MODELING SUSTAINABLE TRAFFIC ASSIGNMENT POLICIES WITH EMISSION FUNCTIONS AND TRAVEL TIME RELIABILITY

by

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Submitted to the Graduate School of Engineering and Natural Sciences
in partial fulfillment of
the requirements for the degree of
Master of Science

Sabancı University

August 2010

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to my family

Acknowledgments

It is a pleasure to thank the people who made this thesis possible. I would like to express my gratitude to my thesis advisor, Assist. Prof. Nilay Noyan for her inspiration, advice and support. I would also like to thank Assoc. Prof. Ş. İlker Birbil and Assoc. Prof. Orhan Feyzioğlu for their guidance and friendly attitude.

I am very thankful to all my friends from Sabancı University for their friendship. I especially thank to Nimet Aksoy, Gizem Kılıçaslan, Elif Özdemir, Merve Şeker and Ezgi Yıldız for their great support, motivation and endless friendship.

I am thankful to the Scientific and Technological Research Council of Turkey (TÜBİTAK) for their financial support on my graduate education.

I am also very grateful to my family for the concern, love and support that they provide throughout my life. Finally, I want to express my deepest gratitude to Özlem Çıtak for her concern, support, motivation and love.

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Industrial Engineering, Master of Science Thesis, 2010

Thesis Supervisors: Assist. Prof. Nilay Noyan

Keywords: Sustainability; urban transportation; traffic assignment; bilevel programming; emission functions; toll pricing; capacity enhancement; stochastic programming; stochastic travel times; risk measure; travel time reliability

Abstract

Urban transport systems play a crucial role in maintaining sustainability. In this study, we focus on two types of sustainability measures; the gas emission and travel time reliability. We propose several bilevel optimization models that incorporate these sustainability measures. The upper level of the problem represents the decisions of transportation managers that aim at making the transport systems sustainable, whereas the lower level problem represents the decisions of network users that are assumed to choose their routes to minimize their total travel cost. We determine the emission functions in terms of the traffic flow to estimate the accumulated emission amounts in case of congestion. The proposed emission functions are incorporated into the bilevel programming models that consider several policies, namely, the toll pricing and capacity enhancement. In addition to the gas emission, the travel time reliability is considered as the second sustainability criterion. In transportation networks, reliability reflects the ability of the system to respond to the random variations in system variables. We focus on the travel time reliability and quantify it using the conditional value at risk (CVaR) as a risk measure on the alternate functions of the random travel times. Basically, CVaR is used to control the possible large realizations of random travel times. We model the random network parameters by using a set of scenarios and we propose alternate risk-averse stochastic bilevel optimization models under the toll pricing policy. We conduct an extensive computational study with the proposed models on testing networks by using GAMS modeling language.

SÜRDÜRÜLEBİLİR TRAFİK ATAMA POLİTİKALARININ EMİSYON FONKSİYONLARI VE YOLCULUK SÜRESİ GÜVENİLİRLİĞİ İLE MODELLENMESİ

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Endüstri Mühendisliği, Yüksek Lisans Tezi, 2010

Tez Danışmanı: Yrd. Doç. Dr. Nilay Noyan

Anahtar Kelimeler: Sürdürülebilirlik; kentsel ulaşım; trafik atama; iki seviyeli programlama; salınım fonksiyonları; geçiş ücretlendirmesi; kapasite arttırımı; rassal programlama; rassal yolculuk süreleri; risk ölçütü; yolculuk süresi güvenilirliği

Özet

Kentsel ulaşım sistemleri sürdürülebilirliğin devam ettirilmesinde önemli bir rol oynamaktadır. Bu çalışmada iki tür sürdürülebilirlik ölçütüne odaklanmaktayız; araç salınımları ve yolculuk süresi güvenilirliği. Belirlenen bu sürdürülebilirlik ölçütlerini içeren çeşitli iki seviyeli eniyileme modelleri önermekteyiz. Problemin üst seviyesi ulaşım sistemlerini sürdürülebilir hale getirmeyi hedefleyen ulaşım ağı yöneticilerinin kararlarını temsil ederken, problemin alt seviyesinde ise toplam yolculuk maliyetlerini en aza indirmeyi hedeflediği varsayılan ağ kullanıcılarının kararları temsil edilmektedir. Sıkışıklıkta biriken salınım miktarlarını tahmin etmek amacıyla salınım fonksiyonları araç akışına bağlı olarak ifade edilmiştir. Bu salınım fonksiyonları geçiş ücretlendirmesi ve kapasite arttırımı yönetim politikalarını içeren iki seviyeli eniyileme modellerine uygun bir biçimde katılmıştır. Araç salınımlarına ek olarak, yolculuk süresi güvenilirliği ikinci sürdürülebilirlik ölçütü olarak kullanılmaktadır. Ulaşım sistemlerinde güvenilirlik sistemin ulaşım ağı değişkenlerinin değerlerindeki belirsiz sapmaları ne ölçüde kaldırabildiğini gösterir. Yolculuk süresi güvenilirliği üzerinde durulmakta ve sayısallaştırılması için de koşullu riske maruz değer (conditional value-at-risk, CVaR) bir risk ölçütü olarak rassal yolculuk sürelerinin alternatif fonksiyonları üzerinde kullanılmaktadır. Temel olarak CVaR olası yüksek yolculuk sürelerini kontrol etmek için kullanılmaktadır. Belirsiz ağ parametreleri bir senaryo kümesi kullanılarak modellenmekte ve geçiş ücretlendirmesi politikası çerçevesinde alternatif riskten kaçınan rassal iki seviyeli eniyileme modelleri önerilmektedir. Önerilen modeller ile örnek test ulaşım ağları için GAMS modelleme dili kullanılarak detaylı bir bilgisayısal çalışma gerçekleştirilmiştir.

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CHAPTER 1

INTRODUCTION AND MOTIVATION

In the last few decades the sustainable development issues have raised a significant interest with the adverse effects of the considerable increase in urban population. Sustainable development can be defined as the concept of meeting the needs of the present generations without compromising the ability of future generations to meet their own needs [175].

Having many potential negative externalities like congestion, high energy consumption and air pollution, urban transport systems play a very crucial role in maintaining sustainability. In this context, a sustainable transportation system is the one that:

- Allows individuals and societies to meet their basic needs safely, healthfully, and equitably.
- Is affordable, offer alternate choices of transportation modes, efficient and encourage a dynamic economy.
- Reduce noise production, air pollution, land use and non-renewable resource consumption.

In other words, for a sustainable transportation system economic, social and environmental issues should be taken into account and strategies that achieve all these objectives should be used. Several strategies are proposed in the literature to make transport systems more sustainable. These strategies involve vehicle and fuel technology changes, road and vehicle operations improvements and demand management [56]. Since all these strategies have their advantages and drawbacks, in 1997 the Transportation Research Board proposes that an effective sustainable urban transportation system requires a mixed use of these strategies [161].

Sustainable urban transportation has become the subject of many recent studies. Traffic congestion (economic impact), air pollution (environmental impact) and reliability (social and environmental impacts) of transportation systems, are always in the center of attention in these studies. Therefore, the main objective of these studies is to reduce congestion, transport emissions and maintain network reliability through use of different methods and policies. Some of the studies involve simulation tools to evaluate the sustainability of different transportation policies and some others utilize the mathematical programming instruments. Although, there are recent studies in literature, there is still a need for optimization models capitalizing on sustainability for transportation networks. The existing approaches mostly propose equilibrium models that are commonly used to predict the traffic patterns on transportation networks. Along this line, bilevel traffic equilibrium models are frequently used. In these models, an upper (system) level involves the decisions about a certain policy to achieve a predetermined objective and the lower (user) level reflects the decisions of the rational network users and their reactions to the upper level decisions [133, 149].

One main indicator of sustainability in transportation networks is the emission amount. Some recent studies use a general optimization model with emission factors per vehicle kilometer. A collection of analytical tools, such as spatial statistics and travel preference functions, which can be used in assessing or maintaining sustainability, are proposed. Nagurney introduced the term of emission pricing, which is defined as the toll price setting to satisfy predetermined emission levels [125]. Nagurney also provides sustainable urban transportation models with basic emission factors and emission constraints [123, 124]. Following Nagurney's influential work, subsequent studies use average emission factors for the sake of computational simplicity. However, this approach prevents models to include real emission amounts, and hence, the resulting observations do not exactly reflect the actual effects of traffic flow on the emission amounts. To this end, we present several bilevel programming models that investigate toll pricing and capacity enhancement policies with emission functions. Presented models can be classified under two groups. First of these include models aim to minimize total network emission. However, it may be equally important to consider high emission accumulations in wider area so we also discuss models with emission dispersion objectives as the second group. As an emission dispersion objective, we first consider pure dispersion case where the main idea is to distribute emission amounts as equitably as possible. On the other hand, preventing high emission accumulations in some parts of the network especially in residential and commercial areas is also important. Thus, as an alternate dispersion objective, we consider to penalize the amount of emissions that exceed the previously determined limits to sweep away the emission from populated areas. In a similar work, Yin and Lawphongpanich [183] also propose model with emission functions. They consider biobjective model, where the objectives are the minimization of congestion as well as the minimization of total emission through toll pricing. In this regard, their model has a similar structure as one of the models that we propose in our study. Nonetheless, they have not considered various traffic management policies through pricing like we extensively study here, neither they have followed the capacity enhancement approach existing in this study.

As we mentioned before, considering reliability is also important in the sustainable transportation framework. In transportation networks, reliability reflects the ability of the system to respond to the variations (uncertainties) in system variables. Several modeling techniques are proposed to quantify impacts on the variable network performance. In this study, we focus on the travel time reliability models, which refers to variability of travel times, in terms of traffic flow values. Several events such as minor accidents, variations in weather conditions, and vehicle breakdowns may lead to the travel time variations on the network. The travel time variations due to non-recurrent events such as weather conditions can be considered by modeling the randomness in the free-flow times whereas vehicle breakdowns and minor accidents can be considered by modeling the randomness in the link capacities. In this study, we use stochastic programming approach and we present the uncertain free-flow times and link capacities by random variables. We characterize these random variables by using a finite set of scenarios where a scenario represents a joint realization of the free-flow times and capacities of all the links in the network. Then, we propose stochastic bilevel programming models that involve the travel time reliability by using the scenario-based approach. In all these proposed bilevel programming models, the upper level problem involves the decisions of transportation managers aim to obtain a sustainable transportation system in terms of the travel time reliability through the toll pricing policy. On the other hand, given the upper level decision, the lower level problem reflects the route choice decisions of the network users based on the expected travel costs. In order to incorporate travel time reliability and find the best pricing policy, we specify some network based performance measures such as the unit travel time summed over all links, the total travel time summed over all links, the maximum unit travel time and the maximum total travel time. In the traditional stochastic programming approach expectation is commonly used as a optimally criterion. However, decisions obtained just according to the expected values may perform poorly under certain realizations of the random data. Thus, in order to model the effects of variability, we decide to incorporate risk measure, conditional value-at-risk (CVaR), into the upper level problem. We develop two types bilevel programming models involving CVaR. The first type include only the risk term, CVaR, whereas the second type of models consider both the expectation and CVaR of the specified random network-based quantity. We also present the risk-neutral versions of these models in order to analyze the effect of incorporating risk measures. Boyles *et al.* [28] also develop a bilevel programming model with the toll pricing policy under stochastic travel times. However, they use the variance as a risk measure and they incorporate reliability in the lower level problem by assuming that all the links in the network are independent. In this study, we relax the link dependency assumption by using the scenario-based approach and we incorporate travel time reliability in the upper level rather than the lower level. In addition, Chen and Zhou [44] model the travel time reliability by using CVaR but they only consider the traffic assignment problem and their models include restrictive distribution assumptions. In contrast to their study, we do not consider any restrictive assumptions.

1.1 Contributions

The main purpose of this study is to develop bilevel programming models to maintain sustainable transportation. The contributions of this study can be summarized as follows:

- We propose several bilevel programming models by using emission functions. These models include toll pricing and capacity enhancement decisions.
- We also develop risk-averse bilevel programming models with toll pricing decisions, where the risk measure is involved in the upper level problem.
- We consider models under elastic demand.
- Using a scenario-based approach for the risk-averse models allows us to model the link dependencies.
- The proposed models can be viewed as implementations of different policies that can be used for sustainable traffic management.
- We provide an extensive numerical study on a well-known test networks to illustrate the effects of different policies and present comparative result with alternate

objectives.

1.2 Outline

This thesis is organized as follows: Chapter 2 includes the literature review. In Chapter 3, we present proposed mathematical programming models including emission functions. We first introduce the traditional mathematical models for transportation and then present derivation of the emission functions. Finally, we introduce the bilevel programming models that involve the proposed emission functions. In Chapter 4, we present stochastic bilevel programming models with the travel time reliability. We first briefly discuss the network and the travel time reliability. Then, we describe how to incorporate the travel time reliability into the toll pricing problem as a sustainability measure in the stochastic bilevel framework. In particular, we consider the conditional value-at-risk (CVaR) as a risk measure on the travel costs to model the travel time reliability. We provide the computational results and analysis in Chapter 5. Finally, in Chapter 6 we present some concluding remarks and possible ideas for future research.

CHAPTER 2

LITERATURE REVIEW

In this chapter, we review the developing sustainable transportation research area which has a important role for maintaining sustainable development. We also introduce traffic assignment problem and how some performance indicators can be expressed in functional form.

2.1 Sustainable Transportation

In 1987, the Brundtland Commission report [175] brought global attention to the sustainability concept. Since then many scholars and policy makers have worked on the sustainability issues raised in the urban and metropolitan context. Having many potential negative consequences like congestion, high energy consumption and air pollution, urban transport systems play a very crucial role in maintaining sustainability. The literature includes many definitions of sustainable transport [95]. In a very compact way, a sustainable transportation system should respond to mobility needs, but at the same time should attend to the habitat, the equity in the society and the economic advancement in the present as well as in the future [56]. Moreover, according to the definition of the World Bank, a sustainable transport policy reaches the balance not by accident but by conscious choices and to this end, it determines points that can be compromised and uses win-win policy tools [173].

There are numerous issues in sustainable transportation that should be taken into account. These issues may be divided into three categories [108]: Economic issues involve business activity, employment and productivity. Some of the social issues are equity, human health, and public involvement. Environmental issues, on the other hand, consist of pollution prevention, climate protection and habitat preservation. The relationship between these categories is depicted in the Figure 2.1 [155].

Our interest is not in sustaining the transport system but in making sure the outputs from the system contribute to the sustainable development of society in terms of its en-

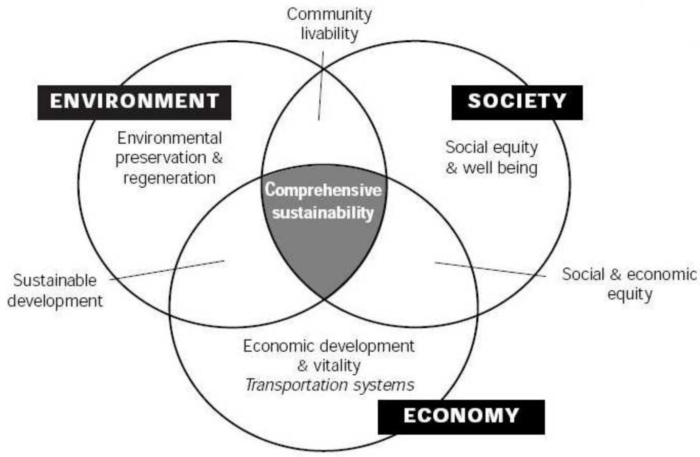


Figure 2.1: Components of sustainability

vironmental, economic, and social dimensions [171]. Moreover, sustainability planning does not always require trade-offs between economical, social and environmental objectives. Hence, policies that achieve all the objectives should be used. Several policies are proposed in the literature to make transport systems sustainable [57, 63, 67, 127, 141]. These policies can be classified as:

- **Pricing policies**: transportation systems and services should be priced by reflecting social and environmental costs so that sources can be appointed in the best way;
- **Technology policies**: technology contributes by making information accessible to users and by reducing environmental destruction;
- **Non-motor transportation policies**: walking and bicycling are at the positive side of sustainability while vehicles with single driver represent the negative side of sustainability. Thus, policies that deter people from motor vehicles are required;
- **Regulatory or prohibitive policies**: some activities may need to be regulated or completely prohibited;
- **Traffic management policies**: traffic flow conditions may be improved by some of the traffic management methods and improved flow contributes to sustainable transportation;
- **Education policies**: drivers should change their existing behavior patterns to create a sustainable transportation system;

- **Land use and transportation policies**: it is difficult to achieve the objective of sustainable transportation without considering integrated land use and transportation.

All of these policies have their advantages and drawbacks. The question is how effective would these policies be in reducing congestion, lowering pollution and cutting fuel use. In 1997, the Transportation Research Board investigated this topic. Their study proposes that an effective sustainable urban transportation system requires a mixed use of these policies [161].

Another difficulty encountered in reality is that a quantitative analysis of transportation sustainability content upon which all stakeholders agree has not been made and even it is not qualitatively explicit [137]. Thus, performance indicators are needed to determine which transportation policies will be more effective in reaching sustainability objectives [74, 129]. Indicators that are traditionally calculated such as road service quality, average speed and delay, convenience of parking, accident per kilometer [92, 93] focus rather on quality of travel with motor vehicle and rule out secondary effects. In addition, most of the existing indicators are digitized based on collective knowledge about vehicles at a certain number. However, many negative effects such as vehicle emissions are not explicitly linear and in such cases, aggregate form of approximation causes serious errors. What's more important is that considering only averages or information may result in ignoring many concepts related to sustainability. In addition to these points, it is recommended that the following principles are taken into consideration during the selection of transportation performance indicators: precision, data quality, comparableness, easy comprehensibility, accessibility, transparency, proper cost, net effect, suitability to determine objectives [88,117]. In the literature and application, there are a considerable number of works that sometimes overlap about which indicators should be included and that sometimes include conflicting propositions [64, 95, 109].

2.2 Optimization Models for Traffic Assignment

In this section we briefly discuss the definition of traffic assignment problem, then we give details of transportation management policies and we provide details of the widely applied bilevel programming approach for discrete and stochastic cases.

2.2.1 Traffic Assignment Problem Definition

The traffic assignment problem (TAP) aims to determine the traffic flows in an urban transportation network resulting from route choice decisions made by the travelers. Each network user chooses a route to travel from an origin to a destination considering the traveling conditions. In other words, a traffic assignment model utilizes origin-destination (O-D) information and the current transportation system conditions as inputs and provides the optimum flow on the transportation network with respect to the demand between all O-D pairs and the associated travel costs.

There are two different formulations of the TAP [60]. First of those formulations is the path formulation which incorporates predetermined routes having specific order of links. The network users then choose which route to use. On the other hand, in the multi-commodity formulation, the modeling structure is based on the numbers of users that are headed to each destination on each link.

There are several ways to model TAP problem as an optimization problem and it is usually modeled in two ways such as the Static Traffic Assignment Problem (STAP) and the Dynamic Traffic Assignment Problem (DTAP).

- In the Static Traffic Assignment Problem (STAP), it is assumed that traffic flows do not depend on time in other words average peak hour demand is considered. [149].
- In the Dynamic Traffic Assignment Problem (DTAP), it is important to consider the demand changes during the day and users' path selection and/or departure time decisions [138].

In this study, we basically focus on the STAP which aims to find a feasible assignment pattern that certain route choice conditions are satisfied. There are two widely applied conditions, namely the User-Equilibrium (UE) condition and System Optimal (SO) condition. These two conditions are widely considered as Wardrop's principles.

UE condition is based on the "Wardrop's first principle" which states that the travel times in all of the used routes are equal and less than those, which would be incurred by a single vehicle on any unused route [172]. The important assumption behind this principle is travelers of the network are expected to choose their routes according to the case in which they minimize their individual traveling times. It is also assumed that all of the travelers have equal traveling times if they have identical traffic conditions. Moreover, all the travelers in the network have the perfect information about all possible used or unused routes. User equilibrium may be a good representation of distribution of existing network traffic, but such distribution of traffic does not suppose to be the best possible use of the network system. This is because user equilibrium considers each traveler individually. As a result this observation, Wardrop states his second principle which describes how to assign all the travelers centrally to minimize the total cost of all users. Wardrop's second principle or the System Optimal (SO) principle is: "the average journey time is a minimum." This implies that each user behaves cooperatively in choosing his own route to ensure the most efficient use for the whole system.

In this study, we focus on the UE condition and as introduced below UE can be handled by two ways under TAP such as the deterministic user equilibrium (DUE) and the stochastic user equilibrium (SUE).

- If it is assumed that all travelers will have perfect information on all possible routes through network, no matter whether the routes are used or not, DUE will be enough to explain user behavior. Beckmann et.al. [17] were the first to transform the user equilibrium principle into a mathematical programming problem for the link flow and has been widely studied since then.
- In the SUE models, it is assumed that users may have different perceptions about their travel times thus, travel selection is made according to the perceived time rather than real time [20, 54, 148].

The number of travels between O-D in the scope of the TAP or in brief, user demand can be handled in three ways:

- In the traditional TAP, it is assumed that the number of network users (drivers) who want to travel from a specific origin to a specific destination do not change under any condition. Then, fixed demand (FD) is in question in this case [17, 52].
- However, in reality the demand between each O-D pair may depend on the congestion level of the transportation network. Then, the type of demand between each O-D pair, which may vary according to the network conditions,(i.e. the travel time between those pairs) is known as the elastic demand (ED) [17, 77, 106, 179].
- If the uncertainty of demand in a long or in a short period is taken into account then stochastic demand (SD) is considered. There are many reasons of demand uncertainty in transportation: a) unexpected developments, b) political and

social-economic changes, c) uncertainties in demand model, d) difficulty of quantifying the performance indicator, e) changes in choices of decision makers. Long-term uncertainty is modeled by assuming that certain demand scenarios exist or the demand complies with multivariate normal distribution [13,75,122,164] while short-term or daily observed uncertainty is usually modeled by assuming that the demand follows a certain continuous or discrete distribution [8, 18, 48, 165]. Naturally, when UE is modeled, expected travel time is considered rather than perceived travel time.

Until now we have discussed details for basic TAP but there are also some widely applied traffic management policies with TAP. In the next section, we provide details for these policies.

2.2.2 Regulation Policies with Traffic Assignment Problem

There are different regulation policies such as the toll pricing policy, the network design policy and the signal setting policy that have been commonly examined in TAP. In this study we have mainly focused on two types of these policies; the toll pricing policy and a special class of network design policies, namely, the capacity enhancement policy. Although we focus on two of these policies, we also discuss the details for signal setting problem in the following parts.

Toll Pricing

As a traffic regulation policy, toll pricing offers a solution for reduction of traffic while it is not feasible to increase the capacity of the transportation network. By using tolls, the network users can be encouraged to follow alternative decisions such as traveling on less congested hours and choosing less directed routes. There are two ways to handle the toll pricing problem namely the first-best and the second-best. In the first-best toll pricing problem, every arc in the network can be tolled, on the other hand, in the second-best toll pricing problem a subset of the roads are subjected to charges.

Marginal social cost pricing (MSCP) is the earliest first-best toll pricing in the literature. This idea was introduced for the first time in the 1920s by Pigou [142]. Marginal social cost pricing (MSCP) offers tolls which are same as the negative externalities enforced on other users (such as congestion, travel delays, air pollution, and accidents) to sustain an efficient utilization of the transportation system [17, 52, 142].

There are also other first-best tolls exist [23,24,89] in the literature. In particular, models and methodologies are offered to gain the first best tolls with different (secondary) objectives [58, 59, 89, 90, 102]. Concept of toll set was first introduced by Bergendorff et al. [24] and is motivated from the alternate first-best tolls. They determine and mathematically show how the toll set will encourage drivers to use the traffic network optimally. By a consequence of the notations and the models of user and system optimal traffic assignment, they provide detailed information about congestion toll pricing and general results about toll sets. An algebraic characterization of the toll set and a procedure known as a toll pricing framework are proposed by Hearn and Ramana [89] for the traffic assignment problems with fixed demand. In this work, toll sets are determined more generally with respect to previous study [24]. Hearn and Ramana [89] also offer many different objectives. Firstly, they propose the model with minimization of the total tolls collected with positive toll values (MINSYS). Then, they minimize the largest nonnegative toll to be collected (MINMAX). Thirdly, targeted revenues (TR) are considered as an alternate objective. In this case, they allow negative toll values and as a result network users gather a credit on some of the links and pay for some others. Then, they consider minimization of the number of the toll booths (MINTB). Lastly, combination of last two models (MINTB/TB) are introduced. In the most of the related studies in the literature all of these objectives are used. As an extension on the these studies Hearn and Yildirim [90] are interested in traffic assignment problems with the elastic demand. They aim to maximize the net benefit of the network users. The set of all tolls are determined and characterized to gain the system optimal solution. Traffic assignment problem with the elastic demand is also studied by Larsson and Patriksson [102]. They present a toll pricing model based on Lagrange multipliers and show that the constant toll revenue property holds for elastic demand problems with side constraints. In their study, systematic solutions are utilized to satisfy the overall traffic management.

There are also several studies based on the second-best toll pricing. Second-best toll pricing problem has tolls with restrictions that do not generally achieve the maximum possible benefit [97]. For strategic traffic management, Patriksson and Rockafellar [134] use traffic management actions the second-best toll pricing problem as congestion pricing. They conceive a (small) number of different model settings and their models include fixed and elastic demands. Brotocorne*et al.* [31] conceive solution for the set of optimal tolls selection problem on a multicommodity transportation network that collects revenues from toll set of arcs of the network. These set of arcs are determined by the shortest path of users traveling on the network and cost of paths are calculated according to the generalized travel costs. In a fixed demand transportation network, while the commuters are assigned to the shortest paths with respect to a generalized cost, private toll highway try to maximize their revenues collected from tolls on a set of multicommodity network arcs. In this model, the rerouting that could be emerged by the introduction of tolls does not effect the congestion. Moreover, two different second-best toll pricing problems are presented by Lawphongpanich and Hearn [103], the first one is proposed with the fixed travel demand and the other with the elastic demand. In this study, the presence situations for optimal toll vectors are determined, and the relation with marginal social cost pricing tolls are given.

Network Design

The Network Design Problem (NDP) involves the optimal decision on the expansion of a street and highway system in response to a growing demand for travel. This problem has been studied with three different versions. These are discrete, continuous and mixed versions. Firstly, the discrete version of the problem which is called as Discrete NDP (DNDP), finds optimal (new) highways added to an existing road network among a set of predefined possible new highways (expressed by 0-1 integer decision variables). On the other hand, continuous NDP (CNDP) tries to find the optimal capacity development of existing highways in the network (expressed as continuous variables). The mixed one (MNDP) unites both CNDP and DNDP in the network. The decisions made by road planers influence the route choice behavior of the network users, which is normally described by the network user equilibrium model.

The DNDP is firstly introduced by Boyce and Janson [27], and by Chen and Alfa [38]. They both take into consideration the minimization of the travel cost but their methodologies are different such as former uses a combined trip assignment and distribution, while the later uses a stochastic incremental traffic assignment approach. Steenbrink [157] also discuss the DNDP. He makes an introduction to modeling the urban road DNDP. He develop a new approach to the network design problem in which user optimal flows are approximated by system optimal flows. One of the other studies is developed by Wu et al. [178]. They study a new version of transportation network design problem by performing the strategy of reversible lanes. They focus on the stochastic user equilibrium assignment with an advanced traveler information system.

Yang and Yagar [181] suggest the problem of the traffic assignment and traffic control in general freeway-arterial corridor systems having flow capacity constraints.

There are also several studies about CNDP. This problem includes the term continuous in its name because the decision variables are continuous. This problem was first introduced by Morlak [120]. Abdulaal and LeBlanc [2] formulate the network design problem with continuous investment variables subject to equilibrium assignment as a nonlinear optimization problem. Another study about CNDP is considered by Friesz [71]. He presents a model for continuous multiobjective optimal design of a transportation network. The model incorporates the user equilibrium constraints and takes the form of a difficult nonlinear, nonconvex mathematical program.

There are also studies about the mixed NDP (MNDP). Bell and Yang [180] propose models with MNDP. They present a general survey of existing literature in this area, and present some new developments in the model formulations. They propose the adaptability of travel demand into NDP and seek economy related objective function for optimization.

Variety of objectives are used in different studies in Network Design Problem. The most commonly used ones are the efficiency objectives. Minimizing the travel time, user cost for a specified budget, investment cost for a given travel demand, and maximization of the user benefit (can be measured according to the consumer surplus) [100,174,180] are the examples of these objectives. Among these objectives, only the last one is consistent with elastic traffic demand since travel time and user cost objectives can be decreased by the decline of the traffic amount. Multiobjective road network design models are also incorporated in some of the studies. As widely applied objectives, user costs and construction costs are tried to be minimized simultaneously [71,72,163]. In addition to an efficiency objective, robustness objectives [50, 164], horizontal and vertical equity objectives [46, 69, 119], environmental objectives (minimization of CO emissions) [36] are also studied in the literature.

