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Theory and evidence on pricing by asymmetric oligopolies

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Abstract

We present an analysis of markets with many asymmetrically positioned retailers that compete for the business of both informed and uninformed customers for a homogenous good, such as software, music, book or a brand-name appliance. We show that two forms of asymmetry, one related to loyal segment sizes of retailers and one related to the positioning of firms, completely explain the observed price dispersion in such markets and the multitude of asymmetrical strategies adopted by retailers. The stochastic dominance of empirical mixed strategy measures is used to test the theory with data on 968 books from 10 online retailers.

JEL classification: L11; D43; L13

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1. Introduction

A new perspective on oligopolies, introduced by electronic commerce, gave rise to the prediction that availability of price comparison agents combined with low entry barriers in

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online markets would result in a Bertrand type competition. As customers comparison-shopped for almost identical goods online, firms would have to cut prices one after another to attract customers. However, online markets of today do not behave as predicted. A dispersion of prices is apparent even for identical products. In this paper we demonstrate that two forms of asymmetry, one related to loyal segment sizes of retailers and one related to the positioning of firms (differentiation), completely explain the observed price dispersion in online markets and the multitude of asymmetrical strategies adopted by retailers.

A recent stream of empirical research on Internet markets reveals the existence of price dispersion in the context of shopping agents that facilitate price comparisons (e.g., Clay et al., 2001, 2002; Smith and Brynjolfsson, 2001; Clemons et al., 2002). However, previous research on the observed price dispersion at most provides a partial explanation to the phenomenon. While differentiation is suggested to cause the observed price dispersion, no complete explanation exists as to how the forces of Bertrand competition and the desire to differentiate resolve into a market equilibrium. Capturing the promotional incentives of many asymmetrical retailers, our research attempts to provide a theoretical operational explanation to this observed price dispersion in these markets.

To provide this operational explanation, we focus on the *asymmetry* between the *many* retailers in a given market. In the online markets for commodities like software, music, movies and books, many firms offer identical products to their customers. When customers compare prices of a homogenous product across retailers, these retailers are in direct competition for the business of that customer. Internet shopping agents further facilitate this price comparison behavior and can increase the competition in a market considerably. Even for small ticket items, the benefits of comparing retailer offerings can exceed the costs. Moreover, low entry barriers allow hundreds of retailers to get listed on the results pages of Internet shopping agents and therefore rise into the consumers' consideration sets. Consequently, in many online markets, Internet shopping agents compare prices across a multitude of asymmetrical retailers and relay this information to the price comparing customers on the demand side.

One would expect that the existence of a multitude of asymmetrically positioned retailers would lead to at least several diverse pricing strategies. However, previous theoretical research on homogenous goods markets focuses predominantly on a duality of strategies, due primarily to the choice of duopoly as the market structure (Narasimhan, 1988; Raju et al., 1990; Rajiv et al., 2002). Even in Varian (1980), the analysis of an oligopoly yields a unique strategy that all firms adopt, because all firms are assumed to be symmetrical. However, a study of an Internet oligopoly with two types of firms reveals that even symmetric firms may adopt asymmetric strategies (Iyer and Pazgal, 2003). We believe a finer analysis of markets where a multitude of asymmetrically positioned retailers develop a variety of strategies is warranted, especially given the rise of electronic commerce and the Internet.

Consequently, for all online retailers that must compete in environments with increasingly informed customers, a finer understanding of the nature of the competition between many asymmetric retailers that sell commodities remains an important topic to be explored. We hope to fill this gap by answering the following research questions: "How do firms compete on price for the business of switcher (informed) and loyal (uninformed) segments of customers given there are many asymmetrically positioned firms from which the customer can buy?", "What is the degree of promotional activity in these asymmetric oligopolies?" and "How do firms differ in their promotional activity, including frequency of promotions as well as depth of promotions, based on their customer base?"

1.1. Brief overview

To address the research questions posed above, we consider a retail market with many asymmetrically positioned retailers. We consider two forms of asymmetry. In the simplest form, we define asymmetry as the diverse loyal segment sizes, a la Varian (1980) and Narasimhan (1988). We use this definition of asymmetry in our base model. We also define asymmetry in terms of the positioning retailers attain in the level of service, convenience, warranty levels, reputation and store ambiance, akin to the differentiation in Clemons et al. (2002) and Clay et al. (2002). This type of asymmetry (differentiation) manifests itself as the price premium customers are willing to pay to buy the commodity from the better-positioned retailer. Consequently, asymmetry in the level of service, convenience and reputation may result in different reservation prices for the offerings of different firms. As our model extension, we incorporate both of these asymmetries in our analysis.

We refer to the customers who can compare prices of certain products across stores as informed customers, or *switchers* as in Salop and Stiglitz (1977), Varian (1980) and Narasimhan (1988). On the other hand, each retailer also enjoys the loyalty of a certain segment of its customer base and can extract monopoly profits from these customers, who remain uninformed about offerings from other retailers. We refer to these customers as the *loyal customers*. However, since the targetability, the ability to predict if the customer is a loyal or a switcher, as well as the addressability, the ability to contact customers individually, is imperfect, retailers have no simple way of price discriminating (Chen et al., 2001; Blattberg and Deighton, 1991).

Both economics and marketing literatures have rich research streams on pricing in an imperfectly competitive environment. Tirole (1988) and Rao (1993) provide surveys of pricing models in economics and marketing, respectively. The game theory-based research streams in pricing in an imperfectly competitive environment seek equilibria in stylized models of profit maximizing firms (Varian, 1980; Narasimhan, 1988; Lal, 1990; Raju et al., 1990; Rajiv et al., 2002). Some key findings of the game theoretical models are that (i) loyal and switcher segments and their sizes affect firms' pricing strategies (Varian, 1980; Narasimhan, 1988); (ii) brands stronger versus weaker in terms of brand/store loyalty and brand/store positioning develop relatively diverse optimal pricing strategies. However, as mentioned before, these studies do not consider markets with many asymmetrical firms. We hope to extend these theoretical results to markets with many asymmetrically positioned firms.

To theoretically and empirically analyze markets with a multitude of retailers affected by comparison shopping, we build a stylized model of price competition in a market for a homogenous good. Our analysis shows there are at least three diverse pricing strategies and there can be as many discernable diverse strategies as the number of firms in the market when there are many asymmetrically positioned retailers.

The basic solution with three diverse strategies is encountered when the reservation price is homogenous across all customers. The retailer with the smallest loyal segment adopts the first strategy and completely randomizes its prices along a continuum. The retailer with the second smallest loyal segment adopts a strategy in which it also randomizes its prices along the same continuum, yet also has the reservation price as a regular price that it quotes frequently. The remaining firms with larger loyal segments exhibit the third and final strategy. These retailers relinquish all price-based promotional activity to the two firms with the smallest loyal segments and consistently quote the reservation price.

When the reservation price varies across retailers, however, this final strategy proliferates into numerous discernable diverse strategies with various resulting average prices in line with the asymmetric positions of the retailers. Therefore, the retailers with large loyal segments continue to abandon price promotions.

Our findings also provide an operational explanation for the observed price dispersion that has received widespread research attention. Research streams in economics and marketing have reported and attempted to explain the sources of dispersed prices where a one-price rule was expected to apply (Salop and Stiglitz, 1977; Burdett and Judd, 1983; Brynjolfsson and Smith, 2000; Clay et al., 2001; Clemons et al., 2002). Our model shows that asymmetrically positioned firms may engage in rivalrous behavior that results in a variety of promotional patterns, which in turn lead to price dispersion. We offer empirical evidence in support of these findings.

