An Axiomatic Analysis of Dynamic Simple Allocation Problems

by

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to my dear family especially to my lovely grandmother

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"An Axiomatic Analysis of Dynamic Simple Allocation Problems"

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Economics,MA Thesis

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Abstract

We look for a "good" solution to the following problem: a perfectly divisible commodity is to be allocated in each period among a set of agents each having an exogenous characteristic vector and a complete, transitive, continuous and monotonic preference relation on his consumption. On this class we analyze the implications of well-known properties such as Pareto optimality, no-envy, strategy proofness and no-manipulation via characteristics. We particularly find that although the Walrasian rule is always manipulable via destruction of characteristics, manipulation can be prevented under the constrainted Walrasian rule if the initial allocation is determined by a dictatorial rule.

Keywords: allocation problem, Pareto Optimality, manipulation via preferences, manipulation via characteristics

" Dinamik Basit Allakasyon Problemlerine Aksiyomatik Analiz"

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Ekonomi, Yüksek Lisans Tezi
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Özet

Egzojen karateristiklere ve tüketim üzerine eksiksiz, geçişli, sürekli ve monoton tercihlere sahip ajanlara sınırsız bölünebilen bir malın her period dağıtılmasıprobleminin güzel çözümlerine baktık. Bu sınıftaki problemlerin bilinen Pareto optimal, kıskanmama, strateji dayanıklılığıve karakteristiklerle manipulasyon yapılamama gibi aksiyomların etkilerini analiz ettik. Özel olarak bulduğumuz sonuçta Walras kuralının her zaman manipule edilebilirken sınırlıWalras kuralıaltındaki manipulasyon ilk allakasyonun diktatörel bir kuralla belirlenmesi ile engellenebilir.

Anahtar Sözcükler: allakasyon problemi, Pareto optimalite, tercihlerle manipulasyon, karakteristiklerle manipulasyon

Contents

1 INTRODUCTION

We look for a “good” solution to the following problem: a perfectly divisible commodity is to be allocated in each period among a set of agents each having an exogenous "characteristic vector" and a complete, transitive, continuous and monotonic preference relation on his consumption in time. The (federal) Environmental Protection Agency (EPA) of the US government faces this problem in each period when allocating pollution permits to the firms. Each permit gives the bearer firm the right to a certain amount of pollution. The permits are allocated according to firms' characteristics. Relevant characteristics depend on the pollutant to be allocated. In case of local and regional pollutants, a firm's right to pollute is bounded by the characteristics imposed by local governments which act as exogenous characteristic for the federal government. For example very strict local limits may be imposed on polluters in densely populated or over-polluted areas for some of the pollutants, called local pollutants, are absorbed in the vicinity of the emission. Another example, some of the pollutants called regional pollutants such as sulfur cannot travel further than 200-600 miles from the source of emission. In case of global pollutants, the characteristic of firms is the past emission level[1].

Permit allocation started in US in 1899 and it is currently used in the regulation of various pollutants.[2] The current rules vary and depend on the industry being regulated. Joskow et al. (1998) explain how Title IV of the 1990 Clean Air Act in the US Acid Rain Program regulates the allocation of SO_2 permits among coal burning electric generating units. Briefly, a firm's share is decided proportionally to an estimate of its profit maximizing emission level or in some cases its past emission level.

There are different mechanisms used in permit allocation. One of the mechanisms that is recently used is called the "cap and trade system" in which a central authority (such as the EPA) sets a limit (cap) on the amount of a pollutant, and allocates it to the firms. If some of the firms want to pollute more, they can buy permits from other firms. Stavins

[1]Pollutants is classified by their zone of influence. While *local pollutants* damage is experienced near the source of emission, the damage from *regional pollutants* is experienced at greater distances from the source of emission. On the other hand, when the damage caused by a *global pollutant* is determined mainly by concetrations of pollutant in the upper atmosphere.

[2]All industrial waste dischargers were required to have permits from the US Army Corps of Engineers by The Refuse Act of 1899[13].

(2001) explains these transactions. One such market is in Chicago. The Illinois EPA set up a trading program for volatile organic compounds, called the Emissions Reduction Market System in 1997. After 2000, eight Illinois counties started trading pollution credits over a hundred major sources of pollution in Chicago.[3]

There are three important features of this example. Firstly, there is a central authority (such as the EPA), which, in each period, allocates that period's endowment among the agents. Secondly, the allocation is repeated in each period. Thus we are dealing with a dynamic problem. Finally, since the (static) allocation of the central authority is dynamically inefficient, firms trade to get an efficient outcome. We examine these features in detail.

This thesis is organized as follows. In **subsection 1.1** we will give some examples and applications of simple allocation problems. In **section 2** we will review the literature on simple allocation problems and manipulation in exchange economies. After we set up our model in **section 3**, we will introduce some axioms in **section 4** and check whether they are satisfied by our mechanisms. Particularly we will describe different manipulation types in **section 4** and inquire as to prevent them. We will conclude our work in **section 5**.

1.1 Examples and Applications

A simple allocation problem for a society N is an $|N|+1$ dimensional nonnegative real vector $(c_1, ..., c_{|N|}, E)$ which is explained as follows. E is the endowment which is perfectly divisible commodity is to be allocated among members of a society and each member of the society $i \in N$ is characterized by an amount c_i of the commodity to be allocated. We will discuss two different applications of the simple allocation problem.

Permit Allocation: The Environmental Protection Agency allocates an amount E of pollution permits among firms in (a society) N (such as CO_2 emission permits allocated among energy producers). Each firm $i \in N$, depending on its location, is imposed by the local authority an emission constraint c_i on its pollution level. For more on this application, see Kıbrıs (2003) and the literature cited therein.

Taxation: A public authority collects an amount E of tax from a society N. Each agent $i \in N$ has income c_i. This is a central and very old problem in public finance. For example, see Edgeworth (1898) and the following literature. Young (1987) proposes a class

[3] http://www.epa.state.il.us/air/erms/overview.html

of "parametric solutions" to this problem.

2 LITERATURE REVIEW

The bankruptcy problem is about how to allocate an estate among creditors when the estate is not enough to satisfy all creditors. It has been studied since very old times. For example the Babylonian Talmud discusses such cases. There are several solutions which were proposed to solve this problem such as the proportional rule, constrained equal awards rule, constrained equal losses rule. These have been axiomatically studied since O'Neill (1982) relates game theoretical models to every bankruptcy problem. He compares such rules which are proposed to different game structures. Aumann and Maschler (1985) study the Contested Garment Consistent rule, which is proposed by Talmud for the two claimant case. To determine which solutions are "good" these rules are controlled whether they satisfy the desirable properties such as Pareto optimality, fairness. Examining such rules lead the characterization of the rules which helps to compare them. A survey on axiomatic analysis of bankruptcy problems is given by Thomson (2003).

Although there is a huge literature on static simple allocation problems, to our knowledge there is only one paper on dynamic simple allocation problems. In this paper Inarra and Skonhoft (2008) consider allocation of fishing rights in the North East Atlantic Sea in a dynamic setting Total Allowable Catch (TAC) as a regulating scheme. Because of overexploiting of fish stocks, in order to sustain ecological and economic considerations an authority sets a TAC for the actual fish (cod fish) stock. Since the total demands of agents, Norway and Russia, exceeds this TAC a simple allocation problem occurs. They consider the characteristics as the previous year fishing levels and analyze which one of the well-known rules, Constrained Equal Awards rule and Proportional rule, which are well-known solution concepts of bankruptcy problems, is better for a sustainable fishing population.

Manipulation is a central topic in economics. For exchange economies, Hurwicz (1972) shows that an agent can achieve a better outcome by misrepresenting his preferences. He also points out that the occurrence of this manipulation is not unique to the competitive process but is common for any reallocation scheme that achieves Pareto optimal and individually rational outcomes. Dasgupta, Hammond and Maskin (1975) change the of individual rationality requirement to non-dictatorship. They also allow the preferences of individuals to be discontinuous and they obtain similar results. Zhou (1991) proves that efficient and strategy-proof allocation rules have to be dictatorial in pure exchange economies and he relates his work with the Gibbard (1973) and Satterhwaite (1975) in social choice theory which proves that any voting scheme is manipulable unless it is dictatorial.

Postlewaite (1979) studies manipulation via endowments in exchange economies. He introduces three different forms of manipulation via endowments: manipulation via withholding, manipulation via coalition and manipulation via destruction. He shows that two of them cannot be prevented, but the manipulation via destruction can be avoided by using the gamma mechanism whose outcome will equally increase the welfare of agents. Rothschild (1981) introduces an "arbitration rule" which makes manipulation via endowments unprofitable. Haller (1988) illustrates that "non-trivial" Nash equilibria of the manipulation game converges to the Walrasian equilibrium allocation as the economy is replicated.

The most related work with our model is Turhan (2009). He studies a special case of our model where the agents' preferences are linear. Turhan (2009) finds that the only rule that satisfies Pareto optimality and strategy proofness in his domain is the dictatorial rule.

