Optimal Skill Distribution in Mirrleesian Taxation

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OPTIMAL SKILL DISTRIBUTION IN MIRRLEESIAN TAXATION

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Abstract

The motivation of our study is how to redistribute income earning skills in a heterogeneous society to reach the social optimum. Leung and Yazıcı (2010) write the first paper that analyzes this issue analytically. In light of their study, we analyze the optimum skill distribution with utilitarian and egalitarian social welfare functions and conduct two analyses. Firstly, we provide numerical simulations to measure the welfare effects of skill distribution choice under different social welfare functions. Secondly, we characterize the optimum skill distribution for different objective welfare functions with different assumptions. Our first result indicates that, it is always optimal to distribute all skills to one type in a society regardless of whether we use egalitarian or utilitarian objective social welfare functions. Secondly, an increase in welfare from Mirrleesian taxation without skill distribution to Mirrleesian taxation with skill distribution is always much more than an increase from laissez faire market to Mirrleesian taxation without skill distribution in both utilitarian and egalitarian problems. Our final result is that the economy with perfectly unequal skill distribution provides a more egalitarian society in terms of how utilities are distributed across agents, in both utilitarian and egalitarian problems.

Keywords: Skill distribution, utilitarian social welfare function, egalitarian social welfare function, Mirrleesian taxation, redistribution.

MİRRLEES YAKLAŞIMI İLE VERGİLENDİRMEDE OPTİMAL YETENEK DAĞILIMI

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Özet

Bizim çalışmamızın motivasyonu farklı yapıda bireyleri olan bir toplumda gelir elde etmemizi sağlayan yetenekleri nasıl dağıtmalıyız ki toplum refahı optimal olsun. Leung ve Yazıcı (2010) bu konuyu analitik olarak analiz yapan ilk çalışmadır. Bu çalışmanın ışığında, optimal yetenek dağıtımını faydacı ve eşitlikçi toplumsal refah fonksiyonlarıyla ayrı ayrı inceledik ve 2 tür analiz yaptık. Öncelikle, farklı toplumsal refah fonksiyonları altında yetenek dağıtım seçimlerinin refaha olan etkilerini ölçen bir nümerik analiz sunduk. İkinci olarak, optimal yetenek dağıtımını farklı refah fonksiyonları için, farklı varsayımlarla karakterize ettik. İlk sonucumuz şunu gösteriyor ki, kullanılan refah fonksiyonun faydacı ya da eşitlikçi olmasından bağımsız olarak, bütün yeteneği toplumda sadece bir tipe dağıtmak her zaman optimaldir. İkinci sonucumuz ise, yetenek dağıtımı yapılmayan Mirrlees yaklaşımı ile vergilendirmeden, yetenek dağıtımı yapan Mirrlees yakalaşımı ile vergilendirmeye geçildiğinde elde edilen refah artışının, laissez faire piyasasından yetenek dağıtımı yapılmayan Mirrlees yaklaşımı ile vergilendirmeye geçildiğinde elde edilen refah artışından hem faydacı hem de eşitlikçi problemlerde her zaman çok daha fazla olmasıdır. Son sonucumuz ise yine hem faydacı hem de eşitlikçi problemlerde, tamamen eşit olamayan bir şekilde yapılan yetenek dağıtımının tipler arasındaki hazların dağılımı açısından daha eşitlikçi bir toplum yapısı sağlıyor olmasıdır.

Anahtar Kelimeler: Yetenek dağıtımı, faydacı toplumsal refah fonksiyonu, eşitlikçi toplumsal refah fonksiyonu, Mirrlees yaklaşımı ile vergilendirme, yeniden dağıtım.

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1 Introduction

A fundamental question in public economics is how to redistribute resources among people. Since people are heterogeneous in their skill levels and their skill levels are private information (as in Mirrlees 1971), people with higher skill levels may find it optimum to mimic lower skilled people unless their hard work is rewarded. This situation is the main problem associated with private information skill levels.

The redistribution among agents is done ex-post by transferring consumption goods across the types in Mirrlees (1971). Leung and Yazıcı (2010) introduce a new channel of redistribution of resources to the Mirrleesian model. There, they redistribute resources via skill distribution ex-ante. A more egalitarian ex-ante skill distribution is equal to transferring skills from high skilled agents to those with lower skills, which implies a more equal consumption distribution with the same amount of ex-post redistribution. They ask how much ex-ante skill distribution is optimal and analyze this question in a static Mirrleesian economy with two types of agents whose fractions in a society is given. The main result of their paper is that it is always optimal to redistribute all skills to only one type, namely to the high skilled type.

To understand the question and the result, it will be helpful to give a real life example, one can think of skill as a type of production capacity that can be taught and learned in schools. According to this interpretation, society has a fixed amount of education force that it can employ to teach different groups of people. By choosing its education policy, society is essentially choosing the skill distribution. With this interpretation, our result says that it is optimal to channel all the education towards one group. It is important to note that this is just an example and one can think of different examples.

In the environment in which Leung and Yazıcı (2010) made their analysis, the utilities the agents receive from their consumption is strictly concave and the disutility they receive from their labor effort is weakly convex. Our contribution to their paper is twofold. First, we prove that in the case of linear utilities and utilitarian social welfare, the result of Leung and Yazıcı (2010) is still true. Therefore, we provide an extension of their theoretical result.

Second, even though Leung and Yazıcı (2010) show that perfectly unequal skill distribution is optimal, they do not provide any numerical results as to the importance of such policy. We provide numerical simulations to measure the welfare effect of the perfectly unequal skill distribution policy. We perform this numerical analysis not only for utilitarian social objective but also egalitarian social objective. Our main numerical finding is that an increase in the social welfare from Mirrleesian taxation without skill distribution to Mirrleesian taxation with skill distribution is much more than an increase from laissez faire market to Mirrleesian taxation without skill distribution. Furthermore, an economy with perfectly unequal skill distribution provides a more egalitarian society in terms of how utilities are distributed across agents. These numerical findings indicates that public policies regarding skill distribution choice can be quite important for social welfare.

It is also important that in our numerical simulations we find that under egalitarian social objective optimal skill distribution is the perfectly unequal one. This result is not a proof, but it suggests that the result of Leung and Yazıcı (2010) may be true under more general social welfare functions than utilitarian form.

Finally, this analysis points out that if governments decide to perform a skill distribution policy and distribute skills to only one type of people, then they need to be careful about income taxation redistribution. If governments skip income taxation, this results in an economy in which one type has no income as a significant levels of inequality. This new situation would be worse in terms of equality compared to a laissez faire market.