Signal Setting

On urban networks; intersections (delays) are the most time consuming points thus effective optimization of signalization of the intersections can clearly improve the performance of the transportation network. In the problem of optimization of signal settings, Signal Setting Design Problem (SSDP), the signal settings (number of phases, cycle length, effective green times, etc.) assume the role of decisional variables. On the other hand, the network topological characteristics (widths, lane number, open or closed link, etc.) are fixed and invariable ones.

Pavese [136] emphasizes the circular interaction between traffic assignment and signal settings for the first time. Then he formulated the node functions related to the performances of the connections to traffic flows of every approach at the downstream intersection. In addition to Pavese [136], this issue is also considered by Cascetta *et al.* [37]. In this study, the SSDP is analyzed according to two approaches: the local and the global [34]. The first idea comes up with the definition of the Local Optimization of Signal Settings (LOSS). In this approach, flow-responsive signals, which are set independently each other either to minimize a local objective function [76] or following a given criterion, like equisaturation [151]. Global Optimization of Signal Settings (GOSS) which tries to minimize the objective function of the global network performances is the formulation of the problem that is used in the global approach [114].

SSD problem is highly interdependent with continuous network design (CND) and traffic assignment problems [76]. Thus, integrated (or combined) model is preferable that provides such mutually consistent solutions. Some of the studies that incorporate combined signal optimization and static user equilibrium problems are as follows. The necessity of combining signal calculation and assignment is emphasized in Allsop [11] and Gartner [79]. According to Wardrop's first principle, the rotation of traffic in a network should depend on signal timings and it should be conceived simultaneously with timing calculations. The general traffic equilibrium network model is considered by Dafermos [53]. In his study the travel cost on each connection of the transportation network may depend on the flow and other connections of the network as well. A detailed study about global signal settings problem, under the constraints of the user equilibrium for traffic flows is provided by Cipriani and Fusco [47].

There are also several studies about the combined signal optimization, continuous network design and static user equilibrium. In the study of Wong and Yang [176], they focus on the optimization of signal timings. They are optimized according to the group-based technique in which the common cycle time, the start time and duration of the time period for each signal group in the network determines the signal timings. Allsop [12] represents a new approach to analyze the traffic capacity of a signalized road junction. By using his new methodology, the capacity is calculated and its results are used for signal settings to maximize capacity. Also, extra capacity amount that is obtained by changing the maximum cycle time, reducing the times which take minimum values, increasing the saturation flows, can be estimated by the engineer according to their methodology.

In terms signal optimization model, five different variables are commonly underlined in most of the studies in literature. This variables are cycle lengths, green splits, time offset, phase sequencing and signal phasing. Sometimes, green splits are the only ones that are optimized and other variables are determined as fixed values [47,107,150,159]. Some of them consider only common cycle length and green splits [1] and another one conceive common cycle length, green splits and time offset simultaneously [78,154,160, 177].

2.2.3 Bilevel Programming

The recent studies in the literature [98,119,156,183] show that there is still a requirement for optimization models obtaining results on sustainability for transportation networks. In these existing studies, proposed equilibrium models generally aim to estimate the traffic patterns on transportation networks. Thus, bilevel traffic equilibrium models are frequently used.

Bilevel programming is a branch of hierarchical mathematical optimization. The relationship between two autonomous and possibly conflicting decision makers is named as hierarchical relationship which is widely related with economic Stackelberg problem [152]. The objective of a bilevel model is to optimize the upper level problem while simultaneously optimizing the lower level problem. To achieve a determined goal (such as reducing the congestion or the investment cost) a typical bilevel traffic equilibrium problem, the upper level involves the decisions about a certain policy (such as toll pricing or network design) whereas the lower level problem models the traffic equilibrium reflecting the decisions of the rational network users and their reactions to the upper level decisions. It is obvious that lower level problem yields a well-known TAP under a given upper level decision.

The general formulation of a bilevel programming problem is

$$\min_{x,y} \quad F(x,y) \tag{2.1}$$

$$\text{s.t.} \quad G(x,y) \leq 0 \tag{2.2}$$

$$\min_{y} \quad f(x,y) \tag{2.3}$$

$$\text{s.t.} \quad g(x,y) \leq 0 \tag{2.4}$$

where $x \in R^n$ is the upper level variable and $y \in R^n$ is the lower level variable. The functions F and f are the upper-level and lower-level objective functions respectively. Similarly, the functions G and g are the upper-level and lower-level constraints respectively. Upper-level constraints may involve variables from both levels.

In many applications the lower-level problem can not be expressed as an optimization problem, but can be described by an equilibrium process, which is given mathematically by a variational inequality problem. These reformulated bilevel programs are often referred as mathematical programming with equilibrium constrains (MPEC) [97, 103].

It is often possible that some of the problem inputs may subject to uncertainties. These uncertainties usually occur in the costs and/or demands, which are usually results of variable external conditions. In such cases, stochastic programming is one of the important approaches to model decision making under uncertainty. This approach develop models to formulate optimization problem in which uncertain quantities are represented by random variables. To consider explicitly the variability of the random inputs a stochastic programming extension of bilevel programming model can be used in such cases [16, 147]. In this case, it is not possible to calculate exactly the vectors x and y, since their values are depend on random parameters. Instead, the values of these vectors can be calculated such that F is optimized on average. Thus, the upper level objective function of deterministic bilevel program (2.1) is replaced with

$$\mathbb{E}_\omega[F(x, y, \omega)], \tag{2.5}$$

and similarly the lower level objective (2.3) function is replaced with

$$f(x, y, \omega). \tag{2.6}$$

Here ω represents the realization of a random variable.

In the following part, we provide the application areas and some solution methodologies for bilevel programming approach. Although, we give some information about solutions and solution methodologies for the bilevel programs, here we also give some important details for the solutions of the stochastic bilevel programming approach. In this case, if the equilibrium solution is not unique then the upper level objective F is not well-defined and as a result the best possible solution can only be obtained by the most favorable equilibrium solution. However, if the lower level decision makers do not necessarily optimize equilibrium exactly, then the upper level decision makers are likely to make a mistake while making their decision.

Here we provide some related studies from literate. Brotcorne *et al.* [32] and Larsson and Patriksson [102] use bilevel programming models for toll optimization. Ben *et al.* [16] also focus on toll pricing policy, but they use stochastic bilevel programming approach. LeBlanc [104] and Marcotte [115] and Chen and Chou [42] use bilevel programming approach in a network design problem. In addition, bilevel programming models can be also used for other real-world problems involving a hierarchical relationship between two decision levels such as management [51, 89, 102], economic planning [15, 167], engineering [131, 132], etc.

Despite the fact that a wide range of applications fit the bilevel programming framework, real-life implementations of the concepts are limited. The main reason is the lack of efficient algorithms for dealing with large-scale problems. For example, the bilevel transportation problems related to the equilibrium problem create a special class and most of the methods developed for the solution of bilevel optimization problems cannot be directly applied [45, 170]. Furthermore, although the problem discussed at the lower level is a convex optimization problem, the network structure to be handled in real problems has a large scale and requires an infrequent data structure causes an extra difficulty. Thus, bilevel programs are intrinsically hard. Even for a "simple" instance, the linear bilevel programming problem can be shown to be NP-hard [87, 96, 169]. Therefore, global optimization techniques such as exact methods [114], heuristics [35, 38, 105, 114, 150] or meta-heuristics [45, 55, 179] have been proposed for its solution in the literature. Although the problem is shown to be NP-Hard, some special cases enable us to solve the problem in polynomial time such as sensitivity based analysis, Karush-Kuhn-Tucker (KKT) based method [115, 168], etc. Some of these conditions are used in various solution methods and algorithms. Descent methods [169], penalty function methods [3, 4] and trust region methods [49] are some examples of these methods.

2.3 Sustainability in Urban Traffic Assignment

There are several issues that decision makers shall take into account to develop and maintain sustainable transportation systems. These issues may be divided into three main categories; environmental, economical and social issues. In this section, we present some of the selected studies incorporating at least one sustainability measure related to one these issues.

2.3.1 Environmental and Economic Issues

Environmental issues consist of pollution prevention, climate protection and habitat preservation and economic issues involve business activity, employment and productivity. There has been a significant interest in considering environmental issues to develop sustainable transportation systems and these environmental issues have also common goals with economic issues. Thus, we focus on both issues in this section. Environmental measures are widely-applied sustainability measures and the studies incorporating the environmental concepts to maintain sustainable transportation usually focus on air pollution, noise pollution, fuel consumption (energy) and car ownership. It is also obvious that in some of these concepts, economic objectives are also considered while focusing on the environmental ones. Note that car ownership may also be considered as an economic and/or a social issue.

Most of the studies that aims to decrease congestion and related emission involve simulation tools to evaluate the sustainability of different transportation policies. TREMOVE is an evaluation tool that is developed to support the European policy making process concerning emission standards for vehicles and fuel specifications [81]. It is an integrated simulation model to study the effects of different transport and environment policies on the emissions of the transport sector.

There are also several studies that exploit mathematical programming instruments. A multi-objective traffic assignment method is introduced by Tzeng and Chen [162]. They use nonlinear programming techniques to solve the introduced models and provide different ways to emit low CO emissions. They incorporate the eigenvector weighting method with pair-wise comparison to estimate the compromised solutions for the flow patterns. The study utilizes a fixed amount of CO emission per link and the emissions are summed up across all vehicles on a link. Rilett and Benedek [22,143] investigate an equitable traffic assignment model with environmental cost functions. They emphasize the impacts of CO emissions when user and system optimum traffic assignments are applied to various networks. These studies utilize a simple macroscopic CO emission model used in the TRANSYT 7F software. Yin and Lawphongpanich [183] also propose a flow versus emission function, where the coefficients are equivalent to those in TRANSYT 7F (see also Rilett and Benedek [143]). In their pioneering work, Yin and Lawphongpanich consider a biobjective model, where the objectives are the minimization of the congestion as well as the minimization of the total emission through toll pricing. Sugawara and Niemeier [153] discuss an emission-optimized traffic assignment model that uses average speed CO emission factors developed by the California Air Resources Board. They report that the emission-optimized assignment is the most effective assignment when the network is under low to moderately congested conditions. Guldmann and Kim [85] concern transportation network design, traffic assignment and pollution emissions, diffusion and concentrations on transportation networks. They offer a nonlinear model which minimizes the sum of costs such as travel time, capacity investment and fuel consumption while considering origin-destination traffic flows, capacity of links, travel speeds and pollution emissions. Jaber and O'Mahony [94] work on travelers' mixed stochastic user equilibrium (SUE) behavior. They consider this behavior under the condition that traveler information provision services with heterogeneous multi-class multi-criteria decision making. Traveler information provision services are formulated as an optimization problem with the route option behavior of equipped and unequipped travelers. In this optimization program, net economic benefit is maximized and the total generated emissions are constrained. Furthermore, environmental impact assessment indices are suggested by Nagurney et al. [126] which interprets the environmental effects of link capacity degradation in transportation network. Environmental link importance indicators are suggested by them. These indicators enable the ranking of links in transportation networks in terms of their environmental importance and suggest if they can be removed or destroyed. Moure *et al.* [121] suggest a total cost minimization model in which system costs are depend on high congestion that is produced by truck operators, barge operators and drivers. The model is presented as a bi-level optimization problem which tries to minimize total cost of the system via pollution emissions and noise pollution constraints in the upper level and user equilibrium model in the lower level. Yang *et al.* [182] try to predict the maximum car ownership that can be carried in a city under environmental conditions. A bilevel programming model is presented where the upper level problem is a maximum car ownership model which aims to maximize zonal car ownership levels subject to environmental load constraint on a link and the lower level problem is the fixed demand user equilibrium assignment model which optimizes travelers' path choice behavior. Tam and Lam [158] also consider car ownership concept. Their aim is to figure out the maximum number of cars in each zone due to parking space and capacity restrictions. They use a bilevel programming approach where the upper level problem is maximizing the sum of zonal car ownership via capacity and parking space constraints whereas the lower level problem is the trip assignment problem.

In the upcoming chapters of this study, we mainly focus on emission minimization objectives by using bilevel programming approach.

2.3.2 Social Issues

There are also various studies in the literature that incorporate social issues to maintain sustainable transportation. These studies mainly focus on accessibility, equity and social welfare. Note that social issues are also directly related with economic issues. Although we do not explicitly consider economic cases in this section, some of the presented concepts in the following part can also be considered with an economical point of view.

In transportation networks, reliability is the ability of system to perform and maintain its functions in routine circumstances, as well as unexpected (variable) circumstances. Several modeling techniques are proposed to quantify impacts on variable network performance and these techniques can be discussed under five main classes [48]:

- **Connectivity reliability models**

  Connectivity reliability focus on the probability that network nodes are remain connected [19].

- **Travel time reliability models**

  Travel time reliability considers the probability of completing a trip within a specified travel time threshold [8, 18, 62].

- **Capacity reliability models**

  Capacity reliability is the probability that the network can handle a certain traffic demand at a required service level while accounting for drivers' route choice behaviors. [43].

- **Behaviorial reliability models**

  Behaviorial reliability focus on how to represent the effect of route choice patterns [110] and other responses such as departure time choice [128].

- **Potential reliability models**

  They are referred as pessimistic models that aim to identify weak points of the transportation network and corresponding effects on the performance.

There is a rich literature on these presented classes of the network reliability. Since we focus on only the travel time reliability in this study, we only provide selected works on this issue in the following part.

Asakura and Kashiwadani [8] introduce measures of travel time reliability and to analyze the changes of road network flow, they modify the traffic assignment problem. Asakura [9] extended the travel time reliability concept to investigate capacity degradation due which are possibly damaged by natural disasters. Another travel time reliability model is suggested by Clark and Watling [48] which shows the effects of stochastic O-D demands on variable network performance. In their model the total time is evaluated as performance measure and it is actually described at the network level. Lo *et al.* [112] present a travel time reliability model as a result of link capacity degradations. To account for the impacts of travel time reliability, they propose probabilistic model in the travel time budget form.In contrast to the TTB models [112] which evaluates only the reliability point of view described by TTB, a new model is suggested by is suggested by Chen and Zhou [44] in which the travelers are willing to minimize their mean-excess travel time (METT), which is defined as the conditional expectation of travel times beyond the TTB. A new α-reliable mean-excess traffic equilibrium model is defined, which assumes both reliability and unreliability point of views of the travel time variability in the route decision process. A bi-level programming model is generated by Boyles *et al.* [28]. They focus on travel time reliability concept via toll optimization policy in a static transportation networks under stochastic supply conditions. On the other hand , Boyles *et al.* [29] focus on the same issue with deterministic demand assumption. Another study based on pricing on the transport network reliability is conducted by Chan and Lam [40] to offer a reliability-based UE model. As a congestion performance measure, they incorporate the ratio of the random travel time and free-flow travel time in their work. In addition to network reliability, there are also studies that focus on social issues by incorporating accessibility, equity and welfare concepts. Accessibility measures for the transportation network are provided by Chen *et al.* [39] to evaluate the vulnerability of degradable transportation network. The network-based accessibility measures consider the consequence of one or more link failures in terms of network travel time or generalized travel cost increase as well as the behavioral responses of users due to the failure in the network. A Simultaneous Transportation Equilibrium Model (STEM) has been presented by Safwat and Magnanti [146] which enable trip generation and distribution, traffic assignment and model split. In the STEM, trip generation depend upon the systems performance by using an accessibility measure based on the random utility theory of users behavior and a logit model is used for trip distribution. Purvis [139] introduce variables related to land use and accessibility to display the improvement of a trip-based travel demand modeling. Models with travel demand are considered with and without accessibility variables and land use density. In addition to studies with accessibility, there are also equity and social welfare based studies in the literature. Meng and Yang [119] consider the equity issue on road network design. They used a critical O-D travel cost ratio to quantify equity issue. They propose a bilevel programming model in which equity constraints are included for network design problem with a bicriterion objective aiming to minimize total system cost. Lo and Szeto [156] focus on social and user equity concepts by using user equilibrium continuous network design problem with elastic demand. Gupta et al. [84] work on the effect of road pricing on traffic, land use and social welfare in the Austin region. They discuss different toll pricing scenarios such as fixed versus variable tolls, time, traffic and distance dependent tolls.

CHAPTER 3

USING EMISSION FUNCTIONS IN MODELING SUSTAINABLE TRAFFIC ASSIGNMENT POLICIES

In this chapter we first present traditional mathematical models for the traffic assignment problem. Then, we focus on gas emission as a environmental sustainability measure. To incorporate emission effects of the congestion into the models properly, we derive the emission functions in terms of traffic flows. We plug these emission functions into the bilevel mathematical programming models that incorporate several policies, namely, toll pricing and network design to assess sustainability in transportation.

3.1 Traditional Mathematical Models for Transportation

Using mathematical programming techniques in sustainable urban transportation is crucial. To model a transportation problem consistent with the real nature of transportation networks, traffic flows should be considered properly. Therefore, traffic assignment problem (TAP) is a important application of mathematical programming in transportation.

In the following sections we first present the basic traffic assignment problem and then we discuss toll optimization and network design problems in the bilevel programming framework where lower level problem corresponds to the traffic assignment problem.

3.1.1 Traffic Assignment Problem

Recall that traffic assignment aims to find a feasible assignment pattern such that certain route choice conditions are satisfied. In this approach, user-equilibrium (UE) and system optimal (SO) conditions are widely employed. Here we focus on the user-equilibrium assignment problem.

As we discussed before, the TAP has two different formulations [60], the path and the multi-commodity. In particular, there are three different types of multi-commodity formulations; the destination based, origin based and the origin-destination based. In the destination based formulation, the flow on each link is determined associated with each destination. Similarly in the origin based one, flow amount on each link is determined according to the emitted origin. On the other hand, in the origin-destination based formulation link flows are disaggregated with respect to both origins and destinations. In our study, we consider only the destination based multi-commodity formulation because of its computational efficiency.

In the traditional TAP, the number of network users (drivers) who want to travel from a specific origin to a specific destination is assumed to be fixed. However, as we discussed before to develop more realistic traffic assignment models, it is crucial to incorporate the elastic demand into the models instead of the fixed demand. In transportation context, the elasticity of a demand between each O-D pair in general is represented by a function of the travel time. In this chapter, we mainly focus on the models with the elastic demand, but in the following chapters we also discuss and present models involving fixed demand.

In the fixed demand case the network will be managed based on the peak-hour demand and it does not change. However, in the elastic case the peak-hour demand is assumed to be variable. For this type of demand, the number of trips from an origin to a destination depends on the minimum travel time between them. Traditionally, it is assumed that the travel demand decreases as the travel time increases. There are many different types of demand functions [14]. In this study we use the linear demand function.

Consider a transportation network defined by a set of nodes $\mathcal{N}$, a set of arcs $\mathcal{A}$ and set of destinations D. A link of the network is designated by $(i,j) \in \mathcal{A},\ i,j \in \mathcal{N}$. If we also denote the flow on link (i,j) in vehicles per hour by f_{ij}, then the travel time or cost in hours is denoted by $c_{ij}(f_{ij})$. Here we use widely applied standard travel cost function introduced by Bureau of Public Roads (BPR) [30],

$$c_{ij}(f_{ij}) = \alpha_{ij} \left(1 + 0.15\ \left(f_{ij}/\beta_{ij}\right)^4\right), \tag{3.1}$$

where α_{ij} is the free flow travel time of link (i,j) in hours and β_{ij} is the capacity of link (i,j) in vehicles per hour. Note that there are two widely used highway capacity estimation methods. One of them is the Highway Capacity Manual (HCM) method in which speed volume density relationship is used [91] and the other one is the statistical method which uses observed traffic volume distribution [41]. In the HCM method first 15 min-base traffic data (speed, volume, density) is detected, then by using the data relationship between speed and volume-density relationship is searched and lastly highway capacity is determined. On the other hand, in the statistical method, peak hour 1 minute base volume and average speed is detected, then 1 minute base data is transferred to the 15 minute base one and by using the average volume, time headway distribution using is found. As the last step, highway capacity is determined when confidence intervals are 99%, 95% and 90%. The variance in the confidence interval obtained from this method greatly affects the result of highway capacity estimation.

We present the destination based multi-commodity formulation that we incorporate in this study as follows:

$$
\begin{aligned}
\text{TAP}: \quad & \min_{\mathbf{x}} \sum_{(i,j)\in\mathcal{A}} \int_0^{f_{ij}} c_{ij}(y)\mathrm{d}y && && (3.2)\\
& \text{s.t.} \sum_{j:(i,j)\in\mathcal{A}} x_{ij}^q - \sum_{j:(j,i)\in\mathcal{A}} x_{ji}^q = d_i^q && i\in\mathcal{N}, q\in\mathcal{D} && (3.3)\\
& \sum_{q\in\mathcal{D}} x_{ij}^q = f_{ij} && (i,j)\in\mathcal{A} && (3.4)\\
& x_{ij}^q \geq 0 && (i,j)\in\mathcal{A}, q\in\mathcal{D}, && (3.5)
\end{aligned}
$$

where x_{ij}^q denotes the amount of flow with destination q on link (i,j) and and d_i^q denotes the demand value between origin i destination q. The objective (3.2) reflects the decisions of the network users based on minimizing the total travel cost. The set of constraints (3.3) is the conservation of flow constraints, the set of constraints (3.4) links the total flow on an arc to the flows resulting from individual destination points and the set of constraints (3.5) ensures that the traffic flows are nonnegative.

To develop this model with elastic demand, we denote travel demand d_i^q as a decision variable and then we propose the demand function as follows:

$$g_{iq}(w_{iq}) = \mu_{iq}w_{iq} + \nu_{iq}, \tag{3.6}$$

where μ_{iq} and ν_{iq} are network specific parameters, $g_{iq}(w_{iq})$ is the demand function and w_{iq} is the minimum travel time between O–D pair (i,q). Furthermore, we obtain TAP with elastic demand by replacing the objective function (3.2) with

$$\min_{\mathbf{x}} \sum_{(i,j)\in\mathcal{A}} \int_0^{f_{ij}} c_{ij}(y)\mathrm{d}y - \sum_{i\in\mathcal{N}} \sum_{q\in\mathcal{D}} \int_0^{d_i^q} g_{iq}^{-1}(v)\mathrm{d}v \tag{3.7}$$

and adding the following set of constraints

$$d_i^q \geq 0, i \in \mathcal{N}, q \in \mathcal{D} \tag{3.8}$$

which ensures that demands are nonnegative.

3.1.2 Toll Optimization Problem

Traffic congestion has become part of everyday life especially in metropolitan areas. If there is not a way to prevent it, imposing appropriate tolls on roads can reduce traffic congestion because tolls can discourage network users using more congested links. It has recently become more practical due to the advent of electronic tolling, and hence, received significant attention from transportation planners and academics.

In toll optimization problem, the main idea is to set the toll prices on a set of links such that the congestion on these links are reduced. Since, it is a bilevel programming approach, the upper level problem usually has the objective of maximizing revenue earned from introduced tolls and the lower level problem corresponds to the traffic assignment problem with the additional travel costs in the objective for tolled links. Marcotte and Savard [116] provides an extensive literature survey on the use of bilevel programming approach to toll optimization problems.

Remember that, there are two classes of toll pricing problems, first-best and second-best. Here, we focus on this later problem.

Let $\bar{\mathcal{A}}$ be the set of tollable links. We assume that the toll price t_{ij} on link (i,j) cannot exceed a prescribed upper bound $t_{ij}^{\max}$ where

$$t_{ij}^{\max} \begin{cases} > 0, & (i,j) \in \bar{\mathcal{A}}, \\ = 0, & (i,j) \in \mathcal{A}/\bar{\mathcal{A}}. \end{cases} \tag{3.9}$$

Based on the definitions above the mathematical model of toll optimization problem with elastic demand can be formulated as

$$\text{TOLL}: \quad \max_{\mathbf{t},\mathbf{x}} \sum_{(i,j)\in\bar{\mathcal{A}}} t_{ij} f_{ij} \tag{3.10}$$

$$\text{s.t. } 0 \le t_{ij} \le t_{ij}^{\max} \qquad (i,j)\in\bar{\mathcal{A}} \tag{3.11}$$

$$\min_{x} \sum_{(i,j)\in\mathcal{A}} \int_0^{f_{ij}} c_{ij}(y)\mathrm{d}y + \sum_{(i,j)\in\bar{\mathcal{A}}} t_{ij} f_{ij} - \sum_{i\in\mathcal{N}}\sum_{d\in\mathcal{D}} \int_0^{d_i^q} g_{iq}^{-1}(v)\mathrm{d}v \tag{3.12}$$

$$\text{s.t. } (3.3)-(3.5) \tag{3.13}$$

$$d_i^q \ge 0 \qquad i\in\mathcal{N}, q\in\mathcal{D}. \tag{3.14}$$

where $\bar{\mathcal{A}} \subseteq \mathcal{A}$ denotes the links that are subject to tolling. In the case $\bar{A} \neq A$ the problem is refereed as second-best toll pricing. The upper level objective function (3.10) is revenue maximization and constraints (3.11) ensure that any toll price t_{ij} can not exceed the maximum allowed value $t_{ij}^{\max}$. Constraints (3.12-3.14) denotes the lower level elastic traffic assignment problem.

In the optimization context, the second-best toll problem is categorized as a mathematical programming problem with equilibrium constraints (MPEC). There are different techniques used to transform the bilevel optimization programming problem to a single level optimization program. These include sensitivity based analysis, Karush-Kuhn-Tucker (KKT) based method and using the system optimal solution to formulate the set of tolls for the second-best case under user equilibrium [89, 90]. In this study we use KKT based method and here we proposed corresponding first order optimality conditions of TAP problem with elastic demand and with toll pricing:

$$x_{ij}^q \left[c_{ij}(f_{ij}) + t_{ij} - \lambda_i^q + \lambda_j^q\right] = 0 \qquad (i,j)\in\mathcal{A}, q\in\mathcal{D} \tag{3.15}$$

$$c_{ij}(f_{ij}) + t_{ij} - \lambda_i^q + \lambda_j^q \ge 0 \qquad (i,j)\in\mathcal{A}, q\in\mathcal{D} \tag{3.16}$$

$$d_i^q \left[\lambda_i^q - g_{id}^{-1}(d_i^q)\right] = 0 \qquad i \in \mathcal{N}, q \in \mathcal{D} \tag{3.17}$$

$$\lambda_i^q - g_{iq}^{-1}(d_i^q) \geq 0 \qquad i \in \mathcal{N}, q \in \mathcal{D} \tag{3.18}$$

$$\sum_{j:(i,j)\in\mathcal{A}} x_{ij}^q - \sum_{j:(j,i)\in\mathcal{A}} x_{j,i}^q = d_i^q \qquad i \in \mathcal{N}, q \in \mathcal{D} \tag{3.19}$$

$$\sum_{q\in\mathcal{D}} x_{ij}^q = f_{ij} \qquad (i,j) \in \mathcal{A} \tag{3.20}$$

$$x_{ij}^q \geq 0 \qquad (i,j) \in \mathcal{A}, q \in \mathcal{D} \tag{3.21}$$

$$d_i^q \geq 0 \qquad i \in \mathcal{N}, q \in \mathcal{D} \tag{3.22}$$

here $\lambda_i^q,\ i \in \mathcal{N},\ q \in \mathcal{D}$, are the dual variables associated with constraint (3.3). At optimality, λ_i^q gives the duration of the shortest path time between i and q. In the rest of this chapter we use these optimality conditions of the TAP to denote the lower level problem and we will present all the bilevel programming models as single level programs.

3.1.3 Network Design Problem

When a transportation network is enhanced in response to some changing conditions, the corresponding bilevel optimization problem is called the network design problem (NDP). Under budgetary constraints, discrete NDPs usually consider link or lane additions, whereas continuous NDPs are limited to network improvements that can be modeled as continuous variables-such as lane and lateral clearance changes and also, other enhancements that produce incremental changes in capacity. We consider the continuous case in this study.

We assume that there are some costs associated with the enhancement of link capacities. The total investment and operating cost function is selected as

$$\sum_{(i,j)\in\mathcal{A}} k_{ij} z_{ij}^2 \ , \tag{3.23}$$

where z_{ij} represents the capacity enhancement on link (i,j) and k_{ij} is the unit cost for link (i,j) [2]. Capacity enhancement naturally affects the travel time on link (i,j) as follows:

$$c_{ij}(f_{ij}, z_{ij}) = \alpha_{ij} \left(1 + 0.15 \left(f_{ij}/(\beta_{ij} + z_{ij})\right)^4\right). \tag{3.24}$$

We now let $\bar{\mathcal{A}}_2$ denote the set of links capacities of which can be enhanced, and set the maximum capacity enhancement on link (i,j), denoted by $z_{ij}^{\max}$, as

$$z_{ij}^{\max} \begin{cases} > 0, & (i,j) \in \bar{\mathcal{A}}_2 \\ = 0, & (i,j) \in \mathcal{A}/\bar{\mathcal{A}}_2. \end{cases} \tag{3.25}$$

Then, the widely used continuous capacity enhancement model aiming at minimizing the total network travel cost [180] with elastic demand is formulated as

$$\text{CDND}: \quad \min_{\mathbf{z},\mathbf{x}} \sum_{(i,j)\in\mathcal{A}} (c_{ij}(f_{ij}, z_{ij}) f_{ij}) \tag{3.26}$$

$$\text{s.t.} \quad \sum_{(i,j)\in\mathcal{A}} k_{ij} z_{ij}^2 \leq B^{\max} \tag{3.27}$$

$$0 \leq z_{ij} \leq z_{ij}^{\max} \qquad (i,j) \in \mathcal{A} \tag{3.28}$$

$$x_{ij}^d \left[c_{ij}(f_{ij}, z_{ij}) - \lambda_i^d + \lambda_j^d \right] = 0 \qquad (i,j) \in \mathcal{A}, d \in \mathcal{D} \tag{3.29}$$

$$c_{ij}(f_{ij}, z_{ij}) - \lambda_i^d + \lambda_j^d \geq 0 \qquad (i,j) \in \mathcal{A}, d \in \mathcal{D} \tag{3.30}$$

$$(3.17) - (3.22). \tag{3.31}$$

Here $B^{\max}$ is the maximum budget that can be allocated for capacity enhancement. Constraints (3.27) ensures that the required expenses can not exceed the maximum budget amount. Constraints (3.29)-(3.31) are optimality conditions for lower level problem as presented in (3.15)-(3.22), where (3.29) and (3.30) are obtained by replacing the travel cost $c_{ij}(f_{ij})$ by $c_{ij}(f_{ij}, z_{ij})$ and by dropping t_{ij}.

