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Sliding-Mode Control of a Flexure Based Mechanism Using Piezoelectric Actuators

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Abstract—**The position control of designed 3 PRR flexure based mechanism is examined in this paper. The aims of the work are to eliminate the parasitic motions of the stage, misalignments of the actuators, errors of manufacturing and hysteresis of the system by having a redundant mechanism with the implementation of a sliding mode control and a disturbance observe. x-y motion of the end-effector is measured by using a laser position sensor and the necessary references for the piezoelectric actuators are calculated using the pseudo inverse of the transformation matrix coming from the experimentally determined kinematics of the mechanism. The effect of the observer and closed loop control is presented by comparing the results with open loop control. The system is designed to be redundant to enhance the position control. In order to see the effects of the redundant system firstly the closed loop control for active 2 piezoelectric actuators experiments then for active 3 piezoelectric actuators experiments are presented. As a result, our redundant mechanism tracks the desired trajectory accurately and its workspace is bigger.**

Keywords-compliant mechanism, sliding mode control, piezoelectric actuator control, flexure based mechanism, observer.

I. INTRODUCTION

The precision engineering concept is derived from mechanical engineering and followed by micro technology and recently nanotechnology. The developments in the field of micro and nano technologies initiated a new market for smaller and high precision positioning devices. These devices are needed in various high precision applications such as cell manipulation, surgery, aerospace, micro fluidics, optical systems and micro assembly applications. The need of increased accuracy and precision requires the development of design and control methods simple enough that can be used in engineering practice. Traditional rigid body mechanisms start not to provide needed accuracy and precision. Then high precision mechanisms with flexible joints are designed in which flexible joints transfer necessary motion or force in the mechanism. The desired motion is provided with the deflection of these flexible joints also called in the literature as “flexures” which provide high resolution, frictionless, smooth and continuous motion. These kinds of mechanisms are also cheaper than the other types of high precision mechanisms which use rigid joints [1].

Despite all the advantages that flexures bring, there are also drawbacks. The major one is the complexity of modeling of these mechanisms which leads to be hard to control its position because of being lack of computing the accurate model. So a usable method should be defined for controlling the mechanism or the control should eliminate the nonlinearities of the mechanism. There are researches going on simplifying the models that can be computed while real time control is running. The most popular one is pseudo rigid body model which computes the stiffness value of the flexures that are equivalent to joints with torsional springs and rest of the mechanism is treated as a rigid body mechanism [1]. Howell and Mathilda have developed loop closure theory which uses the complex number method to model the mechanism [2]. Handley et al. have used this model to make the position control of the mechanism [3]. A linear scheme method is presented by Her and Chang for the displacement analysis of micropositioning stages which linearize the geometric constraint equations of the stages [4]. Zhang et al. developed the work and came up with the idea of constant jacobian method for computing the kinematics of the mechanism [5]. These methods have been used for the PID control of a 3RRR flexure based mechanisms. [6]. Goldfrab has made the position control simulation of a compliant mechanism by using a sliding control [7]. A four bar mechanism is designed for micro/nano manipulation and a robust adaptive control methodology is applied by Liaw et al [8]. Another adaptive control has been used by Shieh and Huang to emulate the unwanted behaviors of the mechanism [9]. Chang et al. have designed a x-y-θ_z piezo micropositioner and used a feedback control to eliminate the hysteresis, nonlinearity and drift of piezoelectric effects [10].

In this work, instead of using a nonlinear model that mimics the behaviour of the flexure based mechanism developed, a control method is proposed to get rid of the unwanted motions. The implemented position control of designed flexure based parallel mechanism is sliding mode control by using end-effector measurement. To eliminate the parasitic motions of the stage, misalignments of the actuators, errors of

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manufacturing and hysteresis of the system, a redundant mechanism and an observer is designed for the position control. The mechanism is actuated using 3 piezoelectric actuators to achieve 2 dof motion having redundancy within itself. We have experimentally determined the kinematics of the system which indicates the relationship between the displacement of the actuators and the end-effector displacements in x-y axes. Closed loop sliding mode control with disturbance observer is designed to eliminate the unwanted and unknown motions. Open loop control for the linear model of the piezoelectric actuators is implemented to compare the results with closed loop control. In the closed loop experiments firstly 2 piezoelectric actuators are used then 3 piezoelectric actuators are controlled to see the effects of redundant control.

