OPTIMAL PROGRESSIVITY OF THE INCOME TAX CODE FOR TURKEY

by

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ABSTRACT

The focus of this paper is to compute the optimal progressivity of the income tax code for Turkish tax system. Following [Conesa and Krueger, 2006], we employ a dynamic general equilibrium model with heterogeneous agents. Labor productivity shocks in the absence of insurance markets create more dispersed income and wealth distribution. A progressive tax system serves as a partial insurance mechanism. Thus, progressivity decreases differences in income that occur during good times and bad times and enhances welfare. On the other hand, progressive taxation distorts incentives for labor supply and saving decisions of private households. The policy maker, thus faces nontrivial trade-offs between the three effects of progressive taxation; social insurance, equity and labor supply efficiency when designing the income tax code.

A flat tax rate of 23% with a fixed deduction of half of the average income, which is roughly 3800TL, maximizes the utilitarian steady state welfare criterion. A tax reform towards this tax system results in welfare gains which is equivalent to 6.2% higher consumption in every possible state of world. Analysis of the Gini coefficients indicates that under the optimal tax system, income, wealth and consumption are more equally distributed.

TÜRKİYE İÇİN OPTİMAL ARTAN ORANLI VERGİLENDİRME

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ÖZET

Türkiye vergilendirme sistemi için gelir vergisinin optimal artan oranlılığının hesaplanması bu tezin odak noktasıdır. Bu çalışmada [Conesa and Krueger, 2006] makalesini takip ederek, heterojen bireyli dinamik genel denge modeli kullandık. Sigorta pazarlarının eksikliğinde, işçi verimliliğine gelen şoklar, gelir ve servet dağılımını daha dengesiz hale getirmektedir. Artan oranlı vergilendirme, kısmen sigorta mekanizması görevini görmektedir. Böylece artan oranlılık, iyi ve kötü zamanlarda oluşan gelir farklılıklarını azaltarak, refahı iyileştirmektedir. Diğer yandan artan oranlı vergilendirme, bireylerin işgücü arzını ve tasarruf kararlarını bozmaktadır. Vergi politikalarını oluşturan birim, gelir vergisi sistemini oluştururken, artan oranlı verginin, sosyal sigorta, hakkaniyet ve işgücü arzı verimliliği üzerindeki etkileri arasında, sonucu kesin olmayan ödünleşmelerle karşılaşmaktadır.

Faydacıl durağan durum sosyal refah kriterini kullanarak, Türkiye için optimal gelir vergisinin, mevcut ortalama gelirin yaklak yarısı, 3800TL, oranında sabit indirim ve %23 oranında düz oranlı bir vergi seviyesi olduğunu bulduk. Bulduğumuz vergi sistemine doğru yapılacak bir vergi reformu, olası her durumda %6.2'lik tüketim artışı şeklinde bir refah artışıyla sonuçlanacaktır. Gini katsayısı analizleri, optimal vergi sistemi altında tüketim, servet ve gelirin daha eşit dağıldığını göstermiştir.

TABLE OF CONTENTS

1 INTRODUCTION **1**

2 THE MODEL **5**

2.1 The economic environment 5

2.1.1 Demographics 5

2.1.2 Endowments and Preferences 5

2.1.3 Technology 7

2.1.4 Government Policy 8

2.1.5 Market Structure 9

2.1.6 Definition of competitive equilibrium 9

3 QUANTITATIVE ANALYSIS **12**

3.1 Functional forms and calibration of the benchmark economy 12

3.1.1 Demographics 13

3.1.2 Preferences 14

3.1.3 Endowments 14

3.1.4 Technology 16

3.1.5 Government policies and the income tax function 17

3.2 The computational experiment 20

4 THE OPTIMAL TAX CODE **21**

5 CONCLUSION **27**

A APPENDIX **28**

A.1 Tax brackets 28

A.2 Normalization 28

A.3 Gini Coefficients 29

List of Tables

1 Demographic Parameters . . . 13
2 Preferences Parameters . . . 14
3 Age-specific productivity levels . . . 15
4 Ability . . . 16
5 Stochastic Productivity . . . 16
6 Technology Parameters . . . 17
7 Turkey individual income tax rates 2006 . . . 18
8 Policy Parameters . . . 20
9 Comparison across tax codes I . . . 22
10 Comparison across tax codes II . . . 23
11 Comparison across tax codes III . . . 24

List of Figures

1 Marginal tax rates under 2 tax regimes . . . 21

1 INTRODUCTION

In the public economics literature much emphasis given to the role of progressive taxes in affecting the consumption, labor supply and saving (capital accumulation) decisions of private households and firms. Progressive income tax systems has both beneficial and undesirable effects. They are beneficial in the sense that they provide more equal distribution of income, and through this, indirect effects can be seen as more equality in wealth distribution and increase in welfare.In addition to this in the presence of idiosyncratic uncertainty, if the economy lacks private insurance markets, progressive taxes may make individuals to smooth their consumption over time. On the other hand, economy faces undesired costs due to reduced incentives for labor supply and saving decisions. Hence the progressive tax system involves trading off the benefits due to equity and social insurance with the costs due to reduced incentives. Thus the decision of the policy maker about the income tax code becomes more complicated.

The literature begins by the pioneering work of [Mirrlees, 1971]. In the paper the economic theory about the trade of between equality and labor supply inefficiency is initialized, and it investigates the question of which principles should govern an income tax code ; how tax schedule would look like; and what degree of inequality would remain once it was established. [Varian, 1980] shows that the optimal redistributive tax involves trading off the benefits due to social insurance with the costs due to reduced incentives by computing some algebraic and numeric examples. On the capital income side of this literature, [Hubbard and Judd, 1986] take policy simulation models that ignore "liquidity constraints" result in flawed tax policy analysis into consideration and analyze the impact of liquidity constraints on consumption functions and conclude that in the presence of capital market imperfections, capital income tax may be optimal. [Aiyagari, 1995] shows that with incomplete markets and borrowing constraints, the optimal tax rate on capital is positive even in the long run, thus setting income tax to zero may result in welfare losses. With this conclusion they opposed to the results that [Judd, 1985] and [Chamley, 1986] arrived by showing optimal tax rate on capital income does tend to zero in the long run.

All those papers investigate just the qualitative implications for the optimal tax code, in order not to lose the analytical tractability. [Conesa and Krueger, 2006] contributes to those papers by investigating quantitative implications of the optimal tax code. In the paper, they take into account an economic environment with the presence of social insurance effect, labor supply efficiency and equity effects of progressive taxes at the same time and quantitatively characterize the optimal progressivity of the income tax code.

In this paper, in a period where tax policy and tax reform are important items on the Turkish policy agenda, we computed the optimal progressivity of the income tax code for Turkey economy in a dynamic general equilibrium model with heterogeneous agents and uninsurable labor productivity risks This paper mainly adopts the model used in [Conesa and Krueger, 2006] and analyze the Turkish tax system using long run statistics of Turkey.

