha > \alpha_b$.

Strategies of the Rational Players: In the first stage of the competitive-bargaining game G, a strategy for rational seller i, μ_i, is a distribution function over the set $\mathcal{C}$. For any $\alpha_i \in \mathcal{C}$, $\mu_i(\alpha_i)$ is the probability that rational seller i announces the demand α_i.

A first-stage strategy for the rational buyer consists of two parts: μ_b and σ_i. Although the strategy μ_b is a function of the sellers' announcements (α_1 and α_2) and σ_i is a function of all three players' announcements, these connections are omitted for notational simplicity. Given that each seller posts α_i, $\mu_b(\alpha_b)$ is the probability that the rational buyer announces the demand $\alpha_b \in \mathcal{C}$ with $\alpha_b \leq \alpha$, where $\alpha = \min\{\alpha_1, \alpha_2\}$. That is, μ_b is a probability measure over $\mathcal{C}_\alpha = \{x \in \mathcal{C} | x \leq \alpha\}$. I require that the game G ends in the first stage, when the rational buyer announces α. That is, the immediate concession of the buyer is represented by the buyer's announcement of α. Moreover, σ_i denotes the probability of the rational buyer visiting seller i first, and so $\sigma_1 + \sigma_2 = 1$.

If the competitive-bargaining game proceeds to the second stage and the first-stage strategies of the players are μ_1, μ_2, σ_1, and μ_b, then Bayes' rule implies the followings: The probability of seller i being obstinate conditional on posting price α_i is

$$\frac{z_s \pi(\alpha_i)}{z_s \pi(\alpha_i) + \mu_i(\alpha_i)(1 - z_s)} := \hat{z}_i(\alpha_i).$$

Furthermore, the probability that the buyer is the commitment type conditional on announcing his demand as $\alpha_b < \alpha$ and visiting seller i first is[13]

$$(1) \qquad \frac{\frac{1}{2}z_b\pi(\alpha_b)}{\frac{1}{2}z_b\pi(\alpha_b) + (1-z_b)\sigma_i\mu_b(\alpha_b)\left[\sum_{x<\alpha}\pi(x)\right]}.$$

Second-stage strategies are relatively more complicated. A nonterminal history of length t (i.e., h_t) summarizes the initial demands chosen by the players in the first stage, the sequence of stores the buyer visits, and the duration of each visit until time t (inclusive). For each $i = 1, 2$, let $\hat{\mathcal{H}}_t^i$ be the set of all nonterminal histories of length t such that the buyer is in store i at time t. Also, let $\mathcal{H}_t^i$ denote the set of all nonterminal histories of length t with which the buyer just enters store i at time t.[14] Finally, set $\hat{\mathcal{H}}^i = \bigcup_{t\geq 0}\hat{\mathcal{H}}_t^i$ and $\mathcal{H}^i = \bigcup_{t\geq 0}\mathcal{H}_t^i$.

The buyer's strategy in the second stage has three parts: The first part determines the buyer's location at any given history. For the other two parts (i.e., $\mathscr{F}_b^i$ for each i), let $\mathbb{I}$ be the set of all intervals of the form $[T,\infty]\,(\equiv [T,\infty)\cup\{\infty\})$ for $T \in \mathbb{R}_+$ and $\mathbb{F}$ be the set of all right-continuous distribution functions defined over an interval in $\mathbb{I}$. Therefore, $\mathscr{F}_b^i : \mathcal{H}^i \to \mathbb{F}$ maps each history $h_T \in \mathcal{H}^i$ to a right-continuous distribution function $F_b^{i,T} : [T,\infty] \to [0,1]$ *representing the probability of the buyer conceding to seller i by time t (inclusive).* Similarly, seller i's strategy $\mathscr{F}_i : \mathcal{H}^i \to \mathbb{F}$ maps each history $h_T \in \mathcal{H}^i$ to a right-continuous distribution function $F_i^T : [T,\infty] \to [0,1]$ *representing the probability of seller i conceding to the buyer by time t (inclusive).*

Player n's reputation $\hat{z}_n$ is a function of histories and n's strategies, representing the probability that the other players attach to the event that n is obstinate. It is updated according to Bayes' rule. At the beginning of the game, we have $\hat{z}_b(\emptyset) = z_b$ and $\hat{z}_i(\emptyset) = z_s$ for each seller i, where $\emptyset$ represents the null history. Given the rational buyer's first-stage strategies and a history h_0, where the buyer announces α_b and visits seller i first, the buyer's reputation at the time he enters store i (i.e., $\hat{z}_b(h_0)$) is given by Equation (1). Following the history h_0, if the buyer plays the concession game with seller i until some time $t > 0$ and the game has not ended yet (call this history h_t), then the buyer's reputation at time t is $\frac{\hat{z}_b(h_0)}{1-F_b^{i,0}(t)}$, assuming that the buyer's strategy in the concession game is $F_b^{i,0}$.

Note that $F_b^{i,0}(t)$ gives the probability that the buyer will accept α_i prior to t. The probability that the buyer will accept α_i prior to t given that he is rational is higher, which is equal to $F_b^{i,0}(t)/(1-\hat{z}_b(h_0))$. Therefore, the upper limit of the distribution function

[13]Given the sellers' announcements α_1 and α_2, the obstinate buyer of type $\alpha_b \geq \alpha = \min\{\alpha_1, \alpha_2\}$ accepts the seller's price α and finalizes the game. Therefore, conditional on the buyer visiting seller i first and demanding some $\alpha_b < \alpha$, the probability that the buyer is obstinate of type α_b should be $\frac{\pi(\alpha_b)}{\sum_{x<\alpha}\pi(x)}$. Moreover, $\frac{1}{2}z_b$ is the probability that the buyer is obstinate and he visits seller i first.

[14]That is, there exits $\epsilon > 0$ such that for all $t' \in [t-\epsilon, t)$, $h_{t'} \notin \hat{\mathcal{H}}_t^i$ but $h_t \in \hat{\mathcal{H}}_t^i$.

$F_b^{i,T}$ is $1-\hat{z}_b(h_T)$, where $\hat{z}_b(h_T)$ is the buyer's reputation at time $T \geq 0$, the time that the buyer (re)visits store i. That is, $\lim_{t\to\infty} F_b^{i,T}(t) \leq 1-\hat{z}_b(h_T)$. The same arguments apply to the sellers' strategies.

Since I will use z_b, z_s, and $\hat{z}_s$ extensively in the paper, it is crucial to emphasize what they refer to. I will denote the buyer's and the sellers' initial reputations by z_b and z_s, respectively. The term $\hat{z}_s$ represents a seller's reputation at the beginning of the second stage conditional on him posting price $\alpha_s \in \mathcal{C}$. Although $\hat{z}_s$ is a function of a rational seller's strategy and his posted price, I will omit this connection only for notational simplicity.

Given $F_b^{i,T}$, the rational seller i's expected payoff of conceding to the buyer at time t (conditional on not reaching a deal before time t where $T \leq t$,) is

$$U_i(t, F_b^{i,T}) := \alpha_i \int_0^{t-T} e^{-r_s y} dF_b^{i,T}(y) + \alpha_b[1-F_b^{i,T}(t)]e^{-r_s(t-T)} + \frac{1}{2}(\alpha_i+\alpha_b)[F_b^{i,T}(t)-F_b^{i,T}(t^-)]e^{-r_s(t-T)} \tag{2}$$

with $F_b^{i,T}(t^-) = \lim_{y\uparrow t} F_b^{i,T}(y)$.

In a similar manner, given F_i^T, the expected payoff of the rational buyer who concedes to seller i at time t is

$$U_b^i(t, F_i^T) := (1-\alpha_b) \int_0^{t-T} e^{-r_b y} dF_i^T(y) + (1-\alpha_i)[1-F_i^T(t)]e^{-r_b(t-T)} + \frac{1}{2}(2-\alpha_i-\alpha_b)[F_i^T(t)-F_i^T(t^-)]e^{-r_b(t-T)} \tag{3}$$

where $F_i^T(t^-) = \lim_{y\uparrow t} F_i^T(y)$.[15]

2 Main Results

In this section, I present the main results of the paper. For this purpose, I fix the values of δ, r_b, and r_s and the set of obstinate types $\mathcal{C}$. Theorem 1 shows that all demands in the set $\mathcal{C}$ can be supported in equilibrium for some values of $z_b, z_s \in (0,1)$. Then by Theorem 2, I prove that a range of prices that includes the monopoly price and 0 are compatible in equilibrium even in the limit where the frictions vanish (i.e., z_b and z_s converge to 0). Finally, Theorem 3 shows that Theorem 2 can be extended to the case where the number of sellers is more than 2.

[15] Expected payoffs are evaluated at time T, and they are conditional on the event that the buyer visits seller i at time $T \geq 0$.

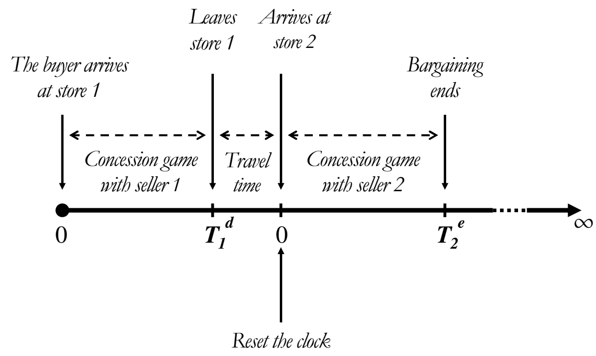

Figure 1: The timeline of the buyer's equilibrium strategy

For any $z_b, z_s \in (0,1)$, let $\mathrm{G}(z_b, z_s)$ denote the competitive-bargaining game G, where the initial reputations of the sellers and the buyer are z_b and z_s, respectively.

Theorem 1. *For all $\alpha_s \in \mathcal{C}$, there exists some small $z_b, z_s \in (0,1)$ and an equilibrium strategy of the game $G(z_b, z_s)$ in which both sellers post α_s in the first stage.*

I defer the proofs of all the results in this section to Appendix A. Note that for any values of z_b and z_s, 0 is an equilibrium price. Theorem 1 shows that any positive demand in $\mathcal{C}$ can be supported in equilibrium if we pick z_s and z_b as follows: For all $\alpha_b \in \mathcal{C}$ with $\alpha_b < \alpha_s$, we have

$$z_b \leq \left(\frac{\hat{z}_s^2}{A}\right)^{\frac{\lambda_b}{\lambda_s}} \tag{4}$$

where $\hat{z}_s = \frac{z_s \pi(\alpha_s)}{z_s \pi(\alpha_s) + 1 - z_s}$, $A = 1 - \frac{1-\delta}{\delta} \frac{1-\alpha_s}{\alpha_s - \alpha_b}$, $\lambda_s = \frac{(1-\alpha_s) r_b}{\alpha_s - \alpha_b}$ and $\lambda_b = \frac{\alpha_b r_s}{\alpha_s - \alpha_b}$. The parameters A, λ_b, and λ_s depend on the sellers' and the buyer's announced demands α_s and α_b, but I omit this connection for notational simplicity.

A short descriptive summary of the equilibrium strategies are as follows: In the first stage, both rational sellers post the demand α_s, and the rational buyer visits each store with equal probabilities and randomly declares a demand $\alpha_b \in \{\alpha \in \mathcal{C} | \alpha < \alpha_s\}$ with probability $\mu(\alpha_b) = \frac{\pi(\alpha_b)}{\sum_{x<\alpha_s} \pi(x)}$. Therefore, if the game does not end in the first stage, then Bayes' rule implies that the posterior probability that seller i is obstinate is $\hat{z}_s$ (as defined above) if he posts α_s and is 1 if he unilaterally deviates and posts a price other than α_s. Similarly, the posterior probability that the buyer is obstinate is z_b if he announces a price that is less than the sellers' price α_s and is 1 otherwise.