3 MODEL

Let $N = \{1, 2\}$ be the set of **agents** and $T = \{1, 2\}$ be the set of **periods**. In each period $t \in T$ a positive **endowment** $E^t \in \mathbb{R}_+$ is allocated. Let $E = (E^1, E^2)$. Each agent $i \in N$ has a characteristic value in each period $t \in T$ $c_i^t \in \mathbb{R}_+$. Let the characteristic vector of agent i be $c_i = (c_i^t)_{t \in T}$ and the characteristic vector at time t be $c^t = (c_i^t)_{i \in N}$. We assume that for each $t \in T$ $\sum_{i \in N} c_i^t \geqq E^t$ Let $c = (c_1^1, c_1^2, c_2^1, c_2^2)$ be the **characteristic vector**. Let $\mathcal{B}$ denotes the class of all (c, E). For each $i \in N$, let R_i be a complete, transitive, continuous and monotonic **preference relation** defined on $\mathbb{R}_+^2$. Let $R = (R_i)_{i \in N}$. Let P_i be the **strict preference relation** of R_i. We use the vector inequalities $\leqq$, $\leq$, $<$. Let $x \vee y = (\max\{x_i^t, y_i^t\})_{i \in N\ t \in T}$ and $x \wedge y = (\min\{x_i^t, y_i^t\})_{i \in N\ t \in T}$. The **simplex** is $\triangle = \{x |\ x \in \mathbb{R}_+^2$ and $\sum_{j=1}^{j=2} x_j = 1\}$.

The triple (R, c, E) is a **dynamic simple allocation problem** for each $(c, E) \in \mathcal{B}$ and for each $i \in N$ R_i is a preference relation. Let $\mathcal{A}$ denote the class of all dynamic simple allocation problems. An **allocation rule** is a correspondence $G : \mathcal{A} \rightrightarrows \mathbb{R}_+^4$ such that for each $(R, c, E) \in \mathcal{A}$, for each $x \in G(R, c, E)$ and for each $t \in T$, we have $\sum_{i \in N} x_i^t = E^t$.

The **feasible set** of a dynamic simple allocation problem is $X(E) = \{x = (x_i)_{i \in N}|$ for all

$x_i \in \mathbb{R}^2_+ \; x_i \leq E$ for all $i \in N\}$ and the **Pareto optimal set** is $PO(R, c, E) = \{x \in X(E)|$ there exists no $x' \in X(E)$ such that $x'_i R_i x_i$ for all $i \in N$ and $x'_j P_j x_j$ for some $j \in N\}$.

3.1 Static Rules

One way to solve a dynamic simple allocation problem is using rules of static problems in each period. A **static rule** for a dynamic simple allocation problems is a function $F : \mathcal{B} \to \mathbb{R}^4_+$ such that for each $t \in T$ we have $\sum_{i \in N} F^t_i(c, E) = E^t$. For example, the **Proportional rule** allocates the endowment proportional to the characteristics in each period: For $t \in T$ and $i \in N$ $PRO^t_i(c^t, E^t) = (c^t_i / \sum_{i \in N} c^t_i)E^t$. We denote $PRO^t(c^t, E^t) = (PRO^t_i(c^t, E^t))_{i \in N}$ and $PRO(c, E) = (PRO^t(c^t, E^t))_{t \in T}$.

Another rule is the **Equal Gains rule** which allocates the endowment equally in each period, subject to no agent receiving more than his characteristic value: For period t for each $i \in N$, $EG^t_i(c^t, E^t) = min\{c^t_i, \lambda^t\}$ where $\lambda^t \in R_+$ satisfies $\sum_{i \in N} min\{c^t_i, \lambda^t\} = E^t$ and $EG^t(c^t, E^t) = (EG^t_i(c^t, E^t))_{i \in N}$ then $EG(c, E) = (EG^t(c^t, E^t))_{t \in T}$.

The **Equal Losses** rule equalizes the losses agents incur in each period, subject to no agent receiving a negative share: For period t for each $i \in N$, $EL^t_i(c^t, E^t) = max\{0, c^t_i - \lambda^t\}$ where $\lambda^t \in R_+$ satisfies $\sum_{i \in N} max\{0, c^t_i - \lambda^t\} = E^t$ and $EL^t(c^t, E^t) = (EL^t_i(c^t, E^t))_{i \in N}$ then $EL(c, E) = (EL^t(c^t, E^t))_{t \in T}$.

We will define the dictatorial rule in which one of the agents is fully satisfied, that is the share of that agent is equal to his characteristics: the **agent i Dictatorial rule** is $D^i_i(c, E) = c_i$ for some $i \in N$.

These rules are independent of R thus almost always violate Pareto optimality.

Let's give an example to violating Pareto optimality by using static rules in each period:

Example 1 *Assume that* $E = (10, 10)$, $c_1 = (7, 7)$ *and* $c_2 = (7, 7)$ *and the preferences of agents are represented with the following utility functions* $u_1(x^1_1, x^2_1) = (x^1_1)^{0.2}(x^2_1)^{0.8}$ *and* $u_2(x^1_2, x^2_2) = (x^1_2)^{0.8}(x^2_2)^{0.2}$. *Note that* $PRO(c, E) = EG(c, E) = EL(c, E) = (x^1_1, x^2_1, x^1_2, x^2_2) = (5, 5, 5, 5)$ *which is not Pareto optimal since* $u_1(2, 8) > u_1(5, 5)$ *and* $u_2(8, 2) > u_2(5, 5)$ *where* $(2, 8, 8, 2) \in X(E)$.

3.2 Market Mechanisms

One way to satisfy Pareto optimality is using a market mechanism and this is at the focus of our analysis. A static rule determines the shares of the agents in each period and this serves as an initial allocation. After the initial allocation, agents trade their shares in the market. An example of these markets is the *Chicago exchange permit market* in the USA. After an initial allocation by *EPA,* firms in the state of Illinois exchange permits in the Chicago exchange permit market.

We will define the Walrasian rule coupled with an initial allocation rule for dynamic simple allocation problems:

Definition 1 *Let F be a static rule. The Walrasian rule from F, $W_F : \mathcal{A} \rightrightarrows \mathbb{R}^4_+$ is defined as follows: for all $x \in W_F(R, c, E)$ there exists $p \in \triangle$ such that for all $i \in N$ $px_i \leqq pF_i(c, E)$ and for all $x_i^{'} \in \mathbb{R}^2_+$ with $px_i' \leqq pF_i(c, E)$ we have $x_i R_i x_i'$.*

In Example 1, under PRO, EA and EL rule $(5, 5, 5, 5)$ is the initial endowment for the Walrasian rule and $\{(2, 8, 8, 2)\} = W_F(R, c, E)$. Note that in the second period, agent 1 gets higher than his characteristic value, that is $8 > c_1^2 = 7$. However, in some simple allocation problems, the agents are not allowed to exceed their characteristics. For example in the permit allocation problem, some pollutants are local and since these pollutants are absorbed in that region, agents are not allowed to pollute more than their local constraints. For such cases, we will introduce two alternative Walrasian rules. In the first one agents maximize their utilities according in a feasible set which is constrained by their own characteristics. In the second version of the Walrasian rule, agents maximize their utilities in a feasible set which is constrained by the characteristic vector. For the alternative Walrasian rules, we define that the **constrained feasible set** is $X(c, E) = \{x = (x_i)_{i \in N} |$ for all $x_i \in \mathbb{R}^2_+$ $x_i \leqslant (c_i \wedge E)$ for all $i \in N\}$.

Definition 2 *Let F be a static rule. The constrained Walrasian rule from F, $W_F^c : \mathcal{A} \rightrightarrows \mathbb{R}^4_+$ is defined as follows: $x \in W_F^c(R, c, E)$ if $x \in X(c, E)$ there exists $p \in \triangle$ such that and for all $i \in N$* ***(i)*** *$px_i \leqq pF_i(c, E)$ and* ***(ii)*** *for all $x_i' \in \mathbb{R}^2_+$ with $x_i' \leq c_i$ and $px_i' \leqq pF_i(c, E)$, we have $x_i R_i x_i'$.*

In Example 1, while $F \in \{PRO, EG, EL\}$ the constrained Walrasian rule is $W_F^c = \{(4, 7, 6, 3), (3, 6, 7, 4), (3, 7, 7, 3)\}$. None of the elements of the W_F^c is Pareto optimal in this example. However, for allocations where the agents' shares are bounded with their characteristics, one of the agents cannot be better off without worsening the other agent. We will thus define constrained optimality for this $X(c,E)$: The **constrained Pareto optimal set** is $CPO(R,c,E) = \{x \in X(c,E)|$ there is no $x' \in X(c,E)$ such that $x'_i R_i x_i$ for all $i \in N$ and $x'_j P_j x_j$ for some $j \in N\}$.

For Example 1, all three allocations chosen by W^c_F are constrained Pareto optimal that is $\{(4,7,6,3),(3,6,7,4),(3,7,7,3)\} \in CPO(R,c,E)$.