People born in an overlapping generations economy with different skills and throughout their working time they are hit by the idiosyncratic, serially correlated income shocks [1] The insurance opportunity of these income shocks is very restricted by assumption, there is just one-period risk-free bond to be traded and cannot be shortened. In each period individuals decide on how to divide their discrete time between labor, leisure and how to allocate their earnings between consumption and savings . Those allocation decisions are affected by the tax code. The government levies taxes on individuals in two forms, one is the proportional consumption taxes, and the other is the income taxes and using the revenue from them, finance the fixed exogenous amount of its expenditures. Since the main focus of this paper is income tax code, we take consumption tax as given. The income tax code is taken from [Berliant and Gouveia, 1993]. It was restricted to lie in a particular class of functional forms and it is relied on the equal sacrifice approach [Berliant and Gouveia, 1993]. The main feature of the income tax code which is very useful to our analysis is, by changing the parameters of the tax code we can face a big spectrum of tax systems, i.e. with the same function, a purely proportional income tax code, progressive tax codes, and regressive tax codes can be created. And although it creates such a big spectrum of tax systems, it is governed by just few parameters, thus numerical optimization over the income tax code becomes feasible.

In order to determine the optimal tax system we need to choose a social welfare function to evaluate policies. The welfare criterion we utilize is ex-ante (before ability is realized) expected (with respect to idiosyncratic shocks) lifetime utility of a newborn in a stationary equilibrium.

[1] See [Huggett, 1993] and [Aiyagari, 1994] for a detailed description.

Progressive taxes play a positive role in achieving a more equal distribution of income and welfare (or in other words, they provide insurance against being born as a low-ability type). They also provide a partial substitute for missing insurance markets against idiosyncratic income shocks during a persons life. On the other hand, labor-leisure and consumption-saving decisions are distorted by the potential presence of tax progressivity. And also with this welfare criterion, the policy maker is assigned to concern for insurance against idiosyncratic shocks and redistribution across households with different ability, since transferring an extra dollar from the highly able to the less able, ceteris paribus, increases social welfare since the value function characterizing lifetime utility is strictly concave in the ability to generate income[2]. The policy maker then has to trade-off this concern against the standard distortions these taxes impose on labor supply and capital accumulation decisions.

The main finding in this paper is that, the optimal tax code for Turkey can be approximated by a marginal tax rate of 23% with a fixed deduction of half of the average taxed income rate of Turkey. Aggregate output is greater of amount 2.33%, in this tax code compared to the benchmark economy. And although average hours worked declined by a huge amount of 1.43%, aggregate labor supply almost increased which is the sign of a shift of labor supply from low-productivity to high-productivity individuals. With the optimal tax system, tax burden on the middle class of the income distribution increase substantially, whereas the lower tail and the upper tail of the income distribution face a substantially lower income tax bill.

Social welfare increases very substantially with the optimal tax code, so that to make people indifferent between being born in a steady state economy with the optimal tax code we found and being born in a benchmark steady state economy, we have to increase consumption uniformly by 6.2% across all agents and all states of world.

Parallel to the findings of [Hall and Rabushka, 1995]in favor of flat tax , our results have mainly the same intuition that is, decreased marginal tax rates that high ability, high productivity individuals face resulted in higher labor supply and saving incentives which is working in the way to decrease the equality of the distribution. But on the other hand, fixed deduction provides the desired redistribution and insurance. We showed the quantitative importance of the fixed deduction by employing a pure flat tax. And the results are dramatic that although aggregate

[2]See,[Conesa et al., 2005].

output increased by 5.2%, welfare reduced by .5% compared to the benchmark economy.

The organization of the remaining part is as follows. In Section 2, the economic environment and definition of the equilibrium are presented. Section 3 presents the description of the functional forms, calibration of the model economy, and computational experiments. Section 4 presents the optimal tax code and the analysis of results. Section 5 concludes.

2 THE MODEL

In this section, following [Conesa and Krueger, 2006], we describe the economic environment and define the equilibrium. The benchmark economy is based on an overlapping generations model, consisting of heterogeneous agents, a representative firm and a government. Individuals get utility from consumption and leisure. They are endowed with one unit of productive time, for either supplying labor or for leisure. The representative firm produces a single good with standard Cobb-Douglas production technology using capital and labor as inputs. Government administers the social security system and levies consumption tax and income tax on individuals to finance its expenditures.

2.1 The economic environment

2.1.1 Demographics

The economy is populated by J overlapping generations. In each period a new generation is born and generations grow with a constant rate n. After the age of retirement individuals face a death probability which is positive all the time. Let the notation for the conditional survival probability from age j to age j_{t+1} is $\psi_j = prob(\text{alive at} j + 1 | \text{alive at} j)$. Agents live up to the age J and die for sure at this age i.e. $\psi_J = 0$. In our economy, the assumption of death probability after retirement results in a part of the population leaving accidental bequests. We denote them by Tr_t. Those bequests accrue to the government budget as general revenue. Agents have a certain retirement age j_r. When they retire,they receive social security payments SS_t at an exogenously specified replacement rate b_t of current average wages. Government levies proportional labor income tax $\tau_{ss,t}$ on individuals to finance social security payments.

2.1.2 Endowments and Preferences

Individuals born with zero assets and during their lifetime they have one unit of productive time in each period. Agents devote this one unit of time to work in the labor market and keep the remaining as leisure. Agents are heterogeneous with respect to three variables which results in high spectrum for labor productivity differences and wage differences. First, labor productivity depends on the ages of the individuals. Different ages have different productivity levels ε_j and after retirement age j_r, age specific labor productivity becomes zero. Second effect to labor productivity comes from types of the agents. Agents are assumed to born with different ability levels α_i and throughout their life they have the same ability level. Therefore people differ in their potential current and future earnings from their birth. The distribution of ability types is determined by the probabilities, i.e. $p_i > 0$ denotes the probability of being born with ability α_i. And last effect to the labor productivity is idiosyncratic uncertainty. Workers realize idiosyncratic uncertainty at the beginning of each period $\eta_t \in \mathbf{E}$. Each worker, independent of their age and ability face the same stochastic process for labor productivity. The stochastic process was assumed to follow a finite state Markov chain with stationary transition over time, i.e.

$$Q_(\eta, E) = Prob(\eta_{t+1} \in E | \eta = \eta) = Q(\eta, E) \tag{1}$$

In the model, it was assumed that all the entries in the Markov transition matrix are strictly positive.[3] In the first year of their life, all individuals face average stochastic productivity i,e $\bar{\eta} = \sum_\eta \eta \Pi(\eta)$ where $\eta \in \mathbf{E}$. In addition to the differences created by different ages and different ability levels to the labor productivity, income and wealth distributions stochastic uncertainty adds further dispersion. At this point progressive tax system can serve as an insurance market for labor productivity risk and give individuals the chance to share this idiosyncratic risk effectively.

