

# Functional Observers for Motion Control Systems

This paper presents a novel functional observer for motion control systems to provide higher accuracy and less noise in comparison to existing observers. The observer uses the input current and position information along with the nominal parameters of the plant and can observe the velocity, acceleration and disturbance information of the system. The novelty of the observer is based on its functional structure that can intrinsically estimate and compensate the un-measured inputs (like disturbance acting on the system) using the measured input current. The experimental results of the proposed estimator verifies its success in estimating the velocity, acceleration and disturbance with better precision than other second order observers.

**Key words:** Motion Control, Disturbance Observer, Estimation, Acceleration Observer

**Croatian translation of the title.** If one of the authors is a native Croatian speaker, then please type the abstract translation in Croatian here. Otherwise, please leave this paragraph as it is. The editors will take care of the translation.

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**Ključne riječi:** ključna riječ 1, ključna riječ 2, ključna riječ 3

## 1 INTRODUCTION

The demand toward better measurement capabilities has been increasing recently with the advances in high precision applications of motion control systems. For any kind of application related to the research areas like force robotic manipulation or transportation and in particular for micro level applications like microassembly, micromachining or micromanipulation, one of the primary needs is to have a clear and accurate measurement of position, velocity and even acceleration of the corresponding system.

High precision position transducers like encoders and resolvers, are widely used as the means of position measurement both in industrial applications and in research. However, they are incapable of measuring the velocity of the system, which is a must in many areas of motion control. Generally in motion control systems, the measurements available to the controller are the input current to the system and position information from the encoder. The problem to obtain the real time velocity and acceleration data with the desired precision and low noise while maintaining a very large bandwidth sits in the middle of all motion control applications that require high performance. The standard approach is to use the first and second order

derivatives of position information of an incremental encoder and process the resulting data through a low pass filter. However this approach brings two disadvantages which are impossible to overcome simultaneously. With this classical structure, one either has to acquire a fast but very noisy data, or has to have a less noisy but sluggish data [1], [2]. The payoff between those two cases is determined by the cut-off frequency of the filter. In either case, the degradation in the performance of controllers might be problematic.

Many researchers analyzed this problem and tried to come up with fast and accurate estimators using different approaches. A primary solution for this problem is usually proposed with the use of a Kalman Filter. In [3] Kalman Filter is used to estimate the velocity and disturbance in low speed range. Although this approach is a good way to clear the noise in estimation, the computational cost might be problematic for cases where fast response in estimation is desired. Another study, which relies on the use of Extended Kalman Filter, implements the velocity estimation with current and DC voltage inputs of an induction motor [4]. A more recent example of Extended Kalman Filtering on velocity estimation can be found in [15]. On the

other hand, the major problem about the tuning of Kalman Filter parameters makes it difficult to use in many applications. The payoff stands between the convergence rate of the filter and ability to clear the noise. So, with Kalman Filter, one should either ignore to observe the very rapid changes and have a clear velocity estimate or to admit a fast response with more noise.

On the other hand, some researchers used the direct output of well known disturbance observer to estimate the velocity. In [5] the disturbance torque and the input current is used to observe the speed of the system. A similar methodology is performed by implementing a disturbance observer based full state observer algorithm to recover the dead time problem in estimation of low speed motion [6]. However, although disturbance observer is proven to be very useful for robust motion control [7], the observer structure intrinsically requires the velocity information of the plant which again requires the precise calculation of the system velocity. Besides, since the disturbance observer gives non-zero value for a scenario where there is non-zero current input and zero position change, this kind of approach might give a non-zero velocity value which can mislead the controllers using this information. In their study, Patten et al. proposed a structure to observe velocity based on optimal state estimation using input torque and position information [8]. Their work basically originates through closing the loop for velocity estimator. This way, even though the estimation result is accurate for low speeds, it is not fast enough to recover rapid fluctuations in velocity. In a recent study by Berducat et al. the speed information is obtained via an adaptive two level observer using estimation of friction model [9]. In [10] a novel approach is tried and the authors used adaptive fuzzy logic to realize the velocity observer. In this method, the fuzzy controller adopts the disturbance acting on the plant and hence it can perform very good in eliminating the noise in the estimation. However, this approach can lose reliability where there is rapid change of disturbance acting on the system. [11] presents another speed estimation method based on a model reference adaptive scheme that can recover mechanical inertia time for changing load. More information about velocity and acceleration estimators can be found in [12], [13], [14], and [16].

In this paper, a novel observer is presented that provides functional structure which, by changing a few parameters, can be used for estimating the velocity or acceleration of a system or the disturbance acting on that system. The presented work is an extension of the study given in [17] providing further proofs over the previously proposed structure. The organization of the paper is as follows. In Section-2 the definition of the problem is given with background information about the system under consideration. In Section-3 the mathematical derivation of the

functional observer is made. In Section-4 the sensitivity analysis of the proposed observer for varying system parameters is handled. Section-5 presents the experimental results. Discussion about the results and concluding remarks are given in Section-6 and Section-7 respectively.