In the rest of this chapter we use the toll pricing and network design problems and propose various bilevel programming models aiming to obtain sustainable transport.

3.2 Proposed Emission Functions and Bilevel Programming Models

The two main indicators of sustainability in transportation networks are the level of congestion and amount of emission. The congestion levels can easily be derived from traffic flows and designed capacities of the links. However, emission cannot be measured easily. To analyze the effect of emission and incorporate them into the models properly, the real relationship between traffic flow and total emission must be specified analytically.

Emission modeling is a wide research area. In one of the early studies, [83] show that vehicle emissions are highly dependent on the vehicle speed. Many researchers have studied the relation between transport emissions and vehicle types, speeds, driving styles, weather or several other factors [66, 80, 82, 99]. Meanwhile, emission factors are usually determined as the average values per vehicle kilometer for each vehicle category. In the literature, several mathematical models and simulation tools using emission factors are proposed to minimize emission [81, 140]. The emission factors that are determined by several institutions give reasonable approximations of the real emission amounts in relatively less congested networks. In case of high congestion, however, the amount of emission committed by the vehicles fluctuate considerably in time, mainly due to the emission during engine start and stop. Therefore, especially in highly congested networks, using emission factors may not fully reflect the real situation. To this end, emission functions with respect to the traffic flow may provide a different angle to evaluate different policies.

In this study, we consider emission functions instead of emission factors. We perform a two-step approach to express the total emission function in terms of the traffic flow. In the first step, we express the emission in terms of the speed. Then, we determine the mathematical relationship between the traffic flow and the average vehicle speed. Using these relationships, a single composite function is created based on the emission-speed and the speed-flow functions. Consequently, we obtain a general function of pollutant emissions with respect to the traffic flow.

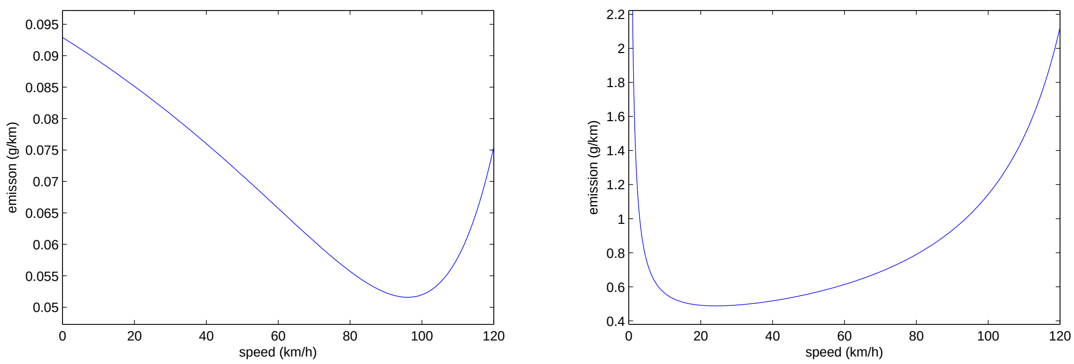
In the following sections, we first give the details of the conducted study for emission function, and then we insert these functions in toll optimization models as an extension. We describe the modifications on the model in details.

3.2.1 Multi-Step Process for Emission Function Determination

With the contribution of 32 member countries, the European Environment Agency (EEA) is a major information source for those involved in developing, adopting, implementing and evaluating environmental policies. Meanwhile, COPERT 4 is a software program which calculates the air pollutant emissions from the road transport and it is financed by the EEA, in the framework of the activities of the European Topic Center for Air and Climate Change. European Commission also defines the acceptable limits for exhaust emissions of new vehicles sold in EU member states as European emission standards (EURO). Currently, for most types of vehicles such as cars, lorries, trains, tractors and similar machinery, barges emissions of nitrogen oxides (NOx), total hydrocarbon (THC), non-methane hydrocarbons (NMHC), carbon monoxide (CO) and particulate matter (PM) are regulated, but they can not be regulated for seagoing ships and aeroplanes. For each vehicle different standards are applicable and non-compliant vehicles cannot be sold within the European Union. However, new standards do not apply to vehicles already on the roads. EURO standards are progressively updated and are refereed as EURO1, EURO2, etc.. As of 2010 EURO5 has been the latest standard and EURO6 standard is planned to be applied starting 2014. COPERT 4 defines the vehicle emission as a function of speed for pre-EURO and EURO class vehicles. The emission *in grams per kilometer* of an EURO class vehicle is expressed as,

$$\frac{a + cv + ev^2}{1 + bv + dv^2}, \tag{3.32}$$

where a, b, c, d and e are parameters specific to a vehicle and pollutant type, and v corresponds to the vehicle speed (kilometers per hour). Figure 3.1 shows the relation between the vehicle speed and the emission of CO and NOx pollutants for a EURO3 gasoline vehicle.



(a) NOxemission

(b) COemission

Figure 3.1: Vehicle emission per kilometer depending on average speed

NOx is known to be one of the major pollutants emitted during the traffic congestion. In fact, the transportation sources are reported to be responsible for a considerable part of all NOx emissions in the US, and moreover, NOx emissions show an increasing trend in the recent years [86, 135]. Similarly, in the UK almost half of all NOx emissions result from the road traffic [65]. Using this information, we focus only on NOx emission in the subsequent part of this section. However, we note that it is possible to follow the same steps here to analyze other major pollutants; such as, carbon monoxide, sulfur dioxide, and so on.

Akçelik [5, 6] has performed extensive studies to show that there is a direct relationship between the vehicle speed and the traffic flow on the link. Using these studies, the general vehicle speed - traffic flow relationship can be demonstrated as in Figure 3.2. The average vehicle speed retains almost constant until capacity is near %70. Afterwards, the average vehicle speed decreases substantially until the link capacity reaches to the designed level. Then the average vehicle speed continues to decrease slowly. We can next obtain the average speed v_{ij} in kilometers per hour on link (i, j) depending on the actual flow such as

$$v_{ij}(f_{ij}) = l_{ij}/c_{ij}(f_{ij}), \tag{3.33}$$

where l_{ij} is the link length in kilometers.

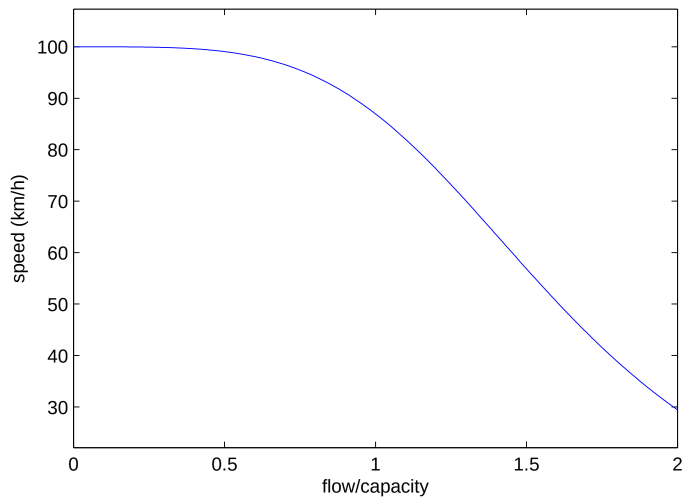


Figure 3.2: Average vehicle speed depending on link flow/capacity ratio

By constructing a composite function with the determined vehicle speed–traffic flow and emission–vehicle speed functions, we are able to express the total emission in terms of the traffic flow.

The total NOx emission function given in Figure 3.3 shows an exponential behavior. In fact, it is not difficult to assess that when the road capacity is reached and congestion occurs, vehicles start to follow stop/go pattern which decreases average vehicle speed and increases the total emission significantly. In sum, we can estimate the total emission of pollutant p in grams per hour on a particular link (i, j) with the following emission function:

$$e_{ij}^p(f_{ij}) = f_{ij} \times l_{ij} \times \frac{a^p + c^p v_{ij}(f_{ij}) + e^p v_{ij}^2(f_{ij})}{1 + b^p v_{ij}(f_{ij}) + d^p v_{ij}^2(f_{ij})}. \tag{3.34}$$

As shown in Figure 3.1, the emission amounts from different pollutants usually have large differences in magnitude. In that case, the emission amount from a particular

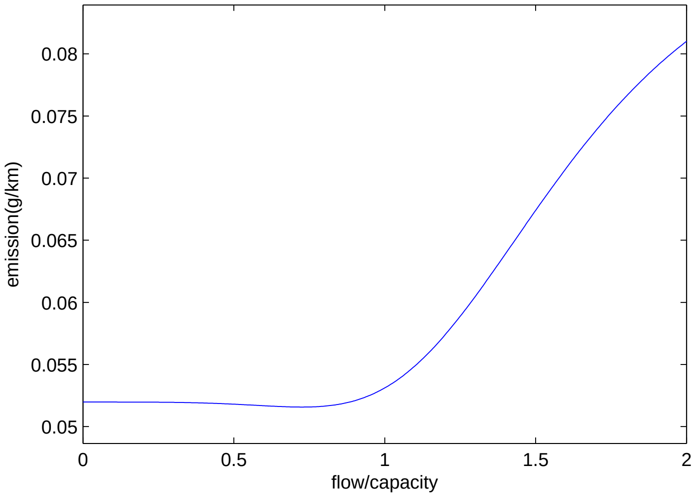


Figure 3.3: Per vehicle and kilometer NOx emission depending on link flow/capacity ratio

pollutant may be scaled by introducing proper coefficients to equation (3.34). To simplify the exposition, we shall simply take the summation over all pollutants using relation (3.34) without introducing such coefficients.

In the subsequent part of this study, we consider emission for a single pollutant,namely NOx and thus drop subscript p for simplicity.

3.2.2 Bilevel Programming Models with Emission Functions

In the following subsections, we discuss several bilevel problems where "upper level objectives involve alternate sustainability measures based on the proposed emission functions (3.34) and (3.50) and the bilevel problem is reformulated using lower level conditions in (3.15)-(3.22).

3.2.2.1 Total Network Emission

In this section, all our models aim to minimize the total network emission. The reduction in the total network emission is accomplished via two policies: (i) toll pricing, and (ii) capacity enhancement.

Toll Pricing

Road pricing is a demand management instrument, which is suitable to use for sustainability purposes. We shall use toll prices as disincentives to discourage network users using more congested links, and consequently, increasing the emissions.

In general, a governmental institution or transportation authority determines the set of tolled links to reduce the emission. Within the general bilevel toll pricing modeling framework, our mathematical model aiming at minimizing the total emission is formulated as

$$
\begin{aligned}
\text{TTE}: \quad & \min_{\mathbf{t},\mathbf{x}} \sum_{(i,j)\in\mathcal{A}} e_{ij}(f_{ij}) && && (3.35)\\
& \text{s.t.} \sum_{(i,j)\in\mathcal{A}} t_{ij} f_{ij} \geq \gamma_1 R^{\max} && && (3.36)\\
& \qquad 0 \leq t_{ij} \leq t_{ij}^{\max} && (i,j)\in\mathcal{A} && (3.37)\\
& \qquad (3.15) - (3.22), && && (3.38)
\end{aligned}
$$

where $R^{\max}$ denotes the maximum revenue that can be received from tolls and $\gamma_1 \in [0,1]$. The parameter $R^{\max}$ can be obtained by solving the model (TOLL). Constraint (3.36) ensures that the collected revenue is above a fraction of the maximum possible revenue. Constraint (3.38) is optimality conditions for lower level problem.

Capacity Enhancement

In addition to the road pricing strategy, capacity enhancement policy may also be a important instrument to decrease the network emission.

Then, our capacity enhancement model aiming at minimizing the total emission is formulated as

$$
\begin{aligned}
\text{CTE}: \quad & \min_{\mathbf{t},\mathbf{x}} \sum_{(i,j)\in\mathcal{A}} e_{ij}(f_{ij}, z_{ij}) && && (3.39)\\
& \text{s.t.} \sum_{(i,j)\in\mathcal{A}} k_{ij} z_{ij}^2 \leq \gamma_2 B^{\max} && && (3.40)\\
& \qquad 0 \leq z_{ij} \leq z_{ij}^{\max} && (i,j)\in\mathcal{A} && (3.41)\\
& \qquad x_{ij}^d \left[c_{ij}(f_{ij}, z_{ij}) - \lambda_i^d + \lambda_j^d \right] = 0 && (i,j)\in\mathcal{A}, d\in\mathcal{D} && (3.42)\\
& \qquad c_{ij}(f_{ij}, z_{ij}) - \lambda_i^d + \lambda_j^d \geq 0 && (i,j)\in\mathcal{A}, d\in\mathcal{D} && (3.43)\\
& \qquad (3.17) - (3.22). && && (3.44)
\end{aligned}
$$

Here $B^{\max}$ is the maximum budget that can be allocated for capacity enhancement and $\gamma_2 \in [0,1]$. $B^{\max}$ can be calculated by solving model (CTE) with constraints (3.40) relaxed. Constraint (3.40) ensures that the required expenses is below a fraction of the budget. Constraints (3.42)-(3.44) are optimality conditions for lower level problem as presented in (3.15)-(3.22).

Simultaneous Toll Pricing and Capacity Enhancement

To observe the combined effect of two traffic management strategies discussed previously, the simultaneous toll pricing and capacity enhancement model (TCTE) is constructed. Only links with variable capacity are allowed to be toll priced in this model. Parameters $R^{\max}$ and $B^{\max}$ are set to the same values selected for (TTE) and (CTE) models, respectively.

$$\text{TCTE}: \quad \min_{\mathbf{t},\mathbf{x}} \sum_{(i,j)\in\mathcal{A}} e_{ij}(f_{ij}, z_{ij}) \tag{3.45}$$

$$\text{s.t.} \quad (3.36), (3.37), (3.40), (3.41), \tag{3.46}$$

$$x_{ij}^{s}\left[c_{ij}(f_{ij}, z_{ij}) + t_{ij} - \lambda_i^s + \lambda_j^s\right] = 0 \qquad (i,j)\in\mathcal{A}, s\in\mathcal{D} \tag{3.47}$$

$$c_{ij}(f_{ij}, z_{ij}) + t_{ij} - \lambda_i^s + \lambda_j^s \geq 0 \qquad (i,j)\in\mathcal{A}, s\in\mathcal{D} \tag{3.48}$$

$$(3.17) - (3.22), \tag{3.49}$$

where (3.47) and (3.48) are obtained by only replacing the travel cost $c_{ij}(f_{ij})$ by $c_{ij}(f_{ij}, z_{ij})$.

3.2.2.2 Emission Dispersion

Directing the vehicle flow to other parts of the transportation network through road pricing may lead to high emission accumulations in wider area. Therefore, it may be preferable to disperse the emission rather than minimizing the total emission on the network. We next discuss the formulation of toll pricing and capacity enhancement policies with the objective of emission dispersion.

As an alternative to the total emission minimization models where link lengths are important, we deal with pollutant concentration in emission dispersion models. By using the analysis in the derivation of equation (3.34), emission concentration on link (i, j) is calculated as

$$\bar{e}_{ij}(f_{ij}) = f_{ij} \times \frac{a + cv_{ij}(f_{ij}) + ev_{ij}^2(f_{ij})}{1 + bv_{ij}(f_{ij}) + dv_{ij}^2(f_{ij})}, \tag{3.50}$$

where $\bar{e}_{ij}^{p}$ is measured in grams per kilometer and hour.

Toll Pricing

The toll pricing model in the context of emission dispersion involves the same constraints (3.36)-(3.38) of model (TTE). The only difference is in the objective function of the upper level problem. We propose two alternate objective functions.

The first objective that we consider is the pure dispersion case. The main idea is to minimize the maximum link emission concentration on the network. Formally, the objective function (3.35) of model (TTE) is replaced with

$$\text{TED1}: \quad \min_{\mathbf{t},\mathbf{x}} \max \{ \bar{e}_{ij}(f_{ij}) | (i,j) \in \mathcal{A} \}. \tag{3.51}$$

Traffic flows with reasonable emission levels in highly dense parts of a network may sum up to excessive amounts. Due to land use characteristics (i.e. residential, commercial, etc.), the network management authorities may determine emission limits for certain parts of the network. Let ζ_{ij} denote the desired emission concentration level on link (i,j). The product of this amount with the link length gives the desired emission level for that link.Along these lines, the second objective that we propose is formulated as

$$\text{TED2}: \quad \min_{\mathbf{t},\mathbf{x}} \sum_{(i,j)\in\mathcal{A}} \max \{ e_{ij}(f_{ij}) - \zeta_{ij} l_{ij},\, 0 \}. \tag{3.52}$$

With this objective, we penalize the amount of emission that exceed the desired level. Introducing (3.52) as the objective function is another way to disperse the total network emission through toll pricing.

Capacity Enhancement

The dispersion of the emission throughout the network may also be attained by capacity enhancement. We keep the set of constraints (3.40)-(3.44) of model (CTE) and consider two alternate upper level objective functions as in the models based on the toll pricing policy. The first one corresponds to the pure dispersion objective in (3.51). The only difference is the inclusion of the capacity enhancements in evaluating the emission amounts

$$\text{CED1}: \quad \min_{\mathbf{z},\mathbf{x}} \max \{ \bar{e}_{ij}(f_{ij}, z_{ij}) | (i,j) \in \mathcal{A} \}. \tag{3.53}$$

Similarly, the second upper level objective function is the same as (3.52) with the only difference of the involved capacity enhancements

$$\text{CED1}: \quad \min_{\mathbf{z},\mathbf{x}} \sum_{(i,j)\in\mathcal{A}} \max\{\, e_{ij}(f_{ij}, z_{ij}) - \zeta_{ij} l_{ij},\, 0\,\}\,. \tag{3.54}$$

Simultaneous Toll Pricing and Capacity Enhancement

With the same line of reasoning that was used in the previous two cases, the constraints (3.46)-(3.49) of model (TCTE) are kept, and the objective function (3.45) is replaced with

$$\text{TCED1}: \quad \min_{\mathbf{t},\mathbf{z},\mathbf{x}} \max\{\, \bar{e}_{ij}(f_{ij}, z_{ij}) | (i,j) \in \mathcal{A}\,\} \tag{3.55}$$

or

$$\text{TCED1}: \quad \min_{\mathbf{t},\mathbf{z},\mathbf{x}} \sum_{(i,j)\in\mathcal{A}} \max\{\, e_{ij}(f_{ij}, z_{ij}) - \zeta_{ij} l_{ij},\, 0\,\} \tag{3.56}$$

to produce new emission dispersion models.

All the proposed models in this chapter, are applied to the testing network in the computation study and analysis chapter.

CHAPTER 4

STOCHASTIC BILEVEL PROGRAMMING WITH TRAVEL TIME RELIABILITY

In this chapter we first briefly discuss the network and the travel time reliability. Then, we describe how to incorporate the travel time reliability into the toll pricing problem as a sustainability measure in the stochastic bilevel framework. In particular, we consider the conditional value-at-risk (CVaR) as a risk measure on the travel time costs to model the travel time reliability. Using such a risk measure allows us to take the effect of the stochastic nature of the system into consideration. We develop alternate risk-averse bilevel programming models involving the CVaR risk measure. We also present the risk-neutral versions of the proposed models in order to analyze the effect of incorporating risk measures.

4.1 Network Reliability

In transportation networks, reliability reflects the ability of the system to respond to the variations in system variables. Several events such as minor accidents, on-street parking violations, variations in weather conditions, road maintenance and traffic signal failures may effect the operation of a network and it is important to quantify the impact of these events to deal with the uncertainty inherent in the transportation systems. There are various approaches to model the network reliability. Here we focus on the travel time reliability, which refers to variability of travel times because of unpredictable underlying conditions over the time, in terms of traffic flow values. When we consider such a stochastic (unreliable) transportation network the travel times are stochastic, and therefore, the travelers do not certainly know whether they will arrive at the destination points on time when they are planning their trips. Thus, the decision makers should take the travel time variations into account while determining their policies. Despite all these disturbances, a network should maintain an acceptable level of service so improving travel time reliability is our main objective in this study. A significant improvement can be achieved by obtaining relatively small travel times under variable conditions.

We can consider the travel time variations due to the non-recurrent events such as weather conditions by modeling the randomness in the free-flow times. Moreover, the travel time variations due to the non-recurrent events such as vehicle breakdowns and minor accidents can be taken into consideration by modeling the randomness in the link capacities. Stochastic programming is one of the important approaches to model decision making under uncertainty. It develops models to formulate optimization problems in which uncertain quantities are represented by random variables. We represent the uncertain free-flow times and link capacities by random variables and we characterize these random variables by using a finite set of scenarios, denoted by $\mathcal{S}$. We assume that the set of scenarios and the associated probabilities, which we denote by $p_s, s \in \mathcal{S}$, are given. A scenario represents a joint realization of the free-flow times and capacities of all the links in the network. Since the travel time is a function of the random link capacities and free-flow times, the travel time is also random. Let us denote the random travel time (cost) when the total flow on link (i, j) equals to f_{ij} by $c_{ij}(f_{ij}, \omega)$. Then the general travel time function for link (i, j) under scenario $s \in \mathcal{S}$, which is basically the realization of $c_{ij}(f_{ij}, \omega)$ under scenario s, is given by:

$$c^s_{ij}(f_{ij}) = \alpha^s_{ij} \left(1 + 0.15 \, \left(f_{ij}/\beta^s_{ij}\right)^4\right). \tag{4.1}$$

Here α^s_{ij} and β^s_{ij} denote the realized free flow time and the realized capacity value of link (i, j) under scenario s, respectively. Thus, scenario s is represented by the deterministic vector $(\alpha^s, \beta^s) \in \mathbb{R}^{2*|A|}$, with components α^s_{ij} and $\beta^s_{ij},\ s \in \mathcal{S},\ (i, j) \in \mathcal{A}$.

Note that one can focus on only the randomness in the link capacities and consider the free-flow times as deterministic parameters. In such cases, we can simply replace α^s_{ij} by the deterministic parameter α_{ij} and a scenario would represent the joint realizations of only the link capacities. Similarly, if one focus on only the randomness in the free-flow times, β^s_{ij} is replaced by the deterministic parameters by β_{ij} in equation (4.1).

In the rest of this chapter using the scenario-based approach we propose stochastic bilevel programming models that involve the travel time reliability.

4.2 Stochastic Bilevel Programming Models

As discussed in Chapter 2, in the transportation framework the bilevel programming problem involves the decisions of the transportation managers in the upper level and, given these decisions, the route choice decisions of network users in the lower level. In all of our proposed models, in the upper level transportation managers aim to obtain a sustainable transportation system in terms of the travel time reliability by using the toll pricing policy. On the other hand, the network users make their traveling decisions based on the expected travel costs. Note that we assume toll pricing decisions do not vary according to the stochastic nature of the network.

In the following sections we first introduce the lower level traffic assignment problem with stochastic travel times under the toll pricing policy. Then we propose upper level models with alternate objectives based on the CVaR risk measure.

4.2.1 Risk-Neutral Traffic Assignment Problem

We assume that network users are unaware of the network conditions when they are making their route choices. Thus, they are not certain about which scenario representing the network conditions will occur. We incorporate the random network conditions by the scenario-based approach and the network users make their traveling decisions based on the expected travel costs. Thus, we use the user equilibrium formulation based on the expected travel times and formulate the risk-neutral user equilibrium formulation with the toll pricing policy as follows:

$$\text{RNTAP}: \quad \min_{\mathbf{x}} \sum_{(i,j)\in\mathcal{A}} \int_0^{f_{ij}} \sum_s p_s c^s_{ij}(y)\mathrm{d}y + \sum_{(i,j)\in\bar{A}} t_{ij} f_{ij} \tag{4.2}$$

$$\text{s.t.} \sum_{j:(i,j)\in\mathcal{A}} x^q_{ij} - \sum_{j:(j,i)\in\mathcal{A}} x^q_{ji} = d^q_i \qquad i\in\mathcal{N}, q\in\mathcal{D} \tag{4.3}$$

$$\sum_{q\in\mathcal{D}} x^q_{ij} = f_{ij} \qquad (i,j)\in\mathcal{A} \tag{4.4}$$

$$x^q_{ij} \geq 0 \qquad (i,j)\in\mathcal{A}, q\in\mathcal{D}. \tag{4.5}$$

The objective (4.2) minimizes the expected total cost. The set of constraints (4.3) represents the flow conservation constraints. Constraints (4.4) link the total flow on an arc to the flows resulting from individual destination points. The rest of the constraints are for the nonnegativity restrictions.

Let $\lambda^q_i,\ i\in\mathcal{N},\ q\in\mathcal{D}$, denote the dual variables associated with constraint (4.3).

Then the first order optimality conditions for the problem RNTAP are

$$x_{ij}^q \left[\sum_s p_s c_{ij}^s(f_{ij}) + t_{ij} - \lambda_i^q + \lambda_j^q \right] = 0 \qquad (i,j) \in \mathcal{A}, q \in \mathcal{D} \tag{4.6}$$

$$\sum_s p_s c_{ij}^s(f_{ij}) + t_{ij} - \lambda_i^q + \lambda_j^q \geq 0 \qquad (i,j) \in \mathcal{A}, q \in \mathcal{D} \tag{4.7}$$

$$\sum_{j:(i,j)\in\mathcal{A}} x_{ij}^q - \sum_{j:(j,i)\in\mathcal{A}} x_{ji}^q = d_i^q \qquad i \in \mathcal{N}, q \in \mathcal{D} \tag{4.8}$$

$$\sum_{q\in\mathcal{D}} x_{ij}^q = f_{ij} \qquad (i,j) \in \mathcal{A} \tag{4.9}$$

$$x_{ij}^q \geq 0 \qquad (i,j) \in \mathcal{A}, q \in \mathcal{D}. \tag{4.10}$$

In the elastic demand case, $d_i^q, i \in \mathcal{N}, q \in \mathcal{D}$, are not given and these demand values are considered as decision variables. We formulate the RNTAP with elastic demand by adding the nonnegativity restrictions on the demand variables and modifying the objective function (4.2) as follows:

$$\min_{\mathbf{x}} \sum_{(i,j)\in\mathcal{A}} \int_0^{f_{ij}} \sum_s p_s c_{ij}^s(y)\mathrm{d}y + \sum_{(i,j)\in\bar{A}} t_{ij} f_{ij} - \sum_{i\in\mathcal{N}} \sum_{q\in\mathcal{D}} \int_0^{d_i^q} g_{iq}^{-1}(v)\mathrm{d}v. \tag{4.11}$$

Here $g_{iq}^{-1}(\cdot)$ denotes the inverse of the demand function on link (i,q) in terms of the travel time. In our study, we assume that the demand value depends on the travel time linearly. However, this assumption is not restrictive and any other type of non-increasing function can be considered in our setup. We refer to the user equilibrium problem with expected total cost and the elastic demand as (ELPNRAP). The first order optimality conditions of the problem (ELPNRAP) consist of the conditions given in (4.6)-(4.10) and the following additional ones involving demand decision variables:

$$d_i^q \left[\lambda_i^q - g_{iq}^{-1}(d_i^q)\right] = 0 \qquad i \in \mathcal{N}, q \in \mathcal{D} \tag{4.12}$$

$$\lambda_i^q - g_{iq}^{-1}(d_i^q) \geq 0 \qquad i \in \mathcal{N}, q \in \mathcal{D} \tag{4.13}$$

$$d_i^q \geq 0 \qquad i \in \mathcal{N}, q \in \mathcal{D}. \tag{4.14}$$

4.2.2 Risk-Neutral Bilevel Programming Models

As discussed at the beginning of this chapter, the random free-flow times and link capacities lead to random travel times. Finding the best toll pricing policy requires approaches to compare the associated random travel times. In traditional stochastic programming approach random variables are compared based on the expected values.

