Sensorless Action-reaction-based Residual Vibration Suppression for Multi-degree-of-freedom Flexible Systems

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***Abstract*—This paper demonstrates the feasibility of controlling motion and vibration of a class of flexible systems with inaccessible or unknown outputs through measurements taken from their actuators which are used as single platforms for measurements, whereas flexible dynamical systems are kept free from any attached sensors. Based on the action reaction law of dynamics, the well-known disturbance observer is used to determine the incident reaction forces from these dynamical systems on the interface planes with their actuators. Reaction forces are considered as feedback-like signals that can be used as alternatives to the inaccessible system outputs. The sensorless action reaction based motion and vibration control technique is implemented on a flexible system with finite modes and all results are verified experimentally.**

I. Introduction

Desire for systems with high speeds, lighter weights and less powerful actuators became significant for majority of nowadays design specifications. On one hand, these flexible systems have numerous advantages over rigid ones. On the other hand, their actuators must attempt to satisfy two demands, namely motion control and active residual vibration suppression. The problem of controlling motion and vibration of flexible systems was extensively studied over the past few decades. However, outputs of these systems are assumed accessible or known. This work is concerned with motion control and active vibration suppression of a class of flexible systems with inaccessible outputs.

Much effort has been expended to suppress flexible system's residual vibration, the most common technique for reducing residual vibration is to pre-filter the control input using either a low-pass or a notch filter [1]-[2] in order to take away any energy at system's resonant frequencies such that the flexible modes will not be excited. Point-to-point motion control is capable of achieving zero residual vibration if the control input waveform succeeded to eliminate any kinetic and potential energy from the elastic elements at the end of the travel [3]-[4]. Introducing additional switching times to the conventional bang-bang control input eliminates the undesired residual vibrations for non-rigid systems [5]-[6]. A novel Pre-shaping technique was proposed in [7] for eliminating residual vibration, based on convolving an arbitrary control signal with a sequence of impulses chosen such that it would not cause residual vibration in the absence of control input. Minimum Energy control of residual vibration was proposed in [9] by imposing additional constrain to the optimal control problem to guarantee the uniqueness of the control law. However, the minimum energy control ignores the higher frequency resonances which would contribute to residual vibration. Cooper and Skaar [10] mentioned that this problem can be alleviated if the control input was to be a minimum "Jerk" rather than a "minimum energy solution".

Each of the previous vibration control technique has special characteristics and drawbacks. None is completely satisfactory under all headings. Furthermore, they all depend on the availability of system states or outputs by either using sensors to measure each state variable or by designing state observers based on availability of system outputs.

A question naturally arises: can we realize motion and vibration suppression control if system outputs are inaccessible and non of them can be measured for certain reasons ? This work presents a motion control and active vibration suppression framework based on realization of the action reaction law of dynamics to control motion and vibration of flexible structures with finite or infinite modes through measurements taken from their actuators.

The proposed algorithm depends on two measurements from the actuator keeping the flexible system free from any additional measurement. Therefore, the word "Sensorless" refers to the flexible plant excluding the actuator, these two measurements are used as inputs to the well-known disturbance observer [12]-[13]. Then disturbance observer structure is modified in order to decouple incident reaction forces from flexible system out of the total disturbance [14]-[15]. Reaction forces are conceptually considered as feedback-like signals which can be used as alternatives to the system inaccessible outputs.

Remainder of this paper is organized as follows, Section II includes a design procedure of an action reaction state observer to estimate system states from measurements taken from the actuator side. Motion and vibration suppression control law is realized in Section III. Experimental results which are conducted on a flexible system with 3-dof are included in Section IV. Eventually, conclusions and final remarks are discussed in Section V.