## 2 PROBLEM DEFINITION

Throughout the analysis presented in the next section, design of the observer will be made on a single degree of freedom (DOF) motion control system. The generalized depiction of a single DOF motion control system is given in Fig. 1. In that structure,  $I_{ref}(s)$  and  $T_{dis}(s)$  stand for the

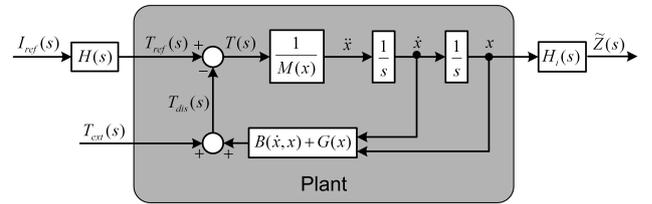


Fig. 1. Structure of a motion control system with ideal observer

Laplace Transformed reference input current and disturbance torque acting on the system respectively. The feedback terms  $B(\dot{x}, x)$  and  $G(x)$  represent the respective actions of viscous friction and gravity over the system. In this generalized structure, the reference input torque  $T_{ref}(s)$  to the system is given by a transfer function from the input current as follows;

$$T_{ref}(s) = H(s)I_{ref}(s)$$

where  $H(s)$  is the transfer function mapping the reference input current to the reference input torque. Ideally, this mapping is given by a constant gain and hence the system input takes the form;

$$T_{ref}(s) = K_n I_{ref}(s) \quad (1)$$

with  $K_n$  being named as the nominal torque constant. The second order plant can be represented with a transfer function  $R(s)$  from the total input torque  $T(s)$  to the generalized coordinate of motion  $X(s)$  by;

$$R(s) = \frac{X(s)}{T(s)} = \frac{1}{M(x)s^2} \quad (2)$$

where,  $M(x)$  stands for the plant inertia. Assuming that the plant inertia shows small variations around a nominal value,  $M(x)$  can be replaced with the nominal inertia value  $M_n$ . In equation (2),  $T(s)$  is the summation of all inputs acting on the system (i.e.  $T(s) = T_{ref}(s) - T_{dis}(s)$ ). So,

the output  $X(s)$  of the structure given in Fig. 1 can be written as;

$$X(s) = R(s) (K_n I_{ref}(s) - T_{dis}(s)) \quad (3)$$

In equation (3), it is assumed that the term  $T_{dis}(s)$  lumps all inputs other than the reference torque  $T_{ref}(s)$ . In that sense,  $T_{dis}(s)$  contains the torques due to; viscous friction  $B(\dot{x}, x)$ , deviations from the nominal values of torque constant  $\Delta K_n I_{ref}(s)$  and inertia  $\Delta M_n s^2 X(s)$ , gravity  $G(x)$  and all other non-modeled external torques  $T_{ext}(s)$ . This way, the content of the disturbance torque can be given as,

$$T_{dis}(s) = \Delta M_n s^2 X(s) + \Delta K_n I_{ref}(s) + B(\dot{x}, x) + G(x) + T_{ext}(s) \quad (4)$$

In order to acquire measurements of the system, one has to incorporate the plant output,  $X(s)$  with a transfer function. In the structure shown in Fig. 1,  $\tilde{Z}(s)$  is the variable of interest that is related to the plant output by the ideal (not necessarily realizable) transfer function  $H_i(s)$  (i.e.;  $\tilde{Z}(s) = H_i(s)X(s)$ ). If the actual value of the variable of interest  $Z(s)$  cannot be directly measured, then  $H_i(s)$  stands for the ideal transfer function of the observer that needs to be designed. However, the content of this observer may not be physically realizable if  $H_i(s)$  is an improper transfer function like  $\eta_1 s^2 + \eta_2 s$  (i.e. a linear combination of acceleration and velocity). Moreover, direct differentiation would yield a correct result only when there was an ideal double integrator system. Since the system is subject to non-ideality (i.e.  $T_{dis}(s) \neq 0$ ) the double integrator assumption is degenerated and the actual value of the variable of interest should contain additional term coming from the disturbance. Without loss of generality, one can assume that the disturbance term is transferred to the actual output by a transfer function  $H_d(s)$  and hence the actual output of the plant gets the following form:

$$Z(s) = \tilde{Z}(s) + H_d(s)T_{dis}(s) \quad (5)$$

As a remedy to the improper structure of the ideal observer, one can make use of the reference current measurement with the ability to observe the variable of interest through integration rather than differentiation. Hence, the reference current measurement can be fused with the position measurement to remove the effect of phase delay in differentiation. Having this in mind, one can utilize an approximate observer structure as shown in Fig. 2 and come up with an estimate of the output  $Z(s)$ . In designing the observer, the main criteria is to select the error between the actual value  $Z(s)$  and the estimation  $\hat{Z}(s)$  to have a desired magnitude of zero.