At each period, types of the individuals are characterized by (a, η, i, j), where a_t are asset holdings (of one-period, risk-free bonds), η_t is stochastic labor productivity status at date t, i is ability type and j is age. The notation for the measure of agents of type (a, η, i, j) at date t is $\Phi(a, \eta, i, j)$

It was assumed that individuals choose their consumption level c_j and and leisure $(1 - l_j)$

[3]With this assumption, we quarantined that there exists a unique invariant distribution associated with Q which is denoted by Π (See[Stokey and Lucas, 1989]

level according to a standard time-separable utility function of the form;

$$E\{\sum_{j=1}\beta^{j-1}\frac{(c_j^{\gamma}(1-l_j)^{1-\gamma})^{1-\sigma}}{1-\sigma}\} \tag{2}$$

where β is the time discount factor, γ determines utility weights household gives to consumption and leisure, and σ is the degree of risk aversion. Expectation sign in front of the utility function is emanated from the stochastic processes governing idiosyncratic labor productivity and the probability of death at each period.

2.1.3 Technology

It was assumed that there is a representative firm and aggregate output is produced according to a standard constant returns to scale Cobb-Douglas technology.

$$Y_t = K_t^{\alpha}(A_t N_t)^{1-\alpha} \tag{3}$$

and the aggregate resource constraint is given by

$$C_t + K_{t+1} - (1-\delta)K_t + G_t \leqslant K_t^{\alpha}(A_t N_t)^{1-\alpha} \tag{4}$$

where Y_t, K_t and C_t stand for aggregate output, aggregate capital stock, aggregate consumption and aggregate labor input in period t respectively and N_t is aggregate labor input measured in efficiency units in period t. It was assumed that technological progress takes the labor augmenting form and here the term A_t stands to capture this assumption i.e. $A_t = (1+g)^{t-1}A_1$. $\alpha \in (0,1)$ is capital share of output and γ is the depreciation rate depreciation rate for physical capital.

2.1.4 Government Policy

The government administers social security system, and finance its spending by levying taxes. In the model each individual after retirement take the same social security benefit, SS_t. Thus retirement benefit doesn't depend on the households earnings history. The balanced budget social security system is satisfied in each period by the social security tax rate $\tau_{ss,t}$. In the model only income tax code is subject to optimization of the policy maker and social security system is taken exogenously.

Government has three fiscal instruments to finance exogenously given government consumption $\{G_t\}_{t=1}^{\infty}$. First, it levies tax on consumption expenditures by a proportional tax rate τ_c, which is taken as exogenously. Second, accidental bequests, the resultant of the death probabilities of individuals in each age, added to the government general revenue. Finally government levies tax on individuals labor income which is $y_t = (1 - .5\tau_{sst})w_t\varepsilon_j\alpha_i\eta l$ for the workers and SS_t for the retired agents and capital income with a constant tax rate τ_k, i.e. $r_t a\tau_k$.[4] Here w_t and r_t are the notations for the wage per efficiency unit of labor and the risk-free interest rate, respectively.

In the model, income tax code is taken as an arbitrary function of individual labor income, in a given period, and denoted by $T(\cdot)$, where $T(y)$ is the total income tax liability if pre-tax income equals y. In addition to this assumption, one more restriction imposed on the tax code that is, anonymity of the tax code is assumed, therefore tax rates doesn't differ for different earning levels.

In the model, government is choosing optimal progressivity of the income tax code as a policy. Since income tax is defined as a function, the problem of the government becomes choosing the optimal tax function $T(\cdot)$, with respect to government budget constraint given the

[4]Here, in order to be consistent with the Turkish tax system we differentiated from the model of Conesa and Krueger which we adapt. In the original model, assumed tax function is levied on individual's labor income and capital income i.e., the government can not condition tax rates on the source of income, but in our model we assume that government can tax labor and capital income at different rates.

stream of government expenditures and consumption tax rate.

2.1.5 Market Structure

In the model there is no explicit insurance markets for labor productivity risk. Agents can only trade one-period risk-free bonds to self-insure against the risk of low labor productivity in the future. Agents are not allowed to sell the bond short. This assumption impose a restriction on the borrowing on all individuals instinctively and prevents agents from leaving debt behind them.

2.1.6 Definition of competitive equilibrium

Here, we will define competitive equilibrium of the economy and a balanced growth path. Individual asset holdings $a \in \mathbf{R}_t$, individual labor productivity status $\eta \in \mathbf{E} = \eta_1, \eta_2, ..., \eta_n$, individual ability type $i \in \mathbf{I} = 1, ..., M$ and age $j \in \mathbf{J} = 1, 2..., J$ define the individual state of the economy at time t. Furthermore aggregate state of the economy is the joint measure Φ_t over individual state variables, i.e. asset positions, labor productivity status, ability and age.

Definition 1 Given a sequence of social security replacement rates $\{b_t\}_{t=1}^{\infty}$ consumption tax rates $\{\tau_c\}_{t=1}^{\infty}$ and government expenditures $\{G_t\}_{t=1}^{\infty}$ and initial conditions K_1 and Φ_1, a competitive equilibrium is a sequence of functions for the household, $\{v_t, c_t, a_t{}', l_t\}$ of production plans for the firm, $\{N_t, K_t\}_{t=1}^{\infty}$, government income tax functions $\{T_t\}_{t=1}^{\infty}$, social security taxes $\{\tau_{ss,t}\}_{t=1}^{\infty}$ and benefits $\{SS_t\}_{t=1}^{\infty}$, prices $\{w_t, r_t\}_{t=1}^{\infty}$, transfers $\{Tr_t\}_{t=1}^{\infty}$ and measures $\{\Phi_t\}_{t=1}^{\infty}$ such that:

1. Given prices, policies, transfers and initial conditions, for each t, v_t solves the policy functions c_t, $a_t{}'$ and l_t.

$$v_t(a,\eta,i,j) = max_{a',l,c}\{u(c,l) + \beta\psi_j \int v_{t+1}(a',\eta',i,j+1)Q(\eta,d\eta') \tag{5}$$

subject to,

$$\begin{gathered}(1+\tau_c)c + a' = (1-\tau_{sst})w_t\varepsilon_j\alpha_i\eta l + (1+r_t)a \quad \text{for } j > j_r \\ -T_t[(1-tau_{sst})w_t\varepsilon_j\alpha_i\eta l] - r_t a\tau_k\end{gathered} \tag{6}$$

$$(1+\tau_c)c + a' = SS_t + (1+r_t)a - T_t[SS_t] - r_t a\tau_k \quad \text{for } j \geqslant j_r \tag{7}$$

$$a' \geqslant 0 \quad c \geqslant 0 \quad 0 \leqslant l \leqslant 1 \tag{8}$$

2. Wage per efficiency unit of labor and w_t and the risk-free interest rate r_t satisfy:

$$r_t = \alpha(\frac{A_t N_t}{K_t})^{1-\alpha} - \delta, \tag{9}$$

$$w_t = (1-\alpha)A_t(\frac{K_t}{A_t N_t})\alpha. \tag{10}$$

3. The social security policies satisfy

$$SS_t = b_t \frac{w_t N_t}{\int \Phi_t(da \times d\eta \times di \times \{1,...,j_r-1\})}. \tag{11}$$

$$\tau_{ss,t} = \frac{SS_t}{w_t N_t} \int \Phi_t(da \times d\eta \times di \times \{j_r,...,J\}). \tag{12}$$