II. Action Reaction State Observer

The class of dynamical flexible system with inaccessible outputs that we consider is modeled by

$$\dot{x} = Ax + Bu \quad , \quad y = Cx \tag{1}$$

where $x \in \mathbb{R}^n$ and $y \in \mathbb{R}^m$ are the state and measurement vectors, respectively. $A \in \mathbb{R}^{n\times n}$, $B \in \mathbb{R}^{n\times 1}$ and $C \in \mathbb{R}^{1\times n}$ are the system matrix, distribution vector of input and observation column vector, respectively. We assume that system (1) has a single input u. It can be shown that system (1) can be partitioned as follows

$$\dot{x}_a = A_a x_a + B_a u_a + B_{reac} f_{reac}(x, \dot{x}) \tag{2}$$
$$\dot{x}_p = A_p x_p + B_p f_{reac}(x, \dot{x})$$

where x_a and x_p are actuator and plant state vectors, respectively. The subscripts (a) and (p) denote the actuator and plant. $f_{reac}(x, \dot{x})$ is the incident instantaneous reaction force on the actuator, B_{reac} is the reaction force distribution vector. For the dynamical system depicted in Fig.1, there exists a flexible element between the plant and the actuator with stiffness k. Therefore, the reaction force is nothing but the product of the elastic element stiffness with its deflection $k(x_a - x_1)$. Furthermore, if there exist an energy storage element with viscous damping coefficient c, the reaction force would be $c(\dot{x}_a - \dot{x}_1) + k(x_a - x_1)$.

$f_{reac}(x, \dot{x})$ is conceptually considered as a feedback-like component which can be used to design state observer to estimate plant states (x_p) through actuator states (x_a). In other words, it is required to estimate plant states (x_p) without measuring any of its outputs that are assumed unknowns or inaccessible. However, actuator states (x_a) are available. For the system depicted in Fig.1, reaction force can be shown to be $c(\dot{x}_a - \dot{x}_1) + k(x_a - x_1)$, it includes a variable (x_1) from the flexible plant. Nevertheless, this variable does not have to be measured in order to determine $f_{reac}(x, \dot{x})$ since reaction force can be estimated from measurements taken from the actuator [13]. First disturbance force has to be estimated using

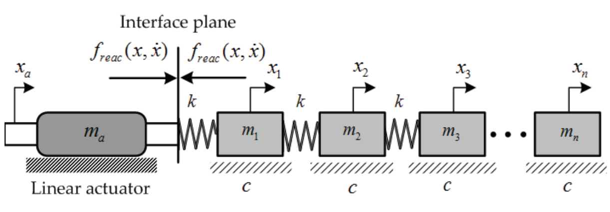


Fig. 1. Dynamical system with n-dof.

the well-known disturbance observer [18]-[19]. Then, reaction force can be decoupled out of the disturbance force and used to design the state observer.

Considering the actuator parameter deviation, (2) can be rewritten as follows

$$\dot{x}_a = (A_{an} + \Delta A_a)x_a + (B_{an} + \Delta B_a)u_a + B_{reac} f_{reac}(x, \dot{x}) \tag{3}$$

$$A_a = A_{an} + \Delta A_a \quad , \quad B_a = B_{an} + \Delta B_a \tag{4}$$

ΔA_a is a deviation of A_a and ΔB_a is the deviation of B_a from the nominal values with the subscript n. Rewriting (3)

$$\dot{x}_a = A_{an}x_a + B_{an}u_a + (\Delta A_a x_a + \Delta B_a u_a + B_{reac} f_{reac}(x, \dot{x})) \tag{5}$$

The third term of (5) is well-known as disturbance (d) with force or torque units [11]-[12]-[13]

$$d \triangleq \Delta A_a x_a + \Delta B_a u_a + B_{reac} f_{reac}(x, \dot{x}) \tag{6}$$