The most significant finding of our analysis pertains to the importance of building loyalty in any market that can be distressed by price comparison shopping. Our analysis shows that, especially in markets that lack entry barriers, price discounting is not a profitable strategy. Those firms with the least to lose offer the deepest discounts to serve the price comparing customers, making it unprofitable for other firms to even compete for the business of switcher customers. Therefore, building loyalty, offering a higher level of service and charging a premium for that service with higher reservation prices seem to be the essential strategy on which to focus.

The rest of the paper is organized as follows. In the next section, we present our model and a model extension with the solutions to both. Then we provide empirical validation for the basic predictions of the models. We further explore some aspects of our models and their empirical validation in the Discussion section. We wrap up with the Conclusions section.

2. Model

Our model is a combination and extension of those by Varian (1980) and Narasimhan (1988). These studies have examined two different supply-side structures; a symmetric oligopoly was analyzed by Varian (1980) and an asymmetric duopoly was analyzed by Narasimhan (1988). Introducing asymmetry into a market with a multitude of retailers, we study an asymmetric oligopoly.

The assumptions and characteristics of our model are similar to those in Varian (1980) and Narasimhan (1988). We assume a market for a homogenous good, such as a book, CD or DVD. On the demand side, there are two segments of customers: loyal customers and comparison shoppers (switchers). Each customer wishes to purchase a single unit of the good. On the supply side of our model. there are $k \geq 3$ firms: firm 1 through firm k. The loyal customers are loyal to only one firm and we represent the number of customers who are loyal to firm i as n_i. Without loss of generality, we assume that $n_i > n_{i+1}$ for all i. The comparison shoppers, s, (switchers) are not loyal to any firm and buy from the firm offering the lowest prices. We normalize the market size to one without loss of generality such that $\sum n_i + s = 1$.

Customers buy from any firm only if the price they are quoted is less than their reservation price, r, which we assume to be homogenous for all customers and all retailers.[1] While the sizes of the segments n_i and s and the reservation price r are common knowledge, firms cannot engage in price discrimination because of imperfect addressability and targetability. All firms face cost functions with fixed and marginal costs that we assume to be zero without any loss of generality. The profit functions of the firms are given by:

$$\pi_i(p_i, p_{-i}) = \begin{cases} p_i(n_i + s) & \text{for } p_i = \min[p_j] \\ p_i(n_i + s/v) & \text{for } p_i \in M = \{m \mid p_m = \min[p_j]\}, N[M] = v \\ p_i n_i & \text{for } p_i > p_j \end{cases} \tag{1}$$

where p_i represents the price quoted by firm i and p_{-i} represents the vector of prices quoted by all other firms. As is evident from the profit function, we assume that, in the event of a tie in prices, the firms in the market serve switchers equally. In the analysis that follows, we first show there is no Nash equilibrium in pure strategies in this game. However, there exists a mixed strategy Nash equilibrium and we sketch this mixed strategy equilibrium later in our analysis.

2.1. The analysis

As usual, we first define the upper and lower boundaries of firms' supports. The upper bound of the feasible price set is r. Prices higher than the reservation price will result in no sales at all, while positive profits are possible when the reservation price is quoted. The lowest price any firm will ever consider charging is given by $p_i^{\min} = n_i r / (n_i + s)$. To see this, note that firm i can have a motivation to reduce its price down to a level where, if it successfully captures all the switchers, it makes at least the same profit it would get from selling only to its loyal customers at r. That is, $(n_i + s) p_i^{\min} = n_i r$. Moreover, bigger loyal segment size will mean higher potential loss in profit due to price reductions and, consequently, the lowest price from which the firm can still benefit serving the switchers will be higher for firms with larger loyal segments. Since $n_i > n_{i+1}$, it follows that $p_i^{\min} > p_{i+1}^{\min}$. We first show the non-existence of a pure strategy Nash equilibrium.

[1] We later relax this assumption by introducing an extension to this base model by allowing for different reservation prices for diversely positioned retailers in the extension. This assumption can be relaxed further to incorporate customers who are distributed over an interval with respect to their reservation prices. However, most of the results are identical in analyses with fixed or distributed reservation prices. The simpler one is presented in this paper for expositional ease.

Proposition 1. *There is no Nash equilibrium in pure strategies in this game with* $k \geq 3$ *firms with asymmetrical loyal segment sizes.*

Proof. See Appendix A. □

While we provide the technical proof of the non-existence of Nash equilibrium in pure strategies in Appendix A, we also provide an intuitive exposition here. Notice that the motivation to undercut the price of other lower-priced firms to capture the switchers results in a downward push in prices. Simultaneously, the motivation to increase the price to the reservation price, if the switchers are not served with a lower price, pushes prices up. The result is a lack of pure strategies. Next, we establish the equilibrium profits of all the firms.

Proposition 2. *The equilibrium profits of all firms in this game will be equal to their reservation utilities (minmax profits), which are the lowest profits that a firm i's opponent can hold it to by any choice of* p_{-i} *(own prices) provided that firm i correctly foresees* p_{-i} *and plays a best response to it. The reservation utilities, or minmax profits are given by:*

$$\pi_i^{\min\ \max} = \min_{p-i}\left[\max_{p_i} \pi_i(p_i, p_{-i})\right] = \begin{cases} n_i r & \text{for } i = 1, 2, 3, \ldots, k-1. \\ (n_i + s)p_{k-1}^{\min} & \text{for } i = k \end{cases} \quad (2)$$

Proof. See Appendix A. □

Having established the characteristics of the supports over the feasible range of prices, we can solve the firms' equilibrium pricing strategies. Any firm will serve its loyal segment with the price it chooses as long as the price is below the reservation price and any firm can capture the switcher segment if the price it quotes is lower than all the prices quoted by the competing firms. Therefore, for any price p, except for any point where firms may have mass points such as $p = r$, the equilibrium conditions for this pricing game for k firms can be written as:

$$E[\pi_i] = n_i r = n_i p + \prod_{j \in -i} \left(1 - F_j(p)\right) ps \quad \text{for } i = 1, 2, 3, \ldots, k-1 \quad (3)$$

$$E[\pi_k] = (n_k + s)p_{k-1}^{\min} = n_k p + \prod_{j \in -1} \left(1 - F_j(p)\right) ps \quad \text{for } k \quad (4)$$

where $p_{k-1}^{\min} \leq p < r$. The next proposition presents the uniqueness of some positions firms hold in this game with many opponents and enable us to solve the set of equations we presented.

