OPTIMAL PRICING STRATEGIES FOR CAPACITY LEASING BASED ON TIME AND VOLUME USAGE IN TELECOMMUNICATION NETWORKS

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ABSTRACT

In this study, we examined optimal pricing strategies for "pay-per-time", "pay-per-volume" and "pay-per both time and volume" based leasing of data networks in a monopoly environment. Conventionally, network capacity distribution includes short/long term bandwidth and/or usage time leasing. When customers choose connection-time based pricing, their rational behavior is to fully utilize the bandwidth capacity within a fixed time period, which may cause network to burst (or overload). Conversely, when customers choose volume-based strategies their rational behaviors is to send only enough bytes (even for time-fixed tasks for real time applications), causing quality of the task to decrease, which in turn creating an opportunity cost for the provider. Choosing pay-per time and volume hybridized pricing scheme allows customers to take advantages of both pricing strategies while lessening the disadvantages of each, since consumers generally have both time-fixed and size-fixed task such as batch data transactions. One of the key contributions of this study is to show that pay-per both time and volume pricing is a viable and often preferable alternative to the only time and/or only volume-based offerings for customers, and that, judicious use of such pricing policy is profitable to the network provider.

Keywords: telecommunications, capacity leasing, time based pricing, volume based pricing, optimization, simulation, sensitivity analysis

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INTRODUCTION AND RELATED LITERATURE

Customers use information networks for a variety of reasons including video conferencing, VoIP (voice over TCP/IP) and business data applications (e.g., EDI), which in this paper we call collectively as tasks. In general, two types of tasks are performed using data networks: time-fixed and size-fixed tasks (Kasap *et* al., 2007). A task is time-fixed (real-time, size compressible) if its size can be changed without disrupting its completion but the transmission time cannot be compressed or extended. Most audio/video tasks are of this type. A task is size-fixed (time compressible) if all bits have to be transmitted but the duration is not fixed. Most data applications such as file transfer, database transactions belong to this type. Both types use some level of network bandwidth during an arbitrary period of time.

Earlier studies showed that the connection billing model has a major effect on how people consume online data and how satisfied they are with the connection and online services. Today there are various billing models for quality-differentiated Internet access as a function of bandwidth, traffic volume, applications, and pricing structure (which is based either on time, volume, a combination of both, or a flat-rate), bandwidth efficiency optimizations guaranteed QoS, uniform pricing, multipart pricing and nonlinear pricing on total call time or the number of packets exchanged (Masuda and Whang, 2006; Kasap *et al.,* 2007). Nonetheless, there are several other pricing strategies (Keon and Anandalingam, 2005) that are examined by a large number of sources including "Paris-metro pricing (similarly time of day pricing)", "priority pricing", "smart-market pricing", "top-percentile pricing" (Levy *et* al, 2006). Some parts of these pricing strategies are usage-based while others are mostly volume-based.

In recent years, the definition of "network service provider" has been changing. In order to manage traffic congestions and scarce resources in the midst of changing consumer preferences for variety of services, network providers have been experimenting with various billing strategies, which differ significantly among countries. Volume-dependent pricing schemas have been commonly used by many providers in the recent past, wherein the costumers are billed based on the volume of the content downloaded or uploaded. There have been some sources claiming that this pricing strategy has lost its appeal in recent years. These volume-based strategies have been decreasing the consumers' interest and usage due to the fact that they may seem too complex to costumers. These complexities may confuse and even frighten consumers (Stiller *et al.*, 2001). Some researches argue that the greatest advantage of volume-based pricing for operators is that the uncertainty and risk of consumption remains with the customer, rather than the operator, which can prorate bills based on the data volumes transferred (Biggs and Kelly, 2006). The pay-per-volume context is evaluated in many respects in recent research studies (Kim, 2006; Bouras and Sevasti, 2004; Altman *et al.,* 1999; Levy *et al.,* 2006; Anderson *et al.,* 2006).

The other common broadband pricing strategies practiced by operators is a time-based pricing strategy, called pay-per-time. According to this point of view, consumers are charged based on per unit of time they spend online. There is a variety of service types to fulfill different classes of customers that need a variety of connection speeds (Jain and Kannan, 2002; Altman *et al.,* 1999; Biggs and Kelly, 2006).

When providers offer connection time based, volume based and both connection time and volume based pricing schemes, each consumer is allowed to select one or the other. If customers have more time-fixed tasks in their agenda, they would select connection time based pricing. Hence, they can plan/schedule their task and increase the volume of the task by increasing transmission rate in order to realize better audio/visual quality. Also they incur the same cost since the connection time has not been changed even though the transmission rate has been increased; therefore, customers can manage costs more effectively. Additionally, when customers choose connection time based pricing, their optimal behavior would be utilizing the bandwidth capacity fully (they can transmit another task simultaneously such as they can perform bulk data/file transfer while carrying out video conference), therefore their behavior can cause the network to burst. Offering the pay-per-volume based pricing to the consumer can prevent bursting. The main advantage of pay-per-volume pricing scheme is independently from how long the task takes to manage; the customer would pay for the total volume used. In addition, if customers have more size-fixed tasks in their agenda, they can plan/schedule their task and manage costs more effectively, because most of the size fixed tasks are divisible, and the customer can schedule a task in different reasonable times. However, the optimal behaviors of the customers who choose the pay-per-volume pricing scheme generally encourages them to send only enough bytes for time-fixed tasks, and this situation can cause quality of the task to decrease, eventually creating an opportunity cost. Choosing pay-per both time and volume based pricing scheme allow customer to take advantages of only pay-per-time and only pay-per-volume pricing strategies while decreasing (minimizing) the disadvantages of each, because consumers generally have both time-fixed and size-fixed task. That is why in this study we examine the following issues: (i) what are the extra benefits to the network provider for providing the time and volume hybridized pricing scheme? and (ii) would this offering schema make an impact on the amount of demand (number of customers enter the market)?

AT&T's CFO said that as data traffic continues to climb in telecommunication networks, operators need to discard all-you-can-eat data plans and explore new pricing models (Connected Planet, April 2010). Moreover, Connected Planet (January 2010) and Telecomasia.net (2011) mentioned that flat rate pricing models simply are not sustainable and differentiated pricing is crucial. In cross-service rating, customers of certain bundles are charged differently. If data services are coupled with voice, then offer better rates on the data plan if the person talks more than 200 minutes per month. In hybrid models, you don’t have to be a postpaid subscriber on all services; you can choose to be postpaid on voice and prepaid on data where you get a monetary allowance and a chance to recharge once a threshold is reached. These pricing options are combinations of time and volume based pricing discussed in this paper.

According to Dr Suk-Chae Lee, chairman of KT Corporation that is one of South Korea's largest telecom providers, telecom companies around the world are struggling to handle the amount of traffic generated by wireless devices, such as smartphones. They move to volume-based data charges to manage their networks better ("Leading Business Intelligence", 2010; “Textio: Phones”, 2010). Similarly, Tim Weingarten, CEO of Visage Mobile, mentioned in Wireless Week (2010) that flat-rate pricing is increasingly being subsumed by tiered or usage-based models. Mark Hopper, Vice President of Smith Micro mentioned in Wireless Week (2010) that one of the biggest challenges in near future will be the movement to metered data plans (volume based pricing). In general, people don't understand the kilobyte currency, they understand minutes, but not data tonnage. So it will be difficult to expose data usage to customers. For example, pricing should be different to watch a video at 320 resolution vs. 720 resolution vs. HD, since the amount of data to watch that same video is massively different. This could be handled by hybrid pricing (both volume and time based) scheme discussed in this paper. Tripod (2011) provided examples of different price-service combinations that may be offered to customers. The customers choose the best suited to their own needs from the different options provided such as different combinations of access and usage price, or different combinations of telecom tariffs for various services. These different options are conceptually quite similar to the hybrid pricing policy discussed in this paper.

Recent articles from trade press and online sources discussed various time and volume based charging schemes (Trademarkia.com, 2011; Turkey's Telecommunication Leader, 2010; Nepal Telecom, 2011; Jows' Telecom Review, 2010; Total Telecom, 2011). According to these sources, time based schema is the one that customers are most familiar with, because it is similar to the way they are charged on voice calls. But when customers are charged with time-based, they are not quite comfortable to use data service because turning on the Internet connection means spending money. Most of the customers however do not have a good familiarity with the volume based pricing since they don't know how many megabytes they would need for usage of different data services. Estimation is not easy because of rich content web pages, video streaming, and browsing the visual images.

In this study, we examined optimal pricing strategies for "pay-per-time", "pay-per-volume" and "pay-per both time and volume" based leasing of data networks in a monopoly environment. As a representative example, we considered the telecommunication market in Turkey, where the network use is increasing exponentially and the provider space has been a monopoly for many years. Until late 2003, Turk Telekom was the only transmission service provider, holding a monopoly over the provision and infrastructure of national and international voice telephone services. Before the liberalization, Turk Telecom was a state company entirely owned and managed by the Secretary of the Treasury. Despite the cessation of Turk Telekom's monopoly, it retained a de facto monopoly (with current market share of 96%), since it still has significant market power as the only operator that owns a nationwide fiber transmission network in Turkey (Turkcell Iletisim Hizmetleri AS, 2008; OpenNetInitiative, 2012).

The rest of the paper is organized as follows: In the next section, we discuss the assumptions and present the model formulation. Then, we present an optimization model and determine the optimal pricing strategy. The following section provides the numerical study and sensitivity analysis of various system parameters. In the next section, a comparison of the optimal pricing policy without the hybrid pricing scheme is given. Finally, we conclude and discuss future research ideas in the following section.

MODEL ASSUMPTIONS AND FORMULATION

In this section, we discuss the model assumptions and formulate the optimization models for different pricing scenarios. In order to examine the optimal pricing strategies for pay-per-time, both pay-per-time and volume, and pay-per-volume leasing of data networks, we adopt the monopoly pricing model originally proposed by Gurnani and Karlapalem (2001). They developed a network pricing model to examine the optimal pricing strategies for selling and pay-per-use licensing of packaged software over the Internet. Unlike their model, our model has three different situations, and in all situations customers can lease the network but in different pricing strategies, and we investigate profit of the network provider per the unit time. Also, we assume that the marginal cost of network services is constant, and therefore all prices are assumed to be net of marginal cost. Pricing decisions with multiple network providers or multiple competing network capacity would result in a game-theoretic problem formulation, which is beyond the scope of this paper. There are other common modeling approaches for duopoly case and competition effects. Iyer (1998) analyzes how manufacturers should coordinate distribution channels when retailers compete in price as well as important non price factors. Einhorn (1992) considers a duopoly in which each firm produces two components of a system and shows that consumers may "mix and match" the firms' components, if the components are compatible.

We will use the following notation while presenting formulations throughout the text:

Ω_1 : Chance of selecting “pay-per-time” pricing scheme for customers.

Ω_2 : Chance of selecting “hybrid” pricing scheme for customers.

Ω_3 : Chance of selecting “pay-per- volume” pricing scheme for customers.

q_1, q_2, q_3 : Demand for Ω_1 , Ω_2 and Ω_3 respectively.

q_4 : Demand for customers prefer not entering to the market.

c^t : Unit price for using the network resources based on total transmission time.

c^x : Unit price for transmission of data carried over the network per byte per unit time.

λ : The coefficient used to set the pricing level of hybrid scheme.

x : Bandwidth usage index (or volume used) of network capacity for the customers. $x \in [0,1]$

r : The reservation price of the consumer. $r \in [0,1]$

t : The effective usage of the transmission time for customers. $t \in [0,1]$

k : The benefit of transmitting unit volume of data.

w : The benefit of using efficient transmission time.

v_1 : Consumer surplus function per unit time for using the network with time based pricing scheme.

v_2 : Consumer surplus function per unit time for using the network with hybrid based pricing.

v_3 : Consumer surplus function per unit time for using the network with volume based pricing.

θ : The coefficient shows ability to carry out multiple tasks simultaneously without paying extra in hybrid pricing scheme.

β : The parameter models the difference in benefit of the using time based against hybrid pricing.

γ : The coefficient shows ability to carry out more than one task simultaneously in volume based pricing scheme.

α : The parameter models the difference in benefit of using volume based pricing against hybrid pricing scheme.

$\prod$: Revenue in unit time.

Let $\lambda(c^t + c^x x)$ be the unit price of using the network resources based on both total transmission time and total volume used, where $0 < \lambda < 1$. The traditional model used to categorize consumers assumes that there is a continuum of consumers indexed by the *reservation prices* $r \in [0,1]$, with r distributed uniformly in the unit interval. The reservation price reflects the maximum amount that a consumer is willing to pay in exchange for a given service (Salop 1979; Dewan *et al.,* 1999; Gurnani and Karlapalem, 2001).

Our model considers the case where the consumer has the choice of pay-per-time, pay-per both time and volume and pay-per-volume based pricing. Therefore, not only consumers' reservation prices affect their network leasing decision but also the benefit derived from network usage in unit time effects their decision. When we talk about network usage, we consider two primary dimensions: time and volume since leasing decision contains contract duration and maximum bandwidth. While using the network, customers occupy some capacity (bandwidth) of the network throughout the connected time. Therefore, we adopt a three dimensional classification schema where each consumer indexed by (x,t,r), where $r \in [0,1]$, as before, represents the reservation price index, $x \in [0,1]$ represents the bandwidth usage index (or volume used) of network capacity for the customers, and upper bound of x shows all available capacity of the network in unit time. The effective usage of the transmission time for customers is represented by $t \in [0,1]$. The upper bound of t represents full utilization of transmission during a given unit connection time. These measures are assumed to be uniformly distributed over the unit time interval. With this new classification, we can consider consumers with different combinations of high/low reservation prices and usage levels of time and volume as depicted in Figure 1.

Sending more bytes during an audio/video conference can increase the quality of the conference, that's why choosing the pay-per-time pricing scheme can be more advantageous for these type of tasks. So, we can assume that the benefit of using the network is higher for the consumers choosing pay-per-time pricing scheme compared to selecting per-per-volume pricing. In addition, the optimal behaviors of the customers who choose the pay-per-time pricing scheme generally encourages them to use all available bandwidth so that they can increase the quality of task by increasing the size of it or they can transmit another task simultaneously, and this situation can cause networks to burst. In order to prevent the bursting of networks, providers generally set pay-per-time unit price higher to discourage some of their customers from choosing pay-per-time pricing scheme.