In our analysis there is a tradeoff between production efficiency and the distribution of consumption. By giving all skills to one type of people, total production increases without increasing the total labor level. In other words, by giving all of the skills to only one type, the same level of production can be acquired with less labor force. If we consider only the production efficiency, then our solution must be at the corner but there is also a distribution aspect and giving all the skills to only one type and naturally making only this type to work increases the utility of pretending the high type as a low skill type. In our analysis, even if this tradeoff is seen in an egalitarian problem type, the effect of productive efficiency is more than the effect of distribution of consumption and as a result the optimal solution is attained at the corner. In the utilitarian problem type, since our assumption on utility function is linear, only the effect of productive efficiency is seen and as expected, all of the skills are assigned to the high skill type.

Having completed our analysis with two types of agents, we generalize our analysis to an arbitrary number of agent types. For this case, we only conduct numerical analysis. With an arbitrary number of types, we always find it optimal to distribute all of the skills to the highest skilled type. By this we show that optimality of perfectly unequal skill distribution is robust to the number of types. Then, we compare the numerical results we obtained for a laissez-faire market, Mirrleesian economy with ex-post consumption redistribution and Mirrleesian economy with both ex-post consumption redistribution and ex-ante skill distribution both for utilitarian and egalitarian problems.

The literature consists of a number of works that follow Mirrlees (1971). These works can be categorized into four main groups. The first group is concentrated in quantitative study. Emmanuel Saez (2001) and Tuomala (1990) can be seen as the most important members of this group. In Emmanuel Saez

(2001), he tries to show that there is a connection between tax methods and earnings. He shows that new results for optimal income taxation can be attained by deriving the optimal income tax rates using elasticity straightly. By this method, he shows how different economic effects become more effective and the significant effects among them in the Mirrleesian optimal income taxation. He represents the optimal nonlinear tax rate formulas as elasticity and the form of the income distribution. Then, the numerical applications of these formulas are obtained.

The second group is concentrated on skill distribution. One of the study is belonging to this group is Hamilton and Pestieau (2005). This study analyzes the effects of changing fractions of types to the individual utilities by using maxmax and maxmin forms of welfare functions. The other member of this group is the Brett and Weymark (2008) who analyze the effect of different skill levels on the social welfare.

In addition, Golosov and Tsyvinski (2006) and Kocherlakota (2005) are the pioneers of the third group that deals with dynamic models. In Kocherlakota (2005), a dynamic economy is considered. Agents' skill levels are private information and change stochastically over time without any restriction. With these assumptions, tax systems that carry out a symmetric constrained Pareto optimal allocation are introduced. As a result, he obtain that wealth taxes in a period depend on the individual's labor income in that period and the former ones. Nevertheless, in any period, there is an expectation that an agent's wealth tax rate in the next period is zero. Besides that, government does not accumulate any net revenue from wealth taxes. In Golosov and Tsyvinski (2006), they introduce a new way of designing a disability insurance system optimally. Their main assumptions are imperfectly observable disability and a dynamic environment. What they do is characterize the social optimum numerically and theoretically. They introduce a tax system that achieves an optimal allocation as a competitive equilibrium. As a result, they suggest that their optimal disability system yields significantly more welfare compared to the existing system.

The final group can be categorized as the relevant to optimal education policies and taxation. In this field De Fraja (2002) has a significant contribution. In his paper, optimal education policy is studied. In the model there are some assumptions such as utilitarian government, different income level of households and different ability level of their children. It is also accepted that private education can be used by households without borrowing to finance it and income taxes can be used by government as a funding of education. In the education policy that they introduced, as a result of this study, the spread among the educational success of the bright and the less bright ones are increased as compared with private provision. In addition, the education obtained by less bright children increases as their parents' income increases. Finally, in their model households with lower income and less bright children contributes more to the education cost fees compared with the ones with more income and brighter children. Besides De Fraja (2002), Hare and Ulph (1979), Bovenberg (2004) and Maldonado (2008) have also contributions in optimal education policies and taxation.

The organization of the paper is as follows. In Section 2, we introduce our model. In Section 3, we characterize the solutions for the utilitarian problem. In Section 4, we characterize the solutions for the egalitarian problem. In Section 5, we compare the optimal values of the objective social welfare functions for the three environments discussed in the preceding paragraph and finally section 6 is the conclusion.

2 Model

There is a unit measure of agents and they produce output individually according to the production function

$$y = wl$$

where y denotes output, w denotes skill level, and l denotes labor effort. Each agent's preferences are given by

$$c - v(l)$$

where c is consumption and v satisfy $v'', v' > 0$.

Following Leung and Yazici (2010) we allow society to choose the distribution of skill. There are n groups of people and all the agents in one group are the same type and also have the same skill level. Each group is represented by index i. For example $i = 1$ represents the first group and also type 1. The measure of type i is p_i where p_i's are exogenous. Since there are n types, there are n skill levels to be distributed, $w_1, w_2, ..., w_n$ where $w_1 \leq w_2 \leq ... \leq w_n$. There are α total of units of skills to be distributed where α is exogenous. Therefore society chooses each w_i subject to

$$\sum_i p_i w_i \leq \alpha$$

and

$$w_i \geq 0 \text{ for all } i$$

The first inequality guarantees that the total amount of skills distributed cannot be any larger than α, and the second inequality states that each skill level must be nonnegative.

An *allocation* in this economy is defined as (w_i, c_i, l_i) where c_i and l_i represent consumption and labor allocation of each type $i = 1, 2, ..., n$ respectively.

An allocation is said to be *socially feasible* if

$$\sum_i p_i c_i \leq \sum_i p_i w_i l_i \tag{1}$$

$$\sum_i p_i w_i \leq \alpha \tag{2}$$

$$w_i, l_i, c_i \geq 0 \text{ for all } i \tag{3}$$

The first inequality says that the total amount of consumption cannot exceed the total amount of production. The second inequality says that the total amount of skills distributed cannot be greater than the total available skill level, α. The third inequality says that an allocation must be nonnegative for each i.

The timing of the events is the same as Leung and Yazıcı (2010) and as follows. First, the society chooses the skill distribution. This information is public. Then, each agent privately draws her skill from this distribution. Finally, society chooses the consumption and labor allocation and agents announce their types and receive the corresponding allocation. This informational friction requires the allocation to satisfy the following familiar *incentive compatibility* conditions:

$$c_i - v(l_i) \geq c_j - v(\frac{w_j l_j}{w_i}) \text{ for all } i, j. \tag{4}$$

A social planner chooses the level of consumption, labor and skill distribution to maximize total welfare subject to social feasibility and incentive compatibility constraints.

An allocation is *utilitarian optimal* if it solves

$$\max_{w_i, c_i, l_i} \sum_i p_i (c_i - v(l_i))$$

st. (1),(2), (3) and (4).