Proposition 3. *Only firm k and firm* $k-1$*, the two firms with the smallest loyal segment sizes and the two lowest* p^{min} *values, offer the deepest discounts.*

Proof. We start our analysis in the left-most interval where only two firms can ever compete, $[p_{k-1}^{\min}, p_{k-2}^{\min}]$. In this interval, there are no pure strategy choices for firm k and firm $k-1$, but both firms have positive support. In this interval, both firms have lower prices than the rest of the firms with probability 1 and so can capture the switcher segment if they can price lower than the other. Firm k has an advantage; it can always price lower than firm $k-1$ with probability 1, if it prices just below $p_{k-1}^{\min}$. However, it will not choose to do so because it can price higher and still capture the switchers with positive probability. Thus, the two firms will randomize their prices in this interval so that their expected profits will be equal to their minmax profits. We can write the equilibrium conditions for the interval $[p_{k-1}^{\min}, p_{k-2}^{\min}]$ except for $p_{k-2}^{\min}$ as:

$$\pi_{k-1}^{\min\max} = n_{k-1}r = n_{k-1}p + [1 - F_k(p)]ps \tag{5}$$

$$\pi_{k}^{\min\max} = (n_k + s)p_{k-1}^{\min} = n_k p + [1 - F_{k-1}(p)]ps \tag{6}$$

where $p_{k-1}^{\min} \le p < p_{k-2}^{\min}$. Note that, in these equations, only the interaction with the other firm is included in the formulation because all other firms have minimum feasible prices above this interval's upper limit. □

The solutions to this set of equilibrium conditions are:

$$F_k(p) = \frac{p(n_{k-1} + s) - rn_{k-1}}{ps} \tag{7}$$

$$F_{k-1}(p) = \frac{(n_k + s)(-rn_{k-1} + p(n_{k-1} + s))}{ps(n_{k-1} + s)}. \tag{8}$$

Note that with this solution $F_k(p_{k-1}^{\min}) = F_{k-1}(p_{k-1}^{\min}) = 0$ as expected and the cumulative probabilities with which firm k and firm $k-1$ will price below $p_{k-2}^{\min}$ are given by:

$$F_k\left(p_{k-2}^{\min}\right) = 1 - \frac{n_{k-1}}{n_{k-2}} \tag{9}$$

$$F_{k-1}\left(p_{k-2}^{\min}\right) = \frac{(n_{k-2} - n_{k-1})(n_k + s)}{n_{k-2}(n_{k-1} + s)}. \tag{10}$$

Note that these cumulative functions represent the mass of prices already lower than the prices of the remaining firms. In the next proposition, we explore how this competition between the firms with the smallest loyal segment sizes extends to other intervals where more firms can compete if they choose to.

Proposition 4. *At most, two firms ever offer discounts or have support in any given interval. Moreover, these two firms that may have positive support in any given interval are the same two firms for all the intervals. These are the firms with the two lowest p^{min} values and two smallest loyal segments: firm k and firm $k-1$.*

Proof. In Proposition 3, we have demonstrated that only firm k and firm $k-1$ have positive support up to the point $p_{k-2}^{\min}$. Moving up from this point, we observe that, with any price in the interval $[p_{k-2}^{\min}, p_{k-3}^{\min}]$, it is not only firm k and firm $k-1$ but also firm $k-2$ that can increase its profits if it can successfully capture the switcher segment.

However, we proceed to show that it is not possible for all three firms to have support over this interval in this game. The lowest price firm $k-2$ will ever quote is $p_{k-2}^{\min}$ and, at this price, the expected profit it will realize is given by:

$$\pi_{k-2}\left(p_{k-2}^{\min}\right) = n_{k-2}p_{k-2}^{\min} + \left[1 - F_k\left(p_{k-2}^{\min}\right)\right]\left[1 - F_{k-1}\left(p_{k-2}^{\min}\right)\right]p_{k-2}^{\min}s. \tag{11}$$

Yet, firm $k-2$ will never price at $p_{k-2}^{\min}$ if $\pi_{k-2}(p_{k-2}^{\min})$ is less than its minmax profit of $n_{k-2}r$. Inserting values from Eqs. (9) and (10) into Eq. (11), we see that the inequality $\pi_{k-2}(p_{k-2}^{\min})<n_{k-2}r$ simplifies to the inequality

$$\frac{r(n_{k-2} - n_{k-1})s(n_{k-2}(n_{k-1} + s) + n_{k-2}(n_k + s))}{n_{k-2}(n_{k-2} + s)(n_{k-1} + s)}>0 \tag{12}$$

which is always true since $n_{k-2}>n_{k-1}$. Therefore, firm $k-2$ will never price at $p_{k-2}^{\min}$ given that firm k and firm $k-1$ are already competing for the switcher segment below this price. Also note that, since it is only firm k and firm $k-1$ that can compete at $p_{k-2}^{\min}$ and possibly above as we have just shown, the cumulative distribution functions presented by Eqs. (7) and (8) will also remain valid above $p_{k-2}^{\min}$. In fact, we can solve for the lowest price point above $p_{k-2}^{\min}$ to which firm $k-2$ will ever reduce its price by solving the equation:

$$n_{k-2}r = n_{k-2}p + [1 - F_k(p)][1 - F_{k-1}(p)]ps \tag{13}$$

The only solution of this equation that is above $p_{k-2}^{\min}$ is r. Hence, given that firm k and firm $k-1$ are already competing for the switcher segment below $p_{k-2}^{\min}$, firm $k-2$ will never price in the interval $[p_{k-2}^{\min}, r]$ but only at r. Moreover, since firms $k-3$ to firm 1 are no different than firm $k-2$ in responding to the price competition between firm k and firm $k-1$, they will also not be competing in any interval but price strictly at r. So, in any given interval, at most two firms will have support. They will be firm k and firm $k-1$, essentially because they are the two firms that can offer the deepest discounts to capture the switcher segment. □

2.2. Model results

Propositions 1–4 demonstrate that, in a market with many asymmetrically positioned retailers, only those with the least to lose from deep price cuts will offer discounts. This severe competition will force the rest of the firms to price at their reservation prices.

This is a striking result indeed. The frequency and depth of discounts firm k and firm $k-1$ offer in order to steal the switcher segment from one and other are so significant that it does not pay for any other firm to even attempt to serve the switcher segment. That is, the price competition between the two firms with the smallest loyal segment sizes is so fierce that even the next firm with the smallest loyal segment size cannot successfully reduce its price to capture the switcher segment profitably.

As we move on considering the firms with larger loyal segments, one by one, all firms lose the competition for the business of the switchers segment against the ongoing competition between the only two firms that can offer the deepest discounts. As a result, only the two firms that have the least to lose by offering the deepest discounts will offer such discounts, forcing the other firms' prices at the reservation price. Therefore, most firms relinquish the price competition to the two firms that can profitably compete with each other in this equilibrium. Solving Eqs. (3) and (4) accordingly, we derive the probability distribution functions representing the equilibrium strategies of the firms.

$$f_i(p) = \begin{cases} 0 & p<r \\ 1 & p=r \qquad i=1,2,\ldots,k-2 \\ 0 & p>r \end{cases}$$

$$f_{k-1}(p) = \begin{cases} 0 & p<p_{k-1}^{\min} \\ \dfrac{rn_{k-1}(n_k+s)}{p^2 s(n_{k-1}+s)} & p_{k-1}^{\min} \le p<r \\ \dfrac{n_{k-1}-n_k}{n_k+s} & p=r \\ 0 & p>r \end{cases}$$

$$f_k(p) = \begin{cases} 0 & p<p_{k-1}^{\min} \\ \dfrac{rn_{k-1}}{p^2 s} & p_{k-1}^{\min} \le p<r\,. \\ 0 & p\ge r \end{cases} \tag{14}$$

Eq. (14) demonstrates the three distinct promotional strategies that firms adopt in a market with many asymmetrically positioned retailers. Firms with large loyal segment sizes never promote in this market.[2] Firm k always discounts and offers the deepest discounts. This firm has the least to lose from promotional price cuts, due to a negligibly small, or perhaps nonexistent, loyal segment size, and so can engage in almost irrational price cuts to serve those informed customers who are willing to search for the lowest available price. In between these two extremes is firm $k-1$, which maintains a regular price, r, with the purpose of maximizing its profits from its loyal customers but also discounts as deep as firm k. Fig. 1 depicts the cumulative distribution functions that represent the mixed strategies firms adopt.