Applying (6) on the following actuator motion equation

$$(m_{an} + \Delta m_a)\ddot{x}_a + f_{reac}(x, \dot{x}) = (k_{fn} + \Delta k_f) i_a \tag{7}$$

where m_a, k_f and i_a are the actuator mass, force constant and current, disturbance force can be written as

$$d = \Delta m_a \ddot{x}_a + f_{reac}(x, \dot{x}) - \Delta k_f i_a \tag{8}$$

where the first and last terms of (d) represents actuator force ripple and actuator self-varied mass [13]. Then, disturbance force can be determined through actuator current and acceleration

$$d = m_{an}\ddot{x}_a - k_{fn} i_a \tag{9}$$

Disturbance force can be estimated through a low-pass filter with a cutoff frequency g_{dist} to avoid the high amplification of noise level due to direct differentiation

$$\widehat{d} = \frac{g_{dist}}{s + g_{dist}}[g_{dist} m_{an}\dot{x}_a + i_a k_{fn}] - g_{dist} m_{an} \dot{x}_a \tag{10}$$

the estimation error $\widetilde{d}$ can be expressed by subtracting (9) and (10) $(\widetilde{d} = d - \widehat{d})$ Therefore, the estimation error dynamics is

$$\dot{\widetilde{d}} + g_{dist}\widetilde{d} = \Omega \tag{11}$$

$$\Omega \triangleq g_{dist}^2 m_{an}\dot{x}_a + g_{dist} i_a k_{fa} + (s + g_{dist})\big[k_f i_a - m_a \ddot{x}_a - g_{dist} m_{an} \dot{x}_a\big]$$

From (11), one can see that selecting $(g_{dist} > 0)$ guarantees that the estimated disturbance force $(\widehat{d})$ would converge to the actual disturbance force (d), estimating disturbance force has to be followed by estimating the reaction force $(f_{reac}(x, \dot{x}))$.

Reaction force $(f_{reax}(x, \dot{x}))$ can be estimated through (8). However, the actuator force ripple $(\Delta k_f i_a)$ and self-varied mass $(\Delta m_a \ddot{x}_a)$ have to be determined. Therefore, (Δm_a) and (Δk_f) have to be identified, rewriting (8)

$$d_{par} = \Delta k_f i_a - \Delta m_a \ddot{x}_a \tag{12}$$

where $\widehat{d}_{par}$ is the estimated disturbance force due to actuator parameter deviation. (Δm_a) and (Δk_f) are actuator's inherent properties. Therefore, they can be identified from the actuator through (12) [14]-[15].

$$\begin{bmatrix} \triangle k_f & -\triangle m_a \end{bmatrix}_{1\times 2} \begin{bmatrix} \underline{i}_a \\ \underline{\ddot{x}}_a \end{bmatrix}_{2\times r} = \begin{bmatrix} \widehat{d}_{par} \end{bmatrix} \tag{13}$$

$$\begin{bmatrix} \widehat{\triangle k_f} & -\widehat{\triangle m_a} \end{bmatrix} = \mathbf{H}^{\dagger} \begin{bmatrix} \widehat{d}_{par} \end{bmatrix}, \ \mathbf{H} \triangleq \begin{bmatrix} \underline{i}_m & \underline{\ddot{x}}_a \end{bmatrix}^T \tag{14}$$

Matrix H consists of two vectors, actuator current ($\underline{i}_a$) and acceleration data points ($\underline{\ddot{x}}_a$). $\mathrm{H}^{\dagger}$ is the pseudo inverse of H. rewriting (8) using estimated disturbance force and identified parameters obtained through (10) and (15), respectively.

$$\widehat{d} = \widehat{\Delta m_a}\ddot{x}_a + f_{reac}(x,\dot{x}) - \widehat{\Delta k_f} i_a \tag{15}$$

Similar to (10), reaction force can be estimated through the following low-pass filter with cutoff frequency g

$$\widehat{f_{reac}}(x,\dot{x}) = \frac{g}{s+g}[g\widehat{\Delta m_a}\dot{x}_a + i_a\widehat{\Delta k_f} + \widehat{d}] - g\widehat{\Delta m_a}\dot{x}_a \tag{16}$$

Similar to the well-known Luenberger observer, structure of the action reaction state observer can be written as follows

$$\dot{\widehat{x}} = A\widehat{x} + Bu + \mathrm{M}\big(\widehat{f_{reac}}(x,\dot{x}) - f_{reac}(\widehat{x},\dot{\widehat{x}})\big) \tag{17}$$