4. Transfers are given by

$$Tr_{t+1} = \int (1-\psi_j) a_t{}' \Phi_t(da \times d\eta \times di \times dj) \tag{13}$$

5. Government budget balance:

$$\begin{aligned} G_t = & \textstyle\int T_t[(1-.5\tau_{sst}) w_t \varepsilon_j \alpha_i \eta l] \times \Phi_t(da \times d\eta \times di \times \{1, ..., j_r - 1\}) \\ & + \textstyle\int T_t[SS_t] \times \Phi_t(da \times d\eta \times di \times \{j_r, ..., J\}) \\ & + \tau_{c,t} \textstyle\int c_t(a, \eta, i, j) \Phi_t(da \times d\eta \times di \times d_j) \\ & + \tau_k \textstyle\int r_t a \Phi_t(da \times d\eta \times di \times d_j) \\ & + (1 + r_t) Tr_t. \end{aligned} \tag{14}$$

6. Market clearing

$$K_t = \int a \Phi_t(da \times d\eta \times di \times dj) \tag{15}$$

$$N_t = \int \varepsilon_j \alpha_i \eta l_t(a, \eta, i, j) \Phi_t(da \times d\eta \times di \times dj) \tag{16}$$

$$\begin{aligned} & \textstyle\int c_t(a, \eta, i, j) \Phi_t(da \times d\eta \times di \times dj) + \int a_t{}'(a, \eta, i, j) \Phi_t(da \times d\eta \times di \times dj) \\ & = K_t^{\alpha} (A_t N_t)^{1-\alpha} + (1-\delta) K_t \end{aligned} \tag{17}$$

7. Law of motion

   (a)

$$\begin{aligned} & \Phi_{t+1}(A \times E \times I \times J) \\ & = \textstyle\int P_t((a, \eta, i, j); A \times E \times I \times J) \Phi_t(da \times d\eta \times di \times dj) \end{aligned} \tag{18}$$

   where

$$P_t((a,\eta,i,j);A\times E\times I\times J)$$
$$= \begin{cases} Q(e,E)\psi_j & \text{if } a_t{}'\in A,\, i\in I, j+1\in J \\ 0 \quad \text{else}, \end{cases} \tag{19}$$

(b)

$$\Phi_{t+1}(A\times E\times I\times J)$$
$$=(1+n)^t \begin{cases} \sum_{i\in k} p_i & \text{if } 0\in A,\, \eta\in E,\, i\in I,\, j\in J \\ 0 \quad \text{else}, \end{cases} \tag{20}$$

Definition 2 A balanced growth path is a competitive equilibrium in which $b_t=b_1$, $\tau_{c,t}=\tau_{c,1}$, $G_t=((1+g)(1+n))^{t-1}G_1$, $a_t{}'(.)=(1+g)^{t-1}a_1{}'(.)$, $c_t(.)=(1+g)^{t-1}c_1(.)$, $l_t(.)=l_1(.)$, $N_t=(1+n)^{t-1}N_1$, $K_t=((1+g)(1+n))^{t-1}K_1$, $T_t=(1+g)^{t-1}T_1$, $\tau_{ss,t}=\tau_{ss,1}$, $SS_t=(1+g)^{t-1}SS_1$, $r_t=r_1$, $w_t=(1+g)^{t-1}w_1$, $Tr_t=(1+g)^{t-1}Tr_1$ for all $t\geqslant 1$ and $\Phi_t((1+g)^{t-1}A,E,I,J)=(1+n)^{t-1}\Phi_1(A,E,I,J)$ for all t and $A\in R_+$. That is, per capita variables and functions grow at constant gross growth rate $1+g$; aggregate variables grow at constant gross growth rate $(1+g)(1+n)$ and all other variables (and functions) are time-invariant.

We work this economy on the computer so first of all, we apply the standard normalization procedure to make the household recursive problem stationary. [5]

3 QUANTITATIVE ANALYSIS

3.1 Functional forms and calibration of the benchmark economy

In this section, we study the calibration of the model economy to the data from the Turkish economy, selection of the parameter values of the model economy, and assumptions about the functional forms. Calibration of the economy is examined through selecting values of demographic, technology and preference parameters.

[5] We follow [Aiyagari and McGrattan, 1998] for the normalization of model, see appendix for detailed equations

3.1.1 Demographics

We have set the demographic parameters in order to have a persistent mimicry between the Turkey economy and stationary demographic structure of the model economy. The entrance age to economy is 20, the retirement age is 60[6], and we assume that agents die for sure at age 85. Each period is set to 5 year, though the model ages consistent with the turning points we mentioned are 1 for the entrance age to the economy, 9 for the retirement age and 13 for the certain death age. The growth rate of population n is assumed to be constant and calculated as the average of the long-run annual data series (between 1985 and 2005 data from the Turkish Statistical Institute, TUIK), and is set to 1.8%.

The population structure in the model is determined together by the maximum age J, the population growth rate and the survival probabilities. We assumed people survive for sure until the retirement age and then until the certain death date they face a constant mortality rate. We set mortality rate after age 60 so that the fraction of population over 60 to population of working age equals 17.6 percent as observed in the data. We found this 17.6 ratio from the data by simply dividing the population over 60 to the number of people total employed for the year 2000.[7] Our demographic parameters are summarized in Table (1).

Table 1: Demographic Parameters

Parameter	Value	Target
Retirement Age	9(60)	Assumed
Maximum Age	13(85)	Certain death(assumed)
Mort Rate after ret.	.2305	Dependency ratio=17.6%
Pop. growth	1.8%	Data

[6]The official retirement age is 58 for woman and 60 for men currently, we take retirement age for men for simplicity

[7]Data is taken from the Turkish Social Insurance Institute(SII) statistics and the Turkish Statistical Institute (TUIK).

3.1.2 Preferences

It was assumed that individuals choose their preferences of consumption level and leisure level according to a standard time-separable utility function of the form:

$$U(c,l) = \frac{(c_j^{\gamma}(1-l_j)^{1-\gamma})^{1-\sigma}}{1-\sigma} \tag{21}$$

We calibrate the preference parameters again using the Turkish data. Following Conesa and Krueger (2005), we fix the coefficient of relative risk aversion σ to 4. Then we chose the discount factor β in order for the equilibrium of our benchmark economy to imply a capital-output ratio of 2.5.[8]. Since we set the period year to five, we divide this ratio to five, i.e the five year counterpart of it is 2.5/5. We choose the share of consumption $\gamma = .4$ in the utility function so that people of working age on average .33 of their discretionary time.