An allocation is *egalitarian optimal* if it solves

$$\max_{w_i,c_i,l_i} \sum_i c_1 - v(l_1)$$

st. (1),(2), (3) and (4).

The main issue in this paper is the optimal skill distributions for both egalitarian and utilitarian objective functions. Therefore, we focus on w_i in both of the problems. To understand the question better, it is helpful to consider the set of distributions that are available to the society for $n = 2$. On the one extreme, we can set $w_1 = 0$ and $w_2 = \alpha/p_2$ or $w_1 = a/p_1$ and $w_2 = 0$. That is to say, one extreme is the perfectly unequal skill distribution. On the other extreme, we can set $w_1 = w_2 = \alpha$. That is to say, the other extreme is the perfectly equal skill distribution, which makes everyone identical. In between, there is a whole range of skill distributions in which both w_1, $w_2 > 0$

We know that for each $i > j$ it is impossible for type j to "mimic" type i. One can see the proof in Leung and Yazıcı (2010). Therefore we can re-write the incentive compatibility constraints as

$$c_i - v(l_i) \geq c_j - v(\frac{w_j l_j}{w_i}) \text{ for all } i, j \text{ such that } i > j.$$

With the following lemma we can simplify the incentive compatibility constraints further.

Lemma 1 *Let (w_i^*, c_i^*, l_i^*) be an egalitarian optimal allocation. Then with this allocation for all $i = 2, .., n$ if $w_i \neq w_{i-1}$, the incentive compatibility constraint between type i and $i-1$ binds and the incentive compatibility constraint between type i and each type $1 \leq j \leq i-2$ do not bind.*

Proof. Let i be given and suppose for a contradiction that the incentive compatibility constraint between type i and $i-1$ do not bind. Then we have

$$c_i^* - v(l_i^*) > c_{i-1}^* - v(\frac{w_{i-1}^* l_{i-1}^*}{w_i^*})$$

Let $c_i' = c_i^* - \epsilon$ and for all $1 \leq j \leq i-1$ let $c_j' = c_j^* + \delta_j$ for some $\epsilon > 0$ and $\delta_j > 0$ so that the social feasibility constraint and the incentive constraint between type i and $i-1$ are still satisfied. Moreover with c_1' egalitarian objective function improves. That contradicts with c^* being optimal.

Now we will show that the incentive compatibility constraints between types i and each type $1 \leq j \leq i-2$ do not bind.

$$\begin{aligned} c_i^* - v(l_i^*) &= c_{i-1}^* - v(\frac{w_{i-1}^* l_{i-1}^*}{w_i^*}) \\ &> c_{i-1}^* - v(l_{i-1}^*) \\ &= c_{i-2}^* - v(\frac{w_{i-2}^* l_{i-2}^*}{w_{i-1}^*}) \end{aligned}$$

Then

$$c_i^* - v(l_i^*) > c_{i-2}^* - v(\frac{w_{i-2}^* l_{i-2}^*}{w_{i-1}^*})$$

Since the incentive compatibility constraint between type i and $i-2$ does not bind, the incentive compatibility constraints between type i and each type $j \leq i-3$ does not bind either. ■

Due to Lemma 1, we can rewrite the incentive compatibility constraints as follows:

$$c_i^* - v(l_i^*) = c_{i-1}^* - v(\frac{w_{i-1}^* l_{i-1}^*}{w_i^*}) \text{ for all } i = 2, .., n$$

Let $\theta_i = \dfrac{w_i}{w_{i+1}}$ for all $i = 1, 2, ..., n-1$. Then we can re-write the skill constraint as

$$p_n w_n + p_{n-1} w_n \theta_{n-1} + p_{n-2} w_n \theta_{n-1} \theta_{n-2} + ... + p_1 w_n \theta_{n-1} \theta_{n-2} ... \theta_1 = \alpha$$

Then

$$
\begin{aligned}
w_n &= \frac{\alpha}{p_n + p_{n-1}\theta_{n-1} + p_{n-2}\theta_{n-1}\theta_{n-2} + ... + p_1\theta_{n-1}\theta_{n-2}...\theta_1} \\
w_{n-1} &= \frac{\alpha\theta_{n-1}}{p_n + p_{n-1}\theta_{n-1} + p_{n-2}\theta_{n-1}\theta_{n-2} + ... + p_1\theta_{n-1}\theta_{n-2}...\theta_1}
\end{aligned}
$$

Or more compactly we have;

$$
w_i = \frac{\alpha \prod_i^{n-1} \theta_i}{\sum_{k=1}^n p_k \prod_{j=k}^{n-1} \theta_j} \text{ for all } i = 1, 2, ...n
$$

Hence the skill constraint becomes,

$$
\sum_i p_i \frac{\alpha \prod_i^{n-1} \theta_i}{\sum_{k=1}^n p_k \prod_{j=k}^{n-1} \theta_j} = \alpha
$$

Note that at one extreme, as θ_i tends to 1, we have $w_i = \alpha$ for all types $i = 1, 2, ...n$. That is, we are at perfectly equal skill distribution. At the other extreme, as θ_i tends to 0 we have $w_i = 0$ for all types $i = 1, 2, ...n-1$ and $w_n = \frac{\alpha}{p_n}$. That is to say, we have all the available skill level α given to one type.

Using the notation of θ_i we can re-write the utilitarian social planner's problem as

$$
\max_{\theta_i, c_i, l_i} \sum_i p_i (c_i - v(l_i))
$$

st.

$$
\begin{aligned}
\sum_i p_i c_i &\leq \sum_i p_i \frac{\alpha \prod_i^{n-1} \theta_i}{\sum_{k=1}^n p_k \prod_{j=k}^{n-1} \theta_j} l_i \\
c_i - v(l_i) &\geq c_j - v(\theta_j l_j) \text{ for all } i, j \text{ such that } i > j \\
\theta_i &\in [0,1] \text{ for all } i = 1, 2, ...n-1 \\
c_i, l_i &\geq 0 \text{ for all } i = 1, 2, ...n
\end{aligned}
$$

Similarly we can re-write the egalitarian social planner's problem as,

$$
\max_{\theta_i, c_i, l_i} c_1 - v(l_1)
$$

st.

$$
\begin{aligned}
\sum_i p_i c_i &\leq \sum_i p_i \frac{\alpha \prod_i^{n-1} \theta_i}{\sum_{k=1}^n p_k \prod_{j=k}^{n-1} \theta_j} l_i \\
c_i - v(l_i) &\geq c_{i-1} - v(\theta_{i-1} l_{i-1}) \text{ for all } i \text{ such that } i = 2, 3, ..n \\
\theta_i &\in [0,1] \text{ for all } i = 1, 2, ...n-1 \\
c_i, l_i &\geq 0 \text{ for all } i = 1, 2, ...n
\end{aligned}
$$

3 Utilitarian Optimal Allocation

In this section we analyze the utilitarian optimal skill distributions. For $n = 2$, in both full information and private information cases we provide an analytical results. For $n \geq 3$, we assume specific forms of utility and disutility functions and provide numerical analysis with both.