$\widehat{f_{reac}}(x,\dot{x})$ is the estimated reaction force which can be obtained through (16) while $f_{reac}(\widehat{x},\dot{\widehat{x}})$ is the reaction force computed using the estimated states ($\widehat{x}$)

$$f_{reac}(\widehat{x},\dot{\widehat{x}}) = c(\dot{x}_a - \dot{\widehat{x}}_1) + k(x_a - \widehat{x}_1) \tag{18}$$

M is the observer gain vector. It is at least intuitively clear that, observability of system (1) has to be analyzed when measurements are only allowed to be taken from the actuator which implies that the observation vector can be written as

$$C = \begin{bmatrix} 1 \mid 1 \mid 0 \ \cdots \ 0 \end{bmatrix} \tag{19}$$

The change of coordinates $x = \mathrm{T}\xi$ transforms the system (1) into the form

$$\begin{aligned} \dot{\xi} &= \mathrm{T}^{-1}A\mathrm{T}\xi + \mathrm{T}^{-1}Bu = \widehat{A}\xi + \widehat{B}u \\ y &= C\mathrm{T}\xi = \widehat{C}\xi \end{aligned} \tag{20}$$

selecting the non-singular matrix T as

$$\mathrm{T} = \begin{bmatrix} 1 & 1 & 1 & \dots & 1 \\ \lambda_1 & \lambda_2 & \lambda_3 & \dots & \lambda_n \\ \lambda_1^2 & \lambda_2^2 & \lambda_3^2 & \dots & \lambda_n^2 \\ \vdots & \vdots & \vdots & \ddots & \vdots \\ \lambda_1^{n-1} & \lambda_2^{n-1} & \lambda_3^{n-1} & \dots & \lambda_n^{n-1} \end{bmatrix} \tag{21}$$

guarantees that system (20) has a diagonal form. Where, $\lambda_1, \dots, \lambda_n$ are the eigenvalues of the system matrix A. Hereafter, system (20) can be written in the following form

$$\left[\begin{array}{c} \dot{\xi}_a \\ \hline \dot{\xi}_p \end{array}\right] = \left[\begin{array}{c|c} \widehat{A}_a & \phi \\ \hline \phi & \widehat{A}_p \end{array}\right] \left[\begin{array}{c} \xi_a \\ \hline \xi_p \end{array}\right] + \left[\begin{array}{c} \widehat{B}_a \\ \hline \widehat{B}_p \end{array}\right] u \tag{22}$$

$\widehat{A}_a$ and $\widehat{A}_p$ are the actuator and plant diagonal system matrices, respectively. $\widehat{B}_a$ and $\widehat{B}_p$ are the actuator and plant input distribution vectors. respectively.

A regular procedure for estimating states of the flexible plant depicted in Fig.1 is to measure some of its states ($x_p = \mathrm{T}\xi_p$), then using these measurements to design state observer. In this work, plant states (x_p) are assumed inaccessible. However, the reaction force $f_{reac}(x,\dot{x})$ is conceptually considered as a natural feedback from the plant on the actuator and therefore used as an alternative to any measurement from the plant required to design the state observer. Computing $\widehat{C}$

$$\widehat{C} = \begin{bmatrix} 1+\lambda_1 \mid 1+\lambda_2 \mid 1+\lambda_3 \mid \dots \mid 1+\lambda_n \end{bmatrix} \tag{23}$$

taking the time derivatives of the output equation of system (20) we obtain the following matrix equation

