A Study on High Accuracy Discrete-Time Sliding Mode Control

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Abstract—**In this paper a Discrete-Time Sliding-Mode based controller design for high accuracy motion control systems is presented. The controller is designed for a general SISO system with nonlinearity and external disturbance. Closed-Loop behavior of the general system with the proposed control and Lyapunov stability is shown and the error of the closed loop system is proven to be within an $o(T^2)$. The proposed controller is applied to a stage driven by a piezo drive that is known to suffer from hysteresis nonlinearity in the control gain. Proposed SMC controller is proven to offer chattering-free motion and rejection of the disturbances represented by hysteresis and the time variation of the piezo drive parameters. As a separate idea to enhance the accuracy of the closed loop system a combination of disturbance rejection method and the SMC controller is explored and its effectiveness is experimentally demonstrated. Closed-loop experiments are presented using PID controller with and without disturbance compensation and Sliding-Mode Controller with and without disturbance compensation for the purpose of comparison.**

I. INTRODUCTION

Piezoelectric actuators have shown a great potential in applications that require submicrometer down to nanometer motion. The advantages that piezoelectric actuators offer are the absence of friction and stiction characteristics that exist in other actuators. Thus, piezoelectric actuators are ideal for very high-precision motion applications. The main characteristics of piezoelectric actuators are: extremely high resolution in the nanometer range, high bandwidth up to several kilo hertz range, a large force up to few tons, and very short travel in the submillimeter range [l]. In all of applications the accuracy of positioning is very important and in many cases the closed loop control is the only answer. Despite this there are many attempts [2], [3] to drive piezoelectric actuators as an open loop system with fine compensation of the hysteresis nonlinearity in one or another way.

Despite the fact that a piezoelectric actuator is a distributed parameters system, modeling for control purposes is based on a lumped parameters system. It is possible to drive piezoelectric actuators with either voltage or charge as input. A piezoelectric actuator driven by voltage as input will exhibit nonlinearity between the input (voltage) and output (position). This nonlinearity is mainly due to the parasitic hysteresis characteristics of piezoelectric crystals. It has been shown in many other works [2] that hysteresis behavior does not exist in the case of a piezoelectric actuator driven by charge and that the actuator exhibits almost linear behavior between charge and position. A major difficulty in using piezo-electric actuators is the hysteresis effect which causes large positioning errors. There are many techniques used in order to handle the nonlinearities brought by this effect such as feedback and model-based feedforward control. Also in [4], iterative method is used in order to find the hysteresis that compensates feedforward input for high-precision positioning. In [5], both the hysteresis and dynamic creep effects are given importance and operator based inverse feedforward controller is applied. It has been shown that this controller works well for highly dynamic operation and that it is simple and inexpensive for mechatronic devices with hysteresis characteristics. There has been also research on the mathematical modeling of hysteresis, such as in [2], [3], [6], [7] and [8] where new results for the modeling of physical hysteresis and its applications in dynamic research are shown. In [2] complex and accurate model of hysteresis is presented, but is hard to implement and too complex for control applications. In [3], [6], and [7] simpler models of hysteresis are proposed, however, those models fail to precisely represent hysteresis behavior throughout the whole range of input voltage of the piezoelectric actuator.

In this paper the sliding mode methods are applied in the design of a high-accuracy piezo actuator position. The solution proposed here combines the sliding mode controller and the disturbance rejection method in order to achieve high accuracy in the actuator trajectory tracking. For the disturbance estimation a sliding mode observer based disturbance compensation method is used here. By manipulating model of a piezo actuator in a form where nonlinearities due to hysteresis are presented as an additive disturbance acting together with external force to the mechanical system a simple second order observer is designed to estimate lumped disturbance.

As a final extension of the work, a disturbance observer based on the lumped parameter model of the piezo-stage proposed in, [2], will be experimentally shown to improve the overall performance of the closed-loop system.

II. CONTROLLER DESIGN AND ANALYSIS

A. Controller Design

Consider the general system defined below

$$\dot{x} = f(x) + Bu \tag{1}$$

Here, it is assumed that f and B are smooth, continuous and bounded. The aim is to drive the states of the system into the set S defined by

$$S = \{x : G(x^r - x) = \sigma(x, x^r) = 0\} \tag{2}$$

Here G is a positive constant, x is the state vector, x_r is the reference vector and it will be assumed to be smooth and continuous, and $\sigma(x, x^r)$ is the function defining the sliding mode manifold.

