# A NON-COOPERATION RESULT IN A REPEATED DISCOUNTED PRISONERS' DILEMMA WITH LONG AND SHORT RUN PLAYERS

by

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Submitted to the Social Sciences Institute in partial fulfillment of the requirements for the degree of Master of Art

Sabancı University

August 2008

**Acknowledgements**

First of all, I would like to thank to my thesis supervisor, Mehmet Barlo for his invaluable guidance throughout this thesis. My work would not have been possible without his motivation and brilliant ideas. I would like to also express my gratitude to him for our enjoyable off-class conversations. I am also appreciative to my thesis jury members, Mehmet Baç and Hakan Orbay for their helpful comments about my thesis.

Special thanks go to my professor Toker Doğanoğlu for his valuable comments about my thesis. All of my other Sabancı University Professors are also deserving of my gratitude for everything they have ever taught me.

I am also thankful to TÜBİTAK ”The Scientific & Technological Research Council of Turkey” for their financial support as a scholarship.

Finally, my family deserves infinite thanks for their encouragement and endless support throughout my education.

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Economics, MA Thesis, 2008
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## Abstract

This study presents a modified version of the repeated discounted prisoners' dilemma with long and short-run players. In our setting a short-run player does not observe the history that has occurred before he was born, and survives into next phases of the game with a probability given by the current action profile in the stage game. Thus, even though it is improbable, a short-run player may live and interact with the long-run player for infinitely long amounts of time. In this model we prove that under a mild incentive condition on the stage game payoffs, the cooperative outcome path is not subgame perfect no matter how patient the players are. Moreover with an additional technical assumption aimed to provide a tractable analysis, we also show that payoffs arbitrarily close to that of the cooperative outcome path, cannot be obtained in equilibrium even with patient players.

**Keywords**: Repeated games, discounting, prisoners' dilemma, short and long run players, Folk theorem.

UZUN ve KISA DÖNEMLİ OYUNCULARLA TEKRARLI İSKONTO EDİLMİŞ TUTUKLU AÇMAZI OYUNUNDA İŞBİRLİKÇİ OLMAYAN SONUÇ

Mustafa Oğuz AFACAN
Ekonomi, Yüksek Lisans Tezi, 2008
Tez Danışmanı: Mehmet BARLO

## Özet

Bu çalışma uzun ve kısa dönemli oyuncularla oynanan tekrarlı iskonto edilmiş tutuklu açmazı oyununun değişiklik yapılmış halidir. Bu çalışmamızda, kısa dönem oyuncu doğmadan önceki geçmişi gözlemlemiyor ve oyunun bir sonraki tekrarı için hayatta kalabilme ihtimali o periyottaki aksiyon profili tarafından belirleniyor. Bu suretle, mümkün görunmesede kısa dönem oyuncu sonsuz zaman diliminde hayatta kalabilir ve uzun dönem oyuncuyla oyunu oynayabilir. Bu modelde, periyot getirileri üzerine bir varsayim altında, oyuncularn sabır seviyelerinden bağımsız olarak işbirlikçi sonuç gidişhatının alt oyun tam Nash dengesi olmadığını ispatladık. Ayrıca, analizimizi kolaylaştırmayı sağlayan teknik bir varsayım ile, sabırlı oyuncularla bile işbirlikçi sonuç gidişhatının sağladığı getirilere rastgele yakın olan getirileri dengede elde edemediğimizi gösterdik.

**Anahtar Kelimeler**: Tekrarlanan oyunlar, iskonto, tutuklu açmazı, kısa ve uzun dönem oyuncular, Folk teorem.

# Contents

# Chapter 1

# Introduction

Repeated games are the standard models used to analyze situations in which some number of decision makers are interacting strategically with each other in a repeated fashion. Thus, these models are essential in the theory of economics as they used almost everywhere in the literature.

In fact, repeated games are a certain type of dynamic games in which players face the same stage game in every period. The stage game can be infinitely or finitely played. Moreover, results obtained from finitely repeating stage game are considerably different those of obtained with in infinite repetitions.

In repeated games literature payoffs obtained at the end of the repeated game are defined in three ways. Payoffs description by limit of the means is considered by Aumann and Shapley (1976), overtaking criterion is due to Rubinstein (1979) and the most common description is the discounting payoff structure in which players' payoffs at the end of the repeated game is the summation of discounted stage game payoffs obtained at each stage. It is worthwhile to note that, similar results have been obtained in repeated game literature under all three payoff structures.

The important feature of the repeated game structure is the ability of players to condition their action at each stage on the prior history up to that stage. This distinctive ability of players allows game theorists to obtain very attractive and different results that cannot be obtained in standard one shot games. One of the most important results is the subgame perfect Folk Theorem with discounting, by Fudenberg and Maskin (1986b), which states that when players can condition their behavior unboundedly on the past and the full dimensionality assumption holds (the dimension of the set of payoffs in the stage game equals to the number of players), every individually rational payoff profile (those that exceed the payoffs from the most serious punishment for every player in the stage game) can be sustained in subgame perfect equilibrium when players are sufficiently patient. Moreover,Barlo, Carmona, and Sabourian (forthcoming) shows that the subgame perfect Folk Theorem with discounting holds with 1 memory strategies (where in each period, players behavior depends only on the action profile of the last period) provided that players possess rich action spaces in the stage game (that is, the action set of each player is given by a non-empty convex and compact set). When each player has a finite action set, Barlo, Carmona, and Sabourian (2008) proves a subgame perfect Folk Theorem with discounting and with finite memory, by showing that every individually rational payoff can be approximated by a finite memory subgame perfect strategy (where the magnitude of memory depends on the approximation measure desired). Moreover, with payoff criteria other than the discounting, subgame perfect Folk Theorems with both limit of means and overtaking payoff criterion have been obtained by Aumann and Shapley (1976) and Rubinstein (1979). Moreover, with limits of the means Sabourian (1998) proves the subgame perfect Folk Theorem with finite memory.

The analysis of the repeated prisoners' dilemma has always been one of the cornerstones in the literature. Recall that, standard prisoners' dilemma is a one shot game between two players with special structure. In that prisoners' dilemma structure, action spaces of players consist of two actions namely, defection and cooperation. Defection is the strictly dominant strategy for each player but when they both choose defection the Pareto inferior outcome arises. Because defection is the strictly dominant strategy for both players, the only equilibrium of the one-period prisoners' dilemma is the action profile in which both players choose defection. On the other hand, the analysis of the repeated discounted prisoners' dilemma reveals that cooperation can be obtained when players are sufficiently patient. Moreover, the perfect Folk Theorem of Fudenberg and Maskin (1986a) implies that this conclusion holds for any strictly individual rational payoff. The main reason for this observation is that a deviating player can be punished effectively in the future stages of the game when all players are sufficiently patient, and gives birth to deviations to more beneficial action in the short run not being profitable in the long run. Consequently, the Folk Theorem reduces the prediction power of game theory, which can be best seen in the repeated prisoners' dilemma. While game theoretical tools attempt to predict the equilibrium outcome of games, the Folk Theorem decreases this power by showing that the set of equilibrium payoffs converge to that of the individual rational payoffs when the discount factor tends to 1.