In section II the flexure based micromotion stage is introduced, the experimental setup is presented in sec. III and the transformation matrix which connects the end-effector x-y motion with the displacement vectors coming from piezoelectric actuators is shown in section IV. The modeling of the piezoactuators and the control design is explained in sec. V. The results for open loop control and the closed loop controls for using 2 and 3 piezoelectric actuators are presented in sec. VI. Finally in sec. VII a conclusion have been made based on the results.

II. Flexure Based Micro Motion Stage Design

We have used a three DOF parallel mechanism called 3PRR in Figure 1a (one prismatic – two revolute joints for each link) concept for our micromotion stage design. Circular notch hinges are used for revolute joints and simple linear spring structures based on circular flexure hinges are used for prismatic joints as shown in Figure 1b. The Stage is actuated by piezoelectric actuators which drives the prismatic joints. The end-effector of the mechanism is the triangular stage which connects the three links and has motion in x-y directions and a rotation about z-axis. A case is also designed outside the mechanisms range so that it can be fixed to the experimental setup properly.

The mechanism is manufactured by using wire electrical discharge machining (Wire EDM) technique by using Aluminium 7075 for the material and the shortest thickness of the flexure is 0.8 mm. The overall thickness of the mechanism in z axis is 10 mm.

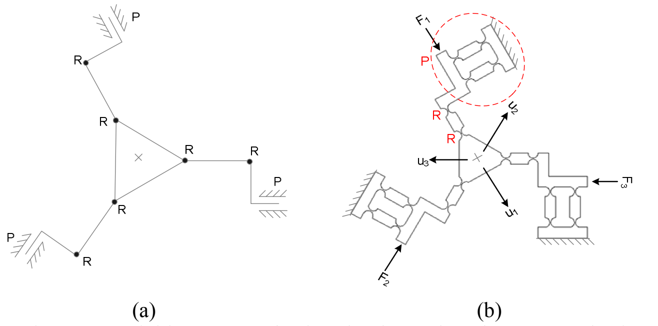


Figure 1. a. Rigid 3 PRR mechanism, b. Flexure based 3 PRR mechanism

III. The Experimental Setup

The setup shown in Figure 3 is composed of the mechanism, three piezoelectric actuators, a base table, three sliding stages with micrometers, a laser position sensor and a middle base. The piezoelectric motor used is piezomechanik's PST 150/5/40 VS10 type which has max stroke 55 µm for semibipolar -30 V/ +150 V activation and 40 µm stroke for unipolar 0V/+150V activation. Piezomechanik's analog amplifier SVR 150/3 is also used for actuating the piezos. PI's P-853 piezoelectric micrometer drives with sliding stages are put in x and y directions according to the links of the mechanisms so that we can manually preload the mechanism and drive the prismatic joints correctly. For the measurement a DL 16-7PCBA3 4mm x 4mm dual axis position sensing diode on a PCB is placed on the triangular effector's center. Meβtechnologie's laser source is assembed on the top of the position sensing diode.

The piezo amplifiers inputs and the laser dual axis position outputs are connected to dSPACE 1103 controller board through DACs and ADCs. Control Desktop is used for CPU calculations for the controller. The schematic diagram of the connections is shown in Figure 2.

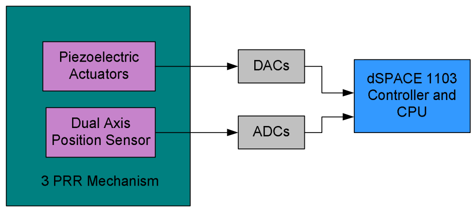


Figure 2. Schematic diagram of the connection of sensors and actuators with the controller and CPU.