In our setup we have multiple random variables to take into consideration; a random travel time on each link of the network. We define alternate ways of obtaining a single random outcome out of the individual random travel times, which is basically a measure for the whole network and focus on that network-based random outcome while finding the best toll pricing policy. In particular, we use the unit travel time summed over all links (AUTT), the total travel time summed over all links (ATTT), the maximum unit travel time (MUTT) and the maximum total travel time (MTTT) to define the network-based measure. Here, the total travel time is obtained by multiplying the unit travel time of a link with the corresponding link flow amount. Note that we also introduce similar network-based measures like the maximum unit emission concentration and the total emission summed over all links in the previous chapter (see Chapter 3). First, to minimize the total network emission in the network, we consider total emission amount in a link by multiplying the emission amount per kilometer with the length of that link. Moreover, to minimize the maximum emission concentration in the network, we use emission amount per kilometer to consider all long and short links equally. Thus, in Chapter 3 link length values enable us to obtain total emission amount in a link rather than the emission amount per kilometer and similarly in this chapter link flow amounts help us to consider the case with all users in a link rather than a single user. In the rest of this section, we first present the risk-neutral bilevel programming models and then their risk-averse versions involving the CVaR as risk measures.

Minimizing Expected Aggregated Unit Travel Time

We refer to the unit travel time per vehicle summed over all the "the aggregated unit travel time" and calculate the point estimator of the expected value of the aggregated unit travel time when the total flow on link (i, j) is equal to f_{ij} as:

$$\mathbb{E}\left[\sum_{(i,j)\in\mathcal{A}} c_{ij}(f_{ij}, \omega)\right] = \sum_{(i,j)\in\mathcal{A}} \sum_{s} p_s c_{ij}^s(f_{ij}). \tag{4.15}$$

Then, the corresponding bilevel programming model reads:

$$\text{EAUTT}: \quad \min_{\mathbf{t},\mathbf{x}} \sum_{(i,j)\in\mathcal{A}} \sum_{s} p_s c_{ij}^s(f_{ij}) \tag{4.16}$$

$$\text{s.t. } 0 \le t_{ij} \le t_{ij}^{\max} \quad (i,j) \in \mathcal{A} \tag{4.17}$$

$$(4.6) - (4.10) \quad \text{(fixed demand case)} \tag{4.18}$$

$$(4.6) - (4.10) \text{ and } (4.12) - (4.14) \quad \text{(elastic demand case)}, \tag{4.19}$$

where the upper level objective (4.16) minimizes the expected aggregated unit travel time per vehicle. Constraints (4.17) ensure that the toll rice on link (i, j) cannot exceed the maximum allowed value $t_{ij}^{\max}$. According to the demand structure either constraints (4.18) or (4.19) are used and these constraints represent the optimality conditions for the lower level problem with the fixed demand and the elastic demand, respectively.

Minimizing Expected Aggregated Total Travel Time

Here we focus on the total travel time on each link and define the network-based measure as the summation of all the total travel time values. Then we can estimate the expectation of the aggregated total travel time as:

$$\mathbb{E}\left[\sum_{(i,j)\in\mathcal{A}} f_{ij} c_{ij}(f_{ij}, \omega)\right] = \sum_{(i,j)\in\mathcal{A}} f_{ij} \sum_{s} p_s c_{ij}^s(f_{ij}). \tag{4.20}$$

Then we replace the objective function of the problem (EAUTT) by

$$\min_{\mathbf{t},\mathbf{x}} \sum_{(i,j)\in\mathcal{A}} f_{ij} \sum_{s} p_s c_{ij}^s(f_{ij}), \tag{4.21}$$

and obtain the risk-neutral model with the aggregated total travel time, which is refereed to as "EATTT".

Minimizing Expected Maximum Unit Travel Time

Here we incorporate the maximum unit travel time as a network-based measure. We deal with the worst possible case for the network by concentrating on the largest travel time of all links. Thus, minimizing the maximum amount help us to minimize all the remaining travel times at the same time. We introduce variables denoted by $e^s, s \in \mathcal{S}$

to represent the maximum unit travel time under each scenario and so that we have

$$\mathbb{E}\left[\max_{(i,j)\in\mathcal{A}} c_{ij}(f_{ij},\omega)\right] = \sum_{s} p_s e^s, \tag{4.22}$$

Then the related formulation of this problem is in the following form

$$\text{EMUTT}: \quad \min_{\mathbf{t},\mathbf{x}} \sum_{s} p_s e^s \tag{4.23}$$

$$\text{s.t.} \ (4.17) \tag{4.24}$$

$$e^s \geq c^s_{ij}(f_{ij}) \quad (i,j)\in\mathcal{A}, s\in\mathcal{S} \tag{4.25}$$

$$(4.6)-(4.10) \quad \text{(fixed demand case)} \tag{4.26}$$

$$(4.6)-(4.10) \text{ and } (4.12)-(4.14) \quad \text{(elastic demand case)}, \tag{4.27}$$

where the upper level objective function (4.23) is used to minimize expected maximum unit travel time and constraints (4.25) is used to obtain the maximum unit travel cost under each scenario. Note that due to the nature of the objective function e^s is exactly equal to the maximum unit travel time value.

Minimizing Expected Maximum Total Travel Time

By replacing the constraints (4.25) in the formulation of (EMUTT) problem with

$$e^s \geq f_{ij}c^s_{ij}(f_{ij}) \quad (i,j)\in\mathcal{A}, s\in\mathcal{S}, \tag{4.28}$$

we obtain the risk-neutral model with the maximum total travel time which is refereed to as "EMTTT". In this formulation $e^s, \in \mathcal{S}$ variables are used to calculate the maximum total travel time instead of the maximum unit travel time under each scenario.

4.2.3 Risk-Averse Bilevel Programming Models

The traditional stochastic programming approaches are based on the expected values. However, decisions obtained just according to the expected values may perform poorly under certain realizations of the random data. Therefore, it is significant to consider also the effect of the inherent variability, which leads to the risk concept. Risk measures can be incorporated into decision making problems in order to model the effects of the variability. Using such a risk-averse approach, we can provide solutions which may perform better under random disruptions than the ones obtained by the risk-neutral approach.

In this study, we propose two types of models which incorporate risk measures. The first type of models consider only the risk terms, whereas the second type of models consider both the expectation and the risk measure of the specified random network-based quantity. While considering both the expectation and the risk measure, we utilize the mean-risk approach, which has been first proposed by Markowitz [118] for a portfolio optimization problem. In the mean-risk approach, the mean represents the expected outcome of interest and some dispersion statistic is used as a measure of risk, and the trade-off between the mean and the risk is considered. Moreover, when the variance is used as a measure of risk, we obtain the classical mean-variance (Markowitz [118]) model. Boyles*et al.* [28] consider the toll pricing model under stochastic travel times. They use the variance as a risk measure in their related study in which they incorporate the reliability into the lower level traffic assignment problem using a scenario-based approach. However, using a symmetric measure such as variance has some drawbacks. One of the drawbacks associated with this measure is that it treats over-performance equally as under-performance which may lead to inferior results. In order to remedy this drawback, models with alternative asymmetric risk measures such as downside risk measures have been proposed (see e.g., Ogryczak and Ruszczyński [130]).

In this thesis, we prefer to incorporate one of the popular downside risk measures, conditional value-at-risk (CVaR) into the proposed risk-averse stochastic programming models. In particular, since we prefer smaller values of travel times in order to improve travel time reliability, we specify CVaR as a risk measure on the specified network-based quantity, which is basically a function of the travel times. Chen and Zhou [44] also model the travel time reliability by using CVaR. Different than our approach, they only consider the traffic assignment problem and they use restrictive distribution assumptions to calculate the CVaR quantities. Our approach does not depend on such restrictive assumptions.

Here we present the definitions of VaR and CVaR and provide some interpretations.

Definition 1 *Let $F_Z(\cdot)$ represent the cumulative distribution function of a random variable Z. In the financial literature, the α-quantile*

$$\inf\{\eta : \ F_Z(\eta) \geq \alpha\}$$

is called the Value at Risk (VaR) at the confidence level α and denoted by $\mathrm{VaR}_\alpha(Z)$*,* $\alpha \in (0, 1]$.

Definition 2 *The Conditional-Value-at-Risk of a random variable Z at the confidence level α is given by*

$$CVaR_\alpha(Z) = \inf_{\eta \in \mathbb{R}} \{\eta + \frac{1}{1-\alpha}\mathbb{E}(\max\{Z - \eta, 0\})\}. \tag{4.29}$$

$\mathrm{CVaR}_\alpha(Z)$ is the expectation of travel cost value exceeding the VaR value at the confidence level α. In the travel cost minimization context, VaR_α is the α-quantile (a high quantile) of the distribution of the travel cost, which provides an upper bound for a cost value that is exceeded only with a small probability of $1-\alpha$. On the other hand, $\mathrm{CVaR}_\alpha(Z)$ is a measure of severity of cost if it is more than $\mathrm{VaR}_\alpha(Z)$ (see [144, 145]). The illustration of CVaR measure and relation with VaR can be seen from the Figure 4.1 explicitly [166].

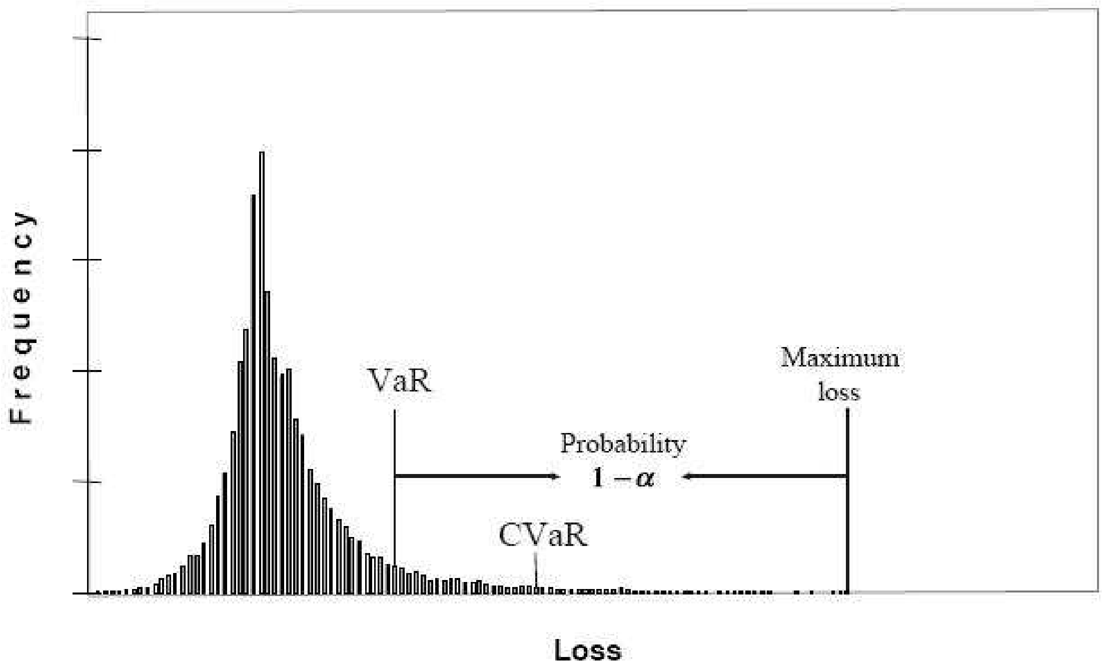


Figure 4.1: Illustration of CVaR measure

In the following parts, as a measure of variability of travel time CVaR is added to the upper level of the bilevel programming models to influence the toll pricing decisions.

4.2.3.1 Proposed Risk-Averse Models with Only Risk Terms

Here, we present risk-averse models with alternate objectives based on the CVaR risk measure. Presented models consider CVaR on the previously stated network-based quantities such as the aggregated unit travel time, the aggregated total travel time, the maximum unit travel time and the maximum total travel time.

Minimizing Aggregated Unit Travel Time (Cost) by Using "CVaR"

In this model, we focus on the random variable, aggregated unit travel time and we denote it as $Z = \sum_{(i,j)\in\mathcal{A}} c_{ij}(f_{ij},\omega)$.

Then the related model which is minimizing $\text{CVaR}_\alpha[\sum_{(i,j)\in\mathcal{A}} c_{ij}(f_{ij},\omega)]$ is proposed as:

$$\text{AUTT\_CVaR}: \quad \min_{\mathbf{t},\mathbf{x}} (\eta + \frac{1}{1-\alpha}\sum_s p_s v^s) \tag{4.30}$$

$$\text{s.t. } (4.17) \tag{4.31}$$

$$v^s \geq \sum_{(i,j)\in\mathcal{A}} c^s_{ij}(f_{ij}) - \eta \quad s \in \mathcal{S} \tag{4.32}$$

$$v^s \geq 0 \qquad s \in \mathcal{S} \tag{4.33}$$

$$(4.6) - (4.10) \quad \text{(fixed demand case)} \tag{4.34}$$

$$(4.6) - (4.10) \text{ and } (4.12) - (4.14) \quad \text{(elastic demand case)}, \tag{4.35}$$

where $v^s, s \in \mathcal{S}$ variables are introduced to specify CVaR on the unit travel time under each scenario. Constraints (4.32) and (4.33) are introduced to linearize the max operator used in the equation (4.29) under each scenario. Note that if the difference in constraint (4.32) is positive then due to the nature of the minimization objective this constraint satisfied as a equality otherwise v^s gets value 0.

Minimizing Aggregated Total Travel Time (Cost) by Using "CVaR"

In this case, the random variable Z is equal to $\sum_{(i,j)\in\mathcal{A}} f_{ij}c_{ij}(f_{ij},\omega)$ and we propose the corresponding model by just replacing the constraints (4.32) of the problem (AUTT_CVaR) with

$$v^s \geq \sum_{(i,j)\in\mathcal{A}} f_{ij}c^s_{ij}(f_{ij}) - \eta \quad s \in \mathcal{S}. \tag{4.36}$$

Here, $v^s s \in \mathcal{S}$ variables are introduced on the aggregated total travel time instead of the aggregated unit travel time. We refer to this model as "ATTT_CVaR"

Minimizing Maximum Unit Travel Time (Cost) by Using "CVaR"

We incorporate $\max_{(i,j)\in\mathcal{A}} c_{ij}(f_{ij}, \omega)$ as the random variable in this model. Here we present the related bilevel programming model as follows:

$$\text{MUTT\_CVaR}: \quad \min_{\mathbf{t},\mathbf{x}} (\eta + \frac{1}{1-\alpha} \sum_s p_s y^s) \tag{4.37}$$

$$\text{s.t. } (4.17) \tag{4.38}$$

$$e^s \geq c^s_{ij}(f_{ij}) \quad (i,j) \in \mathcal{A}, s \in \mathcal{S} \tag{4.39}$$

$$y^s \geq e^s - \eta \quad s \in \mathcal{S} \tag{4.40}$$

$$y^s \geq 0 \qquad s \in \mathcal{S} \tag{4.41}$$

$$(4.6) - (4.10) \quad \text{(fixed demand case)} \tag{4.42}$$

$$(4.6) - (4.10) \text{ and } (4.12) - (4.14) \quad \text{(elastic demand case)}, \tag{4.43}$$

where $e^s, s \in \mathcal{S}$ variables are introduced to represent the maximum unit travel time under each scenario and $y^s, s \in \mathcal{S}$ variables are defined to incorporate CVaR into the model on the maximum unit travel time under each scenario. The objective function (4.37) is minimizing the maximum unit travel time (cost) by using CVaR. Constraints (4.39) are used to determine the maximum unit travel time under each scenario. Constraints (4.40) and (4.41) are used for linearization operation under each scenario.

Minimizing Maximum Total Travel Time (Cost) by Using Using "CVaR"

Here we incorporate the maximum total travel time as a network-based measure by only replacing the constraints (4.39) of the problem (MUTT_CVaR) with

$$e^s \geq f_{ij} c^s_{ij}(f_{ij}) \quad (i,j) \in \mathcal{A}, s \in \mathcal{S}. \tag{4.44}$$

In this formulation $e^s, s \in \mathcal{S}$ and $y^s, s \in \mathcal{S}$ variables are defined on the maximum total travel time and we refer to this model as "MTTT_CVaR"

4.2.3.2 Proposed Risk-Averse Models with Mean-Risk Terms

As we mentioned in the previous parts, the mean-risk approach considers the trade-off between the mean and the risk. Here we show the general formulation of the mean-risk function with CVaR as follows:

$$\mathbb{E}[Z] + \theta \text{CVaR}_\alpha[Z], \tag{4.45}$$

where θ is the trade-off coefficient. We also refer to it as a risk coefficient, which is specified by decision makers according to their risk preferences.

By using the previously stated network-based measures AUTT, ATTT, MUTT, MTTT, four alternate objective functions are developed as follows:

- Mean risk function on aggregated unit travel time (cost) (MRAUTT)

$$\mathbb{E}\left[\sum_{(i,j)\in\mathcal{A}} c_{ij}(f_{ij},\omega)\right] + \theta\text{CVaR}_\alpha\left[\sum_{(i,j)\in\mathcal{A}} c_{ij}(f_{ij},\omega)\right] = \sum_{(i,j)\in\mathcal{A}}\sum_s p_s c_{ij}^s(f_{ij}) + \theta(\eta + \frac{1}{1-\alpha}\sum_s p_s v^s), \tag{4.46}$$

where $v^s, s \in \mathcal{S}$, variables satisfy the constraints (4.32) and (4.33).

- Mean risk function on aggregated total travel time (cost) (MRATTT)

$$\mathbb{E}\left[\sum_{(i,j)\in\mathcal{A}} f_{ij}c_{ij}(f_{ij},\omega)\right] + \theta\text{CVaR}_\alpha\left[\sum_{(i,j)\in\mathcal{A}} f_{ij}c_{ij}(f_{ij},\omega)\right] = \sum_{(i,j)\in\mathcal{A}} f_{ij}\sum_s p_s c_{ij}^s(f_{ij}) + \theta(\eta + \frac{1}{1-\alpha}\sum_s p_s v^s). \tag{4.47}$$

In this case $v^s, s \in \mathcal{S}$, variables satisfy the constraints (4.36) and the nonnegativity constraints.

- Mean risk function on maximum unit travel time (cost) (MRMUTT)

$$\mathbb{E}\left[\max_{(i,j)\in\mathcal{A}} c_{ij}(f_{ij},\omega)\right] + \theta\text{CVaR}_\alpha\left[\max_{(i,j)\in\mathcal{A}} c_{ij}(f_{ij},\omega)\right] = \sum_s p_s e^s + \theta(\eta + \frac{1}{1-\alpha}\sum_s p_s y^s), \tag{4.48}$$

where $e^s, s \in \mathcal{S}$, variables satisfy the constraints (4.39) and $y^s, s \in \mathcal{S}$ variables satisfy the constraints (4.40) and (4.41).

- Mean risk function on maximum total travel time (cost) (MRMTTT)

$$\mathbb{E}\left[\max_{(i,j)\in\mathcal{A}} f_{ij}c_{ij}(f_{ij},\omega)\right] + \theta\text{CVaR}_\alpha\left[\max_{(i,j)\in\mathcal{A}} f_{ij}c_{ij}(f_{ij},\omega)\right] = \sum_s p_s e^s + \theta(\eta + \frac{1}{1-\alpha}\sum_s p_s y^s), \tag{4.49}$$

where $e^s, s \in \mathcal{S}$, variables satisfy the constraints (4.44) and $y^s, s \in \mathcal{S}$ variables satisfy the constraints (4.44) and the nonnegativity constraints.

Then, the corresponding models can be obtained by changing the objective functions of the problems (AUTT_CVaR) with (4.46), (ATTT_CVaR) with (4.47), (MUTT_CVaR) with (4.48) and (MTTT_CVaR) with (4.49).

The effects of risk parameters and comparative results of the proposed models will be analyzed in the computation study and analysis chapter.

CHAPTER 5

COMPUTATIONAL RESULTS AND ANALYSIS

In this chapter we present numerical results for the proposed optimization problems involving sustainability measures. Section 5.1 presents the first main part of the computational study, which is performed for the optimization models with the measurement of gas emissions. The main objective of this section is to analyze the effects of the proposed alternate models on the emission amounts and evaluate the toll pricing and capacity enhancement policies in terms of the specified sustainability measures. Section 5.2 provides the numerical results for the risk-averse models with the travel-time reliability. The associated numerical study focuses on analyzing how the decisions change by incorporating the risk terms, the effects of the risk parameters and comparative results obtained by the alternate objectives.

Although bilevel programs are difficult nonlinear optimization problems, there are very effective methods that reduce the problem to a single level by some reformulations. A particularly powerful implementation exists within GAMS modeling language through the NLPEC package [73]. This package exploits several methodologies for reformulating the mathematical programs with equilibrium constraints as nonlinear programs and calls subsequently several powerful off-the-shelf nonlinear programming solvers for their solution; see [68] for details. We use CONOPT solver [61] in our experiments. All the results are obtained using the following options of the current NLPEC manual: `reftype mult, initmu 1, numsolves 5, finalmu 0`.

Note that all the numerical experiments were performed on a HP Z800 workstation running on Linux with 2 quad-core 3.2GHz CPU, and 32 GB of RAM. All reported CPU times are in seconds.

5.1 Models with Emission Functions

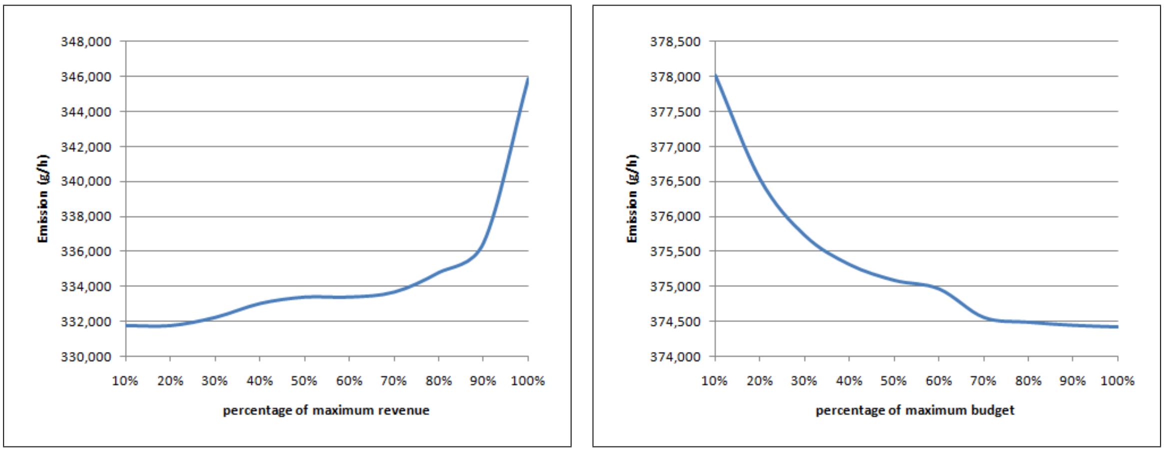
In this part of our computational study, we use the well-known medium-size Sioux Falls network (see Figure 5.2) which consists of 24 nodes and 76 links. Its trip table is nearly symmetric and all the links come in bi-directional pairs with identical characteristics. It is important to note that this map is not to scale, so the length of links is not related to the free flow times between pairs of nodes.

The original Sioux Falls network data includes the fixed peak hour demand for O–D pairs. Modeling the elastic demand requires us to specify the values of the parameters of the linear demand functions given in (3.6). To do this, we first solve TAP formulated in (3.2)-(3.5) for the original fixed demand data. Then based on the optimal link flow values, we calculate the travel time for each link and the shortest path times for each O–D pair. Denoting the duration of the shortest path time and the original fixed demand for O–D pair (i, d) by $\bar{t}_{id}$ and $\bar{d}_{id}$, respectively, the parameters of the elastic demand function in (3.6) are calculated from the linear interpolation of points $(\bar{t}_{id}, \bar{d}_{id})$ and $(\delta\bar{t}_{id}, \bar{d}_{id}/\delta)$, where δ is a random number generated from the uniform distribution on the interval $(2, 3)$ [103].

There are also other parameters specific to the proposed models to be set. We choose the following arcs to charge and/or to enhance: (6,8), (8,6), (10,15), (11,14), (14,11), (15,10), (15,22) (22,15). To solve model (TTE), the maximum revenue parameter $R^{\max}$ should be identified. We solve the model (TOLL) and use its optimum objective function value as the value of the parameter $R^{\max}$. A similar step is taken to find the maximum budget parameter $B^{\max}$ for model (CTE). In fact, the value of this parameter is set to be the total investment and operating cost (3.23) associated with the optimal solution of model (CTE) with the inequality (3.40) being relaxed.

In all our experiments, we consider the accumulated emission only from a single pollutant, namely NOx. The variation of the total NOx emission with respect to γ_1 and γ_2 values are plotted in Figures 5.1(a) and 5.1(b), respectively. Based on these figures, we arbitrarily set γ_1 to 0.70 and γ_2 to 0.80.

The results of our study are presented in Figures 5.2-5.5 and Tables 5.1-5.3. In all of the figures, the network is colored such that least emission values are observed on green links whereas very high emission values are observed on red links. All other colors match intermediate values. On the other hand, the meaning of acronyms used in tables are as follows: value (Val.), difference (Diff.), total network emission (Tot.EM.), average emission concentration (Ave.EC.), minimum emission concentration (Min.EC.), maximum emission concentration (Max.EC.), total network demand (Tot.DM.), single vehicle emission (Veh.EM.), total residential zone emission (Res.EM.), total commercial zone emission (Com.EM.), total industrial zone emission (Ind.EM.), total non-urban



(a) Emission versus maximum revenue.

(b) Emission versus allocated budget.

Figure 5.1: The experiments conducted to determine parameters γ_1 and γ_2.

zone emission (NUr.EM.), total network excess emission (Tot.EE.), total residential zone excess emission (Res.EE.), total commercial zone excess emission (Com.EE.), total industrial zone excess emission (Ind.EE.), total non-urban zone excess emission (NUr.EE.). Ave.EC. is calculated by dividing the total network emission to total network links lengths, while Veh.EM. is found by dividing the total network emission to the total demand.

Here we refer (TAP) with elastic demand as (REG). As model (REG) corresponds to the case where there is no intervention from an upper level authority, its optimal solution is used as a benchmark. Figure 5.2 depicts the emission amounts associated with this optimal solution. As it is common for the city centers, we observe that most of the NOx emission is concentrated at the center. We shall use this result for comparing the outcomes obtained with different policies.

	(REG)	(TTE)		(CTE)		(TCTE)	
		Val.	Diff.	Val.	Diff.	Val.	Diff.
Tot.EM.	378.556	347.668	-8.2%	374.488	-1.1%	344.529	-9.0%
Ave.EC.	1.206	1.107	-8.2%	1.193	-1.1%	1.097	-9.0%
Min.EC.	0.368	0.233	-36.7%	0.382	-3.9%	0.225	-38.7%
Max.EC.	2.802	2.172	-22.5%	2.663	-5.0%	2.244	-19.9%
Tot.DM.	360,608	329,949	-8.5%	369,891	+2.6%	336,552	-6.7%
Veh.EM.	1.050	1.054	+0.4%	1.012	-3.6%	1.024	-2.5%

Table 5.1: Statistics for models with the objective of minimizing the total emission

We start by investigating the results obtained with three models minimizing the total network emission: (TTE), (CTE) and (TCTE). Emission amounts corresponding to the optimum solutions of these models are illustrated in Figure 5.3 and statistics about link emissions are provided in Table 5.1. The main conclusion is that the toll

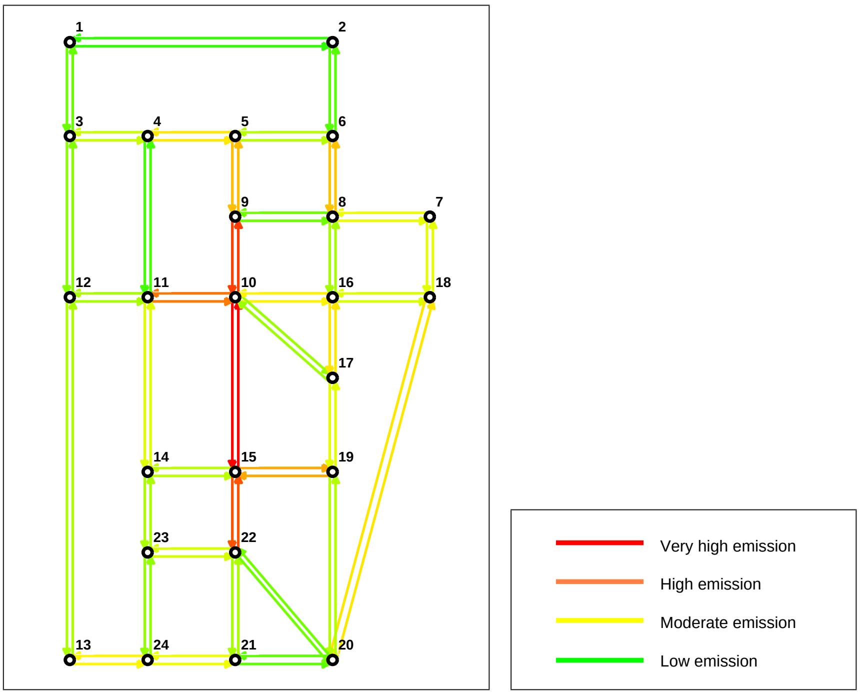
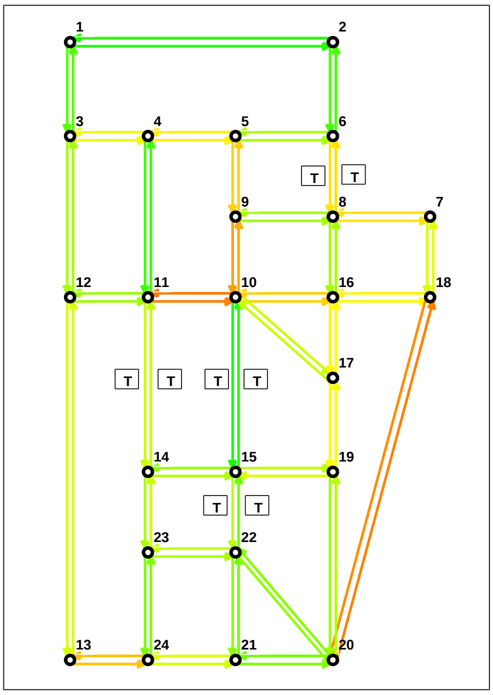
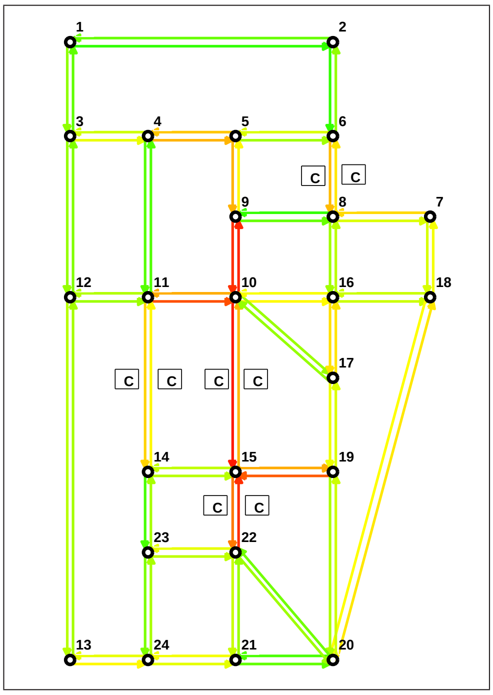


Figure 5.2: Relative emission amounts associated with the solution of the user equilibrium problem (REG).

