Noname manuscript No.
(will be inserted by the editor)

Fuzzy Bi-objective Preventive Health Care Network Design

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This Paper is Accepted by Health Care Management Science: 4 August 2014

Please cite this article in press as: Davari, S., Kilic, K. and Ertek, G., "Health Care Management Science (2014)

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Abstract Preventive health care is unlike health care for acute ailments, as people are less alert to their unknown medical problems. In order to motivate public and to attain desired participation levels for preventive programs, the attractiveness of the health care facility is a major concern. Health economics literature indicates that attractiveness to a facility is significantly influenced by proximity of the clients to it. Hence attractiveness is generally modelled as a function of distance. However, abundant empirical evidence suggests that other qualitative factors such as perceived quality, attractions nearby, amenities, etc. also influence attractiveness. Therefore, a realistic measure should incorporate the vagueness in the concept of attractiveness to the model. The public policy makers should also maintain the equity among various neighborhoods, which should be considered as a second objective. Finally, even though general tendency in the literature is to focus on health benefits, the cost effectiveness is still a factor that should be considered. In this paper, a fuzzy bi-objective model with budget constraints of the problem is developed. Later, by modelling the attractiveness by means of fuzzy triangular numbers and treating the budget constraint as a soft constraint, a modified (and more realistic) version of the model is introduced. Two solution methodologies, namely *fuzzy goal programming* and *fuzzy chance constrained optimization* are proposed as solutions. Both the original and the modified models are solved within the framework of a case study in Istanbul, Turkey. In the case study, the Microsoft Bing Map is utilized in order to determine more accurate distance measures among the nodes.

Keywords Preventive Health Care · Multi-objective Optimization · Fuzzy Goal Programming · Facility Location

Mathematics Subject Classification (2000) 90B80 · 90C29 · 90C70

1 Introduction

Health care constitutes one of the largest shares in economic activities worldwide. US health care spending accounts for slightly more than 17% of the GDP and provides jobs to 11% of the workforce [26]. Increasing spending due to aging population and expensive new technologies used for both diagnostic and treatment purposes, and increasing customer expectations in the quality of health care delivery, magnify the share of health care to levels that are not sustainable. The rate of growth in US health care spending (5% annually in real terms) for the last decade outpaces the average annual growth in GDP and expectation is the continuation of this trend for the coming two decades [26]. Some researchers project that the share of health care spending in GDP of US will rise up to 28% by 2025, and even will be nearly half of the total GDP (∽48%) by 2050 [34]. Turkey, as an example of emerging countries, is not exempt from similar trends. Even though, the proportion of total health expenditure to GDP in 2012 was 6.3% (which is significantly less than the US figures), it is significantly higher than 5.8% which was the case in 2006 and 4.9% which was the case in 2000 [11]. The public health expenditures increased more than four times in a decade, which was around 13.3 BTL in 2002, and became 58.6 BTL in 2012. Therefore, efficient management of health care spending is a major concern of public policy makers, managers of health

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care providers and insurance company managers both in developed countries and emergimng countries like Turkey.

Preventive health care has always been the preferred option for creating awareness and reducing diseases in public. Generally speaking, there are three categories in preventive health care [10]. Primary interventions are those that reduce the risk of disease for healthy individuals (e.g. immunization programs, diet schemes, autism screenings for children, etc.). Secondary interventions are those that are designed for early detection of diseases for the individuals who are in the risk groups (e.g., screening for breast cancer for females over a certain age, cholesterol control, screenings for osteoporosis, colonoscopies, consultancy services provided for pregnant women, audiometric tests). Neither the individuals who receive primary prevention services nor those who receive secondary services have any obvious sign of the disease. On the other hand, tertiary interventions are designed for individuals who have been already diagnosed clinically for a disease and the goal of the preventive health care service is reducing the complications that might be caused by it (e.g., for individuals that have diabetes, regular retinal checks are done).

Health benefits of prevention is intuitive. That is to say, there is little argument against that preventing a disease is more desirable than tackling with the consequences of it. However, there is an ongoing debate on the economical benefits of preventive health care programs. Substantial evidence exists that public health is usually easier to maintain than to fix. Particularly chronic diseases such as cardiovascular diseases, Type 2 diabetes, cancer and chronic respiratory problems which constitutes more that 75% of health expenditures are largely preventable by preventive health care programs [44] [21]. Maciosek et al. [25] examined the costs and benefits of the clinical services recommended for the general population by the U.S. Preventive Services Task Force or the Advisory Committee on Immunization Practices, based on data obtained from an extensive structured literature review. The study claims that greater use of proven clinical preventive services in the United States could avert the loss of more than two million life years annually. Based on their analysis, increasing the use of these services from current levels to 90 percent in 2006 would result in total savings of $3.7 billion, or 0.2 percent of U.S. personal health care spending.

On the other hand, some researchers argue that it might be more cost efficient to treat the few sick patients rather than spending the money to reach the whole population for prevention. Based on a systematic review of 599 articles from the cost effectiveness literature, Cohen et al. [10] concludes that the distributions of cost-effectiveness ratios for preventive measures and treatments are actually very similar. That is to say, the opportunities for efficient investment in health care programs are roughly equal for prevention and treatment. As a result they warn the policy makers to avoid generalizations that argues preventive health care programs are always beneficial in terms of cost effectiveness and careful analysis of the costs and benefits of specific interventions should be a major concern.

Preventive Healthcare Facility Network Design Problem (PHFNDP) deals with where to locate the facilities and determination of their capacities. Since it is among the most significant strategic level decisions in any preventive healthcare program, it should be devised carefully so that the each dollar spent yields substantial health benefit. In order to ensure the economical feasibility of preventive programs the participation of target groups is essential so that the program attains its overall objective, i.e., prevention of diseases. Furthermore, the maximal participation levels also lead to economies of scale in the operational costs of preventive healthcare facilities. Therefore, higher participation levels lower the overall burden of health care spending for the society and increases the expected benefits from the healthcare delivery. Note that, preventive healthcare is inherently different from programs for acute ailments. People who seek preventive services have more flexibility as to when and where to receive preventive healthcare services in contrast to those who are sick and requires immediate medical attention. Ironically, even though the health benefits of preventive services are clear, most of the people are still reluctant to participate. Therefore, the achievement of the desired participation level continues to be a challenge to many preventive healthcare programs.

Due to the strategic nature of PHFNDP, the decision maker is the government who is responsible to spend public money efficiently and fair at the same time. That is to say, the government should locate the facilities and adjust their capacities so that the target groups could have higher participation rate and maintain the equity among different population groups that live in different neighborhoods. Tsou et al. [37] state that the achievement of equity in the distribution of urban public facilities is a goal of paramount importance to urban planners, who must analyze whether and to what degree their distribution is equitable. However, there is a conflict between these two objectives. In the literature the trade off between the "overall good" that would be attained from higher participation rates (i.e., *utilitarianism*) and equity (i.e., *egalitarianism*) is addressed in various ways. For example, in the PHFNDP literature, Gunes et al. [17] incorporate equity in their model as a constraint. Another possible way to address the trade-off is treating the problem as a multi objective optimization problem. In the location literature, earlier examples where equity is treated as part of the objective are available. For example, Feng and Timmermans[12] treats accessibility based equity as an objective where the second objective is mobility in the context of urban planning. Some other references in the context of waste location-routing problem where equity is again treated as part of the objective function in multi objective optimization setting are available in Alumur and Kara [1]. Note that, treating equity as part of the objective rather than a constraint has the advantage of providing multiple solutions to the decision makers and letting them make the last call by incorporating their preferences which is generally hard to assess beforehand.