The second version of the Walrasian rule is the following:

Definition 3 *Let F be a static rule. The restricted constrained Walrasian rule from F, $W^{rc}_F : \mathcal{A} \rightrightarrows R^4_+$ is defined as follows: $x \in W^{rc}_F(R,c,E)$ if $x \in X(c,E)$ and there exists price $p \in \triangle$ such that for all $i \in N$* ***(i)*** *$px_i \leqq pF_i(c,E)$ and* ***(ii)*** *for all $x'_i \in R^2_+$ with $x'_i \in [E - \sum_{N\setminus\{i\}} c_j,$ $c_i]$ and $px'_i \leqq pF_i(c,E)$ we have $x_i R_i x'_i$.*

In Example 1, $\{(3,y,7,10-y),(x,7,10-x,3)|\ y \in [6,7]$ and $x \in [3,4]\} = W^{rc}_F(R,c,E)$ where $F \in \{PRO, EG, EL\}$. Although the intersection of the W^{rc}_F and the $PO(R,c,E)$ is empty, all of the elements of $W^{rc}_F(R,c,E)$ are constrained Pareto optimal that is $W^{rc}_F(R,c,E) \subset CPO(R,c,E)$.

Observe that in Example 1 W^c_F choose a proper subset of W^{rc}_F. This is because the budget set in W^{rc}_F is a subset of the budget set of W^c_F. (See Figure 1 and Figure 2)

The following result shows this relationship in general:

Proposition 1 *For all F, constrained Walrasian rule from F, W^c_F, is a subset of restricted constrained Walrasian rule from F, W^{rc}_F, that is $W^c_F \subseteq W^{rc}_F$.*

Proof. Let $(R,c,E) \in \mathcal{A}$ and $x \in W^c_F(R,c,E)$ with price $p \in \triangle$. Since $x \in X(c,E)$ by definition of W^c_F for all $x'_i \in \mathbb{R}^2_+$ where $x'_i \leq c_i$ with $px'_i \leqq pF_i(c,E)$ we have $x_i R_i x'_i$. So, for all $i \in N$ for all $x''_i \in [E - \sum_{N\setminus\{i\}} c_j,\ c_i]$ with $px''_i \leqq pF_i(c,E)$ we have $x_i R_i x''_i$. Therefore $x \in W^{rc}_F(R,c,E)$. ■

Moreover if the Walrasian rule from F picks an element x of the constrained feasible set, x will also be picked by the constrained Walrasian rule from F and the restricted constrained Walrasian rule from F:

Proposition 2 *If $x \in W_F(R,c,E)$ and $x \in X(c,E)$ then $x \in W^c_F(R,c,E)$ and $x \in W^{rc}_F(R,c,E)$.*

Figure 1: The shaded area is agent 1's budget set for W_F^c. The bundle x maximizes R_1 in the shaded budget set.

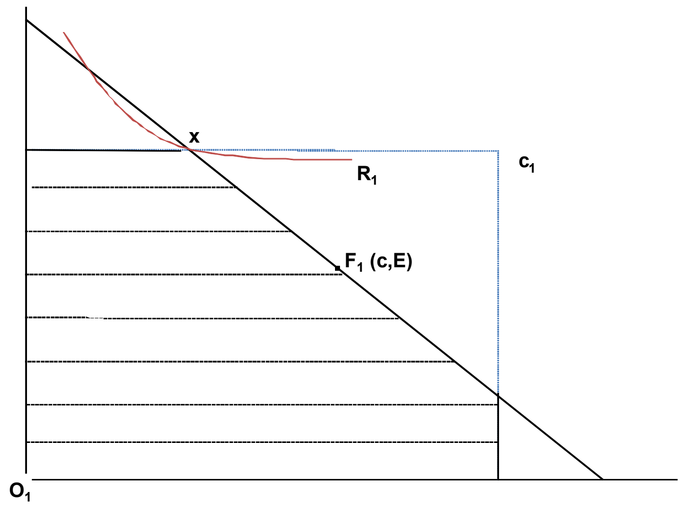
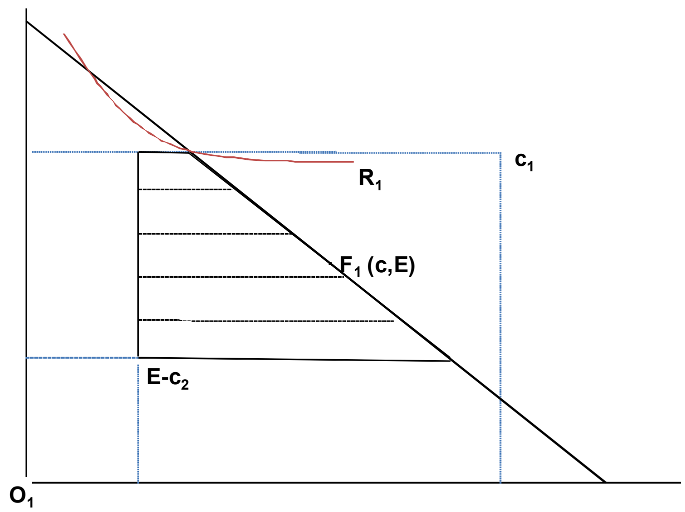


Figure 2: The shaded area is agent 1's budget set for W_F^{rc}. The bundle x maximizes R_1 in the shaded budget set.



Proof. Suppose that $x \in W_F(R, c, E)$. Then there exists $p \in \triangle$ and there exists $F(c, E)$ such that for all $i \in N$, $px_i \leqq pF_i(c, E)$ and for all $x_i^{'} \in \mathbb{R}_+^2$ with $px_i' \leqq pF_i(c, E)$ we have $x_i R_i x_i'$. So, for all $x_i^{"} \in [0,\ c_i]$ with $px_i'' \leqq pF_i(c, E)$ we have $x_i R_i x_i''$. Therefore $x \in W_F^c(R, c, E)$. Finally, since $W_F^c \subseteq W_F^{rc}$ so $x \in W_F^{rc}(R, c, E)$. ■

4 RESULTS

First we will introduce some standard axioms. Then we will check which rules satisfy them.

Since we have already defined Pareto optimality and constrained Pareto optimality, we will not define them here.

Under our assumptions, W_F satisfies Pareto optimality. On the other hand we have a negative result for the other two Walrasian rules.

Proposition 3 *For all F, W_F^c and W_F^{rc} violate Pareto optimality.*

Proof. Suppose $E = (10, 10)$ and $c = (10, 0, 0, 10)$ and consider the utility functions: $u_i = (x_i^1)^{0.5}(x_i^2)^{0.5}$ for each $i \in N$. For any F, since $\sum_{i \in N} c_i^t = E^t$ for each $t \in T$ we have $F(c, E) = (10, 0, 0, 10)$ and also $W_F^c(R, c, E) = W_F^{rc}(R, c, E) = (10, 0, 0, 10)$. Since $u_1(5, 5) > u_1(10, 0)$ and $u_2(5, 5) > u_2(0, 10)$ W_F^c and W_F^{rc} violate Pareto optimality for all F. ■

Since constrained Pareto optimality is a weaker notion of Pareto optimality, Walrasian rule also satisfies this property. Since the optimality is constrained with the feasible set, constrained Walrasian rule and restricted constrained Walrasian rule also satisfy constrained Pareto optimality.

Proposition 4 *For all F, W_F^{rc} and W_F^c satisfy constrained Pareto optimality.*

Proof. Let $(R, c, E) \in \mathcal{A}$ and $x^{rc} \in W_F^{rc}(R, c, E)$ with price $p \in \triangle$. Suppose there exists a $\tilde{x} \in X(c, E)$ such that for all $i \in N$ we have $\tilde{x}_i R_i x_i^{rc}$. Then by continuity, convexity and monotonicity of the preference relation, for all $i \in N$ we must have $p\tilde{x}_i \geq pF_i(c, E)$ and strict inequality for some $j \in N$. Hence, $\sum p\tilde{x}_i > \sum pF_i(c, E)$. However since $\tilde{x} \in X(c, E)$ then $\sum_{i \in N} \tilde{x}_i \leqq \sum_{i \in N} F_i(c, E)$. Contradiction. Therefore x^{rc} is a constrained Pareto optimal. With similar reasoning above W_F^c also satisfy constrained Pareto optimality. ■

One of the most basic property of a simple allocation problem is that an agent should at least 0 or the remaining endowment after all other agents are fully satisfied. We relates this property to our model: An allocation rule G satisfies **respect of minimal rights** for all $(R, c, E) \in \mathcal{A}$ if $x \in G(R, c, E)$ then $x \in X(c, E)$.[4] Since the Walrasian rule is not constrained with the characteristics of agents, Walrasian rule violates respect of minimal rights.