A short descriptive summary of the equilibrium strategies in the second stage is as follows (see *Figure 1*): The buyer visits each store at most once. When the buyer enters store 1 at time 0, the rational buyer plays the concession game with seller 1 until time

$T_1^d = -\log(\hat{z}_s)/\lambda_s > 0$. If the game does not end prior to time T_1^d, the buyer leaves store 1 at this time for sure and goes directly to store 2.

Note that building reputation on inflexibility by negotiating with the first seller is an investment for the buyer, which increases his continuation payoff in the second store. In equilibrium, the rational buyer leaves the first store when his discounted expected payoff in the second store is at least as high as his continuation payoff in the first store. Since z_b is low relative to $\hat{z}_s$ in equilibrium, the rational buyer needs to build up his reputation before leaving the first store.

During the concession game, the rational buyer and seller 1 concede by choosing the timing of acceptance randomly with constant hazard rates λ_b and λ_s respectively. Conditional on the game lasting until time T_1^d, seller 1's reputation reaches 1, and the buyer's reputation reaches $\frac{z_b}{1-F_b^1(T_1^d)}$, where $F_b^1(T_1^d)$ is the probability that buyer 1 concedes to seller 1 prior to time T_1^d. The buyer's posterior probability at time T_1^d is strictly less than 1 because it is the sufficient level of reputation that the rational buyer needs to walk away from the first seller and to search a deal with the second one.

Once the buyer arrives at store 2, the buyer and seller 2 play the concession game until time $T_2^e = -\log(\hat{z}_s/A)/\lambda_s$, the time that both players' reputations simultaneously reach 1. For notational simplicity, I manipulate the subsequent notation and reset the clock once the buyer arrives in store 2. Thus, I define each player's distribution function as if the concession game in each store starts at time 0. In the second store, the rational buyer and seller 2 also concede with constant hazard rates λ_b and λ_s, respectively. The players' concession game strategies are

$$F_b^1(t) = 1 - z_b(A/\hat{z}_s^2)^{\lambda_b/\lambda_s} e^{-\lambda_b t} \quad F_1(t) = 1 - \hat{z}_s e^{\lambda_s(T_1^d - t)}$$

in store 1 and

$$F_b^2(t) = 1 - e^{-\lambda_b t} \qquad F_2(t) = 1 - \hat{z}_s e^{\lambda_s(T_2^e - t)}$$

in store 2 (see Proposition 2.1 and Lemma 2.1 in Appendix A).[16]

In equilibrium, the rational buyer's continuation payoff is no more than $1 - \alpha_s$ if he reveals his rationality.[17] Since the obstinate buyer leaves a seller when he is convinced that his bargaining partner is also obstinate, leaving the first seller "earlier" (or "later") than this time (i.e., T_1^d) would reveal the buyer's rationality. Moreover, since the cost of switching the negotiating partners (i.e., the sellers) is positive, the rational buyer never leaves a seller if there is a positive probability that this seller is rational, and he

[16]For notational simplicity, I skip the superscript T in players' strategies.

[17]Arguments similar to the proof of Lemma 2 in the Online Appendix and the one-sided uncertainty result of Myerson (1991, Theorem 8.4) imply this result.

immediately leaves otherwise. Clearly, the buyer does not revisit a seller once he knows that this seller is obstinate.

The rational players' equilibrium payoffs in the concession games are calculated by Equations (3) and (4). That is, for each seller i

$$v_b^i = F_i(0)(1-\alpha_b) + [1-F_i(0)](1-\alpha_s), \text{and} \\ v_i = F_b^i(0)\alpha_s + [1-F_b^i(0)]\alpha_b. \tag{5}$$

However, the rational players' equilibrium payoffs in the game G is different as they should take into account the buyer's outside option and store selection in the first stage.

In equilibrium, where the buyer first visits seller 1, the rational buyer leaves the first seller when he is convinced that this seller is obstinate. At this moment, walking out of store 1 is optimal for the rational buyer if his discounted continuation payoff in the second store, δv_b^2, is no less than $1-\alpha_s$, which is the payoff to the rational buyer if he concedes to the obstinate seller 1. Let z_b^* denote the level of reputation required to provide the rational buyer enough incentive to leave the first store. Assuming that $z_b < z_b^*$ (i.e., the rational buyer needs to build up his reputation before walking out of store 1), the game ends in store 2 at time $T_2^e = -\log(z_b^*)/\lambda_b$. We find the value of T_2^e by solving the equation $F_b^2(T_2^e) = 1 - z_b^*$, which is implied by the equilibrium: the buyer's reputation reaches 1 at time T_2^e. Thus, given the value of $F_2(0)$ and the rational buyer's discounted continuation payoff in store 2, z_b^* must solve

$$1-\alpha_s = \delta[1-\alpha_b - \hat{z}_s(\alpha_s-\alpha_b)(z_b^*)^{-\lambda_s/\lambda_b}],$$

implying that $z_b^* = \left(\frac{\hat{z}_s}{A}\right)^{\frac{\lambda_b}{\lambda_s}}$, where $A = 1 - \frac{1-\delta}{\delta}\frac{1-\alpha_s}{\alpha_s-\alpha_b}$. Note that z_b^* is well defined (i.e., $z_b^* \in (0,1)$) as A is positive. In fact, A is very close to 1 since the cost of traveling is assumed to be very small.

I call the buyer ***strong*** if the first seller he visits makes an initial probabilistic concession and ***weak*** otherwise.[18] Similarly, seller i is called strong if the rational buyer concedes to him with a positive probability at the time he visits store i first at time 0 and weak otherwise.

In equilibrium, the inequality given in Equation (4) (i.e., $z_b \leq (\hat{z}_s^2/A)^{\lambda_b/\lambda_s}$) implies that the rational buyer's initial reputation is very low, and thus, he needs to spend some time to build up his reputation before leaving the first seller. In this case, $F_1(0) = 0$ (i.e., the buyer does not receive an initial probabilistic gift from seller 1), which implies that the rational buyer is weak, and so the buyer's expected payoff during the concession game with seller 1 (i.e., v_b^1) is $1-\alpha_s$. Therefore, the rational buyer's expected payoff in

[18]Note that the second seller (the one who is visited after the first seller) always makes an initial probabilistic concession in equilibrium.

the game is also $1-\alpha_s$ if he announces any demand in $\mathcal{C}$ that is less than α_s. Thus, the rational buyer has no incentive to deviate from his equilibrium strategies.

In case one of the sellers—say, seller 2—undercuts his opponent and posts a price $\alpha_2 \in \mathcal{C}$ such that $\alpha_2 < \alpha_s$, then there are two scenarios we need to consider: If α_2 is positive, then in the first stage, the rational buyer announces his demand as 0 and visits seller 1 first (with probability 1) to make the "take it or leave it" offer; he leaves store 1 upon his arrival at that store. Conditional on not reaching a deal, the rational buyer goes directly to seller 2 and accepts α_2. On the other hand, rational seller 1 immediately accepts the buyer's demand. Therefore, in case the game does not end in store 1, the buyer infers that seller 1 is the obstinate type with demand α_1. However, if $\alpha_2 = 0$, then the buyer immediately accepts the second seller's posted demand and finishes the game in the first stage (see Proposition 2.2 in Appendix A).

Therefore, if seller 2 deviates from his strategy and price undercuts his opponent, then the buyer infers that seller 2 is obstinate with certainty (as sellers are playing pure strategies in the first stage). Being perceived as an obstinate seller reduces the chance that his offer is accepted by the buyer. This is true because the rational buyer prefers to use the obstinate seller's low price as an "outside option" to increase his bargaining power against seller 1, whom he can negotiate and get a much better deal in expected terms.

On the other hand, if seller 2 unilaterally deviates in the first stage and posts a price $\alpha_2 > \alpha_s$, then the rational buyer visits seller 1 first and never goes to the second store, and the concession game with seller 1 may continue until the time $T_1^e = -\log \hat{z}_s/\lambda_s$ with the following strategies: $F_1(t) = 1-e^{-\lambda_s t}$ and $F_b^1 = 1-z_b(1/\hat{z}_s)^{\lambda_b/\lambda_s}e^{-\lambda_b t}$ (see Proposition 2.2 in the Appendix A).

Therefore, if rational seller i plays according to his prescribed strategies, his expected payoff in the game is greater than $\frac{u}{2}\left[1 - z_b \sum_{\alpha_b \geq \alpha_s} \pi(\alpha_b)\right]$, where $u = \sum_{\alpha_b < \alpha_s} \alpha_b \mu(\alpha_b)$ (see the proof of Theorem 1). But a rational seller i's expected payoff is much less than $z_b + z_s$ if he deviates from his equilibrium strategy (Lemma 2.2 in Appendix A). Hence, for sufficiently small values of z_b and z_s, posting the nonzero price α_s is an optimal strategy for the sellers since the rational sellers' equilibrium payoffs are strictly greater than what they can achieve by price undercutting.

Note that Theorem 1 would still be true in case the buyer is known to be rational but the sellers are not (i.e., $z_b = 0$ and $z_s > 0$). This is true because (1) the buyer would be weak in equilibrium for any values of z_s and α_b and (2) the uncertainty regarding the sellers' actual types still gives rise to lock-in effect, and thus, price undercutting is not optimal for the competing sellers.[19] However, modelling the multilateral bargaining

[19]In fact, the lock-in effect in this case would be much stronger because (in any equilibrium) the buyer

problem as a modified war of attrition game would be a very strong restriction because Proposition B (in Appendix B) would not hold in this case.

The Limiting Case of Complete Rationality

I say the competitive-bargaining game $\mathrm{G}(z_b^m, z_s^m)$ converges to $\mathrm{G}(K)$ when the sequences $\{z_s^m\}$ and $\{z_b^m\}$ of initial priors satisfy

(6) $\lim z_s^m = 0, \lim z_b^m = 0$ as $m \to \infty$ and $\log z_s^m / \log z_b^m = K$ for all $m \geq 0$

Theorem 2. *If the game $G(z_b^m, z_s^m)$ converges to $G(K)$, α_s^m is the equilibrium posted price of the rational sellers in the game $G(z_b^m, z_s^m)$, and if $\alpha_s \in \mathcal{C}$ is a limit point of α_s^m, then we have $2K\alpha r_s \leq (1-\alpha_s) r_b$ holds for all $\alpha \in \mathcal{C}$ with $\alpha < \alpha_s$.*

Theorem 2 indicates that a large set of prices can be supported in equilibrium even when the uncertainties about the players' rationality vanish. Theorem 1 proves that a positive price $\alpha_s \in \mathcal{C}$ can be supported in equilibrium whenever the players' initial priors satisfy the inequality in Equation (4) for all $\alpha \in \mathcal{C}$ with $\alpha < \alpha_s$ (i.e., the buyer is weak). Therefore, for decreasingly small values of the initial priors, the limit of this inequality yields the inequality that is given in the statement of Theorem 2.

Therefore, given the value of $0 < K$, the set of equilibrium prices for the sellers would converge to a subset of $\mathcal{C}$—as z_b, z_s approach to 0—containing all $\alpha_s \in \mathcal{C}$ that satisfy $\alpha_s \leq \frac{r_b}{r_b + 2Kr_s}$. Thus, all prices in $\mathcal{C}$ can be supported in equilibrium with carefully selected and vanishing initial priors. The monopoly price of 1, for example, can be arbitrarily approached if the initial priors are selected so that K is sufficiently close to 0.