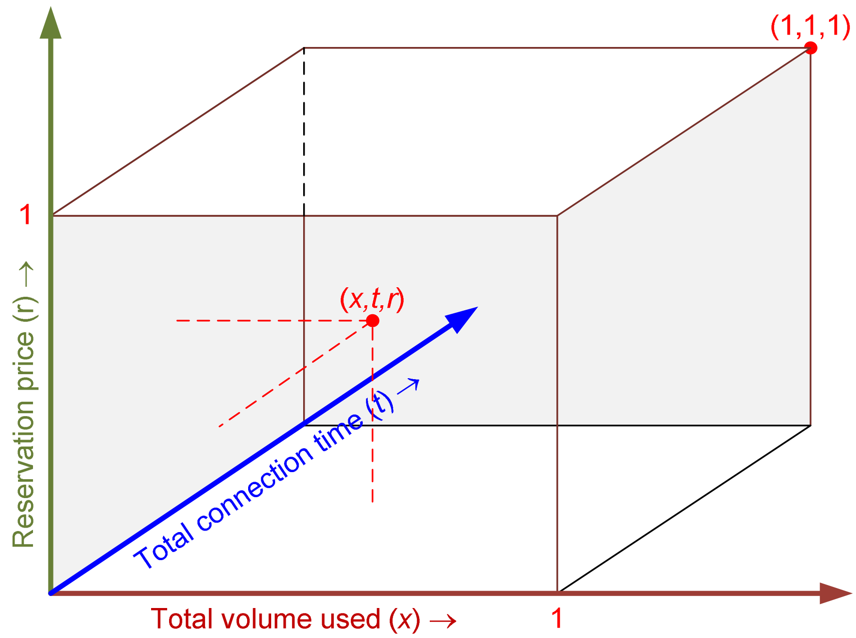

Figure 1: Consumer indexing as per volume, time and reservation price

The optimal behaviors of the customers who choose the pay-per-volume pricing scheme generally encourages them to send only just enough bytes for time fixed tasks, however this situation can cause quality of the task to decrease, and so it can create an opportunity cost. That's why the reservation price of the customers for the pay-per-volume pricing strategy would be lower. Since sending just enough bytes for time fixed tasks can create an opportunity cost, customer would prefer to increase the transmission rate (send more bytes) in order to increase the quality of the task. In order to attract customers having higher reservation price than customers preferring pay-per-volume pricing strategy, hybrid pricing scheme is being offered. By choosing hybrid, in other words, pay-per both time and volume pricing scheme, customers can complete time-fixed tasks with less cost compared to pay per time only pricing scheme since hybrid pricing is cheaper. However, the quality of tasks is lower since customer cannot increase transmission rate too much because of pay per volume part of hybrid scheme. Therefore, the quality of task via hybrid pricing would be higher than the one with pay per volume scheme while it would be lower than the one with pay per time scheme. Hence, customers, whose reservation price is higher than those preferring pay per volume scheme and lower than those preferring pay per time scheme, would prefer hybrid pricing.

Most data applications such as file transfer and database transactions are size-fixed tasks. The optimal behavior of the customer who choose the pay-per-time pricing scheme would push them to perform the task as soon as possible. However, the customer who chooses pay-per-volume pricing has task scheduling flexibility, because most of the size fixed task are divisible, and the customer can schedule a task in different reasonable times. Moreover, the providers will set a lower pay-per-volume unit cost to encourage customers to choose volume based pricing since they generally want to prevent bursting. So, we can easily assume that $c^x x < c^t$.

Choosing pay-per both time and volume based pricing scheme may help customer to take the advantages of only pay-per-time and only pay-per-volume pricing strategies while decreasing the disadvantages of them, because consumers generally have both time-fixed and size-fixed task. So, we can easily assume that $c^x x \le \lambda(c^t + c^x x) \le c^t$, where $0 < \lambda < 1$.

Consumer Surplus Function

For the consumer who leases the network capacity with the pay-per-time pricing strategy (Ω_1) , the total utility is given by $(r + kx + wt)$ for $x > 0$ and $t > 0$. We also assume that consumers with zero usage do not derive any utility. Note that, the consumer leases the network capacity and uses the network for transmitting and completing his tasks; the overall utility depends on the level of total connection time and total volume used. Therefore, the consumer surplus function per unit time for using the network with the pay per connection time pricing strategy is given by utility minus the total charge, that is

$$v_1\left(x,t,r\right) = r + kx + wt - c^t \tag{1}$$

Similarly, for the consumer who uses the network capacity with the hybrid pricing scheme (Ω_2), the total utility in unit time is given by $(\beta r + kx + \theta wt)$ for $x > 0$, $t > 0$, $0 \le \alpha \le 1$ and $0 < \theta < 1$. θ is 1 in v_1 since the consumer chooses pay per time scheme. The reservation price of the hybrid option would be βr , where $0 < \beta < 1$. The smaller the value of β , the higher is the difference in the benefit of the billing choices. Since the consumer who selects the hybrid pricing scheme may not use excess capacity to transmit simultaneous tasks, the utility derived is lower as compared to the consumer who selects connection time based pricing scheme. That is, the parameter β models the consumers' inclination to choose hybrid pricing scheme rather than selecting connection time based pricing. In this case, the consumer pays both per-time and per-volume price $\lambda(c^t + c^x)$, and therefore surplus per unit time is given by

$$v_2\left(x,t,r\right) = \beta r + kx + \theta wt - \lambda(c^t + c^x x) \qquad (2)$$

For the consumer who uses the network capacity with the pay-per-volume strategy (Ω_3), the total utility is given by $(\alpha r + kx + \gamma wt)$ for $x > 0$, $t > 0$, $0 < \alpha < 1$ and $0 < \gamma < 1$. The reservation price of pay per volume option would be αr , where $0 < \alpha < \beta < 1$. The smaller the value of α , the higher is the difference in the benefit of the billing choices. Since the consumer who selects volume based pricing cannot use excess capacity to transmit simultaneous tasks, the utility derived is lower as compared to the consumer who selects connection time based or hybrid pricing schemes. That is, the parameter α models the consumers' inclination to choose volume based pricing rather than selecting the other two pricing schemes. The ability to carry out more than one task simultaneously would be γ , where $0 < \gamma < \theta < 1$. In this case, the consumer pays a per-volume price c^x , and therefore per unit time surplus is given by

$$v_3\left(x,t,r\right) = \alpha r + kx + \gamma wt - c^x x \qquad (3)$$

Depending on the prices set by the provider and their own task profile, consumers would pay per-time price, hybrid price or pay per-volume price or decide not to enter the market. For the network provider the objective is to maximize the revenue by suitably choosing the prices. In the next section, we develop an optimization problem formulation for the network provider.

Optimization Problem Formulation

The objective for the network provider is to maximize revenue by selecting the pay-per-time pricing, c^t, the pay-per-hybrid pricing, $\lambda(c^t + c^x)$, the pay-per-volume pricing, c^x. As a function of these prices, consumers would make their own optimal selection.

Note that q_1 is the demand generated by consumers who prefer to pay per usage time, q_2 is demand due to pay-per-hybrid pricing scheme, q_3 demand due to pay-per-volume. Then, the objective function for the vendor is to maximize the revenue in unit time (normalized revenue), $\prod$:

$$\underset{(c^t, c^x)}{Max} \prod = c^t(q_1 + \lambda q_2) + c^x(q_3 + \lambda q_2) \tag{4}$$

In order to derive the expressions for q_1, q_2 and q_3, we consider the following cases. First, we consider the case when $k \geq c^x$ that is, the per-used volume, the per-hybrid price and the per-used time benefits for the consumer exceeds the access price per-volume usage, so all the customers in the market prefer using the network. In the second case, we assume that $k \leq c^x$, that is some customers neither choose pay-per-time, pay-per-hybrid price nor pay-per-volume pricing scheme, so prefer not to use networks. The optimal solution for the provider is the maximum of these two cases.

Case 1: $k \geq c^x$ - All consumers prefer entering the market.

In this case, it is easy to see from (3) that $v_3(.)$ is nonnegative. As a result, the consumer is better off choosing the pay-per-volume pricing scheme as compared to not to entering the market. Therefore, the entire consumer population is covered in this case, that is, $q_1 + q_2 + q_3 = 1$, see Figure 2.

Most probably the consumers choosing the pay-per-time pricing scheme do not have low reservation prices. Because of this reason, the surface representing the frontier at which the consumer is indifferent between pay-per-time and pay-per-hybrid pricing schemes cannot pass from the points where $r = 0$. Since it is expected that the optimum behavior of pay-per time customers will be transmitting as much task as they can in unit time with full utilization of network, it generally results in network bursting. In order to prevent network bursting, we minimize the number of pay-per-time pricing scheme customers while maximizing the revenue of the network provider. Therefore, in Figure 2, the region representing the demand in consumer index cube for pay-per-time pricing scheme can be a triangular pyramid. And also, we recommend pay-per-hybrid pricing scheme to maintain revenue of the provider. As well, the frontier at which consumer is indifferent between pay-per-time and pay-per-hybrid pricing scheme should be a surface passing from points on the edges at the upper part of the unit cube in order to minimize the number of customers selecting pay-per-time pricing scheme. Consequently, the surface connecting the points are $(x_1,1,1)$, $(1,t_1,1)$ and $(1,1,r_1)$. Using (1) and (2), from three indifference equations, we obtain x_1, t_1, and r_1 as follows.

$$x_1 = \frac{c^t(1-\lambda) - w(1-\theta) - (1-\beta)}{\lambda c^x} \tag{5}$$

$$t_1 = \frac{c^t(1-\lambda) - \lambda c^x - (1-\beta)}{w(1-\theta)} \tag{6}$$

$$r_1 = \frac{c^t(1-\lambda) - \lambda c^x - w(1-\theta)}{(1-\beta)} \tag{7}$$

Then, from the volume of the triangular pyramid shown in Figure 2, we get:

$$q_1 = \frac{1}{2}\frac{1}{3}(1-x_1)(1-t_1)(1-r_1) \tag{8}$$

$$q_1 = \frac{\left[\lambda c^x - c^t(1-\lambda) + w(1-\theta) + (1-\beta)\right]^3}{6w\lambda(1-\theta)(1-\beta)c^x} \tag{9}$$

The providers should set a lower pay-per-volume unit cost to encourage customers to choose volume based pricing, in order to prevent bursting of their networks. Optimal behaviors of the pay-per volume pricing scheme's customers generally encourage them to send only just enough bytes for time-fixed tasks and to take an opportunity cost. Therefore, in order to cover their opportunity cost consumers choosing pay-per-volume pricing schemes would not be intending to pay a premium price, so most probably they have low reservation prices. Thus, the consumers choosing pay-per-volume pricing scheme would be in the lower part of the unit cube in Figure 2.

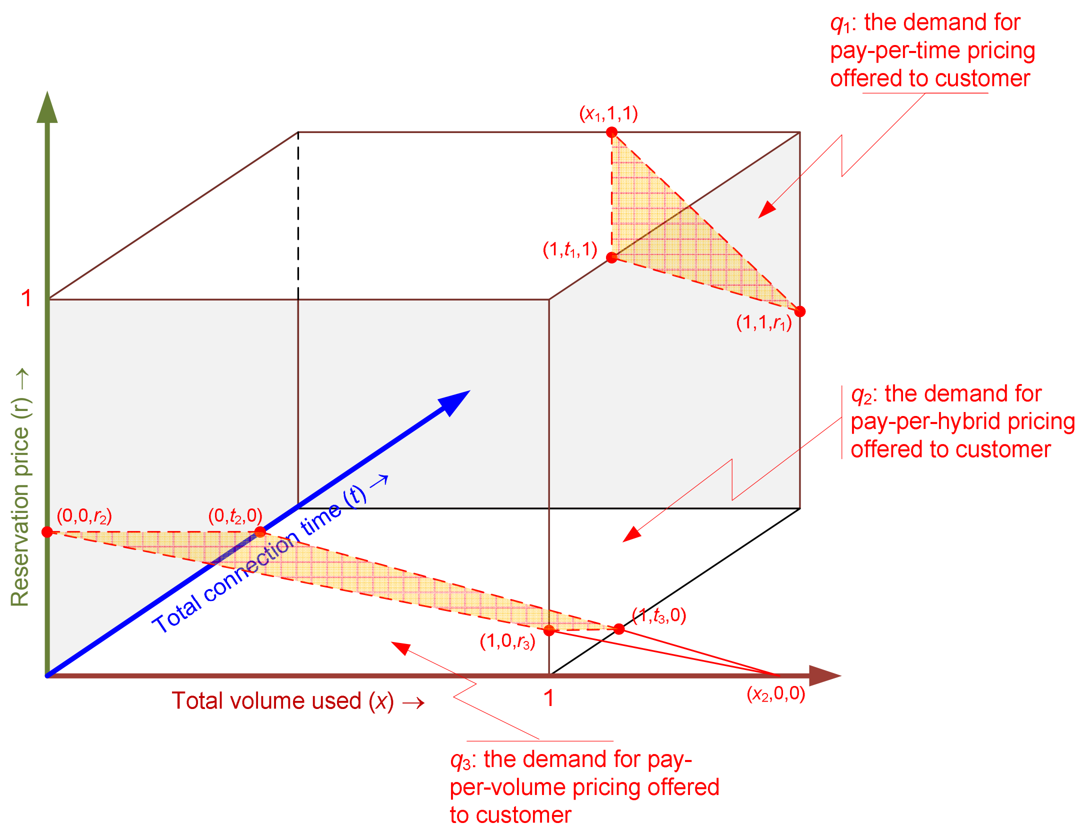

Figure 2: Consumer selections for Case 1

Customers prefer pay-per-hybrid pricing scheme under the conditions as denoted in equation (10)

$$v_2(x,t,r) - v_3(x,t,r) > 0$$

$$(\beta - \alpha)r + (\theta - \gamma)wt + \left(c^x x - \lambda\left(c^t + c^x x\right)\right) > 0 \tag{10}$$

We assume that $c^x \leq \lambda(c^t + c^x x) \leq c^t$, where $0 \leq \lambda \leq 1$, so the frontier surface between pay-per-volume and pay-per-hybrid pricing schemes cannot cross from bottom *x* edges of the cube, where t and r equal to zero since the above equation is always nonnegative due to assumption. Therefore there is no indifference point on the bottom edge of the cube where t and r equal to zero. The indifference point on x axis will be an imaginary point $\left(x_2,0,0\right)$ outside of the cube. As denoted in Figure 2, the surface connecting the points; $\left(0,t_2,0\right)$, $\left(0,0,r_2\right)$, $\left(1,t_3,0\right)$ and $\left(1,0,r_3\right)$, represents the frontier at which the consumer is indifferent between pay-per-volume and pay-per-hybrid pricing schemes. Using (2) and (3), from four indifference equations, we obtain t_2, r_2, t_3, and r_3 as follows.

$$t_2 = \frac{\lambda c^t}{(\theta-\gamma)w} \tag{11}$$

$$r_2 = \frac{\lambda c^t}{(\beta-\alpha)} \tag{12}$$

$$t_3 = \frac{\lambda c^t - (1-\lambda)c^x}{(\theta-\gamma)w} \tag{13}$$

$$r_3 = \frac{\lambda c^t - (1-\lambda)c^x}{(\beta-\alpha)} \tag{14}$$

Then, from the volume of the triangular pyramid shown in Figure 2, we get

$$x_2 = \frac{\lambda c^t}{(1-\lambda)c^x} \tag{15}$$

$$q_3 = \frac{\left[\left(\lambda c^t\right)^3 - \left(\lambda c^t - (1-\lambda)c^x\right)^3\right]}{6w(\beta-\alpha)(\theta-\gamma)(1-\lambda)c^x} \tag{16}$$

and $q_2 = 1-(q_1+q_3)$ (17)

We assume $k \geq c^x$ Also, since $0 \leq x_1 \leq 1$, $0 \leq t_1 \leq 1$, $0 \leq r_1 \leq 1$, $0 \leq t_3 \leq t_2 \leq 1$ and $0 \leq r_3 \leq r_2 \leq 1$ we get:

$$c^t(1-\lambda) \geq w(1-\theta) + (1-\beta) \tag{18}$$

$$c^t(1-\lambda) \leq w(1-\theta) + (1-\beta) + \lambda c^x \tag{19}$$

$$c^t(1-\lambda) \geq (1-\beta) + \lambda c^x \tag{20}$$

$$c^t(1-\lambda) \geq w(1-\theta) + \lambda c^x \tag{21}$$

$$\lambda c^t \geq (1-\lambda)c^x \tag{22}$$

$$(1-\lambda)c^x \geq 0 \tag{23}$$

$$\lambda c^t \leq (\theta-\gamma)w \tag{24}$$

$$\lambda c^t \leq (\beta-\alpha) \tag{25}$$

$$k \geq c^x \tag{26}$$

On substituting q_1, q_2 and q_3 from (9), (16) and (17) respectively into (4), the optimization problem is

$$\underset{(c^t,c^x)}{Max}\prod = c^t\left[\lambda + \frac{(1-\lambda)\left[\lambda c^x - c^t(1-\lambda) + w(1-\theta) + (1-\beta)\right]^3}{6w\lambda(1-\theta)(1-\beta)c^x} - \frac{\lambda\left[\left(\lambda c^t\right)^3 - \left(\lambda c^t - (1-\lambda)c^x\right)^3\right]}{6w(\beta-\alpha)(\theta-\gamma)(1-\lambda)c^x}\right] +$$
$$c^x\left[\lambda + \frac{(1-\lambda)\left[\left(\lambda c^t\right)^3 - \left(\lambda c^t - (1-\lambda)c^x\right)^3\right]}{6w(\beta-\alpha)(\theta-\gamma)(1-\lambda)c^x} - \frac{\lambda\left[\lambda c^x - c^t(1-\lambda) + w(1-\theta) + (1-\beta)\right]^3}{6w\lambda(1-\theta)(1-\beta)c^x}\right]$$

Subject to (18) – (26).