In this paper, we address the bi-objective preventive health care network design problem where the objectives are maximizing the participation level and maximizing the equity. We will also consider budget limitations. In the operations management literature the attractiveness of the facility is used in order to estimate the anticipated level of participation and attractiveness usually is modelled with a nonlinear function of distance to the facility. Empirical research also supports that the distance to the facility significantly influences the attractiveness of the health care facilities. However, some other factors, which are hard to quantify, such as perceived quality, pleasant surroundings, availability of other attractions in the neighborhood, attentive staff etc. also empirically are shown to influence attractiveness [4], [14], [15], [40]. Actually Zhang et al. [47] mentions qualitative factors such as facility type and facility reputation among the factors that influence attractiveness, however, the authors limited their attention again to the distance to the facility in their model. In this research, the attractiveness of a facility will be modelled first as a negative exponential function of distance to travel, in line with the existing literature. We will utilize a fuzzy goal programming approach with two different *and* operators as a solution to this version of the model. Later, the attractiveness will be represented with a triangular fuzzy number which incorporates the vagueness of the attractiveness concept and a modified version of the model will be developed. In the modified version, the budget constraint will also be treated as a soft constraint and modelled with fuzzy chance constraints. In order to solve the modified version a fuzzy chance constrained method will be utilized. The fuzzy chance constrained method will be a modified version of an existing algorithm where *ϵ-constraint method* is incorporated in the solution process. Both of the models will be used in the framework of a case study in Istanbul, Turkey.

The outline of this paper is as follows: It proceeds with a literature review of relevant publications particularly in PHNFDP in section 2. The basic terminology of fuzzy set theory and credibility measures, as well as a brief discussion on multi objective optimization, in particular *goal programming* and *ϵ-constraint method*, is introduced in section 3. The mathematical model of the paper is presented in section 4. The proposed solution methods, i.e., the *fuzzy goal programming* and *modified fuzzy chance constrained method,* and the relevant literature are elaborated in sections 5 and 6. Later, in section 7, the case study and discussion on the results of the algorithms are provided. The paper is finalized by conclusions and some future research areas in section 8.

2 Literature review

The publication of first papers regarding to the design of healthcare networks dates back to early 60s when Hakimi [19] presented the problem of location on a graph with applications in telecommunication, police stations and hospitals. Later, many papers have been published in the context of healthcare network design such as locating organ transplant centers [3], locating healthcare facilities assuming moving populations [30], locating ambulances [35] and real-world case studies in Brazil [13], Burkina Faso [9] and Malaysia [36].

The problem of designing preventive healthcare facilities with congestion considerations is not new to the literature. The first publication in this area was Verter and Lapierre [41], where the problem of locating preventive facilities was presented and case studies in Georgia, USA and Montreal, Canada were reported. Recently, Zhang et al. [48] presented the problem of preventive healthcare network design on a graph with optimal choice allocation and an objective of maximizing participation level. They presented four different heuristic methods for their problem. Later, Zhang et al. [47], addressed a similar problem and modeled it as a bi-level nonlinear optimization model. In order to solve their problem, they developed a lower level problem and an upper level problem and proposed gradient projection method and an efficient tabu search procedure.

Gu et al., developed an accessibility measure for PHNFDP and presented an efficient interchange algorithm to solve it [16]. The impact of client choice behavior on the network and the participation level of people was considered by Zhang et al. [49]. Their decision variables are the location of facilities and also the number of facilities in each location. In Gunes et al. [17], physicians' preferences are also addressed as part of the objective, where the workload, income (due to the practice of fee for service payments to the health care employees), professional service and a collegial environment, and equity are handled by means of various constraints (lower and upper bound on the assigned number of individuals to each physician, lower and upper bounds on the number of physicians in each facility, etc.). Parker and Srinivasan [32] adopt the view point of the individuals and the objective of the PHFNDP is set to maximize the overall utility of the individuals that will receive the service.

To the best of our knowledge, the case of preventive health care network design with equity considerations and budget constraints has not been addressed as a multi objective optimization problem in the literature. On the other hand, Batta et al. [2] points to the gap between model development and analysis, and usage of these models to make actual location decisions. They propose that an appropriate role for the OR/MS analyst should be assisting the decision makers to identify a good set of solutions rather than an optimal solution that may not be practical. We address this gap that exists in the literature and rather than providing an optimal solution, a set of Pareto efficient solutions are obtained as the result of the developed solution methods.

According to Batta et al. [2] other issues that the location theory literature also suffers are lack of data and invalid assumptions regarding to the parameters used in the models. This claim seems to be valid in PHFNDP literature, particularly with respect to the attractiveness measure that is commonly used in order to determine the participation of the public to preventive programs. In order to model, the participation levels in a network, the papers in the literature assume attractiveness as a function of the proximity of the facility (i.e., time to reach to the service). Empirical research in health economics literature deals with the concept of attractiveness of health facilities. It is true that the empirical evidence suggests that the proximity has a significant effect on attractiveness of facilities. For example [29], determines that in urban areas, distance influences the decision on which kind of medical services (e.g. a medical doctor or a hospital) the patients use, whereas in rural areas of developing countries, distance is the decisive factor whether or not to use medical services at all. Other research such as [5], [20] also reveals evidence that supports this argument.

On the other hand, [4], [14], [15], [40] demonstrates that attractiveness of health care facilities is not only influenced by the proximity, but also by other qualitative factors such as quality, availability of other facilities in the neighborhood (e.g., shopping malls, restaurants, etc.), amenities in the facility, etc. That is to say, for a more realistic model, attractiveness of a facility should not be modelled with a crisp number but rather should allow some variance based on the other factors as well. . Therefore, a more realistic approach for incorporating the vagueness in the measure of attractiveness could be achieved by modeling it with fuzzy numbers.

Furthermore, even though budget is always a constraint in the decision making process, it is rather a soft constraint rather than a hard one. That is to say, public policy makers might eventually decide to increase the budget to a degree, based on the anticipated marginal outcome of such an increase in terms of participation level. Another issue that is addressed in this paper, in terms of the modelling aspect of the problem is modelling the budget constraint as a fuzzy constraint. In order to solve the mathematical model with fuzzy variables and fuzzy constraints a modified version of fuzzy chance constrained algorithm is developed. Note that the relevant literature regarding to the solution methods will be provided in section 5, where the details of the proposed algorithms are presented. Next, the basic terminology from fuzzy set and logic theory required for the rest of the paper, as well as the relevant literature regarding to multi-objective optimization approaches utilized in the research, namely, *goal programming* and *ϵ-constraint method,* is presented.

3 Theoretical background

In 1965, Zadeh introduced the concept of fuzzy sets and defined it as: "A collection of objects that might belong to the set to a degree, varying from 1 for full belongingness to 0 for full non-belongingness, through all intermediate values" [46]. A formal definition of fuzzy sets is as follows:

Definition 1 If X is a collection of objects denoted generically by x, then a fuzzy set $\widetilde{A}$ in X is a set of ordered pairs:

$$\widetilde{A} = \left\{ \left(x, \mu_{\widetilde{A}}(x)\right) \mid x \in X \right\}$$

where $\mu_{\widetilde{A}}(x)$ is called the membership function or grade membership (also degree of compatibility truth) of x in $\widetilde{A}$ that maps X to the membership space M.