2.3. An extension on the base model

The asymmetric positioning of the firms in our model has so far been a result of their distinct loyal segment sizes. However, another indication of asymmetric positioning may be the provision of different service levels, store ambiance, convenience and warranty levels associated with different retailers (Rajiv et al., 2002; Smith and Brynjolfsson, 2001). The results of such provisions reveal themselves as price premiums that customers are willing to pay to shop at a particular store.

To incorporate this type of asymmetric positioning, we expand our model by allowing the reservation prices of the loyal customers to vary across stores. We represent the

[2] We would like to remind the reader that our model focuses only on regular products with stable reservation prices. We recognize that retailers, small or large, typically have a variety of other reasons for price promotions, including inventory clearance sales and using loss leaders for traffic generation. In addition, there are numerous product categories, such as fashion items, where downward adjustments in reservation prices over time are a routine practice. Our model does not extend to such promotions.

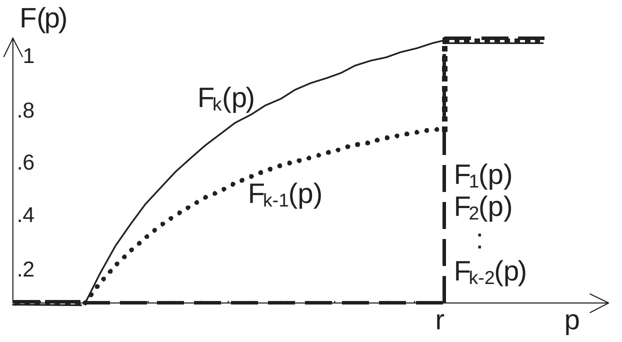

Fig. 1. The cumulative price distribution functions of the firms.

reservation price for firm i as r_i. We do not expect a direct relationship between the loyal segment size and the reservation price of the customers since both of these variables would depend on a multitude of other factors. However, for simplicity purposes, we assume $r_i > r_{i+1}$.[3] We also assume that the reservation price of the switchers is given by $r_s = \min(r_i) = r_k$.

Proposition 5. *Even when the reservation prices are different, at most two firms will offer discounts. These firms are again the firms with the two lowest p^{min} values. However, price dispersion will be greater due to the heterogeneity of the reservation prices.*

Proof. See Appendix A. □

The proof is similar to the combined proofs of Propositions 3 and 4. The equilibrium strategies will follow the distribution functions given by:

$$f_i(p) = \begin{cases} 0 & p < r_i \\ 1 & p = r_i \qquad i = 1, 2, \ldots, k-2 \\ 0 & p > r_i \end{cases}$$

$$f_{k-1}(p) = \begin{cases} 0 & p < p_{k-1}^{\min} \\ \dfrac{rn_{k-1}(n_k + s)}{p^2 s(n_{k-1} + s)} & p_{k-1}^{\min} \le p < r_k \\ 0 & r_k \le p < r_{k-1} \\ \dfrac{n_{k-1} - n_k}{n_k + s} & p = r_{k-1} \\ 0 & p > r_{k-1} \end{cases}$$

$$f_k(p) = \begin{cases} 0 & p < p_{k-1}^{\min} \\ \dfrac{rn_{k-1}}{p^2 s} & p_{k-1}^{\min} \le p < r_k\,. \\ 0 & p \ge r_k \end{cases} \tag{15}$$

[3] This assumption results in a simple model that captures the essence of our modeling effort. Depending on various combinations of the loyal segment sizes and the reservation prices, numerous other equilibria exist that we do not present here.

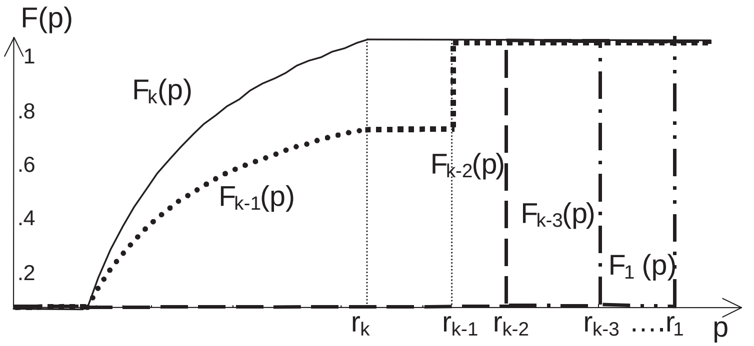

Fig. 2. The cumulative price distribution functions of the firms when reservation prices vary across firms.

The resulting pricing strategies of these firms as represented by their cumulative distribution functions are depicted in Fig. 2. As these distributions show, because of the heterogeneity of the reservation prices, the observed price dispersion will be even greater in this market compared to a market with a homogenous reservation price across customers.

3. Empirical validation

In this section, we use data from the online market for books to test our model predictions. Books are uniquely identified by ISBN (International Standard Book Numbering) numbers, which are widely used and recognized by customers as well as sellers. For a through discussion of the online book market, see Clay et al. (2001). As such, using books identified by ISBN numbers, we were able to make sure that we captured the price competition for identical products across retailers. Our dataset consists of daily prices, and net of shipping and handling for 968 books collected over a period of 6 months (184 days) from 10 online book retailers.

3.1. Model predictions

Our models yield the pricing behavior of firms given asymmetrical loyal segment sizes and varying reservation prices across firms. A retailer's pricing strategy is represented by the empirical distribution of the quoted prices, and hence we attempt to observe if such data could follow from the firms' predicted behavior in testing our model predictions. According to our model results, we can hypothesize that:

H1. The lowest prices quoted by firms with the smallest loyal segment sizes will be less than the lowest prices quoted by the remaining firms.

H2. The average prices of firms with the smallest loyal segment sizes will be less than the average prices of the remaining firms.

H3. The frequency of promotional activity of firms with the smallest loyal segment sizes will be greater than the frequency of promotional activity of the remaining firms.

H4. The depth of promotional activity of firms with the smaller loyal segment sizes will be larger than the depth of promotional activity of the remaining firms.

Since summary statistics, such as the average prices and frequency and depth of discounts, may fall short in completely characterizing price distributions, we also hypothesize that stochastic dominance of price distributions will support our model predictions.

H5. The cumulative distribution of prices for firms with smaller loyal segments will be first order stochastically dominated by the cumulative price distribution of firms with larger loyal segment sizes.

We describe our dataset and define our promotion, price and loyalty measures next.

3.2. Description of the dataset

Our model yields predictions on the firms' promotional behavior based on the size of their relative loyal customer segments, given there is also a switcher segment that is aware of the prices these retailers quote. While one could list all the retailers that are selling the homogenous good to incorporate in such a study, it is also crucial that the switcher segment be aware of all available prices from all these retailers included in the list. Otherwise, the competitive forces that are incorporated in our model would fail to be operational.

To find such a market, in which not only the majority of the retailers selling the homogenous good are identifiable, but also all are included in the comparison set of the switcher segment, we turn to the online market for books. This market has many favorable properties. In addition to enabling the daily collection of price information on many items with little error, the online book market is also served by Internet shopping agents (price comparison engines), tools that price comparing customers use to observe all the retailers and their prices for a particular product.