Now the problem can be formulated as follows: For the system given in Fig. 2, using the nominal plant parameters and measurable outputs (i.e.  $I_{ref}(s)$  and  $X(s)$ ), find

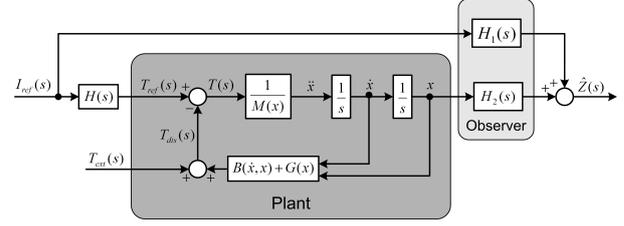


Fig. 2. Proposed observer structure

transfer functions  $H_1(s)$  and  $H_2(s)$  that would approximate variable of interest  $Z(s)$  with error  $H_d(s)T_{dis}(s)$  due to unmeasurable and unknown plant input.

### 3 OBSERVER CONSTRUCTION

Using equation (5) and the structure shown in Fig. 2, one can write the actual and the estimated values of  $Z(s)$  as follows:

$$\begin{aligned} Z(s) &= H_i(s)X(s) + H_d(s)T_{dis}(s) \\ Z(s) &= H_i(s)R(s) \{H(s)I_{ref}(s) - T_{dis}(s)\} + H_d(s)T_{dis}(s) \end{aligned} \quad (6)$$

$$\begin{aligned} \hat{Z}(s) &= H_2(s)X(s) + H_1(s)I_{ref}(s) \\ \hat{Z}(s) &= H_2(s)R(s) \{H(s)I_{ref}(s) - T_{dis}(s)\} + H_1(s)I_{ref}(s) \end{aligned} \quad (7)$$

From (6) and (7), one can write the error in the estimation as follows:

$$\begin{aligned} \Delta Z &= Z - \hat{Z} \\ \Delta Z &= \{RH(H_i - H_2) - H_1\} I_{ref} \\ &\quad - \{R(H_i - H_2) - H_d\} T_{dis} \end{aligned} \quad (8)$$

where, in (8), all terms are functions of  $s$ . The difference between desired output  $Z(s)$  and its estimated value  $\hat{Z}(s)$ , as expressed in (8) depends on both control input and the disturbance. In order to push this estimation error to zero, coefficients of both current ( $I_{ref}(s)$ ) and disturbance ( $T_{dis}(s)$ ) should be imposed to have zero value. Letting those coefficients be equal to zero and solving further, one finds the following two equations for the transfer functions  $H_1(s)$  and  $H_2(s)$ ;

$$\begin{aligned} H_1(s) &= H(s)H_d(s) \\ H_2(s) &= H_i(s) - R^{-1}(s)H_d(s) \end{aligned} \quad (9)$$

The assumption made in (1) saying that the torque can be transmitted to the plant with a constant gain (i.e.  $H(s) = K_n$ ) results in  $H_1(s)$  being equal to a scalar multiple of  $H_d(s)$ . This result is very important since it implies that the error due to disturbance is compensated by

the current input during estimation. In other words, the observer, while using position information and transfer function  $H_2(s)$  to acquire the estimated value, also uses the current information and transfer function  $H_1(s)$  along with the nominal parameters of the plant to cancel the effect of disturbance in estimation.

In order to solve for  $H_1(s)$  and  $H_2(s)$  we can define a generalized transfer function for  $H_d(s)$ . Since the disturbance acting on the system pass through a second order dynamics, we can formulate this generalized transfer function as follows;

$$H_d(s) = \frac{g^2 s(\gamma s + \delta)}{M_n (s + g)^2} \quad (10)$$

where,  $\gamma$  and  $\delta$  are two unknown parameters which need to be solved for the variable of interest to be estimated,  $M_n$  is the nominal inertia of the plant and  $g$  is the cut-off frequency of the low pass filter to be used in realizing the disturbance transfer function. Using this error, the expression for  $R(s)$  from (2) and equation (9), generalized forms for the transfer functions  $H_1(s)$  and  $H_2(s)$  can also be defined;

$$H_1(s) = \frac{K_n g^2 s(\gamma s + \delta)}{M_n (s + g)^2} \quad (11)$$

$$H_2(s) = H_i(s) - \frac{g^2 s^3(\gamma s + \delta)}{(s + g)^2} \quad (12)$$

In both of the equations (11) and (12), the coefficients  $g$ ,  $\gamma$  and  $\delta$  should be selected in design process. In order to design the parameters, we have to refer to the format of the ideal transfer function  $H_i(s)$ . Let the ideal transfer function be  $H_i(s) = \alpha s^2 + \beta s$ ; in other words let us assume that a linear combination of velocity and acceleration is to be estimated. Substituting  $H_i(s)$  into (12), one can obtain;

$$H_2(s) = (\alpha s^2 + \beta s) - \frac{g^2 s^3(\gamma s + \delta)}{(s + g)^2}$$

which can be expanded further as follows,

$$H_2(s) = \frac{C_4 s^4 + C_3 s^3 + C_2 s^2 + C_1 s}{(s + g)^2} \quad (13)$$

where, the coefficients are

$$\begin{aligned} C_4 &= \alpha - g^2 \gamma \\ C_3 &= 2g\alpha - g^2 \delta + \beta \\ C_2 &= 2g\beta + g^2 \alpha \\ C_1 &= g^2 \beta \end{aligned}$$