The final result of this section examines a straightforward extension of the model to the case with $N > 2$ identical sellers. Let $\mathrm{G}^N(z_b^m, z_s^m)$ denote the competitive-bargaining game where the number of sellers is N; it is identical to $\mathrm{G}(z_b^m, z_s^m)$ except for the number of players. Let the convergence of $\mathrm{G}^N(z_b^m, z_s^m)$ to the game $\mathrm{G}^N(K)$ be identical to the convergence of its two-seller counterpart. Therefore,

Theorem 3. *If the game $G^N(z_b^m, z_s^m)$ converges to $G^N(K)$, α_s^m is the equilibrium posted price of the rational sellers in the game $G^N(z_b^m, z_s^m)$, and if $\alpha_s \in \mathcal{C}$ is a limit point of α_s^m, then we have $NK\alpha r_s \leq (1-\alpha_s) r_b$ holds for all $\alpha \in \mathcal{C}$ with $\alpha < \alpha_s$.*

Therefore, for any large but finite number of sellers N, we can find small enough z_b^m relative to z_s^m and $K < 1/N$ such that prices arbitrarily close to 1 can be supported in equilibrium with vanishing uncertainties.

should immediately accept a seller's price α_s and finish the game in stage 1.

3 Some Extensions

In this section, I will analyze various extensions of the model and show that the main conclusions still hold. That is, to be deemed as a commitment type (even if it is a less-greedy type) does not benefit the competing sellers, and so price undercutting is not optimal. Thus, positive prices are consistent with equilibrium even when uncertainties on players' rationality are decreasingly small.

A. The Buyer's Moves Are Unobservable to the Public

In this part, I investigate the case where the buyer's moves and demand announcements are not public. I will show that the sellers' market power will increase further in this case. That is, higher prices can be supported with equilibrium strategies that are similar to those that we used to prove Theorem 1.

I make three modifications on the competitive bargaining game G. First, the rational buyer announces his demand at the sellers' stores and may offer different demands in each store.[20] Second, the buyer's moves, including his arrival to the market, are unknown by the public. That is, sellers can observe the buyer only when he visits their stores. Third, related to the previous one, the buyer arrives at the market according to a Poisson arrival process. Given that the rational buyer plays a strategy in which he visits both sellers with positive probabilities upon his arrival at the market, the last assumption ensures that sellers cannot learn the buyer's actual type and whether they are the first or the second store visited by the buyer.[21]

The next result shows that if z_b is sufficiently small, then the following strategies (which are similar to the ones that we defined in Section 2) support any $\alpha_s \in \mathcal{C} \setminus \{0\}$ in equilibrium. Strategies are as follows: In the first stage, both sellers post α_s. In the second stage, upon his arrival at time $T \geq 0$, the rational buyer (immediately) visits the sellers with equal probabilities. Upon the buyer's entry to store i (at time T), the rational buyer randomly declares his demand $\alpha_b \in \{\alpha \in \mathcal{C} | \alpha < \alpha_s\}$ according to $\mu^T_{\alpha_i}(\alpha_b) = \frac{\pi(\alpha_b)}{\sum_{x<\alpha_s} \pi(x)}$ and starts the concession game with seller i. The players' strategies in the concession games are $F^T_b(t) = 1 - \frac{\hat{z}^{T,i}_b}{\hat{z}^{\lambda_b/\lambda_s}_s} e^{-\lambda_b t}$ and $F^T_i(t) = 1 - e^{-\lambda_s t}$, where $\hat{z}^{T,i}_b$ is the probability

[20] Parallel to the assumptions made in Section 1, the obstinate buyer also announces his demand at the sellers' store if his demand is less than the posted prices. Otherwise, he immediately accepts the lowest posted price and finalizes the game in the first stage.

[21] In the modified game, the rational players' strategies, which may depend on time T indicating the buyer's arrival time, are equivalent to the strategies defined in Section 1 with one exception. Now, $\mu^T_{\alpha_1}, \mu^T_{\alpha_2}$ are parts of the buyer's second-stage strategies and functions of the sellers' posted prices and the arrival time $T \geq 0$. Note that the first stage is time 0, where the sellers announce their demands and the buyer observes these prices. The second stage starts at the time that the buyer arrives at the market.

that the buyer is the commitment type α_b conditional on him visiting seller i at time T and demanding $\alpha_b < \alpha_i$. The rational players' hazard rates λ_b, λ_s are as given in Section 2. The concession game with a seller may last until time $-\log(\hat{z}_s)/\lambda_s + T$ (i.e., the departure time from the first store) at which point both the buyer's and the seller's reputations simultaneously reach 1.

Whenever a seller (say seller 2) deviates to a positive price that is lower than α_s, the rational buyer visits seller 1 first and demands 0. Rational seller 1 immediately accepts the buyer's demand. If he does not, the buyer leaves this seller, goes to store 2, and accepts seller 2's demand. However, if seller 2 deviates and posts 0, then the buyer immediately accepts 0 and finishes the game in the first stage.

According to these strategies, the rational buyer will visit only one seller. Moreover, due to the Poisson arrival process and Bayes' rule, the sellers will be uncertain about the buyer's actual type whenever the buyer arrives at their stores for the first time. In particular, $\hat{z}_b^{T,i}$ (i.e., the probability that the buyer is the commitment type α_b conditional on him visiting seller i at time T and demanding $\alpha_b < \alpha_s$) is independent of i, and it is either equal to z_b or to a number very close to z_b.[22]

In particular, given that the buyer arrives at the market at time T and both the buyer and the first seller are commitment types, the buyer (which is obstinate) leaves the first seller at time $-\log(\hat{z}_s)/\lambda_s + T$ since he will be convinced at this time that his opponent is also obstinate. However, in this case, the rational second seller will play the concession game with the (obstinate) buyer, believing that the buyer is obstinate with probability $\frac{z_b(1+\hat{z}_s)}{1+z_b\hat{z}_s}$.

Proposition 3.1. *For sufficiently small values of z_b and z_s, $\alpha_s \in \mathcal{C}\setminus\{0\}$ can be supported as equilibrium posted price of the rational sellers in the modified game $G(z_b, z_s)$ whenever $z_b \leq \frac{\hat{z}_s^{\lambda_b/\lambda_s}}{1+\hat{z}_s(1-\hat{z}_s^{\lambda_b/\lambda_s})}$ holds for all $\alpha \in \mathcal{C}$ with $\alpha < \alpha_s$, where $\hat{z}_s = \frac{z_s\pi(\alpha_s)}{z_s\pi(\alpha_s)+1-z_s}$, $\lambda_s = \frac{(1-\alpha_s)r_b}{\alpha_s-\alpha}$ and $\lambda_b = \frac{\alpha r_s}{\alpha_s-\alpha}$.*

I defer all the proofs in this section to Appendix A. Proposition 3.1 is the counterpart of Theorem 1 in the modified game. That is, it shows that any price in the set $\mathcal{C}$ can be supported in equilibrium if the initial priors z_b and z_s are carefully selected. Note that when z_b satisfies the inequality given in Proposition 3.1, the buyer is weak in equilibrium for any demand he announces in the sellers' stores. Similar to Theorem 2, the following result shows that a large set of prices can be supported in equilibrium even when the uncertainties on players' rationality vanish.

Proposition 3.2. *If the modified game $G(z_b^m, z_s^m)$ converges to $G(K)$, α_s^m is the equilibrium posted prices of the rational sellers in the modified game $G(z_b^m, z_s^m)$, and if $\alpha_s \in \mathcal{C}$*

[22] I calculate $\hat{z}_b^{T,i}$ formally in the proof of Proposition 3.1.

is a limit point of α_s^m, then we have $K\alpha r_s \leq (1-\alpha_s)r_b$ for all $\alpha \in \mathcal{C}$ with $\alpha < \alpha_s$.

Finally, since the buyer cannot carry his improved reputation when he leaves a seller, the buyer is weak if and only if $z_b \leq \frac{\hat{z}_s^{\lambda_b/\lambda_s}}{1+\hat{z}_s(1-\hat{z}_s^{\lambda_b/\lambda_s})}$, and this is true regardless of the number of sellers in the market. Therefore, the immediate counterpart of Theorem 3 will be as follows:

Corollary 3.1. *If the modified game $G^N(z_b^m, z_s^m)$ converges to $G^N(K)$, α_s^m is the equilibrium posted price of the rational sellers in the modified game $G^N(z_b^m, z_s^m)$, and if $\alpha_s \in \mathcal{C}$ is a limit point of α_s^m, then we have $K\alpha r_s \leq (1-\alpha_s)r_b$ for all $\alpha \in \mathcal{C}$ with $\alpha < \alpha_s$.*

Note that a demand $\alpha_s \in \mathcal{C}$ satisfying the inequality provided in Theorem 2 (or Theorem 3) also satisfies the inequality provided in Proposition 3.2 (or Corollary 3.1), but the converse is not true. Thus, if the buyer's moves are unobservable to the public, then the sellers' market powers may increase as higher prices can be supported in equilibrium of the modified game.

B. The Case with a More Aggressive Obstinate Buyer

The assumption that the obstinate buyer visits each seller at time 0 with equal probabilities is a simplification assumption. It can be generalized with no impact on the main messages of our results. For example, one may assume that there are multiple types for the obstinate buyer (regarding the initial store selection) such that some always choose a fixed seller, and some visit the sellers according to their announcements, while the rest are possibly a combination of these two.

The assumption on the obstinate buyer's departure habit seems a strong one since it eliminates the possibility that the rational buyer would increase his bargaining power by committing to a particular pattern of store choice. In the next two parts, I show that the main message of the paper will not change if the obstinate buyer is "more strategic" in the sense that he commits to immediately switch or leave his bargaining partner in case his demand is not accepted.

I first suppose that the obstinate buyer (of any demand) leaves the first store he visits at time $T = 0$. The next result shows that any $\alpha_s \in \mathcal{C}$ is an equilibrium price for the sellers if the buyer is weak in equilibrium. The equilibrium strategies are as follows: In the first stage, rational sellers post the same demand $0 < \alpha_s$, and the rational buyer visits each seller with equal probabilities and randomly declares his demand $\alpha_b \in \{\alpha \in \mathcal{C} | \alpha < \alpha_s\}$ according to $\mu_b^*(\alpha_b) = \frac{\pi(\alpha_b)}{\sum_{x<\alpha_s}\pi(x)}$. At the beginning of the second stage, assuming that the buyer visits seller 1 first, the rational buyer immediately accepts seller 1's demand at time 0 with probability $P_b = \frac{(\hat{z}_s/A)^{\lambda_b/\lambda_s} - z_b}{(1-z_b)(\hat{z}_s/A)^{\lambda_b/\lambda_s}}$ and immediately leaves store 1 with probability $1 - P_b$. Rational seller 1 never concedes to the buyer. The buyer and seller 2 play the concession game in the second store until time $T_2^e = -\frac{log(\hat{z}_s/A)}{\lambda_s}$ with the following strategies $F_b^2(t) = 1 - e^{-\lambda_b t}$ and $F_2(t) = 1 - Ae^{-\lambda_s t}$, where the terms $\hat{z}_s = \frac{z_s \pi(\alpha_s)}{z_s \pi(\alpha_s)+1-z_s}$, $A = 1 - \frac{1-\delta}{\delta}\frac{1-\alpha_s}{\alpha_s-\alpha}$, $\lambda_s = \frac{(1-\alpha_s)r_b}{\alpha_s-\alpha}$ and $\lambda_b = \frac{\alpha r_s}{\alpha_s-\alpha}$ are equal to ones defined in Sections 1 and 2. Finally, in case one of the sellers deviate in the first stage, then the strategies of the continuation game are given by Proposition 2.2 (in Appendix A).