The derivation of the control law starts with the selection of the Lyapunov function, $V(\sigma)$, and an appropriate form of the derivative of the Lyapunov function, $\dot{V}(\sigma)$. Lyapunov function selection such that it is positive definite

$$V(\sigma) = \frac{\sigma^2}{2} \tag{3}$$

Hence the derivative of the Lyapunov function is

$$\dot{V}(\sigma) = \sigma\dot{\sigma} \tag{4}$$

In order to guaranty the asymptotic stability of solution $\sigma(x, x^r) = 0$, the derivative of the Lyapunov function is selected to be

$$\dot{V}(\sigma) = -D\sigma\zeta(\sigma) \tag{5}$$

Here D is a positive constant and $\zeta(\sigma)$ should be selected such that the sliding mode motion on manifold $\sigma(x, x^r) = 0$ is gurantied and that reaching time to the manifold is finite. Selecting (5) in the form $\dot{V}(\sigma) = -D\sigma^2 - D\eta\sigma/|\sigma| = -D\sigma\zeta(\sigma); \eta > 0$ guaranty required condition where η can be selected as a small positive constant. Hence, if control can be determined from (4) and (5), the stability of the solution (5) will be guaranteed since $V(\sigma) > 0$, $V(0) = 0$ and $\dot{V}(\sigma) < 0$, $\dot{V}(0) = 0$. By combining (4) and (5) the following result is obtained

$$\sigma(\dot{\sigma} + D\zeta(\sigma)) = 0 \tag{6}$$

A solution for (6) is as follows

$$\dot{\sigma} + D\zeta(\sigma) = 0 \tag{7}$$

The derivative of the sliding function is as follows

$$\dot{\sigma} = G(\dot{x}^r - \dot{x}) = G\dot{x}^r - G\dot{x} \tag{8}$$

From (8) and using (1)

$$\dot{\sigma} = \underbrace{G\dot{x}^r - Gf}_{GBu_{eq}} - GBu(t) = GB(u_{eq} - u(t)) \tag{9}$$

If (9) is inserted in (7) and the result is solved for the control

$$u(t) = u_{eq} + (GB)^{-1} D\zeta(\sigma) \tag{10}$$

It can be seen from (9) that u_{eq} is difficult to calculate. Using the fact that u_{eq} is a continuous function since it is a function of x^r and f that are assumed smooth and continuous, (9) can be written in discrete-time form after applying Euler's approximation,

$$\frac{\sigma((k+1)T_s) - \sigma(kT_s)}{T_s} = GB(u_{eq}(kT_s) - u(kT_s)) \tag{11}$$

Here T_s is the sampling time and $k \in Z^+$. It is also necessary to write (10) in discrete-time form just as it was done before

$$u(kT_s) = u_{eq}(kT_s) + (GB)^{-1} D\zeta(\sigma(kT_s)) \tag{12}$$

If (11) is solved for the equivalent control, the following is obtained

$$u_{eq}(kT_s) = u(kT_s) + (GB)^{-1}\left(\frac{\sigma((k+1)T_s) - \sigma(kT_s)}{T_s}\right) \tag{13}$$

Since the system is causal, and control cannot be dependent on a future value of σ, the only way to estimate the current value of the equivalent control is by approximating by a single-step backward value computed from (14) as follows,

$$\hat{u}_{eq\,k} \cong u_{eq\,k-1} = u_{k-1} + (GB)^{-1}\left(\frac{\sigma_k - \sigma_{k-1}}{T_s}\right) \tag{14}$$

Here $\hat{u}_{eq_k}$ (or $\hat{u}_{eq}(kT_s)$) is the estimate of the current value of the equivalent control. If (14) is inserted in (12)

$$u_k = u_{k-1} + (GB)^{-1}\left(D\zeta(\sigma)_k + \frac{\sigma_k - \sigma_{k-1}}{T_s}\right) \tag{15}$$

It easily seen that the above control law is derived from discrete-time approximations based on the continuous-time equations. Hence, it must be shown that the above control satisfies the original conditions based on which it was designed. These conditions are the Lyapunov condition and existence of Sliding Mode.

B. Closed-Loop Behavior with the Approximated Control

As a consequence of the approximations that were made in the derivation of the discrete-time control law some deviations in the sliding surface from the desired sliding manifold may exist. This deviation of the sliding surface from the desired manifold at each sampling instant will be analyzed. Analysis of the inter-sampling behavior of the sliding surface will also be analyzed. Considering (1), the derivative of the sliding surface is given by

$$\dot{\sigma}(t) = G\left(\dot{x}^r - \dot{x}\right) = G\dot{x}^r - Gf - GBu(t) \tag{16}$$

The discrete-time equivalent of the sliding manifold can be obtained by taking the integral on both sides of (16) from kT_s to $(k+1)T_s$

$$\sigma_{k+1} - \sigma_k = \int_{kT_s}^{(k+1)T_s} \left(G\dot{x}^r - Gf - GBu(t)\right)dt \tag{17}$$