In order to measure a fuzzy event, Zadeh introduced the possibility measure [45]. However, the possibility measure was not self-dual and later Liu [23] introduced a self-dual measure referred to as credibility measure which is defined as follows:

Definition 2 Let Θ be a non-empty set, and $\mathcal{P}$ the power set of Θ. Then, each element A in $\mathcal{P}$ is called an event. The set function Cr is called a credibility measure if it satisfies the following axioms [24]:

1- $Cr\{\Theta\} = 1$ (*Normality*)

2- $Cr\{A\} \leq Cr\{B\}$ whenever $A \subset B$ (*Monotonicity*)

3- $Cr\{A\} + Cr\{A^c\} = 1$ (*Self-duality*)

4- $Cr\{\cup_i A_i\} = \sup_i Cr\{A_i\}$ for any events $\sup_i Cr\{A_i\} < 0.5$ (*Maximality*)

Definition 3 A fuzzy variable is defined as a (measurable) function from a credibility space $(\Theta, \mathcal{P}, Cr)$ to the set of real numbers [24].

Definition 4 Let ξ be a fuzzy variable. Then, the expected value of ξ is defined as:

$$E[\xi] = \int_{0}^{+\infty} Cr\{\xi \geq r\}dr - \int_{-\infty}^{0} Cr\{\xi \leq r\}dr$$

provided that at least one of the two integrals are finite [24].

Unlike the real numbers, there is no natural ordering in fuzzy numbers. Therefore, an important problem that exists in fuzzy mathematics is ranking the fuzzy variables. There are various ranking criteria that exists in the literature. The following criterion is referred to as the *Expected Value Criterion:*

Definition 5 Assuming ξ and η to be two fuzzy variables, we say $\xi > \eta$ if and only if $E[\xi] > E[\eta]$, where E is the expected value operator of fuzzy variables. [24].

Note that there are other criteria available to rank fuzzy variables such as *Optimistic Value Criterion, Pessimistic Value Criterion* and also *Credibility Criterion.* For more information, readers may refer to [24].

Another issue in fuzzy set theory is basic set operators, i.e., the conjunction and disjunction operators. Generally speaking $t-norms$ are used as conjunction operators and $t-conorms$ (or $S-norms$) are used as disjunction operators in fuzzy set theory.

3.1 Multi-objective optimization

The general form of a multi-objective optimization problem (MOP) with k objectives, m inequality constraints, and e equality constraints is as follows:

$$\begin{aligned} \max_x f(x) &= [f_1(x), f_2(x), \ldots, f_k(x)]^T \\ g_j(x) &\leq 0 \qquad j = 1, 2, .., m \\ h_l(x) &= 0 \qquad l = 1, 2, .., e \end{aligned} \tag{1}$$

where a set of Pareto-optimal solutions is sought. A Pareto-optimal solution is essentially a solution which cannot be improved in one of the objectives without deteriorating at least one other.

In the literature of MOP, various methods have been proposed to find solutions. Broadly speaking, these procedures can be categorized as *a-priori, a-posteriori* and *interactive* methods depending on at which stage the decision maker's preferences are incorporated to the solution process [22]. Note that, there are a set of advantages associated with all three solution approaches, i.e., a-priori, a-posteriori and interactive methods. While a-priori methods are the most popular ones due to their relative simplicity in implementation, it is usually difficult to elicit preferences of the decision makers accurately before the analysis. Among the latter two which provides more information to the decision makers during the process, the interactive methods are more straightforward whereas a-posteriori methods provide more information to the decision makers by providing a better picture of the non-dominated set of solutions.

In this research fuzzy versions of an interactive approach, namely, *goal programming* [7], [8], [6] and an a-posteriori approach, namely *ϵ-constraint method* [18] which are widely used in the literature are adopted. In the sequel, a brief introduction of the conventional goal programming, *ϵ-constraint* *method* and a modified version, namely the *augmented ϵ-constraint method* will be provided. Note that, the *fuzzy goal programming method* proposed as a solution to the first model (which will be introduced later) is readily available in the literature. However, the *augmented ϵ-constraint method* will be introduced to an existing *fuzzy chance constrained method* and as a result a *modified version* of the *fuzzy chance constrained method* is developed and proposed as a solution to the second model.

In goal programming, the goals, denoted as b_i where $i = 1,..,k$ are defined for each objective function and the overall goal is minimizing the sum of absolute deviations, i.e., $|d_i|$, from these goals. The negative and positive deviations are referred to as d_i^- and d_i^+ respectively. It should be noted that $d_i^-, d_i^+ \geq 0$ and $d_i^+ d_i^- = 0$. The mathematical formulation of a classical goal programming problem is presented as follows:

$$\min_{x \in X, d^-, d^+} \sum_{i=1}^{k} (d_i^+ + d_i^-)$$

$$\begin{aligned} f_i(x) + d_i^- - d_i^+ &= b_i \quad i = 1,2,\ldots,k \\ d_i^+, d_i^- &\geq 0 \quad i = 1,2,\ldots,k \\ d_i^+ d_i^- &= 0 \quad i = 1,2,\ldots,k \\ g_j(x) &\leq 0 \quad j = 1,2,..,m \\ h_l(x) &= 0 \quad l = 1,2,..,e \end{aligned} \tag{2}$$

Note that it is also possible to introduce weighting parameters w_i and u_i in order to prioritize the positive and negative deviations, respectively, for objective i in the formulation. These relative importance parameters as well as the goals (i.e., the target values) are supposedly provided by the decision makers. Therefore, *goal programming* approach was originally developed as an a-priori method. However, later the interactive goal programming approaches are developed and successfully applied in various multi objective problems [27].

On the other hand, the *ϵ-constraint method* belongs to a group of *bounded objective function* methods. These methods try to optimize a single objective $f_p(x)$ while keeping the other objectives as a constraint such as $L_i \leq f_i(x) \leq U_i$; $i = 1,2,\ldots,k$ where $i \neq p$ in which L_i and U_i are the lower and upper bounds of objective i respectively. However, in the *ϵ-constraint method*, either L or U is excluded depending on the type of the objective (*maximization* or *minimization*). Later a systematic, meticulous changing process of bounds brings about finding the efficient frontier. That is to say, the mathematical model of the *ϵ-constraint method* is similar to the model presented in (1) with an extra set of constraints, namely, $f_i(x) \geq F_i$ $i = 1,..,k$ and $i \neq p$. The resulting mathematical model is as follows:

$$\max_x f_p(x)$$

$$\begin{aligned} f_i(x) &\geq F_i \qquad i = 1,..,k \text{ and } i \neq p \\ g_j(x) &\leq 0 \qquad j = 1,2,..,m \\ h_l(x) &= 0 \qquad l = 1,2,..,e \end{aligned} \tag{3}$$

[1] Augmented ϵ-constraint method

where F_i is the threshold on the objective function i.

Recently, Mavrotas [28] presented a modified version of the *ϵ-constraint method* which improves the performance of the traditional *ϵ-constraint method* and called it AUGMECON [1] which avoids generation of weaker Pareto optimal solutions and tries to eliminate some redundant iterations throughout the optimization process. AUGMECON can be stated as follows:

$$\max_x f_p(x) + \kappa(\sum_{i \neq p} \frac{s_i}{r_i})$$

$$\begin{aligned} f_i(x) &\geq F_i \qquad i = 1,..,k \text{ and } i \neq p \\ g_j(x) &\leq 0 \qquad j = 1,2,..,m \\ h_l(x) &= 0 \qquad l = 1,2,..,e \end{aligned} \tag{4}$$

where κ is a relatively small constant, say 10^{-3}, s_i is the slack variable of objective i in the set of constraints and r_i is the range of the objective function i used to avoid scaling problems.

