Resource Dedication Problem in a Multi-Project Environment*

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1 Introduction

Resource dedication problem (RDP) in a multi-project environment is defined as the optimal dedication of resource capacities to different projects within the overall limits of the resources with the objective of minimizing the sum of the weighted tardinesses of all projects. The projects involved are in general multi-mode resource constrained project scheduling problems (MRCPSP) with finish to start zero time lag and nonpreemtive activities. In general, approaches to multi-project scheduling consider the resources as a pool shared by all projects (see, e.g., Kurtuluş and Narula (1985), Speranza and Vercellis (1993), Lawrance and Morton (1993), Kim and Leachman (1993)). When projects are distributed geographically or sharing resources between projects is too costly, then the resource sharing policy may not be appropriate and hence the resources are dedicated to individual projects throughout project durations. To the best of our knowledge, this point of view for resources is not considered in multi-project literature. In the following, we propose a solution methodology for RDP with a new local improvement heuristic by determining the resource dedications to individual projects and solving scheduling problems with the given resource limits.

2 Formulation

General mathematical formulation for RDP is given below.

Parameters:

E_{vj}	Early start of activity j, $j = 1 \dots \|N_v\|$, in project v, $v = 1 \dots \|V\|$
L_{vj}	Late finish of activity j, $j = 1 \dots \|N_v\|$, in project v, $v = 1 \dots \|V\|$
r_{vjkm}	Usage of renewable resource k, $k = 1 \dots \|K\|$, by activity j, $j = 1 \dots \|N_v\|$ with mode m, $m = 1 \dots \|M_{vj}\|$, in project v, $v = 1 \dots \|V\|$
w_{vjim}	Consumption of nonrenewable resource i, $i = 1 \dots \|I\|$, by activity j, $j = 1 \dots \|N\|$, with mode m, $m = 1 \dots \|M_{vj}\|$, in project v, $v = 1 \dots \|V\|$
d_{vjm}	Duration of activity j, $j = 1 \dots \|N_v\|$, operating on mode m, $m = 1 \dots \|M_{vj}\|$, in project v, $v = 1 \dots \|V\|$
R_k	Available renewable resource k, $\forall\ k \in \|K\|$
W_i	Available nonrenewable resource i, $\forall\ i \in \|I\|$
C_v	Weight of project v
DD_v	Due date of project v

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Decision Variables:

$$x_{vjmt} = \begin{cases} 1 \text{ if activity } j, \text{ j=1}\ldots|N_v|, \text{ operating in mode } m, \text{ m=1}\ldots|M|, \\ \quad \text{in project } v,\ v = 1\ldots|V| \text{is finished in period } t\ , \text{ t=1}\ldots|T| \\ 0 \text{ otherwise} \end{cases}$$

BR_{vk} = Maximum level of renewable resource k, $k = 1\ldots|K|$, assigned to project v, $v = 1\ldots|V|$

BW_{vi} = Amount of nonrenewable resource i, $i = 1\ldots|I|$ assigned to project v, $v = 1\ldots|V|$

TC_v = Weighted tardiness of project v, $v = 1\ldots V$

Mathematical Model RDP

$$min.\ z = \sum_{v=1}^{V} TC_v \quad (1)$$

Subject to

$$\sum_{m=1}^{M_{vj}} \sum_{t=E_{vj}}^{L_{vj}} x_{vjmt} = 1 \quad \text{for } \forall\ j \in N_v \text{ and } \forall\ v \in V \quad (2)$$

$$\sum_{m=1}^{M_{vb}} \sum_{t=E_{vb}}^{L_{vb}} (t - d_{vbm}) x_{vbmt} \geq \sum_{m=1}^{M_{va}} \sum_{t=E_{va}}^{L_{va}} t x_{vamt} \quad \forall\ (a,b) \in P_v \text{ and } \forall\ v \in V \quad (3)$$