3.1 Analytical result for $n = 2$

In this section we analyze the utilitarian optimal allocations for $n = 2$. We first characterize the optimal allocations for the case of full information. That is, we find the allocations that maximize utilitarian welfare objective function subject to social feasibility and non-negativity constraints. Secondly, we characterize the private information utilitarian optimal allocations. The only difference in the private information utilitarian social planner's problem is the incentive compatibility constraint between type 2 and type 1. For both in the full and private information utilitarian optimal skill distribution cases we find that $\theta_i^* = 0$, which is the same result as that of Leung and Yazıcı (2010).

3.1.1 Full Information Problem

Following Leung and Yazıcı (2010) we can re-write the feasibility constraint as

$$p_2 c_2 + p_1 c_1 \leq a l_2 + \frac{\alpha \theta_1 p_1 (l_1 - l_2)}{p_2 + p_1 \theta_1}$$

Then, we can re-write the utilitarian social planner's problem as

$$\max_{c_2, c_1, l_2, l_1, \theta_1} p_2 \left[c_2 - v(l_2)\right] + p_1 \left[c_1 - v(l_1)\right]$$

s.t.

$$\begin{aligned} p_2 c_2 + p_1 c_1 &\leq a l_2 + \frac{\alpha \theta_1 p_1 (l_1 - l_2)}{p_2 + p_1 \theta_1} \\ \theta_1 &\in [0, 1] \\ c_2, c_1, l_2, l_1 &\geq 0 \end{aligned}$$

Proposition 1 *In the full information utilitarian optimum $\theta_1^* = 0$.*

Proof. The proof of Proposition 1 is the same as the proof of Theorem 1 in Leung and Yazıcı (2010). Note that even if Leung and Yazıcı (2010) have the concavity of utility assumption, there is no use of this assumption in the proof. ■

The reason behind this result is the lack of the incentive compatibility constraint. Without the incentive compatibility constraint, the social planner does not care about egalitarian distribution so that she considers only the productive efficiency.

3.1.2 Private information problem

Now, we analyze the private information utilitarian optimal allocation. Then, the utilitarian social planner's problem is

$$\max_{c_2,c_1,l_2,l_1,\theta_1} p_2\left[c_2 - v(l_2)\right] + p_1\left[c_1 - v(l_1)\right]$$

$$\text{st.}$$

$$\begin{aligned} p_2c_2 + p_1c_1 &\leq al_2 + \frac{\alpha\theta_1 p_1(l_1 - l_2)}{p_2 + p_1\theta_1} \\ c_2 - v(l_2) &\geq c_1 - v(\theta_1 l_1) \\ \theta_1 &\in [0,1] \\ c_2, c_1, l_2, l_1 &\geq 0 \end{aligned}$$

Theorem 1 *In the private information utilitarian optimum* $\theta_1^* = 0$.

Proof. The proof of Theorem 1 is the same as the proof of Theorem 2 in Leung and Yazıcı (2010). Note that even if, Leung and Yazıcı (2010) have the concavity of utility assumption, there is no use of this assumption in the proof.
■

Theorem 1 states that even in the case of private information, with the utilitarian social planner's problem, it is optimal to distribute all the skill to type 2.

The following is the intuition behind Theorem 1. Since the utility of each type is assumed to be linear, a number of combination of feasible consumption allocations, (c_1, c_2) are admissible for the social planner. Therefore, it becomes easy to satisfy the incentive compatibility constraint.

By Proposition 1 and Theorem 1 we show that the results of Leung and Yazıcı (2010) for full information and private information optimum allocations hold with linear utility as well.

3.2 Analysis for $n \geq 3$

Since we have not derived analytical results for $n \geq 3$, we provide a numerical analysis. In this numerical analysis we assume $v(l) = l^\gamma$ for some $\gamma > 1$. With these functions we have the utilitarian social planner's problem as

$$\max_{\theta_i, c_i, l_i} \sum_i p_i (c_i - l_i^\gamma)$$

st.

$$\begin{aligned} \sum_i p_i c_i &\leq \sum_i p_i \frac{\alpha \prod_i^{n-1} \theta_i}{\sum_{k=1}^n p_k \prod_{j=k}^{n-1} \theta_j} l_i \\ c_i - l_i^\gamma &\geq c_{i-1} - \theta_{i-1}^\gamma l_{i-1}^\gamma \text{ for all } i = 2, .., n \\ \theta_i &\in [0, 1] \text{ for all } i = 1, 2, ...n - 1 \\ c_i, l_i &\geq 0 \text{ for all } i = 1, 2, ...n \end{aligned}$$

Lemma 2 *Let $u(c) = c$ and $v(l) = l^\gamma$. Let R be an associated Lagrange multiplier of the resource constraint and let λ_i be the associated Lagrange multiplier of the incentive compatibility constraint between type i and $i - 1$ for all $i = 2, 3, ..., n$. Then $\lambda_i = 0$ for all $i = 2, 3, ..., n$.*

Proof. The Lagrangian of the problem reads

$$\begin{aligned}\mathcal{L} &= \textstyle\sum_i p_i(c_i - l_i^\gamma) - R(\sum_i p_i(c_i - \frac{\alpha \prod_i^{n-1}\theta_i}{\sum_{k=1}^n p_k \prod_{j=k}^{n-1}\theta_j} l_i)) \\ &\quad -\lambda_n(c_n - l_n^\gamma - c_{n-1} + \theta_{n-1}^\gamma l_{n-1}^\gamma) - \lambda_{n-1}(c_{n-1} - l_{n-1}^\gamma - c_{n-2} + \theta_{n-2}^\gamma l_{n-2}^\gamma) \\ &\quad ... - \lambda_2(c_2 - l_2^\gamma - c_1 + \theta_1^\gamma l_1^\gamma)\end{aligned}$$

Then we have the first order optimality conditions for consumption as

$$\begin{aligned} c_1 &: p_1 - Rp_1 + \lambda_2 = 0 \\ c_i &: p_i - Rp_i + \lambda_{i+1} - \lambda_i = 0 \text{ for all } i = 2, 3, ...n-1 \\ c_n &: p_n - Rp_n - \lambda_n = 0 \end{aligned}$$

Combining these first order conditions we get $\sum_i p_i - \sum_i Rp_i = 0$, which implies $R = 1$. Then plugging R into the first order conditions of each c_i we see that $\lambda_i = 0$ for all $i = 2, 3, ...n$. ■

By Lemma 2, in our numerical analysis we can ignore the incentive compatibility constraints. Our numerical analysis results show that with $n \geq 3$, we have all $\theta_i = 0$. That is to say, all of the skills go to only one type. (i.e. $w_n = \frac{\alpha}{p_n}$ and $w_i = 0$ for all $i = 1, 2, ..., n-1$).