While hundreds of online booksellers exist, we limit our study to the list of retailers that the most popular price comparison engine identified as carrying a majority of the books included in this study. With this method, we eliminated hundreds of marginal or specialized online booksellers not significant for the study, while we retained the significant few that serve the whole market. We utilized MySimon.com as the price comparison site for this study, since during the course of the data collection period, MySimon was the leading price comparison engine with an 80% market share in price comparison site visits with an estimated 14 million unique visitors in a market with a potential size of 35 million (*Mediametrix* and *Nielsen Netratings*; see also Allen and Wu, 2002; Kocas, 2002).[4]

[4] Allen and Wu (2002) researched the reliability of data collected through "price aggregators" such as MySimon and other price comparison sites, collecting data on 459 different books from eight price aggregators daily for 4 months. They conclude that MySimon covers the market best in terms of cross-sectional consistency as well as longitudinal consistency and report that, if a single price aggregator were to be used, that it ought to be MySimon. Since our research requires the coverage of a switcher segment that is informed about the retailer offerings homogenously across the switchers, we were required to work with a single price aggregator and this study by Allen and Wu (2002) confirms our choice.

We initially collected data on a randomly compiled list of 2207 book titles compiled from various booklists publicly available on the Internet, mainly lists of books libraries procured in 2001.[5] The prices were collected over a period of 6 months, as a typical user of the price comparison site would have observed them. A total of 44 retailers were identified as a result of the price searches conducted for all 2207 books.

We preferred to use the portion of data on books that are *not* on the bestseller list, since books on bestseller lists are more likely to go through reservation price adjustments in a given time window, even over a relatively short period of time. Therefore, bestseller books were removed from the dataset to minimize contamination due to reservation price adjustments. Out of the initial 2207 books, after the elimination of bestsellers and duplicates, we chose the 968 books that were carried by most of the retailers for the majority of this time period. We also reduced the number of the retailers in our analysis by including only those that carried the majority of the 968 books during our data collection period. As a result, our final dataset consisted of the prices of 968 books that were carried by 10 booksellers during a course of 184 consecutive days.

Normalizing the prices collected on each of the 968 books across retailers and aggregating these normalized prices over all books and all dates yield enough data points to reveal the possible pricing behavior of the retailers while minimizing data contamination through reservation price adjustments.[6] Price randomizations as mixed pricing strategies are usually assumed to be executed through time and this behavior is usually observed over multiple products (Narasimhan, 1988; Varian, 1980; Iyer and Pazgal, 2003; Raju et al., 1990). Essentially, we use the distribution of prices, which are aggregated over time and multiple products, as a proxy for the probability measure of a mixed strategy.

3.3. Measuring loyalty

As discussed previously, our model yields predictions on the firms' promotional behavior based on the size of their relative loyal customer segments. We also assume that a better positioning in terms of service levels, convenience and reputation translates into a larger loyal segment size for the online booksellers. This assumption follows from the fact that, given the immaturity of online markets, the reputation, size and service levels of sellers are used as signals of trustworthiness and directly shape the size of the loyal segments. Consequently, the online book market displays characteristics inherently consistent with our model assumptions for both our base model as well as our expanded model. We do not have loyalty data to allow us to sort the firms in our dataset by the size

[5] These sources were four booklists publicly available on the internet: (1) 140 books from *Publisher's Weekly* bestsellers list on 6-4-01; (2) 730 books from "One Book List: Collaborative Books to Read", a list compiled by the Usenet newsgroup *rec.arts.books*; (3) 60 books from Latest Acquisitions by Government Environmental Library 2001; and (4) 1277 books bought by Sidney Sussex University Library in 2001.

[6] We normalize prices by dividing the price offered for any book by the highest price offered for that book by any retailer over the 6-month period. The resulting empirical distributions are presented in Fig. 3. We also include in the Discussion section a normalization by dividing a retailer's price of a book by the highest price that retailer offered for the book over the 6-month period. The empirical distributions based on this normalization are presented in Fig. 4.

Table 1
Book link popularities used as a proxy for the loyal segment sizes for the retailers in the dataset

Retailer	Referred to as	Book link popularity
Amazon	Firm 1	9,744,241
Barnes and Noble	Firm 2	264,873
FatBrain	Firm 3	24,463
1BookStreet	Firm 4	13,150
Varsity	Firm 5	9262
A1Books	Firm 6	8343
eCampus	Firm 7	3982
SamGoodys	Firm 8	3348
AllDirect	Firm 9	762
DoubleDiscount	Firm 10	300

of their loyal segments. Therefore, we use *book-related link popularity* as a proxy measure. Link popularity refers to the number of unique links across the Internet pointing to a site. Book-related link popularity of a retailer, in turn, refers to the number of sites with book-related content that links to a given retailer's website. Link popularity is a quantitative measure of the online awareness for a website and a primary determinant of the traffic that a website generates, and therefore a good indicator of the number of customers the website has.[7] Noting that there are orders of magnitude of difference among the link popularities of retailers in our dataset, we expect that the rank order with respect to loyal segment sizes will be considerably similar to the rank order of book link popularity. Therefore, lacking a better measure of loyal segment sizes for the retailers in our dataset, we use link popularities as a proxy to rank these retailers with respect to their *expected* loyal segment sizes. We label each retailer such that firm 1 is the retailer with the highest link popularity, firm 2 is the retailer with the second-highest link popularity, etc., in accordance with the naming convention we have used in our model. Table 1 provides the link popularities and the labels assigned to the 10 retailers in our dataset.

Next, we present the empirical support and the discussions of the hypotheses.

3.4. Testing the hypothesis

To test 1–4, we compare the firm-specific pricing measures with the industry averages. The pricing measures are the average minimum price, the average price deviation from the ongoing price, the average discount and the number of price changes per book. We use *t*-tests to compare each of these statistics with the industry averages and report the result in Table 2.

Hypothesis 1 is that the average lowest prices (the average deepest discounts) are offered by smaller firms. While the average prices along with their standard deviations give us a sense of relative price levels as demonstrated in Table 2, the minimum prices column also demonstrates that the average minimum prices of firms 7, 8, 9 and 10 are

[7] Reach of a website, defined as the number of users who visit a given site, also provides a proxy measure of loyal customer segment sizes. The reach measure shows correlations close to unity with book link popularity, suggesting that measures of traffic are closely correlated to measures of static links.

Table 2
Average prices and depth and frequency of promotions

Retailer			Price levels		Depth of promotions		Frequency of promotions	
Rank	Name	N	Average minimum price	Average price	Deviation from the ongoing price	Average discount (1 − average price)	Number of price changes per book	Instances highest price quoted (%)
Firm 1	Amazon	951	0.767 (0.123)	0.787 (0.114)	0.035 (0.073)	0.213 (0.114)	0.512 (0.798)	13.13
Firm 2	B&N	967	0.793 (0.127)	0.827 (0.123)	0.045 (0.065)	0.173 (0.123)	1.213 (2.236)	16.94
Firm 3	FatBrain	968	0.798 (0.132)	0.824 (0.128)	0.035 (0.061)	0.176 (0.128)	0.672 (1.513)	18.41
Firm 4	1BookStreet	968	0.840 (0.133)	0.889 (0.115)	**0.072*** (0.079)	0.111 (0.115)	0.994 (0.963)	42.42
Firm 5	VarsityBooks	945	0.862 (0.144)	0.871 (0.141)	0.016 (0.039)	0.129 (0.141)	0.256 (0.501)	44.58
Firm 6	A1Books	968	0.715 (0.116)	**0.722****	0.012 (0.039)	**0.278**** (0.113)	0.235 (0.548)	3.15
Firm 7	eCampus	968	**0.665**** (0.142)	**0.711**** (0.107)	0.032 (0.054)	**0.289**** (0.107)	**4.471**** (6.517)	0.06
Firm 8	SamGoodys	942	**0.547**** (0.230)	**0.713**** (0.124)	**0.171*** (0.199)	**0.287**** (0.124)	1.559 (1.064)	1.22
Firm 9	AllDirect	968	**0.618**** (0.098)	**0.626**** (0.097)	0.013 (0.044)	**0.374**** (0.097)	0.180 (0.414)	0.00
Firm 10	DoubleDiscount	968	**0.545**** (0.145)	**0.636**** (0.125)	**0.052**** (0.027)	**0.364**** (0.125)	**22.306**** (15.226)	0.09