In this paper, the modified approach, i.e., AUGMECON is utilized as part of one of the solution methods. Authors kindly ask the readers to refer to [28] for further details of AUGMECON.

4 Problem description

Let $G = (V, A)$ be a network in which $V(|V| = n)$ and A represent vertices and edges respectively as a model for a region with population zones (V) and transportation links (A). The distance matrix is assumed to satisfy the triangular inequality. The total population in the network is P where p_i is the fraction of population living in population zone $i \in V$. The aggregate number of clients requiring services in the whole network is estimated to follow a Poisson distribution with a rate of λ per unit of time. Similarly, the demand from each demand zone i will be λp_i.We also denote the set of potential nodes as $X \in V$. There is a restricted budget of B to spend on establishing facilities. The shortest path from node i to node j is t_{ij}.The attractiveness of a facility is modeled as a negative exponential function of the travel time as $a_{ij} = e^{-\eta t_{ij}}$ where η is a problem-dependent constant parameter. The unit cost of adding a server to a facility is denoted as β, and the fixed establishment cost of a facility at node j is α_j. We assume that each facility j has k servers each providing an exponentially distributed service at a rate of μ service per unit of time. Ω is the set of number of servers which ranges from 1 to K. Additionally $\overline{\lambda_k}$ is the maximum rate of participation possible for a system with no explosion, $\lambda_0 = 0$ and $\nabla\overline{\lambda_k} = \overline{\lambda_k} - \overline{\lambda_{k-1}}, (k \in \Omega)$. There are two objectives for the decision maker. On one hand the decision maker wants to maximize the participation to the preventive program as much as possible, at the same time in order to ensure the fairness, in order to maximize the equity as well.

We defined two decision variables for the model as follows:

$$x_{ij} = \begin{cases} 1 \text{ if client from zone } i \text{ receives service from node } j \\ 0 \text{ otherwise} \end{cases}$$

$$s_{jk} = \begin{cases} 1 \text{ if node } j \text{ has } k \text{ or more servers} \\ 0 \text{ otherwise} \end{cases}$$

Now, the mathematical model of the bi-objective problem is as follows:

$$\max \lambda \sum_{i \in V} p_i \sum_{j \in X} a_{ij} x_{ij} \tag{5}$$

$$\max \min_{i \in V} \sum_{j \in X} a_{ij} x_{ij} \tag{6}$$

$$\sum_{j \in X} \alpha_j s_{j1} + \sum_{j \in X} \sum_{k \in \Omega} \beta s_{jk} \leq B \tag{7}$$

$$\sum_{j \in X} x_{ij} = 1 \qquad \forall i \in V \tag{8}$$

$$x_{ij} \leq s_{j1} \qquad \forall i \in V; j \in X \tag{9}$$

$$t_{ij} x_{ij} \leq t_{ip} + M(1 - s_{p1}) \qquad \forall i \in V; j, p \in X \tag{10}$$

$$\lambda \sum_{i \in V} p_i a_{ij} x_{ij} \leq \sum_{k \in \Omega} \nabla \bar{\lambda}_k s_{jk} \qquad \forall j \in X \tag{11}$$

$$s_{j,(k+1)} \leq s_{jk} \qquad \forall j \in X; k \in \{1, 2, ..., K-1\} \tag{12}$$

$$x_{ij}, s_{jk} \in \{0, 1\} \qquad \forall i \in V; j \in X; k \in \Omega \tag{13}$$

The objective function (5) tries to maximize the aggregate participation level in the network. The objective (6) is the one to maximize the equity in the network. Constraint (7) restricts the total money to be spent. Constraint (8) states that any demand node should be allocated to one and only one open facility. Constraint (9) forbids the assignment to those nodes without any server and constraint (10) enforces the model to allocate any demand node to the nearest facility. Constraint (11) is the constraint which guarantees that assignments satisfy the queuing limit of any facility. Constraint (12) states that the s variable is a non-decreasing function of k and constraint set (13) is the integrality constraint on decision variables. Note that the second objective (6) is a nonlinear one which can be easily transformed into a linear one by assuming

$$Z = \min_{i \in V} \sum_{j \in X} a_{ij} x_{ij} \tag{14}$$

and adding it to the model. Now, the linearized bi-objective model will be as:

$$\max \ \lambda \sum_{i \in V} p_i \sum_{j \in X} a_{ij} x_{ij} \tag{5}$$

$$\max Z \tag{15}$$

$$Z \leq \sum_{j \in X} a_{ij} x_{ij} \qquad \forall i \in D \tag{16}$$

$$\text{and } (7) \ - (13)$$

5 Fuzzy goal programming

Fuzzy goal programming is basically a fuzzified version of the classical goal programming approach which assumes a fuzzy goal $\widetilde{G}$ and a fuzzy constraint $\widetilde{C}$ with a set of alternatives. In fuzzy goal programming a decision $\widetilde{D}$ is defined as the intersection of $\widetilde{G}$ and $\widetilde{C}$. This interaction can be obtained by using $t - norms$ in the fuzzy set theory literature such as the infamous *min* operator originally proposed by Zadeh.

Assuming only fuzzy objectives in the model, Zimmermann [50] defined the following aggregate objective function for a problem with k fuzzy objectives:

$$\max \mu_{\widetilde{D}} = \max \min \{\mu_1(x), \mu_2(x), ..., \mu_{\widetilde{k}}(x)\} \tag{17}$$

where $\mu_{\widetilde{k}}$ is the membership degree of the objective function k, which represents the degree of satisfaction of the objective function value and the target values, i.e., $\widetilde{D_i}$'s and the $\widetilde{G}_i$'s . This model can be easily transformed to the following one:

$$\begin{aligned} &\max \ \zeta \\ &\zeta \leq \mu_i(x) \quad i = 1, 2, ..., k \\ &x \in X \\ &Ax \leq B \\ &x \geq 0 \end{aligned} \tag{18}$$

Although Zimmermann's operator is simple to use, it has the disadvantages of *not* being a compensatory operator and the possibility of not generating Pareto-optimal solutions. In other words, the solution obtained using this operator is affected only by the worst solution and no other objective is able to affect the outcome. Therefore, various scholars have tried to present compensatory operators to solve this issue. In this paper, we will use the Werners' [42] *compensatory and* operator which can be represented as follows:

$$\mu_{\widetilde{D}}(x) = \max\{\gamma \min_i \{\mu_i(x)\} + (1 - \gamma)(\frac{1}{m}) \sum_{i=1}^{m} \mu_i(x)\} \tag{19}$$

in which γ is the compensation magnitude. In other words, the lower the value of γ, the higher the compensation ability of the operator. Needless to say, by assigning a value of $\gamma = 1$, the operator is transformed into the standard *min* operator of Zimmermann's. Note that, the Werners' *compensatory and* operator guarantees generation of Pareto-optimal solutions [31].

Although a decision maker can define the associated membership function of each objective as an arbitrary function. In this paper a monotonically increasing/decreasing (for positive/negative objectives respectively) function for all the objectives is utilized. To this end, assuming lower and upper bounds for an objective Z_i to be Z_i^- and Z_i^+ respectively, the membership degree can be defined as follows for a maximization objective function as follows:

$$\mu_i(Z_i\{x\}) = \begin{cases} 1 & \text{If } Z_i(x) \geq Z_i^+ \\ \frac{Z_j(x) - Z_j^-}{Z_j^+ - Z_j^-} & \text{If } Z_i^- < Z_i(x) \leq Z_i^+ \\ 0 & \text{If } Z_i(x) \leq Z_i^- \end{cases}$$

A sample linear fuzzy membership function as depicted in Figure 1.