$$\sum_{j=1}^{N_v} \sum_{m=1}^{M_{vj}} \sum_{q=t}^{t+d_{vjm}-1} r_{vjkm} x_{vjmq} \leq BR_{vk} \quad \forall\ k \in K, \forall\ t \in T \text{ and } \forall\ v \in V \quad (4)$$

$$\sum_{j=1}^{N_v} \sum_{m=1}^{M_{vj}} \sum_{t=E_{vj}}^{L_{vj}} w_{vjim} x_{vjmt} \leq BW_{vi} \quad \forall\ i \in I \text{ and } \forall\ v \in V \quad (5)$$

$$\sum_{v=1}^{V} BR_{vk} \leq R_k \quad \forall\ k \in K \quad (6)$$

$$\sum_{v=1}^{V} BW_{vi} \leq W_i \quad \forall\ i \in I \quad (7)$$

$$TC_v \geq C_v(t \sum_{m=1}^{M_{vN}} x_{vNmt} - DD_v) \quad \forall\ t = E_{vN}\ldots L_{vN} \text{ and } \forall\ v \in V \quad (8)$$

$x_{vjmt} \in \{0,1\}$ and BR_{vk}, BW_{vi}, $TC_v \in Z^+$

The objective (1) is minimizing the total weighted tardiness of all projects. Constraint set (2) ensures that all activities are scheduled once and only once for all projects. Constraint set (3) implies predecessor relationships P for all activities of all projects. Constraint set (4) specifies the maximum level of renewable resource level that should be dedicated to a project. Constraint set (5) determines the necessary nonrenewable resources for each project. Constraints sets (6) and (7) limit the dedicated renewable and nonrenewable resources, respectively. Constraint set (8) defines the weighted tardiness for each project. Note that when the resource capacities for projects, BR_{vk} and BW_{vi}, are supplied as parameters then the problem becomes a set of binary project scheduling problems one for each project.

3 Solution Methodology

The overall solution procedure is designed with a GA framework. GA searches through different resource dedications for each project (BW_{vi} and BR_{vk}) by using mutation, crossover and a new local improvement heuristic, combinatorial auction (CA) that may yield better results for total weighted tardiness for all projects (Equation 1) with given overall resource limits. An individual in GA encodes the renewable and nonrenewable resource levels (BR_{vk} and BW_{vi}) dedicated to each project v. The fitness is the total weighted tardiness for all projects. Several crossover operations are employed to exchange strings of resource dedications of projects over different individuals whereas mutation operation changes bits of resource dedication to projects in an individual. CA approach is used to improve some of the individuals in the population. The fitness of each individual is calculated as the sum of the weighted tardiness values of each project which can be determined by solving MRCPSP with the given resource capacities. Several authors contributed solution procedures for this problem (see, e.g., Talbot (1982), Hartmann (2001), Bouleimen and Lecocq (2003)). In this study the mathematical model proposed by Talbot (1982) will be solved with *CPLEX 11.2* under the resource dedication that describes the individual. CA has two basic components: guiding direction of the search and approaches to exploit this guiding direction, which are explained in detail below.

The guiding direction for resource dedication can be based on the preferences of the projects for the resources. The first issue is obtaining "good" estimates for the preferences of projects for resources. If the linear relaxation of the given mathematical model is solved for each project with the given level of resources, the solution can be used to determine preferences of projects for the resources. The linear relaxation of the given formulation results in two basic information: dual values and allowable upper and lower bounds for the RHSs of the constraints.

If the resource constraints are binding in the optimal basis, the dual values corresponding to the resource constraints can be used as the preferences for the resources. When this is not the case, which is observed frequently, the RHSs of these constraints will have allowable lower and upper bounds. When these bounds are violated, then the optimal basis will change. If the resource constraints are increased to their allowable upper bounds, the new basis should give at worst as good a feasible solution as the previous basis. The allowable upper bounds for the RHSs of the resource constraints can be used to determine the *sensitivity* of the projects for resources and can be used as preferences for these resources.