Case 2: $k \leq c^x$ - Some consumers prefer not entering the market.

In this case, we note that some consumers (with low reservation price r and low usage w) who choose the pay-per-volume pricing scheme could have negative surplus and would therefore not enter the market. Therefore for this case, we could have $q_1 + q_2 + q_3 < 1$, see Figure 3.

Under the conditions as denoted in equation (27) some customers prefer not entering to the market, because their reservation prices and their benefit of using network are not high enough to cover their bills.

$$v_3\left(x,t,r\right) < 0 \tag{27}$$

From the above equation we realize that because of the assumption, it is always negative on *x* edge at the lower end of the cube when *t* and *r* equals to zero. Therefore, the surface connecting $(0,0,0)$, $(1,t_4,0)$ and $(1,0,r_4)$ in Figure 3, represents the frontier at which the consumer is indifferent between choosing pay-per-volume pricing scheme and not entering the market. Then from two indifference equations, we obtain t_4, and r_4 as follows.

$$t_4 = \frac{c^x - k}{\gamma w} \tag{28}$$

$$r_4 = \frac{c^x - k}{\alpha} \tag{29}$$

Then, from the volume of the triangular pyramid shown in Figure 3, we get:

$$q_4 = \frac{(c^x - k)^2}{6\alpha\gamma w} \tag{30}$$

Also, q_1 and q_2 for case 2 is the same as defined in (8) and (17) in case 1, and therefore, q_3 for case 2 will be calculated as follows.

$$q_3 = 1 - q_1 - q_2 - q_4 \tag{31}$$

We assume $k \le c^x$ Also, since $0 \le x_1 \le 1$, $0 \le t_1 \le 1$, $0 \le r_1 \le 1$, $0 \le t_3 \le t_2 \le 1$, $0 \le r_3 \le r_2 \le 1$, $0 \le t_4 \le t_3 \le 1$ and $0 \le r_4 \le r_3 \le 1$, we get:

$$c^x \ge k \tag{32}$$

$$c^x w\left(\theta - \lambda\gamma\right) \le \lambda c^t \gamma w + kw\left(\theta - \gamma\right) \tag{33}$$

$$(1-\lambda)c^x \ge \lambda c^t - (\theta - \gamma)w \tag{34}$$

$$(\beta - \alpha\lambda)c^x \le \alpha\lambda c^t + (\beta - \alpha)k \tag{35}$$

$$(1-\lambda)c^x \ge \lambda c^t - (\beta - \alpha) \tag{36}$$

On substituting (9), (17) and (31) into (4), the optimization problem is

$$\underset{(c^t,c^x)}{Max}\prod = c^t\left[\lambda+\frac{(1-\lambda)\left[\lambda c^x-(1-\lambda)c^t+(1-\theta)w+(1-\beta)\right]^3}{6w\lambda(1-\theta)(1-\beta)c^x}-\lambda\frac{\left[\left(\lambda c^t\right)^3-\left(\lambda c^t-(1-\lambda)c^x\right)^3\right]}{6w(\beta-\alpha)(\theta-\gamma)(1-\lambda)c^x}\right]+$$

$$c^x\left[\lambda+\frac{(1-\lambda)\left[\left(\lambda c^t\right)^3-\left(\lambda c^t-(1-\lambda)c^x\right)^3\right]}{6w(\beta-\alpha)(\theta-\gamma)(1-\lambda)c^x}-\frac{\left(c^x-k\right)^2}{6\alpha\gamma w}-\frac{\lambda\left[\lambda c^x-(1-\lambda)c^t+(1-\theta)w+(1-\beta)\right]^3}{6w\lambda(1-\theta)(1-\beta)c^x}\right]$$

Subject to (18) – (26) and (32) – (36)

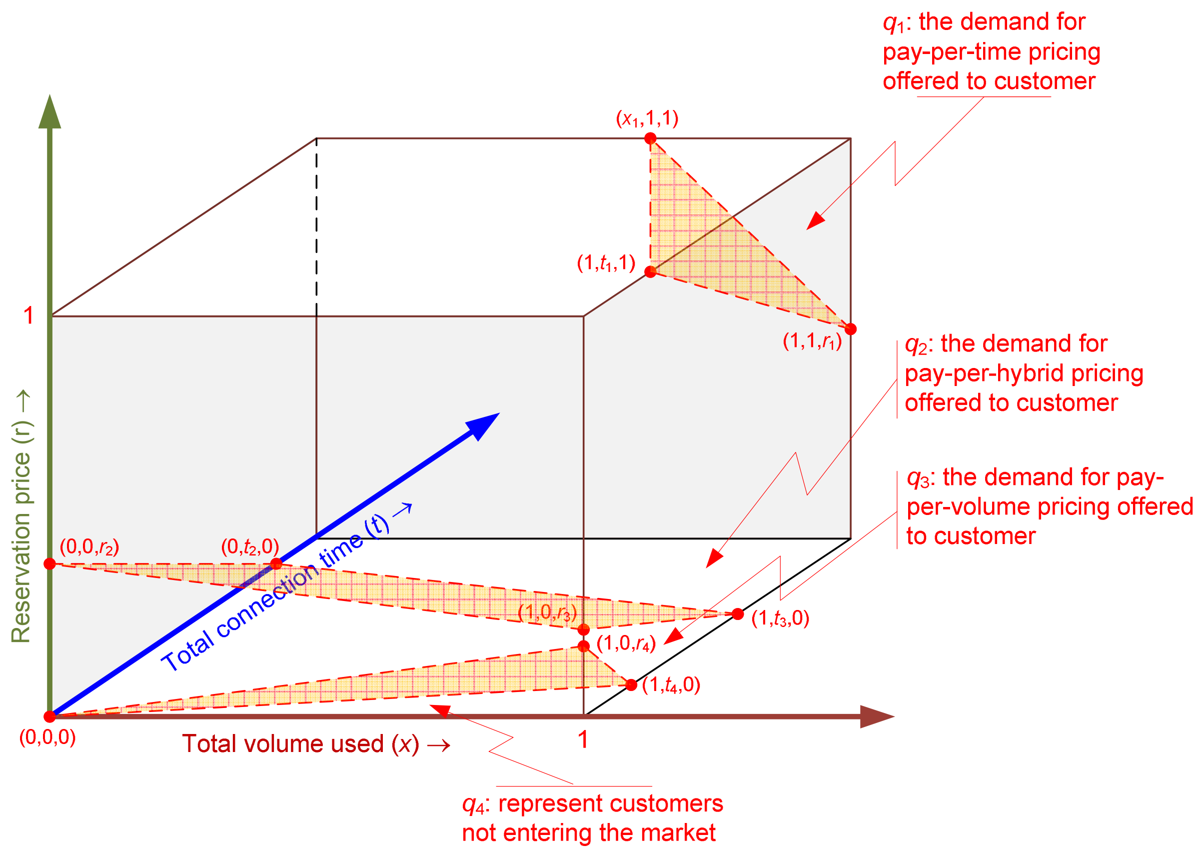

Figure 3: Consumer selections for Case 2

NUMERICAL STUDY AND SENSITIVITY ANALYSIS

It is difficult to tell what would be the optimal c^t and c^x when consumer perceptions change, since the provider could decrease price levels and maximize revenue by covering more population, or increase price levels by preventing the customers passing from a higher level price scheme to a lower level price scheme. We performed numerical study in order to understand the behavior of the provider against changing consumer perceptions, under the assumption that provider wants to minimize the number of pay-per time customers while increasing profit. Because, rational consumer behavior for pay-per time leasing may result in network bursting.

The numerical study is performed with a program written in GAMS. Firstly, we performed sensitivity analysis to asses the benefits of byte transmission per unit time, k. We fix w = 1 and select $\lambda = 0.48$, $\alpha = 0.10$, $\beta = 0.90$, $\gamma = 0.1$ and $\theta = 0.90$ as a base scenario. Then, analyze the trend of *k,* for different values of $\lambda = 0.50$, $\alpha = 0.17$, $\beta = 0.86$, $\gamma = 0.2$ and $\theta = 0.85$. Results of the scenario analyses for two different values of *k* (0.3 and 1.3) are shown in Table 1. Note that λ controls the price variation in hybrid scheme. Pricing will be close to volume based scheme when λ value is low. It will be close to time based scheme when λ value is high.

The managerial implications for the base scenario can be described as follows. In Table 1, we notice that $c^x = k$ in Case 1 for both values of k. In other words, the optimal price for transmitting unit volume equals to the benefits of transmitted volume for pay per volume pricing scheme for Case 1. Since all customers enter the market, it is optimal for the network provider to charge the highest per volume price without having the risk of losing any of the customers. In Figure A1, we observe that optimal prices and total revenue of the provider increases with the increasing values of k in both cases. In Figures A2 and A3, it is seen that while the number of customers choosing pay-per hybrid pricing schemes is decreasing, the number of customers choosing pay-per-volume pricing scheme increases until reaching the threshold value of k. After k reaches the threshold value (k = 1.4), some customers change their billing preferences from pay-per volume pricing scheme to pay-per hybrid or pay-per time pricing schemes. However, after a threshold value revenue acceleration rate goes down and converges to c^x (see Figure A1).

The results are similar to the base scenario when we increase the value of λ from 0.48 to 0.50. The revenues for Case 2 is higher than revenues for Case 1 in both scenarios (see Figures A1 and A4).

This means that for a monopoly, serving only to the customers with higher reservation prices can be more profitable than covering all the market. We also observe that q_{total} reaches 1 (100% of the consumer market) for lower values of k $\left(k = 1.4\right)$.

Table 1: Numerical results for *k*

$w = 1.0,\ \gamma = 0.10,\ \theta = 0.90$													
	λ	α	β	γ	θ	k	c^t	c^x	q_1	q_2	q_3	q_{total}	Revenue
Case 1	0.48	0.10	0.90	0.10	0.90	0.3	0.662	0.300	0.000	0.954	0.046	1.000	0.454
Case 1	0.48	0.10	0.90	0.10	0.90	1.3	1.585	1.300	0.000	0.831	0.169	1.000	1.370
Case 2	0.48	0.10	0.90	0.10	0.90	0.3	0.680	0.320	0.000	0.952	0.041	0.993	0.470
Case 2	0.48	0.10	0.90	0.10	0.90	1.3	1.594	1.310	0.000	0.829	0.169	0.998	1.378
Case 1	0.50	0.10	0.90	0.10	0.90	0.3	0.700	0.300	0.000	0.939	0.061	1.000	0.488
Case 1	0.50	0.10	0.90	0.10	0.90	1.3	1.600	1.300	0.003	0.793	0.204	1.000	1.420
Case 2	0.50	0.10	0.90	0.10	0.90	0.3	0.725	0.325	0.000	0.936	0.053	0.990	0.509
Case 2	0.50	0.10	0.90	0.10	0.90	1.3	1.600	1.313	0.005	0.793	0.199	0.997	1.423
Case 1	0.48	0.17	0.90	0.10	0.90	0.3	0.662	0.300	0.000	0.949	0.051	1.000	0.453
Case 1	0.48	0.17	0.90	0.10	0.90	1.3	1.521	1.300	0.001	0.835	0.164	1.000	1.345
Case 2	0.48	0.17	0.90	0.10	0.90	0.3	0.680	0.320	0.000	0.947	0.049	0.996	0.470
Case 2	0.48	0.17	0.90	0.10	0.90	1.3	1.521	1.306	0.001	0.835	0.163	1.000	1.348
Case 1	0.48	0.10	0.86	0.10	0.90	0.3	0.738	0.300	0.000	0.936	0.064	1.000	0.486
Case 1	0.48	0.10	0.86	0.10	0.90	1.3	1.583	1.300	0.001	0.821	0.178	1.000	1.369
Case 2	0.48	0.10	0.86	0.10	0.90	0.3	0.761	0.325	0.000	0.933	0.056	0.990	0.505
Case 2	0.48	0.10	0.86	0.10	0.90	1.3	1.583	1.310	0.002	0.822	0.175	0.998	1.372
Case 1	0.48	0.10	0.90	0.20	0.90	0.3	0.662	0.300	0.000	0.947	0.053	1.000	0.453
Case 1	0.48	0.10	0.90	0.20	0.90	1.3	1.458	1.300	0.008	0.841	0.151	1.000	1.321
Case 2	0.48	0.10	0.90	0.20	0.90	0.3	0.680	0.320	0.000	0.945	0.052	0.997	0.470
Case 2	0.48	0.10	0.90	0.20	0.90	1.3	1.458	1.303	0.008	0.841	0.150	1.000	1.320
Case 1	0.48	0.10	0.90	0.10	0.85	0.3	0.758	0.300	0.000	0.930	0.070	1.000	0.493
Case 1	0.48	0.10	0.90	0.10	0.85	1.3	1.562	1.300	0.004	0.823	0.173	1.000	1.362
Case 2	0.48	0.10	0.90	0.10	0.85	0.3	0.781	0.326	0.000	0.928	0.061	0.989	0.513
Case 2	0.48	0.10	0.90	0.10	0.85	1.3	1.562	1.309	0.005	0.823	0.171	0.999	1.363

By comparing Figures A2 and A5, and Figures A3 and A6, we observe that while λ is having higher values, number of pay-per volume customers decreases, but the number of pay-per time and pay-per hybrid customers increases. This means that existence of hybrid pricing scheme gives the opportunity for increasing both c^x and c^t, and total revenue, while preventing too much customers passing to the pay-per time pricing scheme and network bursting. In other words, the existence of hybrid pricing prevents network bursting.

For $\alpha = 0.17$, the results are given in Figures A7, A8 and A9 in appendix. The results are similar to the base scenario where $\lambda = 0.48$ and $\alpha = 0.1$. By comparing Figures A1 and A7, we observe that q_{total} reaches 1 for lower values of k $(k = 1.3)$. Because consumers' inclination to choose hybrid pricing scheme and pay-per time pricing strategy is higher as α gets higher. Also, by comparing Figures A2 and A8, and Figures A3 and A9, we observe that for higher values of α, decreasing price levels results in some pay-per volume customers switch to the pay-per hybrid and pay-per time pricing schemes, and network provider gaining more customers from the entire population and market share increases since more nonusers enters the market. However, for most of the customers changing their billing choice, pay-per hybrid pricing scheme is more beneficial than pay-per time pricing scheme. As a consequence it can be said that, when the level of c^t decreased, the existence of hybrid pricing scheme prevents network bursting by preventing too much customers choosing pay-per time pricing scheme against pay-per volume pricing scheme.