$$\underbrace{\begin{bmatrix} y \\ \dot{y} \\ \ddot{y} \\ \dddot{y} \\ \vdots \end{bmatrix}}_{\aleph(y)} = \underbrace{\begin{bmatrix} \widehat{C} \\ \widehat{C}\widehat{A} \\ \widehat{C}\widehat{A}^2 \\ \widehat{C}\widehat{A}^3 \\ \vdots \end{bmatrix}}_{\mathcal{O}} \underline{\xi} + \underbrace{\begin{bmatrix} 0 & 0 & 0 & 0 & \cdots \\ \widehat{C}\widehat{B} & 0 & 0 & 0 & \cdots \\ \widehat{C}\widehat{A}\widehat{B} & \widehat{C}\widehat{B} & 0 & 0 & \cdots \\ \widehat{C}\widehat{A}^2\widehat{B} & \widehat{C}\widehat{A}\widehat{B} & \widehat{C}\widehat{B} & 0 & \cdots \\ \vdots & \vdots & \vdots & \vdots & \ddots \end{bmatrix}}_{\Gamma} \underbrace{\begin{bmatrix} u \\ \dot{u} \\ \ddot{u} \\ \dddot{u} \\ \vdots \end{bmatrix}}_{\aleph(u)} \tag{24}$$

Putting (24) in the following compact form

$$\aleph(y) = \mathcal{O}\ \underline{\xi} + \Gamma\ \aleph(u) \tag{25}$$

where $\aleph(y)$ and $\aleph(u)$ are the Nordsieck vectors of the output and the input, respectively.

Computing the observability matrix $\mathcal{O}$ using (23) and (24), we obtain

$$\mathcal{O} = \begin{bmatrix} 1+\lambda_1 & 1+\lambda_2 & 1+\lambda_3 & \dots & 1+\lambda_{n-1} \\ \lambda_1+\lambda_1^2 & \lambda_2+\lambda_2^2 & \lambda_3+\lambda_3^2 & \dots & \lambda_{n-1}+\lambda_{n-1}^2 \\ \lambda_1^2+\lambda_1^3 & \lambda_2^2+\lambda_2^3 & \lambda_3^2+\lambda_3^3 & \dots & \lambda_{n-1}^2+\lambda_{n-1}^3 \\ \vdots & \vdots & \vdots & \ddots & \vdots \\ \lambda_1^{n-1}+\lambda_1^n & \lambda_2^{n-1}+\lambda_2^n & \lambda_3^{n-1}+\lambda_3^n & \dots & \lambda_n^{n-1}+\lambda_n^n \end{bmatrix} \tag{26}$$

It can be easily shown that $\mathcal{O}$ is full ranked if all eigenvalues of the system matrix A are distinct. Therefore, plant states (x_p) can be observed from measurements taken from the actuator (x_a) under the condition that all eigenvalues of A are distinct. The similarity transformation $x_p = \mathrm{T}\xi_p$ allows obtaining the unique structure of the observability matrix $\mathcal{O}$. Now from (1) and (17), estimation error is $e = x - \widehat{x}$, thus error dynamics is governed by

$$\dot{e} = (I - c\mathrm{ML})^{-1}(A + k\mathrm{ML})e = \mathcal{A}e \tag{27}$$

$$\mathrm{L} = [1\ 0\ \cdots\ 0]$$

Therefore, estimation error (e) will converge to zero if all eigenvalues of ($\mathcal{A} = (I - c\mathrm{ML})^{-1}(A + k\mathrm{ML})$) lie on the left-half plane. Selection of the observer gain (M) is a regular pole placement problem. It can be shown now that the state observer (17) does not necessitate taking any measurement