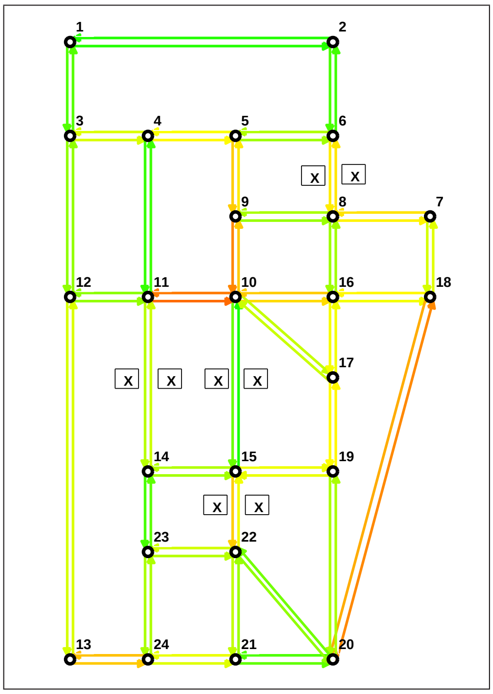
pricing based policies are more effective in reducing the total emission. Compared to (REG), models (TTE) and (TCTE) achieve an emission decrease about 8.2% and 9.0%, respectively. Meanwhile, only 1.1% decrease was possible with the capacity enhancement model (CTE). A close examination shows that the success of the toll pricing policies can be attributed to their demand reducing potentials. As the demand is assumed to be variable and depending on the travel time, the pricing policies shift some of the demand to the alternative transportation means, which in turn inherently leads to a reduction in the emission level. The reverse is true for the capacity enhancement policies, where the additional capacity clearly reduces the traffic congestion, but also generates additional demand on its own. For example, the travel demand in the optimal solution of (CTE) model is 2.6% higher than the one obtained by the (REG) model. This behavior limits their effectiveness in decreasing the total emission. Meanwhile, (CTE) model is only superior in per vehicle emission statistic as the total network emission slightly decreases and the total travel demand increases compared to (REG). As the demand decrease is restricted while the emission decrease is substantial, the solution associated with the mix strategy implemented in (TCTE) model can be assumed to be the most efficient.



(a) Toll pricing (TTE).



(b) Capacity enhancement (CTE).



(c) Toll pricing and capacity enhancement (TCTE).

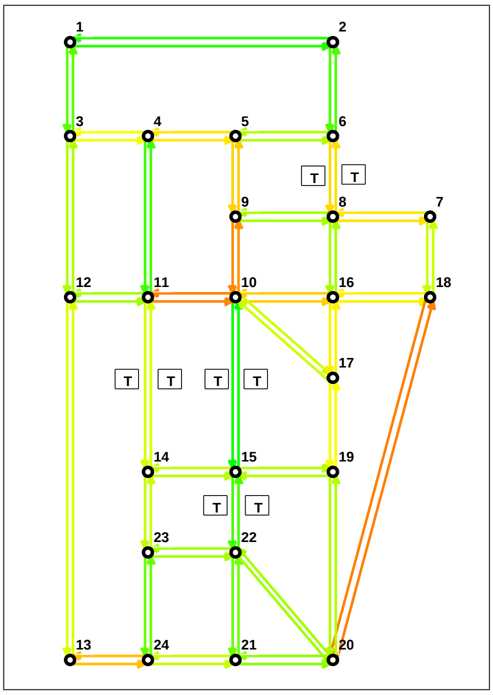
Figure 5.3: Minimizing the total emission.

	(REG)	(TED1)		(CED1)		(TCED1)	
		Val.	Diff.	Val.	Diff.	Val.	Diff.
Tot.EM.	378.556	349.941	-7.6%	381.123	+0.7%	357.545	-5.6%
Ave.EC.	1.206	1.114	-7.6%	1.214	+0.7%	1.139	-5.6%
Min.EC.	0.368	0.122	-66.8%	0.412	+12.0%	0.228	-37.9%
Max.EC.	2.802	2.138	-23.7%	2.472	-11.8%	2.059	-26.5%
Tot.DM.	360,608	325,325	-9.8%	365,614	+1.4%	340,235	-5.6%
Veh.EM.	1.050	1.076	+2.5%	1.042	-0.7%	1.051	+0.1%

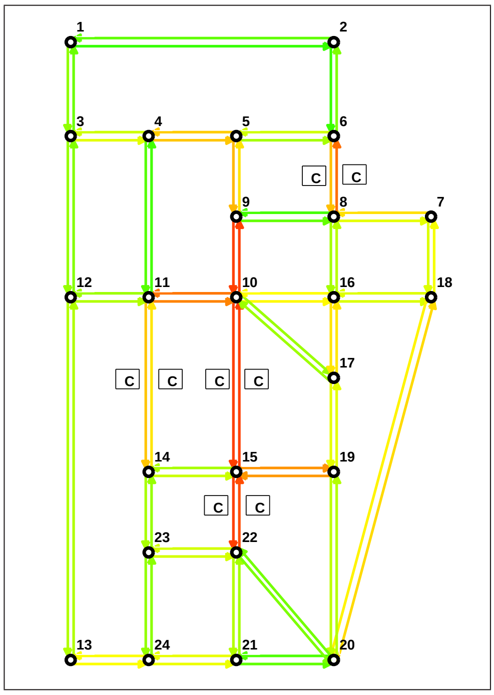
Table 5.2: Statistics for models with the objective of minimizing the maximum emission concentration

In the next step, we contrast (TED1), (CED1) and (TCED1) models having a common objective to minimize the maximum emission concentration. The optimum solutions are illustrated in Figure 5.4 and the derived outcomes are summarized in Table 5.2. Inferences similar to those made for the total emission minimization models are also valid here. First of all, the maximum link emission concentrations are significantly lowered for all three models thanks to the change in the objective. (TED1) model provides a solution with the least total emission, and also the least travel demand and the highest per vehicle emission concentration. (CED1) model solution results in a total emission and demand almost equal to that of (REG). Moreover, it can be noticed from the numbers that (CED1) requires concentration increase on some links to reduce the concentration of others, which is not really a desirable effect. Finally, the mix strategy model (TCED1) solution is moderate in terms of the total emission and the demand decrease, and also leads to a higher decrease in the maximum emission.

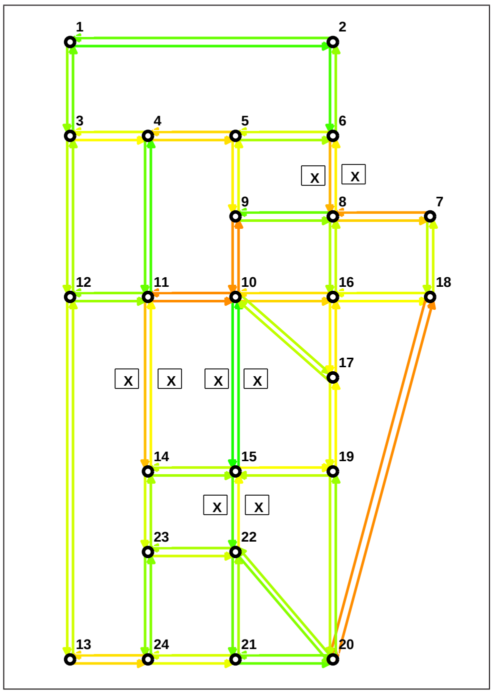
As a final step, we compare the remaining models (TED2), (CED2) and (TCED2) with each other based on the results given in Figure 5.5 and Table 5.3. In terms of both total emission and total excess emission, strategy imposed on model (TCED2) is the most efficient. It seems that by successfully diverting the actual traffic, the undesirable excess emission in a relatively populated commercial zone is significantly reduced and shifted to non-urban areas. Excess emission is also reduced in residential and industrial zones but not as high as observed in the commercial zone. (TED2) model solution produces quite similar outcomes as (TCED2) model solution but is less efficient. Last model (CED2) solution has an almost equal total emission with (REG). Both total and excess emissions are highly increased for the non-urban areas, and the excess emission is importantly reduced in the commercial area. In sum, the capacity enhancement is not efficient as the two former pricing strategies but accomplishes its emission dispersion mission when compared to do nothing strategy (REG).



(a) Toll pricing (TED1).

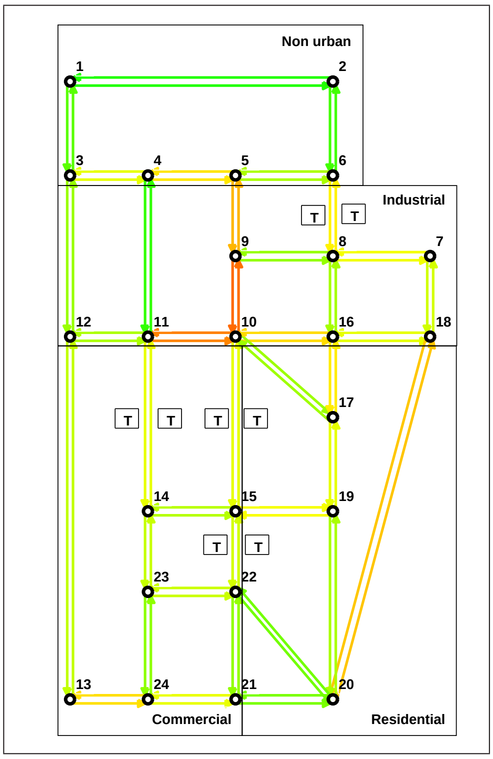


(b) Capacity enhancement (CED1).

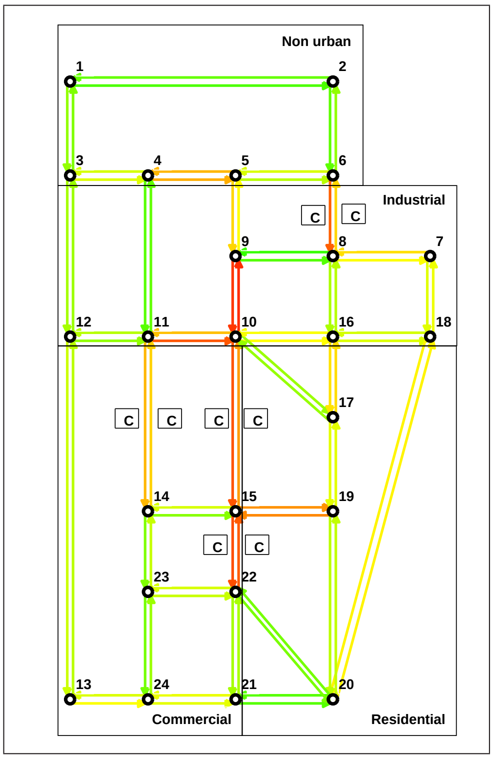


(c) Toll pricing and capacity enhancement (TCED1).

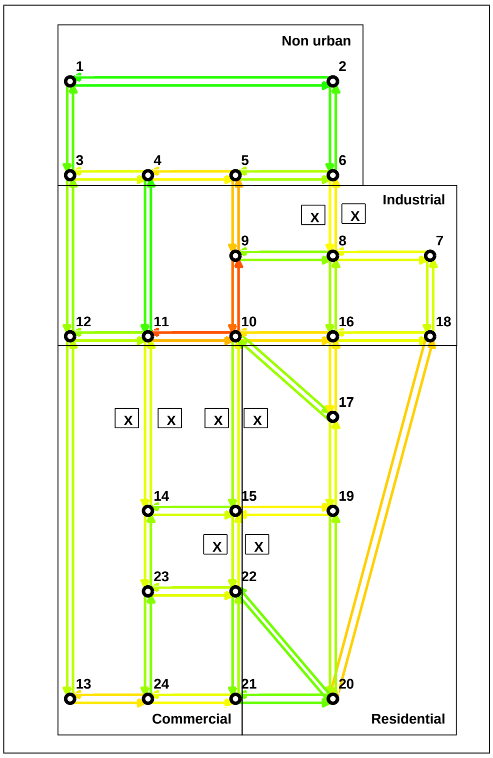
Figure 5.4: Minimizing the maximum emission concentration



(a) Toll pricing (TED2).



(b) Capacity enhancement (CED2).



(c) Toll pricing and capacity enhancement (TCED2).

Figure 5.5: Minimizing the excess emission

	(REG)	(TED2)		(CED2)		(TCED2)	
		Val.	Diff.	Val.	Diff.	Val.	Diff.
Tot.EM.	378.556	356.686	-5.8%	378.659	+0.0%	352.712	-6.8%
Res.EM.	73.907	73.951	+0.1%	74.452	+0.7%	72.921	-1.3%
Com.EM.	124.636	102.332	-17.9%	119.131	-4.4%	99.803	-19.9%
Ind.EM.	140.079	142.239	+1.5%	136.837	-2.3%	141.591	1.1%
NUr.EM.	39.934	38.164	-4.4%	48.239	+20.8%	38.397	-3.8%
Tot.EE.	75.080	49.765	-33.7%	71.272	-5.1%	48.640	-35.2%
Res.EE.	24.402	22.593	-7.4%	25.015	+2.5%	22.211	-9.0%
Com.EE.	25.389	2.428	-90.4%	20.108	-20.8%	2.382	-90.6%
Ind.EE.	21.631	20.146	-6.9%	19.931	-7.9%	19.808	-8.4%
NUr.EE.	3.659	4.599	+25.7%	6.218	+69.9%	4.239	+15.9%
Tot.DM.	360,608	346,826	-3.8%	369,634	+2.5%	349,377	-3.1%
Veh.EM.	1.050	1.028	-2.0%	1.024	-2.4%	1.010	-3.8%

Table 5.3: Statistics for models with maximum emission concentration minimization objective

5.2 Models with Travel Time Reliability

In this section, we present the numerical results for the risk-averse models with the travel-time reliability. As discussed in Chapter 4, these models are formulated based on a set of scenarios representing the conditions of the stochastic network. First we discuss the details of generating the set of scenarios for a given transportation network. In order to provide illustrative results we consider the well-known small-size Nine Node network, which consists of 9 nodes and 18 arcs. We also consider the larger Sioux Falls network to obtain more elaborative results. Using the generated problem instances, we analyze the effects of incorporating the risk terms on the toll pricing decisions, the effects of the risk parameters and the alternate objectives.

5.2.1 Generating Problem Instances

In order to test our models, we consider several problem instances of different sizes. In this computational study, we focus three cases. In the first case we focus only on the randomness in the link capacities and the free flow times are assumed to be deterministic. In the second one, the free-flow times are assumed to be random and the link capacities are assumed to be deterministic. In the last case, we focus on randomness in both of these system variables. Thus, a scenario represents a joint realization of the link capacities in the first case, joint realization of the link free-flow times in the second case and combination of these two cases in the last one. We generate two groups of data sets to show the effectiveness of the proposed models.

Group I

The data sets with *random link capacities* are generated according to the following ordered steps:

- We obtain the UE solution of the deterministic TAP for the specified network.
- We calculate the unit travel time for each link according to the flow values under the UE.
- We find the shortest paths for each O-D pair according to the calculated unit travel time values.
- We determine the critical link set $\widehat{A}$ based on the shortest paths. We say that a link is "critical" if it appears on the shortest paths of several (more than one) different origin-destination pairs.
- The realizations of the capacities of the critical links, β_{ij}^s, $s \in S$, $(i,j) \in \widehat{\mathcal{A}}$, are generated from the original capacity values by multiplying each of them with a random coefficient. This random coefficient is sampled from a uniform distribution on the interval [0.3,0.7]. The realized capacity values for the noncritical links (links belonging to the set $\mathcal{A}/\widehat{\mathcal{A}}$) are set as their original values. Thus, there is no disruption associated with the noncritical links under the generated scenarios.
- Scenario probabilities p_s, $s \in S$, are set to be equal or sampled from the uniform distribution on the interval [0.2,0.6] and then normalized.

Data sets with *random link free-flow times* are generated according to the following ordered steps:

- We follow the same first four steps listed above.
- We examine the critical link set and we select nodes that are the intersection of two or more critical links. We also name these nodes as "critical".
- We assume that the set of incoming and outgoing links, $(i,j) \in \breve{\mathcal{A}}$, of the critical nodes are subject to degradation so there is a increases in the free-flow times of these link.
- The realizations of the free-flow times of these links, α_{ij}^s, $s \in S$, $(i,j) \in \breve{\mathcal{A}}$, are generated from the original free-flow time values by multiplying each of them

with a random coefficient. This random coefficient is sampled from a uniform distribution on the interval [2,4]. Since a link connect two different nodes and so it is incoming link for one of those and outgoing link for the other, it is subjected to degradation two times. Thus, we select the maximum one as the realization for these kind of links. The realized free-flow time values for the remaining links (links belonging to the set $\mathcal{A}/\breve{\mathcal{A}}$) are set as their original values.

- Scenario probabilities $p_s,\ s \in S$, are set to be equal or sampled from the uniform distribution on the interval [0.2,0.6] and then normalized.

Data sets with *random link capacities and free-flow times* are generated as follows: In the first data set we focus on only the capacity degradations and the free flow times are assumed to be deterministic. On the other hand, in the second data set we focus on only the free-flow time degradations and the link capacities are assumed to be deterministic. Here we combined the random link capacities of the first data set with the random free-flow times of the second data set and obtained the mixed case.

Group II

To generate this family of data sets with *random link capacities*, we follow the same steps listed above for the random link capacities, but different that the previous one, the realized capacities of the critical links are generated using random multipliers sampled from the uniform distribution on the interval [0.2,0.6]. Thus, the capacity degradation is more likely to be worse under this type of scenarios.

Similarly, to generate this kind of data set with *random free-flow times*, we again follow the same steps listed for the random free-flow times, but this time the realized free-flow times are generated using random multipliers sampled from the uniform distribution on the interval [2.5,5]. Thus, the free-flow time degradation is also more likely to be worse under this type of scenarios.

For the case with random link capacities and free-flow times, we again combine the previous two cases.

Here we elaborate on why we utilize the critical links while constructing the scenarios. In real-life applications the travelers generally take the potential capacity and free-flow time degradations and the associated variability of the travel times into consideration. Therefore, the travelers may make different route choices compared to the setup with the deterministic travel times. In order to illustrate such different traveling behaviors under the stochastic setup, it is crucial to generate scenarios where the capacities and free-flow times of some of the crucial links are degraded. In a transportation network it is common that some of links are widely used, since they are on several reasonably short paths between some O–D pairs. It is clear that any capacity or free-flow time degradation on such a widely-used link is more likely to result in different traveling behaviors. Therefore, we select the links that appear more than one shortest path and we refer them as "critical" links.

In the subsequent part, we number the data set using the notation "xW_y", where "x" denotes the group number, "y" denotes the data set number within the group and "W" denotes the randomness type (link capacity, free-flow time or both link capacity and free-flow time). For example, "1A_2" indicates the second data set of Group I with random link capacities. Similarly, if the instance is generated according to the random free-flow time values, we denote it by "B" instead of "A" and if the randomness is due to both the link capacities and the free-flow times then it is denoted by "C".

5.2.2 Risk-Averse Models with Only Risk Terms

In this section, we present the expectation and CVaR values of the random outcomes of interest associated with the risk-neutral models and the models with only risk terms. We first present results on the random aggregated unit travel times (MUTT) and the random maximum unit travel times (MTTT) for the Nine Node (NN) and the Sioux Falls (SF) networks. Then we extend our study for the Sioux Falls network by presenting results on two additional random outcomes of interest; the random maximum unit travel time (AUTT) the random maximum total travel time (ATTT). In other words, we present results on

- the random aggregated unit travel times (MUTT) associated with the solutions obtained by solving the risk-neutral model (EMUTT) and the risk-averse model (MUTT_CVaR) (for the NN and SF networks)

- the random maximum unit travel times (MTTT) associated with the solutions obtained by solving the risk-neutral model (EMTTT) and the risk-averse model (MTTT_CVaR) (for the NN and SF networks).

- the random aggregated total travel times (AUTT) associated with the solutions obtained by solving the risk-neutral model (EAUTT) and the risk-averse model (AUTT_CVaR) (for the SF network).

- the random maximum total travel times (ATTT) associated with the solutions obtained by solving the risk-neutral model (EATTT) and the risk-averse model (ATTT_CVaR) (for the SF network).

Please see Appendix A for the dimension of these problems.

Note that we also provide the relative differences of the expectation and CVaR values of the random outcomes associated with the risk-averse models with respect to the outcomes obtained by the risk-neutral models. Thus, for the CVaR values we define the relative difference (RD) as follows:

$$RD = \frac{(\mathrm{CVaR}_\alpha^1 - \mathrm{CVaR}_\alpha^2)}{\mathrm{CVaR}_\alpha^2}, \tag{5.1}$$

where CVaR_α^1 and CVaR_α^2 correspond to the risk-averse and the risk-neutral models, respectively. The relative differences of the expectation values are also found similarly.

Results for the Nine Node Network

Here, we present results for one data set from Group 1 with random link capacities. The results presented in this part are obtained with equal and different scenario probabilities.

1. Comparative results with MUTT

$\alpha = 0.8$							
Type of Probability	Number of Scenarios	Risk-averse Model		Risk-neutral Model		Relative Difference	
		CVaR_α	Exp.	CVaR_α	Exp.	CVaR_α	Exp.
Equal	N=10	42.78	42.40	55.52	29.99	-22.9522%	41.3738%
	N=100	42.87	37.51	52.44	30.29	-18.2445%	23.8536%
Different	N=10	40.28	40.22	57.57	28.21	-30.0306%	42.5460%
	N=100	44.40	37.48	52.97	30.39	-16.1809%	23.3441%
$\alpha = 0.9$							
Type of Probability	Number of Scenarios	Risk-averse Model		Risk-neutral Model		Relative Difference	
		CVaR_α	Exp.	CVaR_α	Exp.	CVaR_α	Exp.
Equal	N=10	42.96	42.87	87.43	29.99	-50.8579%	42.9543%
	N=100	47.92	40.17	80.14	30.29	-40.2008%	32.6493%
Different	N=10	40.28	40.28	92.31	28.21	-56.3609%	42.7795%
	N=100	48.64	43.30	81.19	30.39	-40.0932%	42.4786%

Table 5.4: Comparative results with MUTT with fixed demand for the NN network

$\alpha = 0.8$							
Type of Probability	Number of Scenarios	Risk-averse Model		Risk-neutral Model		Relative Difference	
		$CVaR_\alpha$	Exp.	$CVaR_\alpha$	Exp.	$CVaR_\alpha$	Exp.
Equal	N=10	40.15	27.61	46.10	21.74	-12.9085%	26.9988%
	N=100	36.34	26.43	71.06	20.93	-48.8612%	26.2803%
Different	N=10	41.81	27.22	52.27	22.14	-20.0159%	22.9380%
	N=100	37.01	26.64	72.39	21.82	-48.8701%	22.0711%
$\alpha = 0.9$							
Type of Probability	Number of Scenarios	Risk-averse Model		Risk-neutral Model		Relative Difference	
		$CVaR_\alpha$	Exp.	$CVaR_\alpha$	Exp.	$CVaR_\alpha$	Exp.
Equal	N=10	65.66	27.83	88.12	21.74	-25.4900%	28.0490%
	N=100	49.15	26.40	126.96	20.93	-61.2848%	26.1440%
Different	N=10	60.93	27.34	89.93	22.14	-32.2507%	23.4526%
	N=100	51.48	26.80	129.69	21.82	-60.3027%	22.8064%

Table 5.5: Comparative results with MUTT with elastic demand for the NN network

As seen from Tables 5.4 and 5.5, incorporating the risk measure, CVaR, help us to obtain more travel time reliable policies with respect to the risk-neutral case. It is also seen that, increasing the α parameter result in increases in the corresponding $CVaR_\alpha$ values. As a result $CVaR_\alpha$ accounts for risk for larger realizations. Here, we are able to achieve up to 56% reduction amounts in the fixed demand case and 61% reduction amounts in the elastic demand case in the CVaR values with respect to risk-neutral case.

2. Comparative results with MTTT

$\alpha = 0.8$							
Type of Probability	Number of Scenarios	Risk-averse Model		Risk-neutral Model		Relative Difference	
		$CVaR_\alpha$	Exp.	$CVaR_\alpha$	Exp.	$CVaR_\alpha$	Exp.
Equal	N=10	1207.78	1199.78	1450.58	825.26	-16.7379%	45.3827%
	N=100	1210.70	975.85	1380.86	807.47	-12.3226%	20.8523%
Different	N=10	1253.73	1207.78	1436.17	818.54	-12.7030%	47.5534%
	N=100	1211.97	925.34	1370.23	805.55	-11.5501%	14.8710%
$\alpha = 0.9$							
Type of Probability	Number of Scenarios	Risk-averse Model		Risk-neutral Model		Relative Difference	
		$CVaR_\alpha$	Exp.	$CVaR_\alpha$	Exp.	$CVaR_\alpha$	Exp.
Equal	N=10	1226.83	1201.11	2232.23	825.26	-45.0403%	45.5444%
	N=100	1405.61	985.94	2097.60	807.47	-32.9895%	22.1026%
Different	N=10	1272.95	1233.97	2030.58	818.54	-37.3109%	50.7529%
	N=100	1445.05	1173.07	2076.08	805.55	-30.3952%	45.6237%

Table 5.6: Comparative results with MTTT with fixed demand for the NN network

$\alpha = 0.8$							
Type of	Number of	Risk-averse Model		Risk-neutral Model		Relative Difference	
Probability	Scenarios	$CVaR_\alpha$	Exp.	$CVaR_\alpha$	Exp.	$CVaR_\alpha$	Exp.
Equal	N=10	838.94	688.03	867.34	663.01	-3.2739%	3.7742%
	N=100	830.76	693.22	846.66	651.36	-1.8778%	6.4274%
Different	N=10	815.34	707.65	867.34	661.32	-5.9954%	7.0055%
	N=100	834.26	744.76	844.75	651.29	-1.2409%	14.3521%
$\alpha = 0.9$							
Type of	Number of	Risk-averse Model		Risk-neutral Model		Relative Difference	
Probability	Scenarios	$CVaR_\alpha$	Exp.	$CVaR_\alpha$	Exp.	$CVaR_\alpha$	Exp.
Equal	N=10	1064.25	731.81	1122.74	663.01	-5.2097%	10.3774%
	N=100	1005.08	693.82	1090.78	651.36	-7.8570%	6.5195%
Different	N=10	935.66	686.98	1096.62	661.32	-14.6781%	3.8794%
	N=100	1007.40	694.17	1086.56	651.29	-7.2855%	6.5841 %

Table 5.7: Comparative results with MTTT with elastic demand for the NN network

Tables 5.6 and 5.7 present the results for the maximum total travel time, MTTT. As presented in these tables, we observe higher $CVaR_\alpha$ and Expectation values. Since we concentrate on the total number of travelers in a link rather than a single one, observing high values is not surprising. Similar to the previous results obtained for the MUTT case, here we also obtain significant reductions amounts up to 45%.