Comparing Figures A1 and A10, we observe that for the low values of k, price levels and total revenue of the provider increases while β ($\beta = 0.86$) getting smaller. Again by comparing Figure A2 and A11, and Figure A3 and A12, we see that as a result of increasing prices some customers switch from pay-per volume pricing scheme to pay-per hybrid pricing scheme. If the provider would not have hybrid pricing scheme, probably these customers would maintain their billing policy and the profit of the provider would decrease. However, for the high values of k, the provider chooses decreasing the level of c^t, while maintaining c^x in order to minimize its profit lost by letting some customers switch from pay-per volume pricing scheme to others. In this situation, preferred behavior of the provider is taking some risk of network bursting. However, for the smaller values of k, q_3 is higher and, q_2 is lower in both cases. This is intuitive since the smaller the value of β, the smaller the difference in the benefit of pay-per hybrid pricing and pay-per-volume pricing. As a consequence, more customers choose the cheapest option. The optimal behavior of the provider would be to decrease the levels of c^t and c^x in Case 2, for the consumers' profile having low k values and high tendency to choose volume based pricing rather than selecting the other two pricing schemes.

By comparing Figures A1 and A13, we observe that price levels in both base scenario and $\gamma = 0.2$ scenario are same for low k values. However, for high k values, price levels decrease, when γ gets higher. When we compare Figures A2 and A14, and Figures A3 and A15, we observe that when γ gets higher; provider tries to cover all the market. Moreover, the provider has tendency to increase the number of its pay-per volume customers when k gets lower. However, when k gets higher the provider has tendency to increase its pay-per hybrid and pay-per time pricing schemes customers by decreasing its price levels and tries to minimize its profit loss.

When we compare Figures A2 and A17, and Figures A3 and A18, we observe that when θ gets lower; the number of pay-per hybrid customers decrease while others increase. Since pay-per hybrid pricing scheme is more profitable than pay-per volume pricing scheme and has less network bursting risk than pay-per time pricing scheme, the provider has to adjust its price levels in order to prevent its revenue loss, decrease it bursting risk. By comparing Figures A1 and A16, we observe that, for low k values, optimum behavior for the provider would be increasing its price levels in order to increase its revenues. However, for high k values, optimum behavior for the provider would be decreasing its price levels in order to decrease its network bursting risk.

We also performed sensitivity analysis to assess the benefits of unit connection time, w. We fixed k = 0.1 and select $\lambda = 0.48$, $\alpha = 0.10$, $\beta = 0.90$, $\gamma = 0.1$ and $\theta = 0.90$ as a base scenario. Then, analyzed the trend of *w,* for different values of $\lambda = 0.50$, $\alpha = 0.17$, $\beta = 0.86$, $\gamma = 0.2$ and $\theta = 0.85$. Results of the scenario analyses for different two values of *w* (0.5 and 5) are shown in Table 2.

Table 2: Numerical results for *w*

$k = 0.1$													
	λ	α	β	γ	θ	w	c^t	c^x	q_1	q_2	q_3	q_{total}	Revenue
Case 1	0.48	0.10	0.90	0.10	0.90	0.50	0.381	0.100	0.000	0.961	0.039	1.000	0.226
Case 1	0.48	0.10	0.90	0.10	0.90	5.00	1.246	0.100	0.000	0.949	0.051	1.000	0.618
Case 2	0.48	0.10	0.90	0.10	0.90	0.50	0.396	0.116	0.000	0.960	0.032	0.992	0.239
Case 2	0.48	0.10	0.90	0.10	0.90	5.00	1.309	0.168	0.000	0.946	0.038	0.984	0.667
Case 1	0.50	0.10	0.90	0.10	0.90	0.50	0.400	0.100	0.000	0.952	0.048	1.000	0.243
Case 1	0.50	0.10	0.90	0.10	0.90	5.00	1.300	0.100	0.000	0.939	0.061	1.000	0.663
Case 2	0.50	0.10	0.90	0.10	0.90	0.50	0.419	0.119	0.000	0.949	0.039	0.988	0.260
Case 2	0.50	0.10	0.90	0.10	0.90	5.00	1.375	0.175	0.000	0.935	0.046	0.981	0.733
Case 1	0.48	0.17	0.90	0.10	0.90	0.50	0.381	0.100	0.000	0.957	0.043	1.000	0.225
Case 1	0.48	0.17	0.90	0.10	0.90	5.00	1.246	0.100	0.000	0.944	0.056	1.000	0.616
Case 2	0.48	0.17	0.90	0.10	0.90	0.50	0.396	0.116	0.000	0.956	0.039	0.995	0.239
Case 2	0.48	0.17	0.90	0.10	0.90	5.00	1.309	0.168	0.000	0.941	0.050	0.991	0.675
Case 1	0.48	0.10	0.86	0.10	0.90	0.50	0.458	0.100	0.000	0.938	0.062	1.000	0.257
Case 1	0.48	0.10	0.86	0.10	0.90	5.00	1.323	0.100	0.000	0.939	0.061	1.000	0.647
Case 2	0.48	0.10	0.86	0.10	0.90	0.50	0.477	0.121	0.000	0.935	0.050	0.985	0.274
Case 2	0.48	0.10	0.86	0.10	0.90	5.00	1.390	0.172	0.000	0.936	0.047	0.983	0.710
Case 1	0.48	0.10	0.90	0.20	0.90	0.50	0.381	0.100	0.000	0.956	0.044	1.000	0.225
Case 1	0.48	0.10	0.90	0.20	0.90	5.00	1.246	0.100	0.000	0.942	0.058	1.000	0.614
Case 2	0.48	0.10	0.90	0.20	0.90	0.50	0.396	0.116	0.000	0.954	0.042	0.996	0.239
Case 2	0.48	0.10	0.90	0.20	0.90	5.00	1.309	0.168	0.000	0.939	0.053	0.992	0.674
Case 1	0.48	0.10	0.90	0.10	0.85	0.50	0.429	0.100	0.000	0.946	0.054	1.000	0.245
Case 1	0.48	0.10	0.90	0.10	0.85	5.00	1.667	0.100	0.001	0.899	0.100	1.000	0.774
Case 2	0.48	0.10	0.90	0.10	0.85	0.50	0.446	0.119	0.000	0.944	0.044	0.988	0.261
Case 2	0.48	0.10	0.90	0.10	0.85	5.00	1.667	0.188	0.010	0.896	0.068	0.974	0.822

The managerial implications for the results are as follows. In Table 2, we notice similar results for the analysis of k where $c^x = k$ in Case 1. Since all customers enter the market, it is optimal for the network provider to charge the highest per volume price without the risk of losing any of the customers. We also observe that the revenue for the provider increases with the increasing values of w. This is intuitive because as the benefit in per-connected time to the consumer gets higher, the provider would charge a higher price and generate more revenue.

The results of base scenario are given in Figures A19, A20 and A21 in appendix. Differently from k analysis, we notice that for all values of w, $c^x > k$ and $q_1 + q_2 + q_3 < 1$ in Case 2. In Figure A20 and A21, we see that the number of pay-per volume and pay-per hybrid customers in case 2 are lower than the number of pay-per volume and pay-per hybrid customers in case 1. For very low values of k, it does not matter what the value of w is since populations having low reservation prices also have low benefit in per-connection time. However, for larger values of *w*, the number of pay-per time customers in case 2 is bigger than the number of pay-per time customers in case 1. It is interesting, because if the price levels are high such as in case 2, the customers having high *w* value still prone choosing pay-per time option, despite their low reservation prices.

The results are given in Figures A22, A23 and A24 in appendix for $\lambda = 0.50$. By comparing base scenario with the scenario, where $\lambda = 0.50$, we observe that when λ gets higher, price levels and revenues increase in both cases. While λ is getting higher in Case 2, in spite of losing some customers, total revenue of the provider would also increase because of increasing percentage of pay-per volume customers as well as the increase in pay-per time and pay-per volume price levels. This is an interesting result because pay-per volume price level is lower than pay-per hybrid price level. However, a higher value of λ implies that the hybrid pricing will be close to the time based scheme; hence serving pay-per hybrid pricing option gives the provider an opportunity to increase its price levels in order to prevent profit loss caused by decreasing number of total customers.

The results are given in Figures A25, A26 and A27 for $\alpha = 0.17$. The results are similar to the base scenario. However, we observe that even the price levels don't change, the value of $\left(q_1 + q_2 + q_3\right)$ gets bigger. Because, the consumers' inclination to pay per volume pricing strategy (as compared to other pricing strategies) increases, and the provider would use this chance to gain new customers from the whole population by maintaining price levels. Hence, having more pay-per volume customers hedges the provider against network bursting.

In Figures A28, A29 and A30 in appendix, we observe that for smaller values of β increasing price levels of c^t and c^x results in customers to change their pricing schemes to the cheaper one. The optimal behavior of the provider is to take some risk of losing some customers with low profitability in order to increase profit margin of all billing options, and hence to increase its total revenue. However, for the smaller values of β, q_3 gets bigger and, q_1 and q_2 gets smaller in both cases. While value of β getting smaller, the difference in the benefit of pay-per hybrid pricing and pay-per-time pricing choices gets higher. However, the smaller is the value of β, the smaller would the difference be in the benefit of pay-per hybrid pricing and pay-per-volume pricing choices. Thus, customers shift to cheaper billing choice, and most customers choose the cheapest option in both Cases. In order to increase its revenue, for larger values of *w*, the preferred behavior of the provider would be to increase the levels of c^t and c^x, which contradicts the behavior observed in the sensitivity analysis of large k values with respect to the decrease in β. In spite of the decrease in consumers' inclination in pay-per hybrid pricing scheme, having such an alternative price option that is indexed to both c^x and c^t enables the provider to take the advantage of increasing its revenue. Since, there were no hybrid price options when c^t increase, the pay-per time customers would have to choice pay-per volume option. Therefore, the provider would have to set c^x in a much higher level in order to preserve its revenue.

The results are given in Figures A31, A32 and A33 in appendix for $\gamma = 0.20$. The optimum price levels are same with the base scenario. However, since γ is larger in this scenario, we observe that the value of $\left(q_1 + q_2 + q_3\right)$ is bigger, in Case 2. Intuitively, if the pay-per volume consumer's benefit of unit connection time increases, the provider would use this chance to gain new customers from the whole population. However, when the consumer's benefit of unit connection time increases for pay-per volume pricing option, it gets closer to the unit connection time benefit of pay-per hybrid billing option. And, if the provider does not change the price levels, some pay-per hybrid billing customers would shift to the pay-per volume pricing option. But still remaining price levels can be an optimum behavior for the provider since, having more pay-per volume customers hedges the provider against network bursting.

In Figures A34, A35 and A36 in appendix, we observe that for smaller values of θ, increasing price levels of c^t and c^x results in pay per hybrid customers (having high w value) to change their pricing schemes to the other two options, in both Case 1 and Case 2. The optimal behavior of the provider is to take some risk of network bursting, that is to have some pay-per-time customers who have highest profit margin to increase its revenue. However, for the smaller values of w, q_3 is higher and, q_2 is lower in this scenario compared to base scenario in both cases. This is intuitive since for the smaller value of θ, the smaller is the difference in the benefit of unit connection time for pay-per hybrid pricing and pay-per-volume pricing choices, and as a consequence of this situation more customers choose the cheapest option. For the consumers' profile having low w values and high tendency to choose volume based pricing rather than selecting the other two pricing schemes, again, the optimal behavior of the provider would be to increase the levels of c^t and c^x. Because unit connection time benefit of the consumers is low, not many nonusers would be interested on entering the market despite the low price levels. Therefore, increasing price levels in order to prevent too much migration from pay-per hybrid pricing scheme to pay-per volume pricing scheme is more profitable for the provider, than decreasing price levels in order to gain more customers.

COMPARISON THE PRICING MODEL WITH NO HYBRID PRICING SCHEME

In this section we compare the results for the case where the provider offers only the pay per time and pay per volume pricing schemes. Let q_1 again be the total demand of the consumers choosing pay per time billing option, and q_4 again be the volume of the customers not entering the market. Note that the consumer selection for this scenario is similar to the ones in Figure 2 and Figure 3 for the Case 1 and 2 with q_3 only (since there is no hybrid scheme, there is no q_2). Hence, q_3 becomes $q_3 = 1 - q_1$ for Case

1, and $q_3 = 1 - q_1 - q_4$ for Case 2. Let the surface connecting the three points $(x_1, 1, 1)$, $(1, t_1, 1)$ and $(1, 1, r_1)$ represent the frontier at which the consumer is indifferent between pay-per-time and pay-per-volume pricing schemes. Using (1) and (3), from three indifference equations, we obtain x_1, t_1, and r_1 as follows.

$$x_1 = \frac{c^t - w(1-\theta) - (1-\alpha)}{c^x} \tag{37}$$

$$t_1 = \frac{(c^t - c^x) - (1-\alpha)}{w(1-\theta)} \tag{38}$$

$$r_1 = \frac{(c^t - c^x) - w(1-\theta)}{(1-\alpha)} \tag{39}$$

Then, from the volume of the triangular pyramid shown in Figure 2, we get:

$$q_1 = \frac{[c^x - c^t + w(1-\theta) + (1-\alpha)]^3}{6w(1-\theta)(1-\alpha)c^x} \tag{40}$$

and $q_3 = 1 - q_1$ (41)

Also, since $0 \le x_1 \le 1$, $0 \le t_1 \le 1$ and $0 \le r_1 \le 1$, we get:

$$c^t \ge w(1-\theta) + (1-\alpha) \tag{42}$$

$$c^t \le w(1-\theta) + (1-\alpha) + c^x \tag{43}$$

$$c^t \ge (1-\alpha) + c^x \tag{44}$$

$$c^t \ge w(1-\theta) + c^x \tag{45}$$

For Case 1, substituting q_1 and q_3 from (40) and (41) respectively into (4), the optimization problem becomes

$$\underset{(c^t, c^x)}{Max} \prod = \frac{(c^t - c^x - 1 + \alpha)^3}{6c^x(1-\alpha)} + \frac{6c^t c^x (1-\alpha) - (c^t - c^x - 1 + \alpha)^3}{6c^t(1-\alpha)} \tag{46}$$

Subject to (42) – (45)

The surface connecting $(0,0,0)$, $(1,t_4,0)$ and $(1,0,r_4)$ represents the frontier at which the consumer is indifferent between choosing pay-per-volume pricing scheme and not entering the market (see Figure 3). Then from two indifference equations, we obtain t_4, and r_4 as follows.

$$t_4 = \frac{c^x - k}{\theta w} \tag{47}$$

$$r_4 = \frac{c^x - k}{\alpha} \tag{48}$$

Then, from the volume of the triangular pyramid shown in Figure 3, we get:

$$q_4 = \frac{(c^x - k)^2}{6\alpha\theta w} \tag{49}$$

Also, q_1 is the same as defined in (40), and therefore

$$q_3 = 1 - q_1 - q_4 \tag{50}$$

Since $0 \le t_4 \le 1$ and $0 \le r_4 \le 1$, the following conditions must hold:

$$c^x \ge k \tag{51}$$

$$c^x \le k + \theta w \tag{52}$$

$$c^x \le k + \alpha \tag{53}$$

For Case 2, substituting (40) and (50) into (4), the optimization problem becomes

$$\underset{(c^t,c^x)}{Max} \prod = \left(c^t - c^x\right)\frac{[c^x - c^t + w(1-\theta) + (1-\alpha)]^3}{6w(1-\theta)(1-\alpha)c^x} + c^x\left(1 - \frac{(c^x - k)^2}{6\alpha\theta w}\right)$$

Subject to (42) – (45) and (51) – (53)

In order to make a comparison between with and without hybrid pricing scheme, we present the results in Table 3 which are partially obtained from numerical examples presented previously where, $\alpha = 0.10$, $\gamma = 0.10$ for the values of $k = 0.4$, $k = 0.8$ and $k = 1.4$.