Since, for a physical system, the estimator will have at most second degree derivative, we can set the coefficients

of  $s^4$  and  $s^3$  terms ( $C_4$  and  $C_3$ ) be equal to zero, which gives;

$$\alpha - g^2 \gamma = 0 \implies \gamma = \frac{\alpha}{g^2} \quad (14)$$

$$2g\alpha - g^2 \delta + \beta = 0 \implies \delta = \frac{\beta + 2g\alpha}{g^2} \quad (15)$$

Substituting (14) and (15) into (11) and (12) gives the following set of transfer functions:

$$\begin{aligned} H_1(s) &= \frac{K_n \alpha s^2 + (\beta + 2g\alpha)s}{M_n (s + g)^2} \\ H_2(s) &= g s \frac{(g\alpha + 2\beta)s + g\beta}{(s + g)^2} \\ H_i(s) &= \alpha s^2 + \beta s \end{aligned} \quad (16)$$

Now, the only design parameters are  $\alpha$  and  $\beta$  which is determined from the structure of the ideal observer  $H_i(s)$ . Due to the selected structure of disturbance transfer function ( $H_d(s)$ ), the functional observer can be realized using just two first order filters as depicted in Fig. 3.

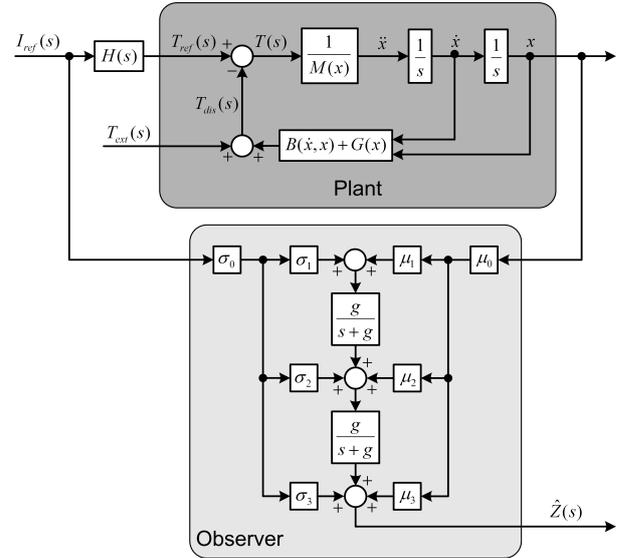


Fig. 3. Block diagram of functional observer

This structure mathematically imposes the following two equations.

$$H_1(s) = K_n \frac{\alpha s^2 + (\beta + 2g\alpha)s}{M_n (s + g)^2} = \sigma_0 \left( \sigma_3 + \frac{\sigma_2 g}{(s + g)} + \frac{\sigma_1 g^2}{(s + g)^2} \right) \quad (17)$$

$$H_2(s) = \frac{(g^2 \alpha + 2g\beta)s^2 + g^2 \beta s}{(s + g)^2} = \mu_0 \left( \mu_3 + \frac{\mu_2 g}{(s + g)} + \frac{\mu_1 g^2}{(s + g)^2} \right) \quad (18)$$

The values for gains  $\sigma_i$  and  $\mu_i$  ( $i = 0, 1, 2, 3$ ) can be found by substituting the necessary numbers for  $\alpha$  and  $\beta$  to

the ideal observer  $H_i(s)$ . A summary of the coefficients for velocity, acceleration and disturbance estimation is given in Table 1

Table 1. Parameters of the Functional Observer for Different Configurations

$H_i$	$0s^2 + s$	$s^2 + 0s$	$K_n I_{ref} - M_n s^2$
$(\hat{z})$	$(\hat{x})$	$(\hat{x})$	$(\tau_{dis})$
-----	-----	-----	-----
$\sigma_0$	$\frac{K_n}{gM_n}$	$\frac{K_n}{M_n}$	$-K_n$
$\sigma_1$	$-1$	$-1$	$-1$
$\sigma_2$	$1$	$0$	$0$
$\sigma_3$	$0$	$1$	$0$
$\mu_0$	$g$	$g^2$	$-M_n g^2$
$\mu_1$	$1$	$1$	$1$
$\mu_2$	$-3$	$-2$	$-2$
$\mu_3$	$2$	$1$	$1$

#### 4 PARAMETER VARIATION ANALYSIS

In order to have a complete analysis of the given structure, it is important to analyze the response of the observer with respect to the variations in the system parameters. Recalling from equation (8),  $H_i(s)$  and  $H_d(s)$  are the transfer functions which map the input and the disturbance to the output and hence does not include any system dependent parameters. Moreover, transfer functions  $H_1(s)$  and  $H_2(s)$  are derived based on the zero error solution of the proposed estimator (offline) using the nominal system parameters, which means that they also do not show variation. The only remaining source of variation in the system parameters exist either from  $R(s)$  or from  $H(s)$ . We can now proceed to analyze them further.