In the repeated discounted prisoners' dilemma literature, one of the important modifications consists of the formulation where the repeated games is played with a long lived (long run) player and countably many short lived (short run) players (who die at the end of every period) given in the studies of Fudenberg, Kreps, and Maskin (1990) and Fudenberg and Levine (2006). In that setting, the long run player plays the stage game in every period with one of the short run players whereas each short run player plays with the long run player in only one period. Under this modification, the conclusion derived drastically changes in contrast to that of standard repeated discounted prisoners' dilemma as is pointed out in Fudenberg, Kreps, and Maskin (1990). In fact, the set of equilibrium outcomes reduces to the repetition of the non-cooperative (defection) action. This is because the short run players do not have any incentives to do an action other than the defection, due to not being able to consider returns from the future phases of the strategic interaction. As a result of this, short run players repeatedly playing deviation is their only best response, and hence the long run player plays defection at each period no matter what the history is.

Therefore, we consider a model that is somewhat in between of these two extreme models, the standard repeated discounted prisoners' dilemma (in which the Folk Theorem holds) and the repeated discounted prisoners' dilemma with long and short run players (in which the defection is the only equilibrium outcome and payoff). In such a model to see if cooperation can be sustained, we think, is an appealing question.

In this study we consider a repeated prisoners' dilemma with a long-run player (player 1), and countably many short run players. In order to provide an easier reading, we describe the model thinking of a soccer club, where player 1 is the owner of the club and the short run players, the coaches. Time is discrete, and every period a coach is born and we refer to a coach born in period $t$ by player $(2.t)$. In the first period, player 1 faces player (2.1), and they both choose an action in $\{C, D\}$. Then, in the same period a public signal in $\{0, 1\}$ is realized, where 0 is to be interpreted as *failure* and 1 to be *success*. The probability distribution on the set of public signals is determined by the action profile chosen in that period. In case of failure, they both receive zero payoffs and coach $(2.t)$ gets fired, and in the next period player 1 faces coach (2.2). On the other hand, in case of success, they receive the prisoners' dilemma payoffs (all strictly positive) and coach (2.1) does not get fired, and is active also in period 2. Thus, in any period $t$, player 1 may face (with some probabilities) any one of players $(2.\tau)$, $\tau \leq t$. We assume that a coach born in period $t$ dies in that period when he is not employed. A $t$ length history in this setting is given by $t$ actions and $t$ public signals, all of which player 1 observes. But player $(2.\tau)$, $\tau \leq t$, does not observe the history (both public and private) before his birth, but the $t - \tau$ tail of a $t$

length history (we work with the convention that the 0 tail of any history is the empty set). All the players discount future payoffs, but not necessarily with the same discount rate.

We show that when payoffs from short run deviations are sufficiently high, sustaining cooperation turns out to be impossible even with patient players. In particular, this study proves that when short run deviation payoffs are sufficiently high, then the cooperative outcome path (which consists of the cooperative action regardless of failures and successes) is not subgame perfect for any discount factor. Moreover, we also show that payoffs arbitrarily close to that of the cooperative outcome path cannot be obtained in equilibrium even with patient players.

Hence, our results indicate that the sheer reason for not obtaining cooperation in prisoners' dilemma with long and short run players is not only because short run players die at the end of every period with probability 1. We identify another important reason: Punishing the long run player might be difficult, which happens when the long run player's stage game payoffs from deviating from the cooperation action is sufficiently high, when the new born short run players cannot observe the past. The main cause of this observation is that, when cooperation is considered, a deviation by player 1 is more difficult to be punished because: (1) a coach born in period $t$ must cooperate on the first day that he is active (note that such players do not observe that player 1 has deviated in the past), and (2) only coaches who faced a deviation by player 1 and became successful are able to punish him.

While our result can be immediately used to provide a simple explanation for why soccer coaches get fired very frequently[1], an important application includes an infinitely repeated version of Kyle's market model with a long run and many short run traders, where all are *informed*.

Kyle's Market Model, due to S.Kyle (1989), is a one shot financial economics model for asset pricing in which there are four parties involved in the trade of an asset. The real value of the asset is only known to one of these traders, the *informed* trader, while the other uninformed but rational player is trying guess the value from the total demand for this asset in order to maximize his returns. The third party involved is the noise trader (also called hedger) who is not rational, and has to demand some random amount of this asset, and this provides noise into the model. The final player is the market maker, who does not observe the real value of the asset (and hence is uninformed), and tries to get the efficiency of the market by determining the price based on the total demand he observes. In the equilibrium of this model, the informed trader achieves a surplus, which can be thought as a rent for his information.

In a simple repeated version of Kyle's model with long and short run players, we imagine that every period there are two informed traders, first one is the long run informed investor and the second is the short run informed

[1] It is appropriate to point out that in the first 10 weeks of the 2007-2008 season of the Turkcell Superlig (the Premiere Turkish Soccer League) 10 coaches have been fired.

investor. Indeed, while the first player can be thought of as an investor with high job security, the second players can be viewed as fresh graduates from good economics/business programs being hired as traders, but their jobs are not secure and their continued employment critically depends on their performance.

We, then, consider a simplified version of this interaction where the information rent of these players depend on their actions which only take 2 values, high (defection) or low (cooperation). In particular we do not have an uninformed but rational player and the noise traders. Instead, we simplify the model by assuming that the market maker is myopic (discounts future returns with discount rate of zero) and cannot condition his behavior on the past. He can identify the real value of the asset with a probability which depends on the action choices of the two traders. Indeed, when the market maker identifies the real value of the asset, it can be considered as a *failure* for the two traders and in this case they obtain zero excess returns. Moreover in this contingency the short run informed investor loses his job, and is replaced by another in the next period. On the other hand, if the market maker cannot identify the real value of the asset, then the excess return that the informed traders obtain (the information rent) depends again on their action choices, and is in the form of a standard prisoners' dilemma. Furthermore, in this state of the world, particular short run informed investor continues to be employed in the next period.

Our results then imply that even with patient investors, cooperation can not be obtained when deviations in the short run are sufficiently beneficial. Recall that the main reason for this observation is not only because of the lack of incentives of the short run investors, but also because of the lack of efficient punishments for the long run investors under the assumption that short run investors do not observe histories that happened before their birth.