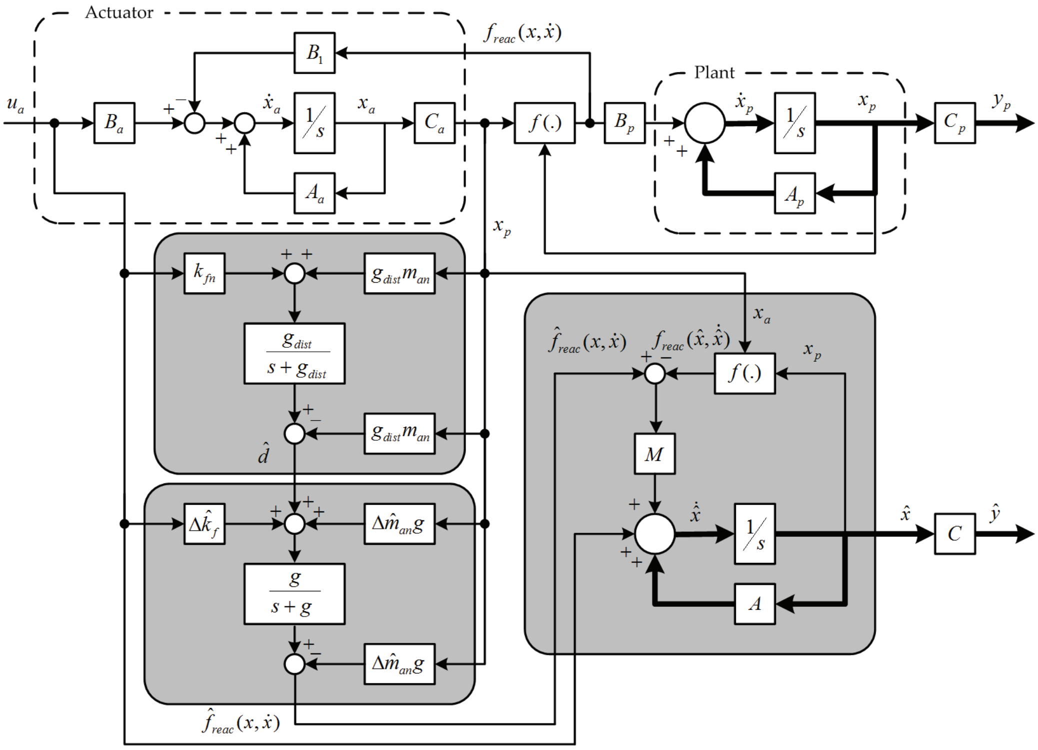


Fig. 2. Action reaction state observer.

from the plant side, plant states (x_p) are not measured at all. However, the incident reaction force is conceptually considered as a natural feedback from the sensorless plant. Thus, used to design the state observer (17) which only requires two measurements from the actuator to estimate disturbance and reaction forces through (10) and (16), respectively. The action reaction state observer is depicted in Fig.2 where flexible plant is kept free from any measurement except the reaction force that is estimated through actuator variables and considered as a single feedback from the flexible plant.

III. MOTION AND VIBRATION SUPPRESSION CONTROL

In order to achieve vibrationless point-to-point motion control of the flexible lumped system depicted in Fig.1, the end conditions have to be

$$\xi\,|_{t=T}= \begin{bmatrix}\xi_a\\0\\\vdots\\0\end{bmatrix} \quad , \quad \dot{\xi}\,|_{t=T}= \begin{bmatrix}0\\0\\\vdots\\0\end{bmatrix} \tag{28}$$

which indicates that in order to achieve vibrationless motion control manoeuvre, potential and kinetic energy have to vanish at the end of the travel ($t = T$). Solution of (20) can be expressed in terms of the following convolution integral [1]

$$\xi(t) = \xi(0)e^{\widehat{A}}t + \int_0^t e^{\widehat{A}(t-\tau)}.\widehat{B}u(\tau)d\tau \tag{29}$$

assuming zero initial condition and replacing the actual state ($x = \mathrm{T}\xi$) with the estimated ones ($\widehat{x} = \mathrm{T}\widehat{\xi}$) obtained through (17)

$$e^{-\widehat{A}t}\widehat{\xi}(t) = \int_0^t e^{\widehat{A}(-\tau)}.\widehat{B}u(\tau)d\tau \tag{30}$$

Using constrain equation (30) to determine the control u which minimizes the energy content of the flexible plant. Therefore, the performance index can be expressed as

$$\mathrm{J} = \int_0^t |u|^2 d\tau + \lambda^T \left[e^{-\widehat{A}t}\widehat{\xi}(t) - \int_0^t e^{-\widehat{A}\tau}.\widehat{B}u(\tau)d\tau \right] \tag{31}$$