If we assume the demand in the transportation network is elastic then depending on the travel time and pricing policy, some of the network users may shift to alternate transportation means (see Appendix E for the percentage of shifted total demand). As a consequence of this behavior, most of the time depending on the amount of shifted demand, it is reasonable to observe decreases in the performance measures, $CVaR_\alpha$ and Expectation (see Table 5.7). However, it is not possible to say that using the elastic demand instead of the fixed one lead greater or smaller RD amounts. In the rest of this study we will present results with only fixed demand, for more results with elastic demand see Appendix C.

In all of the tables above, we present results for different number of scenarios and using different sizes of problem instances leads different outcomes. Note that, increasing the number of scenarios does not have to result in better reduction amounts. The only claim that we can make is, larger sizes of scenarios helps us to observe more realistic cases.

In order to analyze the effect of different network type, instance type and instance size, we present results with Sioux Falls network for different instances with different scenario sizes.

Results for the Sioux Falls Network

In this section we comparatively analyze the risk-averse versus risk-neutral models by using the Sioux Falls network. We present some selected result sets for three different sizes of scenarios of two different data sets and we only consider equal scenario probabilities. For more results with fixed and elastic demand, see Appendix B and Appendix C respectively. In some of the tables below, we also provide CPU times for illustrative purposes.

1. Comparative results with MUTT

$\alpha = 0.8$							
Instance Number	Number of Scenarios	Risk-averse Model		Risk-neutral Model		Relative Difference	
		$CVaR_\alpha$	Exp.	$CVaR_\alpha$	Exp.	$CVaR_\alpha$	Exp.
1A_1	N=50	436.56	209.60	494.50	200.88	-11.7165 %	4.3443 %
1A_2		426.10	207.04	484.58	195.65	-12.0671%	5.8220 %
1A_1	N=100	436.55	208.18	488.21	198.35	-10.5820 %	4.9558%
1A_2		414.89	196.20	461.63	189.31	-10.1245%	3.6421 %
1A_1	N=200	449.36	211.48	503.9735	200.32	-10.8364%	5.5727%
1A_2		419.46	205.33	474.68	192.85	-11.6341 %	6.4713 %
$\alpha = 0.9$							
Instance Number	Number of Scenarios	Risk-averse Model		Risk-neutral Model		Relative Difference	
		$CVaR_\alpha$	Exp.	$CVaR_\alpha$	Exp.	$CVaR_\alpha$	Exp.
1A_1	N=50	720.26	210.20	861.53	200.88	-16.3975%	4.6430%
1A_2		703.97	206.89	845.74	195.65	-16.7628 %	5.7440%
1A_1	N=100	818.81	198.92	850.54	198.35	-3.7305%	0.2897%
1A_2		702.12	195.15	802.03	189.31	-12.4574%	3.0853%
1A_1	N=200	790.43	206.12	883.54	200.32	-10.5383%	2.8963%
1A_2		681.38	206.18	826.97	192.85	-17.6047%	6.9133%

Table 5.8: Comparative results with MUTT with fixed demand for the SF network

It can be seen from Table 5.8 that there is a great increase in the values of CVaR and expectation. Since the SF network utilize more demand than NN network, observing higher values of these performance measures are expected. Moreover, RD amounts that we illustrate in this table is smaller than the one presented for the NN network (see Table 5.4). This is a consequence of using relatively bigger network which contains more alternative paths. Even though we have smaller RD amounts with respect to NN network, we again attain our main objective and obtain 17% relative difference amount by the risk-averse model with respect to the risk-neutral case.

$\alpha = 0.9$									
Instance	Number of	Risk-averse Model		Risk-neutral Model		Relative Difference		CPU Times	
Number	Scenarios	$CVaR_\alpha$	Exp.	$CVaR_\alpha$	Exp.	$CVaR_\alpha$	Exp.	R-averse	R-neutral
1B_2	N=100	72.04	70.16	129.48	65.11	-44.3569%	7.7539%	330	323
2B_1		95.44	76.26	135.67	74.34	-29.6518%	2.5857%	343	325
1B_2	N=200	86.52	70.56	126.77	65.66	-31.7523%	7.4635%	611	692
2B_1		81.13	79.70	131.93	74.72	-38.5059%	6.6599%	760	651

Table 5.9: Comparative results with MUTT with fixed demand for the SF network

In the Table 5.8, we display results for the random link capacities. On the other hand, Table 5.9 provides results for the random free-flow times. As seen from this table, we are again successful to obtain significant amount of decreases in the CVaR values. It is also seen that we obtain better reduction amounts with respect to the case that utilize the random link capacities, but note that all these results are network and data dependent thus, in a different network structure or with a different problem instance it is also possible to observe the reverse case.

2. Comparative results with MTTT

$\alpha = 0.8$							
Instance	Number of	Risk-averse Model		Risk-neutral Model		Relative Difference	
Number	Scenarios	$CVaR_\alpha$	Exp.	$CVaR_\alpha$	Exp.	$CVaR_\alpha$	Exp.
1A_2	N=50	5272.01	2300.71	6218.13	2208.95	-15.2154%	4.1541%
2A_1		12901.43	5588.30	15093.65	5332.40	-14.5241%	4.7989%
1A_2	N=100	4907.46	2225.28	6004.63	2149.38	-18.2721%	3.5314%
2A_1		11842.98	5302.15	14095.10	5068.80	-15.9780%	4.6035%
1A_2	N=200	4964.55	2302.68	6346.38	2182.43	-21.7736%	5.5099%
2A_1		12291.73	5471.76	14734.07	5233.06	-16.5761%	4.5614%
$\alpha = 0.9$							
Instance	Number of	Risk-averse Model		Risk-neutral Model		Relative Difference	
Number	Scenarios	$CVaR_\alpha$	Exp.	$CVaR_\alpha$	Exp.	$CVaR_\alpha$	Exp.
1A_2	N=50	9226.08	2292.46	11229.59	2208.95	-17.8413%	3.7806%
2A_1		22167.85	5588.32	27295.20	5332.40	-18.7848%	4.7993%
1A_2	N=100	8649.16	2212.97	10823.70	2149.38	-20.0905%	2.9585%
2A_1		20027.20	5301.76	25377.96	5068.80	-21.0843%	4.5959%
1A_2	N=200	8751.57	2275.62	11551.31	2182.43	-24.2374%	4.2697%
2A_1		20832.67	5471.05	26610.33	5233.06	-21.7121%	4.5477%

Table 5.10: Comparative results with MTTT with fixed demand for the SF network

Here we display results for the case that we focus on the total number of users in a link. As presented in Table 5.10, using the risk-averse model with MTTT enables us to obtain up to 24% more reliable policies than the risk-neutral case. We also observe from this table that when we increase the number of scenarios, we obtain better RD amounts. This means that for the more realistic cases, the risk-averse model which incorporate MTTT yields better results. Note that, it does not always have to be the case, we may not always obtain better results with large number of scenarios.

Here we present the following table (Table 5.11) to show the effect of a different instance type. The results are obtained according to random link capacities and free-flow times and it is seen that we again achieve significant amount of improvements.

$\alpha = 0.9$									
Instance Number	Number of Scenarios	Risk-averse Model		Risk-neutral Model		Relative Difference		CPU Times	
		$CVaR_\alpha$	Exp.	$CVaR_\alpha$	Exp.	$CVaR_\alpha$	Exp.	R-averse	R-neutral
1C_2	N=100	23757.61	5469.00	26448.58	5454.39	-10.1743%	0.2679%	364	353
2C_1		62408.67	14664.52	69805.71	14566.64	-10.5966%	0.6719%	395	347
1C_2	N=200	22717.94	5690.36	26134.65	5557.35	-13.0735%	2.3935%	771	715
2C_1		65245.98	15156.48	71929.47	15084.90	-9.2917%	0.4746%	745	736

Table 5.11: Comparative results with MTTT with fixed demand for the SF network

3. Comparative results with AUTT

$\alpha = 0.8$							
Instance Number	Number of Scenarios	Risk-averse Model		Risk-neutral Model		Relative Difference	
		$CVaR_\alpha$	Exp.	$CVaR_\alpha$	Exp.	$CVaR_\alpha$	Exp.
1A_1	N=50	1805.80	1122.52	1816.17	1122.21	-0.5712%	0.0278%
1A_2		1637.38	1065.20	1637.48	1064.91	-0.0059%	0.0269%
1A_1	N=100	1875.68	1102.29	1878.90	1101.44	-0.1713 %	0.0773%
1A_2		1659.70	1065.28	1660.34	1065.23	-0.0386%	0.0046%
1A_1	N=200	1822.53	1089.41	1831.14	1089.00	-0.4701 %	0.0378%
1A_2		1735.88	1075.38	1741.49	1072.79	-0.3224 %	0.2414%
$\alpha = 0.9$							
Instance Number	Number of Scenarios	Risk-averse Model		Risk-neutral Model		Relative Difference	
		$CVaR_\alpha$	Exp.	$CVaR_\alpha$	Exp.	$CVaR_\alpha$	Exp.
1A_1	N=50	3340.10	1123.21	3363.74	1122.21	-0.7030 %	0.0890%
1A_2		2630.65	1069.19	2634.69	1064.91	-0.1536 %	0.4019%
1A_1	N=100	3158.34	1102.19	3161.28	1101.44	-0.0930 %	0.0683%
1A_2		2644.89	1066.31	2646.11	1065.23	-0.0458 %	0.1014%
1A_1	N=200	3097.83	1089.50	3106.21	1089.00	-0.2698 %	0.0460%
1A_2		2915.21	1071.48	2922.70	1071.06	-0.2561 %	0.0399 %

Table 5.12: Comparative results with AUTT with fixed demand for the SF network

Table 5.12 shows that, although there is some improvement with respect to the risk-neutral case, here we do not achieve such good improvement amounts that we obtain for the previous models. Since we are dealing with link degradations, we may observe relatively large travel times on some of the effected links. In the previously described models, we concentrate on the maximum of those travel times and by minimizing this value we are able to minimize all the remaining ones automatically. On the other hand, here we consider all unit travel times one by one and we aggregate them and so larger travel time values are balanced by the smaller ones. As a consequence, we obtain smaller RD amounts with respect to the previous models incorporating MUTT and MTTT. This result holds for most of the time but sometimes it is also possible to observe the reverse case depending on the network type, instance type or scenario size.

4. Comparative results with ATTT

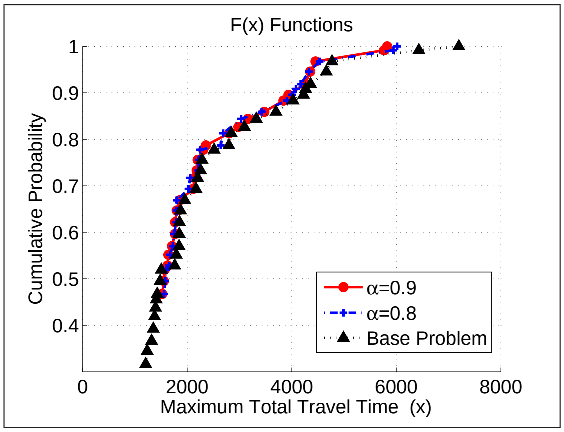
$\alpha = 0.8$									
Instance	Number of	Risk-averse Model		Risk-neutral Model		Relative Difference		CPU Times	
Number	Scenarios	$CVaR_\alpha$	Exp.	$CVaR_\alpha$	Exp.	$CVaR_\alpha$	Exp.	R-averse	R-neutral
1A_2	N=50	13939.70	7594.75	14287.99	7542.43	-2.4377%	0.6938%	150	116
2A_1		32055.50	15448.59	33379.61	15238.10	-3.9668%	1.3813%	137	117
1A_2	N=100	14452.57	7539.14	14550.91	7489.31	-0.6758%	0.6653%	326	216
2A_1		32759.06	15153.72	33130.28	14952.57	-1.1205%	1.3452%	322	249
1A_2	N=200	15016.96	7678.26	15230.92	7590.83	-1.4048%	1.1519%	788	466
2A_1		34377.91	15660.63	36018.16	15387.74	-4.5540%	1.7734%	813	499
$\alpha = 0.9$									
Instance	Number of	Risk-averse Model		Risk-neutral Model		Relative Difference		CPU Times	
Number	Scenarios	$CVaR_\alpha$	Exp.	$CVaR_\alpha$	Exp.	$CVaR_\alpha$	Exp.	R-averse	R-neutral
1A_2	N=50	26816.70	7615.76	27205.62	7542.43	-1.4296%	0.9722%	130	116
2A_1		58565.02	15540.90	63527.50	15238.10	-7.8115%	1.9871%	120	117
1A_2	N=100	26400.56	7563.09	27343.55	7489.31	-3.4487%	0.9851%	292	216
2A_1		58676.74	15246.06	62740.14	14952.57	-6.4766%	1.9628%	298	249
1A_2	N=200	27265.51	7677.69	28150.04	7590.83	-3.1422%	1.1443%	658	466
2A_1		62930.72	15660.15	66667.89	15387.74	-5.6057%	1.7703%	751	499

Table 5.13: Comparative results with ATTT with fixed demand for the SF network

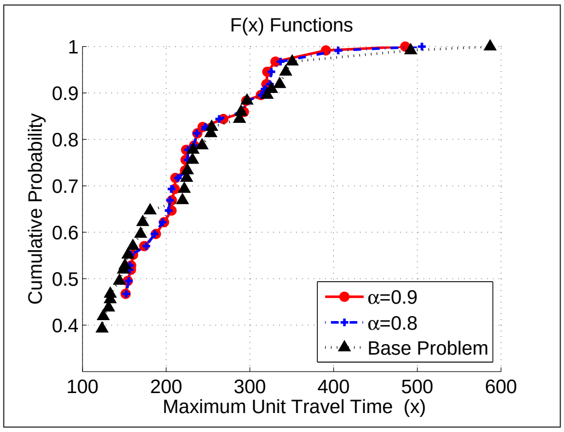
Table 5.13 display results for the models minimizing the aggregated total travel times on each link instead of aggregated unit travel times. As seen from table, we obtain better reduction amounts than the previous model employing AUTT. We are able to obtain improvements up to 6% in the CVaR values with respect to the risk-neutral problem.

Up to now we present results with tables to show the relative differences of risk-averse models with respect to risk-neutral models, here we also provide figures for some of the selected models with different problem instances, to shows the cumulative distributions of the random travel times associated with the risk-neutral problems (the “Base problems”) and the risk-averse problems for $\alpha = 0.8$ and $\alpha = 0.9$. As seen from Figure 5.6, the α parameter helps us to shape the cumulative distribution according to the preferences of the decision maker. Larger α helps us to shift the right tail of the cumulative distribution function to the left.

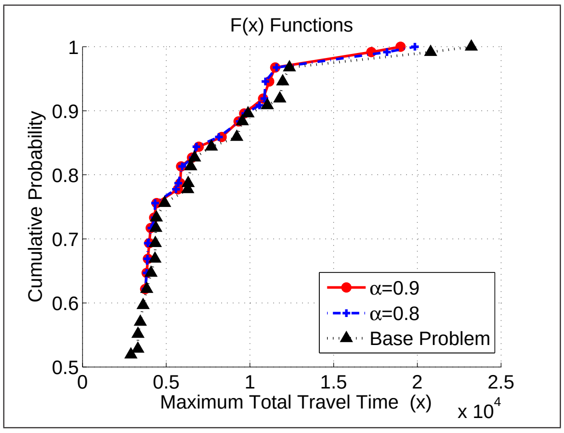
As a result of the conducted computational study we observe that the proposed risk-averse models are successful to achieve network reliability. We observe that models employing a function of maximum travel time yields better results. We also observe that using different problem instances is crucial to see the performance of the proposed models. For more results with different problem instances see the Appendix B and Appendix C.



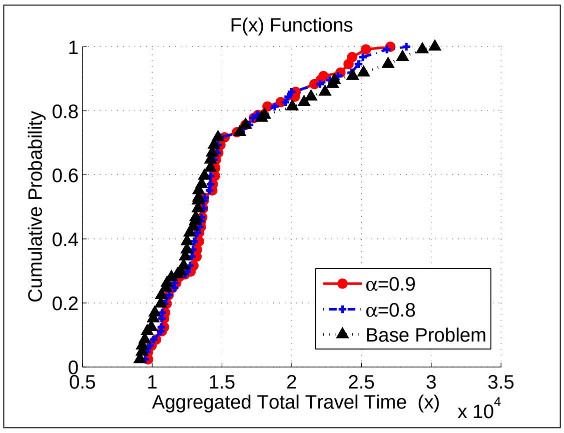
(a) The Instance number=1_2 for N=50



(b) The Instance number=1_2 for N=50



(c) The Instance number=2_1 for N=50



(d) The Instance number=2_1 for N=50

Figure 5.6: Cumulative distribution functions

5.2.3 Risk-Averse Models with Mean-Risk Terms

In this section, we present results for the models with mean-risk terms. We present results on the aggregated unit travel (AUTT) and maximum unit travel time (MUTT) for the Nine Node (NN) and Sioux Falls (SF) networks.

The mean-risk approach quantifies the problem for two criteria: the mean and CVaR. In addition to the α parameter that we discussed in the risk-averse models with only risk terms, mean-risk functions utilize another risk parameter, θ. It is refereed as a risk coefficient, which is specified by decision makers according to their risk preferences. Here we discuss how these risk parameters effect the optimal solutions and we report the expected travel times versus CVaR_α for different values of risk parameters α and θ.

Results for the Nine Node Network

Here, we provide results for the NN network. We use one data set (belongs to Group 1 with random link capacities) and we obtain results with equal and different scenario probabilities.

1. Comparative Results with AUTT

$\alpha = 0.8$ and Number of Scenarios=100									
Type of	Trade-off Par.	Fixed				Elastic			
Probability	θ	CVaR_α	Exp.	Relative Difference		CVaR_α	Exp.	Relative Difference	
Equal	0	261.06	186.13	CVaR_α	Exp.	230.32	154.95	CVaR_α	Exp.
	0.1	259.74	186.14	-0.5076%	0.0070%	229.59	154.99	-0.3169%	0.0275%
	1	256.34	186.38	-1.8087%	0.1365%	224.55	155.00	-2.5073%	0.0320%
	10	256.03	186.87	-1.9268%	0.3990%	219.92	155.00	-4.5156%	0.0324%
Different	0	271.28	186.56	CVaR_α	Exp.	231.69	155.12	CVaR_α	Exp.
	0.1	270.35	186.94	-0.3433%	0.2031%	231.57	155.91	-0.0492%	0.5090%
	1	268.59	187.31	-0.9944%	0.3994%	227.29	156.26	-1.8966%	0.7334%
	10	265.53	187.63	-2.1197%	0.5719%	223.70	159.56	-3.4487%	2.8610%
$\alpha = 0.9$ and Number of Scenarios=100									
Type of	Trade-off Par.	Fixed				Elastic			
Probability	θ	CVaR_α	Exp.	Relative Difference		CVaR_α	Exp.	Relative Difference	
Equal	0	395.24	186.13	CVaR_α	Exp.	345.51	154.95	CVaR_α	Exp.
	0.1	390.61	186.17	-1.1719%	0.0251%	344.08	155.02	-0.4147%	0.0476%
	1	377.58	187.03	-4.4695%	0.4863%	334.33	155.10	-3.2370%	0.0932%
	10	373.44	187.67	-5.5155%	0.8303%	299.10	155.13	-13.4322%	0.1180%
Different	0	414.34	186.56	CVaR_α	Exp.	349.65	155.120	CVaR_α	Exp.
	0.1	410.12	186.97	-1.0189%	0.2211%	339.30	155.124	-2.9615%	0.0026%
	1	402.58	187.32	-2.8400%	0.4076%	330.53	155.125	-5.4687%	0.0028%
	10	383.44	187.97	-7.4575%	0.7567%	305.12	155.13	-12.7351%	0.0088%

Table 5.14: Results for the mean-risk models with AUTT for the NN network and $N = 100$

2. Comparative Results with MUTT

$\alpha = 0.8$ and Number of Scenarios=100									
Type of	Trade-off Par.	Fixed				Elastic			
Probability	θ	CVaR_α	Exp.	Relative Difference		CVaR_α	Exp.	Relative Difference	
Equal	0	52.44	30.29	CVaR_α	Exp.	71.06	20.93	CVaR_α	Exp.
	0.1	52.44	30.29	0.0000%	0.0000%	48.04	21.19	-32.3965%	1.2647%
	1	48.06	31.11	-8.3552%	2.7352%	47.83	21.21	-32.6874%	1.3670%
	10	44.60	37.38	-14.9542%	23.4280%	36.34	26.43	-48.8612%	26.2803%
Different	0	52.97	30.39	CVaR_α	Exp.	72.39	21.82	CVaR_α	Exp.
	0.1	52.90	30.42	-0.1326%	0.0805%	47.33	22.19	-34.6109%	1.7075%
	1	49.16	32.22	-7.1940%	6.0311%	46.89	22.46	-35.2160%	2.9148%
	10	45.62	36.48	-13.8781%	20.0414%	37.01	26.64	-48.8701%	22.0711%
$\alpha = 0.9$ and Number of Scenarios=100									
Type of	Trade-off Par.	Fixed				Elastic			
Probability	θ	CVaR_α	Exp.	Relative Difference		CVaR_α	Exp.	Relative Difference	
Equal	0	80.14	30.29	CVaR_α	Exp.	126.96	20.93	CVaR_α	Exp.
	0.1	80.14	30.29	0.0000%	0.0000%	81.49	21.19	-35.8157%	1.2663%
	1	71.92	30.88	-10.2541%	1.9674%	76.62	21.299	-39.6503%	1.7722%
	10	64.22	33.71	-19.8628%	11.3120%	60.30	23.42	-52.5009%	11.8860%
Different	0	81.19	30.39	CVaR_α	Exp.	129.69	21.82	CVaR_α	Exp.
	0.1	80.91	30.41	-0.3453%	0.0655%	87.52	22.99	-32.5142%	5.3721%
	1	75.46	31.89	-7.0646%	4.9209%	85.75	23.232	-33.8850%	6.4711%
	10	63.25	34.99	-22.1023%	15.1214%	58.43	24.79	-54.9472%	13.6063%

Table 5.15: Results for the mean-risk models with MUTT for the NN network and $N = 100$

Tables 5.14 and 5.15 provide results to show how risk parameter effect the solutions of the mean-risk models employing AUTT and MUTT. As presented in these tables, increasing θ increases the the relative importance of the risk term. In other words, increasing θ result in smaller CVaR values and larger expected values. Thus, larger θ values lead more risk averse policies. On the other hand, larger α values leads to higher CVaR and mean-risk function values, but do not always result in higher expected cost.

Note that when θ=0 we obtain the risk-neutral model with expected travel time functions. All of the relative difference amounts presented in these tables are obtained according to the risk-neutral case (θ=0) by using the equation 5.1. For a given θ, CVaR value of the mean-risk function is denoted by CVaR_α^1, whereas CVaR_α^2 is used to denote the CVaR value of the risk-neutral case. The relative differences for the expectation values are also found in a similar manner.

Results for the Sioux Falls Network

In the following parts, we display results for Sioux Falls network. We use different data sets with equal scenario probabilities to generate the following results. Tables presented in this section include instances with only 200 scenarios. For more results with different number of scenarios and with different models please see Appendix D.

1. Comparative Results with AUTT

$\alpha = 0.8$ and Number of Scenarios=200									
Instance	Trade-off Par.	Fixed				Elastic			
Number	θ	$CVaR_\alpha$	Exp.	Relative Difference		$CVaR_\alpha$	Exp.	Relative Difference	
1A_1	0	1831.14	1089.00	$CVaR_\alpha$	Exp.	517.07	422.75	$CVaR_\alpha$	Exp.
	0.1	1826.67	1089.02	-0.2444 %	0.0020 %	516.78	422.76	-0.0561 %	0.0022 %
	1	1826.67	1089.02	-0.2444%	0.0020 %	515.31	422.80	-0.3406 %	0.0119 %
	10	1822.63	1089.34	-0.4646 %	0.0316 %	514.39	422.88	-0.5189 %	0.0315 %
1A_2	0	1741.49	1072.79	$CVaR_\alpha$	Exp.	509.70	423.21	$CVaR_\alpha$	Exp.
	0.1	1736.19	1075.36	-0.3042 %	0.2398 %	509.55	423.21	-0.0294 %	0.0001 %
	1	1736.06	1075.36	-0.3118 %	0.2402 %	509.31	423.26	-0.0777 %	0.0121 %
	10	1735.91	1075.37	-0.3204 %	0.2411 %	509.21	423.32	-0.0976 %	0.0255 %
$\alpha = 0.9$ and Number of Scenarios=200									
Instance	Trade-off Par.	Fixed				Elastic			
Number	θ	$CVaR_\alpha$	Exp.	Relative Difference		$CVaR_\alpha$	Exp.	Relative Difference	
1A_1	0	3106.21	1089.00	$CVaR_\alpha$	Exp.	679.85	422.75	$CVaR_\alpha$	Exp.
	0.1	3099.19	1089.06	-0.2258%	0.0057%	679.68	422.99	-0.0250%	0.0567%
	1	3097.84	1089.09	-0.2695%	0.0082 %	679.00	423.01	-0.1242%	0.0616%
	10	3097.83	1089.10	-0.2697 %	0.0089%	678.60	423.04	-0.1827%	0.0685%
1A_2	0	2922.70	1071.06	$CVaR_\alpha$	Exp.	647.85	423.21	$CVaR_\alpha$	Exp.
	0.1	2922.54	1071.06	-0.0053 %	0.0001 %	647.85	423.21	0.0000%	0.0000%
	1	2918.85	1071.22	-0.1317 %	0.0151 %	647.25	423.30	-0.0929%	0.0214%
	10	2915.23	1071.48	-0.2556%	0.0394 %	647.25	423.30	-0.0929%	0.0214 %

Table 5.16: Results for the mean-risk models with AUTT for the SF network and $N = 200$

2. Comparative Results with MUTT

$\alpha = 0.8$ and Number of Scenarios=200									
Instance	Trade-off Par.	Fixed				Elastic			
Number	θ	CVaR$_\alpha$	Exp.	Relative Difference		CVaR$_\alpha$	Exp.	Relative Difference	
1A_1	0	503.97	200.32	CVaR$_\alpha$	Exp.	56.51	25.95	CVaR$_\alpha$	Exp.
	0.1	500.84	200.36	-0.6208%	0.0187 %	54.66	25.95	-3.2674%	0.0136 %
	1	486.08	201.73	-3.5514 %	0.7066 %	54.58	25.97	-3.4223%	0.0956 %
	10	449.36	211.48	-10.8364 %	5.5727 %	54.55	25.98	-3.4666%	0.1149 %
1A_2	0	474.68	192.85	CVaR$_\alpha$	Exp.	49.71	23.94	CVaR$_\alpha$	Exp.
	0.1	473.29	192.90	-0.2932%	0.0225 %	49.71	23.94	0.0000 %	0.0000 %
	1	458.86	195.36	-3.3328 %	1.2995 %	49.66	23.98	-0.1006 %	0.1671 %
	10	419.46	205.33	-11.6341 %	6.4713 %	49.61	24.00	-0.2000 %	0.2596 %
$\alpha = 0.9$ and Number of Scenarios=200									
Instance	Trade-off Par.	Fixed				Elastic			
Number	θ	CVaR$_\alpha$	Exp.	Relative Difference		CVaR$_\alpha$	Exp.	Relative Difference	
1A_1	0	883.54	200.32	CVaR$_\alpha$	Exp.	84.79	25.95	CVaR$_\alpha$	Exp.
	0.1	876.46	200.36	-0.8020%	0.0187%	84.79	25.95	0.0000%	0.0000%
	1	852.70	201.25	-3.4905%	0.4630%	81.74	26.20	-3.5964%	0.9634%
	10	811.56	204.18	-8.1464%	1.9269%	79.43	26.92	-6.3231%	3.7380%
1A_2	0	826.97	192.85	CVaR$_\alpha$	Exp.	81.92	23.94	CVaR$_\alpha$	Exp.
	0.1	767.91	196.97	-7.1421%	2.1330%	81.92	23.94	0.0000%	0.0000%
	1	767.91	196.97	-7.1421%	2.1330%	81.92	23.94	0.0000%	0.0000%
	10	683.72	205.83	-17.3222%	6.7295%	76.85	24.25	-6.1950%	1.3048 %

Table 5.17: Results for the mean-risk models with MUTT for the SF network and $N = 200$

The interpretation of Tables 5.16 and 5.17 are similar with the previous discussion. In comparison to the tables presented for the NN network, here we observe lower RD amounts. Although we have smaller RD amounts, we can still observe the effect of the risk parameter explicitly.