Let us suppose that the original value of plant transfer function is  $\bar{R}(s) + \Delta R(s)$  while the observer assumes it as  $\bar{R}(s)$  with bar representing the assumed nominal value. Inserting this original value into equation (8), the error in estimation becomes;

$$\begin{aligned} \Delta Z_R(s) &= \{ \Delta R(s) \bar{H}(s) (\bar{H}_i(s) - \bar{H}_2(s)) \} I_{ref}(s) \\ &- \{ \Delta R(s) (\bar{H}_i(s) - \bar{H}_2(s)) \} T_{dis}(s) \end{aligned} \quad (19)$$

where, in (19) the transfer functions with bar represent the ones constructed assuming the nominal system parameters. Looking at the structure of this equation, it is obvious that the variations in the plant inertia are reflected both in mapping from input current and from disturbance to the output.

Now let us suppose that the original value of transfer function that maps current to the plant is  $\bar{H}(s) + \Delta H(s)$  while the observer assumes it as  $\bar{H}(s)$  with bar representing the assumed nominal value. Inserting this original value into equation (8), the error in estimation becomes;

$$\Delta Z_H(s) = \{ \Delta H(s) \bar{R}(s) (\bar{H}_i(s) - \bar{H}_2(s)) \} I_{ref}(s) \quad (20)$$

For the selection of  $H(s)$  and  $R(s)$ , there are two possible sources of uncertainty. Either one or both of the two nominal plant parameters (i.e.  $K_n$  and/or  $M_n$ ) might be assumed different from their respective true values. The following subsections analyze the independent effects of variations in any of those two parameters.

##### 4.1 Response with respect to fluctuations in nominal inertia

Assuming that the original value of nominal system inertia is  $M_n + \Delta M$  while the observer assumes the system has nominal inertia  $M_n$ , one can write down;

$$\Delta R(s) = - \frac{\Delta M}{M_n(M_n + \Delta M)s^2}$$

The effect of this difference in the estimation can best be seen on a bode plot which reflects the transfer function  $\Delta Z_R(s)/Z(s)$ , where  $Z(s)$  is the actual output of the estimator given in (6). Equation (19) is a function of both input current and disturbance. Hence, the plotted response is a mapping from those two inputs to the output (i.e. the change in the response of the variable of interest). The responses are obtained with a variation of %10 in the nominal inertia and with the selection of cut-off frequency  $g = 1000$  Rad/s.

The bode plots given in Fig. 4 show that %10 change in parameters is reflected to the output only for frequencies higher than the cut off frequency. For range of operation with lower frequencies than the selected cut-off frequency, the variation of system inertia from its respective nominal value is tolerated by the observer and is not reflected in the output for the estimation of velocity. On the other hand, the bode plots shown in Fig. 5 points out a similar situation for the estimation of acceleration. One important indication in both bode plots is that, for applications over selected cut-off frequency, the effect of disturbance on the estimation is augmented. Hence, in order to get the best performance out of the proposed observer, the frequency  $g$  of the proposed observer should be selected as high as possible.

##### 4.2 Response with respect to fluctuations in nominal torque constant

A similar analysis can be carried out to see the effect of changes in the nominal torque constant. Supposing that the original value of nominal torque constant is

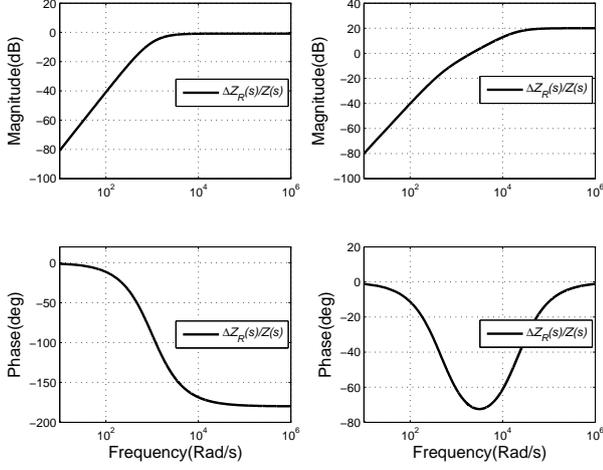


Fig. 4. Effect of %10 change in the nominal inertia on the estimation of velocity. Mapping from input current is shown on the left column while mapping from disturbance is shown on the right column