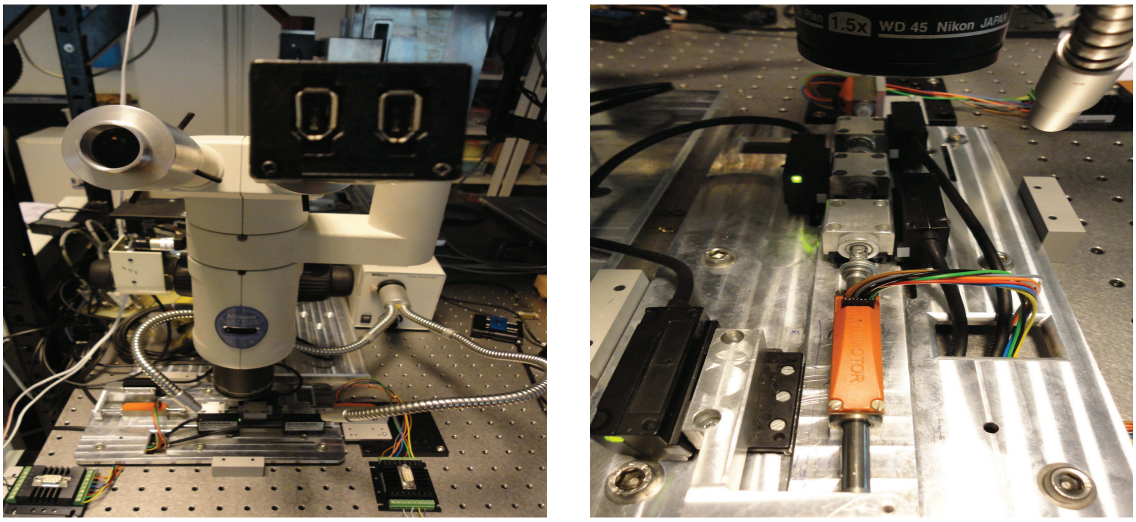
where λ is a vector of Lagrange multipliers. Variation $\delta\mathrm{J}$ can be shown to be

$$\delta\mathrm{J} = \int_0^t \left[2u^* - \lambda^T e^{-\widehat{A}\tau}.\widehat{B}\right]\delta u \; d\tau \tag{32}$$

the variation $\delta\mathrm{J}$ will vanish if the integrand of (32) is zero. Therefore, the control u is

$$u = \frac{1}{2}\lambda^T e^{-\widehat{A}^* t}.\widehat{B} \tag{33}$$

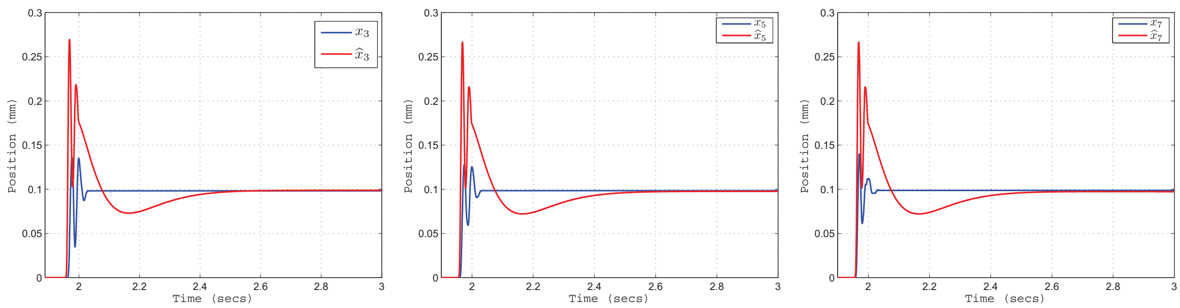
Equation (33) along with the estimated states obtained through (17) allow controlling motion and vibration of any non-collocated mass along the flexible plant from measurements taken from its actuator. In (33), the control law is depending on the Lagrange multipliers which can be computed from the estimated states obtained through (17).



(a) Microsystems workstation

(b) Flexible plant with 3-dof

Fig. 3. Experimental setup.