It needs to be emphasized that restricting a player 2 born in period $t > 1$ not to observe histories (actions and public signals) prior to his birth is an important one. Indeed, the formulation with player $(2.t)$ being able to observe the public signals (but not past action profiles) prior to his birth is more appealing. We need to say that it is not known to us whether or not some versions of our result can be extended to such situations. On the other hand, it needs to be emphasized that such endeavors are not trivial. To see this, consider player $(2.t)$ being able to observe the public signals that has occurred before his birth, and to see some implications and related complications consider the following strategy: On the first day he is born, he cooperates if the public signals are all consisting of 1's (i.e. the past has been nothing but success); otherwise, he defects. And, in later periods he cooperates only when player 1 cooperated in the periods that are observable to player $(2.t)$; otherwise, he defects. With this strategy a deviation of player 1 from the cooperative path can be punished more effectively, because his deviation would increase the probability of failure, and thus, the probability of his deviation being punished by a player 2 who was born in a period after player 1's deviation. Thus, a general formulation with player $(2.t), t > 1$ observing public signals in periods $\tau = 1, \ldots, t-1$, would call for checking whether or not we can adopt techniques presented in Abreu, Pearce, and Stachetti (1990), and analyzing equilibrium payoffs. In particular, whether or not the principle of one–deviation holds in this setting is the main obstacle that one has to deal with, when such generalizations are to be considered.

Our analysis of the repeated prisoners' dilemma differs from the standard (complete information) versions in the following ways: As in Fudenberg, Kreps, and Maskin (1990) and Fudenberg and Levine (2006) our model features short and long-run players with the important distinction that short-run players may survive with some probability into future phases of the game, whereas in Fudenberg, Kreps, and Maskin (1990) and Fudenberg and Levine (2006) long run player plays with different short run player in each period, and this imposes an important constraint by requiring that each equilibrium outcome must lie in short run players' best responses. On the other hand, our study does not involve such a constraint since there is always a positive probability of survival of short run players in each period.

Moreover, the second important difference is the additional restriction that short-run players do not observe the history prior to their birth. In that regard, our setup shares some similarities with the analysis of limited memory in the context of the repeated prisoners' dilemma with complete information and long-run players. We refer the reader to Aumann (1981), Neyman (1985), Rubinstein (1986), Kalai and Stanford (1988), Sabourian (1998), Barlo, Carmona, and Sabourian (2008), and Barlo and Carmona (2007) for more on the subject. On the other hand, Cole and Kocherlakota

(2005) delivers a similar conclusion to ours in the context of a repeated prisoners' dilemma with imperfect monitoring and finite memory: They prove that for some parameter settings the only strongly symmetric public perfect equilibrium consists of the repetition of the non-cooperative action profile regardless of the discount factor. Meanwhile, other studies of the repeated prisoners' dilemma with imperfect monitoring include Bashkar and Obara (2002), Mailath and Morris (2002), Mailath, Obara, and Sekiguchi (2002), and Piccione (2002).

# Chapter 2

# The Model

We consider a similar game to the partnership game of Radner, Myerson, and Maskin (1986). Every period, a long-run (infinitely lived) player, henceforth to be referred to as the first player, is interacting with one of countably many short-run players. Every period a short-run player is born, and the one born in period $t$ will be referred to as player $(2.t)$.

The period interaction is a modified version of the standard prisoners' dilemma: each action profile in $A \equiv \{C, D\} \times \{C, D\}$ is followed by a public signal $\theta$ in $\{0, 1\}$, where $\theta = 1$ signals the "success" of the interaction between player 1 and 2 in period $t$. When $\theta = 0$, both agents do not obtain any payoffs from their interaction. Moreover, player $(2.t')$, $t' \leq t$ that player 1 has played against in period $t$, is "fired" and player 1 will interact with player $(2.(t+1))$ in the next period. On the other hand, in case of success the players obtain the following payoffs:

$$\begin{array}{ccc} & C & D \\ C & (1,1) & (c,b) \\ D & (b,c) & (d,d) \end{array} \tag{2.1}$$

where $b > 1 > d > c > 0$ and $\frac{b+c}{2} < 1$. Moreover, then $(2.t')$, $t' \leq t$, is not to be fired and will be the player 2 that player 1 will play against in period $t+1$.

We let $\Pr(\theta = 0|CC) = p_1$, $\Pr(\theta = 0|CD) = \Pr(\theta = 0|DC) = p_2$ and $\Pr(\theta = 0|DD) = p_3$, with $0 < p_1 < p_2 < p_3 < 1$. It is worthwhile to point out that the probability of success decreases with defection.

Consequently, in period $t$ players obtain the following (short-run) returns:

| $Pl1/Pl(2.t')$ | $C$ | $D$ |
|---|---|---|
| $C$ | $(1-p_1),(1-p_1)$ | $(1-p_2)c,(1-p_2)b$ |
| $D$ | $(1-p_2)b,(1-p_2)c$ | $(1-p_3)d,(1-p_3)d$ |

for $t' \leq t$, where $t'$ is the period in which the player 2 that player 1 faces was born. In order to ensure that the short-run payoffs is given by a prisoners' dilemma, we have the following assumption:

**Assumption 1** *$(1-p_2)b > (1-p_1)$, $(1-p_3)d > (1-p_2)c$, $(1-p_1) > (1-p_3)d$, $b > 1 > d > c > 0$ and $\frac{b+c}{2} < 1$.*

We denote the set of histories by $H$, any $t-1$ length history consists of signals and action played up to $t$ period, $h_t = ((a_1,\theta_1),(a_2,\theta_2),....,(a_{t-1},\theta_{t-1}))$.

Let $f_1$ be the pure strategy of player 1 such that $f_1(h_t) \in \{C, D\}$ for each period $t$. We denote the set of all pure strategies of first player by $F_1$.

The following assumption will play a critical role in our analysis:

**Assumption 2** *Assume that player* $(2.t)$*, a second player born in period* $t$*, is restricted to use pure strategies that do not depend on the history (both public and private) that has happened before he was born.*

Consequently, for any $t' \geq t \geq 1$ and for any $t' - 1$ length history $h$, let $T^{t'-t}(h)$ be the $t' - t$ tail of $h$. That is, given that $h = ((a_\tau, \theta_\tau)_{\tau=1}^{t'-1})$, $T^{t'-t}(h) \equiv ((a_\tau, \theta_\tau)_{\tau=t}^{t'-1})$. Obviously, if $t' = t$, $T^0(h) = e$ for all $t' - 1$ length history $h$. We let $f_{(2.t)}$ be the pure strategy of player $(2.t)$ so that for any $t' \geq t$ and $h_{t'}$ any $t' - 1$ length history, $f_{(2.t)}(h_{t'}) : T^{t'-t}(h_{t'}) \to \{C, D\}$, thus, $f_{(2.t)} = \{f_{(2.t)}(h_{t'})\}_{t' \geq t}$. Denote the set of all pure strategies for second players by $F_2$.[1]

An outcome path $\pi = \{\pi_t\}_{t \in \mathbb{N}}$ where for any $t$, $\pi_t = ((a_{1,t}, a_{2,t}), \theta_t)$. We denote the set of outcome paths by $\Pi$. Moreover, a strategy pair $f = (f_1, f_2)$ induces the set of possible outcome paths $\pi(f) \in A^\infty \equiv A \times A \times \ldots$ as follows:

[1]Note that since in period $t'$ the exact identity of the second player is one of $\cup_{\tau=1}^{t'}(2.\tau)$ depending on previous signals that are already included in the $t' - 1$ length history $h_{t'}$. Thus, the birth-period of the particular second player that player 1 faces in period $t'$, is already given by the history. Hence, the behavior of a second player born in period $t$, for a $t' - 1$ length history $h_{t'}$, with $t' \geq t$, is described by $f_{(2.t)}(h_{t'}) : T^{t'-t}(h) \to \{C, D\}$.