Proposition 5 *For all F, W_F violates respect of minimal rights.*

Proof. Suppose $E = (10, 10)$ and $c = (10, 0, 0, 10)$ and consider the utility functions: $u_i = (x_i^1)^{0.5}(x_i^2)^{0.5}$ for each $i \in N$. For any F, since $\sum_{i \in N} c_i^t = E^t$ for each $t \in T$ we have $F(c, E) = (10, 0, 0, 10)$ and also $W_F(R, c, E) = (5, 5, 5, 5)$. However the minimal right of agent 1 is $(10, 0)$ and the minimal right of agent 2 is $(0, 10)$. Then W_F violates respect of minimal rights for all F. ■

On the other hand, the other two Walrasian rules satisfies this property:

Proposition 6 *For all F, W_F^{rc} and W_F^c respect minimal rights.*

Proof. Let $(R, c, E) \in \mathcal{A}$ and $x^{rc} \in W_F^{rc}(R, c, E)$ with price $p \in \triangle$. Since $x^{rc} \in X(c, E)$, W_F^{rc} satisfies respect of minimal rights for all F. With similar reasoning above W_F^c also satisfies respect of minimal rights. ■

Now we will introduce some axioms on fairness: An allocation rule G satisfies **no-envy** if no agent wants to change his share with any other agent, that is for all $(R, c, E) \in \mathcal{A}$ if $x \in G(R, c, E)$ is envy free then for all $i, j \in N$ we have $x_i R_i x_j$. All of the Walrasian rule violates this property.

Proposition 7 *For all F, W_F, W_F^c and W_F^{rc} violate no-envy.*

Proof. Let's look at the following example. Let $E = (10, 10)$ and $c = (9, 9, 1, 1)$ and the utility functions: $u_i = (x_i^1)^{0.5}(x_i^2)^{0.5}$ for each $i \in N$. For any F, since $\sum_{i \in N} c_i^t = E^t$ for each $t \in T$ we have $F(c, E) = (9, 9, 1, 1)$ and also $W_F(R, c, E) = W_F^c(R, c, E) = W_F^{rc}(R, c, E) = (9, 9, 1, 1)$. Since $u_2(9, 9) > u_2(1, 1)$, W_F, W_F^c and W_F^{rc} violate no-envy for all F. ■

[4] The minimal right of an agent is the maximum of 0 and the remaining amount after the other agents are fully satisfied with their claims, that is the shares of other agents are equal to their claims. For all $i \in N$, $x_i^t \geq \max\{E^t - \sum_{N \setminus \{i\}} c_j^t, 0\}$ for all $t \in T$. Since this should hold for all $i \in N$, $x \in X(c, E)$.

As discussed earlier, in some dynamic simple allocation problems agents are not allowed to be allocated more than their characteristics. We introduce this constraint to the no-envy axiom: An allocation rule G satisfies **constrained no-envy** if an agent does not want to change his share with any other agents' share which is constrained with his own characteristics, that is for all $(R, c, E) \in \mathcal{A}$ if $x \in G(R, c, E)$ is constrained envy free if for all $i, j \in N$ we have $x_i R_i (c_i \wedge x_j)$.

Proposition 8 *For $F \in \{PRO, EL\}$, W_F ,W_F^c and W_F^{rc} violate constrained envy freeness.*

Proof. Let $E = (8, 8)$ and $c = (7, 7, 3, 3)$ and for each $i \in N$ the utility functions: $u_i = (x_i^1)^{0.5}(x_i^2)^{0.5}$. $W_{PRO}(R, c, E) = W_{PRO}^c(R, c, E) = W_{PRO}^{rc}(R, c, E) = (5.6, 5.6, 2.4, 2.4)$. Since $(5.6, 5.6) > (2.4, 2.4)$, $\hat{x}_2 = (3, 3)$ and $\hat{x}_2 P_2 x_2$. If $F = EL$ then $W_{EL}(R, c, E) = W_{EL}^c(R, c, E) = W_{EL}^{rc}(R, c, E) = (6, 6, 2, 2)$. Since $(6, 6) > (2, 2)$ $\hat{x}_2 = (3, 3)$ and $\hat{x}_2 P_2 x_2$. Therefore for $F = \{PRO, EL\}$, W_F, W_F^c and W_F^{rc} violate constrained envy freeness. ■

Kıbrıs (2003) introduces a property which requires that an agent can only have the right to envy others whose characteristics are not greater than his characteristics: An allocation rule G satisfies **hierarchical no-envy** for all $(R, c, E) \in \mathcal{A}$ if $x \in G(R, c, E)$ and for all $i, j \in N$, if $c_i \geq c_j$ we have $x_i R_i x_j$. Before showing whether W_F, W_F^c and W_F^{rc} satisfy or not this property, we give define another axiom in which the shares of agents are ranked due to their characteristic values. A static rule F satisfies **order preservation** for all $(c, E) \in \mathcal{B}$ for all $i, j \in N$ if $c_i \geq c_j$, we have $F_i(c, E) \geq F_j(c, E)$. Most of the static rules such as Proportional Rule, Equal Gains and Equal Losses satisfy order preservation.

Proposition 9 *If F satisfies order preservation then W_F, W_F^c and W_F^{rc} satisfy hierarchical no-envy.*

Proof. Assume that $c_j \leqq c_i$ for some $i, j \in N$. Without loss of generality assume that $c_2 \leq c_1$. So only Agent 1 has a right to envy agent 2.

$\mathbf{W}_F$: Assume that $x \in W_F(R, c, E)$ with the price vector $p \in \triangle$. Since $c_2 \leq c_1$ by order preservation of F we have $F_2(c, E) \leqq F_1(c, E)$. Therefore $pF_2(c, E) \leqq pF_1(c, E)$. By definition of W_F we have $px_2 \leqq pF_2(c, E)$. Binding these two inequalities we have $px_2 \leqq pF_1(c, E)$. Again by definition of W_F for all $x_1' \in \mathbb{R}_+^2$ with $px_1' \leqq pF_1(c, E)$ we have $x_1 R_1 x_1'$. Therefore $x_1 R_1 x_2$.

$\mathbf{W}_F^{rc}$: Suppose that $x^{rc} \in W_F(R, c, E)$ with the price $p \in \triangle$. Since $c_2 \leq c_1$ by order preservation of F we have $F_2(c, E) \leqq F_1(c, E)$. Therefore $pF_2(c, E) \leqq pF_1(c, E)$. By definition of W_F^{rc} we have $px_2^{rc} \leqq pF_2(c, E)$. Binding these two inequalities we have $px_2^{rc} \leqq pF_1(c, E)$. Again by definition of W_F^{rc} for all $\tilde{x}_1 \in \mathbb{R}_+^2$ and $\tilde{x}_1 \in [E - c_2, c_1]$ with $p\tilde{x}_1 \leqq pF_1(c, E)$ we have $x_1^{rc} R_1 \tilde{x}_1$. Moreover since $x_2^{rc} \in [E - c_2, c_1]$ or $x_2^{rc} \leqq E - c_2$ we have $x_1^{rc} R_1 x_2^{rc}$. Note that if $x_2^{rc} \leqq E - c_2$ by monotonicity of R_1 since we have $x_1^{rc} R_1 (E - c_2)$ therefore $x_1^{rc} R_1 a$ for all $a \leqq E - c_2$.

Since with similar reasoning above, if F satisfies order preservation W_F^c also satisfies Hierarchical Envy Free. ■

In the following table, we summarize our general findings so far:

Axioms	$\mathbf{W}_F$	$\mathbf{W}_F^c$	$\mathbf{W}_F^{rc}$
Pareto Optimality	✓	×	×
Constrained Pareto Optimality	✓	✓	✓
Respect of Minimal Rights	×	✓	✓
No − envy	✓	×	×
Hierarchical No − envy[5]	✓	✓	✓

4.1 Manipulation

In this subsection, we study manipulability of dynamic rules. In our setting, there can be two types of manipulation, manipulation via preferences and manipulation via characteristics. Manipulation via preferences or strategy-proofness is studied by Hurwicz (1972) in the exchange economies. Turhan (2009) looks for strategy proof allocations in dynamic bankruptcy problems in which preferences are linear.

An allocation rule satisfies **strategy-proofness** if an agent cannot be better of by misrepresenting his preferences, that is for all $(R, c, E) \in \mathcal{A}$ if $x \in G(R, c, E)$ and for all $i \in N$ and for all $x' \in G(R', c, E)$ where $R' = (R_i', R_j)$ we have $x_i R_i x_i'$. As Hurwicz (1972) proves that agents can be better off with misrepresenting preferences in exchange economies, we have similar result for Walrasian rule case:

Proposition 10 *For all F, W_F violates strategy-proofness.*

[5] F satisfies order preservation for hierarchical no-envy property.