4 Egalitarian Optimal Allocation

In Section 3, we analyzed the optimal skill distribution when the objective welfare function is utilitarian with linear utility and strictly convex disutility functions. In the utilitarian optimal allocation we always have all the skills allocated to the highest skilled type, regardless of the number of the types in the society. Therefore, we ask "what if the available skill level in the society is distributed in a more egalitarian manner?" That is, "what if the net utility of the lowest skilled type, type 1, is maximized subject to the incentive compatibility constraints of the other types?" Formally, the social planner's problem with egalitarian objective welfare function is

$$\max_{\theta_i, c_i, l_i} c_1 - v(l_1)$$

s.t.

$$\begin{aligned} \textstyle\sum_i p_i c_i &\leq \textstyle\sum_i p_i w_i l_i \\ \textstyle\sum_i p_i w_i &\leq \alpha \\ c_i - v(l_i) &\geq c_{i-1} - v(\theta_{i-1} l_{i-1}) \text{ for all } i = 2, .., n \\ l_i, c_i &\geq 0 \text{ for all } i \\ \theta_i &\in [0, 1] \end{aligned}$$

Since we cannot derive analytical results for egalitarian optimal allocations, we provide numerical analysis with $u(c) = c$ and $v(l) = l^\gamma$ for some $\gamma > 1$. The striking result is that in egalitarian optimal allocation, type n has all the skills. Plugging u and v into the egalitarian social planner's problem we can re-write it as

$$\max_{\theta_i, c_i, l_i} c_1 - l_1^\gamma$$

st.

$$
\begin{aligned}
\sum_i p_i c_i &\leq \sum_i p_i \frac{\alpha \prod_i^{n-1} \theta_i}{\sum_{k=1}^n p_k \prod_{j=k}^{n-1} \theta_j} l_i \\
c_i - l_i^\gamma &\geq c_{i-1} - (\theta_{i-1}^\gamma l_{i-1}^\gamma) \text{ for all } i = 2, .., n \\
l_i, c_i &\geq 0 \text{ for all } i = 1, 2, ...n \\
\theta_i &\in [0, 1] \text{ for all } i = 1, 2, .., n
\end{aligned}
$$

Since the incentive compatibility constraints bind at the optimum we have

$$c_i = l_i^\gamma + c_{i-1} - (\theta_{i-1}^\gamma l_{i-1}^\gamma) \text{ for all } i = 2, 3, ..., n$$

$$
\begin{aligned}
c_n &= l_n^\gamma + c_{n-1} - (\theta_{n-1}^\gamma l_{n-1}^\gamma) \\
c_{n-1} &= l_{n-1}^\gamma + c_{n-2} - (\theta_{n-2}^\gamma l_{n-2}^\gamma) \\
&. \\
&. \\
&. \\
c_2 &= l_2^\gamma + c_1 - (\theta_1^\gamma l_1^\gamma)
\end{aligned}
$$

Since the resource constraint binds at the optimum as well, we have,

$$\sum_i p_i c_i = c_1 + \sum_{i=2}^n p_i . \sum_{j=2}^i [l_i^\gamma - \theta_{j-1}^\gamma l_{j-1}^\gamma] = \sum_i p_i w_i l_i$$

Hence,

$$c_1 - l_1^\gamma = \sum_i p_i w_i l_i - \sum_{i=2}^n p_i . \sum_{j=2}^i [l_i^\gamma - \theta_{j-1}^\gamma l_{j-1}^\gamma] - l_1^\gamma$$

Therefore, we can re-write the egalitarian social planner's problem as

$$\max_{\theta_i \in [0,1]} \sum_i p_i w_i l_i - \sum_{i=2}^n p_i . \sum_{j=2}^i [l_i^\gamma - \theta_{j-1}^\gamma l_{j-1}^\gamma] - l_1^\gamma$$

where

$$w_i = \frac{\alpha \prod_{j=i}^{n-1} \theta_j}{\sum_i p_i \prod_{j=i}^{n-1} \theta_j}$$

Having this functional form which depends only on $\theta = (\theta_1, ..., \theta_{n-1})$, we conduct a numerical analysis.

5 Comparisons

In this section we compare overall utilities obtained in a laissez-faire market structure, Mirrleesian taxation without skill distribution and Mirrleesian taxation with skill distribution via numerical analysis for utilitarian and egalitarian social welfare functions. We assume $v(l) = l^\gamma$ for our numerical analysis and calculate the social welfare functions in all cases for various fractions p_i, skill level α and γ.

5.1 Laissez Faire Market

In a laissez faire market, each type of agents solves their own problem. That is, each type of agents maximizes her net utility subject to what she produces. Formally each type of agents solves the following problem.

$$\max c_i - l_i^\gamma$$

$$s.t.$$

$$c_i = w_i l_i \text{ for all } i \epsilon \{1, 2, ..., n\}$$

From the first order optimality conditions we have

$$\begin{aligned} l_i^* &= (\frac{w_i}{\gamma})^{\frac{1}{\gamma-1}} \\ c_i^* &= w_i(\frac{w_i}{\gamma})^{\frac{1}{\gamma-1}} \end{aligned}$$

Hence, we have the utilitarian optimal welfare function as

$$\sum_i p_i(w_i(\frac{w_i}{\gamma})^{\frac{1}{\gamma-1}} - (\frac{w_i}{\gamma})^{\frac{\gamma}{\gamma-1}})$$

and the egalitarian optimal social welfare function reads

$$w_1(\frac{w_1}{\gamma})^{\frac{1}{\gamma-1}} - (\frac{w_1}{\gamma})^{\frac{\gamma}{\gamma-1}}$$

5.2 Mirrleesian Taxation without Skill Distribution

We analyze both utilitarian optimal and egalitarian optimal allocations for Mirrleesian taxation without skill distribution. That is, we take each type's skill level as given.