$\pi_1(f) = ((f_1(e), f_{(2.1)}(e)), \theta_1)$. Note that if $\theta_1 = 1$, the player 2 that player 1 faces in period 2 is (2.1). Thus, $\pi_2(f) = (f_1(a_1, \theta_1), f_{(2.1)}(a_1, \theta_1), \theta_2)$. But, if $\theta_1 = 0$, then player 1 faces player (2.2), thus, $\pi_2(f) = (f_1(a_1, \theta_1), f_{(2.2)}(e), \theta_2)$. An inductive argument can be used to formalize $\pi_t(f)$ as above. To elaborate on this process formally, we need to define the following function: For any $t-1$ length history $h_t$, let $\theta^t = (\theta_1, \ldots, \theta_{t-1})$, and define $\iota : \{0,1\}^{t-1} \to \{1, \ldots, t\}$ by

$$\iota(h_t) = \begin{cases} \tau & \text{if there exists } t' \leq t-1 \text{ with } \theta_{t'} = 0, \\ & \quad \text{and } \tau = \arg\max_{\{t'' \leq t\}} \{t'' : \theta_{t''-1} = 0\} \\ 1 & \text{otherwise.} \end{cases}$$

In words, $\iota(h_t) = \tau \in \{2, \ldots, t\}$ if $\theta^{\tau-1}$ is the last 0 entry in $\theta^t$, and $\iota(h_t) = 1$ if $\theta^t$ does not contain any 0 entry. Thus, for any $t-1$ length history $h_t$, $(2.\iota(h_t))$ with $\iota(h_t) \leq t$ is the player whom player 1 faces in period $t$. Thus, for any $t \in \mathbb{N}$,

$$\pi_t(f) = \left( \left( f_1(\{\pi_\tau(f)\}_{\tau=1}^{t-1}), f_{(2.\iota(\{\pi_\tau(f)\}_{\tau=1}^{t-1}))}(T^{t-\iota(\{\pi_\tau(f)\}_{\tau=1}^{t-1})}) \right), \theta_t \right).$$

Players discount future payoffs with the discount factor $\delta_i \in [0,1)$, $i \in \{1\} \cup \{(2.t) : t \in \mathbb{N}\}$. In particular, for every given outcome path $\pi$ the payoffs are given by:

$$
\begin{aligned}
U_1(\pi) &= (1-\delta_1)\sum_{t=1}^{\infty}\delta_1^{t-1}u_1(\pi_\tau), \qquad (2.2)\\
U_{(2.t)}(\pi) &= \begin{cases}
(1-\delta_{(2.1)})(u_2(\pi_1) & \text{if } t=1;\\
\quad +\sum_{\tau=2}^{\infty}\delta_{(2.1)}^{\tau-1}\prod_{k=1}^{\tau-1}\Pr(\theta_k=1\mid a_k)u_2(\pi_\tau)) & \\
(1-\delta_{(2.t)})(u_2(\pi_t) & \text{if } t>1\\
\quad +\sum_{\tau=t+1}^{\infty}\delta_{(2.t)}^{\tau-t}\prod_{k=t}^{\tau-1}\Pr(\theta_k=1\mid a_k)u_2(\pi_\tau)) & \text{ and } \theta_{t-1}=0;\\
0 & \text{otherwise.}
\end{cases}
\end{aligned}
$$

where $u_i(\pi_\tau) = \Pr(\theta_\tau = 1 \mid a_\tau)u_i(a_\tau)$, and $u_i(a_\tau)$ is as given in the prisoners' dilemma described by equation 2.1.

# Chapter 3

# Sustaining Cooperation is Difficult

Our main observation is that sustaining cooperative behavior in subgame perfection is impossible under a critical condition which guarantees that deviation payoff from cooperation is sufficiently high, condition 3.1 given in the statement of Proposition 1. Under that condition, deviation return is sufficiently high for the first player, and along with the difficulty at punishing the deviating first player by second players (due to their inability to see the past histories and their possible death after each period) makes the cooperative behavior not sustainable in equilibrium. Moreover, the same conclusion holds for payoffs arbitrarily close to that of the cooperative behavior when players are sufficiently patient.

Define the *cooperative outcome path* to be an outcome path $\pi$ such that $\pi_t = ((C, C), \theta_t)$ for any $\theta_t \in \{0, 1\}$ and $t \in \mathbb{N}$. Notice that the cooperative outcome path calls for cooperation independently of the signals.

In the following, we show that under Assumptions 1 and 2 and a condition on stage game payoffs, the cooperative outcome path, cannot be supported as a *Nash equilibrium* for all $\delta_i \in [0,1)$, $i \in \{1\} \cup \{(2.t) : t \in \mathbb{N}\}$.

**Proposition 1** *Suppose that*

$$(1-p_1)(1-p_3)d + p_1(1-p_2)b > (1-p_1). \tag{3.1}$$

*Then the cooperative outcome path $\pi$ is not Nash Equilibrium path for any $\delta_i \in [0,1)$, $i \in \{1\} \cup \{(2.t) : t \in \mathbb{N}\}$.*

**Proof.** Under the cooperative path $\pi$, $U_1(\pi)$ equals $(1-p_1)$, because, $U_1(\pi) = (1-\delta_1)\sum_{t=1}^{\infty} \delta_1^{t-1} u_1(\pi_t)$, and $u_1(\pi_t) = \Pr(\theta_t = 1 \mid a_t) = (1-p_1)$, for every $t \in \mathbb{N}$.