Proof. Let's look at the following example: Let $E = (10, 10)$ and $c = (10, 0, 0, 10)$ and the utility functions of agents: $u_1(x_1^1, x_1^2) = (x_1^1)^{0.8}(x_1^2)^{0.2}$ and $u_2(x_2^1, x_2^2) = (x_2^1)^{0.2}(x_2^2)^{0.8}$. For all F, $F(c, E) = (10, 0, 0, 10)$ and $W_F(R, c, E) = (8, 2, 2, 8)$. If agent 1 misrepresents his preferences as: $\hat{u}_1(x_1^1, x_1^2) = (x_1^1)^{0.2}(x_1^2)^{0.8}$ then $W_F(\tilde{R}, c, E) = (8, 8, 2, 2)$. So, $u_1(8, 8) = 8 > 5 = u_1(5, 5)$. ■

Although the Walrasian rule violates strategy-proofness, manipulation via preferences for the other Walrasian rules can be prevented by using the dictatorship rule for initial allocation:

Proposition 11 *W_F^c and W_F^{rc} satisfy strategy-proofness if and only if for every $(c, E) \in \mathcal{B}$ there exists $i \in N$ such that $F(c, E) = D^i(c, E)$.*

Proof. $\Leftarrow$ For all $(R, c, E) \in \mathcal{A}$ the share of agent i by the dictatorial rule is his characteristic value, that is $D_i^i(c, E) = c_i$. Then $W_{D^i}^c(R, c, E) = W_{D^i}^{rc}(R, c, E) = (c_i, E - c_i)$. This is because u_i attains maximum at c_i in the $X(c, E)$ and W_F^c and W_F^{rc} satisfy individually rationality. Since the final allocation is independent of the agents' preferences, agents cannot be better off by manipulating preferences.

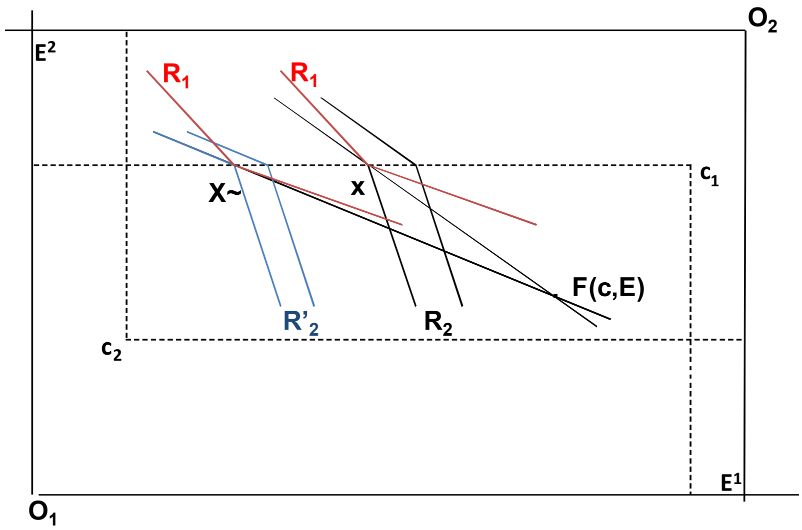
$\Rightarrow$ Assume W_F^c and W_F^{rc} satisfy strategy-proofness and suppose for every $i \in N$ $F \neq D^i$. Then there exists an $(c, E) \in \mathcal{B}$ such that $F(c, E) = w$. Suppose $x^{rc} \in W_F^{rc}(R, c, E)$ and $x^{rc} \in \partial X(c, E)$. Then there exists a price $p \in \triangle$ such that for all $i \in N$ we have $px_i^{rc} \leqq pw_i$ and for all $x_i' \in [E - c_{-i},\ c_i]$ with $px_i' \leqq px_i^{rc}$ we have $x_i^{rc} R_i x_i'$. Assume that agent j misrepresents his preference relation such that $R' = (R_i, R_j')$. For this preference relation, we can find a price vector $p' \in \triangle$ such that for each $i \in N$ for all $x_i'' \in \mathbb{R}_+^2$ and $x_i'' \in [E - c_{-i}, c_i]$ with $p'x_i'' \leqq p'w_i$ we have $\tilde{x}_i^{rc} R_i x_i''$ such that $\tilde{x}_j^{rc} P_j x_j^{rc}$ (look at the Figure 3). So W_F^{rc} violates strategy-proofness if $F \neq D^i$.

With same reasoning above W_F^c satisfy strategy-proofness if and only if for every $(c, E) \in \mathcal{B}$ there exists $i \in N$ such that $F(c, E) = D^i(c, E)$. ■

The second type of manipulation can be via characteristics. We will show some three types of manipulation. These manipulation types are related with the Postlewaite (1979).

In the first manipulation type, agents can make a coalition and in a fixed period exchange their characteristic value, that is $\sum_{i \in N} c_i^t = \sum_{i \in N} \hat{c}_i^t$ for all $t \in T$. An allocation rule G satisfies **no–manipulation via static coalition** for all $(R, c, E) \in \mathcal{A}$ and for all $S \subsetneqq N$, if $x \in G(R, c, E)$ then for all $\hat{c} \in \mathbb{R}_+^4$ such that $\hat{c} = (\hat{c}_S, c_{N \setminus S})$ with for all $t \in T$ $\sum_{i \in S} c_i^t = \sum_{i \in S} \hat{c}_i^t$ and for all $x' \in G(R, \hat{c}, E)$ we have $x_i R_i x_i'$ for each $i \in S$.

Figure 3: Agent 2 is better off by misrepresenting his preference in a special case where agents' preferences are linear.



Proposition 12 *If $|N| = 2$, Walrasian rule satisfies no-manipulation via static coalition.*

Proof. Since the allocation by Walrasian rule is Pareto optimal agents cannot be better off by manipulation via static coalition. ■

However if $|N| \geq 3$ we have a negative result:

Remark 1 *If $|N| \geq 3$ for all F, W_F violates no–manipulation via static coalition. For a dynamic simple allocation problem $(R, c, E) \in \mathcal{A}$ assume that $\sum_{i \in N} c_i^t = E^t$ for all $t \in T$. Then for all F, $F_i(c, E) = c_i$ for all $i \in N$ which implies that characteristics of the agents have become the initial endowment. Then characteristic values are the initial endowments of agents and summation of the characteristic values is the total amount for an exchange economy. Postlewaite (1979) proves that any mechanism for an exchange economy is manipulable via coalition.*[6] *Therefore for all F, W_F violates no–manipulation via static coalition.*

Since the allocation by constrained Walrasian rule and restricted constrained Walrasian rules is constrained with the characteristics vector, agents can exchange their characteristics and be better off:

Proposition 13 *For all F, W_F^c and W_F^{rc} violate no–manipulation via static coalition.*

Proof. Consider the following example: Suppose $E = (10, 10)$ and $c = (10, 0, 0, 10)$ and the utility functions $u_1 = (x_1^1)^{0.5}(x_1^2)^{0.5}$ and $u_2 = (x_2^1)^{0.5}(x_2^2)^{0.5}$. Since $\sum_{i \in N} c_i^t = E^t$ for all $t \in T$, for all F we have $F(c, E) = (10, 0, 0, 10)$. Then $W_F^{rc}(R, c, E) = W_F^{rc}(R, c, E) = (10, 0, 0, 10)$. So $u_1(10, 0) = u_1(0, 10) = 0$. Now if agents make a coalition in which the characteristic vector is $\tilde{c} = (5, 5, 5, 5)$ then they will be better off since with the same reason above for all F, $F(\tilde{c}, E) = (5, 5, 5, 5)$ and $W_F^{rc}(R, \tilde{c}, E) = W_F^{rc}(R, \tilde{c}, E) = (5, 5, 5, 5)$ that is $u_1(5, 5) = u_2(5, 5) = 5 > 0$. ■

The following axiom is weaker than no–manipulation via static coalition. An allocation rule G satisfies **no–manipulation via dynamic coalition** for all $(R, c, E) \in \mathcal{A}$ and for all $S \subsetneqq N$ if $x \in G(R, c, E)$ then for all $\hat{c} \in \mathbb{R}_+^4$ such that $\hat{c} = (\hat{c}_S, c_{N \setminus S})$ satisfies $\sum_{t \in T} \sum_{i \in S} c_i^t =$

[6] Postewaite (1979) discusses a mechanism is C-manipulable if there exists a subset (coalition) of agents in which all agents gain by trading their initial endowment before they enter the exchange economy. Here since claims are fixed in a time, it can be interpreted that C-manipulable is the same argument with manipulation via static coalition.