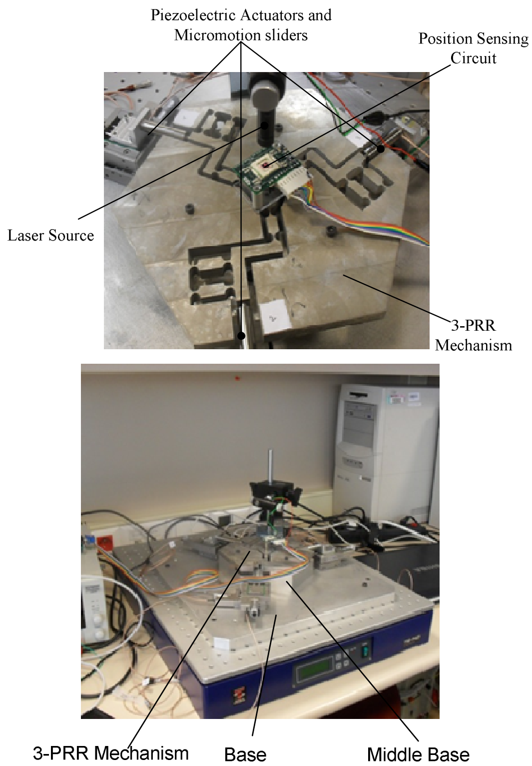


Figure 3. Manufactured 3 PRR mechanism and the experimental Setup

IV. KINEMATICS OF THE MECHANISM

T. S Smith says in his work that no matter how crude the machining the displacement characteristics of compliant mechanisms will remain linear, the axis of the motion will change [11]. So we have to determine the direction of the displacement vectors from the links which are u1, u2 and u3 to have the kinematics of the mechanism shown in Figure 4. After calibration of laser position sensor, we have applied respectively 30, 60, 90, 120 and 150 Volts to the piezoelectric actuators when all the piezoelectric actuators are assembled to the mechanism and preloaded before starting actuation. The results of the experiments are shown in Figure 5. We have observed that u_1 vector has 64° u_2 vector has 65° and u_3 vector has 1.5° angle with the x axis. Now we can write a transformation matrix **A** for the displacements as:

$$\begin{bmatrix} x \\ y \end{bmatrix} = \underbrace{\begin{bmatrix} -\cos(65°) & -\cos(64°) & \cos(1.5°) \\ -\sin(65°) & \sin(64°) & \sin(1.5°) \end{bmatrix}}_{A} \cdot \begin{bmatrix} u_1 \\ u_2 \\ u_3 \end{bmatrix} \quad (1)$$

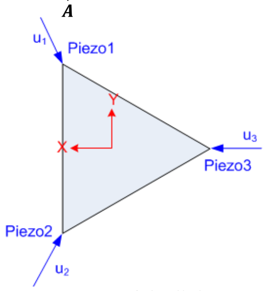
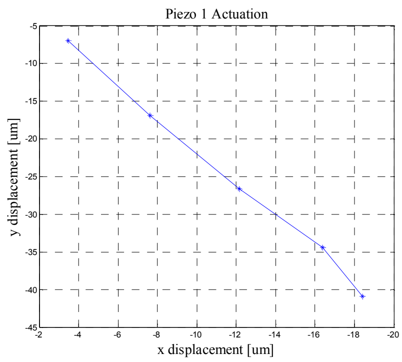


Figure 4. The displacement vectors of the links connected to the triangular stage.



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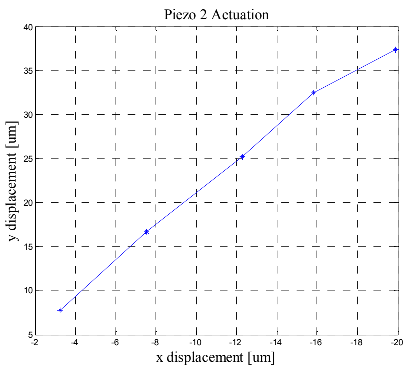

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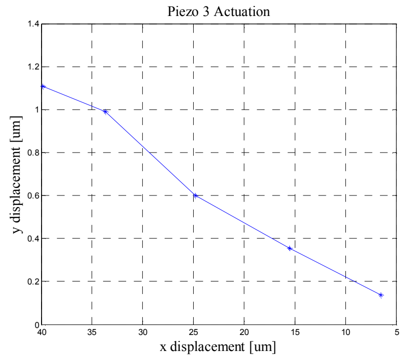


(c)

Figure 5. a. u_1 displacement vector for only piezo 1 actuation, b. u_2 displacement vector for only piezo 2 actuation, c. u_3 displacement vector for only piezo 3 actuation.