Applying a sample and hold to the control input between consecutive samples $u(t) = u_k$ for $kT_s \le t < (k+1)T_s$

$$\sigma_{k+1} - \sigma_k = \int_{kT_s}^{(k+1)T_s} \left(G\dot{x}^r - Gf\right)dt - T_s GBu_k \tag{18}$$

Using the assumptions that $\dot{x}^r$ and f are smooth and bounded, the integrations in (18) can be approximated by using Euler's integration

$$\sigma_{k+1} = \sigma_k + T_s G\left(\dot{x}_k^r - f_k\right) - T_s GBu_k + O(T_s^{\,2}) \tag{19}$$

Here $O(T_s^2)$ is the error introduced due to Euler's integration, [3]. If the control defined by (15) is introduced into (19)

$$\begin{aligned} \sigma_{k+1} &= \sigma_k + T_s G\left(\dot{x}_k^r - f_k\right) \\ &\quad - T_s GBu_{k-1} - T_s D\sigma_k - \sigma_k + \sigma_{k-1} + O(T_s^{\,2}) \end{aligned} \tag{20}$$

After some simplifications (20) is reduced to

$$\begin{aligned} \sigma_{k+1} &= T_s G\left(\dot{x}_k^r - f_k\right) - T_s GBu_{k-1} \\ &\quad - T_s D\sigma_k + \sigma_{k-1} + O(T_s^{\,2}) \end{aligned} \tag{21}$$

If $T_s G\left(\dot{x}_{k-1}^r - f_{k-1}\right)$ is added and subtracted from the r.h.s of (21), the following is obtained

$$\begin{aligned} \sigma_{k+1} &= T_s G\left(\dot{x}_k^r - f_k\right) - T_s G\left(\dot{x}_{k-1}^r - f_{k-1}\right) - T_s D\sigma_k \\ &\quad + \underbrace{T_s G\left(\dot{x}_{k-1}^r - f_{k-1}\right) - T_s GBu_{k-1}}_{\sigma_k - \sigma_{k-1} + O(T_s^{\,2})} + \sigma_{k-1} + O(T_s^{\,2}) \end{aligned} \tag{22}$$

After some simplifications, (22) becomes

$$\sigma_{k+1} = \sigma_k - T_s D\sigma_k + T_s G\left(\Delta\dot{x}_k^r - \Delta f_k\right) + O(T_s^{\,2}) \tag{23}$$

Here $\Delta\dot{x}_k^r = \dot{x}_k^r - \dot{x}_{k-1}^r$ and $\Delta f_k = f_k - f_{k-1}$. Note that if $D = 1/T_s$, then the r.h.s of (23) is of order $O(T_s^2)$, keeping in mind that $\dot{x}^r$ and f are smooth and continuous. Hence,

$$\sigma_{k+1} = O(T_s^{\,2}) \tag{24}$$

Hence, it is shown that the maximum deviation from the sliding surface at each sampling instant is of order $O(T_s^2)$. Next, it will be shown that the inter-sampling deviation of the sliding surface from the desired manifold is also of order $O(T_s^2)$.

Consider the inter-sampling instant of $t = kT_s + \tau$ where $0 \le \tau \le T_s$. If (16) is integrated on both sides from kT_s to $kT_s + \tau$

$$\sigma(kT_s + \tau) - \sigma_k = \int_{kT_s}^{kT_s + \tau} \left(G\dot{x}^r - Gf - GBu(t)\right)dt \tag{25}$$

Applying the sample and hold to the control and Euler's integration to the remaining integral gives

$$\sigma(kT_s + \tau) = \sigma_k + \tau G\left(\dot{x}_k^r - f_k\right) - \tau GBu_k + O(\tau^2) \tag{26}$$

If the control defined by (15) is introduced into (26)

$$\begin{aligned} \sigma(kT_s + \tau) &= \sigma_k + \tau G\left(\dot{x}_k^r - f_k\right) - \tau GBu_{k-1} \\ &\quad - \tau D\sigma_k - \frac{\tau}{T_s}\left(\sigma_k - \sigma_{k-1}\right) + O(\tau^2) \end{aligned} \tag{27}$$