Next notice that any strategy of player $(2,t)$, $t \in \mathbb{N}$, inducing the cooperative outcome path $\pi$, must require player $(2,t)$ to choose $C$ if either $t = 1$, or $t > 1$ and $\theta_{t-1} = 0$. Let $(f_{(2,t)_{t\in\mathbb{N}}})$ be such a pure strategy profile. Define $f_1'$ be the strategy for the first player such that $f_1'(h_t) = D$ for all $h_t$ and $t$. Then, below we show that

$$U_1(f_1', f_2) > (1-p_1)(1-p_3)d + p_1(1-p_2)b.$$

This is because the payoff of player 1 in period $t > 1$ is greater or equal to $(1-p_1)u_1(D,D) + p_1 u_1(D,C) = (1-p_1)(1-p_3)d + p_1(1-p_2)b$. Moreover, this relation holds with strict inequality when $t = 1$. This is because, $\Pr(\theta_{t-1} = 0 \mid a_{t-1}) \geq p_1$ for all $a_{t-1} \in A$, thus, $\Pr(\theta_{t-1} = 1 \mid a_{t-1}) \leq (1-p_1)$ for all

$t > 1$; and, in the first period player 2 chooses $C$ while player 1 goes for $D$. Hence, $U_1(f_1', f_2) > (1-\delta_1)\sum_{t=1}^{\infty}\delta_1^{t-1}(1-p_1)u_1(\{D,D\}) + p_1 u_1(\{D,C\})$, showing that $U_1(f_1', f_2) > (1-p_1)(1-p_3)d + p_1(1-p_2)b$.

Because that

$$U_1(f_1', f_2) > (1-p_1)(1-p_3)d + p_1(1-p_2)b > (1-p_1) = U_1(f_1, f_2),$$

$f_1'$ defined as above is a profitable deviation for player 1 from the cooperative outcome path, thus, the result follows. ■

The intuition behind this result is as follows: Due to Assumption 2, the strategies of the short run players do not depend on the past histories (both public and private) before their birth. Therefore, when the long run player, player 1, deviates from the cooperative outcome path, he can be punished by only the short run player who experienced that deviation, and not by short run players whom player 1 may face in the later phases of the game. Hence, condition 3.1 guarantees that no matter what the value of the discount factors are, player 1 cannot be punished (because of his deviation from the cooperative path) effectively.

In the rest of this section, we analyze whether or not payoffs arbitrarily close to that of the cooperative outcome path can be obtained in subgame perfection.

It is important to point out that for any player $(2.t)$, $t \in \mathbb{N}$, the constant outcome paths $\pi_{(a)}$, $a \in \{C,D\}^2$, defined by $\pi_{\tau,(a)} = (a, \theta_\tau)$ for all $\tau \in \mathbb{N}$ and

$\theta_\tau \in \{0, 1\}$, deliver returns given by:

$$
\begin{aligned}
U_{(2.t)}(\pi_{(CC)}) &= \left(1 - \frac{p_1}{1-(1-p_1)\delta_{(2.t)}}\right), \\
U_{(2.t)}(\pi_{(DC)}) &= \left(1 - \frac{p_2}{1-(1-p_2)\delta_{(2.t)}}\right) b, \\
U_{(2.t)}(\pi_{(CD)}) &= \left(1 - \frac{p_2}{1-(1-p_2)\delta_{(2.t)}}\right) c, \\
U_{(2.t)}(\pi_{(DD)}) &= \left(1 - \frac{p_3}{1-(1-p_3)\delta_{(2.t)}}\right) d.
\end{aligned}
$$

Therefore, even with the normalization by multiplying period returns with $(1 - \delta_{(2.t)})$, the returns of player $(2.t)$ from constant outcome paths depends on his discount factor $\delta_{(2.t)}$. Consequently, the following restriction helps to obtain a trackable analysis.

**Assumption 3** *Suppose that* $p_1 = p_2 = p_3$.

When assumption 3 holds, let $p = p_1 = p_2 = p_3$ and for simplicity, we consider

$$\tilde{U}_{(2.t)} = \left(\frac{1 - \delta_{(2.t)} + \delta_{(2.t)}p}{1 - \delta_{(2.t)}}\right) U_{(2.t)},$$

and because $p \in (0, 1)$, $\tilde{U}_{(2.t)}$ is a linear transformation of $U_{(2.t)}$.

Then, $\tilde{U}_{(2.t)}(\pi_{CC}) = (1-p)$, $\tilde{U}_{(2.t)}(\pi_{DC}) = (1-p)b$, $\tilde{U}_{(2.t)}(\pi_{CD}) = (1-p)c$, and $\tilde{U}_{(2.t)}(\pi_{DD}) = (1-p)d$.

The following Proposition proves that payoffs sufficiently close to the one of the cooperative outcome path cannot be obtained in subgame perfection.

**Proposition 2** *Suppose that Assumptions 1, 2, 3, and the condition given in inequality 3.1 hold. Then there exists* $\epsilon > 0$ *and* $\bar{\delta}_1 < 1$*, such that for all* $\epsilon' < \epsilon$*, every payoff* $u = (u_1, (u_{(2.t)})_{t\in\mathbb{N}})$ *with* $\|(u_1, u_{(2.1)}) - ((1-p), (1-p))\| \leq \epsilon'$*, and* $\|(u_1, u_{(2.t)}) - ((1-p), p(1-p))\| \leq \epsilon'$ *for all* $t \in \mathbb{N}$*, cannot be obtained with a Nash equilibrium pure strategy profile for all* $\delta_1 \geq \bar{\delta}_1$*.*

**Proof.** Let $\epsilon > 0$ be sufficiently small so that $\epsilon < \left(1 - \frac{b+c}{2}\right)$, a condition needed in order to use Assumption 1 in the following analysis. Define $r_\delta$ on $\{0,1\}^\infty$ for $\delta \in [0,1)$ by $r_\delta(\zeta) = (1-\delta)\sum_{t\in\mathbb{N}} \delta^{t-1}\zeta_t$, for $\zeta \in \{0,1\}^\infty$. Note that $r_\delta$ is continuous on $\{0,1\}^\infty$, and $\{0,1\}^\infty$ is compact (due to Tychonoff's Theorem) with the product topology. Moreover, it is clear that $r_\cdot(\zeta)$ is continuous for every $\delta \in [0,1)$. Thus, because that for all $\delta \in [0,1)$, $r_\delta(\zeta) \in [0,1]$, we have: For any $\delta_n \to \delta \in [0,1)$ and $\zeta_n \to \zeta$, $\lim_{n\to\infty} r_{\delta_n}(\zeta_n) = r_\delta(\zeta)$. Moreover, when $\delta_n$ tends to 1 and $\zeta_n \to \zeta$, we have, $\lim_{\delta_n\to 1} r_{\delta_n}(\zeta_n) = \liminf_{T\to\infty} \frac{1}{T}\sum_{t=1}^{T} \zeta_t \equiv r(\zeta)$, that is, the fraction of 1's in $\zeta$. This is because, (1) Theorem 7.9 of Rudin (1976) shows that $r_{\delta_n}$ converges uniformly on $r$ on $\{0,1\}^\infty$ since $\lim_n r_{\delta_n}(\zeta) = r(\zeta)$ for every $\zeta \in \{0,1\}^\infty$ and $\sup_{\zeta\in\{0,1\}^\infty} |r_{\delta_n}(\zeta) - r(\zeta)|$ converges 0 as $n$ tends to infinity. (2) because $r_{\delta_n}$ is a sequence of continuous functions converging uniformly to $r$ on $\{0,1\}^2$, following the same arguments needed to solve exercise 9 from chapter 7 of Rudin (1976), suffices to establish that for any $\delta_n \to 1$ and $\zeta_n \to \zeta$, $\lim_{\delta_n\to 1} r_{\delta_n}(\zeta_n) = r(\zeta)$.