V. PIEZO ACTUATOR MODELING AND POSITION CONTROL

A. Piezoelectric Actuator Model

Piezoelectric actuators electromechanical lumped model can be defined by the equations (2)-(7) [12]. v is the total voltage across the actuator, v_p is the piezoelectric voltage and v_h is the hysteresis voltage. T is the electromechanical transformation ratio that connects electrical part and mechanical part of the model. q is the total charge in the actuator, q_p is the charge transduced due to mechanical motion, H is the hysteresis function that depends on q, F_p is the force of the piezoelectric effect and F_{ext} is the external force on the actuator. According to equation (7) u is the displacement, m_p, c_p and k_p are respectively the equivalent mass, damping and stiffness of the piezoelectric actuator. F_c is the control force and F_{dis} is the disturbance force.

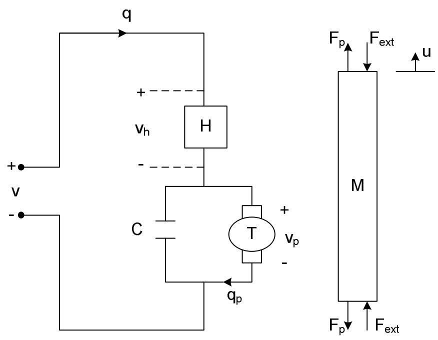


Figure 6. Piezoelectric actuator model [12].

$$v_p = v - v_h \quad (2)$$
$$v_h = H(q) \quad (3)$$
$$q = Cv_p + q_p \quad (4)$$
$$q_p = Tu \quad (5)$$
$$F_p = Tv_p \quad (6)$$
$$m_p\ddot{u} + c_p\dot{u} + k_p u = \underbrace{Tv}_{F_c} - \underbrace{Tv_h - F_{ext}}_{F_{dis}} \quad (7)$$

B. Disturbance Observer

We are able to eliminate disturbances by modeling an observer so a linear model is defined by using nominal parameters of actuator as in equation (7). The displacement u

for every piezo actuator can be measurable by using laser position sensor and inverse of the transformation matrix. The supply voltage is also measurable. The linear model of the piezoelectric actuator is:

$$m_n\ddot{u} + c_n\dot{u} + k_n u = T_n v - F_d \quad (8)$$

We can define F_d as hysteresis force, external force and the uncertainties of the plant parameters which are Δm, Δc, Δk and ΔT. These parameters are assumed as bounded and continuous.

$$F_d = T_n v_h + F_{ext} + \Delta T(v + v_h) + \Delta m\ddot{u} + \Delta c\dot{u} + \Delta k u \quad (9)$$

The observer can be designed as a position tracking system in which F_d is replaced with an observer control $T_n v_{obsx}$ because u and v_{in} can be measured and the observer transfer function is written as following equation.

$$m_n\ddot{\hat{u}} + c_n\dot{\hat{u}} + k_n\hat{u} = T_n v_{in} - T_n v_{obsc} \quad (10)$$

$\hat{u}$ is the estimated position, v_{in} is the plant control input, v_{obsc} is the observer control input. When $\hat{u}$ tracks u, F_d equals to $T_n v_{obsc}$. A sliding manifold is selected for that purpose which is $\sigma = \dot{u} - \dot{\hat{u}} + C_{obs}(u - \hat{u})$. The Lyapunov function is taken as $v_L = \sigma^2/2$ which is positive definite and the derivative of Lyapunov function is taken as $-D_{obs}\sigma^2$ which is negative definite. We will get equation (11) by equating the above results and simplfying:

$$L = \sigma\dot{\sigma} = -D_{obs}\sigma^2 \Rightarrow \dot{\sigma} + D_{obs}\sigma = 0 \quad (11)$$

If we insert sliding mode manifold into the equation (11):

$$(\ddot{u} - \ddot{\hat{u}}) + (C_{obs} + D_{obs})(\dot{u} - \dot{\hat{u}}) + C_{obs}D_{obs}(u - \hat{u}) = 0 \quad (12)$$

When we subtract the equations (10) from (9) and insert the result into the above equation (12) we can find the equivalent control v_{eqc} which keep system motion in manifold $\sigma + D\dot{\sigma} = 0$.

$$v_{ceq} = \frac{1}{T_n}\{F_d + [c_n - m_n(C_{obs} + D_{obs})](\dot{u} - \dot{\hat{u}})[k_n - m_n C_{obs}D_{obs}](u - \hat{u})\} \quad (13)$$