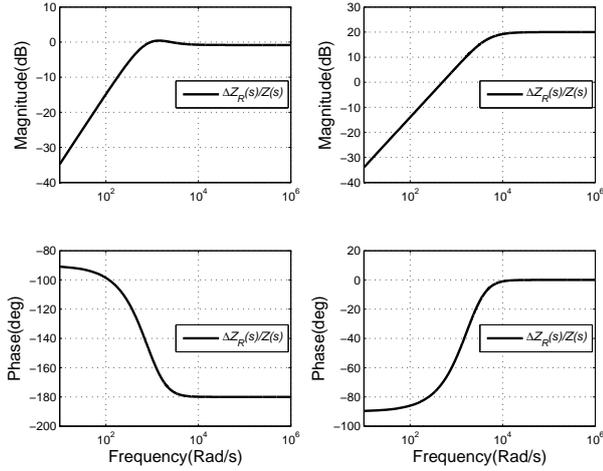


Fig. 5. Effect of %10 change in the nominal inertia on the estimation of acceleration. Mapping from input current is shown on the left column while mapping from disturbance is shown on the right column

$K_n + \Delta K$  while the observer assumes the system has a nominal torque constant value of  $K_n$ , one can write down;

$$\Delta H(s) = \Delta K$$

Once again, frequency response is used to visualize the difference in the estimation. The transfer function used in the bode plots given below is  $\Delta Z_H(s)/Z(s)$ , where  $Z(s)$  is the actual output of the estimator given in (6). Since equation (20) is only a function of the input current, the plotted response is a mapping only from input current to

the output. The frequency responses shown below is obtained with a variation of %10 in the nominal torque constant. Results obtained for the relative changes in the estimation of velocity and relative changes in the estimation of acceleration is given in Fig. 6 and Fig. 7 respectively.

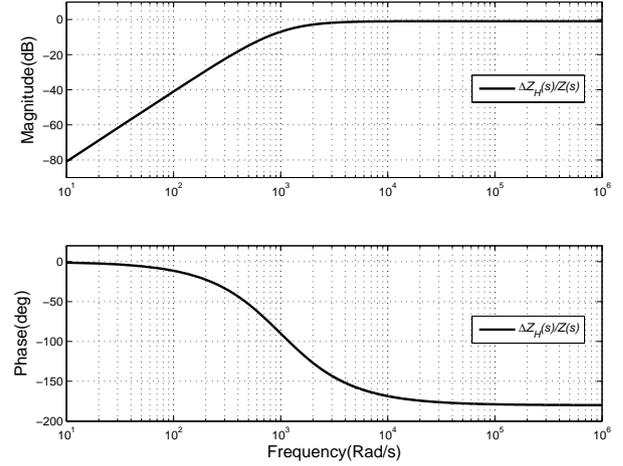


Fig. 6. Effect of %10 change in the nominal torque constant on the estimation of velocity

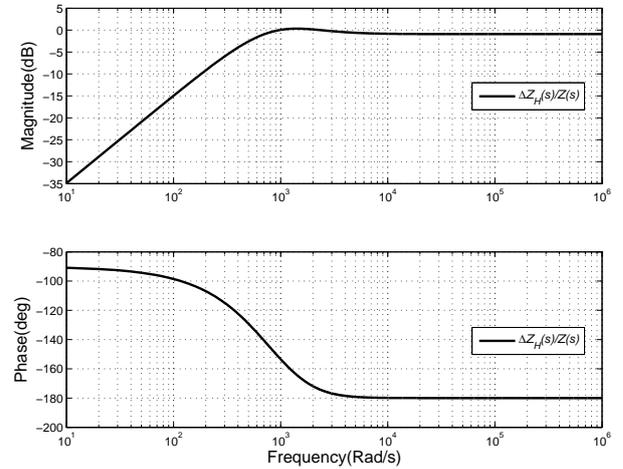


Fig. 7. Effect of %10 change in the nominal torque constant on the estimation of acceleration

The bode plots indicate that similar to the responses obtained based on the variation of inertia, the changes in the nominal torque constant is tolerated for the operational frequencies lower than the cut-off frequency.

## 5 EXPERIMENTS

Series of experiments were conducted in order to verify the proposed functional observer. As an experimental

setup one Hitachi-ADA series linear motor and driver stage was used. The stage prepared for the setup provides motion in single dimension and is designed using brushless, high-precision direct drive linear servomotors. Position feedback to the motion stage is obtained from an incremental optical encoder with a resolution of  $1\mu m$ . The stage is controlled by the modular Dspace control system DS1005 that features a PowerPC 750GX processor running at 1 GHz. Control system features the 24-bit encoder signal processing card and 16-bit DA card. MATLAB-Simulink environment is used for the implementation of the functional observer algorithms. Picture of experimental setup is shown in Fig. 8. The verification of the proposed estimator is done

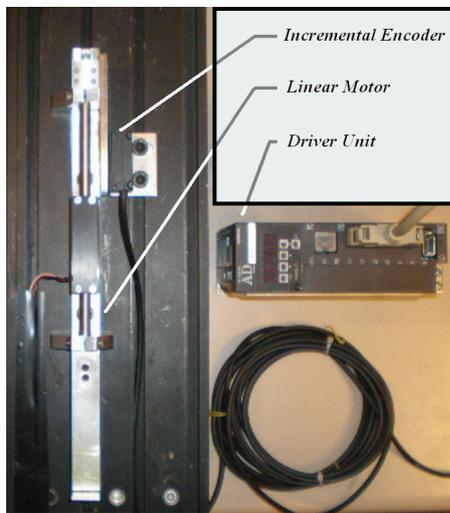


Fig. 8. Picture of the Experimental Setup

with different experiments for velocity, acceleration and disturbance. The following subsections discuss the details and results of the experiments for different observer configurations.