Table 3. Comparison of the results for the pricing strategies with and without pay-per hybrid pricing

		α	γ	k	c^t	c^x	q_1	q_2	q_3	q_{total}	Revenue
$w = 1.0$											
3 Pricing Option	Case 1	0.10	0.10	0.4	0.754	0.400	0.000	0.945	0.055	1.000	0.545
	Case 1	0.10	0.10	0.8	1.123	0.800	0.000	0.903	0.097	1.000	0.911
	Case 1	0.10	0.10	1.4	1.667	1.400	0.000	0.817	0.183	1.000	1.459
	Case 2	0.10	0.10	0.4	0.771	0.419	0.000	0.944	0.050	0.994	0.560
	Case 2	0.10	0.10	0.8	1.137	0.815	0.000	0.901	0.095	0.996	0.922
	Case 2	0.10	0.10	1.4	1.667	1.408	0.000	0.818	0.181	0.999	1.462
2 Pricing Option	Case 1	0.10	0.10	0.4	1.800	0.400	0.033	-	0.967	1.000	0.446
	Case 1	0.10	0.10	0.8	1.800	0.800	0.132	-	0.868	1.000	0.932
	Case 1	0.10	0.10	1.4	2.300	1.400	0.107	-	0.893	1.000	1.496
	Case 2	0.10	0.10	0.4	1.800	0.472	0.046	-	0.868	0.914	0.492
	Case 2	0.10	0.10	0.8	1.800	0.841	0.146	-	0.826	0.972	0.957
	Case 2	0.10	0.10	1.4	2.320	1.420	0.106	-	0.888	0.993	1.506
3 Pricing Option	Case 1	0.17	0.10	0.4	0.754	0.400	0.000	0.940	0.060	1.000	0.545
	Case 1	0.17	0.10	0.8	1.123	0.800	0.000	0.894	0.106	1.000	0.910
	Case 1	0.17	0.10	1.4	1.521	1.400	0.013	0.834	0.153	1.000	1.403
	Case 2	0.17	0.10	0.4	0.771	0.419	0.000	0.938	0.058	0.996	0.561
	Case 2	0.17	0.10	0.8	1.137	0.815	0.000	0.892	0.106	0.998	0.922
	Case 2	0.17	0.10	1.4	1.521	1.400	0.013	0.834	0.153	1.000	1.398
2 Pricing Option	Case 1	0.17	0.10	0.4	1.730	0.400	0.036	-	0.964	1.000	0.447
	Case 1	0.17	0.10	0.8	1.730	0.800	0.143	-	0.857	1.000	0.686
	Case 1	0.17	0.10	1.4	2.300	1.400	0.091	-	0.909	1.000	1.482
	Case 2	0.17	0.10	0.4	1.730	0.500	0.056	-	0.846	0.902	0.520
	Case 2	0.17	0.10	0.8	1.749	0.849	0.150	-	0.826	0.976	0.964
	Case 2	0.17	0.10	1.4	2.333	1.433	0.089	-	0.900	0.989	1.498

We can notice that for smaller values of k offering pay-per hybrid billing option with pay-per time and pay-per volume billing options is more profitable for the provider for both Case 1 and Case 2. Also, market size for the provider increases by offering pay-per hybrid pricing scheme for smaller values of k in Case 2. For larger values of k, revenue of the provider increases because, some of the pay-per hybrid pricing customers with high benefit of transmitting unit volume shift to the pay-per time pricing option in the absence of this pricing option. However, market size for the provider decreases when it does not offer pay-per hybrid pricing scheme to the customers. Therefore, by offering the pay-per hybrid pricing option, the provider is able to get more consumers to enter the market (see Figure A6). This illustrates that judicious use of pay-per hybrid scheme is not only beneficial to the provider, but it also increases the market size.

For the higher values α, we notice that offering pay-per hybrid billing option with pay-per time and pay-per volume billing options is more profitable for the wider range of k in Case 1. Also, market size for the provider increases by offering pay-per hybrid pricing scheme for wider range of the values of k in Case 2. For larger values of k revenue of the provider increases, because more customers have a tendency to choose pay-per time pricing scheme than the pricing strategy without pay-per hybrid pricing scheme since their benefit of transmitting unit volume increases as the value of α gets higher. Hence, the provider increases levels of c^x and c^t. As a result of high prices, fewer customers enter the market.

Also, in Tables 1 and 3 it can be seen that market size for the provider decreases when it does not offer pay-per hybrid pricing scheme to the customers while α getting higher. Therefore by offering the pay-per hybrid pricing option, the provider is able to get more revenue and more consumers to enter the market for wider range of the values of k. This illustrates that judicious use of pay-per hybrid scheme is not only beneficial to the provider, but it also increases the market size.

In order to make a comparison between with and without hybrid pricing scheme, we present the results in Table 4 which are obtained from the numerical example presented previously where $\alpha = 0.10$, $\gamma = 0.10$ for the values of $w = 0.35$, $w = 1.0$ and $w = 3.5$.

We notice that for all values of w, offering hybrid billing option is more profitable for the provider for both Case 1 and Case 2. Also, market size for the provider increases by offering hybrid pricing scheme for all values of w in Case 2.

For the higher values α, we also notice that offering hybrid billing option is more profitable for all values of w, in both Case 1 and 2. Also, market size for the provider increases by offering pay-per hybrid pricing scheme in Case 2. Revenue of the provider increases, because as it can be seen in Table 4 more customers would have a tendency to choose pay-per time pricing scheme than the pricing strategy without pay-per hybrid pricing scheme at $\alpha = 0.1$, since their benefit of transmitting unit volume increases as α gets bigger. Thus the provider maintains level of c^x and decreases level of c^t. Hence, more customers enter the market s α gets bigger.

Also, in Tables 2 and 4 it can be seen that market size for the provider decreases when it does not offer pay-per hybrid pricing scheme to the customers as α gets bigger. Therefore by offering the pay-per hybrid pricing option, the provider is able to get more revenue and more consumers to enter the market for all values of w. This also illustrates that judicious use of pay-per hybrid scheme is not only beneficial to the provider, but it also increases the market size.

CONCLUSIONS AND FUTURE RESEARCH

Hybrid and pay per volume pricing schemes are the next generation strategies that have the promise to prevent network busting and quality related customer dissatisfaction. These strategies facilitate leasing excess capacity to other customers, hence maximizing network usage. From customer perspective, the hybrid pricing scheme is useful for those having fewer tasks with low total volume and relatively high reservation price (as compared to customers who choose pay per volume scheme). The pay per volume pricing scheme is useful for those customers having generally size fixed tasks and/or low reservation price. Thus, the provider can increase revenue by setting higher prices by offering pay-per-time pricing scheme to those customers who use data networks too often, having tasks with more volume or having more time-fixed tasks.

The main focus of the study is to show that hybrid scheme is a viable and often preferable pricing option to the network provider. This point is illustrated by the optimization models, maximizing provider's revenue when hybrid pricing strategy exists, as it compared to the situation where it does not exist.

Table 4. Comparison of the results of the pricing strategies with pay-per hybrid pricing and without pay-per hybrid pricing option for different values of w

$k = 0.1$											
		α	γ	w	c^t	c^x	q_1	q_2	q_3	q_{total}	Revenue
3 Pricing Option	Case 1	0.10	0.10	0.35	0.370	0.100	0.000	0.942	0.058	1.000	0.227
	Case 1	0.10	0.10	1.00	0.500	0.100	0.000	0.960	0.040	1.000	0.292
	Case 1	0.10	0.10	3.50	1.000	0.100	0.000	0.950	0.050	1.000	0.527
	Case 2	0.10	0.10	0.35	0.387	0.117	0.000	0.930	0.063	0.993	0.242
	Case 2	0.10	0.10	1.00	0.525	0.125	0.000	0.952	0.043	0.995	0.315
	Case 2	0.10	0.10	3.50	1.056	0.156	0.000	0.939	0.054	0.993	0.578
2 Pricing Option	Case 1	0.10	0.10	0.35	1.215	0.100	0.006	-	0.994	1.000	0.107
	Case 1	0.10	0.10	1.00	1.800	0.100	0.002	-	0.998	1.000	0.103
	Case 1	0.10	0.10	3.50	4.050	0.100	0.001	-	0.999	1.000	0.102
	Case 2	0.10	0.10	0.35	1.215	0.135	0.011	-	0.931	0.942	0.139
	Case 2	0.10	0.10	1.00	1.800	0.200	0.008	-	0.825	0.833	0.180
	Case 2	0.10	0.10	3.50	4.050	0.200	0.002	-	0.950	0.952	0.200
3 Pricing Option	Case 1	0.20	0.10	0.35	0.370	0.100	0.000	0.934	0.066	1.000	0.226
	Case 1	0.20	0.10	1.00	0.500	0.100	0.000	0.955	0.045	1.000	0.291
	Case 1	0.20	0.10	3.50	1.000	0.100	0.000	0.942	0.058	1.000	0.524
	Case 2	0.20	0.10	0.35	0.387	0.117	0.000	0.930	0.063	0.993	0.242
	Case 2	0.20	0.10	1.00	0.525	0.125	0.000	0.952	0.043	0.995	0.315
	Case 2	0.20	0.10	3.50	1.056	0.156	0.000	0.939	0.054	0.993	0.578
2 Pricing Option	Case 1	0.20	0.10	0.35	1.115	0.100	0.007	-	0.993	1.000	0.107
	Case 1	0.20	0.10	1.00	1.700	0.100	0.002	-	0.998	1.000	0.104
	Case 1	0.20	0.10	3.50	3.950	0.100	0.001	-	0.999	1.000	0.103
	Case 2	0.20	0.10	0.35	1.115	0.135	0.012	-	0.959	0.971	0.143
	Case 2	0.20	0.10	1.00	1.700	0.200	0.009	-	0.907	0.917	0.197
	Case 2	0.20	0.10	3.50	3.950	0.300	0.006	-	0.899	0.905	0.293

In conclusion, this paper shows that (i) the offering hybrid pricing option can increase the profitability of the network provider, (ii) it can increase the overall market size, and (ii) when all three strategies are offered, it is possible to formulate an optimal pricing strategy that maximizes the provider's revenue. A natural extension of this research is to study pricing decisions with multiple network providers in a game-theoretic problem formulation. Hence, how the competition affects the prices set by the providers could be examined.

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APPENDIX

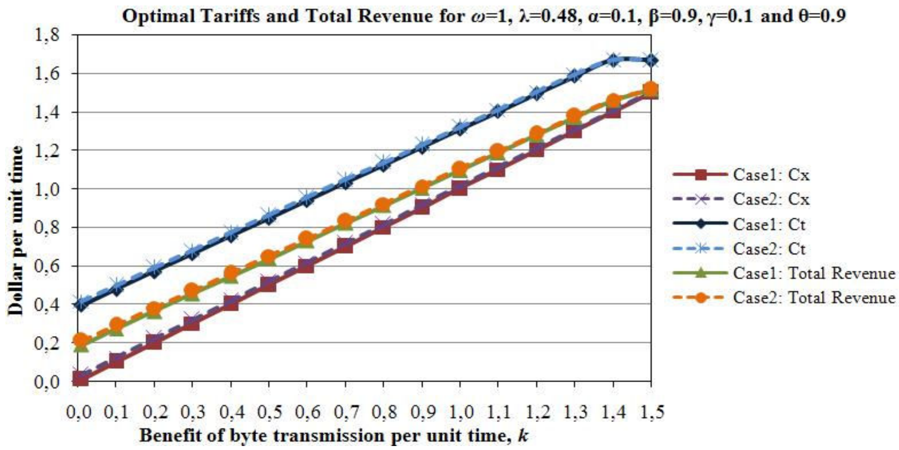

Figure A1: Optimal tariffs and optimal revenues for $\lambda = 0.48$, $\alpha = 0.1$, $\beta = 0.9$, $\gamma = 0.1$ and $\theta = 0.9$

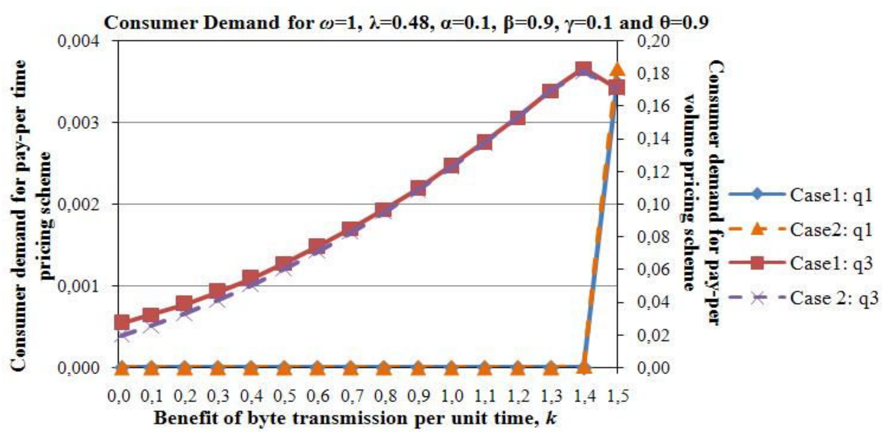

Figure A2: Consumer demands for $\lambda = 0.48$, $\alpha = 0.1$, $\beta = 0.9$, $\gamma = 0.1$ and $\theta = 0.9$

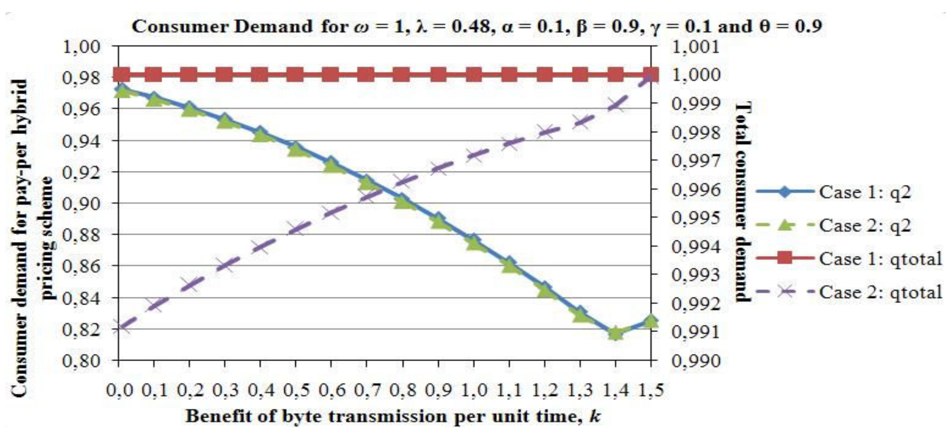

Figure A3: Consumer demands and total demands for $\lambda = 0.48$, $\alpha = 0.1$, $\beta = 0.9$, $\gamma = 0.1$ and $\theta = 0.9$

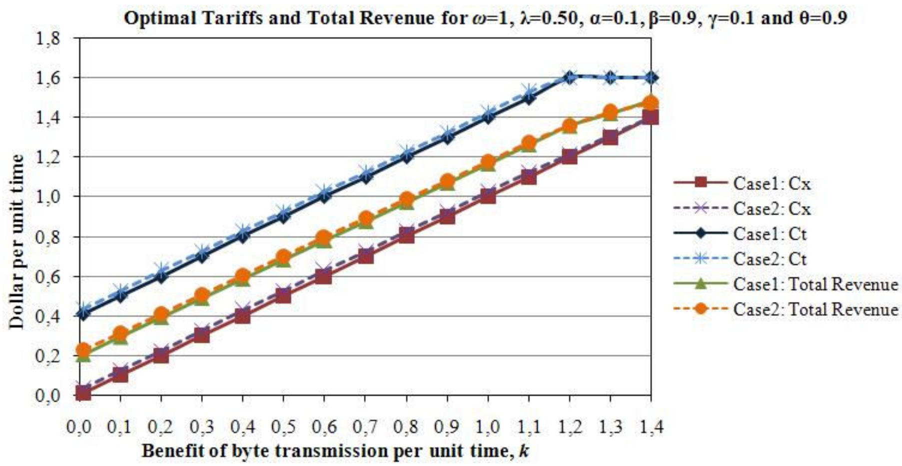

Figure A4: Optimal tariffs and optimal revenues for $\lambda = 0.50$, $\alpha = 0.1$, $\beta = 0.9$, $\gamma = 0.1$ and $\theta = 0.9$