CHAPTER 6

CONCLUSIONS AND FUTURE WORK

In this study, we have developed several new optimization models to support the management of sustainable urban transportation systems. First, we focus on the measurement of the gas emissions as an environmental sustainability criterion. To better reflect the emission in the congested networks, we have derived emission functions with respect to the traffic flow and used these functions in the proposed bilevel programming models with fixed or elastic demand. In the models with the emission functions, we have considered two main policies: the toll pricing and the capacity enhancement. We have both focused on the total network emission and the emission dispersion with alternate objectives. In this study, we have also considered the network reliability as a sustainability measure. Several events have impact on network parameters such as the link capacities and the free-flow times and so lead travel times to be random outcomes. However, despite of all random disturbances a transportation system should maintain an acceptable level of service. In particular, we consider the travel time reliability in terms of traffic flow values to quantify the network reliability. We represent the uncertain parameters of the network by random variables and characterize the associated randomness by using a set of scenarios. A scenario represents the joint realization of link capacities and free-flow times of all links in the network. Then using the scenario-based approach we develop several stochastic bilevel programming formulations where the travel time reliability is incorporated into the toll pricing problem. Moreover, we quantify the travel time reliability by employing the risk measure CVaR on the alternate network-based quantities, which are basically functions of the individual random travel times. We introduce models with only risk terms and mean-risk terms on the network-based quantities. We also present the risk-neutral models in order to analyze the effect of incorporating the risk measures. Finally, we conduct a comprehensive computational study to analyze the effects of different policies and present comparative results for the proposed alternate models. As a consequence, for the models incorporating the emission functions we observe that the toll pricing strategies lead to a significant decrease in the emission amounts but they also significantly decrease the total travel demand. On the other hand, the capacity enhancement strategies lead to an increase in the travel demand so they are not very efficient in terms of the emission reduction. When two strategies are applied simultaneously, we observe compromised results, in other words both the emission and total demand decrease. For the models incorporating the travel time reliability, we succeed to obtain significant amount of improvements in terms of the travel time reliability with respect to the risk-neutral models. We also observe that models with the maximum unit and total travel times yield better solutions than the models with aggregated unit and total travel times.

For the future research, we determine numerous research paths to follow. Since the emission functions include the link capacity and free-flow time terms, variations on these system variables lead emission amounts to be random outcomes. Therefore, CVaR can also be introduced on the random emission values to obtain risk-averse policies. Incorporating a risk measure into the lower level traffic assignment problem is also an another important research problem. Such a model would be more realistic in representing the travelers' route choice behaviors, since the travelers make their route choice decisions not only based on the expected travel time values but also based on the travel time variations. Furthermore, we may also develop different scenario generation techniques to analyze the performance of the proposed models. Another important thing is how to identify the links to be tolled or enhanced. In this case, the resulting models should involve integer decision variables. Thus, the problems become highly difficult to solve. Since the users of a transportation network drive different types of vehicles or commute by means of public transport, it would be significant to extend the proposed models by considering the multi-modality of the flows. This will also increase the accuracy of the models with the emission functions, since the different vehicles have different emission profiles. Moreover, the road types (belt lines, highways, etc.) could also have an impact on these emission profiles. Introducing multiple, mostly conflicting, objectives within the proposed models leads to multi-objective optimization models. This is also another possible research path. Since a typical real-life problem has a very large scale, we intend to investigate fast solution methods that make use of the special structure of the involved models unlike the of-the-shelf solvers.

Appendix A

Dimensions of the Problems

	EAUTT		AUTT_CVaR		MRAUTT		EATTT		ATTT_CVaR		MRATTT	
Number of Scenarios	Constraints	Continuous Variable	Constraints	Continuous Variable	Constraints	Continuous Variable	Constraints	Continuous Variable	Constraints	Continuous Variable	Constraints	Continuous Variable
N=50	2377	2383	2427	2434	2427	2434	2377	2383	2427	2434	2427	2434
N=100	2377	2383	2477	2484	2477	2484	2377	2383	2477	2484	2477	2484
N=200	2377	2383	2577	2584	2577	2584	2377	2383	2577	2584	2577	2584

	EMUTT		MUTT_CVaR		MRMUTT		EMTTT		MTTT_CVaR		MRMTTT	
Number of Scenarios	Constraints	Continuous Variable	Constraints	Continuous Variable	Constraints	Continuous Variable	Constraints	Continuous Variable	Constraints	Continuous Variable	Constraints	Continuous Variable
N=50	6177	2433	6227	2484	10027	2534	6177	2433	6227	2484	10027	2534
N=100	9977	2483	10077	2584	17677	2684	9977	2483	10077	2584	17677	2684
N=200	17577	2583	17777	2784	32977	2984	17577	2583	17777	2784	32977	2984

Table A.1: Dimensions of the problem with fixed demand for the Sioux Falls network

Number of Scenarios	EAUTT		AUTT_CVaR		MRAUTT		EATTT		ATTT_CVaR		MRATTT	
	Constraints	Continuous Variable	Constraints	Continuous Variable	Constraints	Continuous Variable	Constraints	Continuous Variable	Constraints	Continuous Variable	Constraints	Continuous Variable
N=50	2573	2935	2623	2986	2623	2986	2573	2935	2623	2986	2623	2986
N=100	2573	2935	2673	3036	2673	3036	2573	2935	2673	3036	2673	3036
N=200	2573	2935	2773	3136	2773	3136	2573	2935	2773	3136	2773	3136

Number of Scenarios	EMUTT		MUTT_CVaR		MRMUTT		EMTTT		MTTT_CVaR		MRMTTT	
	Constraints	Continuous Variable	Constraints	Continuous Variable	Constraints	Continuous Variable	Constraints	Continuous Variable	Constraints	Continuous Variable	Constraints	Continuous Variable
N=50	6373	2986	6423	3036	10223	3086	6373	2986	6423	3036	10223	3086
N=100	10173	3036	10273	3136	17873	3236	10173	3036	10273	3136	17873	3236
N=200	17773	3136	17973	3336	33173	3536	17773	3136	17973	3336	33173	3536

Table A.2: Dimensions of the problem with elastic demand for the Sioux Falls network

	EAUTT		AUTT_CVaR		MRAUTT		EATTT		ATTT_CVaR		MRATTT	
Number of Scenarios	Constraints	Continuous Variable	Constraints	Continuous Variable	Constraints	Continuous Variable	Constraints	Continuous Variable	Constraints	Continuous Variable	Constraints	Continuous Variable
N=50	71	75	81	86	81	86	71	75	81	86	81	86
N=100	71	75	121	126	121	126	71	75	121	126	121	126
N=200	71	75	171	176	171	176	71	75	171	176	171	176

	EMUTT		MUTT_CVaR		MRMUTT		EMTTT		MTTT_CVaR		MRMTTT	
Number of Scenarios	Constraints	Continuous Variable	Constraints	Continuous Variable	Constraints	Continuous Variable	Constraints	Continuous Variable	Constraints	Continuous Variable	Constraints	Continuous Variable
N=50	251	85	261	96	441	106	251	85	261	96	441	106
N=100	971	125	1021	176	1921	226	971	125	1021	176	1921	226
N=200	1871	175	1971	276	3771	376	1871	175	1971	276	3771	376

Table A.3: Dimensions of the problem with fixed demand for the Nine Node network

	EAUTT		AUTT_CVaR		MRAUTT		EATTT		ATTT_CVaR		MRATTT	
Number of Scenarios	Constraints	Continuous Variable	Constraints	Continuous Variable	Constraints	Continuous Variable	Constraints	Continuous Variable	Constraints	Continuous Variable	Constraints	Continuous Variable
N=50	75	91	85	102	85	102	75	91	85	102	85	102
N=100	75	91	135	142	135	142	75	91	135	142	135	142
N=200	75	91	175	192	175	192	75	91	175	192	175	192

	EMUTT		MUTT_CVaR		MRMUTT		EMTTT		MTTT_CVaR		MRMTTT	
Number of Scenarios	Constraints	Continuous Variable	Constraints	Continuous Variable	Constraints	Continuous Variable	Constraints	Continuous Variable	Constraints	Continuous Variable	Constraints	Continuous Variable
N=50	225	101	265	112	445	122	225	101	265	112	445	122
N=100	975	141	1025	192	1925	242	975	141	1025	192	1925	242
N=200	1875	191	1975	292	3775	392	1875	191	1975	292	3775	392

Table A.4: Dimensions of the problem with elastic demand for the Nine Node network

Appendix B

Additional Comparative Results for the Models with Only Risk Terms with Fixed Demand

Results for the Nine Node (NN) Network

The results presented in this part are obtained according to the problem instance belongs to the Group 1 with the random link capacities.

$\alpha = 0.8$							
Type of Probability	Number of Scenarios	Risk-averse Model		Risk-neutral Model		Relative Difference	
		$CVaR_\alpha$	Exp.	$CVaR_\alpha$	Exp.	$CVaR_\alpha$	Exp.
Equal	N=10	263.87	186.25	270.77	185.68	-2.5507%	0.3077%
	N=100	265.53	187.63	271.28	186.56	-2.1197%	0.5719%
Different	N=10	244.44	187.78	248.90	187.37	-1.7955%	0.2166%
	N=100	256.03	186.87	261.06	186.13	-1.9268%	0.3990%
$\alpha = 0.9$							
Type of Probability	Number of Scenarios	Risk-averse Model		Risk-neutral Model		Relative Difference	
		$CVaR_\alpha$	Exp.	$CVaR_\alpha$	Exp.	$CVaR_\alpha$	Exp.
Equal	N=10	411.74	186.64	426.13	185.68	-3.3783%	0.5169%
	N=100	379.99	188.31	414.34	186.56	-8.2922%	0.9374%
Different	N=10	363.44	188.03	380.90	187.37	-4.5831%	0.3522%
	N=100	373.41	187.67	395.24	186.13	-5.5242%	0.8315%

Table B.1: Comparative results with AUTT with fixed demand for the NN network

The following results are obtained according to the problem instance belongs to the Group 1 with the random free-flow times.

$\alpha = 0.9$							
Type of Probability	Number of Scenarios	Risk-averse Model		Risk-neutral Model		Relative Difference	
		$CVaR_\alpha$	Exp.	$CVaR_\alpha$	Exp.	$CVaR_\alpha$	Exp.
Equal	N=50	35.29	34.98	68.27	31.37	-48.3103%	11.5211%
	N=100	35.54	35.17	76.06	31.44	-53.2709%	11.8681%
Different	N=50	34.64	33.20	71.30	31.57	-51.4151%	5.1655%
	N=100	35.67	34.62	76.52	31.37	-53.3836%	10.3708%

Table B.2: Comparative results with MUTT with fixed demand for the NN network

$\alpha = 0.9$							
Type of	Number of	Risk-averse Model		Risk-neutral Model		Relative Difference	
Probability	Scenarios	CVaR$_\alpha$	Exp.	CVaR$_\alpha$	Exp.	CVaR$_\alpha$	Exp.
Equal	N=50	798.83	798.72	851.92	794.42	-6.2310%	0.5410%
	N=100	802.91	801.46	862.39	797.13	-6.8978%	0.5426%
Different	N=50	799.37	799.22	847.52	795.60	-5.6814%	0.4561%
	N=100	801.18	800.86	866.89	795.38	-7.5800%	0.6893%

Table B.3: Comparative results with MTTT with fixed demand for the NN network

The results presented in this part are obtained according to the problem instance belongs to the Group 1 with the random free-flow times and link capacities.

$\alpha = 0.9$							
Type of	Number of	Risk-averse Model		Risk-neutral Model		Relative Difference	
Probability	Scenarios	CVaR$_\alpha$	Exp.	CVaR$_\alpha$	Exp.	CVaR$_\alpha$	Exp.
Equal	N=50	193.45	107.18	413.79	75.65	-53.2491%	41.6824%
	N=100	174.95	105.33	375.72	70.78	-53.4360%	48.8188%
Different	N=50	190.55	106.89	375.58	71.31	-49.2652%	49.8938%
	N=100	182.49	106.08	356.12	71.14	-48.7564%	49.1119%

Table B.4: Comparative results with MUTT with fixed demand for the NN network

$\alpha = 0.9$							
Type of	Number of	Risk-averse Model		Risk-neutral Model		Relative Difference	
Probability	Scenarios	CVaR$_\alpha$	Exp.	CVaR$_\alpha$	Exp.	CVaR$_\alpha$	Exp.
Equal	N=50	8115.79	4324.91	14779.17	2641.42	-45.0863%	63.7342%
	N=100	6944.81	4207.81	13657.69	2559.34	-49.1509%	64.4100%
Different	N=50	7781.43	4291.47	14779.17	2627.64	-47.3487%	63.3206%
	N=100	7059.65	4219.29	13892.22	2575.37	-49.1827%	63.8328%

Table B.5: Comparative results with MTTT with fixed demand for the NN network

Results for the Sioux Falls (SF) Network

$\alpha = 0.9$									
Instance Number	Number of Scenarios	Risk-averse Model		Risk-neutral Model		Relative Difference		CPU Times	
		$CVaR_{\alpha}$	Exp.	$CVaR_{\alpha}$	Exp.	$CVaR_{\alpha}$	Exp.	R-averse	R-neutral
1B_2	N=100	2027.22	655.50	2037.54	643.96	-0.5067%	1.7925%	329	349
2B_1		2211.62	725.11	2238.06	715.20	-1.1811%	1.3852%	346	337
1B_2	N=200	2146.37	645.36	2146.57	645.33	-0.0089%	0.0037%	683	642
2B_1		2278.82	728.51	2395.45	727.22	-4.8690%	0.1773%	650	653

Table B.6: Comparative results with MTTT with fixed demand for the SF network

$\alpha = 0.9$									
Instance Number	Number of Scenarios	Risk-averse Model		Risk-neutral Model		Relative Difference		CPU Times	
		$CVaR_{\alpha}$	Exp.	$CVaR_{\alpha}$	Exp.	$CVaR_{\alpha}$	Exp.	R-averse	R-neutral
1C_2	N=100	2334.25	486.55	2347.90	486.05	-0.5815%	0.1030%	349	337
2C_1		6422.52	1159.48	6456.75	1158.78	-0.5302%	0.0602%	346	335
1C_2	N=200	2350.20	489.12	2358.06	489.10	-0.3333%	0.003%	712	632
2C_1		6491.29	1188.06	6546.86	1186.69	-0.8488%	0.1158%	763	690

Table B.7: Comparative results with MUTT with fixed demand for the SF network

Appendix C

Additional Comparative Results for the Models with Only Risk Terms with Elastic Demand

Results for the Nine Node (NN) Network

The results presented in this part are obtained according to the problem instance belongs to the Group 1 with the random link capacities.

$\alpha = 0.8$							
Type of Probability	Number of Scenarios	Risk-averse Model		Risk-neutral Model		Relative Difference	
		$CVaR_\alpha$	Exp.	$CVaR_\alpha$	Exp.	$CVaR_\alpha$	Exp.
Equal	N=10	221.07	159.10	224.63	155.76	-1.5861%	2.1463%
	N=100	206.37	155.07	230.32	154.95	-10.4021%	0.0743%
Different	N=10	219.30	158.79	223.74	155.62	-1.9820%	2.0395%
	N=100	223.70	159.56	231.69	155.12	-3.4487%	2.8610%
$\alpha = 0.9$							
Type of Probability	Number of Scenarios	Risk-averse Model		Risk-neutral Model		Relative Difference	
		$CVaR_\alpha$	Exp.	$CVaR_\alpha$	Exp.	$CVaR_\alpha$	Exp.
Equal	N=10	325.11	158.72	336.49	155.76	-3.3837%	1.9044%
	N=100	299.10	155.15	345.51	154.95	-13.4322%	0.1283%
Different	N=10	335.04	158.68	362.26	155.62	-7.5117%	1.9684%
	N=100	302.01	155.14	349.65	155.12	-13.6250%	0.0152%

Table C.1: Comparative results with AUTT with elastic demand for the NN network

The following results are obtained according to the problem instance belongs to the Group 1 with the random free-flow times.

$\alpha = 0.9$							
Type of Probability	Number of Scenarios	Risk-averse Model		Risk-neutral Model		Relative Difference	
		$CVaR_\alpha$	Exp.	$CVaR_\alpha$	Exp.	$CVaR_\alpha$	Exp.
Equal	N=50	34.89	32.42	75.27	31.17	-53.6486%	4.0208%
	N=100	33.97	31.20	74.48	30.78	-54.3876%	1.3690%
Different	N=50	35.30	34.98	68.49	31.26	-48.4525%	11.9201%
	N=100	35.54	35.16	75.47	31.13	-52.9151%	12.9313%

Table C.2: Comparative results with MUTT with elastic demand for the NN network

$\alpha = 0.9$							
Type of	Number of	Risk-averse Model		Risk-neutral Model		Relative Difference	
Probability	Scenarios	CVaR$_\alpha$	Exp.	CVaR$_\alpha$	Exp.	CVaR$_\alpha$	Exp.
Equal	N=50	213.97	213.89	243.66	211.18	-12.1857%	1.2807%
	N=100	228.00	227.92	268.94	224.36	-15.2208%	1.5877%
Different	N=50	215.63	215.53	247.54	213.10	-12.8902%	1.1430%
	N=100	227.92	227.58	268.85	224.00	-15.2247%	1.6003%

Table C.3: Comparative results with MTTT with elastic demand for the NN network

The results presented in this part are obtained according to the problem instance belongs to the Group 1 with random free-flow times and link capacities.

$\alpha = 0.9$							
Type of	Number of	Risk-averse Model		Risk-neutral Model		Relative Difference	
Probability	Scenarios	CVaR$_\alpha$	Exp.	CVaR$_\alpha$	Exp.	CVaR$_\alpha$	Exp.
Equal	N=50	151.54	41.39	154.66	41.35	-2.0173%	0.1043%
	N=100	149.42	41.26	154.13	41.24	-3.0503%	0.0719%
Different	N=50	154.37	41.61	157.63	41.59	-2.0719%	0.0533%
	N=100	152.69	41.37	155.17	41.34	-1.5989%	0.0696%

Table C.4: Comparative results with MUTT with elastic demand for the NN network

$\alpha = 0.9$							
Type of	Number of	Risk-averse Model		Risk-neutral Model		Relative Difference	
Probability	Scenarios	CVaR$_\alpha$	Exp.	CVaR$_\alpha$	Exp.	CVaR$_\alpha$	Exp.
Equal	N=50	3124.58	1499.62	6961.64	1298.53	-55.1172%	15.4860%
	N=100	3039.95	1434.31	6189.76	1276.48	-50.8874%	12.3643%
Different	N=50	3190.81	1488.39	7145.68	1298.57	-55.3463%	14.6176%
	N=100	3105.13	1439.46	6361.76	1290.91	-51.1908%	11.5073%

Table C.5: Comparative results with MTTT with elastic demand for the NN network

Results for the Sioux Falls (SF) Network

$\alpha = 0.8$							
Instance	Number of	Risk-averse Model		Risk-neutral Model		Relative Difference	
Number	Scenarios	CVaR$_\alpha$	Exp.	CVaR$_\alpha$	Exp.	CVaR$_\alpha$	Exp.
1A_1	N=50	519.88	427.90	538.59	427.65	-3.4739%	0.0573%
1A_2		498.61	423.72	500.42	422.26	-0.3615%	0.3456%
1A_1	N=100	518.50	427.60	533.26	427.39	-2.7669%	0.0505 %
1A_2		502.39	423.60	510.11	422.82	-1.5119 %	0.1827 %
1A_1	N=200	514.35	422.91	517.07	422.75	-0.5271%	0.0360%
1A_2		509.21	423.32	509.70	423.21	-0.0976 %	0.0255%
$\alpha = 0.9$							
Instance	Number of	Risk-averse Model		Risk-neutral Model		Relative Difference	
Number	Scenarios	CVaR$_\alpha$	Exp.	CVaR$_\alpha$	Exp.	CVaR$_\alpha$	Exp.
1A_1	N=50	698.70	427.97	702.68	427.65	-0.5663 %	0.0742%
1A_2		626.34	423.61	631.35	422.26	-0.7941 %	0.3202 %
1A_1	N=100	674.78	427.77	694.33	427.39	-2.8164%	0.0889%
1A_2		647.38	424.36	654.28	422.82	-1.0545%	0.3632%
1A_1	N=200	678.61	423.04	679.85	422.75	-0.1824%	0.0684%
1A_2		647.25	423.30	647.85	423.21	-0.0929%	0.0214%

Table C.6: Comparative results with AUTT with elastic demand for the SF network

$\alpha = 0.8$							
Instance	Number of	Risk-averse Model		Risk-neutral Model		Relative Difference	
Number	Scenarios	CVaR$_\alpha$	Exp.	CVaR$_\alpha$	Exp.	CVaR$_\alpha$	Exp.
1A_2	N=50	1480.71	1080.76	1516.75	1080.50	-2.3760%	0.0243%
2A_1		1489.17	1042.89	1498.81	1040.04	-0.6434%	0.2737%
1A_2	N=100	1499.66	1078.53	1540.13	1078.48	-2.6280%	0.0051%
2A_1		1517.96	1039.12	1527.46	1036.65	-0.6223%	0.2386%
1A_2	N=200	1491.47	1078.50	1533.52	1078.21	-2.7421 %	0.0266%
2A_1		1520.41	1037.76	1529.30	1036.56	-0.5813%	0.1155%
$\alpha = 0.9$							
Instance	Number of	Risk-averse Model		Risk-neutral Model		Relative Difference	
Number	Scenarios	CVaR$_\alpha$	Exp.	CVaR$_\alpha$	Exp.	CVaR$_\alpha$	Exp.
1A_2	N=50	2045.60	1096.67	2195.56	1080.50	-6.8300%	1.4972 %
2A_1		2224.22	1044.01	2268.64	1040.04	-1.9581%	0.3814%
1A_2	N=100	2212.61	1079.99	2288.47	1078.48	-3.3151%	0.1399%
2A_1		2338.18	1039.79	2383.10	1036.65	-1.8849%	0.3028%
1A_2	N=200	2192.23	1078.78	2280.34	1078.21	-3.8641%	0.0528%
2A_1		2342.42	1038.78	2376.35	1036.56	-1.4277%	0.2143%

Table C.7: Comparative results with ATTT with elastic demand for the SF network

$\alpha = 0.8$								
Instance	Number of	Risk-averse Model		Risk-neutral Model		Relative Difference		
Number	Scenarios	CVaR$_\alpha$	Exp.	CVaR$_\alpha$	Exp.	CVaR$_\alpha$	Exp.	
1A_1	N=50	45.26	26.31	50.65	25.25	-10.6417%	4.1980%	
1A_2		52.14	26.08	52.66	25.60	-0.9854%	1.8383%	
1A_1	N=100	52.71	25.96	54.13	25.62	-2.6233 %	1.3271%	
1A_2		45.80	23.99	46.25	23.89	-0.9691 %	0.4302%	
1A_1	N=200	54.55	26.62	56.51	25.95	-3.4684 %	2.5819%	
1_2		49.61	24.00	49.71	23.94	-0.2000 %	0.2596%	
$\alpha = 0.9$								
Instance	Number of	Risk-averse Model		Risk-neutral Model		Relative Difference		
Number	Scenarios	CVaR$_\alpha$	Exp.	CVaR$_\alpha$	Exp.	CVaR$_\alpha$	Exp.	
1A_1	N=50	75.2708	26.28	82.3194	25.25	-8.5625%	4.0629%	
1A_2		76.73	25.63	82.60	25.60	-7.1049%	0.0806%	
1A_1	N=100	77.29972	26.61	82.5283	25.62	-6.3355 %	3.8570%	
1A_2		73.96	23.96	74.21	23.89	-0.3362%	0.3030%	
1A_1	N=200	79.4281	26.92	84.7894	25.95	-6.3231 %	3.7380%	
1A_2		76.85	24.25	81.92	23.94	-6.1950 %	1.3048%	

Table C.8: Comparative results with MUTT with elastic demand for the SF network

$\alpha = 0.8$							
Instance	Number of	Risk-averse Model		Risk-neutral Model		Relative Difference	
Number	Scenarios	CVaR$_\alpha$	Exp.	CVaR$_\alpha$	Exp.	CVaR$_\alpha$	Exp.
1A_2	N=50	305.72	141.30	313.85	140.07	-2.5918%	0.8822%
2A_1		350.70	173.96	396.37	166.95	-11.5214%	4.1984%
1A_2	N=100	288.22	137.89	289.98	136.66	-0.6073%	0.9041%
2A_1		344.65	161.75	349.51	156.46	-1.3904%	3.3822%
1A_2	N=200	260.04	133.64	272.82	128.72	-4.6867%	3.8276%
2A_1		335.26	165.80	354.78	157.04	-5.5002%	5.5784%
$\alpha = 0.9$							
Instance	Number of	Risk-averse Model		Risk-neutral Model		Relative Difference	
Number	Scenarios	CVaR$_\alpha$	Exp.	CVaR$_\alpha$	Exp.	CVaR$_\alpha$	Exp.
1A_2	N=50	515.02	140.79	531.08	140.07	-3.0256%	0.5183%
2A_1		636.62	176.38	683.14	166.95	-6.8097%	5.6460 %
1A_2	N=100	479.63	138.88	480.39	136.66	-0.1571%	1.6218%
2A_1		572.72	163.53	590.83	156.46	-3.0656%	4.5196%
1A_2	N=200	425.14	133.10	452.95	128.72	-6.1403%	3.4057%
2A_1		597.19	165.17	601.95	157.04	-0.7915%	5.1807%

Table C.9: Comparative results with MTTT with elastic demand for the SF network

$\alpha = 0.9$									
Instance	Number of	Risk-averse Model		Risk-neutral Model		Relative Difference		CPU Times	
Number	Scenarios	CVaR$_\alpha$	Exp.	CVaR$_\alpha$	Exp.	CVaR$_\alpha$	Exp.	R-averse	R-neutral
1B_2	N=100	85.13	33.39	88.80	33.37	-4.1380%	0.0547%	67	83
2B_1		103.26	38.84	103.53	38.81	-0.2612%	0.0684%	77	64
1B_2	N=200	84.92	33.28	84.93	33.28	-0.0103%	-0.0050%	144	127
2B_1		104.19	38.73	104.354	38.71	-0.1544%	0.0416%	115	120

Table C.10: Comparative results with MUTT with elastic demand for the SF network

$\alpha = 0.9$									
Instance	Number of	Risk-averse Model		Risk-neutral Model		Relative Difference		CPU Times	
Number	Scenarios	CVaR$_\alpha$	Exp.	CVaR$_\alpha$	Exp.	CVaR$_\alpha$	Exp.	R-averse	R-neutral
1B_2	N=100	131.57	89.78	136.00	89.76	-3.2567%	0.0143%	78	113
2B_1		100.84	94.95	101.28	94.33	-0.4313%	0.6597%	61	73
1B_2	N=200	114.08	92.33	119.75	90.74	-4.7416%	1.7580%	147	147
2B_1		96.39	95.30	96.41	93.95	-0.0247%	1.4381%	146	162

Table C.11: Comparative results with MTTT with elastic demand for the SF network

$\alpha = 0.9$									
Instance	Number of	Risk-averse Model		Risk-neutral Model		Relative Difference		CPU Times	
Number	Scenarios	$CVaR_\alpha$	Exp.	$CVaR_\alpha$	Exp.	$CVaR_\alpha$	Exp.	R-averse	R-neutral
1C_2	N=100	114.04	91.87	118.41	89.60	-3.6856%	2.5373%	103	101
2C_1		108.81	105.71	109.55	103.04	-0.6771%	2.5924%	96	101
1C_2	N=200	109.31	91.60	113.67	90.36	-3.8334%	1.3710%	210	227
2C_1		100.94	98.65	101.41	98.64	-0.4638%	0.0088%	256	228

Table C.12: Comparative results with MTTT with elastic demand for the SF network

$\alpha = 0.9$									
Instance	Number of	Risk-averse Model		Risk-neutral Model		Relative Difference		CPU Times	
Number	Scenarios	$CVaR_\alpha$	Exp.	$CVaR_\alpha$	Exp.	$CVaR_\alpha$	Exp.	R-averse	R-neutral
1C_2	N=100	85.58	33.57	86.15	33.40	-0.6603%	0.5274%	82	77
2C_1		89.12	34.22	89.49	34.17	-0.4124%	0.1541%	83	88
1C_2	N=200	85.32	33.32	85.64	33.29	-0.3740%	0.0785%	173	172
2C_1		88.04	34.19	88.16	34.16	-0.1321%	0.0981%	226	145

Table C.13: Comparative results with MUTT with elastic demand for the SF network

Appendix D

Additional Comparative Results for the Mean-Risk Models

Results for the Nine Node (NN) Network

The results presented in this part are obtained according to the problem instance belongs to the Group 1 with random link capacities.