Proposition 3.3. *For sufficiently small values of z_b and z_s, $\alpha_s \in \mathcal{C}\setminus\{0\}$ can be supported as equilibrium posted prices of the rational sellers in the modified game $G(z_b, z_s)$ (where the obstinate buyer leaves the first store he visits immediately following his arrival) whenever $z_b \leq \frac{(\hat{z}_s/A)^{\lambda_b/\lambda_s}(\alpha_s-\alpha)}{\alpha_s+\alpha}$ holds for all $\alpha \in \mathcal{C}$ with $\alpha < \alpha_s$.*

Parallel to our results in Section 2, Proposition 3.3 shows that if z_b and z_s are selected carefully, then all prices in the set $\mathcal{C}$ can still be supported in equilibrium.

C. The Case with the Most Aggressive Obstinate Buyer

Now suppose that the obstinate buyer (of any demand) leaves all stores immediately following his arrival. The following strategies ensure that all demands in the set $\mathcal{C}$ can be supported in equilibrium for small values of z_b and z_s. Rational sellers post the price of $0 < \alpha_s \in \mathcal{C}$, and the rational buyer visits each seller with equal probabilities and declares his demand as $\alpha_b < \alpha_s$ according to μ_b^* that is given above. At the beginning of the second stage, assuming that the buyer visits seller 1 first, the rational buyer immediately accepts seller 1's demand at time 0 with probability $P_b = \frac{\alpha_s(1-z_b)-\alpha_b}{(1-z_b)(\alpha_s-\alpha_b)}$ and immediately leaves store 1 with probability $1 - P_b$. Rational seller 1 never concedes to the buyer. In store 2, rational seller 2 accepts the buyer's demand upon his arrival with probability $P_s = \frac{(1-\alpha_s)(1-\delta)}{\delta(1-\hat{z}_s)(\alpha_s-\alpha_b)}$ and never concedes to the buyer with probability $1-P_s$.[23] The rational buyer does not leave store 2 immediately. Instead, he waits for the seller's concession. However, if the game does not end at time 0 by seller 2's concession, the rational buyer concedes to the buyer immediately. Finally, in case one of the sellers deviate in the first stage, then the strategies of the continuation game are given in Proposition 2.2 (in Appendix A).

Proposition 3.4. *For sufficiently small values of z_b and z_s, $\alpha_s \in \mathcal{C} \setminus \{0\}$ can be supported as equilibrium posted prices of the rational sellers in the modified game $G(z_b, z_s)$ (where the obstinate buyer leaves both stores immediately following his arrival) whenever $z_b \leq \frac{(\alpha_s-\alpha)^2}{\alpha_s(\alpha_s+\alpha)}$ holds for all $\alpha \in \mathcal{C}$ with $\alpha < \alpha_s$.*

[23] Note that P_s is in $(0,1)$ as $\hat{z}_s < \frac{(1-\alpha_s)(1-\delta)}{\delta(\alpha_s-\alpha_b)} < 1$.

D. Different Initial Reputations for the Sellers

Suppose for now that the probability distribution π_i over $\mathcal{C}$ is different for each seller i and the sellers' initial reputations are not equal (i.e., $z_1 \neq z_2$). These assumptions would not change the essence of our results as long as z_1 and z_2 are small enough. Similar to Theorem 1, in equilibrium, rational sellers post the same price α_s whenever the buyer is weak, which would mean $z_b \leq \left(\frac{\hat{z}_1\hat{z}_2}{A}\right)^{\lambda_b/\lambda_s}$ for all $\alpha \in \mathcal{C}$ with $\alpha < \alpha_s$, where $\hat{z}_i = \frac{z_i\pi_i(\alpha_s)}{z_i\pi_i(\alpha_s)+1-z_i}$, $A = 1 - \frac{1-\delta}{\delta}\frac{1-\alpha_s}{\alpha_s-\alpha_b}$, $\lambda_s = \frac{(1-\alpha_s)r_b}{\alpha_s-\alpha}$ and $\lambda_b = \frac{\alpha r_s}{\alpha_s-\alpha}$. As the rational buyer is weak, his expected payoff is independent of the sellers' initial reputations, and so these particular sources of heterogeneity do not change the fundamentals of the competition between the sellers.

E. Sequential Price Quoting

Suppose now that the price announcement in the game G is sequential. Seller 1 announces his demand first. Then the second seller posts his price after observing the first seller's announcement. Finally, the buyer declares his demand after observing the sellers' prices, and the rest of the game follows as before. Note that this change in the first stage does not alter the equilibrium strategies of the players in the concession game. Therefore, the continuation strategies provided in Section 2 still constitute an equilibrium of the game G in the second stage.

Similar to the previous arguments, if the buyer is weak (i.e., $z_b \leq (\hat{z}_s/A)^{\lambda_b/\lambda_s}$), then the rational sellers' expected payoff in the game increases with the price they post if z_b and z_s are sufficiently small.[24] Hence, in equilibrium, both sellers will post the same price, which will be the highest price available in the set $\mathcal{C}$. As a result, given that the number of sellers is $N \geq 2$ and the buyer is weak, the unique equilibrium price will converge to $\frac{r_b}{r_b+NKr_s}$ (the upper bound we found in Theorem 3) when z_b and z_s vanish at the same rate K.

[24] See the rational sellers' expected payoff, for example, in the proof of Theorem 1.

4 Conclusion

This paper investigated the impacts of reputation on competitive search markets where the sellers announce their initial demands prior to the buyer's visit and the buyer directs his search for a better deal. Facing multiple sellers, the buyer can negotiate with only one at a time and can switch his bargaining partner with some delay. A modified war of attrition structure is derived in the equilibrium (see Appendix B).

In equilibrium, if the sellers' posted prices are the same, then the buyer will never visit one seller more than once. In Sections 2 and 3, I show that the range of prices including the monopoly price and 0 are compatible in equilibrium even when frictions vanish. This is mainly due to the fact that (1) reputational concerns of the buyer has a lock-in effect, which forces the buyer to share a significant portion of the surplus with the sellers, and that (2) being known to be a tough (obstinate) bargainer is not an advantage for the competing sellers, and so price undercutting may not be advantageous. Further extensions of the model show that the main message of the paper and the crucial dynamics of the game are robust in many aspects.

Appendix A

Proposition 2.1. *In any (sequential) equilibrium of the competitive-bargaining game G, the rational buyer visits each store at most once. Moreover, the rational buyer leaves the first store at some finite time for sure, given that the game does not end before, and directly goes to the other store if and only if the first seller is obstinate. Finally, in an equilibrium where the rational buyer visits seller 1 first with probability* $1/2$*, leaves store* 1 *at time* T_1^d *and finalizes the game in store* 2 *at time* T_2^e *if the game has not yet ended before, the players' concession game strategies must be*

$$F_b^1(t) = 1 - c_b^1 e^{-\lambda_b t} \quad F_1(t) = 1 - \hat{z}_s e^{\lambda_s (T_1^d - t)}$$
$$F_b^2(t) = 1 - e^{-\lambda_b t} \qquad F_2(t) = 1 - \hat{z}_s e^{\lambda_s (T_2^e - t)}$$

satisfying

$$F_b^1(0)F_1(0) = 0 \qquad \text{and} \qquad F_b^2(T_2^e) = 1 - \frac{z_b}{1 - F_b^1(T_1^d)}$$

where $\lambda_s = \frac{(1-\alpha_s)r_b}{\alpha_s - \alpha_b}$ *and* $\lambda_b = \frac{\alpha_b r_s}{\alpha_s - \alpha_b}$.

Proof of Proposition 2.1. First, I will study the properties of equilibrium strategies (distribution functions) in concession games. For this purpose, take any $i \in \{1, 2\}$ and history $h_{T_i} \in \mathcal{H}^i$, and consider a pair of equilibrium distribution functions $(F_b^{i,T_i}, F_i^{T_i})$ defined over the domain $[T_i, T_i']$ where $T_i' \leq \infty$ depends on the buyers' equilibrium strategy. Proofs of the following results directly follow from the arguments in Hendricks, Weiss and Wilson (1988) and are analogous to the proof of Lemma 1 in Abreu and Gul (2000), so I skip the details.

Lemma A.1. *If a player's strategy is constant on some interval $[t_1, t_2] \subseteq [T_i, T_i')$, then his opponent's strategy is constant over the interval $[t_1, t_2 + \eta]$ for some $\eta > 0$.*

Lemma A.2. *F_b^{i,T_i} and $F_i^{T_i}$ do not have a mass point over $(T_i, T_i']$.*

Lemma A.3. *$F_i^{T_i}(T_i)F_b^{i,T_i}(T_i) = 0$*

Therefore, according to Lemma A.1 and A.2, both $F_i^{T_i}$ and F_b^{i,T_i} are strictly increasing and continuous over $[T_i, T_i']$. Recall that

$$U_i(t, F_b^{i,T_i}) = \int_{T_i}^{t} \alpha_s e^{-r_s y} dF_b^{i,T_i}(y) + \alpha_b e^{-r_s t}(1 - F_b^{i,T_i}(t))$$

denote the expected payoff of rational seller i who concedes at time $t \geq T_i$ and

$$U_b(t, F_i^{T_i}) = \int_{T_i}^{t} (1 - \alpha_b) e^{-r_b y} dF_i^{T_i}(y) + (1 - \alpha_s) e^{-r_b t}(1 - F_i^{T_i}(t))$$

denote the expected payoff of the rational buyer who concedes to seller i at time $t \geq T_i$. Therefore, the utility functions are also continuous on $[T_i, T_i']$.

Then, it follows that $\mathcal{D}^{i,T_i} := \{t | U_i(t, F_b^{i,T_i}) = \max_{s \in [T_i, T_i']} U_i(s, F_b^{i,T_i})\}$ is dense in $[T_i, T_i']$. Hence, $U_i(t, F_b^{i,T_i})$ is constant for all $t \in [T_i, T_i']$. Consequently, $\mathcal{D}^{i,T_i} = [T_i, T_i']$. Therefore, $U_i(t, F_b^{i,T_i})$ is differentiable as a function of t. The same arguments also hold for $F_i^{T_i}$. The differentiability of $F_i^{T_i}$ and F_b^{i,T_i} follows from the differentiability of the utility functions on $[T_i, T_i']$. Differentiating the utility functions and applying the Leibnitz's rule, we get $F_i^{T_i}(t) = 1 - c_i e^{-\lambda_s t}$ and $F_b^{i,T_i}(t) = 1 - c_b^i e^{-\lambda_b t}$ where $c_i = 1 - F_i^{T_i}(T_i)$ and $c_b^i = 1 - F_b^{i,T_i}(T_i)$ such that $\lambda_b = \frac{\alpha_b r_s}{\alpha_s - \alpha_b}$ and $\lambda_s = \frac{(1-\alpha_s) r_b}{\alpha_s - \alpha_b}$.

Therefore, the rational buyer's expected payoff of playing the concession game with seller i during $[T_i, T_i']$ is $[F_i^{T_i}(T_i))(1 - \alpha_b) + (1 - F_i^{T_i}(T_i))(1 - \alpha_s)]$. Moreover, by Lemma A.3, we know that if the buyer is strong in a concession game with seller i (starting at time T_i), then seller i is weak. Hence, there is no sequential equilibrium of the game G such that the buyer visits a store multiple times. Suppose on the contrary that there is a strategy in which, without loss of generality, the buyer visits store 1 twice. Then, the buyer must be strong in his second visit to seller 1. Otherwise the buyer would prefer to concede to seller 2 and finish the game before making the second visit to store 1 (because $\delta < 1$). Thus, since seller 1 is weak, his expected payoff is α_b when the buyer visits his store for the second time. However, in equilibrium, this continuation payoff contradicts the optimality of seller 1's strategy because seller 1 would prefer to accept the buyer's offer (for sure) when the buyer first attempts to leave his store to eliminate a further delay.