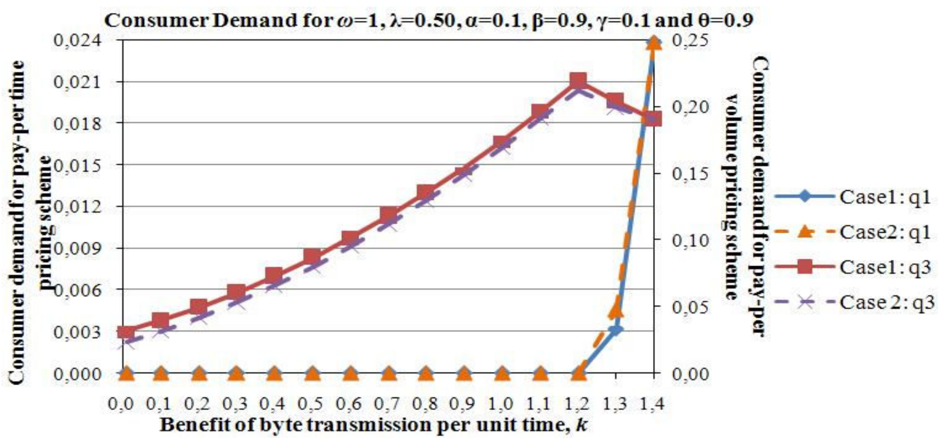

Figure A5: Consumer demands for $\lambda = 0.50$, $\alpha = 0.1$, $\beta = 0.9$, $\gamma = 0.1$ and $\theta = 0.9$

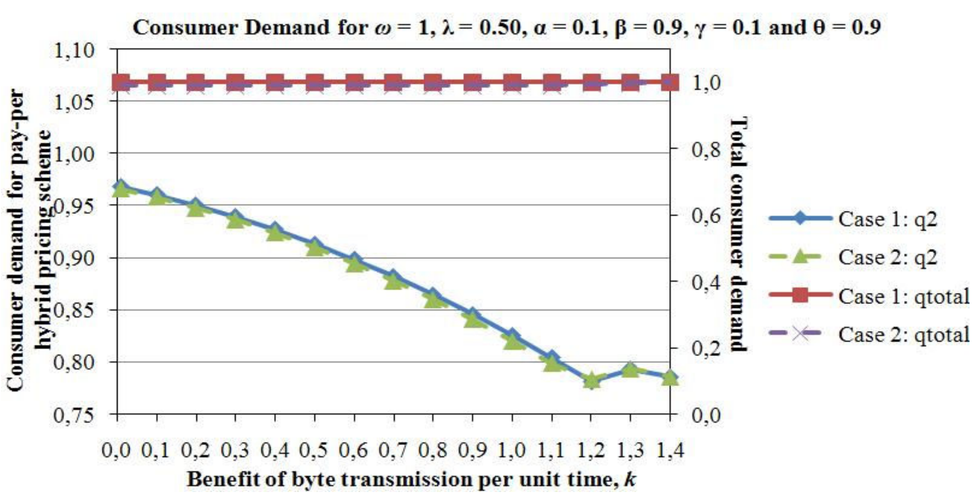

Figure A6: Consumer demands and total demands for $\lambda = 0.50$, $\alpha = 0.1$, $\beta = 0.9$, $\gamma = 0.1$ and $\theta = 0.9$

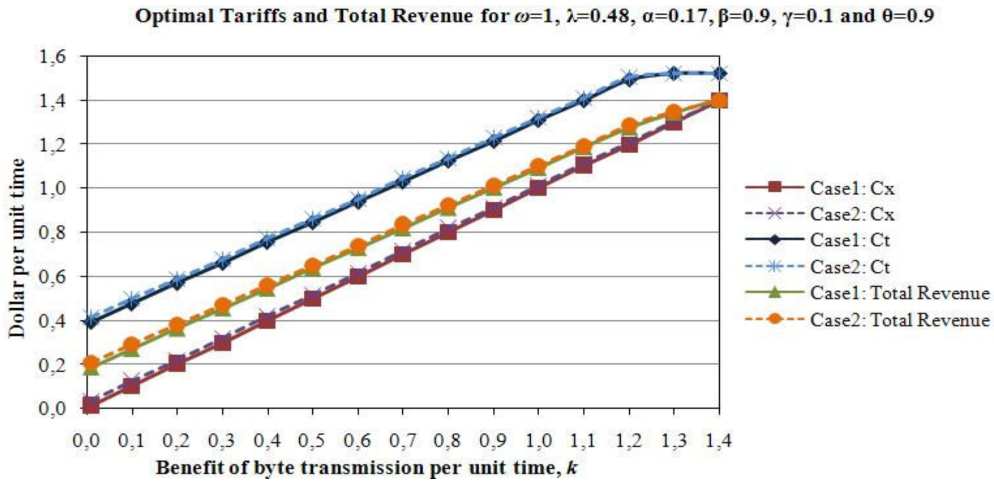

Figure A7: Optimal tariffs and optimal revenues for $\lambda = 0.48$, $\alpha = 0.17$, $\beta = 0.9$, $\gamma = 0.1$ and $\theta = 0.9$

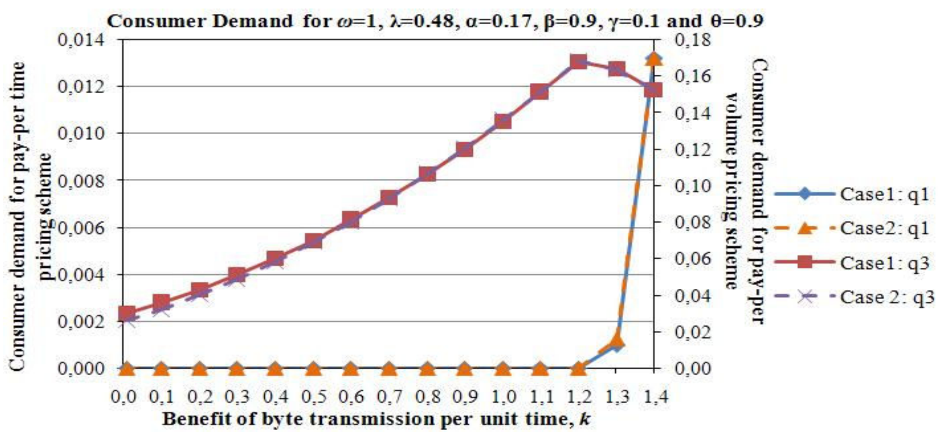

Figure A8: Consumer demands for $\lambda = 0.48$, $\alpha = 0.17$, $\beta = 0.9$, $\gamma = 0.1$ and $\theta = 0.9$

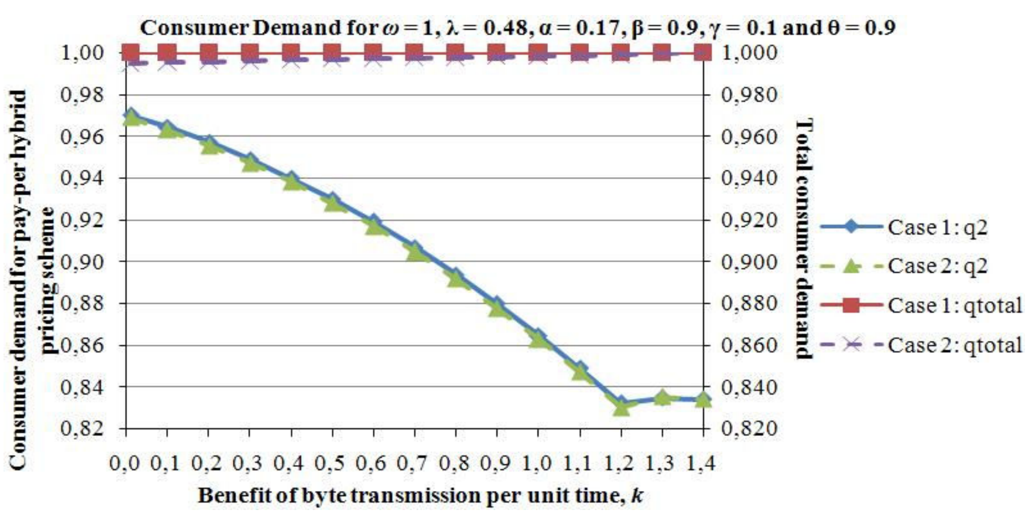

Figure A9: Consumer demands and total demands for $\lambda = 0.48$, $\alpha = 0.17$, $\beta = 0.9$, $\gamma = 0.1$ and $\theta = 0.9$

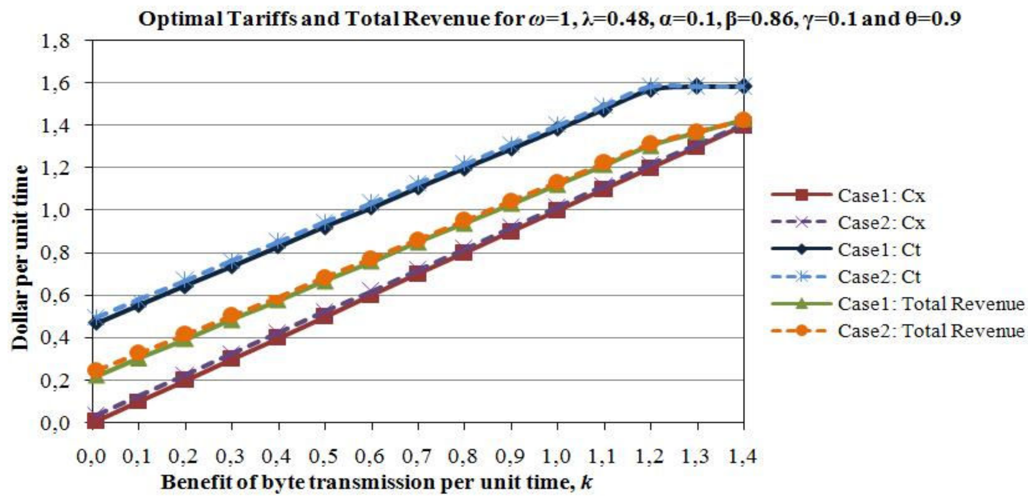

Figure A10: Optimal tariffs and optimal revenues for $\lambda = 0.48$, $\alpha = 0.1$, $\beta = 0.86$, $\gamma = 0.1$and $\theta = 0.9$

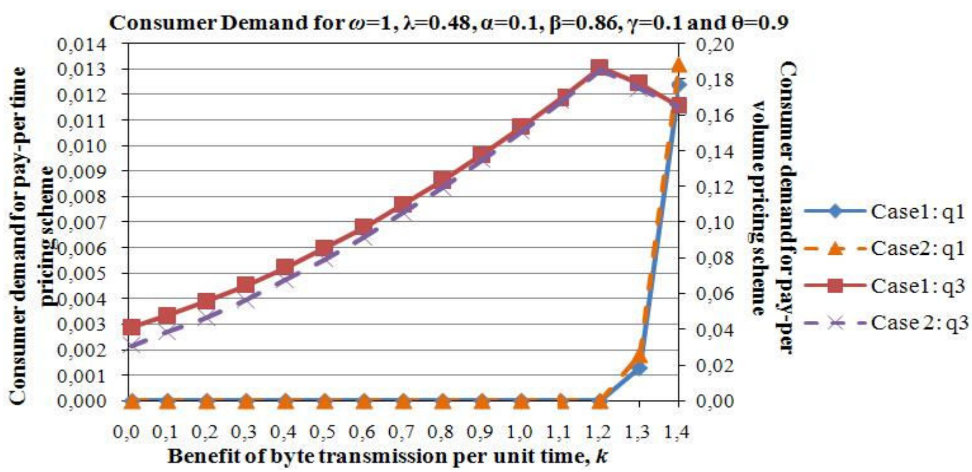

Figure A11: Consumer demands for $\lambda = 0.48$, $\alpha = 0.1$, $\beta = 0.86$, $\gamma = 0.1$and $\theta = 0.9$

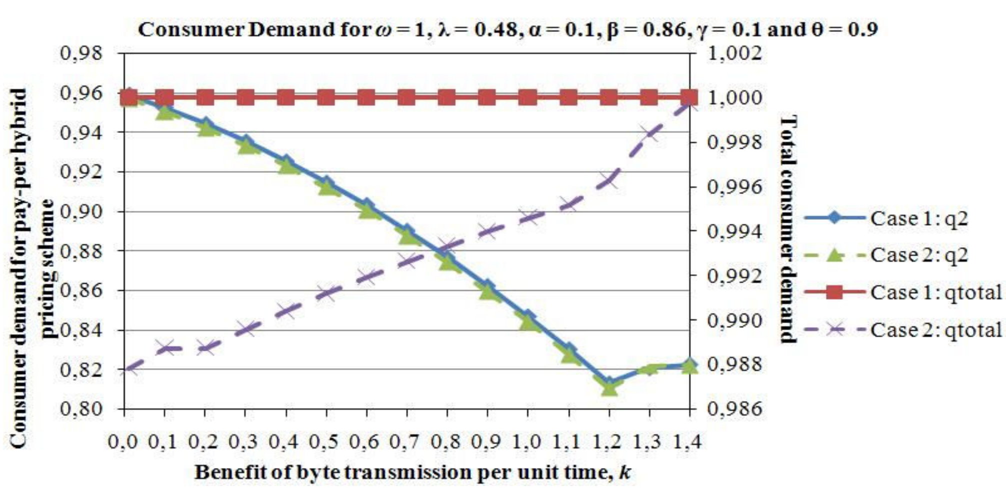

Figure A12: Consumer demands and total demands for $\lambda = 0.48$, $\alpha = 0.1$, $\beta = 0.86$, $\gamma = 0.1$ and $\theta = 0.9$

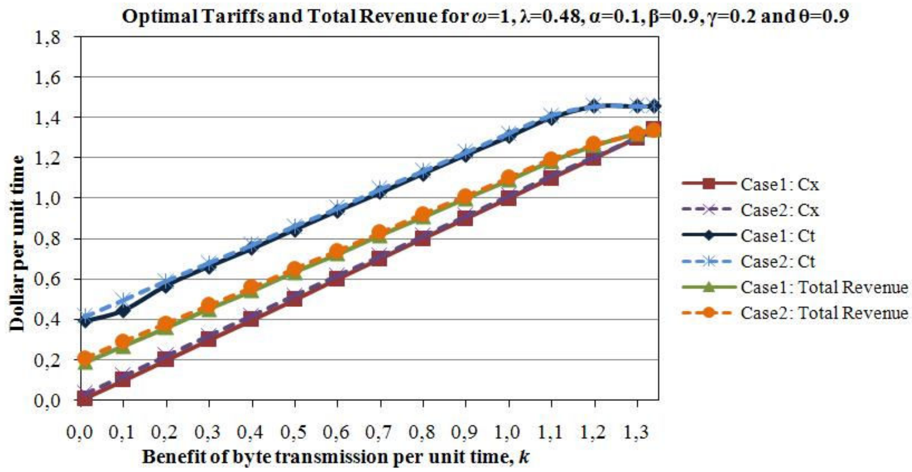

Figure A13: Optimal tariffs and optimal revenues for $\lambda = 0.48$, $\alpha = 0.1$, $\beta = 0.9$, $\gamma = 0.2$ and $\theta = 0.9$