Numbers in parentheses are standard deviations.
Bold numbers are values higher or lower than the industry averages in the predicted directions.
* The difference from the industry average is statistically significant at the 0.01 level (one-sided).
** The difference from the industry average is statistically significant at the 0.001 level (one-sided).

significantly lower than the industry average. This finding supports our claim that firms with smaller loyal segment sizes have less to lose with deeper discounts and can offer deeper discounts than the firms with larger loyal segment sizes.

For the purpose of validating Hypothesis 2, we need to demonstrate that the average prices of the firms with the smallest loyal segment sizes are lower than the industry average. Those averages that are significantly lower than the industry average are depicted in bold in Table 2. As Hypothesis 2 predicts, we see that half of the firms, firms with smaller loyal segment sizes, have lower average prices than the industry average.

Hypothesis 3 asserts that the frequency of promotional activity of firms with the smallest loyal segment sizes will be greater than the frequency of promotional activity of the remaining firms. We operationalize frequency of promotions as the number of changes in the prices quoted for a book. Price changes are defined as differences in listed prices from one date to the next date in the same store. Dates with missing prices are not considered as price changes. Due to the nature of our data, any same-day price changes are not accounted for.

As presented in Table 2, the results of the one sample t-tests show that two firms, firm 7 and firm 10, have changed the prices of their offerings more frequently than the average frequency across all the retailers over the course of 6 months. These differences are both significant at the 0.001 level. Similarly, all other eight firms have changed their prices less frequently than the industry average.

We also tally the instances in which each firm quoted the maximum price in any day for any book. As Table 2 shows, smaller firms had significantly less instances in which they quoted the maximum price. In fact, firms 7, 9 and 10 each quote the highest price for a book in less than 1% of the instances.

These results provide empirical support for our hypothesis that the frequency of promotional activity of a few firms with the smaller loyal segment sizes are greater than the frequency of promotional activity of the remaining firms.

To empirically validate Hypothesis 4, we need to establish what constitutes a promotion. Our dataset shows that the most frequently quoted price of any book by a firm is observed 99% of the time for almost all books and for all firms but firms 7 and 10. However, this most frequently quoted price of any book by a firm is different for all firms. That is, the dataset indicates the existence of a different reservation price for each firm. In fact, this finding supports the structure of our model extension, in which we assume that each firm has a different reservation price. However, with this assumption, a promotion would be defined as a reduction in price from this firm-specific reservation price. We refer to this definition of a promotion as a *deviation from the ongoing price* (first column under depth of promotions).

On the other hand, if we assume that all customers have the same reservation price, we see that a price promotion is defined as the deviation of the price from the maximum quoted price, which is almost always the book's MSRP (Manufacturer's Suggested Retail Price). Since this is the view taken by our base model, we refer to this definition of a promotion as the *discount on price* for that book.

Also note that Hypothesis 4 applies to both of these definitions of a price promotion as stated, that is, we expect the average discount (deviation from the overall maximum) as well as the deviation from the ongoing price, to be deeper for smaller firms. In Table 2, we present the averages for both of these definitions of a price promotion; *deviation from the ongoing price* is presented in the first column under depth of promotions and *average discount on price* is presented in the second column. We observe from Table 2 that the *average discounts on price* are significantly larger than the industry average for the firms with smaller loyal segment sizes, while these discounts are significantly smaller than the industry average for the firms with larger loyal segment sizes. As we calculate the changes in normalized prices on a day-by-day basis, we observe the *deviations from the ongoing prices*. That is, if we assume that each firm has a different reservation price for a book, we observe the price promotions from this firm-specific reservation price. The result of this analysis is also presented in Table 2. As presented, firms 4, 8 and 10 have significantly deeper deviations from their firm-specific reservation prices than the industry average, while all other firms have significantly less depth in their deviations than the industry average.

These results provide empirical support for our hypothesis that the depth of promotional activity of a few firms with the smaller loyal segment sizes are greater than the depth of promotional activity of the remaining firms.

3.5. Stochastic dominance

Summary statistics, such as the mean and the standard deviations as we have used above, may fall short in completely characterizing price distributions. To validate the effect that the ranking of firms have on the level of promotional activity they engage in, we borrow from the stochastic dominance literature. Stochastic dominance is fundamentally used in comparisons of distributions to observe if one distribution offers generally lower or higher values than the other. Stochastic dominance has been used in evaluating the distribution of security returns (Hadar and Russell, 1969), in ranking income distributions (Saposnik, 1981, 1983) and in ranking nutrition distributions of individuals (Kakwani, 1989). We use stochastic dominance to evaluate the relationship between rankings of firms with respect to their loyal segment sizes and the promotional activities in which they engage.

To see the overall relative distribution of prices, we use the most stringent dominance criterion, *first order stochastic dominance* (Hadar and Russell, 1969; Saposnik, 1981). For any two price distributions $f_1(p)$ and $f_2(p)$ with cumulative distribution functions $F_1(p)$ and $F_2(p)$, if $F_1(p)$ lies nowhere above and at least somewhere below $F_2(p)$, that is $F_1(p) \leq F_2(p)$ for all p, then distribution $f_1(p)$ is said to display first order stochastic dominance over distribution $f_2(p)$. Hence, in distribution $f_1(p)$, there are no more observed prices, which are less than a given level than in distribution $f_2(p)$, for all levels of prices.

To test Hypothesis 5, we compare the empirical cumulative price distribution functions of the retailers used in our study. Hypothesis 5 asserts that price distributions of firms with smaller loyal segments will be first order stochastically dominated by the price distribution of firms with larger loyal segment sizes. Therefore, we expect to see that the cumulative distribution functions of smaller firms lie above the distribution functions of larger firms in our comparison. To this end, we present the cumulative distribution functions of the normalized prices for the 10 firms in Fig. 3.