As a result, in equilibrium, rational sellers will not allow the buyer to leave their stores. On the other hand, the rational buyer will eventually leave the first store he visits if that seller is obstinate. The reason for this is clear. Since the players' concession game strategies are increasing and continuous, the seller's reputation will eventually converge to one at some finite time. The rational buyer has no incentive to continue the concession game with an obstinate seller, and so he must either concede to the seller at that time or leave the store. However, Lemma A.2 implies that concession game strategies must be continuous in their domain, eliminating the possibility of mass acceptance at the time that the seller's reputation reaches one.

Next, for notational simplicity, I reset the clock each time the buyer arrives at a store, and denote the buyer's concession game strategy against seller i by F_b^i and i's strategy by F_i. Now, consider an equilibrium where the rational buyer visits seller 1 first with probability σ_1, leaves store 1 at time T_1^d and finalizes the game in store 2 at time T_2^e if the game has not yet ended before. Then, rational buyer visits seller 2 only if $F_2(0) > 0$ is true. Suppose $F_2(0) = 0$. Then, the rational buyer's discounted continuation payoff in store 2, $\delta[F_2(0)(1-\alpha_b) + (1-F_2(0))(1-\alpha)]$, will be $\delta(1-\alpha)$. In this case, the rational buyer prefers to concede to seller 1 instead of traveling store 2, yielding the required contradiction. By lemma A.3., as $F_2(0) > 0$, we must have $F_b^2(0) = 0$, implying that $c_b^2 = 1$. That is, $F_b^2(t) = 1 - e^{-\lambda_b t}$. Furthermore, assuming that the rational buyer leaves store 1 at time T_1^d and the concession game in store 2 ends at time T_2^e, we must have $F_1(T_1^d) = 1 - z_s$ and $F_1(T_2^e) = 1 - z_s$. Thus we have $c_1 = z_s e^{\lambda T_1^d}$ and $c_2 = z_s e^{\lambda T_2^e}$ as required.

Finally, Lemma A.3 implies that $F_b^1(0)F_1(0) = 0$. Since seller 2's reputation reaches 1 at time T_2^e, then the rational buyer will not continue the game G after this time. Thus, his reputation must also reach 1 at that time, implying that $F_b^2(T_2^e) = 1 - z_b^*$ where $z_b^* = \frac{z_b}{1-F_b^1(T_1^d)}$ is the buyer's reputation at the time he arrives at store 2 and z_b is the buyer's reputation at the time he arrives at store 1.

Lemma 2.1. *In equilibrium where the rational buyer visits seller 1 first with probability 1/2 and* $z_b \leq z_b^*(\hat{z}_s/A)^{\lambda_b/\lambda_s} = (\hat{z}_s^2/A)^{\lambda_b/\lambda_s}$ *holds, the buyer leaves store 1 at time* $T_1^d = -\log(\hat{z}_s)/\lambda_s$ *for sure, if the game has not yet ended, and goes directly to store 2. The concession game with seller 2 may continue until the time* $T_2^e = -\log(\hat{z}_s/A)/\lambda_s$*. The players' concession game strategies are* $F_b^1(t) = 1 - z_b(A/\hat{z}_s^2)^{\lambda_b/\lambda_s}e^{-\lambda_b t}$ *and* $F_1(t) = 1 - e^{-\lambda_s t}$ *in store 1, and* $F_b^2(t) = 1 - e^{-\lambda_b t}$ *and* $F_2(t) = 1 - Ae^{-\lambda_s t}$ *in store 2.*

Proof of Lemma 2.1. Consider an equilibrium where the rational buyer visits seller 1 first with probability 1/2 and $z_b \leq (\hat{z}_s^2/A)^{\lambda_b/\lambda_s} < z_b^*$. Then, the rational buyer prefers to play the concession game with seller 1 over going to store 2 at time 0. Since the buyer leaves store 1 if and only if seller 1 is obstinate, seller 1's reputation reaches one at time $T_1^d = \tau_1 = \min\{\tau_b^1, \tau_1\}$ where $\tau_b^1 = \inf\{t \geq 0 | \ F_b^1(t) = 1 - z_b\} = -\frac{\log z_b}{\lambda_b}$ and $\tau_1 = \inf\{t \geq 0 | \ F_1(t) = 1 - \hat{z}_s\} = -\frac{\log \hat{z}_s}{\lambda_s}$ denote the times that the buyer's and seller 1's reputations reach 1, respectively.

However, leaving 1 is optimal for the rational buyer if and only if the buyer's reputation at time T_1^d reaches z_b^*, implying that

$$c_b^1 e^{-\lambda_b T_1^d} = \frac{z_b}{z_b^*} \tag{7}$$

Given the value of T_1^d, solving the last equality yields the buyer's equilibrium strategy in store 1. Finally, the game ends in store 2 at time $T_2^e = \tau_b^2 = \min\{\tau_b^2, \tau_2\}$ for sure where $\tau_b^2 = -\frac{\log z_b^*}{\lambda_b}$ and $\tau_2 = -\frac{\log \hat{z}_s}{\lambda_s}$, at which points both players' reputation simultaneously reach one. Given the value of T_2^e, Proposition 2.1 implies the concession game strategies in the second store.

Proposition 2.2. *Consider a history at which sellers post the prices α_1 and α_2 with $\alpha_1 \neq \alpha_2$, seller 2 is known to be obstinate whereas the true types of seller 1 and the buyer are unknown. Then following continuation strategies form a sequential equilibrium of the continuation game followed by this history:*

(i) If $\alpha_1 > \alpha_2 > 0$, then the rational buyer announces his demand as 0 and visits seller 1 first (with probability one) to make the take it or leave it offer; he leaves store 1 upon his arrival at that store. Conditional on not reaching a deal, the rational buyer goes directly to seller 2 and accepts α_2. On the other hand, rational seller 1 immediately accepts the buyer's demand.

(ii) If $\alpha_1 > \alpha_2 = 0$, then the buyer immediately accepts the second seller's posted demand and finishes the game in the first stage.

(iii) If $\alpha_2 > \alpha_1$, then the buyer never visits store 2 and plays the concession game with seller 1 until time $\frac{-\log \hat{z}_s}{\lambda_s}$ with the following strategies: $F_1(t) = 1 - e^{-\lambda_s t}$ and $F_b^1 = 1 - z_b(1/\hat{z}_s)^{\lambda_b/\lambda_s} e^{-\lambda_b t}$.

Proof of Proposition 2.2. First note that $1 - \alpha_1 < \delta(1 - \alpha_2)$ because the search friction is assumed to be sufficiently small. Therefore, it is optimal for the rational buyer to go to store 2 and to accept α_2 instead of accepting α_1. Moreover, regardless of the buyer's announcement α_b, postponing concession or not accepting α_b is not optimal for rational seller 1 since the buyer will never accept α_1 in equilibrium. Thus, it is a best response for rational seller 1 to accept the buyer's demand upon his arrival at store 1, and so it is a best response for the rational buyer to choose $\alpha_b = 0$.

For the last part, if $\alpha_2 > \alpha_1$, then the buyer never visits seller 2. Therefore, in any equilibrium, the continuation game is identical to the Abreu and Gul (2000) setup and the equilibrium strategies are characterized by the following three conditions: (i) $F_b^1(t) = 1 - c_b^1 e^{-\lambda_b t}$ and $F_1(t) = 1 - c_1 e^{-\lambda_s t}$ for all $t \leq T^e = \min\{\frac{-\log \hat{z}_s}{\lambda_s}, \frac{-\log z_b}{\lambda_b}\}$, (ii) $(1 - c_b^1)(1 - c_1) = 0$, and (iii) $F_b^1(T^e) = 1 - z_b$ and $F_1(T^e) = 1 - \hat{z}_s$. Note that these strategies form an equilibrium for small values of z_s, in particular for the values of z_s such that $z_s < A$. The rest of the strategies are optimal given the belief that seller 2 is known to be obstinate.

Lemma 2.2. *Consider the strategy profile σ^G described above where both sellers post price $\alpha_s > 0$. Suppose that rational seller 2 deviates and posts α_2 in the first stage. Then, his continuation payoff in the game will be 0 if $\alpha_2 > \alpha_s$ and $\alpha_2 \left[z_b \sum_{\alpha_b \geq \alpha_2} \pi(\alpha_b) + \hat{z}_s(1 - z_b)\right]$, which is strictly less than $(z_b + z_s)\alpha_2$, otherwise.*

Proof of Lemma 2.2. Recall that rational sellers' price posting strategies are pure in σ^G. Therefore, if rational seller 2 deviates to α_2 at time 0, then other players will conclude that seller 2 is obstinate of type α_2. Given the assumptions on obstinate types, the rational buyer's expected payoff of posting $\alpha_2 > \alpha_s$ is 0. Proposition 2.2 gives the strategies of the continuation game following a history where seller 2 price undercuts his opponent. Deviation to $\alpha_2 = 0$ clearly implies expected payoff of 0. However, if $\alpha_2 > 0$, then the second seller's expected payoff will be $\alpha_2 \left[z_b \sum_{\alpha_b \geq \alpha_2} \pi(\alpha_b) + \hat{z}_s(1 - z_b)\right]$ where $z_b \sum_{\alpha_b \geq \alpha_2} \pi(\alpha_b)$ is the probability that the buyer is an obstinate type with demand higher than or equal to α_2. Finally, note that $\hat{z}_s = \frac{z_s \pi(\alpha_s)}{z_s \pi(\alpha_s) + 1 - z_s} < z_s$.

Proof of Theorem 1. Note that 0 is equilibrium for any values of $z_b, z_s \in (0, 1)$. Next, I will prove that any $\alpha_s \in \mathcal{C} \setminus \{0\}$ can be supported in equilibrium whenever we have $z_b \leq (\hat{z}_s^2/A)^{\lambda_b/\lambda_s}$ for all $\alpha_b \in \mathcal{C}$ with $\alpha_b < \alpha_s$. Therefore, fix the value of $\alpha_s \in \mathcal{C}$ and suppose that $z_b \leq (\hat{z}_s^2/A)^{\lambda_b/\lambda_s}$ holds for all $\alpha_b \leq \alpha_s$. Given that both sellers choose α_s, the equilibrium strategies of the rational buyer in the first stage, σ_i^* and μ_b^*, must satisfy the followings.

1. σ_i^* is the probability of visiting seller i first with $\sigma_1^* + \sigma_2^* = 1$ and μ_b^* is a probability distribution over the set $\mathcal{D} \subset \mathcal{C}_{\alpha_s} = \{\alpha_b \in \mathcal{C} | \alpha_b \leq \alpha_s\}$ with $\sum_{x \in \mathcal{D}} \mu_b^*(x) = 1$.

2. For all $i \in \{1, 2\}$ and $\alpha_b \in \mathcal{D}$ we must have $V_b^i(\alpha_b) = V$. By Lemma 2.1 and by the assumption that $z_b \leq (\hat{z}_s^2/A)^{\lambda_b/\lambda_s}$, we have $V_b^i(\alpha_b) = 1 - \alpha_s$.

3. $V \geq 1 - \min\{\mathcal{C} \setminus \mathcal{D}\}$. That is, the rational buyer should have no incentive to deviate and declare some other demand α_b' which is not in the support of μ_b^*.

Therefore, in equilibrium μ_b^* and σ_i^* are solutions of $\#\mathcal{D}+1$ (nonlinear) equations for $\#\mathcal{D}+1$ unknowns. For small values of z_b (relative to $\hat{z}_s$), existence of these strategies is easy to show. Consider the following strategy profile σ^G:

(a) In the first stage, rational sellers post the same demand α_s (i.e., $\mu_i^*(\alpha_s) = 1$ and $\mu_i^*(\alpha_s') = 0$ for all $\alpha_s' \in \mathcal{C} \setminus \{\alpha_s\}$), the rational buyer visits each seller with equal probabilities (i.e., $\sigma_1^* = 1/2$) and declares a demand $\alpha_b < \alpha_s$ according to $\mu_b^*(\alpha_b) = \frac{\pi(\alpha_b)}{\sum_{x<\alpha_s} \pi(x)}$.