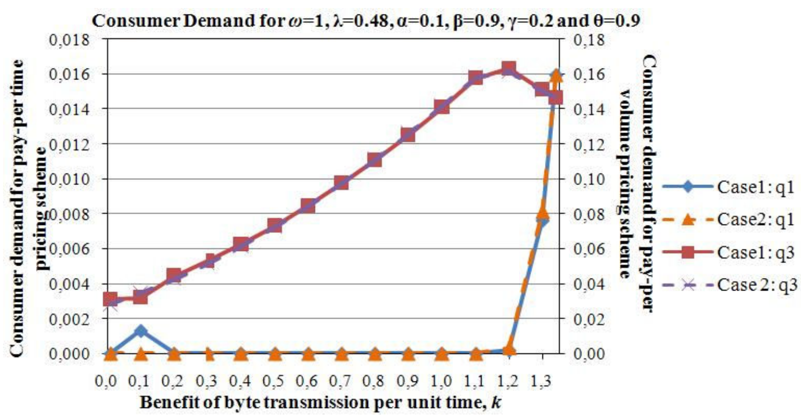

Figure A14: Consumer demands for $\lambda = 0.48$, $\alpha = 0.1$, $\beta = 0.9$, $\gamma = 0.2$ and $\theta = 0.9$

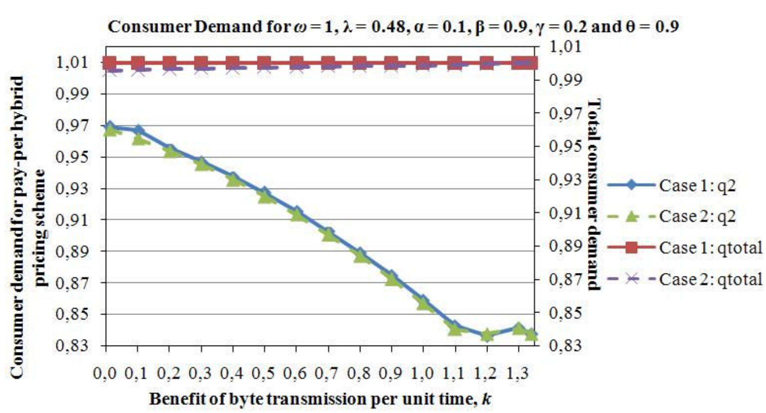

Figure A15: Consumer demands and total demands for $\lambda = 0.48$, $\alpha = 0.1$, $\beta = 0.9$, $\gamma = 0.2$ and $\theta = 0.9$

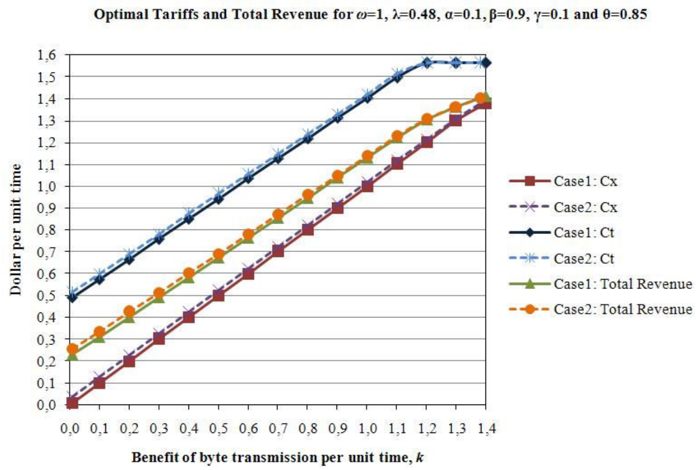

Figure A16: Optimal tariffs and optimal revenues for $w = 1$, $\lambda = 0.48$, $\alpha = 0.1$, $\beta = 0.9$, $\gamma = 0.1$ and $\theta = 0.85$

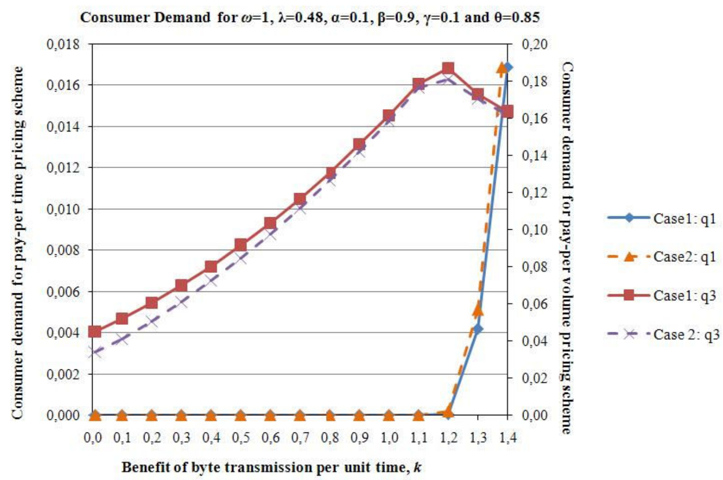

Figure A17: Consumer demands for $w = 1$, $\lambda = 0.48$, $\alpha = 0.1$, $\beta = 0.9$, $\gamma = 0.1$ and $\theta = 0.85$

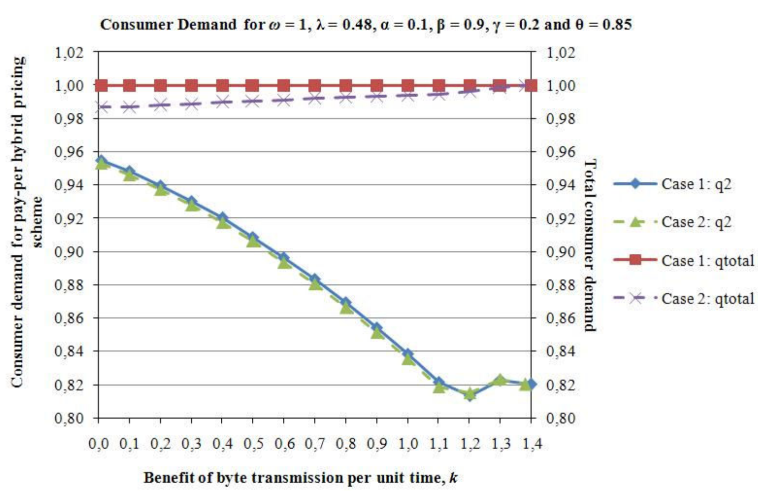

Figure A18: Consumer demands and total demands for $w = 1$, $\lambda = 0.48$, $\alpha = 0.1$, $\beta = 0.9$, $\gamma = 0.1$ and $\theta = 0.85$

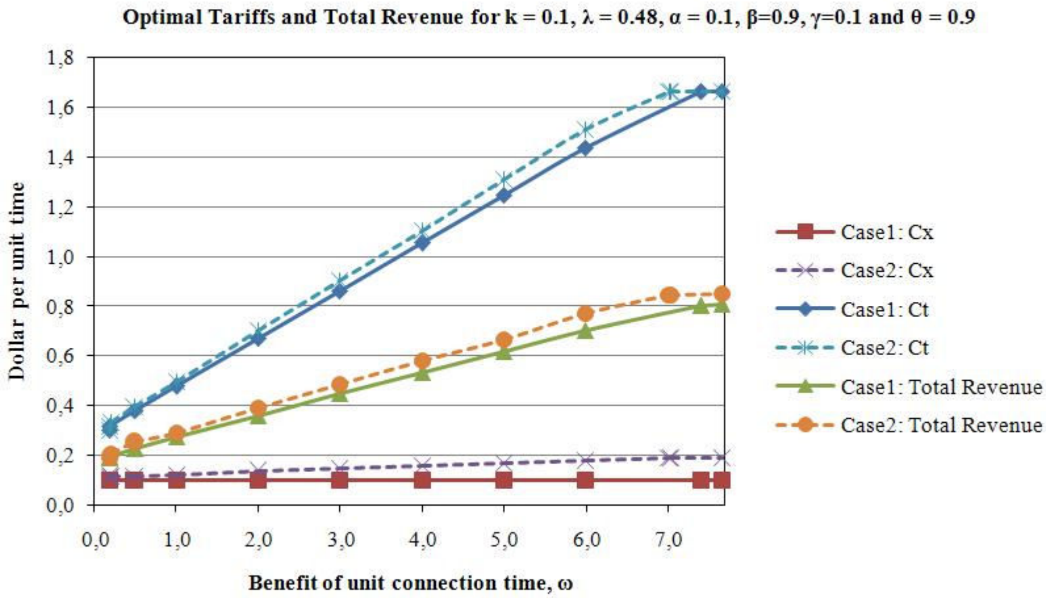

Figure A19: Optimal tariffs and optimal revenues for k =0.1, λ = 0.48, α = 0.1, β = 0.9, γ = 0.1 and θ = 0.9

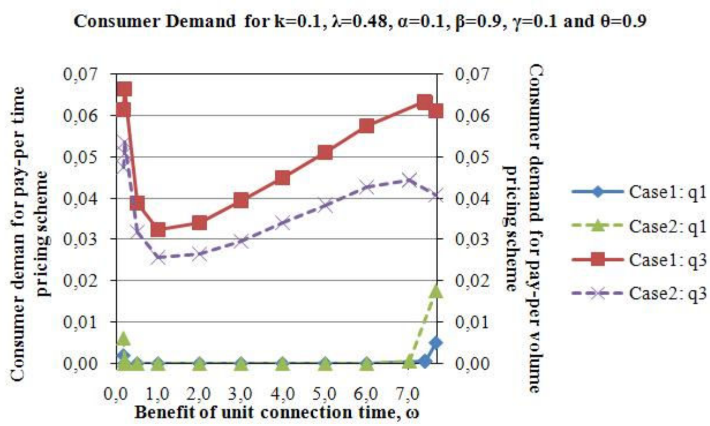

Figure A20: Consumer demands for k = 0.1, λ = 0.48, α = 0.1, β = 0.9, γ = 0.1 and θ = 0.9

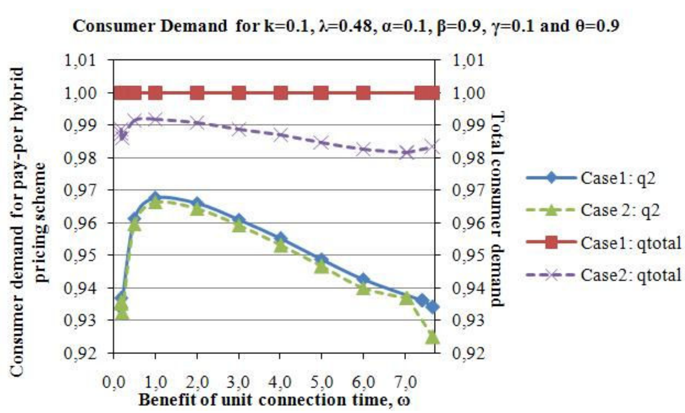

Figure A21: Consumer demands and total demands for k = 0.1, λ = 0.48, α = 0.1, β = 0.9, γ = 0.1 and θ = 0.9

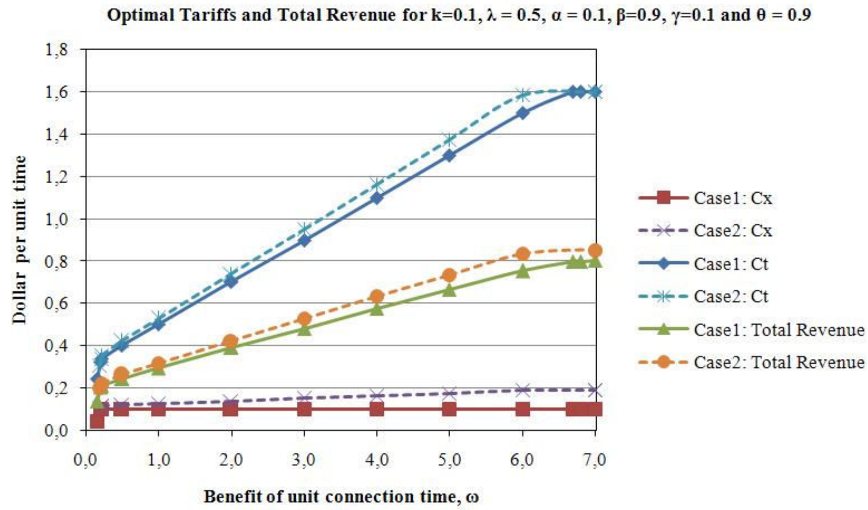

Figure A22: Optimal tariffs and optimal revenues for k =0.1, λ = 0.5, α = 0.1, β = 0.9, γ = 0.1 and θ = 0.9

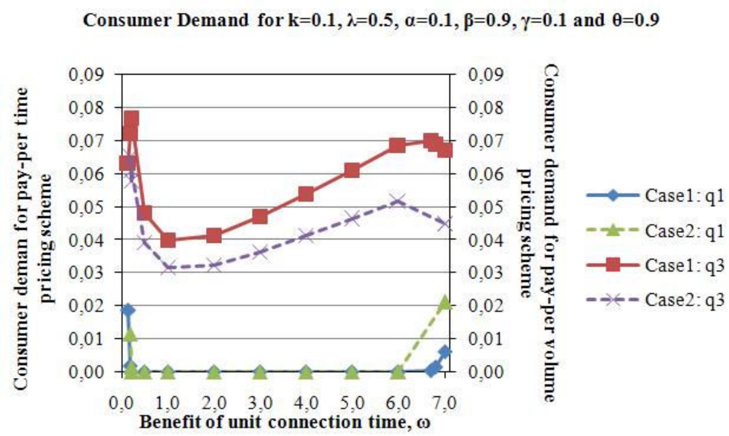

Figure A23: Consumer demands for k = 0.1, λ = 0.5, α = 0.1, β = 0.9, γ = 0.1 and θ = 0.9

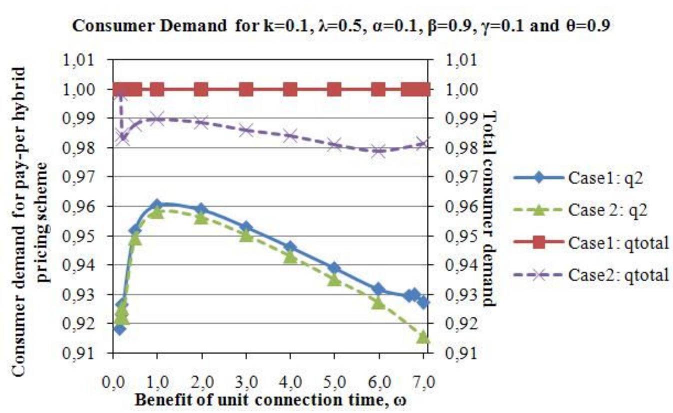

Figure A24: Consumer demands and total demands for k = 0.1, λ = 0.5, α = 0.1, β = 0.9, γ = 0.1 and θ = 0.9

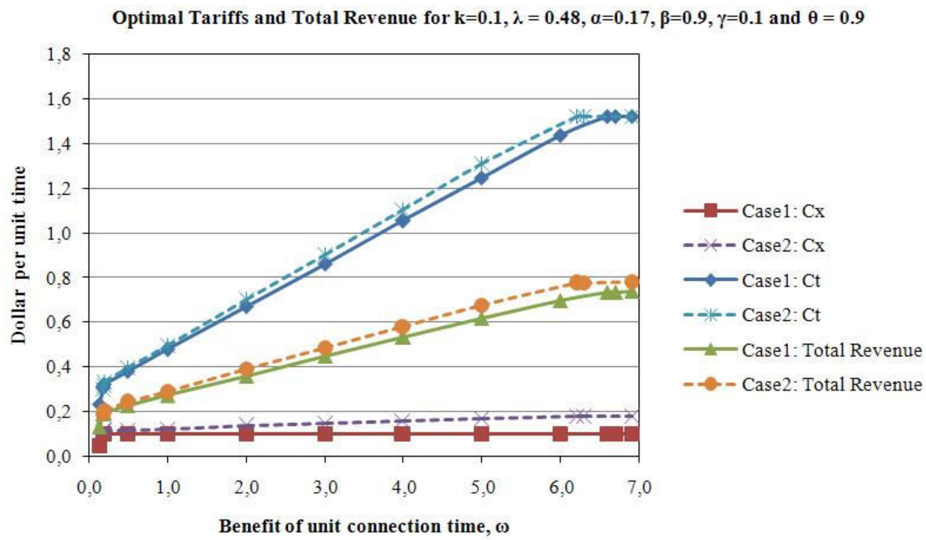

Figure A25: Optimal tariffs and optimal revenues for k =0.1, λ = 0.48, α = 0.17, β = 0.9, γ = 0.1 and θ = 0.9