We can write the utilitarian social welfare function as

$$\max_{c_i, l_i} \sum_i p_i(c_i - l_i^\gamma)$$

$$s.t$$

$$\begin{aligned} \sum_i p_i c_i &\leq \sum_i p_i w_i l_i \\ c_i - l_i^\gamma &\geq c_{i-1} - \theta_{i-1}^\gamma l_{i-1}^\gamma \text{ for all } i \text{ such that } i = 2, 3, ..n-1 \\ c_i, l_i &\geq 0 \text{ for all } i = 1, 2, ...n \end{aligned}$$

From the first order optimality conditions we have

$$l_i^* = (\frac{w_i}{\gamma})^{\frac{1}{\gamma-1}} \text{ for all } i = 1, 2, ..., n.$$

Hence, the optimal production becomes

$$y^* = \sum_i p_i (\frac{w_i}{\gamma})^{\frac{1}{\gamma-1}} w_i = \sum_i p_i c_i$$

Then the utilitarian social welfare function reads at the optimum

$$\sum_i p_i c_i - \sum_i p_i (l_i^*)^\gamma = \sum_i p_i (\frac{w_i}{\gamma})^{\frac{1}{\gamma-1}} w_i - \sum_i p_i \left[(\frac{w_i}{\gamma})^{\frac{1}{\gamma-1}} \right]^\gamma$$

Note that the objective functions of utilitarian optimal Mirrleesian taxation without skill distribution and the utilitarian laissez-faire market is the same.

The egalitarian social planner's problem for Mirrleesian taxation without skill distribution is written as

$$\max_{c_i, l_i} c_1 - l_1^\gamma$$

st.

$$\begin{aligned} \sum_i p_i c_i &\leq \sum_i p_i w_i l_i \\ c_i - l_i^\gamma &\geq c_{i-1} - \theta_{i-1}^\gamma l_{i-1}^\gamma \text{ for all } i \text{ such that } i = 2, 3, ..n - 1 \\ c_i, l_i &\geq 0 \text{ for all } i = 1, 2, ...n \end{aligned}$$

We note that the resource constraint binds. Moreover, the incentive compatibility constraints bind as well. Hence, combining the objective function and these constraints, the egalitarian social planner's becomes

$$\max_{l_i} \sum_{i=1}^n p_i w_i l_i - \sum_{i=2}^n p_i . \sum_{j=2}^i [l_i^\gamma - \theta_{j-1}^\gamma l_{j-1}^\gamma] - l_1^\gamma$$

$$s.t$$
$$l_i \geq 0 \text{ for all } i = 1, 2, ...n$$

The first order conditions yield

$$l_1^* = \left(\frac{p_1 w_1}{\gamma(1 - (1 - p_1)\theta_1^\gamma)}\right)^{\frac{1}{\gamma - 1}}$$
$$l_i^* = \left(\frac{p_i w_i}{\gamma(\sum_{k=i}^n p_k - \theta_i^\gamma \sum_{k=i+1}^n p_k)}\right)^{\frac{1}{\gamma - 1}} \text{ for all } i = 2, ...n$$

Hence the maximized social welfare function reads

$$c_1^* - l_1^* = \sum_{i=1}^n p_i w_i l_i^* - \sum_{i=2}^n p_i . \sum_{j=2}^i [(l_i^*)^\gamma - \theta_{j-1}^\gamma (l_{j-1}^*)^\gamma] - (l_1^*)^\gamma$$

5.3 Comparisons with Utilitarian and Egalitarian Social Planner's Problem

In this section, we compare the results of the numerical analysis we obtained for three different environments, laissez-faire market, Mirrleesian taxation without skill distribution and Mirrleesian taxation with skill distribution. We compare the results for $n = 2$ and $n = 3$.

Results for n=2 In this part, we compare the values of both utilitarian and egalitarian social welfare functions that are obtained in different environments for $n = 2$. We assume $p_1 = 0.5$, $p_2 = 0.5$ and $\alpha = 1$. With these values, we conduct our analysis for five different values of γ. That is, $\gamma \in \{1.1, 1.5, 2.0, 2.7, 4.0\}$. Given γ, p_1, p_2 and α, we compare the values of the optimal social welfare functions in these three environment for three different values of w_1. That is $w_1 \in \{0, 0.5, 1\}$.

Figure 1 and Figure 3 summarize our results for the utilitarian and egalitarian social planner's problem respectively for $n = 2$.

Our first result is, it is always optimal to distribute all of the skills to one type in a society, the type with highest skill, regardless of whether egalitarian or utilitarian social welfare functions are used. This result shows that the results of Leung and Yazıcı (2010) hold with different assumptions and with different social welfare functions.

Secondly, as expected, the value of the utilitarian and egalitarian social welfare functions in the Mirrleesian taxation with skill distribution environment always yields a better result than its value in the other two environments. This finding is because, while we allow skill distribution, we are treating one of the parameter as a variable and choosing the value of that variable which maximizes our objective function. In the other environments we take this parameter as a given. An increase in welfare from Mirrleesian taxation without skill distribution to Mirrleesian taxation with skill distribution is always more than an increase from a laissez faire market to Mirrleesian taxation without skill distribution in both utilitarian and egalitarian problems. The result shows, the importance of skill distribution numerically.

Our final result is that an economy with perfectly unequal skill distribution provides a more egalitarian society in terms of how utilities are distributed across agents, in both utilitarian and egalitarian problems. This result shows that more equal distribution can be obtained by distributing skills.

Another observation is about the value of γ. As seen in the figure 1, as the value of γ increases, that is as the value of disutility increases, the difference among the values of the welfare function in these three environments get smaller. For instance, when $\gamma = 1.1$ and $w_1 = 0.5$, the value of the welfare function in the Mirrleesian taxation without skill distribution environment is 1.516, whereas the value of the welfare function is 35.891 when skill distribution is allowed. However, when $\gamma = 4.0$ and $w_1 = 0.5$, the values becomes 0.499 and 0.595 respectively. This observation demonstrates that as the cost of labor effort in terms of disutility increases, the incentive for the social planner to implement skill redistribution decreases.

Results for $n = 3$ Numerical analysis for $n = 3$ similar to the one we made for $n = 2$ are done. Figure 2 and 4 summarize our results. As opposed to Figures 1 and 3, in Figures 2 and 4, we did not calculate the net utilities of the types $1, 2$ and 3. We only calculated the overall objective welfare.[1] All of arguments we made for $n = 2$ apply to $n = 3$ as well.

6 Conclusion

The motivation of our study is how to redistribute income earning skills in a heterogeneous society to reach the social optimum. By social optimum we mean both equity and efficiency. By equity we mean each type has utility as close to each other as possible and by efficiency we mean producing more with the same level of labor effort. To find a solution to this question we analyzed the optimal skill distribution with utilitarian and egalitarian social welfare functions.