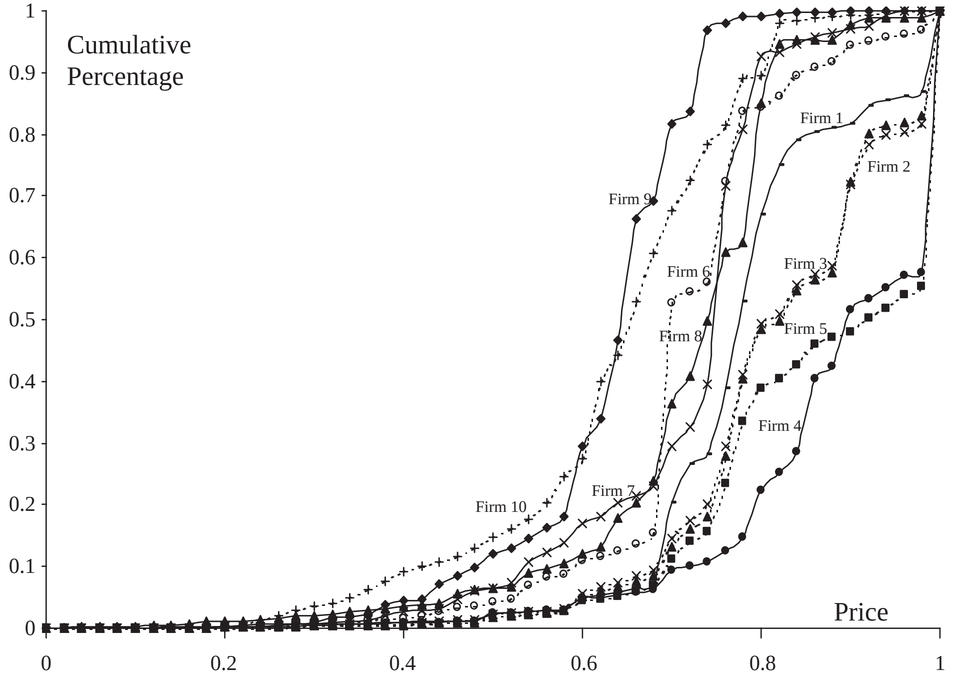

Fig. 3. The comparison of cumulative normalized price distribution functions of retailers.

These distribution functions are formed across all books for each retailer. To observe promotional activity on a book for a given retailer, the maximum price ever quoted by any retailer for that particular book is used for normalization and these normalized prices are aggregated across all books separately for each retailer. Hence, each cumulative distribution function depicted in Fig. 3 is made up of at least 116,000 observations on prices from a particular retailer.

Fig. 3 provides empirical support for the predictions of Hypothesis 5 that cumulative price distributions of firms with smaller loyal segments stay above the cumulative price distributions of firms with larger loyal segments. Specifically, firm 10 and firm 9 have distribution functions, which are strictly first order dominated by all firms but firm 7 and firm 8.[8] The chi-square statistic of 4.00 shows the prediction that the two smallest firms being strictly dominated by all other 8 firms produces significantly better results ($p<0.05$) than by chance alone.[9] These findings provide overall empirical support to our models, showing that the promotional activity engaged in by smaller firms is more extensive then the promotional activities of the larger firms.

[8] The deficit functions (the integral of the cumulative distribution functions) of firms 10 and 9 lie everywhere above the deficit functions of all other firms, hence showing that firm 10 and firm 9 are second order stochastically dominated by all the remaining firms.

[9] Moreover, as a weaker comparison of small versus large firms, firms 10, 9, 8, 7 and 6 are strictly first order dominated by firms 5, 4, 3, 2 and 1.

4. Discussion

Our objective in building and testing a model of price competition in an oligopolistic market with asymmetrically positioned firms was to provide theoretical and empirical insight into markets in which price comparison shopping was possible. From a theoretical point of view, our model demonstrates the existence of at least three distinct pricing strategies, indicating the trade-offs different firms make to balance the desire to serve the price comparing switcher segment and the desire to maximize the profit from their loyal segments. Moreover, the three distinct strategies also provide a theoretical explanation to the observed price dispersions in the homogenous goods markets.

One theoretical finding that we would like to discuss further relates to the abandonment of price promotions by the firms with larger loyal segments. As both our base model and the model extension demonstrates, all but the two firms with the smallest loyal segment sizes relinquish price promotions to the two firms with the least to lose from the deepest price promotions. In order to observe if this is really the case, we present the cumulative price distributions of all firms, where prices are normalized with respect to the maximum price quoted per book by the retailer itself, a distribution which demonstrates just *deviations from the ongoing price*. Hence, if a retailer has a price point at which it always prices, that is lower than the maximum price quoted by some other retailer, that price would be normalized to be 1 with this approach. Therefore, such a normalization would reveal whether the majority of the firms relinquish competition to a few firms with the deepest and most frequent discounts. We present these cumulative price distributions of firms in Fig. 4.

As Fig. 4 demonstrates, all firms but firms 10, 8, 7 and 4 have almost no promotional activity in terms of price cuts from their firm- and book-specific reservation prices.[10] This observation supports our model prediction that the majority of firms, especially those with the larger loyal segment sizes, relinquish competition to the few firms with smaller loyal segments and engage in almost no promotional activity.

Moreover, we also see that it is only firm 10 that has almost no mass point at the reservation price, that is, firm 10 almost always offers price cuts. This behavior is also in line with our model expectation that the firm with the smallest loyal segment size has no mass point and always discounts. Meanwhile, we also observe that firms 8, 7 and 4 all have varying levels of deep discounts and mass points at their specific reservation prices.[11] To summarize, Fig. 4 exhibits pricing behavior very similar to that predicted by our model: (i) the firm with the smallest loyal segment has all discounted prices; (ii) there are a few firms with relatively small loyal segment sizes with discounted as well as non-discounted

[10] While firms 9 ,6, 5, 3, 2 and 1 seem to have at least some price randomization, we expect the true degree of price randomization to be less than this observed level. The level of price variation is probably due to the inclusion of systematic price shocks in our dataset that spans six months. A similar analysis with just 1 month of the dataset reveals the amount of price promotions offered by these firms is much less than that observed from Fig. 4, evidenced by almost zero price variations by these firms.

[11] While our model expects only one firm to engage in such an activity, it is not unreasonable to think that similar firms may form a pooling equilibria in which they collectively engage in a strategy, a la Varian (1980).

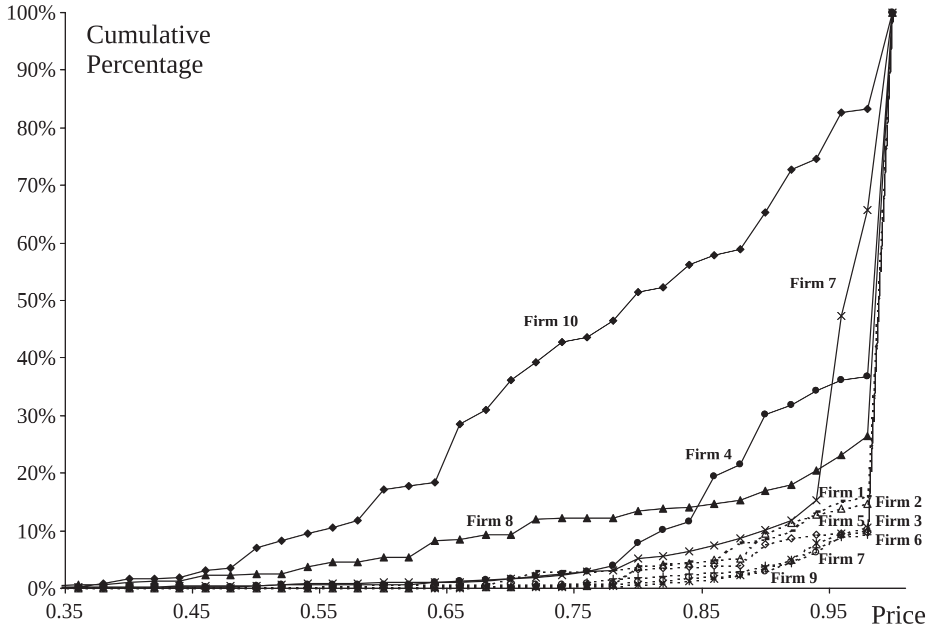

Fig. 4. Cumulative distribution of prices where each book price for a retailer is normalized with respect to the highest price quoted by that retailer for that book.

prices; and (iii) the remaining firms in the market price at their firm- and book-specific reservation prices, having relinquished price promotion to the firms with smaller loyal segments.