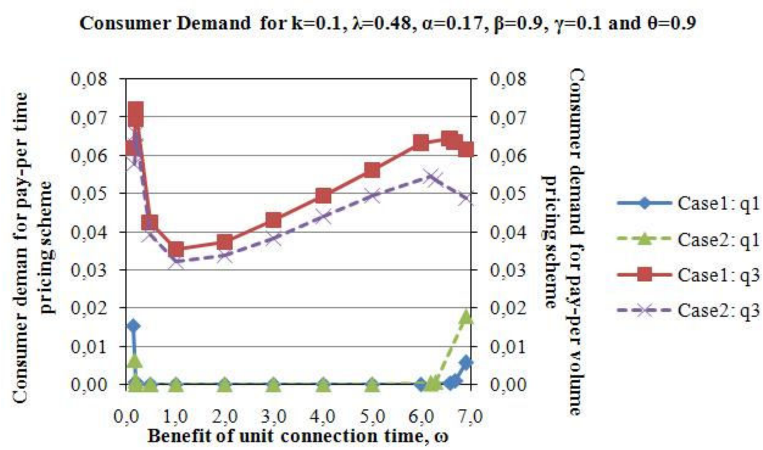

Figure A26: Consumer demands for k = 0.1, λ = 0.48, α = 0.17, β = 0.9, γ = 0.1 and θ = 0.9

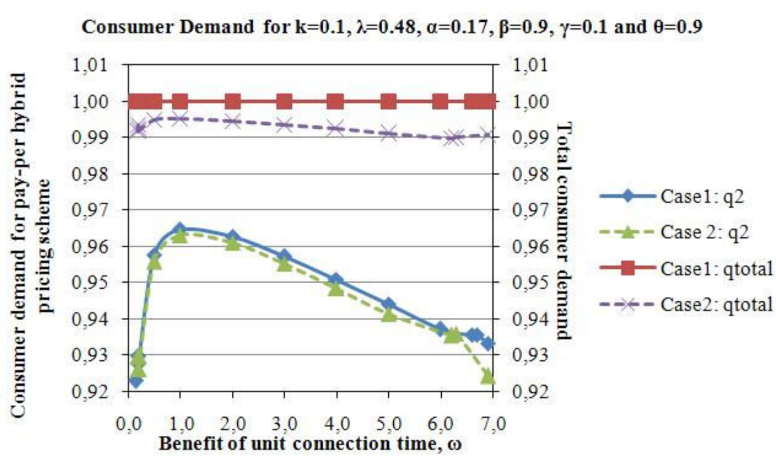

Figure A27: Consumer demands and total demands for k = 0.1, λ = 0.48, α = 0.17, β = 0.9, γ = 0.1 and θ = 0.9

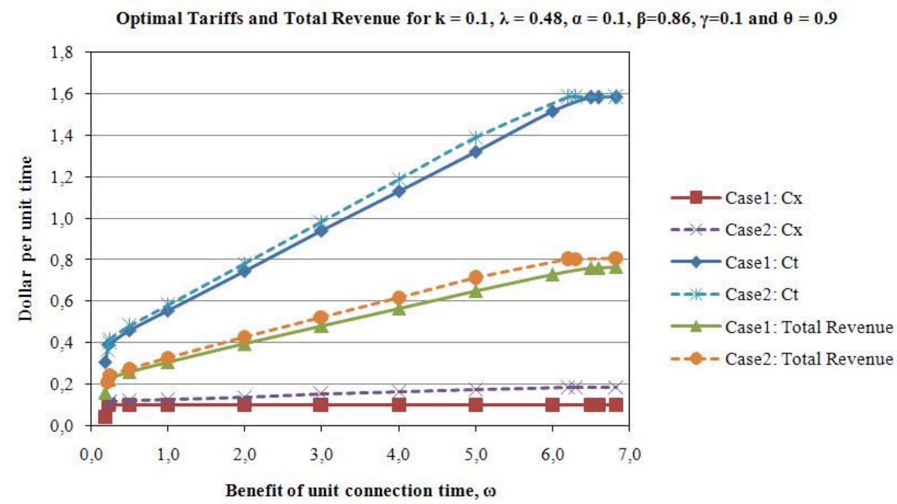

Figure A28: Optimal tariffs and optimal revenues for k =0.1, λ = 0.48, α = 0.1, β = 0.86, γ = 0.1 and θ = 0.9

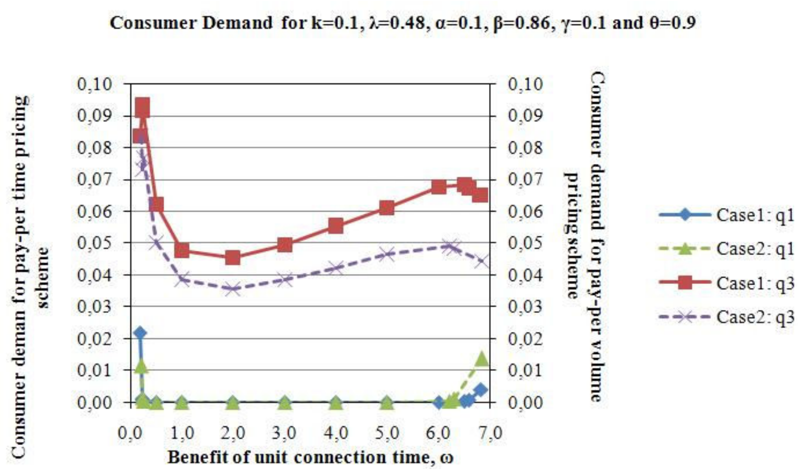

Figure A29: Consumer demands for k = 0.1, λ = 0.48, α = 0.1, β = 0.86, γ = 0.1 and θ = 0.9

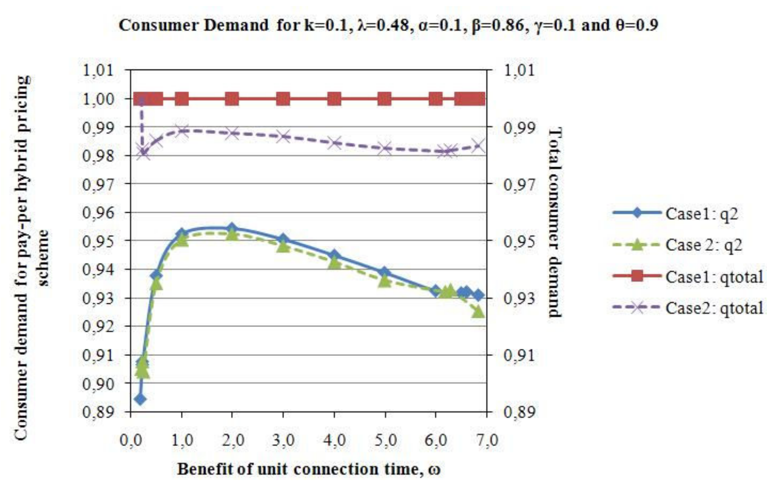

Figure A30: Consumer demands and total demands for k = 0.1, λ = 0.48, α = 0.1, β = 0.86, γ = 0.1 and θ = 0.9

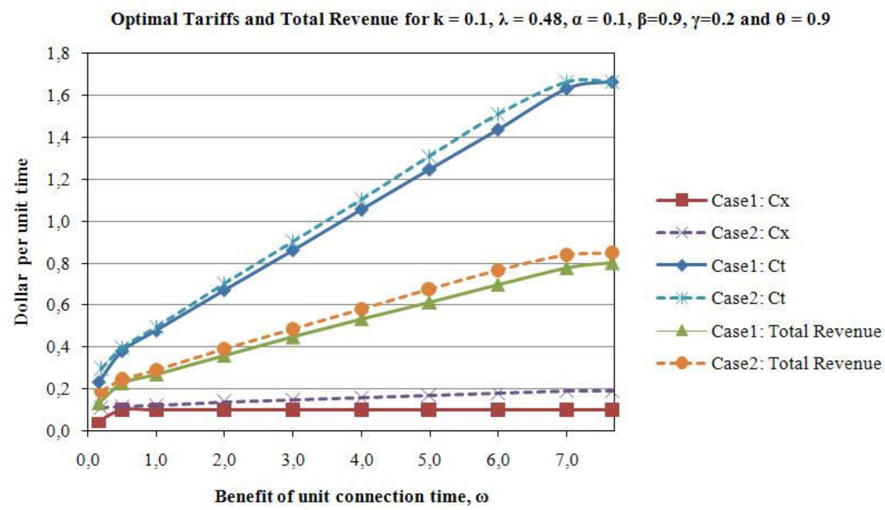

Figure A31: Optimal tariffs and optimal revenues for k =0.1, λ = 0.48, α = 0.1, β = 0.9, γ = 0.2 and θ = 0.9

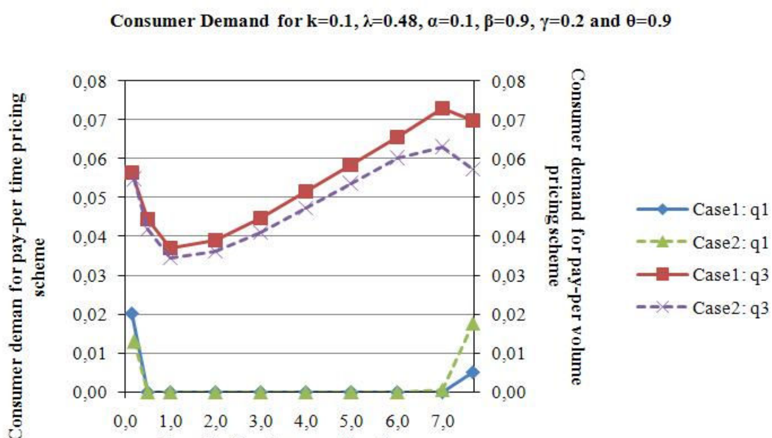

Figure A32: Consumer demands for k = 0.1, λ = 0.48, α = 0.1, β = 0.9, γ = 0.2 and θ = 0.9

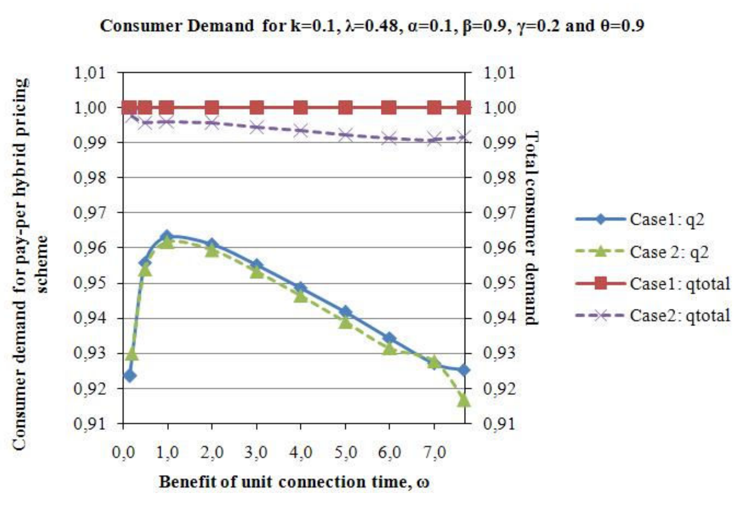

Figure A33: Consumer demands and total demands for k = 0.1, λ = 0.48, α = 0.1, β = 0.9, γ = 0.2 and θ = 0.9

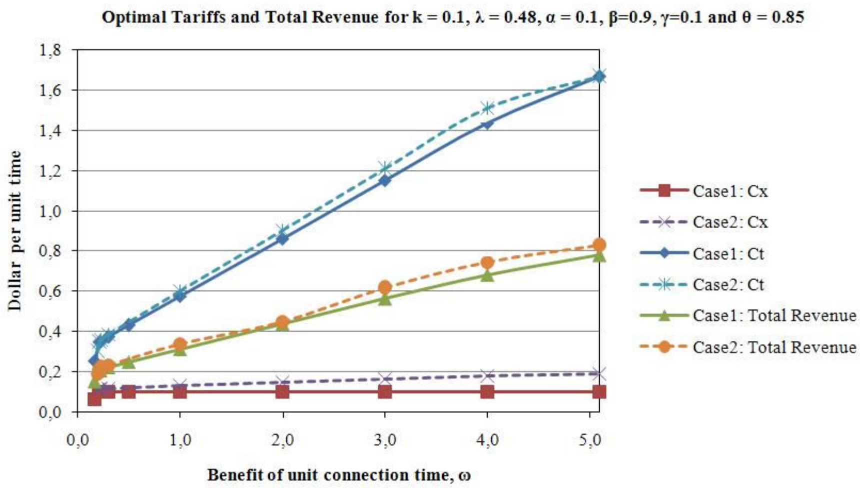

Figure A34: Optimal tariffs and optimal revenues for k =0.1, λ = 0.48, α = 0.1, β = 0.9, γ = 0.1 and θ = 0.85

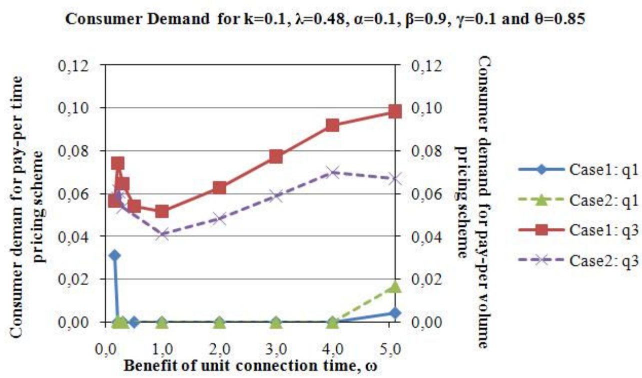

Figure A35: Consumer demands for k = 0.1, λ = 0.48, α = 0.1, β = 0.9, γ = 0.1 and θ = 0.85

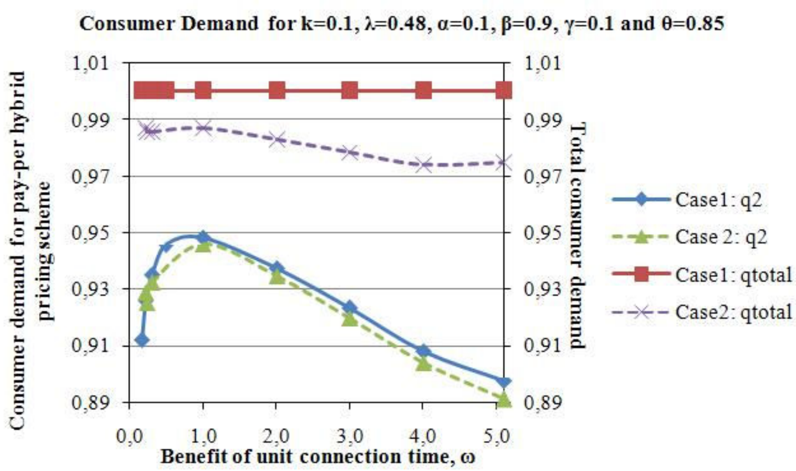

Figure A36: Consumer demands and total demands for k = 0.1, λ = 0.48, α = 0.1, β = 0.9, γ = 0.1 and θ = 0.85