{Insert Figure 1 Around Here}

{Insert Table 1 Around Here}

The solution methodology for the fuzzy goal programming model is as follows:

Algorithm 1 - Fuzzy Goal Programming

Step 1. The problem is solved for each of the k objective functions one by one with all of the constraints. The payoff table is constructed which is an asymmetric matrix where the matrix elements represent the optimal values. Table 1 tabulates sample payoffs with k objectives in which x_k^* is the optimal solution for objective k, and z_{ik} is the value of objective function k when objective i is optimized. On the other hand, Z_k^* is the vector of solution values when objective k is optimized. It should be noted that the values on the diagonal are the optimal objective values of the corresponding objective function.

Step 2. The lower and upper bounds of each objective function are determined based on the payoff table and membership functions are defined.

Step 3. Using equation (19), a secondary problem $\wp$ with a single objective function and the identical constraints as (16, 7 - 13) is constructed.

Step 4. The secondary problem $\wp$ is solved using different values of γ and results are reported.

6 Fuzzy chance constrained optimization

Recall that the empirical evidence suggests that attractiveness is not only influenced by a quantitative measure of distance to facility but also qualitative factors. Therefore, a more realistic way of representing attractiveness of the facilities would be with fuzzy numbers. Furthermore, budget is rather a soft constraint than a hard one. These two observations yield a modified version of the mathematical model. However, the fuzzy goal programming approach is not suitable for the modified version of the model.

Liu [24] presents a *fuzzy expected value model* which allows fuzzy parameters and fuzzy constraints as follows:

$$\begin{aligned} &\max\ E(f_1(x,\xi)), E(f_2(x,\xi)), ..., E(f_k(x,\xi)) \\ &E(g_j(x,\xi)) \le 0 \quad j = 1,2,..,m \\ &x \ge 0 \end{aligned}$$

where ξ is a fuzzy vector, $f_i(x,\xi)$ is the i^{th} objective function $i = 1,2,..,k$ and $g_j(x,\xi)$ are fuzzy constraint functions $j = 1,2,..,m$. Since the fuzzy constraints $g_j(x,\xi) \le 0$, $j = 1,2,..,m$ do not define a deterministic feasible set, a natural idea is to provide a confidence level α which defines the level of satisfaction of the constraint. This transformation leads to a separate chance constraints $Cr\left\{g_j(x,\xi) \le 0\right\} \ge \alpha_j, \quad j = 1,2,..,m$.

In line with the above formulations the bi-objective preventive health care network design problem is transformed as follows in which τ is a user-defined parameter:

$$\max\{E[\lambda \sum_{i\in V} p_i \sum_{j\in X} a_{ij}x_{ij}], E[Z]\} \tag{20}$$

$$Z \le \sum_{j\in N} a_{ij}x_{ij} \qquad \forall i \in D \tag{15}$$

$$Cr\{\sum_{j\in X} \alpha_j s_{j1} + \sum_{j\in X}\sum_{k\in \Omega} \beta s_{jk} \le B\} \ge \tau \tag{21}$$

$$\text{and } (8) \ -(13)$$

In order to solve the above formulation a simulation-based optimization procedure developed by Liu [24] is utilized. In this approach, two separate fuzzy simulation procedures executes simultaneously in order to estimate the expected value of each objective and also the feasibility of a solution related to the budget constraint.

Liu [24] proposed simulation algorithms in order to simulate $U_1 : x \to E[f(x,\xi)]$ and $U_2 : x \to Cr\{g_j(x,\xi) \le 0; j = 1,2,...,p\}$. The pseudo-codes of the algorithms are presented below. For the sake of limited space, the interested readers are kindly referred to Liu [24] for further details.

Algorithm 2 - Fuzzy Simulation for U_1

Step 1. Set $e = 0$.

Step 2. Generate θ_k from the credibility space $(\Theta, \mathcal{P}, Cr)$, determine $\nu_k = (2Cr\{\theta_k\}) \wedge 1$ and produce $\xi_k = \xi(\Theta_k)$, $k = 1,2,...,N$, respectively, where $\wedge$ is a $t-norm$ operator.

Step 3. Set two numbers $a = f(x,\xi_1) \wedge f(x,\xi_2) \wedge \ldots \wedge f(x,\xi_N)$ and $b = f(x,\xi_1) \vee f(x,\xi_2) \vee \ldots \vee f(x,\xi_N)$, where $\vee$ is a $t-conorm$ operator.

Step 4. Randomly generate r from $[a,b]$.

Step 5. If $r \ge 0$, then $e \leftarrow e + Cr\{f(x,\xi) \ge r\}$

Step 6. If $r < 0$, then $e \leftarrow e - Cr\{f(x,\xi) \le r\}$

Step 7. Repeat the fourth to sixth steps for N times.

Step 8. $U_1(x) = a \vee 0 + b \wedge 0 + e\frac{b-a}{N}$

Algorithm 3 - Fuzzy Simulation for U_2

Step 1. Randomly generate θ_k from the credibility space $(\Theta, \mathcal{P}, Cr)$, determine $\nu_k = (2Cr\{\theta_k\}) \wedge 1$ and produce $\xi_k = \xi(\Theta_k)$, $k = 1,2,...,N$ respectively.

Step 2. Estimate the value of $U_2(x) = \frac{1}{2}(\max_{1\ \le k \le N}\{\nu_k | g_j(x,\xi_k)_{j=1,2,...,p} \le 0\} + \min_{1\ \le k \le N}\{1 - \nu_k | g_j(x,\xi_k)_{j=1,2,...,p} > 0\})$

Note that, in order to generate the efficient frontiers, the modified *ϵ-constraint method* which is introduced earlier and referred to as AUGMECON [28] is utilized. In other words, the simulated solution (after running algorithms 2 and 3) is fed to AUGMECON in order to obtain the efficient frontier. Note that, the fuzzified version of AUGMECON by embedding the Liu's simulation-optimization procedure in it, is a novel solution framework in the literature.

7 Case study

The health care status in Turkey has improved in the last decade with the implementation of Health Transformation Program since 2003. Preventive health care programs receive a significant share of public health care budget and the total amount of expenditure to these programs have reached to 9 BTL in 2013 [39]. The preventive health care programs include Alcohol Control Programs, the Healthy Nutrition, Obesity Control, the Reduction of Excessive Salt Consumption, the Strategic Asbestos Control, the National Tobacco Control, etc. Note that, in particular the National Tobacco Control program, which was started in 2008, sets a good example for the effectiveness of these programs. As a result of this program, tobacco consumption rates decreased from %31.2 in 2008 to %27.1 in 2012, and Turkey became the first country in the world to implement all of the five tobacco control measures [43].

Particularly before the implementation of the Health Transformation Program a major problem of the Turkish network was to provide access to health services to all the people, including those living in remote places. For instance in 2002 Turkey had about 25 hospital beds per 10.000 population on the average, and the variation accross the country was from 3 to 60 beds per 10.000 population [33]. As of 2012 the average of number of beds per 10.000 population is increased to 28, and more noticeably the regional variance is reduced drastically and the range accross the country has become 13 to 54 [38]. This improvement is partly due to the strategic goals that are set in the Health Transformation Program regarding to maintain the spatial equity (SH.2.9) and partly due to the governing law of the health care services (No. 3359) which imposes that the health care institutions should deliver, plan and coordinate their services considering the spatial equity, quality and efficiency (Article 3.a). As a result, any decision to be made should consider the equity, participation level and budget in Turkey. Therefore, in this research we developed a model with these considerations on hand and a framework is proposed as a solution.