$\alpha = 0.8$ and Number of Scenarios=10									
Type of	Trade-off Par.	Fixed				Elastic			
Probability	θ	$CVaR_\alpha$	Exp.	Relative Difference		$CVaR_\alpha$	Exp.	Relative Difference	
Equal	0	248.90	187.37	$CVaR_\alpha$	Exp.	224.63	155.76	$CVaR_\alpha$	Exp.
	0.1	248.38	187.378	-0.2110	0.0021	224.02	155.78	-0.2742	0.0158
	1	246.33	187.48	-1.0329	0.0584	221.65	157.118	-1.3301	0.8733
	10	244.74	187.71	-1.6743	0.1813	221.07	159.10	-1.5861	2.1463
Different	0	270.77	185.68	$CVaR_\alpha$	Exp.	223.74	155.62	$CVaR_\alpha$	Exp.
	0.1	268.19	185.991	-0.9521	0.1665	223.16	155.85	-0.2599	0.1509
	1	267.43	185.03	-1.2346	-0.3496	222.21	156.176	-0.6803	0.3601
	10	265.42	185.13	-1.9766	-0.2971	220.73	157.35	-1.3461	1.1139
$\alpha = 0.9$ and Number of Scenarios=10									
Type of	Trade-off Par.	Fixed				Elastic			
Probability	θ	$CVaR_\alpha$	Exp.	Relative Difference		$CVaR_\alpha$	Exp.	Relative Difference	
Equal	0	380.90	187.37	$CVaR_\alpha$	Exp.	336.49	155.76	$CVaR_\alpha$	Exp.
	0.1	378.90	187.381	-0.5253%	0.0038%	336.49	155.76	0.0000%	0.0000%
	1	371.01	187.56	-2.5963%	0.0996%	332.15	157.056	-1.2918%	0.8331%
	10	364.67	187.93	-4.2599%	0.2973%	325.11	158.72	-3.3837%	1.9044%
Different	0	426.13	185.68	$CVaR_\alpha$	Exp.	362.26	155.62	$CVaR_\alpha$	Exp.
	0.1	421.06	185.848	-1.1906%	0.0896%	361.94	155.78	-0.0865%	0.1030%
	1	417.13	186.28	-2.1126%	0.3227%	356.70	157.557	-1.5348%	1.2475%
	10	412.33	186.47	-3.2393%	0.4246%	345.18	157.63	-4.7125%	1.2957%

Table D.1: Results for the mean-risk models with AUTT for the NN network for N=10

$\alpha = 0.8$ and Number of Scenarios=10									
Type of	Trade-off Par.	Fixed				Elastic			
Probability	θ	CVaR$_\alpha$	Exp.	Relative Difference		CVaR$_\alpha$	Exp.	Relative Difference	
Equal	0	55.52	29.99	CVaR$_\alpha$	Exp.	46.10	21.74	CVaR$_\alpha$	Exp.
	0.1	55.07	30.81	-0.8006%	2.7355%	46.10	21.74	0.0000%	0.0000%
	1	54.39	30.98	-2.0310%	3.2821%	42.12	27.606	-8.6407%	26.9988%
	10	49.58	34.63	-10.6989%	15.4810%	40.15	28.00	-12.9085%	28.8090%
Different	0	53.56	28.21	CVaR$_\alpha$	Exp.	46.77	22.14	CVaR$_\alpha$	Exp.
	0.1	53.07	28.46	-0.9085%	0.8716%	46.57	22.54	-0.4328%	1.7862%
	1	51.39	29.58	-4.0510%	4.8236%	44.22	24.560	-5.4548%	10.9086%
	10	47.58	31.63	-11.1687%	12.1197%	42.52	26.81	-9.0946%	21.0778%
$\alpha = 0.9$ and Number of Scenarios=10									
Type of	Trade-off Par.	Fixed				Elastic			
Probability	θ	CVaR$_\alpha$	Exp.	Relative Difference		CVaR$_\alpha$	Exp.	Relative Difference	
Equal	0	87.43	29.99	CVaR$_\alpha$	Exp.	88.12	21.74	CVaR$_\alpha$	Exp.
	0.1	84.73	30.01	-3.0889%	0.0692%	86.56	21.81	-1.7687%	0.3468%
	1	81.98	30.47	-6.2262%	1.5959%	84.54	22.106	-4.0635%	1.6968%
	10	68.26	34.63	-21.9248%	15.4810%	74.62	25.42	-15.3187%	16.9423%
Different	0	84.29	28.21	CVaR$_\alpha$	Exp.	78.94	22.14	CVaR$_\alpha$	Exp.
	0.1	82.33	28.32	-2.3275%	0.3860%	76.55	22.57	-3.0211%	1.9308%
	1	79.98	29.17	-5.1071%	3.3927%	74.52	23.630	-5.5990%	6.7085%
	10	65.26	32.41	-22.5767%	14.8808%	67.82	25.56	-14.0873%	15.4375%

Table D.2: Results for the mean-risk models with MUTT for the NN network for N=10

Results for the Sioux Falls (SF) Network

$\alpha = 0.8$ and Number of Scenarios=50									
Instance	Trade-off Par.	Fixed				Elastic			
Number	θ	CVaR$_\alpha$	Exp.	Relative Difference		CVaR$_\alpha$	Exp.	Relative Difference	
1A_1	0	1816.17	1122.21	CVaR$_\alpha$	Exp.	538.59	427.65	CVaR$_\alpha$	Exp.
	0.1	1805.80	1122.52	-0.5712%	0.0280%	538.59	427.65	0.0000%	0.0000%
	1	1805.80	1122.52	-0.5712%	0.0280%	537.12	427.729	-0.2734%	0.0179%
	10	1805.80	1122.52	-0.5712%	0.0280%	520.10	427.87	-3.4321%	0.0506%
1A_2	0	1637.48	1064.91	CVaR$_\alpha$	Exp.	500.42	422.26	CVaR$_\alpha$	Exp.
	0.1	1637.46	1064.91	-0.0011%	0.0001 %	500.42	422.26	0.0000 %	0.0000%
	1	1637.39	1064.95	-0.0053%	0.0042%	500.42	422.26	0.0000%	0.0000%
	10	1637.38	1065.20	-0.0059%	0.0269 %	498.61	423.72	-0.3615 %	0.3456%
$\alpha = 0.9$ and Number of Scenarios=50									
Instance	Trade-off Par.	Fixed				Elastic			
Number	θ	CVaR$_\alpha$	Exp.	Relative Difference		CVaR$_\alpha$	Exp.	Relative Difference	
1A_1	0	3363.74	1122.21	CVaR$_\alpha$	Exp.	702.68	427.65	CVaR$_\alpha$	Exp.
	0.1	3357.89	1122.24	-0.1739%	0.0021	702.67	427.85	-0.0002%	0.0468%
	1	3340.10	1122.52	-0.7030%	0.0278	699.22	427.89	-0.4917%	0.0550%
	10	3340.10	1122.52	-0.7030%	0.0278	698.70	427.97	-0.5663%	0.0743%
1A_2	0	2634.69	1064.91	CVaR$_\alpha$	Exp.	631.35	422.26	CVaR$_\alpha$	Exp.
	0.1	2634.40	1065.42	-0.0112%	0.0482	630.51	422.37	-0.1333%	0.0262%
	1	2630.65	1069.19	-0.1536%	0.4019	629.94	422.84	-0.2235%	0.1361%
	10	2630.65	1069.19	-0.1536%	0.4019	626.34	423.61	-0.7941%	0.3202%

Table D.3: Results for the mean-risk models with AUTT for the SF network for N=50

$\alpha = 0.8$ and Number of Scenarios=100									
Instance	Trade-off Par.	Fixed				Elastic			
Number	θ	CVaR$_\alpha$	Exp.	Relative Difference		CVaR$_\alpha$	Exp.	Relative Difference	
1A_1	0	1878.90	1101.44	CVaR$_\alpha$	Exp.	533.25	427.39	CVaR$_\alpha$	Exp.
	0.1	1878.90	1101.44	0.0000%	0.0000%	519.73	427.42	-2.5353%	0.0059%
	1	1875.68	1102.29	-0.1713%	0.0770%	518.57	427.47	-2.7530%	0.0176 %
	10	1875.68	1102.29	-0.1713%	0.0770 %	518.52	427.51	-2.7622%	0.0270%
1A_2	0	1660.34	1065.23	CVaR$_\alpha$	Exp.	510.11	422.82	CVaR$_\alpha$	Exp.
	0.1	1660.30	1065.23	-0.0028%	0.0000%	508.02	422.86	-0.4082%	0.0089%
	1	1660.06	1065.24	-0.0171 %	0.0009%	505.51	423.13	-0.9009%	0.0719%
	10	1659.77	1065.27	-0.0343%	0.0037 %	502.55	423.55	-1.4820%	0.1706%
$\alpha = 0.9$ and Number of Scenarios=100									
Instance	Trade-off Par.	Fixed				Elastic			
Number	θ	CVaR$_\alpha$	Exp.	Relative Difference		CVaR$_\alpha$	Exp.	Relative Difference	
1A_1	0	3161.28	1101.44	CVaR$_\alpha$	Exp.	694.33	427.39	CVaR$_\alpha$	Exp.
	0.1	3159.73	1101.46	-0.0489%	0.0020%	684.78	427.57	-1.3761%	0.0421%
	1	3158.34	1102.19	-0.0930%	0.0683%	679.03	427.67	-2.2046%	0.0655 %
	10	3158.34	1102.19	-0.0930%	0.0683%	674.78	427.77	-2.8164%	0.0889%
1A_2	0	2646.11	1065.23	CVaR$_\alpha$	Exp.	654.28	422.82	CVaR$_\alpha$	Exp.
	0.1	2646.11	1065.23	0.0000%	0.0000 %	654.28	422.82	0.0000%	0.0000%
	1	2644.89	1066.31	-0.0458 %	0.1014%	650.57	423.51	-0.5665%	0.1630%
	10	2644.89	1066.31	-0.0458%	0.1014 %	648.00	423.60	-0.9596%	0.1834%

Table D.4: Results for the mean-risk models with AUTT for the SF network for N=100

$\alpha = 0.8$ and Number of Scenarios=50									
Instance	Trade-off Par.	Fixed				Elastic			
Number	θ	CVaR$_\alpha$	Exp.	Relative Difference		CVaR$_\alpha$	Exp.	Relative Difference	
1A_2	0	14287.99	7542.43	CVaR$_\alpha$	Exp.	1516.75	1080.50	CVaR$_\alpha$	Exp.
	0.1	14210.74	7543.90	-0.5407 %	0.0195%	1481.75	1080.61	-2.3077%	0.0102%
	1	14072.12	7561.85	-1.5109%	0.2575%	1480.95	1080.753	-2.3606%	0.0238%
	10	13957.76	7588.87	-2.3112%	0.6157%	1480.71	1080.76	-2.3760%	0.0243%
$\alpha = 0.9$ and Number of Scenarios=50									
Instance	Trade-off Par.	Fixed				Elastic			
Number	θ	CVaR$_\alpha$	Exp.	Relative Difference		CVaR$_\alpha$	Exp.	Relative Difference	
1A_2	0	27205.62	7542.43	CVaR$_\alpha$	Exp.	2195.56	1080.50	CVaR$_\alpha$	Exp.
	0.1	27205.62	7542.43	0.0000 %	0.0000%	2152.02	1080.51	-1.9829%	0.0009%
	1	26939.65	7557.91	-0.9776%	0.2053%	2152.02	1080.91	-1.9829%	0.0378%
	10	26896.70	7623.76	-1.1355%	1.0783 %	2130.25	1081.20	-2.9745%	0.0649%

Table D.5: Results for the mean-risk models with ATTT for the SF network for N=50

$\alpha = 0.8$ and Number of Scenarios=100									
Instance	Trade-off Par.	Fixed				Elastic			
Number	θ	CVaR$_\alpha$	Exp.	Relative Difference		CVaR$_\alpha$	Exp.	Relative Difference	
1A_2	0	14550.91	7489.31	CVaR$_\alpha$	Exp.	1540.13	1078.48	CVaR$_\alpha$	Exp.
	0.1	14550.91	7489.31	0.0000%	0.0000 %	1505.13	1078.51	-2.2723%	0.0025%
	1	14548.05	7489.43	-0.0197%	0.0016 %	1501.61	1078.51	-2.5013%	0.0032%
	10	14491.36	7533.08	-0.4093%	0.5844 %	1499.92	1078.52	-2.6108%	0.0040%
$\alpha = 0.9$ and Number of Scenarios=100									
Instance	Trade-off Par.	Fixed				Elastic			
Number	θ	CVaR$_\alpha$	Exp.	Relative Difference		CVaR$_\alpha$	Exp.	Relative Difference	
1A_2	0	27343.55	7489.31	CVaR$_\alpha$	Exp.	2288.47	1078.48	CVaR$_\alpha$	Exp.
	0.1	27329.84	7489.52	-0.0502%	0.0028%	2243.49	1078.49	-1.9656%	0.0006%
	1	26591.53	7536.54	-2.7503%	0.6305%	2225.45	1078.50	-2.7540%	0.0019%
	10	26450.08	7554.39	-3.2676%	0.8689%	2214.62	1079.51	-3.2272%	0.0958%

Table D.6: Results for the mean-risk models with ATTT for the SF network for N=100

$\alpha = 0.8$ and Number of Scenarios=200									
Instance	Trade-off Par.	Fixed				Elastic			
Number	θ	$CVaR_\alpha$	Exp.	Relative Difference		$CVaR_\alpha$	Exp.	Relative Difference	
1A_2	0	15230.92	7489.31	$CVaR_\alpha$	Exp.	1533.52	1078.21	$CVaR_\alpha$	Exp.
	0.1	15214.17	7591.54	-0.1100%	1.3650 %	1495.39	1078.26	-2.4864%	0.0042%
	1	15049.55	7639.04	-1.1908 %	1.9992 %	1495.00	1078.34	-2.5121%	0.0121 %
	10	15016.96	7678.26	-1.4048%	2.5229 %	1494.95	1078.50	-2.5153 %	0.0266 %
$\alpha = 0.9$ and Number of Scenarios=200									
Instance	Trade-off Par.	Fixed				Elastic			
Number	θ	$CVaR_\alpha$	Exp.	Relative Difference		$CVaR_\alpha$	Exp.	Relative Difference	
1A_2	0	28150.04	7590.83	$CVaR_\alpha$	Exp.	2280.34	1078.21	$CVaR_\alpha$	Exp.
	0.1	28098.88	7591.85	-0.1817%	0.0135%	2198.03	1078.69	-3.6099 %	0.0446%
	1	27520.44	7638.20	-2.2366 %	0.6241 %	2193.15	1078.70	-3.8238 %	0.0457%
	10	27394.58	7668.95	-2.6837%	1.0292 %	2192.23	1078.78	-3.8641 %	0.0525%

Table D.7: Results for the mean-risk models with ATTT for the SF network and $N = 200$

$\alpha = 0.8$ and Number of Scenarios=50									
Instance	Trade-off Par.	Fixed				Elastic			
Number	θ	$CVaR_\alpha$	Exp.	Relative Difference		$CVaR_\alpha$	Exp.	Relative Difference	
1A_1	0	494.50	200.88	$CVaR_\alpha$	Exp.	50.65	25.25	$CVaR_\alpha$	Exp.
	0.1	491.06	201.01	-0.69504%	0.0648%	50.64	25.254	-0.0148%	0.0162%
	1	475.86	201.91	-3.77006%	0.5166%	50.64	25.254	-0.0148%	0.0162%
	10	436.56	209.60	-11.7165%	4.3443%	48.18	26.03	-4.8813%	3.0825%
1A_2	0	484.58	195.65	$CVaR_\alpha$	Exp.	52.66	25.60	$CVaR_\alpha$	Exp.
	0.1	484.57	195.65	-0.0016%	0.0001%	52.66	25.60	0.0000%	0.0000%
	1	474.29	196.32	-2.1232%	0.3458%	52.22	25.75	-0.8341%	0.5541%
	10	426.10	207.04	-12.0671%	5.8220%	52.14	26.08	-0.9854%	1.8383%
$\alpha = 0.9$ and Number of Scenarios=50									
Instance	Trade-off Par.	Fixed				Elastic			
Number	θ	$CVaR_\alpha$	Exp.	Relative Difference		$CVaR_\alpha$	Exp.	Relative Difference	
1A_1	0	861.53	200.88	$CVaR_\alpha$	Exp.	82.32	25.25	$CVaR_\alpha$	Exp.
	0.1	845.13	201.19	-1.90355%	0.15388%	81.37	25.40	-1.15382 %	0.58467%
	1	819.46	201.85	-4.88295%	0.48455%	81.37	25.40	-1.1541%	0.58503%
	10	783.79	204.29	-9.02323%	1.70136%	76.85	26.42	-6.64405 %	4.61446%
1A_2	0	845.74	195.65	$CVaR_\alpha$	Exp.	82.60	25.60	$CVaR_\alpha$	Exp.
	0.1	845.74	195.65	0.0000%	0.0000%	82.60	25.60	0.0000 %	0.0000%
	1	785.55	195.91	-7.1170%	0.1356%	82.60	25.60	0.0000 %	0.0000%
	10	703.97	206.89	-16.7628%	5.7440%	76.73	25.63	-7.1049 %	0.0806%

Table D.8: Results for the mean-risk models with MUTT for the SF network for N=50

$\alpha = 0.8$ and Number of Scenarios=100									
Instance	Trade-off Par.	Fixed				Elastic			
Number	θ	$CVaR_\alpha$	Exp.	Relative Difference		$CVaR_\alpha$	Exp.	Relative Difference	
1A_1	0	488.21	198.35	$CVaR_\alpha$	Exp.	54.13	25.62	$CVaR_\alpha$	Exp.
	0.1	471.86	199.03	-3.34962%	0.34513%	54.06	25.62	-0.12359%	0.00062%
	1	471.86	199.03	-3.34962%	0.34513%	52.72	25.62	-2.61051%	0.00361%
	10	436.55	208.18	-10.582%	4.95585%	52.72	25.62	-2.61051%	0.00361%
1A_2	0	461.63	189.31	$CVaR_\alpha$	Exp.	46.25	23.89	$CVaR_\alpha$	Exp.
	0.1	454.51	189.45	-1.5423%	0.0748%	46.19	23.95	-0.1251%	0.2882%
	1	441.20	190.89	-4.4246%	0.8330%	46.18	23.96	-0.1528%	0.3030%
	10	416.56	195.78	-9.7636%	3.4199%	45.80	23.99	-0.9691%	0.4302%
$\alpha = 0.9$ and Number of Scenarios=100									
Instance	Trade-off Par.	Fixed				Elastic			
Number	θ	$CVaR_\alpha$	Exp.	Relative Difference		$CVaR_\alpha$	Exp.	Relative Difference	
1A_1	0	850.54	198.35	$CVaR_\alpha$	Exp.	82.53	25.62	$CVaR_\alpha$	Exp.
	0.1	822.43	198.71	-3.30529%	0.1814%	77.73	25.83	-5.81237%	0.81509%
	1	821.14	198.75	-3.45636%	0.20083%	77.30	26.62	-6.33549%	3.91789%
	10	818.81	198.92	-3.73032%	0.28966%	77.30	26.62	-6.33549%	3.91789%
1A_2	0	802.03	189.31	$CVaR_\alpha$	Exp.	74.21	23.89	$CVaR_\alpha$	Exp.
	0.1	783.30	189.50	-2.3351%	0.1014%	73.96	23.96	-0.3362%	0.3030%
	1	764.19	190.14	-4.7176%	0.4386%	73.96	23.96	-0.3362%	0.3030%
	10	702.12	195.15	-12.4574%	3.0853%	73.96	23.96	-0.3362%	0.3030%

Table D.9: Results for the mean-risk models with MUTT for the SF network for N=100

Appendix E

Percentages of the Total Shifted Demand to the Other Transportation Means

Results for the models with only the risk terms for the Nine Node (NN) Network

The results presented in this part are obtained according to the problem instance belongs to the Group 1 with random link capacities.

Probability Type	Number of Scenarios	$\alpha = 0.8$ Risk-averse Model	$\alpha = 0.8$ Risk-neutral Model	$\alpha = 0.9$ Risk-averse Model	$\alpha = 0.9$ Risk-neutral Model
Equal	N=10	34%	33%	35%	33%
	N=100	20%	32%	20%	32%
Different	N=10	34%	33%	35%	33%
	N=100	32%	32%	21%	32%

Table E.1: Average percentages of the total shifted demand to the other transportation means for the model with AUTT for the NN network

Probability Type	Number of Scenarios	$\alpha = 0.8$ Risk-averse Model	$\alpha = 0.8$ Risk-neutral Model	$\alpha = 0.9$ Risk-averse Model	$\alpha = 0.9$ Risk-neutral Model
Equal	N=10	26%	18%	29%	18%
	N=100	25%	29%	25%	29%
Different	N=10	27%	19%	28%	19%
	N=100	25%	29%	25%	29%

Table E.2: Average percentages of the total shifted demand to the other transportation means for the model with MUTT for the NN network

Probability Type	Number of Scenarios	$\alpha = 0.8$ Risk-averse Model	$\alpha = 0.8$ Risk-neutral Model	$\alpha = 0.9$ Risk-averse Model	$\alpha = 0.9$ Risk-neutral Model
Equal	N=10	24%	23%	26%	23%
	N=100	24%	22%	24%	22%
Different	N=10	28%	23%	24%	23%
	N=100	25%	23%	24%	23%

Table E.3: Average percentages of the total shifted demand to the other transportation means for the model with MTTT for the NN network

The following results are obtained according to the problem instance belongs to the Group 1 with random free-flow times.

Probability Type	Number of Scenarios	$\alpha = 0.9$ Risk-averse Model	Risk-neutral Model
Equal	N=50	36%	36%
	N=100	36%	33%
Different	N=50	44%	44%
	N=100	44%	44%

Table E.4: Average percentages of the total shifted demand to the other transportation means for the model with MUTT for the NN network

Probability Type	Number of Scenarios	$\alpha = 0.9$ Risk-averse Model	Risk-neutral Model
Equal	N=50	58%	58%
	N=100	57%	57%
Different	N=50	58%	58%
	N=100	57%	57%

Table E.5: Average percentages of the total shifted demand to the other transportation means for the model with MTTT for the NN network

The results presented in this part are obtained according to the problem instance belongs to the Group 1 with random free-flow times and link capacities.

Probability Type	Number of Scenarios	$\alpha = 0.9$ Risk-averse Model	Risk-neutral Model
Equal	N=50	50%	48%
	N=100	50%	50%
Different	N=50	50%	50%
	N=100	49%	49%

Table E.6: Average percentages of the total shifted demand to the other transportation means for the model with MUTT for the NN network

Probability Type	Number of Scenarios	α = 0.9 Risk-averse Model	Risk-neutral Model
Equal	N=50	50%	45%
	N=100	50%	47%
Different	N=50	50%	45%
	N=100	50%	46%

Table E.7: Average percentages of the total shifted demand to the other transportation means for the model with MTTT for the NN network

Results for the mean-risk models for the Nine Node (NN) Network

The following results are obtained according to the problem instance belongs to the Group 1 with only random link capacities.

Probability	Trade-off Par.	$\alpha = 0.8$		$\alpha = 0.9$	
Type	θ	N=10	N=100	N=10	N=100
Equal	0	33%	32%	33%	32%
	0.1	33%	32%	33%	32%
	1	33%	32%	34%	31%
	10	34%	32%	35%	20%
Different	0	33%	32%	33%	32%
	0.1	33%	32%	33%	32%
	1	34%	32%	33%	28%
	10	34%	32%	35%	23%

Table E.8: Average percentages of the total shifted demand to the other transportation means for the mean-risk model with AUTT for the NN network

Probability	Trade-off Par.	$\alpha = 0.8$		$\alpha = 0.9$	
Type	θ	N=10	N=100	N=10	N=100
Equal	0	18%	29%	18%	29%
	0.1	18%	28%	19%	28%
	1	25%	28%	20%	28%
	10	26%	25%	26%	27%
Different	0	19%	29%	19%	29%
	0.1	19%	28%	20%	27%
	1	22%	28%	22%	27%
	10	26%	25%	26%	26%

Table E.9: Average percentages of total shifted demand to other transportation means for the mean-risk model with MUTT for the NN network

Results for the models with only the risk terms for the Sioux Falls (SF) Network

Instance Number	Number of Scenarios	$\alpha = 0.8$		$\alpha = 0.9$	
		Risk-averse Model	Risk-neutral Model	Risk-averse Model	Risk-neutral Model
1A_1	N=50	34%	38%	34%	38%
	N=100	33%	38%	33%	38%
	N=200	34%	34%	33%	34%
1A_2	N=50	32 %	33%	34%	33%
	N=100	32%	33%	32%	33%
	N=200	33%	33%	33%	33%

Table E.10: Average percentages of the total shifted demand to the other transportation means for the model with AUTT for the SF network

Instance Number	Number of Scenarios	$\alpha = 0.8$		$\alpha = 0.9$	
		Risk-averse Model	Risk-neutral Model	Risk-averse Model	Risk-neutral Model
1A_2	N=50	36%	38%	34%	38%
	N=100	37%	38%	36%	38%
	N=200	37%	38%	37%	38%
2A_1	N=50	42%	43%	42%	43%
	N=100	43%	44%	43%	44%
	N=200	43%	44%	43%	44%

Table E.11: Average percentages of the total shifted demand to the other transportation means for the model with ATTT for the SF network

Instance Number	Number of Scenarios	$\alpha = 0.8$		$\alpha = 0.9$	
		Risk-averse Model	Risk-neutral Model	Risk-averse Model	Risk-neutral Model
1A_1	N=50	32%	32%	32%	32%
	N=100	33%	32%	34%	32%
	N=200	33%	32%	34%	32%
1A_2	N=50	36%	36%	37%	36%
	N=100	34%	34%	34%	34%
	N=200	35%	34%	34%	34%

Table E.12: Average percentages of the total shifted demand to the other transportation means for the model with MUTT for the SF network

Instance Number	Number of Scenarios	$\alpha = 0.8$		$\alpha = 0.9$	
		Risk-averse Model	Risk-neutral Model	Risk-averse Model	Risk-neutral Model
1A_2	N=50	31%	33%	31%	33%
	N=100	34%	35%	33%	35%
	N=200	36%	36%	35%	36%
2A_1	N=50	37%	37%	37%	37%
	N=100	38%	35%	38%	35%
	N=200	38%	39%	38%	39%

Table E.13: Average percentages of the total shifted demand to the other transportation means for the model with MTTT for the SF network

Instance Number	Number of Scenarios	$\alpha = 0.9$	
		Risk-averse Model	Risk-neutral Model
1B_2	N=100	65%	65%
	N=200	62%	62%
2B_1	N=100	68%	68%
	N=200	68%	68%

Table E.14: Average percentages of the total shifted demand to the other transportation means for the model with MUTT for the SF network

Instance Number	Number of Scenarios	$\alpha = 0.9$	
		Risk-averse Model	Risk-neutral Model
1B_2	N=100	64%	64%
	N=200	65%	65%
2B_1	N=100	65%	65%
	N=200	66%	66%

Table E.15: Average percentages of the total shifted demand to the other transportation means for the model with the MTTT for the SF network

Instance Number	Number of Scenarios	$\alpha = 0.9$	
		Risk-averse Model	Risk-neutral Model
1C_2	N=100	65%	67%
	N=200	68%	68%
2C_1	N=100	67%	68%
	N=200	68%	68%

Table E.16: Average percentages of the total shifted demand to the other transportation means for the model with MUTT for the SF network

Instance Number	Number of Scenarios	$\alpha = 0.9$	
		Risk-averse Model	Risk-neutral Model
1C_2	N=100	68%	67%
	N=200	68%	67%
2C_1	N=100	68%	68%
	N=200	66%	66%

Table E.17: Average percentages of the total shifted demand to the other transportation means for the model with the MTTT for the SF network

Results for the mean-risk models for the Sioux Falls (SF) Network

Instance Number	Trade-off Par. θ	$\alpha = 0.8$			$\alpha = 0.9$		
		N=50	N=100	N=200	N=50	N=100	N=200
1A_1	0	38%	38%	34%	38%	38%	34%
	0.1	38%	33%	34%	38%	33%	34%
	1	38%	33%	34%	38%	33%	33%
	10	34%	33%	34%	34%	33%	33%
1A_2	0	33%	33%	33%	33%	33%	33%
	0.1	33%	33%	33%	34%	33%	33%
	1	33%	32%	33%	34%	33%	33%
	10	32%	32%	33%	34%	32%	33%

Table E.18: Average percentages of the total shifted demand to the other transportation means for the mean-risk with AUTT for the SF network

Instance Number	Trade-off Par. θ	$\alpha = 0.8$			$\alpha = 0.9$		
		N=50	N=100	N=200	N=50	N=100	N=200
1A_2	0	38%	38%	38%	38%	38%	38%
	0.1	36%	38%	37%	38%	37%	37%
	1	36%	37%	37%	38%	37%	37%
	10	36%	37%	37%	36%	36%	37%

Table E.19: Average percentages of the total shifted demand to the other transportation means for the mean-risk model with ATTT for the SF network

Instance Number	Trade-off Par. θ	$\alpha = 0.8$			$\alpha = 0.9$		
		N=50	N=100	N=200	N=50	N=100	N=200
1A_1	0	32%	32%	32%	32%	34%	32%
	0.1	32%	32%	32%	33%	34%	32%
	1	32%	32%	32%	33%	34%	34%
	10	33%	32%	32%	33%	34%	34%
1A_2	0	36%	34%	34%	36%	34%	34%
	0.1	36%	34%	34%	36%	34%	34%
	1	36%	34%	35%	36%	34%	34%
	10	36%	34%	35%	37%	34%	34%

Table E.20: Average percentages of the total shifted demand to the other transportation means for the mean-risk model with MUTT for the SF network

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