Determining the amount of resources that will be exchanged between projects can be handled by calculating the slack values for the resource constraints in the solution for MRCPSP. With the preferences of projects for resources and the total slack of resources at hand, one can determine a resource exchange between projects which distributes the slack resources according to the preferences. A continuous knapsack problem is used to distribute the calculated slack resources for maximizing the total gain for all projects. This procedure is called here the *combinatorial auction for resource dedication*. To summarize, with given resource capacities MRCPSP is solved for each project. The results are used to calculate the slacks for the resource constraints. After this, the linear relaxation of MRCPSP is solved and allowable upper bounds for the RHSs of the resource constraints are used as preferences of projects for resources. Finally, the slack resources are distributed to projects by using these preferences. This procedure can be used in a GA either as an improvement heuristic applied till an improvement is not seen or as a single step move or it can be used as a solely applied heuristic to a number of initial solution.

4 Experimental Study and Conclusions

12 multi-project test problems each with 4-6 projects are created combining different problems from c15, j20 and j30 sets in PSPLIB (http://129 .187.106.231/psplib/). The due dates of the projects and general resource capacities, R_k and W_i, in the multi-project prob-

lems are determined by solving individual project scheduling problems without resource constraints. The due dates of projects are set either as no-delay due dates or below no-delay to obtain a positive total weighted tardiness value for multi-project problems. The general resource capacities are determined by decreasing no-delay due date resource requirement by 15 percent. Results are presented in Table 1, where GA-1 and GA-2 refer to GA where is used as an improvement heuristic and as a single step move, respectively. Columns GA and CA refer to sole GA and CA implementations and exact solution is obtained using *CPLEX 11.2*. The optimal column shows the optimal objective values for the problems. The values in parenthesis show the execution time of the algorithms in minutes. To secure a fair comparison of the three GA applications common random numbers are used. All experiments are carried out with an *Intel Core2 Duo 2.33GHz* processor.

P. No	**Projects**	**R1**	**R2**	**W1**	**W2**	**GA-1**	**GA-2**	**GA**	**CA**	**Exact**	**Opt.**
P1	18/18/18/18	74	72	195	176	14(58)	12(104)	45(121)	12(44)	12(13)	12
P2	22/22/22/22	57	48	222	165	18(156)	28(234)	83(205)	35(52)	12(16)	12
P3	32/32/32/32	75	91	385	642	12(73)	12(156)	103(246)	24(58)	12(28)	12
P4	15/22/22/30	70	79	310	323	14(143)	32(242)	110(234)	12(41)	12(19)	12
P5	18/18/18/18/18	78	133	420	186	69(206)	78(230)	89(192)	90(27)	16(236)	16
P6	22/22/22/22/22	87	91	261	242	20(231)	20(162)	56(134)	36(58)	16(203)	16
P7	32/32/32/32/32	88	197	807	858	24(214)	39(203)	111(123)	140(61)	NA(>240)	16
P8	15/15/22/22/32	95	143	499	380	16(106)	19(147)	253(127)	356(43)	16(216)	16
P9	18/18/18/18/18/18	78	92	223	285	88(128)	111(212)	198(205)	642(29)	20(231)	20
P10	22/22/22/22/22/22	95	187	549	618	28(185)	81(197)	184(103)	206(56)	NA(>240)	20
P11	32/32/32/32/32/32	136	218	992	941	20(206)	92(222)	336(114)	121(49)	NA(>240)	20
P12	15/15/22/22/32/32	99	145	541	442	31(104)	52(207)	223(129)	452(44)	NA(>240)	20

Table 1. Experimental results

As seen in the results, exact solution with *CPLEX 11.2* has a clear advantage in solution time when number of projects is small and sole GA and CA fail to cope with other approaches. In addition to this, when the problem size increases GA/CA-1 gives competitive results with exact solution approach in both solution time and solution quality. Note that when problem size reaches 6 projects GA/CA-1 dominates all approaches in both solution time and quality.

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