$\sum_{t\in T}\sum_{i\in S}\hat{c}_i^t$ and for all $x' \in G(R,\hat{c},E)$ we have $x_i R_i x_i'$ for each $i \in S$. Since no-manipulation via coalition is a weaker axiom of no–manipulation via static coalition we have the following results:

Proposition 14 *For all F, W_F satisfies no–manipulation via dynamic coalition for $|N| = 2$.*

Proof. For $|N| = 2$ the proof is similar with the proof of proposition 12. ■

Remark 2 *For all F, W_F violates this property for $|N| \geq 3$. The argument is similar with remark 1. The manipulation via static coalition in the example of remark 1 is also manipulation via dynamic coalition. Therefore we have the similar result.*

Proposition 15 *For all F, W_F^c and W_F^{rc} violate no–manipulation via dynamic coalition.*

Proof. Consider the same example with proposition 14. The manipulation via static coalition in that example is also manipulation via dynamic coalition. Therefore we have the similar result. ■

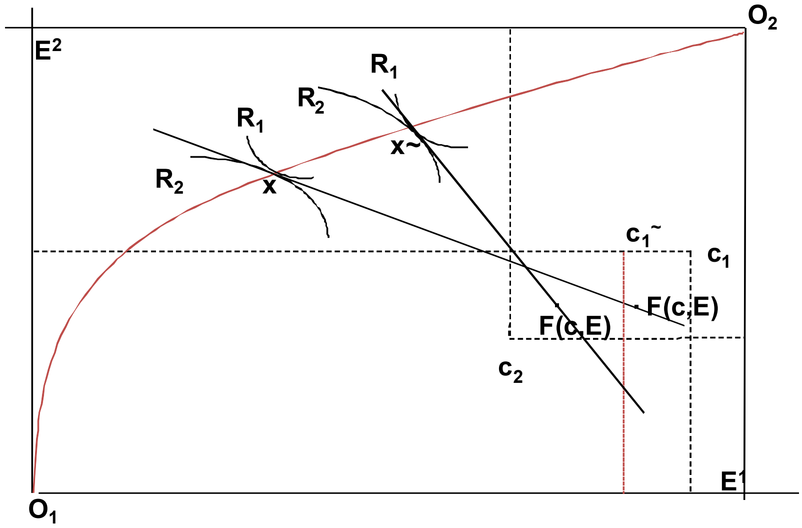
In the following manipulation type, since the destruction of characteristics affects the initial allocation an agent can destroy some of his characteristic value to be better off: An allocation rule G satisfies **no–manipulation via destruction of characteristics** for all $(R,c,E) \in \mathcal{A}$ if $x \in G(R,c,E)$ then for all $\hat{c} \in \mathbb{R}_+^4$ such that for some $i \in N$ and $t \in T$ $\hat{c}_i^t \leq c_i^t$ and for all $x' \in G(R,\hat{c},E)$ for all $i \in N$ we have $x_i R_i \hat{x}_i$.

For the exchange economies, an agent can be better off via transferring his initial endowment to the other agents. This is called **transfer paradox**. For the Walrasian rule case we have similar result with transfer paradox:

Proposition 16 *For all F, W_F violates no-manipulation via destruction of characteristics.*

Proof. Suppose there exists an F such that W_F satisfies no-manipulation via destruction of characteristics. There exists $(R,c,E) \in \mathcal{A}$ such that $F(c,E) = w$. Suppose $x \in W_F(R,c,E)$. Suppose agent 1 destroys some of his characteristic value so that $\tilde{c}_1 = (\tilde{c}_1^1, c_1^2)$ satisfies $\tilde{c}_1^1 < w_1^1$ then the characteristic vectors becomes $\tilde{c} = (\tilde{c}_1, c_2)$ and for $(R,\tilde{c},E) \in \mathcal{A}$ the initial allocation becomes $F(\tilde{c},E) = \tilde{w}$ and we can find a price vector $\tilde{p} \in \triangle$ such that for all $i \in N$ for all $\tilde{x}_i' \in \mathbb{R}_+^2$ with $p\tilde{x}_i' \leqq pw_i$ we have $\tilde{x}_i R_i \tilde{x}_i'$ therefore $\tilde{x} \in W_F(R,\tilde{c},E)$ and $\tilde{x}_1 P_1 x_1$. (see Figure 4) ■

Figure 4: Agent 1 is better of by destruction of his first characteristic value.



This means that the EPA cannot prevent manipulation via destruction of characteristics for the global pollutants.

Let's give an example:

Example 2 *(Original example is in MWG[14]). Suppose* $E = (2+r, 2+r)$ *where* $r = 2^{8/9} - 2^{1/9} \simeq 0.7717$*. Let* $c = (E, E, r, 2)$ *and let the utilities of agents be* $u_1(x_1^1, x_1^2) = x_1^1 - 1/8(x_1^2)^{-8}$ *and* $u_2(x_2^1, x_2^2) = -1/8(x_2^1)^{-8} + x_2^2$*. Suppose* $F = D^2$ *so* $F(c, E) = (E - c_2, c_2)$*. Then there exists 3 Walras equilibria in which the allocations and relative prices are:*

	$x_1 \simeq$	$u_1(x_1) \simeq$	$x_2 \simeq$	$u_2(x_2) \simeq$
$p = 2$	$1.8458, 1.0801$	1.7783	$0.9259, 1.6916$	1.4602
$p = 1$	$1.7717, 1$	1.6467	$1, 1.7717$	1.6467
$p = 0.5$	$1.6916, 0.9259$	1.4602	$1.0801, 1.8458$	1.7783

Now, assume that agent 2 destroys 10^{-4} *amount of his second characteristics such that* $\tilde{c}_2^2 = (r, 2 - 10^{-4})$*. After initial allocation there exists also 3 Walras equilibria:*

	$x_1 \simeq$	$u_1(x_1) \simeq$	$x_2 \simeq$	$u_2(x_2) \simeq$
$p = 2.01$	$1.8463, 1.0807$	1.7791	$0.9254, 1.6910$	1.4586
$p = 0.9835$	$1.7698, 0.9982$	1.6430	$1.0019, 1.7735$	1.6504
$p = 0.506$	$1.6931, 0.9271$	1.4641	$1.0786, 1.8446$	1.7764

With $p = 1$ his utility is $u_2(x_2) \simeq 1.6467$ and when he destroys some of his characteristics then with $p = 0.9835$ his utility becomes $u_2(x_2) \simeq 1.6504$. So we can see that agent 2 can gain by destroying some of his characteristic value even he is fully satisfied by the initial allocation rule.

There is a large literature on manipulation. For exchange economies, Postlewaite (1979) constructs a mechanism, γ, in order to prevent manipulation via destruction of initial endowment. Since destruction of characteristics directly affects the initial allocation, we relate this γ mechanism to our work. Let's consider the following a dynamic simple allocation problem: $A = (R, c, E)$ and for all $i \in N$ let R_i be represented by the continuous utility function u_i and let $w = F(c, E)$. Since $X(E)$ is compact and utility functions are continuous, then there exists $\bar{x} \in X(E)$ such that $V(x) = \min_{i \in N} u_i(x_i) - u_i(w_i)$ achieves a maximum at $\bar{x}$ and $u_i(\bar{x}_i) - u_i(w_i) = u_j(\bar{x}_j) - u_j(w_j)$ for all $i, j \in N$. Since $V(x)$ achieves a maximum at $\bar{x}$ and $u_i(\bar{x}_i) - u_i(w_i) = u_j(\bar{x}_j) - u_j(w_j)$ for all $i, j \in N$ there is no $\hat{x} \in X(E)$ such that both agents will be better of with $\hat{x}$, that is a Pareto optimal allocation. Finally, since utility functions are strictly concave, there exists unique $\bar{x}$. The mechanism $\gamma_F : \mathcal{A} \to X(E)$ such that $\gamma_F(R, c, E) = \bar{x}$.

For some dynamic simple allocation problems in which agents are not allowed to be allocated more than characteristics. For others we define constrained mechanism is $\gamma_F^c : \mathcal{A} \to X(c, E)$ such that $\gamma_F^c(R, c, E) = \bar{x}$ where $\bar{x} \in X(c, E)$ maximizes $V(x)$ in the constrained feasible set $X(c, E)$.

We show that under some conditions these mechanisms can prevent manipulation via destruction of characteristics:

Proposition 17 *Suppose F satisfies order preservation then γ_F and γ_F^c satisfy no manipulation via destruction of characteristics.*

Proof. Assume that F satisfies order preservation and suppose γ_F is manipulable via destruction of characteristics. Let $F(c, E) = w$ and $\gamma_F(R, c, E) = \bar{x}$. Assume one of the agents destroys some of his characteristics (without loss of generality agent 1) such that

$\hat{c}_1 \leq c_1$ then let $F(\hat{c}, E) = \hat{w}$ and $\gamma_F(R, \hat{c}, E) = \hat{x}$ and suppose $u_1(\hat{x}_1) > u_1(\bar{x}_1)$. Note by order preservation of F we have $\hat{w}_1 \leq w_1$ and since $E - w_1 \leq E - \hat{w}_1$ therefore $\hat{w}_2 \geq w_2$. Then $u_1(\hat{x}_1) - u_1(w_1) > u_1(\bar{x}_1) - u_1(w_1)$ by equal gain property of γ_F we have $u_1(\bar{x}_1) - u_1(w_1) = u_2(\bar{x}_2) - u_2(w_2)$. Since γ_F satisfies Pareto optimality then $\hat{x}, \bar{x}$ are a Pareto optimal allocations and since $u_1(\hat{x}_1) > u_1(\bar{x}_1)$ we have $u_2(\hat{x}_2) < u_2(\bar{x}_2)$.Therefore $u_2(\bar{x}_2) - u_2(w_2) > u_2(\hat{x}_2) - u_2(w_2)$. So, $u_1(\hat{x}_1) - u_1(w_1) > u_2(\hat{x}_2) - u_2(w_2)$ which violates the equal gain property of γ_F.