For any given strategy profile $f_2 = \{f_{(2.t)}\}_{t\in\mathbb{N}}$, let $\zeta(f_2) \in \{0,1\}^\infty$ be defined by

$$\zeta_t(f_2) = \begin{cases} 1 & \text{if } t = 1 \text{ and } f_{(2.1)}(e) = C, \\ & \text{or } t > 1 \text{ and for any } h \text{ with length } t-1 \text{ and } \theta_{t-1} = 0,\ f_{(2.t)}(h) = C, \\ 0 & \text{otherwise.} \end{cases}$$

Let $\epsilon' < \epsilon$, and consider any payoff $u = (u_1, (u_{(2.t)})_{t\in\mathbb{N}})$ as described in the statement of the Proposition. We restrict attention to strategy profiles $f$ and $\delta \in [0,1)$ with

$$\left\| \left(U_1^\delta(f) - U_{(2.t)}^\delta(f)\right) - (u_1, u_{(2.t)}) \right\| < \frac{\epsilon'}{2}, \tag{3.2}$$

for all $t \in \mathbb{N}$.

Consider a deviation for player 1, $f_1'$, in which player 1 chooses $D$ independent of the past, i.e. $f_1'(h) = D$ for all $h$. Then,

$$\begin{aligned} U_1^\delta(f_1', f_2) \;\geq\; & (1-\delta)(1-p)\left(1_{\zeta_1(f_2)=1}u_1(D,C) + 1_{\zeta_1(f_2)=0}u_1(D,D)\right) \\ & +(1-\delta)\delta(1-p)((1-p)u_1(D,D) + \\ & p\left(1_{\zeta_2(f_2)=1}u_1(D,C) + 1_{\zeta_2(f_2)=0}u_1(D,D)\right)) \\ & +(1-\delta)\delta^2(1-p)(\left((1-p)^2 + p(1-p)\right)u_1(D,D) + \\ & \quad \left(1 - \left((1-p)^2 + p(1-p)\right)\right) \\ & \qquad \left(1_{\zeta_3(f_2)=1}u_1(D,C) + 1_{\zeta_3(f_2)=0}u_1(D,D)\right)) \\ & + \ldots \end{aligned} \tag{3.3}$$

Note that in 3.3, the player 2 experiencing and surviving after player 1's deviation, is choosing $D$. If this were not the case, player 1's deviation would be even more profitable. Moreover, because for all $t > 1$, the probability of the opponent of the first player being born before the time period $t$ is given by $(1-p)^{t-1}+p(1-p)^{t-2}+p(1-p)^{t-3}+\ldots+p(1-p)^{t-(t-2)}+p(1-p)^{t-(t-1)} = (1-p)$, inequality 3.3 is reduced to

$$U_1^\delta(f_1', f_2) \geq (1-\delta)(1-p)\left(g_1 + p\sum_{t=2}^{\infty}\delta^{t-1}g_t\right) + \delta(1-p)^2 u_1(D,D), \quad (3.4)$$

where $g_t \equiv 1_{\zeta_t(f_2)=1}u_1(D,C) + 1_{\zeta_t(f_2)=0}u_1(D,D)$.

Due to Assumptions 1 and 3, $r_\delta(\zeta(f_2))$ increases to 1 in a continuous manner when $\epsilon \to 0$ and $\delta \to 1$ for $f$ such that $f$ satisfies inequality 3.2. Thus, the right hand side of inequality 3.4 tends to $p(1-p)b + (1-p)^2 d$ which is strictly greater than $(1-p)$ due to the condition given in inequality 3.1.

Thus, when $\epsilon \to 0$ and $\delta \to 1$, for any strategy profile $f$ satisfying 3.2, $U_1^\delta(f_1', f_2) > U_1^\delta(f_1, f_2)$. Hence, there exists $\epsilon > 0$ and $\bar{\delta}_1 < 1$ such that the conclusion of the Proposition holds, an observation finishing the proof. ■

# Chapter 4

# An Application to Financial Markets

As an important application of our repeated prisoners' dilemma model, we can consider the simplified version of the repeated Kyle's market model where there is one informed long run (infinitely lived), to be referred as the first player, and informed short run traders, to be referred as second players. Every period a short run informed trader is born, and the short run player born in period $t$ will be referred to as player $(2.t)$.

Action spaces for stage games consists of two actions, high demand (defection) and low demand (cooperation). That is, let $A$, denoting the action space for each player in each period, be given by $\{C, D\}$. Apart from those first and second players, there is a market maker who tries to identify the real price of the asset with a probability depending on the action choices of the players at that stage. Assume that there are two state of the world, the first state is realized when the market maker determines the real value of the traded asset and players obtain zero excess profits, whereas the other state corresponds the situation in which market maker cannot identify this real value and players get excess returns (for their information rent) depending their action choices. So let us denote the state space by $\Theta$ and $\theta \in \Theta \equiv \{0, 1\}$. Suppose that the state $\theta = 0$ refers to state in which the market maker identifies real value of the asset, to which we refer to as *failure* since players get zero profits at that state, and furthermore, *failure* makes the previously active short run player be fired, and in the next period the first player faces the new born second player. On the other hand, the state $\theta = 1$ refers to other state, and call for this state of the world as *success* due to strictly positive returns. Furthermore, the *success* state makes the short run player keep his job in the next period as well. In case of *success* the players obtain the following payoffs:

$$\begin{array}{ccc} & C & D \\ C & (1,1) & (c,b) \\ D & (b,c) & (d,d) \end{array} \tag{4.1}$$

where $b > 1 > d > c > 0$ and $\frac{b+c}{2} < 1$.

The First player plays repeatedly this stage game with one of the second player who is determined accordingly the set of rules that we describe above. Repeated game payoffs of the players discounted with the discount factor which may not be same for any player.

We assume that $\Pr(\theta = |CC) = p_1, \Pr(\theta = 0|CD) = \Pr(\theta = 0|DC) = p_2$ and $\Pr(\theta = 0|DD) = p_3$, with $0 < p_1 < p_2 < p_3 < 1$. These probabilities reveal that with the cooperative behavior taken by the players, the probability of realizing *failure* state decreases, whereas as the players choose defection the probability of realizing *failure* state increases.