Table 2: Preferences Parameters

Parameter	Value	Target
β	.127	K/Y=2.5
σ	4	Fixed
γ	.4	Avg hours=1/3

3.1.3 Endowments

Agents have one unit of time to allocate between leisure and work. Their labor productivity at work depends on three factors; type of the agent α_i, age of the agent ϵ_j and idiosyncratic stochastic component η_t. These three factors with the proportion of the unit time agent devoted to work determines the effective labor force, in a multiplicative fashion. i.e. $\varepsilon_j \alpha_i \eta l$. The age specific component of the efficiency units is taken from [Bağış, 2009]. [9] They were summarized

[8]The capital-output ratio is taken from [Şeref Saygılı et al., 2005]

[9]In his thesis, he used weekly hours and wages from 1985 to 2005 for each age group of agents. Then, he evaluated hourly wages for each individual and mean hourly wages for each age group and mean hourly wage of all individuals.(To find the mean hourly wages, he simply divided hourly wages by 4(weekly payments) and then divided by working hours per week, which is average hourly wages for those working over 30 hours a week, that

in Table (3).

Table 3: Age-specific productivity levels

Age	Productivity
1	0.570
2	0.808
3	1.012
4	1.129
5	1.201
6	1.232
7	1.134
8	0.858
9	0.697

For the productivity levels depending on the ability and the stochastic idiosyncratic productivity, we use the findings of the [Conesa and Krueger, 2006]. They assumed two types for ability, high productivity and low productivity, $M = 2$ with equal probabilities, $p_i = 0.5$ for $i = 1, 2$. They were presented in Table (4). And finally for the stochastic shocks, they assumed individuals enter the economy with average productivity level $\bar{\eta}$. Then through their lives they hit by shocks which is changing according to a seven state Markov chain.[10]

We used these shock values and the distribution and create Markov chain using [Tauchen, 1986] The states and the values are stated in Table (5).

is of full-time workers). Finally he found the age specific efficiency by dividing mean hourly wages of each age group to the mean hourly wage of all.

[10]For more detailed description, see[Conesa and Krueger, 2006]

Table 4: Ability

Parameter	Value	p_i
α_1	0.6115	0.5
α_2	1.6354	0.5

Table 5: Stochastic Productivity

Parameter	Value	Π
η_1	.447	0.034
η_2	.589	0.135
η_3	.749	0.214
$\eta_4 = \bar{\eta}$	.942	0.236
η_5	1.185	0.214
η_6	1.508	0.135
η_7	1.986	0.034

3.1.4 Technology

Aggregate output is produced according to a standard constant returns to scale Cobb-Douglas technology:

$$F(K_t, N_t) = K_t^{\alpha}(A_t N_t)^{1-\alpha} \tag{22}$$

The capita share parameter $\alpha = 0.57$ is taken from the paper [Şeref Saygılı et al., 2005] The constant growth trend for the aggregate variables is $(1+n)(1+g)$, where n is the growth rate of adult population and g is the growth rate of per capita GDP. We choose $g = 2.23\%$ in accordance with the long-run growth rate of the per capita GDP for the Turkish data.

$$K_{t+1} = I_t + (1-\delta)K_t \tag{23}$$

The depreciation rate δ is computed from the equation for capital accumulation by using $K_{t+1} = ((1+g)(1+n))K_t$ where investment to output ratio is 0.226. [11] Technology parameters

[11] Investment output ratio is taken as the long run average investment share for the Turkish economy (from

are summarized in Table (6).

Table 6: Technology Parameters

Parameter	Value	Target
α	0.57	Data
δ	5.3%	I/Y=22.6%
g	2.23%	Data

3.1.5 Government policies and the income tax function

The regulations about determination of taxable income have been regulated in tax legislation in Turkey. Valuation measurements are included in Tax Procedures Law (TPL). In Income Tax Law (ITL) and Corporation Tax Law (CTL), there are incomes and expenses that should be taken into account while determining taxable income. Turkey's actual tax function is a progressive tax function. Marginal tax rates for 2006 is described as follows; for income less than 7,000 TL, tax rate is 15 percent, meanwhile, someone earning more than 7,000 TL would face a more complicated calculation i.e. for income levels between 7,000 and 18,000; for the first 7000 TL, 1050 TL is taken and for more than 7,000 until 18,000, the tax rate is 20 percent, for income levels between 18,000 and 44,000, for the first 18.000, 3.250 is taken and for more than 18,000 until 44,000, the tax rate is 27 percent, for income levels higher than 44,000, for the first 44,000, 9,190 TL is taken and for more than 44,000, the tax rate is 35 percent. These were summarized in Table (7). In order to use this tax law in our model, first of all we take the annual average income for 5,477$ per year from statistics of Maliye Bakanligi for the year 2006.[12], then convert the brackets according to the income levels in our calibrated economy.[13]

The replacement rate is chosen according to the actual social security law. We assumed individuals work 30 years. Income replacement rate for retirement pension was 3.5 percent for

TUIK statistics)

[12]Maliye Bakanligi, "Genel Faaliyet Raporu - 2006", www.maliye.gov.tr - June 2007.

[13]See appendix for a detailed description.

Table 7: Turkey individual income tax rates 2006

The Tax Base (TL)	Tax
0-7000	15
7001 - 18000	20
18001- 44000	27
44001 and over	35

the first ten year, 2 percent for the second ten year and 1.5 percent for the third ten year periods. Accordingly we choose the replacement rate as 70 per cent.

In order to satisfy the assumption of a balanced budget for the social security system, and our assumptions about demographics, the required social security tax is $\tau_{ss} = .128$. The calibrated rate is less than the actual social security tax which is roughly %30 on average. This is so because in our model we ignore the possibility of early retirement which is a big case for Turkish economy.[14] Even with the 2006 reform, the minimum retirement age could not have been restored to the official retirement age (which is 58 for woman and 60 for men currently). Therefore there are many retirees even in their late-30's. And for the OECD reports Turkey is coming first between the OECD countries in terms of paying social security benefits for longest years.[15] The proportional consumption tax rate is set to 13.6%. The principal focus of this paper is the income tax code, so we simply fix the consumption tax according to the Turkish data.[16]

For the main subject of our study, income tax code, we use functional form based on the modern developments of the theory of equal sacrifice (see Gouveia and Strauss, 1994). The tax function has beautiful features such that it yields a flexible functional form. It performs a wide range of functional form, nesting from proportional tax code, to a variety of regressive and progressive tax codes. Finally this functional form serves us to find the optimal tax code with an

[14]For example, the 62% of the retirees from SSK retired before the minimum official retirement age can explain us the huge difference between the actual social security tax and the calibrated one.

[15]For more detailed analysis of Turkish Social Security System see [Brook et al., 2006]

[16]Maliye Bakanligi, "Genel Faaliyet Raporu - 2006", www.maliye.gov.tr - June 2007.

assumed welfare function form by varying the parameters of it. The tax code is in the form,

$$T(t) = a_0(y - (y^{-a_1} + a_2)^{-1/a_1}), \tag{24}$$

where (a_0, a_1, a_2) are parameters, $T(y)$ denotes the total tax paid by an individual, and y denotes the pre-tax labor income.