For γ_F^c, the proof is similar with the γ_F. ■

If F violates order preservation then γ_F mechanism can violate manipulation via destruction of characteristics, because violation of order preservation leads an increase the level of wealth of the agent who destroys his characteristics.

We then inquire whether these mechanisms can prevent other manipulation types:

Proposition 18 *If $|N| = 2$, for all F γ_F satisfies no manipulation via dynamic coalition and no–manipulation via static coalition.*

Proof. Since $\gamma_F(R, c, E)$ is a Pareto optimal allocation then an agent cannot gain without worsening the other agent and since no–manipulation via static coalition is a weaker axiom of no–manipulation via dynamic coalition. ■

Proposition 19 *If $|N| \geq 3$, for all F, γ_F violates manipulation via static coalition and no–manipulation via dynamic coalition.*

Proof. Suppose $E = (9, 9)$. Let $c_1 = (6, 6)$, $c_2 = (0, 3)$ and $c_3 = (3, 0)$. The utility functions are for all $i \in N$: $u_i(x_i^1, x_i^2) = (x_i^1)^{0.5}(x_i^2)^{0.5}$. Since $\sum_{i \in N} c_i^t = E^t$ for all $t \in T$, for all F we have $F(c, E) = w = (6, 6, 0, 3, 3, 0)$. The outcome of the mechanism is $\gamma_F(R, c, E) = (x_1, x_2, x_3)$ where $x_1 = (7, 7)$, $x_2 = (1, 1)$ and $x_3 = (1, 1)$. The gain is $u_i(w_i) - u_i(x_i) = 1$ for all $i \in N$. Suppose $S = \{2, 3\}$ and they make a coalition and exchange their characteristics such that $\tilde{c}_2 = (3/2, 3/2)$ and $\tilde{c}_3 = (3/2, 3/2)$. Then $F(c, E) = (6, 6, 3/2, 3/2, 3/2, 3/2)$ and the outcome of the mechanism becomes $\gamma_F(R, c, E) = (\tilde{x}_1, \tilde{x}_2, \tilde{x}_3)$ where $\tilde{x}_1 = (6, 6)$, $\tilde{x}_2 = (3/2, 3/2)$ and $\tilde{x}_3 = (3/2, 3/2)$ in which there is no gain for all $i \in N$. However, $u_j(\tilde{x}_j) = 3/2 > 1 = u_j(x_j)$ for $j \in \{2, 3\}$. Therefore γ_F violates manipulation via static coalition for all F. Since no–manipulation via static coalition is a weaker axiom of no–manipulation via dynamic coalition γ_F violates manipulation via dynamic coalition. ■

Proposition 20 *For all F γ_F^c violates no-manipulation via static coalition and no-manipulation via dynamic coalition.*

Proof. Consider the following example: Suppose $E = (10, 10)$ and $c = (10, 0, 0, 10)$ and the utility functions $u_1 = (x_1^1)^{0.5}(x_1^2)^{0.5}$ and $u_2 = (x_2^1)^{0.5}(x_2^2)^{0.5}$. Since $\sum_{i \in N} c_i^t = E^t$ for all $t \in T$ for all F, we have $F(c, E) = (10, 0, 0, 10)$ and the outcome of the mechanism is $\gamma_F^c(R, c, E) = (10, 0, 0, 10)$. Now if agents make a coalition in which $\tilde{c} = (5, 5, 5, 5)$ then For all F with the same reason above we have $F(\tilde{c}, E) = (5, 5, 5, 5)$ and the outcome of the mechanism becomes $\gamma_F^c(R, \tilde{c}, E) = (5, 5, 5, 5)$ So, $u_1(5, 5) = u_2(5, 5) = 5 > 0 = u_1(10, 0) = u_2(0, 10)$. Therefore γ_F^c violate no–manipulation via static coalition for all F. Since no–manipulation via static coalition is a weaker axiom of no–manipulation via dynamic coalition γ_F^c violates no–manipulation via dynamic coalition. ■

Since the outcomes of the γ_F and γ_F^c mechanisms are directly affected by utility functions, we would have cardinality problem.[7] Therefore we want to find an initial allocation rule to prevent manipulation for constrained and restricted constrained Walrasian rules. We firstly characterize the dictatorship rule:

Lemma 21 *F is continuous[8] and $F_i(c, E) = c_i$ for some $i \in N$ if and only if $F = \mathcal{D}^i$ for some $i \in N$.*

Proof. $\Leftarrow$: Trivially holds.

$\Rightarrow$: Assume E is fixed. For $c^* = (E, E)$ we have $F_i(c^*, E) = c_i$ for some $i \in N$. Without loss of generality let $i = 1$ and $F_1(c^*, E) = c_1$. There exists $\tilde{c}_2 < c_2^*$ and for $\tilde{c} = (c_1^*, \tilde{c}_2)$ we have $F_1(\tilde{c}, E) = c_1^*$. Otherwise if $F(c, E) = (E - \tilde{c}_2, \tilde{c}_2)$ we will have disconnected path in the range of the F which will violate the continuity of F. Now, let there exists $\bar{c}_1 \in \mathbb{R}_+^2$ with $\bar{c}_1 < c_1^*$. For $\bar{c} = (\bar{c}_1, \tilde{c}_2)$ we have $F_1(\bar{c}, E) = \bar{c}_1$, otherwise if $F(\tilde{c}, E) = (E - \bar{c}_2, \tilde{c}_2)$ with the same reason above the continuity of F will be violated. So, for all $c \in \mathbb{R}_+^4$ with fixed $E \in \mathbb{R}_+^2$ where $c_1 + c_2 \geq E$, we have $F = \mathcal{D}^i$.

Now assume that $c \in \mathbb{R}_+^4$ is fixed. The domain of E is $[0, c_1 + c_2]$. Let $E = c_1 + c_2$ then $F(c, E) = (c_1, c_2)$. For $\tilde{E} = (E^1, c_2^2)$ without loss of generality let $F_1(c, \tilde{E}) = c_1$ then for all

[7] A preference relation can be represented with many utility functions. Because of that, if an outcome of a mechanism is affected by utility functions, we would have a cardinatility problem, that is the outcome will change by using different utility functions for the same preference relation.

[8] F is continuous for all $(c, E) \in \mathcal{B}$ if it is continuous for all $c' \in \mathbb{R}_+^4$ while E is fixed and for all $E' \in \mathbb{R}_+$ while c is fixed.

$\bar{E} \in [\tilde{E}, E]$ we have $F_1(c, \bar{E}) = c_1$ otherwise if there exists $\hat{E} \in [\tilde{E}, E]$ and $F_2(c, \hat{E}) = c_2$ the continuity of F will be violated. ■

We prove that we can prevent the manipulation via destruction of characteristics by using the dictatorship rule for the initial allocation of the constrained Walrasian rule:

Theorem 1 *Suppose F is continuous. W_F^c satisfies no manipulation via destruction of characteristics if and only if $F_i(c, E) = c_i$ for some $i \in N$.*

Proof. $\Leftarrow$: By the previous lemma $F = D^i$. Since for all $(R, c, E) \in \mathcal{A}$ there exists $i \in N$ such that $F = \mathcal{D}^i$, the outcome of constrained Walrasian rule is $W_F^c(R, c, E) = \mathcal{D}^i(c, E)$. Since for all $j \in N \backslash \{i\}$ the outcome is not affected and the best outcome of the constrained Walrasian rule for a dynamic simple allocation problem is $\mathcal{D}_i^i(c, E)$ for $i \in N$, W_F^c satisfies no manipulation via destruction of characteristics.

$\Rightarrow$: Assume W_F^c satisfies no manipulation via destruction of characteristic vectors and suppose $F_i(c, E) \neq c_i$ for all $i \in N$. Therefore $F_i(c, E) < c_i$. Without loss of generality suppose agent 1 destroys $\epsilon \in \mathbb{R}_+$ amount of his characteristics such that $\tilde{c}_1 \in \mathbb{R}_+^2$ with $\tilde{c}_1 = (c_1^1 - \epsilon, c_1^2)$ and for $\tilde{c} = (\tilde{c}_1, c_2)$ let $F(\tilde{c}, E) \neq F(c, E)$. We will have two cases about $F(\tilde{c}, E)$:

Case1: $F_1^1(\tilde{c}, E) > F_1^1(c, E)$

Let $R = \bar{R}$ be such that: $x \in W_F^c(\bar{R}, c, E)$ and $x \in \partial X(c, E)$. For $(R, \tilde{c}, E)$ we can find a price vector $\tilde{p} \in \Delta$ such that for all $i \in N$ for all $\bar{x} \in X(\tilde{c}, E)$ with $\tilde{p}\bar{x}_i \leqq \tilde{p}F_i(\tilde{c}, E)$ we have $\tilde{x}_i \bar{R}_i \bar{x}_i$ and $\tilde{x} \in \partial X(\tilde{c}, E)$ where $\tilde{x}_1 \geq x_1$. Therefore $\tilde{x} \in W_F^c(\bar{R}, \tilde{c}, E)$ and since $\tilde{x}_1 \geq x_1$ by monotonicity of $\bar{R}$, we have $\tilde{x}_1 \bar{P}_1 x_1$. (Look at the Figure 5).