(b) (Proposition 2.1 and Lemma 2.1) In the second stage, following a history where the buyer visits seller 1 first, the buyer leaves store 1 at time $T_1^d = -\log(\hat{z}_s)/\lambda_s$ for sure, if the game has not yet ended, and goes directly to store 2. The concession game with seller 2 may continue until the time $T_2^e = -\log(\hat{z}_s/A)/\lambda_s$. The players' concession game strategies are $F_b^1(t) = 1 - z_b(A/\hat{z}_s^2)^{\lambda_b/\lambda_s} e^{-\lambda_b t}$ and $F_1(t) = 1 - e^{-\lambda_s t}$ in store 1, and $F_b^2(t) = 1 - e^{-\lambda_b t}$ and $F_2(t) = 1 - Ae^{-\lambda_s t}$ in store 2. Symmetric strategies would work following a history where the buyer visits seller 2 first.

(c) (Proposition 2.2) In case, one of the sellers, say, seller 2 undercuts his opponent and posts a price $\alpha_2 \in \mathcal{C}$ such that $\alpha_2 < \alpha_s$, then there are two possible scenarios:

- (i) If $\alpha_2 > 0$, then the rational buyer announces his demand as 0 and visits seller 1 first (with probability one) to make the take it or leave it offer; he leaves store 1 upon his arrival at that store. Conditional on not reaching a deal, the rational buyer goes directly to seller 2 and accepts α_2. On the other hand, rational seller 1 immediately accepts the buyer's demand. Therefore, in case the game does not end in store 1, the buyer infers that 1 is the obstinate type with demand α_1.
- (ii) If $\alpha_2 = 0$, then the buyer immediately accepts the second seller's posted demand and finishes the game in the first stage.

(d) (Proposition 2.2) If seller 2 deviates and posts a price $\alpha_2 > \alpha_s$, then the buyer visits seller 1 first and never goes to the second store, and the concession game with seller 1 may continue until the time $T_1^e = \frac{-\log \hat{z}_s}{\lambda_s}$ with the following strategies: $F_1(t) = 1 - e^{-\lambda_s t}$ and $F_b^1 = 1 - z_b(1/\hat{z}_s)^{\lambda_b/\lambda_s} e^{-\lambda_b t}$.

Note that the strategies μ_b^* and σ_i^* satisfy the requirements 1-3. Moreover, by Lemma 2.1 and Proposition 2.2, the second stage strategies also form an equilibrium.

Lastly, we need to show that the first stage strategies μ_1^* and μ_2^* are optimal. That is, I will show that posting the demand α_s at time 0 is an optimal strategy for a seller if the other seller also posts α_s. For this reason, I will first calculate each sellers expected payoff under the strategy profile σ^G. Let V_i denote seller i's expected payoff under the strategy profile σ^G. Since a deviating seller's equilibrium payoff is less than $(z_b + z_s)$ (by Lemma 2.2), I will argue that price undercutting is not optimal if we choose z_b and z_s sufficiently small. Moreover, following the assumptions on obstinate types, if a seller deviates and posts a price above α_s, then his expected payoff in the game will be simply 0.

Under the strategy σ^G, we have $V_i = p\alpha_s + (\frac{1}{2} - p)(a+b)$ and we calculate it as follows:

Case 1. The buyer picks store i first and he is obstinate of type $\alpha_b \geq \alpha_s$. Probability to this event is $\frac{1}{2} z_b \sum_{\alpha_b \geq \alpha_s} \pi(\alpha_b) := p$. Rational seller i's expected payoff in this case is α_s.

Case 2. The buyer picks store i second and he is obstinate of type $\alpha_b \geq \alpha_s$. Probability to this event is p and rational seller i's expected payoff in this case is 0.

Case 3. The buyer picks store i first and he is either rational or obstinate of type $\alpha_b < \alpha_s$. Probability to this event is $\frac{1}{2} - p$, $[\frac{1}{2}(1 - z_b) + z_b \frac{1}{2} - p]$, and rational seller i's expected payoff in this case is $\sum_{\alpha_b < \alpha_s} [\frac{\pi(\alpha_b)}{\sum_{x < \alpha_s} \pi(x)}][\alpha_b + F_b^i(0)(\alpha_s - \alpha_b)] := a$ where $F_b^i(0) = 1 - z_b (A/\hat{z}_s^2)^{\frac{\alpha_b r_s}{(1-\alpha_s) r_b}}$.

Case 4. The buyer picks store i second and he is either rational or obstinate of type $\alpha_b < \alpha_s$. Probability to this event is $\frac{1}{2} - p$ and rational seller i's expected payoff in this case is $\frac{e^{-\Delta r_s} \hat{z}_s}{\sum_{x < \alpha_s} \pi(x)} \sum_{\alpha_b < \alpha_s} \hat{z}_s^{\frac{r_s(\alpha_s - \alpha_b)}{(1-\alpha_s) r_b}} \alpha_b \pi(\alpha_b) := b$. Note that the buyer will visit the second store only if the first seller is obstinate and the rational buyer announces $\alpha_b < \alpha_s$. Therefore, seller i's expected payoff in this case is discounted by the travel time $e^{-\Delta r_s}$ and $\hat{z}_s^{\frac{r_s(\alpha_s - \alpha_b)}{(1-\alpha_s) r_b}}$ - the discount due to the delay in the first store j, i.e. T_j^d.

Note that V_i is strictly greater than $(\frac{1}{2} - p)u$ where u is the convex combination of the demands in $\mathcal{C}_{\alpha_s} \setminus \{\alpha_s\}$, i.e., $u = \sum_{\alpha_b < \alpha_s} \alpha_b \mu_b(\alpha_b)$, and it is much higher than $(z_b + z_s)$ if z_b and z_s are sufficiently small. Hence, posting α_s is optimal for each seller. This completes the proof.

Proof of Theorem 2. Recall that Theorem 1 implies that for any given z_b^m and z_s^m small enough the demand $\alpha_s^m \in \mathcal{C}$ can be supported as an equilibrium posted price of the sellers in the game $\mathrm{G}(z_b^m, z_s^m)$ whenever $z_b^m \leq [(\hat{z}_s^m)^2/A]^{\frac{\alpha_b r_s}{(1-\alpha_s^m) r_b}}$ for all $\alpha \in \mathcal{C}$ with $\alpha < \alpha_s^m$ where $\hat{z}_s^m = \frac{z_s^m \pi(\alpha_s^m)}{z_s^m \pi(\alpha_s^m) + 1 - z_s^m}$. Taking the log of both sides we have

$$\log z_b^m \leq \frac{\alpha r_s}{(1 - \alpha_s^m) r_b} (2 \log \hat{z}_s^m - \log A)$$

dividing both sides by $\log z_b^m$ and taking the limit as $m \to \infty$ we get $2K\alpha r_s \le (1-\alpha_s) r_b$ for all $\alpha \in \mathcal{C}$ with $\alpha < \alpha_s$, yielding the desired inequality.

Proof of Theorem 3. Recall that the proof of Theorem 2 relies solely on the fact that the buyer must be weak for each α_b in the support of μ_b^*. Same arguments in the proof of Theorem 1 shows that if there are N identical sellers and the buyer is weak in equilibrium, then we can support positive prices in equilibrium. Next, I will show that being weak in equilibrium with N sellers means $z_b \le \left(\hat{z}_s^N / A^{N-1}\right)^{\lambda_b/\lambda_s}$.

For the ease of exposition, I will derive this condition for the 3-sellers case, which can be extended to N-sellers case by iterating the same process. For this reason, suppose now that there are three sellers all of which choose the same demand α_s in the first stage and the buyer declares his demand as $\alpha_b < \alpha_s$. Without loss of generality, I assume that the buyer visits seller 1 first and seller 3 last (if no agreement have been reached with the sellers 1 and 2). The following arguments are straightforward extensions of the approach that I use in the proof of Proposition 2.1. Therefore, let T_i^d denote the time that the buyer leaves seller $i \in \{1,2\}$ and $\hat{z}_b(T_i^d)$ denote the buyer's reputation at the time he leaves store i.

The rational buyer leaves seller 2 when his discounted continuation payoff in store 3, i.e. $\delta[1-\alpha_b - \hat{z}_s[\hat{z}_b(T_2^d)]^{-\lambda_s/\lambda_b}(\alpha_s-\alpha_b)]$, equals to $1-\alpha_s$. This equality implies that $\hat{z}_b(T_2^d) = (\hat{z}_s/A)^{\lambda_b/\lambda_s}$. As a result, the buyer's expected payoff in store 2 at the time he enters this store is $v_b^2 = 1-\alpha_b - \hat{z}_s\left[\frac{(\hat{z}_s/A)^{\lambda_b/\lambda_s}}{\hat{z}_b(T_1^d)}\right]^{\lambda_s/\lambda_b}(\alpha_s-\alpha_b)$. Similarly, the buyer leaves seller 1 when his discounted continuation payoff in store 2, i.e. δv_b^2, equals to $1-\alpha_s$. Then we have $\hat{z}_b(T_1^d) = (\hat{z}_s^2/A^2)^{\lambda_b/\lambda_s}$.

Also, note that we have $\hat{z}_b(T_1^d) = \frac{\hat{z}_b^1}{1-F_b^1(T_1^d)}$, $F_b^1(T_1^d) = 1 - c_b^1 e^{-\lambda_b T_1^d}$ and $c_b^1 = 1$ because the buyer is weak. Thus, it must be true that $T_1^d = -\frac{\log(\hat{z}_b^1/(\hat{z}_s^2/A^2)^{\lambda_b/\lambda_s})}{\lambda_b} \ge \frac{-\log \hat{z}_s}{\lambda_s}$ again because the buyer is weak. The last inequality implies $\hat{z}_b^1 \le \left(\hat{z}_s^3/A^2\right)^{\lambda_b/\lambda_s}$. In equilibrium, the last inequality must hold for all $\hat{z}_b^i$ with $i=1,2,3$, implying that it must hold for z_b as well. The rest directly follows from the parallel arguments of the proof of Theorem 2. Iterating the above arguments suffice to prove the claim for any finite N.

Proof of Proposition 3.1. Suppose that the Poisson arrival rate of the buyer is κ. First, if the players play the strategies described in the main text, then the Bayes' rule implies that the probability of the buyer being the commitment type α_b conditional on him visiting seller i during the period of $[T, T+dt]$ and demanding $\alpha_b < \alpha_i$ is

$$\hat{z}_b^{(T+dt),i} = \frac{\frac{1}{2} z_b \pi(\alpha_b)\kappa dt + \frac{1}{2} z_b \hat{z}_s \pi(\alpha_b)\kappa dt}{\frac{1}{2} z_b \pi(\alpha_b)\kappa dt + \frac{1}{2} z_b \hat{z}_s \pi(\alpha_b)\kappa dt + (1-z_b)\mu_{\alpha_i}^T(\alpha_b)\sigma_i\left(\sum_{x<\alpha_i}\pi(x)\right)\kappa dt}$$

The first term in the numerator corresponds to the probability that the obstinate buyer with demand α_b is visiting seller i first and arriving at the market in a short period dt.