Hence, stage game payoff structure at any period $t$ becomes as follows;

| $Pl1/Pl(2.t')$ | $C$ | $D$ |
|---|---|---|
| $C$ | $(1-p_1),(1-p_1)$ | $(1-p_2)c,(1-p_2)b$ |
| $D$ | $(1-p_2)b,(1-p_2)c$ | $(1-p_3)d,(1-p_3)d$ |

where $t' \leq t$, and the $(2.t')$ is the short run player borned at period $t'$ and plays the stage game at period $t$ against first player.

Under the Assumption 1, making sure that the stage game is a standard prisoners' dilemma, and Assumption 2, making sure that every second player is not able to see past histories prior to his birth, and the condition given by inequality 3.1, we can say that in this financial market cooperative outcome path can not be sustainable in equilibrium no matter how patients players are.

It needs to be mentioned that the critical assumption for this result is the inability of second players to observe past histories. This, in turn, makes it more difficult to punish the first player after his deviation, which is unlike in the standard repeated prisoners' dilemma. In addition, condition3.1 makes payoff that he obtains by deviating, is sufficiently high.

Considering arbitrarily close payoffs to that of cooperative outcome path, under the additional Assumption 3(in addition to Assumptions1,2 and condition given in inequality 3.1), we can conclude that in financial markets with sufficiently patient players, any payoff which is arbitrarily close to that of cooperative outcome path cannot be obtained as in equilibrium. Note that in this setting, the probability of success/failure (the probability with which the market maker can find out the real value of the asset) does not depend on players' action choices, but is constant.

# Chapter 5

# Concluding Remarks

In this thesis, we consider a simple repeated prisoners' dilemma with long and short run players with the additional feature that the short run players do not necessarily die at the end of every period. Indeed, a short run player may survive with a positive probability which depends on the action choices of the agents. In this model, an important restriction that we have imposed, is not to allow short run players to observe histories (both public and private) that happened before their birth. In this setting, we investigate the cooperative behavior between the long run player and the short run players, and prove that cooperative outcome path is not an equilibrium no matter how patient players are. Furthermore, with the additional assumption that makes the probability of success be independent of the action choices (which is made to simplify the analysis), we also show that with sufficiently patient players payoffs arbitrarily close to that of cooperative outcome path can not be obtained in equilibrium.

The main reasons behind these results are that it is getting more difficult to punish the deviation by the first player due to the possible death of second player after each stage, and their inability of observing past histories, and sufficiently high one shot deviation payoff.

It is well worth to investigate this model with short run players who are able to observe only the public histories that happened prior to their birth.

Note that, a $t$ length public history consists of the record of the public signals up to that point in time, reflecting the state of the world in each stage. With this modification, we find a condition under which, cooperative behavior can be obtained in equilibrium between the (sufficiently patient) first player and (sufficiently patient) second player born in the first period. Moreover, we also observe that, the parameter space which satisfies both that condition and the critical condition3.1 is non-empty. This shows the importance of our Assumption 2.

Our model remains the same apart from the modification that for each second player $(2.t)$ is able to condition his action on the past realized $t-1$ length public history of the game.

Let $f_1$ be the strategy of the first player such that:

$$f_1(h_t) = \begin{cases} C & \text{if } \pi_{t'} = ((C,C), \theta_{t'} = 1) \text{ for all } t' \leq t-1, \\ D & \text{otherwise.} \end{cases}$$

Let $f_{2.t'}$ be the strategy of short run player born in period $t'$ of the game which is given by for any $t$ length history $h$:

$$f_{2.t'}(h) = \begin{cases} C & \text{if } \theta_\tau = 1 \text{ for all } \tau = 1, \ldots, t-1, \\ & \quad \text{and } \pi_\tau = ((C,C), \theta_\tau = 1) \text{ for all } \tau \text{ with } t' \leq \tau \leq t-1, \\ D & \text{otherwise.} \end{cases}$$

Note that $t = 1$ denotes the strategy of (2.1), in which this player will choose $C$ in every period that he is active provided that player 1 has also chosen $C$.

The payoff of the long run player under the above strategy profile:

$$\begin{aligned} U_1(f_1, f_2) &= (1-\delta_1)(1-p_1) \\ &\quad + (1-\delta_1)\delta_1 \left( \frac{p_1(1-p_3)d}{1-\delta_1} + (1-p_1)^2 \right) \\ &\quad + (1-\delta_1)\delta_1^2 \left( \frac{p_1(1-p_1)(1-p_3)d}{1-\delta_1} + (1-p_1)^3 \right) \\ &\quad + (1-\delta_1)\delta_1^3 \left( \frac{p_1(1-p_1)^2(1-p_3)d}{1-\delta_1} + (1-p_1)^4 \right) \\ &\quad + \ldots \\ &= \frac{(1-\delta_1)(1-p_1)}{1-\delta_1(1-p_1)} + \frac{p_1(1-p_3)d\delta_1}{1-\delta_1(1-p_1)}. \end{aligned}$$

Clearly the long run player does not deviate from the $f_1$ if the game is in defection phase. If the long run player deviates from $f_1$ in the cooperation phase in period $t$, the past must be such that in every previous periods the public signal is 1 and the action profile is $(C, C)$) then his continuation payoff starting period $t$ is

$$(1-\delta_1)(1-p_2)b + \delta_1(1-p_3)d.$$

Short run players born after the first period have no incentive to deviate from the strategy $f_2$ given the strategy of the long run player $f_1$, because the short run players' action is clearly given by $D$ in such situations. So it is enough to look at incentives of the short run player born in the first period. If he conforms the strategy $f_{2.1}$ his payoff is

$$U_{2.1}(f_1, f_{2.1}) = \frac{(1-\delta_{2.1})(1-p_1)}{1-\delta_{2.1}(1-p_1)}.$$

On the other hand, his continuation payoff starting from period $t$, if he were to deviate from $f_2$ in period $t$ is:

$$(1-\delta_{(2.1)})(1-p_2)b + (1-\delta_{(2.1)})(1-p_2)\delta_{(2.1)}(1-p_3)d\sum_{\tau=0}^{\infty}((1-p_3)\delta_{(2.1)})^{\tau}$$
$$= (1-\delta_{(2.1)})(1-p_2)b + \frac{(1-\delta_{(2.1)})(1-p_2)(1-p_3)\delta_{(2.1)}d}{1-\delta_{(2.1)}(1-p_3)}$$

Therefore we observe that if the following condition holds, then the above given strategy profile $(f_1, (f_{2.t})_{t\in N})$ is subgame perfect:

$$\frac{(1-\delta_1)(1-p_1)}{1-\delta_1(1-p_1)}+\frac{p_1(1-p_3)d\delta_1}{1-\delta_1(1-p_1)} \geq (1-\delta_1)(1-p_2)b+\delta_1(1-p_3)d, \quad (5.1)$$

$$\frac{(1-\delta_{(2.1)})(1-p_1)}{1-\delta_{(2.1)}(1-p_1)} \geq (1-\delta_{(2.1)})(1-p_2)b + \frac{(1-\delta_{(2.1)})(1-p_2)(1-p_3)\delta_{(2.1)}d}{1-\delta_{(2.1)}(1-p_3)}.$$

Note that when condition 5.1 holds, then the strategy profile $(f_1, (f_{2.t})_{t\in N})$ being subgame perfect implies that cooperative behavior can be sustained between the first and the second player who is born in the beginning of the game as long as the public outcomes turn out to be successes.