Leung and Yazıcı (2010) is the first to study this question in the Mirrleesian environment. In their paper they introduce a new channel of redistribution

[1] We generalize this procedure for arbitrary n . Due to the memory constraints of Matlab, we made the numerical analysis up to $n = 10$. In all cases, we see that the objective welfare function is maximized when all $\theta_i = 0$. All the comments in the Results sections are valid for arbitrary n as well.

in a static Mirrleesian economy, which is to let the planner choose an ex-ante distribution of skills. Given a constant level of total skill between two types of agents, they show that the planner always finds it optimal to choose the perfectly unequal skill distribution. As opposed to Leung and Yazıcı (2010), our contribution to the literature is providing numerical simulations to measure the welfare effects of skill distribution choice under different social welfare functions and the characterization of optimal skill distributions for different objective welfare functions with different assumptions.

Our first result showed that, it is always optimal to distribute all of the skills to one type in a society (the type with highest skill) regardless of whether we used egalitarian or utilitarian social welfare functions. By this, we show that the results of Leung and Yazıcı (2010) hold with different assumptions and with different welfare functions. Secondly, the increase in welfare from Mirrleesian taxation without skill distribution to Mirrleesian taxation with skill distribution is always much more than the increase from laissez faire market to Mirrleesian taxation without skill distribution in both utilitarian and egalitarian problems. With this result, we show the importance of the skill distribution numerically. Since in our numerical analysis the increase with the skill distribution is significant, it is a good sign for policy makers to consider this seriously. Our final result is that an economy with perfectly unequal skill distribution provides a more egalitarian society in terms of how utilities are distributed across agents, again in both utilitarian and egalitarian problems. This result shows that with skill distribution not only we increase total welfare but also make a more equal distribution.

We present three possible extensions for our study. The first possible extension is analyzing the optimum skill distribution for other social welfare functions. Secondly, for the egalitarian problem we did not present an analytical solution, which can be investigated. Finally, a more extensive numerical analysis with real data where parameters are estimated can be performed.

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8 Appendices

8.1 A.1 Numerical Analysis

Since the numerical analysis for $n = 2$ and $n = 3$ depends only on $\theta_1 = \dfrac{w_1}{w_2}$, $\theta_2 = \dfrac{w_2}{w_3}$ respectively it is easy to perform a numerical analysis for these cases. However for $n > 3$, the analysis becomes more complex. For instance, when $n = 4$, the objective function becomes a function of $\theta_1 = \dfrac{w_1}{w_2}$, $\theta_2 = \dfrac{w_2}{w_3}$ and $\theta_3 = \dfrac{w_3}{w_4}$. In this case, we need a four dimensional space to visualize the values of the objective function. To simplify our analysis we represent this four dimensional space by a two dimensional matrix. The main tool we use is the Kronecker product of two vectors. Let $kron(A, B)$ denote the Kronecker product of matrices A and B.

Let $n = 4$ and let $\theta_i = [0\ 1/m\ 2/m\ ...1]_{1\times(m+1)}$ be a $m + 1$ dimensional vector for $i = 1, .., 3$. Then we can generate all possible combinations of θ_1,θ_2 and θ_3 with the following procedure.

First calculate the Kronecker product θ_1 and the transpose of θ_2, θ_2'. Then calculate the Kronecker product of θ_3 and this resulting Kronecker product.

That is,

$$kron(\theta_1, \theta_2') = \begin{bmatrix} & & & \theta_2 & & & \\ & & 0 & 1/m & . & . & 1 \\ & 0 & 0 & 0 & . & . & 0 \\ \theta_1 & 1/m & 0 & 1/m^2 & & & 1/m \\ & . & . & . & & & \\ & . & . & . & & & \\ & 1 & 0 & 1/m & & & 1 \end{bmatrix}$$

$$kron(\theta_3, kron(\theta_1, \theta_2')) = [0{*}kron(\theta_1, \theta_2') \mid 1/m{*}kron(\theta_1, \theta_2') ... \mid 1{*}\ kron(\theta_1, \theta_2')]$$

which is an $m \times m^2$ dimensional matrix.

Then we generalize this procedure for arbitrary n and perform our numerical analysis. Due to the memory constraints of Matlab, we made the numerical analysis up to $n = 10$. In all cases, we see that the objective welfare function is maximized when all $\theta_i = 0$.

8.2 A.2 Tables

gamma= 1.1, weigth of high= 0.50, weigth of low= 0.50, a= 1.00

	market			mirrlees			mirrlees+skill		
	u2	u1	total	u2	u1	total	u2	u1	total
w1=0.00, w2=2.00	71.781	0.000	35.891	35.891	35.891	35.891	35.891	35.891	35.891
w1=0.50, w2=1.50	3.032	0.000	1.516	1.516	1.516	1.516	35.891	35.891	35.891
w1=1.00, w2=1.00	0.035	0.035	0.035	0.035	0.035	0.035	35.891	35.891	35.891

gamma= 1.5, weigth of high= 0.50, weigth of low= 0.50, a= 1.00

	market			mirrlees			mirrlees+skill		
	u2	u1	total	u2	u1	total	u2	u1	total
w1=0.00, w2=2.00	1.185	-0.000	0.593	0.593	0.593	0.593	0.593	0.593	0.593
w1=0.50, w2=1.50	0.500	0.019	0.259	0.274	0.244	0.259	0.593	0.593	0.593
w1=1.00, w2=1.00	0.148	0.148	0.148	0.148	0.148	0.148	0.593	0.593	0.593

gamma= 2.0, weigth of high= 0.50, weigth of low= 0.50, a= 1.00

	market			mirrlees			mirrlees+skill		
	u2	u1	total	u2	u1	total	u2	u1	total
w1=0.00, w2=2.00	1.000	-0.000	0.500	0.500	0.500	0.500	0.500	0.500	0.500
w1=0.50, w2=1.50	0.563	0.062	0.313	0.340	0.285	0.312	0.500	0.500	0.500
w1=1.00, w2=1.00	0.250	0.250	0.250	0.250	0.250	0.250	0.500	0.500	0.500

gamma= 2.7, weigth of high= 0.50, weigth of low= 0.50, a= 1.00

	market			mirrlees			mirrlees+skill		
	u2	u1	total	u2	u1	total	u2	u1	total
w1=0.00, w2=2.00	1.055	0.000	0.528	0.528	0.528	0.528	0.528	0.528	0.528
w1=0.50, w2=1.50	0.668	0.117	0.393	0.425	0.360	0.393	0.528	0.528	0.528
w1=1.00, w2=1.00	0.351	0.351	0.351	0.351	0.351	0.351	0.528	0.528	0.528

gamma= 4.0, weigth of high= 0.50, weigth of low= 0.50, a= 1.00

	market			mirrlees			mirrlees+skill		
	u2	u1	total	u2	u1	total	u2	u1	total
w1=0.00, w2=2.00	1.191	0.000	0.595	0.595	0.595	0.595	0.595	0.595	0.595
w1=0.50, w2=1.50	0.811	0.187	0.499	0.530	0.469	0.499	0.595	0.595	0.595
w1=1.00, w2=1.00	0.472	0.472	0.472	0.472	0.472	0.472	0.595	0.595	0.595