Equation (13) tells us that when $\sigma \to 0$ then $u \to 0$ and $T_n v_{ceq} \to F_d$. For the implementations discrete form of sliding mode control is used as:

$$v_{(k)} = v_{(k-1)} + K_{uobs}\left(D_{obs}\sigma_{(k)} + \frac{\sigma_{(k)} - \sigma_{(k-1)}}{dT}\right) \quad (14)$$

K_{uobs} is a design parameter that optimize the controller and dT is the sampling interval for discrete time control. The system and the observer can be summarized as in equations (15-17):

$$m_n\ddot{u} + c_n\dot{u} + k_n u = T_n v_{in} - F_d \quad (15)$$

$$m_n\ddot{\hat{u}} + c_n\dot{\hat{u}} + k_n\hat{u} = T_n v_{in} - T_n v_{obsc} \quad (16)$$

$$v_{in} = v_c + \frac{\alpha}{T_n}v_{obsc} \quad (17)$$

C. Position Control

A closed loop control is applied for the position control of the mechanism. The end-effector reference is given in x-y coordinates and with the pseudo inverse of the transformation matrix A showed in equation (1), corresponding reference positions u_i (i=1,2,3) for piezoelectric actuators are calculated as in equation (18).

The sliding manifold is selected to be as in equation (19) and when the sliding manifold is reached the closed loop control showed in equation (20) and the system is described by equation (21).

$$\boldsymbol{u_i} = \boldsymbol{A}^{\dagger}[x\ y] \quad (18)$$

$$\sigma_x = (\dot{u}_{ref} - \dot{u}) + C_x(u_{ref} - u) \quad (19)$$

$$v_{(k)} = v_{(k-1)} + K_{ux}\left(D_x\sigma_{x(k)} + \frac{\sigma_{x(k)} - \sigma_{x(k-1)}}{dT}\right) \quad (20)$$

$$(\ddot{u}_{ref} - \ddot{u}) + (C_x + D_x)(\dot{u}_{ref} - \dot{u}) + C_x D_x(u_{ref} - u) = 0 \quad (21)$$

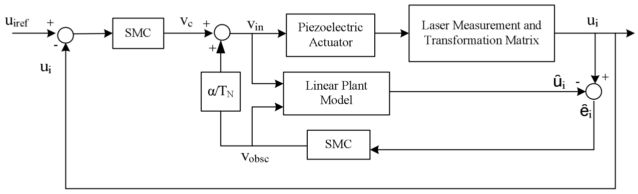
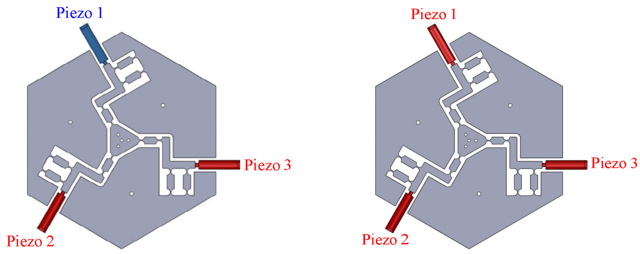


Figure 7. Closed loop control block diagram

VI. RESULTS

The experiments are done for open loop and closed loop control of the 3 PRR mechanism to compare and see clearly the effects of observer and sliding mode control. Figure 8 shows the experiments done for seeing the advantage of redundancy. The red colored piezoelectric actuators are the active ones (working) and the blue one is passive (not working). First 2 piezoelectric actuators are controlled as shown in Figure 8a then all piezoelectric actuators are controlled as shown in Figure 8b to give an answer to the question "is it possible to achieve the same motion by using 2 and 3 actuations?".



(a) (b)

Figure 8 a. 2 piezoelectric actuators are activated, b. 3 piezoelectric actuators are activated

The parameters of the piezo are presented in Table 1. These parameters are used for nominal parameters in linear model of the piezoelectric actuators for open and closed loop experiments. A circular trajectory having 20µm diameter of circle is given as a reference to the mechanism by setting references as $x_{ref} = 10 + 10\sin(0.2\pi t)$ and $y_{ref} = 10 + 10\cos(0.2\pi t)$ for all of the experiments. The pseudo inverse of transformation matrix **A** as in equation (18) is used for calculating the necessary references for the u_1, u_2 and u_3 displacement vectors. The control input voltages is saturated between 0V to 150V to use the bipolar actuation property of the piezoelectric actuators.