The reason behind this result is that with the public monitoring ability of the second players, they can punish the deviating first player more effectively than it was the case in our original model.

Whether or not conditions 3.1 and 5.1 are compatible with each other is a question that we have to address. Indeed, they are. And that is why this exercise reveals the importance of the ability of second players to observe the public history. To see that these two conditions are compatible, the following specific values for our parameters can be considered: $\delta_1 = \delta_{(2.t)} = \frac{9}{10}$ for all $t \in \mathbb{N}$, $p_1 = \frac{3}{5}, p_2 = \frac{7}{10}, p_3 = \frac{4}{5}, b = \frac{9}{5}, d = \frac{3}{5}, c = \frac{1}{10}$. Then under these values, conditions in Assumptions 1, 3.1 and 5.1 all hold.

Hence, the above reveals the importance of Assumption 2 to some extent. When the short run players are given abilities to punish player 1 more effectively, even though short run deviations are still providing high levels of

excess returns, player 1 would not deviate from the cooperative behavior as long as he faces the short run player born in the first period.
However, this does not provide a counter example for our results, because in this example, the payoff that player 1 obtains is not in the vicinity of the cooperative return given by $(1 - p_1)$.

As we have remarked in the introduction, whether or not cooperative behavior (and its payoff) can be (approximately) obtained in equilibrium when short run players observe only the public histories before their birth (note that when they can also observe the private ones, cooperation can be obtained when all parties are sufficiently patient), is a very interesting question that was not answered in this thesis. Such an attempt, though, would require us to consider whether or not techniques developed by Abreu, Pearce, and Stachetti (1990) can be used, and if so, whether or not cooperative behavior can be(approximately) obtained in public perfect equilibrium. In particular, one has to check if some version of the principle of one–deviation holds in this setting.

Our final remark related tothese issues is that our results are similar to those of Cole and Kocherlakota (2005): Under a certain set of restrictions of the parameters of the stage game payoffs and probabilities, the cooperative outcome path is not an equilibrium. Although, results are so similar we do not know the relationship of this thesis with their study. Moreover, in that setting exploring the whole equilibrium payoffs with and without the key condition 3.1 constitutes a future avenue for research.

# Bibliography

Abreu, D., D. Pearce, and E. Stachetti (1990): "Toward a Theory of Discounted Repeated Games with Imperfect Monitoring," *Econometrica*, 58(5), 1041–1063.

Aumann, R. (1981): "Survey of Repeated Games," in *Essays in Game Theory and Mathematical Economics in Honor of Oskar Morgenstern*. Bibliographisches Institut, Mannheim.

Aumann, R., and L. Shapley (1976): "Long Term Competition: A Game Theoretic Analysis," unpublished manuscript,Stanford University.

Barlo, M., and G. Carmona (2007): "Folk Theorems for the Repeated Prisoners' Dilemma with Limited Memory and Pure Strategies," Sabancı University, Universidade Nova de Lisboa and University of Cambridge.

Barlo, M., G. Carmona, and H. Sabourian (2008): "Bounded Memory with Finite Action Spaces," Sabancı University, Universidade Nova de Lisboa and University of Cambridge.

——— (forthcoming): "Repeated Games with One Memory," *Journal of Economic Theory*.

BASHKAR, V., AND I. OBARA (2002): "Belief-Based Equilibria in the Repeated Prisoners' Dilemma with Private Monitoring," *Journal of Economic Theory*, 102(1), 40–69.

COLE, H., AND N. KOCHERLAKOTA (2005): "Finite Memory and Imperfect Monitoring," *Games and Economic Behavior*, 53, 59–72.

FUDENBERG, D., D. M. KREPS, AND E. MASKIN (1990): "Repeated Games with Long-run and Short-run Players," *Review of Economic Studies*, 57(4), 555–73.

FUDENBERG, D., AND D. LEVINE (2006): "A Dual-Self Model of Impulse Control," *American Economic Review*, 96(5), 1449–1476.

FUDENBERG, D., AND E. MASKIN (1986a): "The Folk Theorem in Repeated Games with Discounting and with Incomplete Information," *Econometrica*, 54, 533–554.

——— (1986b): "The Folk Theorem in Repeated Games with Discounting or with Incomplete Information," *Econometrica*, 54, 533–554.

KALAI, E., AND W. STANFORD (1988): "Finite Rationality and Interpersonal Complexity in Repeated Games," *Econometrica*, 56, 397–410.

MAILATH, G., AND S. MORRIS (2002): "Repeated Games with Almost-Public Monitoring," *Journal of Economic Theory*, 102, 189–228.

MAILATH, G., I. OBARA, AND T. SEKIGUCHI (2002): "Maximum Efficient Equilibrium Payoff in the Repeated Prisoners' Dilemma," *Journal of Economic Theory*, 102(1), 96–122.

Neyman, A. (1985): "Bounded Complexity Justifies Cooperation in the Finitely Repeated Prisoner's Dilemma," *Economic Letters*, 9, 227–229.

Piccione, M. (2002): "The Repeated Prisoner's Dilemma with Imperfect Private Monitoring," *Journal of Economic Theory*, 102(1), 70–83.

Radner, R., R. Myerson, and E. Maskin (1986): "An Example of a Repeated Partnership Game with Discounting and Uniformly Inefficient Equilibria," *Review of Economic Studies*, 53, 59–70.

Rubinstein, A. (1979): "Equilibrium in Supergames with the Overtaking Criterion," *Journal of Economic Theory*, 21, 1–9.

Rubinstein, A. (1986): "Finite Automata Play the Repeated Prisoner's Dilemma," *Journal of Economic Theory*, 39, 83–96.

Rudin, W. (1976): *Principles of Mathematical Analysis*. Mc Graw Hill, 3rd edn.

Sabourian, H. (1998): "Repeated Games with $M$-period Bounded Memory (Pure Strategies)," *Journal of Mathematical Economics*, 30, 1–35.

S.Kyle, A. (1989): "Informed Speculation with Imperfect Competition," *The Review of Economic Studies*, 56(3), 317–355.