Figure 1: Utilitarian Social Planner's Problem for $n = 2$.

gamma= 1.1, a= 1.00, p1= 0.33

	laissez faire market	mirrlees w/o skill	mirrlees with skill distribution
wi=0.00, wn=3.00	2069.631	2069.631	2069.631
wi=0.50, wn=2.00	23.927	23.927	2069.631
wi=1.00, wn=1.00	0.035	0.035	2069.631

gamma= 1.5, a= 1.00, p1= 0.33

	laissez faire market	mirrlees w/o skill	mirrlees with skill distribution
wi=0.00, wn=3.00	1.333	1.333	1.333
wi=0.50, wn=2.00	0.407	0.407	1.333
wi=1.00, wn=1.00	0.148	0.148	1.333

gamma= 2.0, a= 1.00, p1= 0.33

	laissez faire market	mirrlees w/o skill	mirrlees with skill distribution
wi=0.00, wn=3.00	0.750	0.750	0.750
wi=0.50, wn=2.00	0.375	0.375	0.750
wi=1.00, wn=1.00	0.250	0.250	0.750

gamma= 2.5, a= 1.00, p1= 0.33

	laissez faire market	mirrlees w/o skill	mirrlees with skill distribution
wi=0.00, wn=3.00	0.678	0.678	0.678
wi=0.50, wn=2.00	0.413	0.413	0.678
wi=1.00, wn=1.00	0.326	0.326	0.678

gamma= 4.0, a= 1.00, p1= 0.33

	laissez faire market	mirrlees w/o skill	mirrlees with skill distribution
wi=0.00, wn=3.00	0.681	0.681	0.681
wi=0.50, wn=2.00	0.522	0.522	0.681
wi=1.00, wn=1.00	0.472	0.472	0.681

Figure 2: Utilitarian Social Planner's Problem for $n = 3$.

gamma= 1.1, weigth of high= 0.50, weigth of low= 0.50, a= 1.00

		market			mirrlees			mirrlees+skill	
	u2	u1	total	u2	u1	total	u2	u1	total
w1=0.00, w2=2.00	71.781	0.000	0.000	35.891	35.891	35.891	35.891	35.891	35.891
w1=0.50, w2=1.50	3.032	0.000	0.000	1.516	1.516	1.516	35.891	35.891	35.891
w1=1.00, w2=1.00	0.035	0.035	0.035	0.035	0.035	0.035	35.891	35.891	35.891

gamma= 1.5, weigth of high= 0.50, weigth of low= 0.50, a= 1.00

		market			mirrlees			mirrlees+skill	
	u2	u1	total	u2	u1	total	u2	u1	total
w1=0.00, w2=2.00	1.185	-0.000	-0.000	0.593	0.593	0.593	0.593	0.593	0.593
w1=0.50, w2=1.50	0.500	0.019	0.019	0.258	0.253	0.253	0.593	0.593	0.593
w1=1.00, w2=1.00	0.148	0.148	0.148	0.148	0.148	0.148	0.593	0.593	0.593

gamma= 2.0, weigth of high= 0.50, weigth of low= 0.50, a= 1.00

		market			mirrlees			mirrlees+skill	
	u2	u1	total	u2	u1	total	u2	u1	total
w1=0.00, w2=2.00	1.000	-0.000	-0.000	0.500	0.500	0.500	0.500	0.500	0.500
w1=0.50, w2=1.50	0.563	0.062	0.062	0.313	0.298	0.298	0.500	0.500	0.500
w1=1.00, w2=1.00	0.250	0.250	0.250	0.250	0.250	0.250	0.500	0.500	0.500

gamma= 2.7, weigth of high= 0.50, weigth of low= 0.50, a= 1.00

		market			mirrlees			mirrlees+skill	
	u2	u1	total	u2	u1	total	u2	u1	total
w1=0.00, w2=2.00	1.055	0.000	0.000	0.528	0.528	0.528	0.528	0.528	0.528
w1=0.50, w2=1.50	0.668	0.117	0.117	0.396	0.374	0.374	0.528	0.528	0.528
w1=1.00, w2=1.00	0.351	0.351	0.351	0.351	0.351	0.351	0.528	0.528	0.528

gamma= 4.0, weigth of high= 0.50, weigth of low= 0.50, a= 1.00

		market			mirrlees			mirrlees+skill	
	u2	u1	total	u2	u1	total	u2	u1	total
w1=0.00, w2=2.00	1.191	0.000	0.000	0.595	0.595	0.595	0.595	0.595	0.595
w1=0.50, w2=1.50	0.811	0.187	0.187	0.505	0.480	0.480	0.595	0.595	0.595
w1=1.00, w2=1.00	0.472	0.472	0.472	0.472	0.472	0.472	0.595	0.595	0.595

Figure 3: Egalitarian Social Planner's Problem for $n = 2$.

gamma= 1.1, a= 1.00, p1= 0.33

	laissez faire market	mirrlees w/o skill	mirrlees with skill distribution
wi=0.00, wn=3.00	0.000	2069.631	2069.631
wi=0.50, wn=2.00	0.000	23.927	2069.631
wi=1.00, wn=1.00	0.035	0.035	2069.631

gamma= 1.5, a= 1.00, p1= 0.33

	laissez faire market	mirrlees w/o skill	mirrlees with skill distribution
wi=0.00, wn=3.00	0.000	1.333	1.333
wi=0.50, wn=2.00	0.019	0.403	1.333
wi=1.00, wn=1.00	0.148	0.148	1.333

gamma= 2.0, a= 1.00, p1= 0.33

	laissez faire market	mirrlees w/o skill	mirrlees with skill distribution
wi=0.00, wn=3.00	0.000	0.750	0.750
wi=0.50, wn=2.00	0.063	0.365	0.750
wi=1.00, wn=1.00	0.250	0.250	0.750

gamma= 2.5, a= 1.00, p1= 0.33

	laissez faire market	mirrlees w/o skill	mirrlees with skill distribution
wi=0.00, wn=3.00	0.000	0.678	0.678
wi=0.50, wn=2.00	0.103	0.401	0.678
wi=1.00, wn=1.00	0.326	0.326	0.678

gamma= 4.0, a= 1.00, p1= 0.33

	laissez faire market	mirrlees w/o skill	mirrlees with skill distribution
wi=0.00, wn=3.00	0.000	0.681	0.681
wi=0.50, wn=2.00	0.188	0.509	0.681
wi=1.00, wn=1.00	0.472	0.472	0.681

Figure 4: Egalitarian Social Planner's Problem for $n = 3$.