Likewise, the second term denotes the probability that the obstinate buyer visits seller i second, implying that the buyer should have arrived at the market $-\log(\hat{z}_s)/\lambda_s + \Delta$ units of time ago during the short period dt.[25]

Given the strategies of the players, if the buyer arrives at the market at the period $0+dt$, then the obstinate buyer's arrival time at the second store is $\bar{T} = -\log(\hat{z}_s)/\lambda_s + \Delta + dt$. Therefore, the second term in the numerator does not exists if $T < \bar{T}$. Moreover, the limiting case where dt approaches 0 implies that $\hat{z}_b^{T,i}$ equals to z_b for all $T < \log(\hat{z}_s)/\lambda_s + \Delta$ and to $\frac{z_b(1+\hat{z}_s)}{1+z_b\hat{z}_s}$ otherwise.

Second, for any $0 < \alpha_b < \alpha_s$, we have $\hat{z}_b^{T,i} < \hat{z}_s^{\lambda_b/\lambda_s}$ because $z_b < \frac{\hat{z}_s^{\lambda_b/\lambda_s}}{1+\hat{z}_s(1-\hat{z}_s^{\lambda_b/\lambda_s})}$. Moreover, according to the strategies, the rational buyer never leaves the sellers' stores. This implies that the buyer and the seller will play the concession game according to the strategies F_b and F_i's until the time $-\frac{\log(\hat{z}_s)}{\lambda_s} = \min\{-\frac{\log \hat{z}_s}{\lambda_s}, -\frac{\log \hat{z}_b^{T,i}}{\lambda_b}\}$ (this directly follows from Abreu and Gul (2000), Proposition 1.) As a result, the buyer's expected payoff in each store is $1 - \alpha_s$ because independent of the buyer's arrival time at either store, the buyer will be weak in both. Hence, visiting each seller with equal probabilities is an optimal strategy for the rational buyer. Furthermore, if the rational buyer leaves his current bargaining partner at any point of time and goes to the other seller, then his continuation payoff will be $\delta(1 - \alpha_s)$. Hence, not leaving a seller's store and playing the concession game until the time $-\log(\hat{z}_s)/\lambda_s$ are also optimal strategies.

Third, independent of α_b ($\leq \alpha_s$), the rational buyer's expected payoff is $1 - \alpha_s$ in each store. Thus, the mixed strategy $\mu_{\alpha_s}^T(\alpha_b) = \frac{\pi(\alpha_b)}{\sum_{x<\alpha_s}\pi(x)}$ is an optimal strategy for the rational buyer.

Finally, I will show that posting the demand α_s at time 0 is an optimal strategy for a seller if the other seller also posts α_s. For this person, I will first calculate each seller's expected payoff under the strategies given in the main text. Let $V_i(T)$ denote seller i's expected payoff in the game (evaluated in time T) given that the buyer arrives at the market at time $T \geq 0$. Then, I calculate a deviating seller's equilibrium payoff (again evaluated in time T assuming that the buyer arrives at the market at T) and argue that it is smaller than $V_i(T)$ if we choose z_b and z_s sufficiently small. Thus, $V_i(T) = [p\alpha_s + (\frac{1}{2} - p)(a + b)]$ where

Case 1. The buyer picks store i first and he is the obstinate type with demand $\alpha_b \geq \alpha_s$. Probability to this event is $\frac{1}{2}z_b \sum_{\alpha_b \geq \alpha_s} \pi(\alpha_b) := p$ and seller i's expected payoff is α_s.

Case 2. The buyer picks the other store j first and he is the obstinate type with demand

[25] Recall that $-\log(\hat{z}_s)/\lambda_s$ is the length of the concession game in the stores where $\lambda_s = \frac{(1-\alpha_s)r_b}{\alpha_s - \alpha_b}$, and Δ is the time required to travel between the stores.

$\alpha_b \geq \alpha_s$. Probability to this event is p and i's expected payoff is 0.

Case 3. The buyer picks store i first and he is either rational or the obstinate type with demand $\alpha_b < \alpha_s$. Probability to this event is $\frac{1}{2} - p$, $[\frac{1}{2}(1 - z_b) + z_b \frac{1}{2} - p]$, and seller i's expected payoff is $\sum_{\alpha_b < \alpha_s} [\frac{\pi(\alpha_b)}{\sum_{x<\alpha_s} \pi(x)}][\alpha_b + F_b^T(T)(\alpha_s - \alpha_b)] := a$ where $F_b^T(T) = 1 - \hat{z}_b^{T,i} \hat{z}_s^{-\frac{\alpha_b r_s}{(1-\alpha_s) r_b}}$.

Case 4. The remaining case is that the buyer picks store j first and he is either rational or the obstinate type with demand $\alpha_b < \alpha_s$. Probability to this event is $\frac{1}{2} - p$ and i's expected payoff is $\frac{e^{-r_s \Delta} z_b \hat{z}_s}{\sum_{x<\alpha_s} \pi(x)} \sum_{\alpha_b < \alpha_s} \alpha_b \hat{z}_s^{r_s/\lambda_s} \pi(\alpha_b) \int_0^{\frac{-\log(\hat{z}_s)}{\lambda_s}} e^{-r_s t} \frac{dF_s(t)}{1-\hat{z}_s} := b$ where $F_s(t) = 1 - e^{-\lambda_s t}$.

On the other hand, if seller i price undercuts j and posts α_i such that $0 < \alpha_i < \alpha_s$, then rational seller i's expected payoff is $\left(\left[z_b \sum_{\alpha_b \geq \alpha_i} \pi(\alpha_b)\right] + \hat{z}_s \left[1 - z_b \sum_{\alpha_b \geq \alpha_i} \pi(\alpha_b)\right]\right) \alpha_i$, and it is less than $(z_b + z_s)\alpha_i$ (see Lemma 2.2). This is true because in any equilibrium following the history where seller i price undercuts j, the rational buyer visits seller j first with certainty, makes a "take it or leave it" offer 0, which will be accepted by the rational seller j, and immediately leaves if seller j does not accept 0. Then, the rational buyer immediately visits seller i to accept α_i. It is clear that $(z_b + z_s)\alpha_i < V_i(T)$ for sufficiently small values of z_b and z_s.

Proof of Proposition 3.2. Recall that Proposition 3.1 implies that for any given z_b^m and z_s^m small enough the demand α_s^m is the equilibrium posted price of the sellers in the game $\mathrm{G}(z_b^m, z_s^m)$ whenever $z_b^m \leq \frac{(\hat{z}_s^m)^{\lambda_b/\lambda_s}}{1+(\hat{z}_s^m)[1-(\hat{z}_s^m)^{\lambda_b/\lambda_s}]}$ for all $\alpha \in \mathcal{C}_{\alpha_s^m}$, where $\hat{z}_s^m = \frac{z_s^m \pi(\alpha_s^m)}{z_s^m \pi(\alpha_s^m) + 1 - z_s^m}$. Taking the log of both sides we have

$$\log z_b^m \leq \frac{\alpha r_s}{(1 - \alpha_s^m) r_b} \left(\log \hat{z}_s^m - \log \left[1 + \hat{z}_s^m [1 - (\hat{z}_s^m)^{\lambda_b/\lambda_s}]\right]\right)$$

dividing both sides by $\log z_b^m$ and taking the limit as $m \to \infty$ we get $K \alpha r_s \leq (1 - \alpha_s) r_b$ for all $\alpha \in \mathcal{C}_{\alpha_s}$.

Proof of Proposition 3.3. I will show that the strategies given in the main text constitute and equilibrium. Suppose that the rational buyer announces $\alpha_b < \alpha_s$ in the first stage and consider the second stage. First, at time 0, the rational buyer and seller 1 has two options; accept and reject. Rejection for the buyer means leaving the store. I assume that if the buyer chooses to leave but the seller accepts, then the game will end with the seller's acceptance. If the rational buyer does not leave the first store at time 0, he reveals his rationality, in which case the buyer's expected payoff will be no more than $1 - \alpha_s$ (since the buyer is discounting time). Hence, in equilibrium, the rational buyer will either concede or leave the store at time 0.

Second, if the rational buyer finishes the game in store 1 with probability P_b, then the buyer's reputation conditional on him arriving store 2 after visiting 1 is $(\hat{z}_s/A)^{\lambda_b/\lambda_s}$ as calculated by $\frac{z_b}{z_b+(1-z_b)(1-P_b)}$. Therefore, the buyer and seller 2 will play the concession game until time $T_2^e = \min\{-\frac{log(\hat{z}_s/A)}{\lambda_s}, -\frac{log\hat{z}_s}{\lambda_s}\}$ which is equal to $-\frac{log(\hat{z}_s/A)}{\lambda_s}$ as $A < 1$. Thus, the equilibrium concession game strategies in store 2 must be as given in the main text. As a result, the rational buyer's expected payoff in the second store is $\frac{1-\alpha_s}{\delta}$.

Third, the rational buyer's expected payoff of accepting α_s in store 1 is

$$V_b(accept) = \hat{z}_s(1-\alpha_s) + (1-\hat{z}_s)\left[\frac{1}{2}P_s(2-\alpha_s-\alpha_b) + (1-P_s)(1-\alpha_s)\right]$$

whereas

$$V_b(reject) = \hat{z}_s\delta V + (1-\hat{z}_s)[P_s(1-\alpha_b) + (1-P_s)\delta V]$$

where $V = \frac{1-\alpha_s}{\delta}$ is the buyer's continuation payoff when he leaves the first seller at time 0. Note that if $P_s = 0$, then $V_b(accept) = V_b(reject) = 1-\alpha_s$, implying that the buyer's strategy P_b is a best response. Moreover, since the rational buyer's expected payoff in each store and in the game, regardless of his announcement $\alpha_b < \alpha_s$, is $1-\alpha_s$, visiting each seller with probability $1/2$ and announcing α_b according to μ_b^* are also best response strategies.

Similarly, rational seller i's expected payoff is

$$V_i(accept) = z_b\alpha_b + (1-z_b)\left[\frac{1}{2}P_b(\alpha_s+\alpha_b) + (1-P_b)\alpha_b\right]$$

whereas

$$V_i(reject) = z_b 0 + (1-z_b)\left[P_b\alpha_s + (1-P_b)0\right]$$

Therefore, given the value of P_b and $z_b \leq \frac{(\hat{z}_s/A)^{\lambda_b/\lambda_s}(\alpha_s-\alpha_b)}{\alpha_s+\alpha_b}$, we have $V_i(accept) < V_i(reject)$. Hence, $P_s = 0$ is a best response as well.

Finally, I will show that posting the demand α_s at time 0 is an optimal strategy for a seller if the other seller also posts α_s. For this reason, I will first calculate each sellers expected payoff in the game for the second stage strategies given in the main text. Let V^i denote seller i's expected payoff in the game. Since a deviating seller's equilibrium payoff is less than (z_b+z_s) (by Lemma 2.2), I will argue that price undercutting is not optimal if we choose z_b and z_s sufficiently small. We have $V^i = \alpha_s\left[p + \frac{(1-\hat{z}_s)}{2}[P_b + e^{-r_s\Delta}(1-P_b)]\right]$ and calculate it as follows:

Case 1. The buyer picks store i first and he is obstinate of type $\alpha_b \geq \alpha_s$. Probability to this event is $\frac{1}{2}z_b\sum_{\alpha_b\geq\alpha_s}\pi(\alpha_b) := p$. Rational seller i's expected payoff in this case is α_s.

Case 2. The buyer picks store i second and he is obstinate of type $\alpha_b \geq \alpha_s$. Probability to this event is p and rational seller i's expected payoff in this case is 0.

Case 3. The buyer is obstinate of type $\alpha_b < \alpha_s$. Probability to this event is $z_b - 2p$ and rational seller i's expected payoff in this case is 0.

Case 4. The buyer picks store i first and he is rational. Probability to this event is $(1-\hat{z}_s)\frac{1}{2}$ and rational seller i's expected payoff in this case is $P_b\alpha_s$.

Case 5. The buyer picks store i second and he is rational. Probability to this event is $(1-\hat{z}_s)\frac{1}{2}$ and rational seller i's expected payoff in this case is $(1-P_b)e^{-r_s\Delta}\alpha_s$.