TABLE I. Parameters of piezoelectric actuator

Parameter	Quantitiy	Value
m_p	Mass	6.16 x 10^{-4} kg
c_p	Damping	1027.5 Ns/m
k_p	Stiffness	12 x 10^6 N/m
T	Transformation ratio	4.738 N/V

A. Open Loop Results

Inverse of the linear model of the plant is used for estimating the necessary voltage input to the piezoelectric actuators as shown in Figure 9. The errors in x direction and y direction are shown in Figures 10a and 10b. The x-y motion is presented in Figure 10c.

It can be seen from the results that open loop control with the inverse of linear models of the piezoelectric actuators doesn't have enough accuracy for the end-effector motion of our flexure based mechanism. The error in x direction is between 3 µm and -8 µm and the error in y direction is between 2 µm and -8 µm. And when we look at the motion result we can examine that the reference trajectory is shifted and it's not in a circular shape.

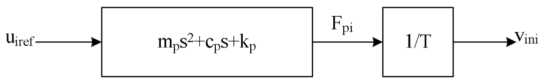
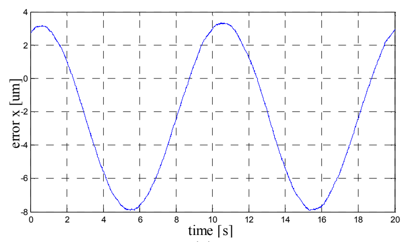


Figure 9. Open loop control block diagram



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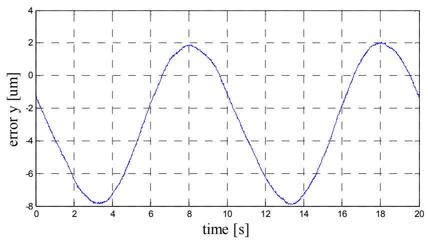

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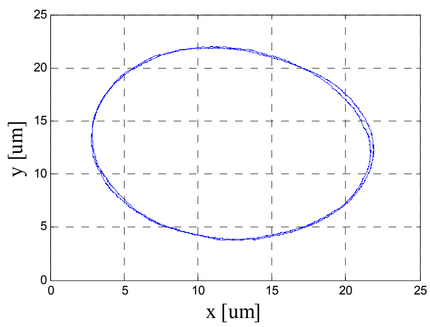


(c)

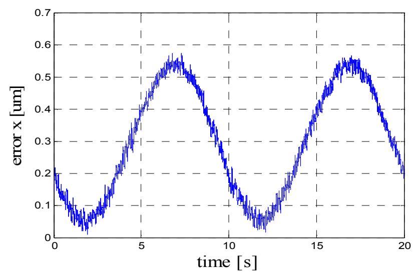
Figure 10. a. errors in x direction for open loop control, b. errors in y direction for open loop control, c. the measured x-y motion of the end-effector for open loop control.

B. Closed Loop Results

Closed loop control is applied to the mechanism and the block diagram of each of the piezoelectric actuator is shown ing Figure 7. The linear model with the nominal parameters as presented in table 1 is used for the observer with sliding mode control to kill the hysteresis and unwanted disturbances of the system and another sliding mode control is used for tracking the reference positions. As in open loop control the necessary reference positions of piezoelectric actuators for tracking reference x-y motion is calculated by using pseudo inverse of transformation matrix **A**. Firstly 2 piezoelectric actuators (the 2nd and 3rd ones according to the Figure 4) are in action while the other piezo (the 1st one) is attached to the mechanism as a rigid support. Then all of the actuators are in action and controlled with the same manner. The sampling time is taken as 100 µsec.

1) 2 Actuators

The errors in x direction as showed in Figure 11a are between 0.05 µm and 0.55 µm and the errors in y direction shown in Figure 11b are between 0.05 µm and 0.5 µm. The measured x-y motion of the end-effector is presented in Figure 11c. The errors are smaller than open loop control of 3 piezoelectric actuators but there is still a shift from the reference trajectory. When looking at the control outputs (piezoelectric actuator inputs) the piezoelectric actuator that creates u_2 displacement vector reaches to 150V which is the maximum stroke voltage. This means that the workspace is almost limited to a circle having 20µm.