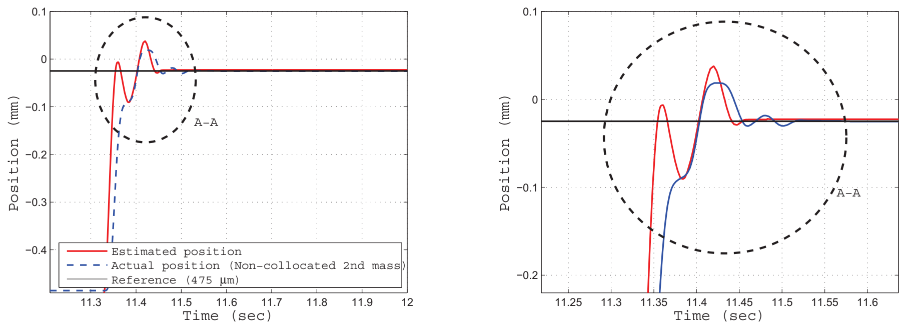


(a) x_1 vs. $\hat{x}_1$

(b) x_3 vs. $\hat{x}_3$

(c) x_5 vs. $\hat{x}_5$

Fig. 4. Experimental states estimation results of a dynamical system with 3-dof (x_3, x_5 and x_7 represent first, second and third masses positions, respectively).



(a) Response of the 2^{nd} non-collocated mass to a $475\mu m$ reference input

(b) Magnified plot

Fig. 5. Sensorless vibrationless motion control experimental result of the 2^{nd} non-collocated mass.

IV. Experimental Results

Experiments are conducted on a flexible system with 3-dof as depicted in Fig.3. The experimental setup consists of a microscope mounted on the top of the 3-dof flexible system. A linear actuator is used as a single platform for measurements.

Optical encoders are attached to each lumped mass as depicted in Fig.3-b in order to verify validity of the action reaction state observer (17) by comparing the actual measurements with the estimated ones. The nominal experimental parameters are included in table.I.

The estimated states obtained through the action reaction state observer are compared with the actual measured ones in Fig.4. The third, fifth and seventh states represent positions of each lumped mass along the flexible system. Convergence time of the estimated states to the actual ones is 0.5 Seconds which can be shortened by placing the poles of the observer (17) in different locations of the s-plane through the observer vector gain M. Furthermore, convergence time of the estimated states can be shortened by changing the positive gains of both the disturbance and the reaction force observers, g_{dist} and g, respectively. Figure 5 illustrates the sensorless vibrationless control result of the second non-collocated mass for a step reference input of $475\mu m$. The previous experiments are conducted on a microsystems workstation where sensor utilization is costly or impractical due to the limited workspace size along with the associated problems with sensors utilization.

V. Conclusion

The problem of motion control and active vibration suppression of a class of flexible structures with inaccessible outputs is addressed in this work. The action reaction law of dynamics is interpreted in a way that allows determination of a feedback-like signal which is conceptually considered as natural feedback from these systems. The natural feedback or the reaction force is then used in the design of a state observer. Flexible plant dynamical states are then estimated through the action reaction state observer. The proposed observer allows estimating flexible system states from measurement taken from its actuator if the system matrix has distinct eigenvalues.

Experimental results showed 0.5 Seconds convergence time due to the phase lag induced by both disturbance and reaction force observers. However, observer poles can be located such that they are twice faster than the controller poles. The estimated states are then used in the sensorless vibrationless control law (33). The control law is satisfying the boundary condition which guarantee that flexible system contains zero potential and kinetic energy in its energy storage elements at the end of the travel.

Experimental results demonstrated the validity of the proposed control framework which can be easily repeated. Applications of the proposed framework are oriented toward control of dynamical system with inaccessible outputs, microsystems, micromanipulation operation and other operation at which sensors utilization is costly or even impractical.

TABLE I
Experimental parameters.

Parameters	Value	Parameter	Value
k_{fn}	6.43 N/A	g_{dist}	100 Hz
m_{an}	59 gm	g	100 Hz
$m_{1n,2n,3n}$	19 gm	k_{bn}	5.25 mv/rpm

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