### 5.1 Estimation of Velocity

In order to present the velocity estimation results three different observers were implemented and tested with the same reference. Trapezoidal velocity reference is imposed to the plant and the response is recorded. The rising and falling edges of the reference have  $0.02m/s^2$  slope with a peak constant velocity of  $0.01m/s$ . The velocity estimation results for this experiment are provided in Fig. 9. Among the given velocity responses; (a) is the response of filtered differentiation using  $2^{nd}$  order low pass filter (i.e. two cascaded first order low pass filters), (b) is the response of filtered differentiation using using a Butterworth filter and (c) is the response of proposed functional observer. All of the observers have cutoff frequency of

159.24 Hz (i.e. 1000 Rad/s). As the graphs show, the performance of the proposed functional observer in estimating the velocity is much better than filtered differentiation. Moreover, although internally the structure of the proposed functional observer includes two cascade filters, it can still outperform the estimation results obtained via using a Butterworth type second order filter with direct differentiation.

The reduction in the noise level is also measured numerically for the experiments. In that sense, the signal to noise ratio (SNR) is calculated for the acquired velocity profiles. In the calculation of SNR, the ratio of mean to standard deviation of the measured response (normalized to the given reference) is used. The calculated SNRs came out to be 13.305, 19.380 and 21.879 for the experiments given in parts (a), (b) and (c) respectively. Numerical results for the improvement in signal power proves the success of functional observer.

### 5.2 Estimation of Acceleration

The acceleration estimation results are tested with a different experiment. In acceleration experiment, consecutive positive and negative pulse references are given to the system and the estimation responses are recorded. The amplitude of the pulse reference was selected to be  $15m/s^2$ . The results of the proposed observer are compared to the results obtained from the double differentiation using using Chebyshev 0.5dB filter. In order to have a better comparison of the observed accelerations, one needs the actual acceleration response of the system. For that purpose, the position data obtained from the optical encoder is double differentiated in an offline setting and shown on the same plot. For offline numerical differentiation, the three-point estimation approach is utilized.

The acceleration estimates of the functional observer and filtered double differentiator are given in Fig. 10 along with the actual acceleration response. For both observers, the low-pass filter gains are selected to be 159.24 Hz. When the results are compared, it becomes obvious that the tracking performance of the functional observer is much better than that of the double differentiation using Chebyshev 0.5dB filter. Those graphs show the effectiveness of the implemented methodology, namely using current input in estimation to eliminate the unmeasured disturbances.

### 5.3 Estimation of Disturbance

For comparison of disturbance estimation responses, a constant velocity reference tracking experiment is done. During the experiment, output of classical disturbance observer is compared to that of functional observer. Fig. 11 show the disturbance estimation results for the proposed functional observer and classical disturbance observer respectively. Like the velocity observers, the functional disturbance observer is capable of making the same estimation with less noise in comparison to classical disturbance