In order to analyze the performance of the proposed framework a case in Istanbul, Turkey is studied. Istanbul is a megacity with a population count of more than 13 million residents. Geographical coordinates for the 38 districts (all but the Princess Islands, which have no road connection to the other districts) of Istanbul were obtained through a commercial database. The geographical data was obtained as (X, Y) pairs where X is the latitude of the potential district and Y is the longitude of this location. Then the population of each district (namely, the results of the 2012 general population count), were obtained from the Turkish Statistical Institute.

{Insert Figure 2 Around Here}

{Insert Figure 3 Around Here}

The distance matrix was created from obtained data using a software developed particularly for this purpose using the *C#* programming language. A snapshot of this software is included in Figure 2. The software acquires the road distances and puts them into the distance matrix by automatically querying them from the Microsoft Bing Maps through the appropriate API[2]. It was crucial for our problem that the road distances to be used instead of the Euclidean distances especially due to the Boshporus strait, i..e, the strait that flows through the middle of Istanbul and separates the city into two. Any model built on Euclidean distances for both the European and Asian sides of Istanbul at the same time would be highly inaccurate due to the existence of the strait. Figure 3 illustrates the need to use road distances through showing the Euclidean distance and the road distance, where the latter is much longer.

7.1 Results of fuzzy goal programming

In order to assess the performance of the fuzzy goal programming approach for the problem on hand an experimental analysis was conducted. Even though Werners' *compensatory and* was preferred as the $t - norm$ by the authors, in order to compare the results of the methodology, the same problem is also solved by using the Zimmermann's operator. The problem parameters in the analysis were generated as Table 2 where $U(a, b)$ is a uniform random variable between a and b.

{Insert Table 2 Around Here}

{Insert Table 3 Around Here}

{Insert Figure 4 Around Here}

Table 3 and Figure 4 present the results of algorithm using both Zimmermann's and Werners' operators. Clearly, the results of the two operators are pretty different. While the trend of change with Zimmermann's operator is an almost linear function of γ, the compensatory operator of Werners is quite robust for different γ values. Note that even though Zimmermann's operator doesn't utilize the γ parameter since for different γ levels the bounds of the objective function changes, the optimal value of the goal programming approach that utilizes Zimmermann's operator also changes. Recall that a major disadvantage of the interactive approaches such as goal programming is their reliance on decision makers subjective preferences, such as the γ parameter, due to the difficulties in the process of elicitation. Therefore, the robustness of the Werners' operator (as demonstrated by the Figure 4) is a highly desirable property and supports its preference in fuzzy goal programming.

{Insert Table 4 Around Here}

{Insert Figure 5 Around Here}

Figure 5 depicts the resulting network layouts and Table 4 demonstrates the payoff table of running test problems with different values of budget and for three objectives as Participation-only, Equity-only, and the Werner-based biobjective with $\gamma = 0.4$. In Table 4, each block (which are associated with different budget levels), the values on the diagonal report the optimal values of each problem. Results clearly show that all of the three objectives are all sensitive to a change in budget. For the tight budget case ($B = 20,000$) Werners' *compensatory and* yields the same results with Equity-only objective. On the other hand for medium and relaxed budget cases Werners' results are kind of compromised solution that handle both of the objectives as expected. Moreover, due to the special nature of districts in Istanbul, where some districts are remote from others, in those cases where the objective is to maximize the Participation-only, the value of equity is zero which corresponds to the most unfair network design with respect to the definition of the adopted equity measure.

Results plotted in Figure 5, demonstrate the resulting network layouts for the three different budget levels and also each of the three objective functions. The figure clearly highlights the difference between the solutions with different objective functions. While the layouts of regarding to the problem which optimizes the participation level only, almost all the facilities are located in the center of the plane. The dispersion of the facilities are more fair in the other two models. As expected, the higher the available budget, the substantial increases in all three objective functions which is more clear for the equity objective function owing to its special characteristic. Another interesting point in the figure is the level of similarity among solutions with different values of budget for the same objective. Results show that almost two-thirds of facilities are

[2] Application Programming Interface

identical among these set of problems which means that the solutions are almost robust to more investment in establishing facilities. This is of a high practical importance, as relocating facilities are really costly. Hence, establishment plans may start with a lower budget availability without a need for drastic changes in near future.

7.2 Results of the chance constrained optimization approach

As discussed earlier, in reality the attractiveness of a health care facility is not a function of a single parameter, namely, travel distance. Therefore, a more realistic approach would be modeling the attractiveness as the outcome of highly complex relations among various factors such as the proximity (i.e., time to travel), availability of other attractions in the neighborhood, perceived quality, etc. Therefore it is decided to measure the attractiveness with linguistic variables which can be represented with fuzzy sets rather than a crisp number. Likewise, as facility location is a strategic decision problem which needs major investments, there is a degree of uncertainty in the total cost to incur in the future. Furthermore, usually policy makers have some flexibility in terms of money spending in such major decisions. Hence, the budget constraint is actually a soft constraint rather than a hard constraint. Therefore the budget constraint is modeled as a chance constraint in the second phase of the research. The modified mathematical model presented in Section 6 is solved by using the *fuzzy chance constrained method* which is discussed earlier.

In order to examine the behavior of the new model, we generated a set of test problems as follows. For any element of the attractiveness matrix, say a_{ij}, which was computed in the previous section, a triangular membership function is generated as triangular membership function with parameters $(\varphi_1 a_{ij}, a_{ij}, \varphi_2 a_{ij})$ where φ_1 and φ_2 were selected to equal 0.2 and 0.3 respectively. Note that, in reality experts would provide the appropriate triangular fuzzy numbers based on the factors that would influence the attractiveness of the node. However, for demonstrative purposes the simplified approach is considered. After the preprocessing stage, the simulation-based optimization procedure is executed and the efficient frontier, which should be regarded as a fuzzy efficient frontier owing to the fuzziness of parameters, is obtained.

{Insert Figure 6 Around Here}

Figure 6 depicts the Pareto-frontier of the two objectives. Each point in the curve depicts a different solution which has been found using the simulation-optimization as a result of 1000 iterations. The more one goes to the right in the figure, an increase in participation level accompanied with a decrease in equity levels is observed. One should also observe that, initially the participation level increases significantly with a relatively low deterioration of the equity. One of the interesting observation in the resulting set of Pareto efficient solutions is its cardinality. That is to say, a lower number of Pareto solutions are obtained than expected. Second observation is the sharp change in the right hand side of the Pareto efficient frontier. Both of these observations can be attributed to the special spread of nodes in Istanbul. Note that in a megacity like Istanbul, even though most of the population lives in districts that are usually close to one another, there are exceptional districts, which are located in relatively remote places and smaller population (in the case of Istanbul, the Sile district). Hence, the equity objective is almost robust throughout a large proportion of its range and sharply reduces as soon as the facility moves out of such a remote place in order to increase the participation levels.

8 Concluding remarks and future research

In this paper, we have addressed the preventive health care network design problems with budget constraints. The objectives of the decision makers, in this case the public policy makers, are maximizing the participation level as well as maximizing the equity among the populations living in different neighborhoods. On one hand, maximization of the participation level is crucial, since it increases the health benefits that are anticipated from such preventive programs as well as the cost effectiveness due to the economies of scale. On the other hand, the public policy makers should also maintain the fairness among the public. The dilemma among these two conflicting objectives are common in public decision making, i.e., the maximization of overall goodness (i.e., *utilitarianism*) and fairness (i.e., *egalitarianism*). As a result the problem is treated as a bi-objective problem.