The technical properties of this function are, first of all marginal and average tax rate become a_0 as y goes to infinity i.e. $lim_{y\to\infty}T(y)/y = lim_{y\to\infty}T'(y) = a_0$. Secondly with $a_1 = -1$, tax code turns to be constant value $T(y) = -a_0a_1$, and doesn't depend on the income level. Additionally when a_1 goes to infinity tax code reflects a purely proportional system with tax rate a_0, $T(y) = a_0y$. Finally, for strictly positive values of a_1, the tax code becomes a progressive system since,

$$t(y) = \frac{T(y)}{y} = a_0(1 - (1 + a_2y^{a_1})^{-1/a_1}), \tag{25}$$

$$T'(y) = a_0(1 - (1 + a_2y^{a_1})^{-1/a_1-1}) \tag{26}$$

and thus the average and marginal taxes are strictly increasing function of income y. Here the parameter a_2 is used to satisfy the assumption of balanced budget in the balanced growth path and one more thing about it is that it depends on the units of the measurements, in case all variables are scaled by a fixed factor, a_2 has to be adjusted for the sake of having the same tax function.[17] Technology parameters are summarized in Table (8).

[17]The parameter a_2 has to be adjusted in the following way. When we scale income by a factor$\gamma > 0$, in order to have the same function we should change a_2 according to the following equation: $a_2y^{a_1} = \bar{a}_2(\gamma y)^{a_1}$ and therefore $(\bar{a}_2 = a_2\gamma^{-a_1})$

Table 8: Policy Parameters

Parameter	Value
τ_c	13.6%
τ_k	15%
τ_{ss}	12.8%
b	.7

3.2 The computational experiment

The optimal tax code is chosen so that with this tax code ex ante steady state expected utility of a newborn is maximum. The social welfare function is

$$\begin{aligned} SWF(T) &= \int_{\{(a,\eta,i,j):a=0,j=1\}} v_t(a,\eta,i,j)d\Phi_T \\ &= \sum_{i\in\mathbf{I}} p_i v_t(a=0,\eta=\bar{\eta},i,j=1) \end{aligned} \tag{27}$$

where T is the given tax code with parameters (a_0, a_1, a_2), $\Phi_T(a,\eta,i,j)$ is the invariant measure of the corresponding balanced growth path, and $v_T(a,\eta,i,j)$ specifies the value function, from here we are going to search for the parameters a_0 and a_1 which maximizes the utilitarian social welfare function we choose;

$$T^\star = arg \max_{(a_0,a_1)} SWF(T) \tag{28}$$

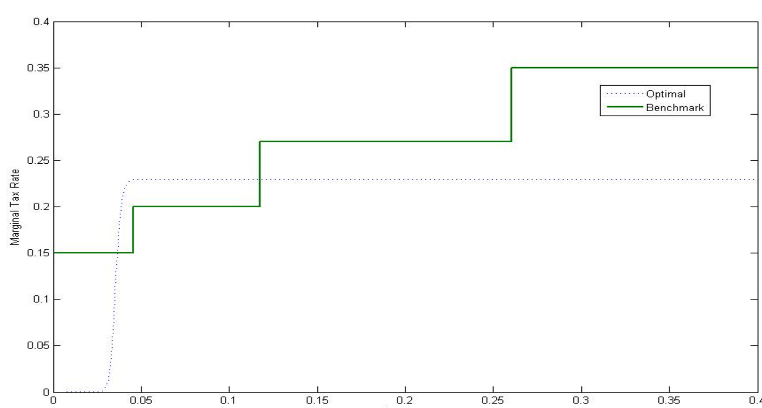
In order to find the parameters (a_0, a_1) so that the tax code that is maximizing our chosen welfare function, we specify grids for these parameters. Then, we find the associated steady state equilibrium for each parameter combination, and choose one combination that maximize the welfare. Then we compare the benchmark economy and the economy associated with the optimal tax code in terms of total efficient labor supply N, total capital stock K, total output GDP. We looked at the average hours worked in order to analyze if the optimal tax code create any disincentives to work, and also we looked at the equity effects of the chosen tax code on the distribution of pre-tax income, after-tax income, consumption and wealth.

4 THE OPTIMAL TAX CODE

After representing the economy on computer, we find that the optimal tax code have parameters of $a_0 = .23$ and $a_1 = 22$. When we approximate it with a proportional tax code, the tax rate will be roughly 23% with a fixed deduction of about half of the average labor income which is roughly 3800TL.

In figure (1), we plot the marginal tax rates of the benchmark economy against the marginal tax rates of the optimal tax code we have found. From the results we conclude for the optimal tax code that marginal tax rates are lower for the upper and lower tails of the income distribution. And we can say for the optimal tax code that for labor incomes under about half of the average income, marginal tax rates are roughly zero.

Figure 1: Marginal tax rates under 2 tax regimes



In Tables (9), (10) and (11), we listed the macroeconomic aggregates for benchmark economy and for associated optimal tax economies in order to understand the economic forces un-

Table 9: Comparison across tax codes I

Variable	Benchmark	Optimal(small open economy)
Parameter a_0	-	.23
Parameter a_1	- 22	
Interest rate r	12.8%	12.59%
Wages w	-	1.8%
Average hours worked	0.300	-1.43 %
Total labor supply N	-	.68%
Capital stock	-	3.84%
GDP Y	-	2.33%
Aggregate consumption C	-	2.76%
Gov. share in GDP	24.02%	23.47%
Total income tax as % of Y	5.1%	4.63%
Gini coefficient for pre-tax income	0.3733	0.3605
Gini coefficient for after-tax income	0.3848	0.3624
Gini coefficient for wealth	0.5236	0.4926
Gini coefficient for consumption	0.3328	0.3188
ECV	-	6.24%

derlying the results after the optimal tax code. In our first and main exercise we optimize the welfare keeping wages and interest rates fixed in order to reflect the small open economy character of the Turkish economy. In the second exercise we employ a pure proportional tax system without exemption level and find the associated parameters optimizing the welfare. This exercise helps us to isolate the efficiency from insurance and redistribution effect. We also optimize the welfare function assuming closed economy and let the prices change accordingly. All the associated macroeconomic aggregates are stated in Table (11) column three. And finally, we take the parameters of the optimal tax code we found under the assumption of closed economy, but this time we don't let the prices change. This exercise is done for the sake of isolating the effects of higher steady state capital stock and wages. The results are presented in Table (11).

Beside the understanding we get from the comparison of macroeconomic aggregates, we compute the welfare differences between different tax codes. The tool we utilize for comparison is CEV(consumption equivalent variation) which measures how much we should increase consumption uniformly, in each labor leisure allocation, in order to equate the welfare of the benchmark economy to the associated optimal tax system. The CEV values we found are positive which is the sign of increase in the welfare with the new tax code compared to the benchmark system.