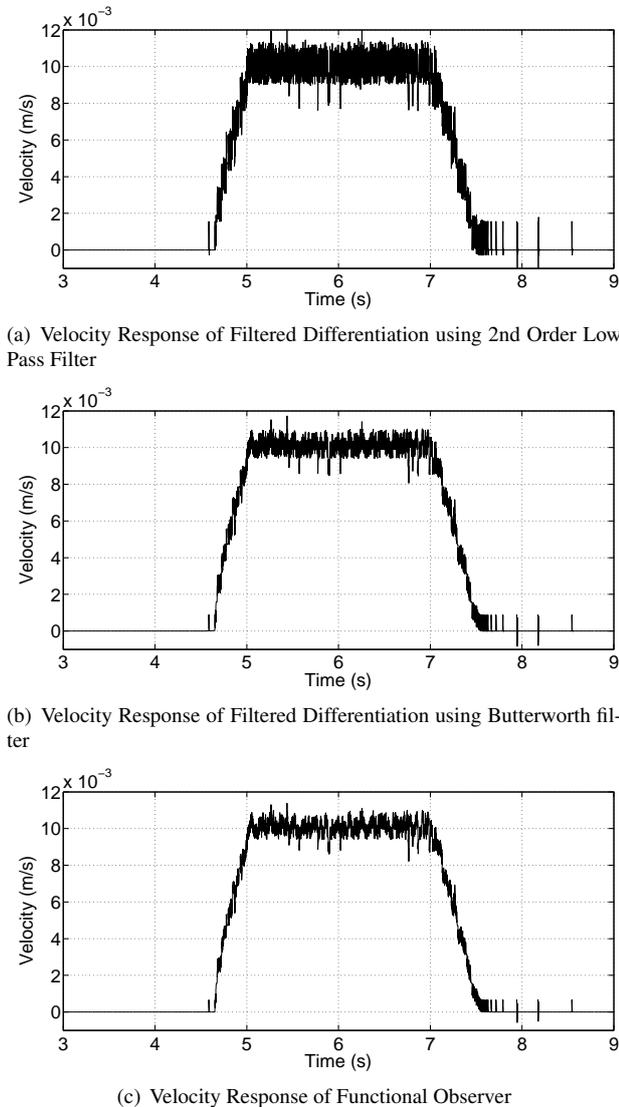


Fig. 9. Comparison of Velocities from (a) Filtered Differentiation using 2<sup>nd</sup> Order LPF, (b) Filtered Differentiation using Butterworth Filter and (c) Functional Observer, Under Trapezoidal Velocity Reference

observer. The SNRs for estimated disturbances are calculated to be 6.625 and 7.15 for classical disturbance observer and functional disturbance observer respectively.

## 6 DISCUSSION

Proposed functional observer is useful for obtaining accurate and low noise level velocity estimation. These characteristics make the functional observer preferable over conventional filtered derivative methods. Smoother velocity estimation brings the advantage of acquiring higher precision in many motion control systems. Moreover, the estimation in velocity is as fast as the classical estimators. In

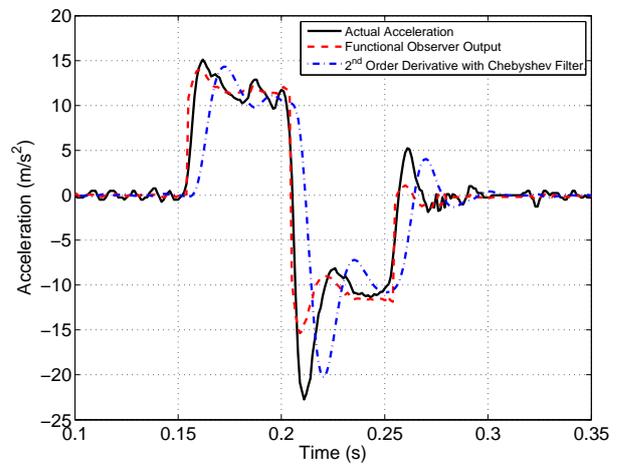


Fig. 10. Comparison of Accelerations from Functional Observer and Double Differentiation using Chebyshev 0.5dB Filter Under Constant Acceleration Reference

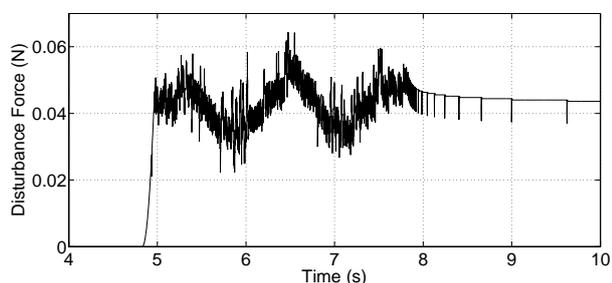
other words, noise in estimation is reduced considerably while the bandwidth of operation remains the same.

Besides velocity, much faster and more accurate acceleration estimation can be made with the proposed functional observer in comparison to filtered double differentiators. Although the acceleration information is usually not directly used in motion control systems, in many settings it is used as the feed forward term. Having faster response in acceleration estimation would decrease the integration error resulting in a better controller performance.

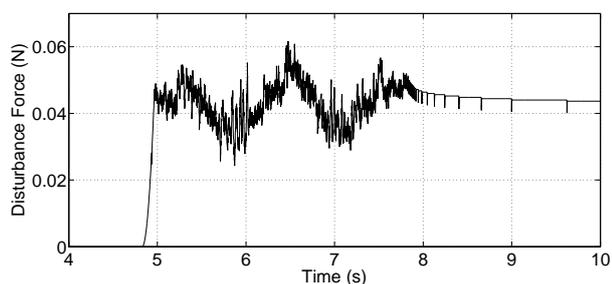
Concerning the disturbance observer in motion control systems, usually wide bandwidth operation is very crucial for the robustness of the system. Instead of using a double filtered estimation, use of classical disturbance observer might still perform better in control loop due to having a single filter and hence a little faster response time. However, smoother disturbance estimation from the functional observer can be a better candidate for external torque/force reconstruction.

## 7 CONCLUSION

In this paper, a functional observer is presented. The observer is capable of estimating the velocity, acceleration and disturbance information of a motion control system only by a change in the configuration parameters. In addition to the position measurement, the estimator benefits from estimating and eliminating the disturbance effects by using the measured input current and plant's nominal parameters. The theoretical development of the estimator has been validated through experiments.



(a) Disturbance Estimation Response of Classical Observer



(b) Disturbance Estimation Response of Functional Observer

Fig. 11. Comparison of Disturbances from (a) Standard Disturbance Observer and (b) Functional Observer, Under Constant Velocity Reference

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