Note that for small values of z_b and z_s, the value of V^i is greater than $(z_b + z_s)$ which concludes the proof.

Proof of Proposition 3.4. Similar arguments in the proof of Proposition 3.3 will prove our claim. Note that given the value of P_b, as in the main text, the buyer's reputation conditional on him announcing α_b and arriving store 2 after visiting store 1 is $z_b^* = 1 - \frac{\alpha_b}{\alpha_s}$. The value of z_b^* makes rational seller 2 indifferent between immediate concession, with payoff of α_b, and rejection with payoff of $(1-z_b^*)\alpha_s$. Since rational seller 2 is indifferent, immediate concession with probability P_s (as given in the main text) is optimal. Moreover, P_s ensures the expected payoff of $\frac{(1-\alpha_s)}{\delta}$ to the rational buyer, and it makes the buyer indifferent between conceding to seller 1 and leaving for seller 2. Finally, with the value of P_b and $z_b \leq \frac{(\alpha_s-\alpha_b)^2}{\alpha_s(\alpha_s+\alpha_b)}$, rational seller 1's expected payoff of rejecting the buyer's demand is higher than conceding to him as $V_1(accept) = z_b\alpha_b + (1-z_b)[\frac{1}{2}P_b(\alpha_s+\alpha_b) - (1-P_b\alpha_b)]$ and $V_1(reject) = (1-z_b)P_b\alpha_s$.

Appendix B

The Discrete-Time Model and Convergence

Here, I consider the competitive-bargaining game in discrete time and investigate the structure of its equilibria as players can make their offers increasingly frequent. I show that given the symmetric obstinate types, the second stage equilibrium outcomes of the competitive-bargaining game in discrete-time converge to a unique limit, independent of the exogenously given bargaining protocols, as time between offers approach to 0, and this limit is equivalent to the unique outcome of the continuous-time game partially investigated in Section 2. I characterize the second stage equilibrium strategies of the game G (given that both sellers post the same demand $0 < \alpha_s \in \mathcal{C}$) in Online Appendix.

To be more specific, I suppose that each player has a single commitment type; some $\alpha_s \in \mathcal{C}$ for the sellers and $\alpha_b \in \mathcal{C}$ for the buyer where $0 < \alpha_b < \alpha_s$. In the first stage, first the sellers and then the buyer announces their types. Then the buyer chooses a store to visit first. Upon the buyer's arrival at store i, beginning of the second stage, the buyer and seller i bargain in discrete time according to some protocol g^i that generalizes Rubinstein's alternating offers protocol. A bargaining protocol g^i between the buyer and seller i is defined as $g^i : [0, \infty) \to \{0, 1, 2, 3\}$ such that for any time $t \geq 0$, an offer is made by the buyer if $g^i(t) = 1$ and by seller i if $g^i(t) = 2$.[26] Moreover, $g^i(t) = 3$ implies a simultaneous offer whereas $g^i(t) = 0$ means no offer is made at time t. An infinite horizon bargaining protocol is denoted by $g = (g^1, g^2)$. The bargaining protocol g is discrete. That is, for any seller i and for all $\bar{t} \geq 0$, the set $\mathcal{I}^i := \{0 \leq t < \bar{t} | g^i(t) \in \{1, 2, 3\}\}$ is countable. Notice that this definition for a bargaining protocol is very general and accommodates non-stationary, non-alternating protocols.

In the first stage, the rational players are free to choose any offer from the set $[0, 1]$. An offer $x \in [0, 1]$ denotes the share the seller is to receive. If the proposer's opponent accepts his offer, the game ends with agreement x where xe^{-tr_s} denotes the payoff to seller i, 0 is the payoff to seller j and finally $(1 - x)e^{-tr_b}$ is the payoff to the buyer. If the proposer's opponent rejects his offer, the game continues. Prior to the next offer, the rational buyer decides whether to stay or leave the store. If the rational buyer decides to stay, the next offer is made at time $t' := \min\{\hat{t} > t | \hat{t} \in I^i\}$, for example, by the buyer if $g^i(t') = 1$. The two-stage competitive-bargaining game in discrete-time is denoted by $\mathrm{G}\langle g, (z_n, r_n)_{n \in \{b,s\}} \rangle$ (or $\mathrm{G}(g)$ in short). The competitive-bargaining game $\mathrm{G}(g)$ ends if the offers are compatible. In the event of strict compatibility the surplus is split equally. Throughout the game, both sellers can perfectly observe the buyer's moves. Thus, the players' actual types remain to be the only source of uncertainty.

I am particularly interested in equilibrium outcome(s) of the competitive-bargaining game $\mathrm{G}(g)$ in the limit where the players can make sufficiently frequent offers. Therefore, for $\epsilon > 0$ small enough, let $\mathrm{G}(g_\epsilon)$ denote discrete-time competitive-bargaining game where the buyer and the sellers bargain, in the second stage, according to the protocol $g_\epsilon = (g^1_\epsilon, g^2_\epsilon)$ such that for all $t \geq 0$ and i, both seller i and the buyer have the chance to make an offer, at least once, within the interval $[t, t + \epsilon]$ in the bargaining protocol g^i_ϵ.[27] In this sense, the discrete-time competitive-bargaining game $\mathrm{G}(g_\epsilon)$ converges to continuous time as $\epsilon \to 0$.[28]

Now, let σ_ϵ denote a sequential equilibrium of the discrete-time competitive-bargaining game $\mathrm{G}(g_\epsilon)$ and σ_i be the rational buyer's equilibrium strategy for store selection at time 0. Given σ_i, the random outcome corresponding to σ_ϵ is a random object $\theta_\epsilon(\sigma_i)$ which denotes any realization of an agreed division as well as a time and store at which

[26] Time 0 denotes the beginning of the bargaining phase.

[27] More formally, either $g^i(\hat{t}) = 3$ for some $\hat{t} \in [t, t + \epsilon]$, or $g^i(t') = 1$ and $g^i(t'') = 2$ for some $t', t'' \in [t, t + \epsilon]$.

[28] One may assume that the travel time is discrete and consistent with the timing of the bargaining protocols so the buyer never arrives a store at some non-integer time.

agreement is reached.

The next result shows that in the limit as ϵ converges to 0 $\theta_\epsilon(\sigma_i) \to \theta(\sigma_i)$ in distribution, where $\theta(\sigma_i)$ is the unique equilibrium distribution of the continuous-time game G, that is fully characterized in the online appendix for $\sigma_1 = 1/2$. Therefore, the outcome of the discrete-time competitive-bargaining game, independent of the bargaining protocol g_ϵ, converge in distribution to the unique (given the buyer's initial choice of store) equilibrium outcome of the competitive-bargaining game analyzed in Section 2.

Proposition B. *As ϵ converges to 0, $\theta_\epsilon(\sigma_i)$ converges in distribution to $\theta(\sigma_i)$.*

I defer the proof to the online appendix.

References

[1] Abreu, Dilip, and Faruk, Gul. 2000."Bargaining and Reputation." *Econometrica* 68(1): 85-117.

[2] Abreu, Dilip, and Rajiv, Sethi. 2003. "Evolutionary Stability in a Reputational Model of Bargaining." *Games and Economic Behavior* 44(2): 195-216.

[3] Atakan, E., Alp, and Mehmet, Ekmekci. 2014. "Bargaining and Reputation in Search Markets." *The Review of Economic Studies* 81(1): 1-29.

[4] Arrow, J., Kenneth, Robert, H., Mnookin, Lee, Ross, Amos, Tversky, and Robert, B., Wilson. 1995. *Barriers to Conflict Resolution*. Norton & Company, Inc. New York.

[5] Bester, Helmut. 1988. "Bargaining, Search Costs and Equilibrium Price Distributions." *The Review of Economic Studies* 55(2): 201-214.

[6] Bester, Helmut. 1989. "Noncooperative Bargaining and Spatial Competition." *Econometrica* 57(1): 97-113.

[7] Binmore, G., Kenneth and M., J., Herrero. 1988. "Matching and Bargaining in Dynamic Markets." *The Review of Economic Studies* 55(1): 17-31.

[8] Caruana, Guillermo, and Liran, Einav. 2008. "A Theory of Endogenous Commitment." *The Review of Economic Studies* 75(1): 99-116.

[9] Caruana, Guillermo, Liran, Einav, and Daniel, Quint. 2007. "Multilateral bargaining with concession costs." *Journal of Economic Theory* 132(1): 147-166.

[10] Chatterjee, Kalyan, and Larry, Samuelson. 1987. "Bargaining with Two-sided Incomplete Information: An Infinite Horizon Model with Alternating Offers." *The Review of Economic Studies* 54(2): 175-192.

[11] Compte, Olivier, and Philippe, Jehiel. 2002. "On the Role of Outside Options in Bargaining with Obstinate Parties." *Econometrica* 70(4): 1477-1517.

[12] Crawford, P., Vincent 1982. "A Theory of Disagreement in Bargaining." *Econometrica* 50(3): 607-637.

[13] Gale, Douglas. 1986 a. “Bargaining and Competition Part I: Characterization.” *Econometrica* 54(4): 785-806.

[14] Gale, Dougles. 1986 b. “Bargaining and Competition Part II: Existence.” *Econometrica* 54(4): 807-818.

[15] Hendricks, Ken, Andrew, Weiss, and Charles, Wilson. 1988. “The War of Attrition in Continuous-Time with Complete Information.” *International Economic Review* 29(4): 663-680.

[16] Kambe, Shinsuke. 1999. “Bargaining with Imperfect Commitment.” *Games and Economic Behavior* 28(2): 217-237.

[17] Klemperer, Paul. 1987. “Markets with Consumer Switching Costs.” *The Quarterly Journal of Economics* 102(2): 375-394.

[18] Kreps, M., David, and Robert, Wilson. 1982. “Reputation and Imperfect Information.” *Journal of Economic Theory* 27(2): 253-279.

[19] Milgrom, Paul, and John, Roberts. 1982. “Predation, Reputation, and Entry Deterrence.” *Journal of Economic Theory* 27(2): 280-312.

[20] Muthoo, Abhinay. 1996. “A Bargaining Model Based on the Commitment Tactic.” *Journal of Economic Theory* 69(1): 134-152.

[21] Myerson, B., Roger. 1991. *Game Theory: Analysis of Conflict.* Cambridge, MA: Harvard University Press.

[22] Osborne, J., Martin, and Ariel, Rubinstein, 1990 *Bargaining and Markets.* Academic Press, Inc.

[23] Rubinstein, Ariel 1982. “Perfect Equilibrium in a Bargaining Model.” *Econometrica* 54(1): 97-109.

[24] Rubinstein, Ariel, and Asher, Wolinsky. 1985. “Equilibrium in a Market with Sequential Bargaining.” *Econometrica* 53(5): 1133-1150.

[25] Rubinstein, Ariel, and Asher, Wolinsky. 1990. “Decentralized Trading, Strategic Behaviour and the Walrasian Outcome.” *The Review of Economic Studies* 57(): 63-78.

[26] Samuelson, Larry. 1992. “Disagreement in Markets with Matching and Bargaining.” *The Review of Economic Studies* 59(1): 177-185.

[27] Satterthwaite, Mark, and Artyom, Shneyerov. 2007. “Dynamic Matching, Two-Sided Incomplete Information and Participation Costs: Existence and Convergence to Perfect Competition.” *Econometrica* 75(1): 155-200.

[28] Schelling, C., Thomas. 1960. *The Strategy of Conflict.* Harvard University Press.

[29] Shaked, Avner, and John, Sutton. 1984. “Involuntary Unemployment as a Perfect Equilibrium in a Bargaining Model.” *Econometrica* 52(6): 1351-1364.