Table 10: Comparison across tax codes II

Variable	Benchmark	Proportional
Parameter a_0	-	.09
Parameter a_1	-	0
Interest rate r	12.8%	13%
Wages w	-	-2%
Average hours worked	0.300	4.46%
Total labor supply N	-	7.56%
Capital stock	-	3.6%
GDP Y	-	5.32 %
Aggregate consumption C	-	7.93%
Gov. share in GDP	24.02%	22.83%
Total income tax as % of Y	5.1%	3.85%
Gini coefficient for pre-tax income	0.3733	.3775
Gini coefficient for after-tax income	0.3848	.3856
Gini coefficient for wealth	0.5236	.5345
Gini coefficient for consumption	0.3328	.3563
ECV	-	-0.5%

In our first exercise, we noticed that total labor supply and capital accumulation increased

Table 11: Comparison across tax codes III

Variable	Benchmark	Optimal(closed economy)	Fixed(w,r)
Parameter a_0	-	.27	.27
Parameter a_1	-	22	22
Interest rate r	12.8%	12.58%	12.58%
Wages w	-	2.1%	0%
Average hours worked	0.300	-1.93 %	-2.83%
Total labor supply N	-	.01%	-.01%
Capital stock	-	3.03%	3.13%
GDP Y	-	1.43%	.96%
Aggregate consumption C	-	1.35%	1%
Gov. share in GDP	24.02%	23.68%	23.58%
Total income tax as % of Y	5.1%	4.92%	4.63%
Gini coefficient for pre-tax income	0.3733	0.3599	0.3606
Gini coefficient for after-tax income	0.3848	0.3613	0.3623
Gini coefficient for wealth	0.5236	0.4942	0.493
Gini coefficient for consumption	0.3328	0.3121	0.3085
ECV	-	7.3%	7.1%

by .68%, 3.84% respectively in the optimal tax code, compared to the benchmark economy. Thus, by the nature of production function, these two effects cause GDP per capita to increase by 2.33%. In the light of this result and the numerous decrease we noticed in the marginal tax rates for the high end of the income distribution, we can conclude that optimal tax system reduced disincentives to save and work for upper tail of the income distribution. Besides this, although there is a vastly decrease of 1.43% in the average hours worked, total labor supply increases which is caused by the high ability, high productivity agents. They respond with a higher labor supply to the decrease in marginal tax rates for their income brackets. Finally, the share of government expenditure and so average tax rate required to fund government expenditures goes down. With higher GDP after the new income tax code and lower fractions devoted to the government outlays, there is an increase in fractions of private consumption and investment which is an increase about 2.76% for aggregate consumption. Column 3 of Table (9) presents the results from the exercise.

The optimal tax system not only increases the incentives to work and save but also creates a more equal income, wealth and consumption distribution. We can see it with the decrease in the amounts of Gini coefficients for income, consumption and wealth. For the after-tax income Gini coefficients, firstly under the optimal tax system, the lower tail of proportion 30% faces increase in their incomes, with almost 83% of them pay zero tax, and the remaining 17% have marginal tax rate lower than 15% of their income that they have to pay under the benchmark tax system. Secondly the individuals in the middle class of 60% of the income distribution which corresponds to the second interval of the current tax system have to pay more income tax under the optimal tax system. These two effects make the incomes of the lower class and the middle class come close to each other. But on the other hand, under the optimal tax system since the most upper class, high productivity and high ability agents have to pay less, they save and work more under the new tax code, and their income increase disproportionately. This effect is in the way to increase the dispersion of the income. Since their proportion in the population is low compared to the middle class, the first two effects dominate the negative effect of the increase in the incomes of the upper class. It should also be noticed that after tax income Gini increases more proportionately than the before tax income Gini, this stems from the deduction of about half of the average income. The decrease in consumption Gini is the result of more equally distributed income.

We compare the benchmark economy with a pure proportional tax system without any deduction. All the macroeconomic aggregates increase with high proportions but the welfare of the agents and equality in income distributions suffer with the pure proportional tax code. In this economy, fixed deduction is absent, so that we cannot realize increase in the equality of income and thus consumption. Both consumption Gini and income Gini increased compared to the benchmark economy. Although GDP per capita increases by a huge amount 5.32% and aggregate consumption increases by 7.93% under purely proportional tax system, social welfare is lower. In purely proportional tax system, insurance against idiosyncratic income shocks and insurance against being born as a low type does not exist. The increase in the welfare of the high ability individuals is dominated by the decrease in the welfare losses of low ability agents. The results can be seen in Table (10).

Finally we assumed closed economy, and allow prices change accordingly and, in order to isolate the effects of higher steady state capital stock and wages, we fixed the wage rate and interest rate to the benchmark balanced growth path levels. First of all, We found the optimal tax system for the closed economy, which has the parameters of $a_0 = .27$ and $a_1 = 22$. For the closed economy exercise, the results about the directions of the macroeconomic aggregates, equality of distributions and welfare are qualitatively similar with those of we found for the optimal tax system under the assumption of small open economy. Then we calculate a new steady state with these tax parameters and the fixed prices. By doing this we wanted to present pure effects of new tax system. We wanted to see the portion of welfare gains caused by higher capital and higher wages and the portion resulted from the efficiency gains from decreased disincentives on labor supply, having partial insurance against income shocks and more equity in the distribution of income. We see that, when we reset the effect on welfare gains of the increase in steady state capital and wages, welfare gains decreased slightly from 7.3 to 7.1. This reflects the fact that a huge portion of the welfare gains is due to the decreased disincentives on labor supply by lower marginal tax rates and more equity in the distribution of income. In column 3 and 4 of Table (11), we present the associated results.

5 CONCLUSION

In this paper, it has been shown that, the optimal progressive income tax for Turkey can be well approximated by a flat tax rate of 23% with a fixed deduction of about half of the average income for Turkish citizens which is roughly 3800TL. This results are found under a stochastic dynamic general equilibrium model with a utilitarian welfare function. With this tax system upper tail of the income distribution faces lower marginal tax rates compared to the actual Turkish tax system which reflects that the disincentives on labor supply and capital accumulation stemming from high marginal taxes is shortened by the new tax system. And also the lower tail of the income distribution is benefited from the deduction of this new tax system. Thus the optimal tax system compensates for the absent insurance markets and provides insurance against idiosyncratic labor income uncertainty. This system implies lower tax burdens for the two ends of the income distribution, but on the other hand the tax burden middle class faces increase compared to the current tax system. Although we did not analyze, the welfare gains of the lower and the upper class, and the welfare losses middle class face, may suggest that if there is such a reform towards the optimal tax system, the middle class may be the biggest opponent to the proposed tax reform.

Welfare gains are very substantial with this tax reform, in order to make people indifferent between the actual tax system and optimal tax system, consumption should increase 6.2% for all agents and for all states of the world. The population structure of Turkey of about the population weights for the income levels, significantly effects the results concerning the equity in distributions of wealth, pre-tax income, after-tax income, and thus consumption. All the Gini coefficients decrease showing there is more equal distribution between the different types of population after the optimal tax code.

For further work, also the transition path induced by a reform of the current towards the optimal tax system that is desired in terms of equity and efficiency of labor supply can be analyzed in order to see if the reform is feasible or not.