In order to solve the bi-objective PHFNDP, two multi objective approaches are utilized. First, for a conventional version of the model, a fuzzy goal programming approach based on the Zimmermann's operator and Werners' operators are developed. Later a modified version of the problem is proposed. In the modified version, the budget constraint is treated as a soft constraint and modelled as a chance constraint. Furthermore, since in reality the attractiveness measure is not a function of proximity to the facility but influenced by other qualitative factors, it is represented with triangular fuzzy numbers in the modified version. In order to solve the modified version of the problem a fuzzy chance constrained optimization model is developed. The developed methodologies are applied to Istanbul, Turkey in a case study. In order to be more accurate the traveling distance between the 38 nodes were generated using the Microsoft Bing Maps.

This paper is the first bi-objective modelling with objectives of participation and equity applied to PHFNDP. Note that, the resulting pareto efficient solutions obtained from the *ϵ-constraint method* allow the decision makers to make informed decisions. That is to say, the model acts as a prescriptive decision support tool which has various advantages to normative approaches. Note that, even though many facility location papers are published in the literature, usually these papers had only scientific contribution but not practical value. One of the major reasons for this gap between the theory and practice is the developed models usually can't incorporate some concerns of the decision makers such as future development plans, tendency to promote their local voters, etc. Therefore, a prescriptive multi objective approach that yields multiple pareto efficient solutions would be preferable for the public policy makers as opposed to a normative solution that dictates the best solution. Modeling the budget constraint as a soft constraint and representation of the atrractiveness with fuzzy numbers also serve to the same purpose. A novel methodological framework that incorporates simulation-optimization to the AUGMECON is developed in order to handle the modified model. Finally, the paper is among the pioneers in PHFNDP by utilizing the more accurate travelling distance as opposed to the studies that utilizes only Euclidean flight distance.

Since almost all of the parameters in real-world are uncertain, a possible future research area would be considering stochastic variables such as travel times in the network. Note that the preventive health care network design is inherently intertwined with some other problems such as staff rostering. Integration of these problems in the current model can also be a promising future research area. Finally, the model can be enriched considering cultural and demographic issues such as the population of each age group in a neighborhood or using cultural aspects in defining the attractiveness functions.

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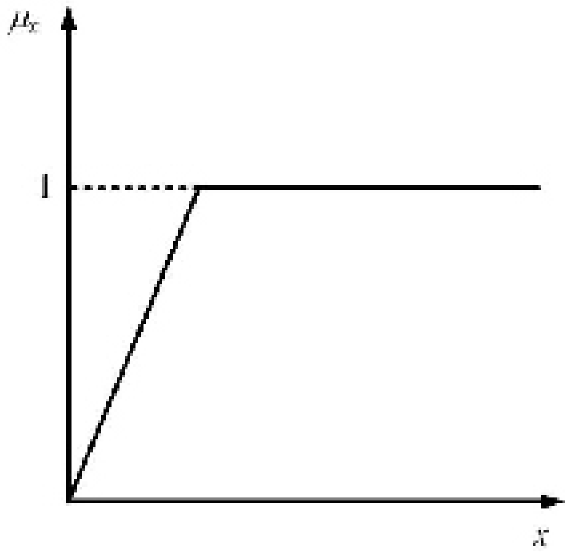

Fig. 1: A sample linear fuzzy membership function

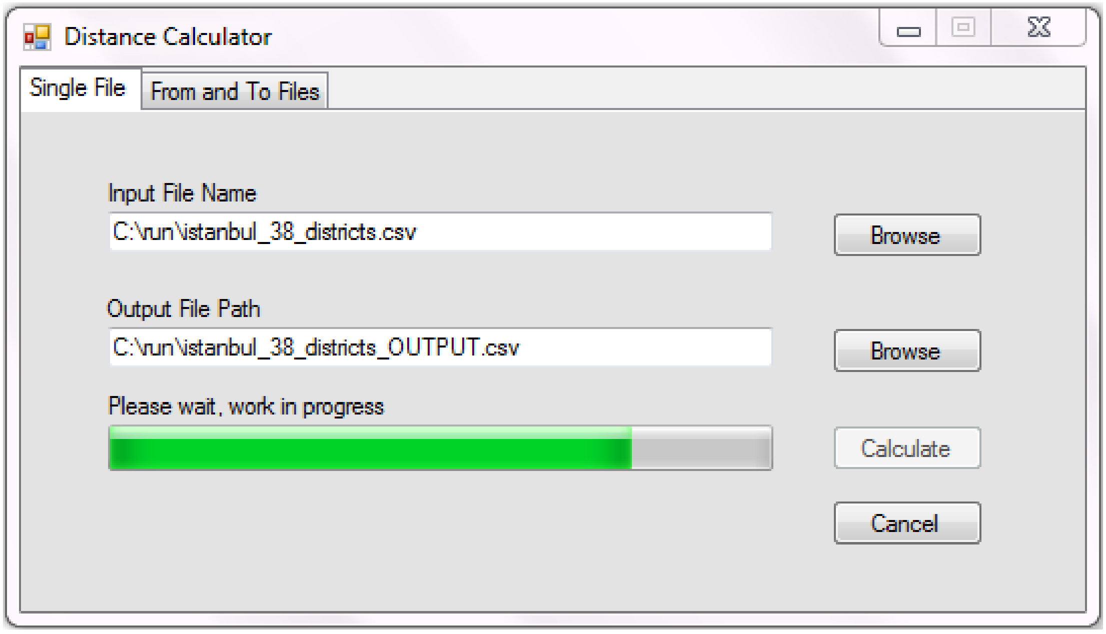

Fig. 2: A snapshot of the distance calculator software

Table 1: Payoff Table for Fuzzy Goal Programming Algorithm

	x_1^*	x_2^*	$\cdots$	x_k^*
Z_1^*	z_{11}	z_{12}	$\cdots$	z_{1k}
Z_2^*	z_{21}	z_{22}	$\cdots$	z_{2k}
$\vdots$	$\vdots$	$\vdots$	$\vdots$	$\vdots$
Z_k^*	z_{k1}	z_{k2}	$\cdots$	z_{kk}