Case2: $F_1^1(\tilde{c}, E) \leq F_1^1(c, E)$ and $F_1^2(\tilde{c}, E) \neq F_1^2(c, E)$

Let $R = \bar{R}$ be such that: $x \in W_F^c(\bar{R}, c, E)$ and $x \in \partial X(c, E)$. For $(\bar{R}, \tilde{c}, E) \in \mathcal{A}$ we can find a price vector $\tilde{p} \in \Delta$ such that for all $i \in N$ and for all $\bar{x} \in X(\tilde{c}, E)$ with $\tilde{p}\bar{x}_i \leqq \tilde{p}F_i(\tilde{c}, E)$ we have $\tilde{x}_i \bar{R}_i \bar{x}_i$ and $\tilde{x} \in \partial X(\tilde{c}, E)$ where $\tilde{x}_1 \geq x_1$. Therefore $\tilde{x} \in W_F^c(\bar{R}, \tilde{c}, E)$ and since $\tilde{x}_1 \geq x_1$ by monotonicity of $\bar{R}$, we have $\tilde{x}_1 \bar{P}_1 x_1$. (Look at the figure).

Let $R = R'$ be such that: $x \in W_F^c(R', c, E)$ and $x \in \partial X(c, E)$. For $(R', \tilde{c}, E) \in \mathcal{A}$ we can find a price vector $\tilde{p} \in \Delta$ such that for all $i \in N$ and for all $\bar{x} \in X(\tilde{c}, E)$ with $\tilde{p}\bar{x}_i \leqq \tilde{p}F_i(\tilde{c}, E)$ we have $\tilde{x}_i \bar{R}_i \bar{x}_i$ and $\tilde{x} \in \partial X(\tilde{c}, E)$ where $\tilde{x}_1 \geq x_1$. Therefore $\tilde{x} \in W_F^c(R', \tilde{c}, E)$ and since $\tilde{x}_1 \geq x_1$ by monotonicity of R', we have $\tilde{x}_1 P_1' x_1$. (Look at the Figure 6).

Therefore W_F^c violates no manipulation via destruction of characteristic vectors if $F \neq \mathcal{D}^i$ for some $i \in N$. ■

Figure 5: Case 1

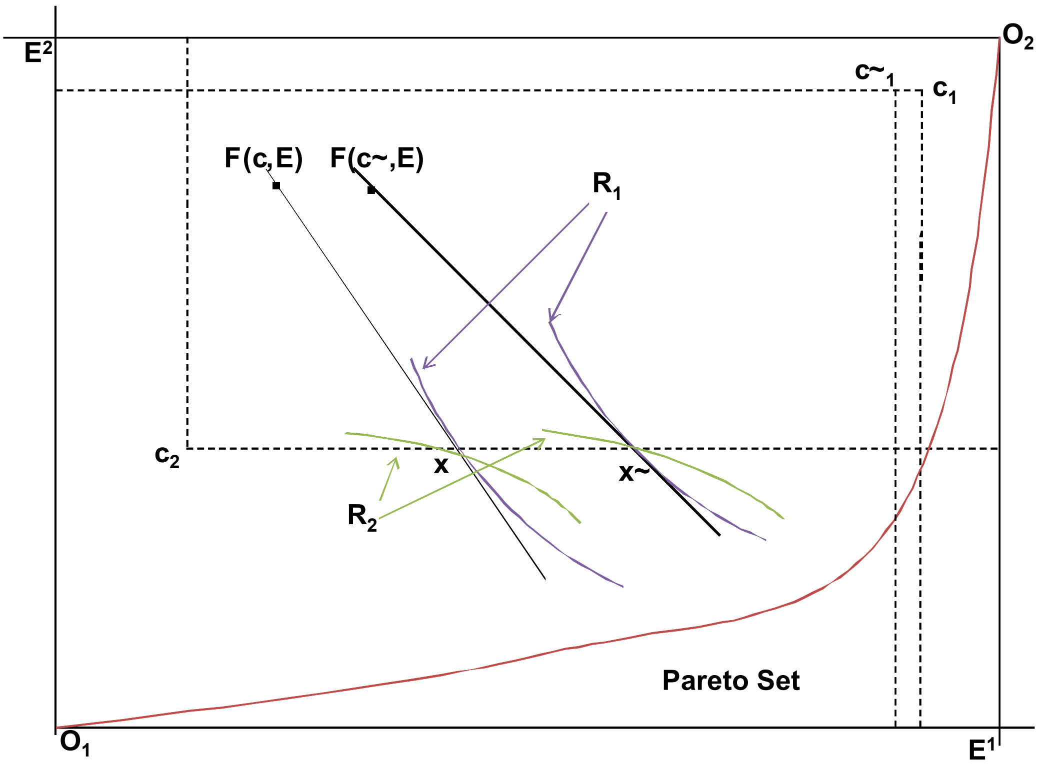
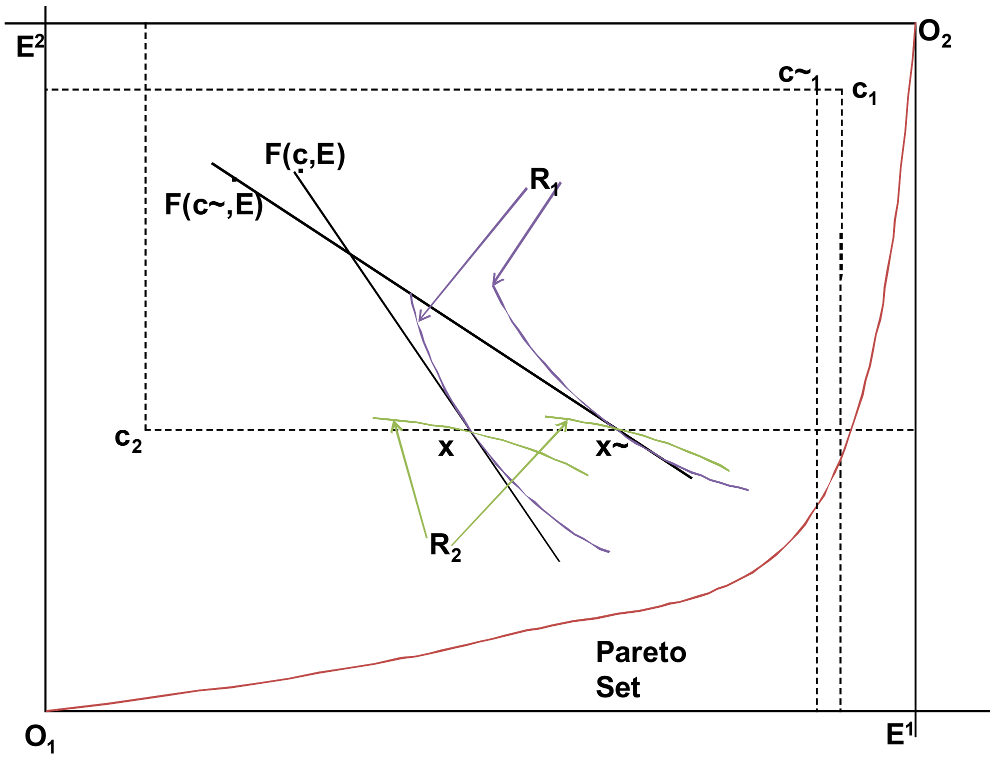


Figure 6: Case 2



So in order to prevent manipulation via destruction characteristics for the local pollutant case, EPA should use the dictatorial rule for the initial allocation.

5 CONCLUSION

We see that for global pollutants the agents can always manipulate the final allocation via destruction of their characteristics. However, for the local pollutants the manipulation via destruction of characteristic vectors is only prevented by allocating the endowment with an initial rule of the dictatorial rule. Moreover, although agents can manipulate the final allocation in the global pollutant market via preferences, in the local pollutant market the manipulation via preferences can be prevented by using the dictatorial rule in the initial allocation.

Inarra et. al. (2008) argues that future endowment can be a function of today's allocation. Either the total endowment allocated today or the way of allocation of today's endowment affects tomorrow. For example as in Inarra et. al. (2008), in the North East Atlantic Sea after allocating the endowment of today, remaining fishes breed and the endowment of next day increases. Therefore today's endowment directly affects tomorrow. On the other hand the initial allocation rule also affects the endowment of tomorrow. In their study, they show that using constrained equal awards rule is better than proportional rule for tomorrow. This is left for future research.

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