A APPENDIX

A.1 Tax brackets

We convert tax brackets as follows. First of all, since the income tax brackets is revised according to the average income of the individuals of the previous year, income tax brackets change over almost every year. Thus we take 2006 tax brackets for our analysis. Average income for the year 2006 is 5,477$, we convert it to TL by simply multiplying it with the average exchange rate. We will describe it for just the bottom limit of the second tax bracket and the others are found accordingly. The ratios of the actual economies bottom level bracket BA_2 to the average income of the actual economy AM should be same for our calibrated economy BC_2, so we follow this path; We find the mean income for our calibrated economy MC. Then we multiply it to the ratio we described before. The corresponding tax brackets is found by;

$$BC_2 = MC\frac{BA_2}{AM} \tag{29}$$

A.2 Normalization

In a balanced growth equilibrium, all per capita variables and functions grow at a constant gross growth rate $(1+g)$, aggregate variables grow at a constant gross growth rate $(1+r)(1+g)$. Note that r_t, τ_t, $\tau_{ss,t}$ and τ_c are constant.

We transform the model into a stationary form by dividing the utility function and budget constraint by A_t. Let define our new variables as; $\tilde{c} = c/At$, $\tilde{a} = a/At$, $\tilde{a'} = a'/At$, $\tilde{SS}_t = SS_t/At$, $\tilde{w} = w/At$, $\tilde{T}_t = T_t/At$.

Here for the tax function, we can think in the way that an agent with income y in period one, faces the same average and marginal tax rate as an agent with income $(1+g)^{t_1}y$ in period t, that is;

$$T_t[(1+g)^{t_1}y] = T_1[y](1+g)^{t-1} \tag{30}$$

Now we can rewrite the model as follows. First, we divide through the consumer's budget constraint by A_t and rewrite the consumer's preferences by using the definition of $\tilde{c}$. Thus, the consumer's problem becomes

$$E[A_0^{\gamma(1-\sigma)}\sum_{j=1}[\tilde{\beta}^{j-1}\frac{(\tilde{c}_j^{\gamma}(1-l_j)^{1-\gamma})^{1-\sigma}}{1-\sigma}] \tag{31}$$

subject to

$$\begin{gathered}(1+\tau_c)\tilde{c}+\tilde{a'}=(1-\tau_{sst})\tilde{w}_t\varepsilon_j\alpha_i\eta l+(1+r_t)\tilde{a} \quad \text{for } j>j_r \\ -\tilde{T}_t[(1-.5\tau_{sst})w_t\varepsilon_j\alpha_i\eta l]-r_t\tilde{a}\tau_k\end{gathered} \tag{32}$$

$$(1+\tau_c)\tilde{c}+\tilde{a'}=\tilde{SS}_t+(1+r_t)\tilde{a}-\tilde{T}_t[SS_t]-r_t\tilde{a}\tau_k \quad \text{for } j\geqslant j_r \tag{33}$$

$$\tilde{a'}\geqslant 0 \quad \tilde{c}\geqslant 0 \quad 0\leqslant l\leqslant 1 \tag{34}$$

where $\tilde{\beta}=\beta(1+g)^{\gamma(1-\sigma)}$

A.3 Gini Coefficients

Since we have discrete probability distribution, finding the gini coefficients we follow the following way. We reset points with zero probabilities in the discrete distribution $f(y_i)$ and indexed the matter of gini coefficient in increasing order i.e.($y_i < y_{i+1}$):

$$G=1-\frac{\sum_{i=1}^{n}f(y_i)(S_{i-1}+S_i)}{S_n} \tag{35}$$

where

$$S_i=\sum_{j=1}^{i}f(y_j)y_j \quad \text{and} \quad S_0=0 \tag{36}$$

In our calculations, we apply same procedure to consumption, pre-tax income, after-tax income and wealth of the population and fin all corresponding gini coefficients.

References

[Aiyagari, 1994] Aiyagari, R. R. (1994). Uninsured idiosyncratic risk and aggregate saving. *The Quarterly Journal of Economics*, 109(3):659–684.

[Aiyagari, 1995] Aiyagari, R. S. (1995). Optimal capital income taxation with incomplete markets, borrowing constraints, and constant discounting. *The Journal of Political Economy*, 103(6):1158–1175.

[Aiyagari and McGrattan, 1998] Aiyagari, S. R. and McGrattan, E. R. (1998). The optimum quantity of debt. *Journal of Monetary Economics*, 42(3):447–469.

[Bağış, 2009] Bağış, B. (2009). Social security reform: A macroeconomic approach to the recent reform. Master's thesis, Sabanci University, Istanbul.

[Berliant and Gouveia, 1993] Berliant, M. and Gouveia, M. (1993). Equal sacrifice and incentive compatible income taxation. *Journal of Public Economics*, 51(2):219–240.

[Brook et al., 2006] Brook, A., Çıplak, U., Gönenç, R., and Yılmaz, G. (2006). Making the pension system less of an obstacle to formalisation. In *OECD Economic Surveys: Turkey*, chapter 4. OECD.

[Chamley, 1986] Chamley, C. (1986). Optimal taxation of capital income in general equilibrium with infinite lives. *Econometrica*, 54(3):607–622.

[Conesa et al., 2005] Conesa, J. C., Kitao, S., and Krueger, D. (2005). Taxing capital: not a bad idea after all. *Mimeo.*

[Conesa and Krueger, 2006] Conesa, J. C. and Krueger, D. (2006). On the optimal progressivity of the income tax code. *Journal of Monetary Economics*, 53(7):1425–1450.

[Şeref Saygılı et al., 2005] Şeref Saygılı, Cihan, C., and Yurtoğlu, H. (2005). Türkiye ekonomisinde sermaye birikimi, verimlilik ve büyüme (1972-2003): Uluslararası karşılaştırma ve avrupa birliği'ne yakınsama süreci (2014). *TÜSÍAD Araştırma Raporları Serisi*, 12.

[Hall and Rabushka, 1995] Hall, R. and Rabushka, A. (1995). *The Flat Tax, second ed.* Stanford University Press.

[Hubbard and Judd, 1986] Hubbard, G. and Judd, K. (1986). Liquidity constraints, fiscal policy, and consumption. *Brookings Papers on economic activity*, pages 1–59.

[Huggett, 1993] Huggett, M. (1993). The risk-free rate in heterogeneous-agent incomplete-insurance economies. *Journal of Economic Dynamics and Control*, 17(5-6):953–969.

[Judd, 1985] Judd, K. L. (1985). Redistributive taxation in a simple perfect foresight model. *Journal of Public Economics*, 28(1):59–83.

[Mirrlees, 1971] Mirrlees, J. (1971). An exploration in the theory of optimal income taxation. *Review of Economic Studies*, 38:175–208.

[Stokey and Lucas, 1989] Stokey, N. L. and Lucas, R. E. (1989). *Recursive Methods in Economic Dynamics*. Harvard University Press.

[Tauchen, 1986] Tauchen, G. (1986). Finite state markov-chain approximations to univariate and vector autoregressions. *Economics Letters*, 20(2):177–181.

[Varian, 1980] Varian, H. R. (1980). Redistributive taxation as social insurance. *Journal of Public Economics*, 14(1):49–68.