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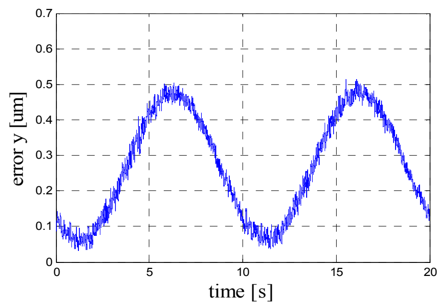

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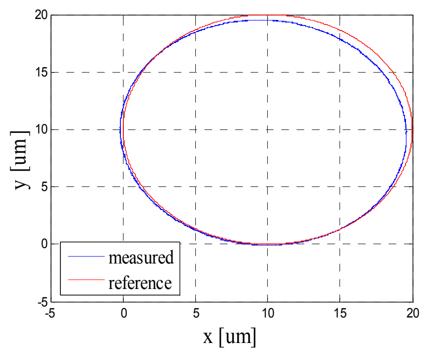
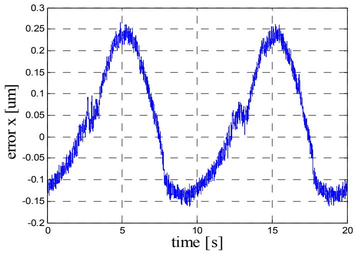


(c)

Figure 11. a. errors in x direction for 2 actuation, b. errors in y direction for 2 actuation, c. the measured x-y motion of the end-effector for 2 actuation.

2) 3 Actuators

The errors in x direction is between -0.15 µm and 0.25 µm as shown in Figure 12a and the errors in y direction is between 0.06 µm and 0.025 µm as shown in Figure 12b. When looking at the errors and measured x-y motion results compared with the reference motion shown in Figure 12c. The maximum control input voltages for piezoelectric actuators are respectively 90 V, 50 V and 50 V. This shows us that the voltages that are coming to the piezoelectric actuators are not close to 150V so the workspace is bigger than a circle with 20 µm diameter. So this means that the redundancy allows us to extend the workspace when compared to the results of 2 actuators results.



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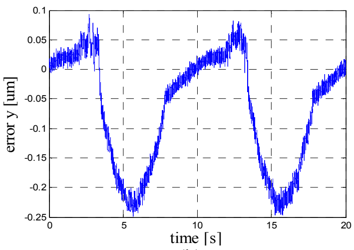

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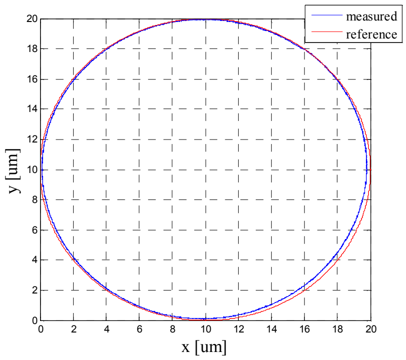

(c)

Figure 12 a. errors in x direction for 3 actuation, b. errors in y direction for 3 actuation, c. the measured x-y motion of the end-effector for 3 actuation.

VII. CONCLUSION

The position control of a designed flexure based mechanism called 3 PRR is succeeded by using sliding mode control with disturbance observer. The mechanism is designed as to be redundant to eliminate the manufacturing errors, hysteresis, assembling errors etc. In order to see the effects of closed loop control and having 3 actuators for x-y position, firstly we have implemented the open loop control and then respectively 2 piezoelectric actuators and 3 piezoelectric actuators are controlled for closed loop control. Open loop control with a linear model results show that it can't be used for precise position control of the stage because of being lack of accuracy. The proposed closed loop control eliminates the nonlinearities of the system. We have 2 piezoelectric actuators active; the maximum position errors are in 550 nm whereas when 3 piezoelectric actuators are active the maximum error is decreased to 250 nm. When we examine the control output voltages, our redundant mechanism provides a larger workspace because the voltages are not close to 150V which is the limitation of piezoelectric actuators. Shortly, the experimental results tells us that making our mechanism redundant gives us better results for tracking the reference motion and the work space of the redundant mechanism is bigger.

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