5. Conclusion

In this paper, we analyze price competition in markets where customers have the chance to choose one retailer among many, such as the online markets for commodities. Our model findings provide an operational explanation to the previous empirical evidence and also lead to hypotheses that we empirically validate using data from the online market for books.

Specifically, our model demonstrates that, among the many firms competing to sell a homogenous product, a small number of firms with the least to lose will engage in price promotions more actively than the remaining firms. The rest of the firms, with larger loyal segments, will choose to price at higher and rather stable prices, where these prices are determined by the reservation prices of their loyal customer segment based on the positioning of the store.

Compared to symmetric oligopolies and asymmetric duopolies modeled in the literature, our model is unique because it deals with an asymmetric oligopoly. Moreover, compared to the literature, our paper is the first to consider multiple sources of asymmetry. Our model extension provides an explanation to the observed price dispersion that has been reported especially for online markets based on the two sources of asymmetry. While asymmetry in terms of loyal segment sizes determines whether a firm will price promote, asymmetry in terms of positioning of the brand determines the level of price premium that will correspond to the level of services and trustworthiness offered by the firm. Some key findings of our analysis can be summarized as follows.

Asymmetry, not only in terms of different loyal segment sizes, but also in terms of the positioning of firms, leads to a multiplicity of strategies.

The firms with the smallest loyal segments are responsible for almost all price promotions, while the firms with larger loyal segment sizes do not price promote at all.

Firms that differentiate themselves with the level of services that they offer and charge a premium for that differentiated positioning can coexist profitably with heavy promoters in markets where information on prices is widely available and used.

The strategy adapted by the majority of firms that dictates the abandonment of price promotions when smaller firms compete fiercely for switchers indicate the importance of strategies to build loyalty in markets with low search costs, such as those mediated by Internet shopping agents.

We consider this paper a unifying extension to models of price comparison by Varian (1980) and Narasimhan (1988) and related empirical research (Clay et al., 2001; Smith and Brynjolfsson, 2001; Clemons et al., 2002). However, we also recognize limitations of the theoretical and empirical components of our research. For example, we intentionally omit traffic-building concerns in our modeling effort and high-traffic items in our dataset. We hope to include traffic issues in our future research. Also, having access to just some proxy measure of overall loyal customer sizes, we could only distinguish large firms from smaller firms. With detailed data on the loyal segment sizes, future research could scrutinize the drivers of price promotions in further detail.

Appendix A

Proof of Proposition 1. Let us denote the jth lowest price as p^j. Note though, some of these quoted prices can be the same, and hence $p^1 \leq p^2 \leq p^3 \leq \ldots \leq p^k$. Now, assume that there is a strategy set, which is a pure Nash equilibrium. There are two possibilities: either $p^1 = p^2$, that is, the two lowest prices quoted are the same, or $p^1 < p^2$, they are different. Note that, if $p^1 = p^2 = p^3$, the case is covered by the first possibility or, if $p^1 < p^2 = p^3$ or $p^1 < p^2 < p^3$, the case is covered by the second possibility. Let us assume the first possibility that the two prices are the same. Let us denote the two firms with these prices as firm z and firm v so that $p_z = p^1 = p_v = p^2$. Since this unique price point is higher than either $p_v^{\min}$ or $p_z^{\min}$, either firm z or firm v can undercut the price of its rival with an infinitesimal reduction in price and capture the entire switcher segment and increase its profits, Hence, there cannot be a pure strategy Nash equilibrium where $p^1 = p^2$.

Now assume the second possibility, that $p^1 < p^2$. Let us again denote the two firms with these two lowest prices as firm z and firm v so that $p_z = p^1 < p_v = p^2$. If $p_z = p^1 < p_v = p^2 < r$, then firm v can increase its price to r and profit from this move, because it sells to its loyal segment at a higher price. On the other hand, if $p_z = p^1 < p_v = p^2 = r$, then firm z can increase its price while still serving the switchers and profit from the sale to both segments at a high price. Hence, there cannot be a pure strategy Nash equilibrium where $p^1 < p^2$. □

Proof of Proposition 2. To see that the equilibrium profits of these firms will be equal to their minmax profits, note that all k firms have a loyal segment that will buy from them regardless of the price as long as the price they quote is lower than or equal to the reservation price. Hence, all firms can guarantee the profit $n_i r$ by choosing to price at r. However, in terms of undercutting all other firms and serving the switcher segment, only one firm has an absolute advantage: firm k. Therefore, only firm k can improve its minmax profit above $n_k r$, by pricing at the minimum price any other firm can ever feasibly pull its price back, $p_{k-1}^{\min}$, which results in a minmax profit of $(n_i+s)p_{k-1}^{\min}$ for firm k. Also note that $(n_i+s)p_{k-1}^{\min} > n_k r$, that is, firm k increases its profit by lowering its price to $p_{k-1}^{\min}$ because by definition $(n_i+s)p_k^{\min} = n_k r$ and $p_{k-1}^{\min} > p_k^{\min}$, which implies that (n_i+s) $p_{k-1}^{\min} > n_k r$. Therefore, $k-1$ firms with the largest loyal segment sizes will have the reservation price within their supports in equilibrium and firm k will have the point $p_{k-1}^{\min}$ in its support. Moreover, since playing the minmax response is within the mixed strategy set of each firm, the equilibrium profits of each firm must equal their reservation utilities.[12] □

Proof of Proposition 5. The proof is similar to the combined proofs of Propositions 3 and 4. Given $n_i > \mathrm{n}_{i+1}$ and $r_i > r_{i+1}$, remembering that $p_i^{\min} = n_i r_i / (n_i + s)$, these two firms with the lowest $p^{\min}$ values will once more be firm k and firm $k-1$. That is, $p_k^{\min} < p_{k-1}^{\min} < p_{-\mathrm{i}}^{\min}$. Since firm k and firm $k-1$ again have the absolute advantage of pricing in the left-most interval, they will compete in this interval with cumulative distribution functions identical to those derived in the Proof of Proposition 3.

Moreover, moving up from point $p_{k-2}^{\min}$, the increasing cumulative function values will be deterrent to other firms. So, even when the reservation prices vary for firms, only firm k and firm $k-1$ have support over any interval. Given the competition between the two firms that can pull their prices lowest, it does not pay for any other firm to compete with them. Only firm k and firm $k-1$ will randomize their prices while other firms will prefer to remain at their respective reservation prices and serve their loyal segments. Additionally, since $r_k < r_{k-1}$, there will be no support for any firm in the interval $[r_k, r_{k-1}]$ and firm $k-1$ will have its mass, which equals to $\frac{n_{k-1} - n_k}{n_k + s}$ at its reservation price, r_{k-1}. Moreover, the remaining firms will not share the same regular price at which they always price. Each firm will have a separate regular price, which is equal to its reservation price. □

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[12] However, also note that, although the pure strategy price vector, $(r, r, r, \ldots, r, p_{k-1}^{\min})$ may correspond to the minmax profit vector, $(n_1 r, n_2 r, n_3 r, \ldots, n_{k-1} r, (n_k+s)p_{k-1}^{\min})$, this pure strategy is not a Nash equilibrium. Firm k can increase its profit by increasing its price unilaterally.

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