Fig. 3: The actual road distance vs. the Euclidean distance

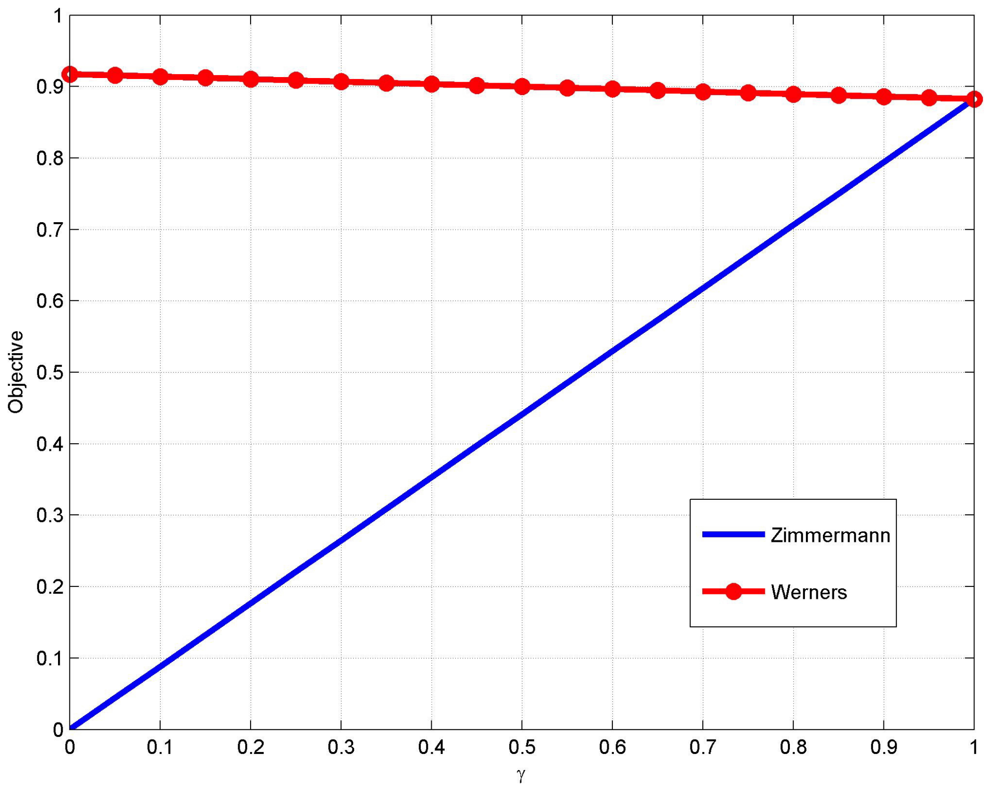

Fig. 4: Comparison of the results of Zimmermann's norms and Werners'

Table 2: Parameters used in experimental analysis

Parameter	Value
α_j	$U(2000, 10000)$
β	1000
Budget	$\{20000, 50000, 80000\}$
M	1000
λ	100
μ	4

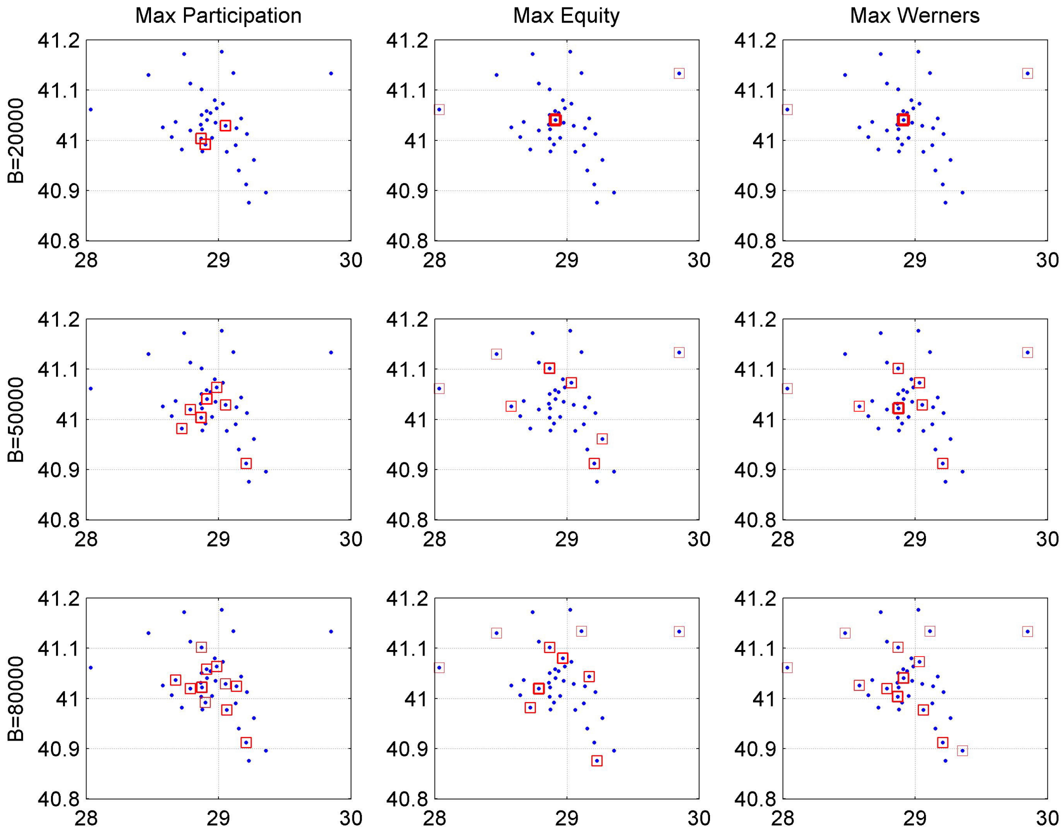

Fig. 5: Comparing solutions with the different objectives

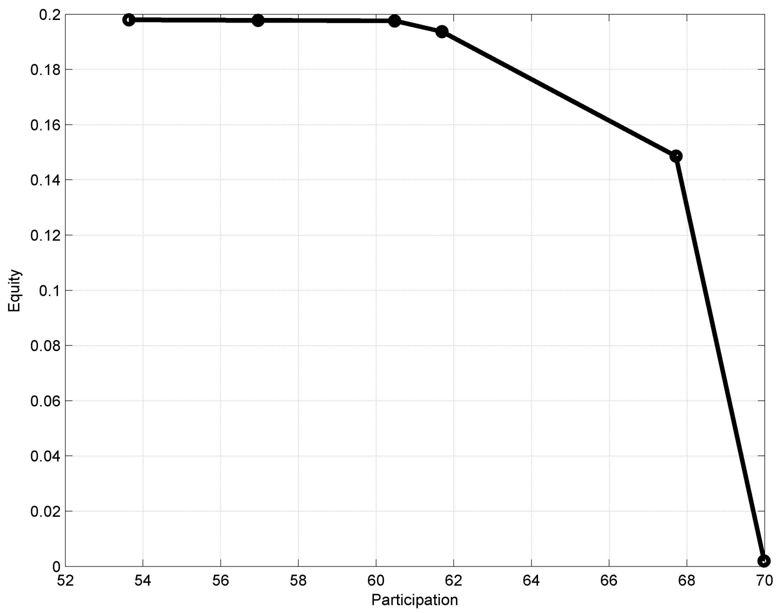

Fig. 6: Pareto frontier for the chance constrained solution

Table 3: Results with Zimmerman's and Werners' operators

γ	Zimmermann's	Werners'	γ	Zimmermann's	Werners'
0	0	0.917			
0.05	0.044	0.915	0.55	0.485	0.898
0.1	0.088	0.913	0.6	0.529	0.896
0.15	0.132	0.912	0.65	0.573	0.894
0.2	0.176	0.910	0.7	0.617	0.892
0.25	0.220	0.908	0.75	0.661	0.891
0.3	0.264	0.906	0.8	0.705	0.889
0.35	0.308	0.905	0.85	0.750	0.887
0.4	0.352	0.903	0.9	0.794	0.885
0.45	0.397	0.901	0.95	0.838	0.884
0.5	0.441	0.899	1	0.882	0.882

Table 4: Optimal values with respect to different budget levels

Budget		Participation	Equity	Werners'
20000	Participation	35.998	0	0.3
	Equity	26.307	0.006	0.811
	Werners'	26.307	0.006	0.811
50000	Participation	57.891	0	0.3
	Equity	38.489	0.156	0.765
	Werners'	51.088	0.148	0.902
80000	Participation	69.982	0	0.3
	Equity	51.775	0.198	0.817
	Werners'	61.695	0.194	0.911