If $\tau G\left(\dot{x}_{k-1}^r - f_{k-1}\right)$ is added and subtracted from the r.h.s of (21) and $D = 1/T_s$, the following is obtained

$$\begin{aligned} \sigma(kT_s + \tau) &= \sigma_k + \frac{\tau}{T_s} G\left(T_s\left(\Delta\dot{x}_k^r - \Delta f_k\right)\right) - \frac{\tau}{T_s}\sigma_k - \frac{\tau}{T_s}\sigma_k \\ &\quad + \frac{\tau}{T_s}\underbrace{G\left(T_s\left(\dot{x}_{k-1}^r - f_{k-1}\right) - T_s Bu_{k-1}\right)}_{\sigma_k - \sigma_{k-1} + O(T_s^{2})} + \frac{\tau}{T_s}\sigma_{k-1} + O(\tau^2) \end{aligned} \tag{28}$$

Further simplifications lead to

$$\sigma(kT_s+\tau)=\sigma_k-\frac{\tau}{T_s}\sigma_k+\frac{\tau}{T_s}G\left(T_s\left(\Delta\dot{x}_k^r-\Delta f_k\right)\right)+O(\tau^2) \quad (29)$$

If $\dot{x}^r$ and f are smooth and continuous then

$$\sigma(kT_s+\tau)=\sigma_k-\frac{\tau}{T_s}\sigma_k+O(\tau^2) \quad (30)$$

Note that if $\sigma_k=O(T_s{}^2)$ as was shown previously then the maximum intersampling value of the sliding function is $O(T_s^2)$. Hence,

$$\sigma(kT_s+\tau)=O(T_s{}^2) \quad (31)$$

C. Lyapunov Stability Analysis of the Closed-Loop System

In this section it will be shown that with discrete-time control defined by (15) it is possible to satisfy the Lyapunov conditions (4) and (5) in discrete-time.

Starting with the definition of the Lyapunov function in discrete-time

$$V_k=\sigma_k^2 \quad (32)$$

The difference of two consecutive values of the Lyapunov function in discrete-time can be given by

$$V_{k+1}-V_k=\sigma_{k+1}^2-\sigma_k^2 \quad (33)$$

Here it is required that $V_{k+1}-V_k<0$ for $\sigma_k\neq 0$. However, it will be shown that $V_{k+1}-V_k<0$ for $|\sigma_k|>O(T_s^2)$. The condition $V_{k+1}-V_k<0$ means that

$$\sigma_{k+1}^2-\sigma_k^2<0 \quad (34)$$

If (24) is inserted into (34),

$$V_{k+1}-V_k=O(T_s^4)-\sigma_k^2 \quad (35)$$

Note that if $|\sigma_k|>O(T_s^2)$ then $V_{k+1}-V_k<0$. Thus, (35) shows that σ_k is always converging towards a boundary of $O(T_s^2)$ around the desired sliding-manifold and (31) shows that once σ_k reaches $O(T_s^2)$ boundary it will tend to stay in that boundary.

III. Implementation on a Piezo-Stage

In this work a piezo-stage that consists of a piezo-drive integrated with a sophisticated flexure structure for motion amplification is used. The flexure structure is wire-EDM-cut and is designed to have zero stiction and friction. Figure 1 shows the piezo-drive integrated flexure structure. In addition to the absence of internal friction, flexure stages exhibit high stiffness and high load capacity. Flexure stages are also insensitive to shock and vibration. However, since the piezo-drive exhibits non-linear hysteresis behavior, the overall system will also exhibit the same behavior.

The dynamics of the piezo-stage can be represented by the following second-order differential equation coupled with hysteresis in the presence of external forces

$$m_{eff}\ddot{y}+c_{eff}\dot{y}+k_{eff}y=T\left(u(t)-h(y,u)\right)-F_{ext} \quad (36)$$

Here m_{eff} denotes the effective mass of the stage, y denotes the displacement of the stage, c_{eff} denotes the effective damping of the stage, k_{eff} denotes the effective stiffness of the stage, T denotes the electromechanical transformation ratio, u denotes the input voltage and $h(y,u)$ denotes the non-linear hysteresis that has been found to be a function of y and u, [1], and F_{ext} is the external force acting on the stage.

The structure of model (36) is showing that, from the mechanical motion the hysteresis may be perceived as a disturbance force that satisfy matching conditions. This means that the sliding mode based control should be able to reject the influence of the hysteresis nonlinearity on the mechanical motion. At the same time it is obvious that the lumped disturbance consisting of the external force acting on the system and the hysteresis can be estimated, thus allowing the application of the disturbance rejection method in the overall system design.

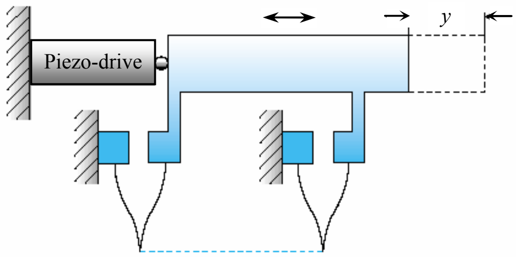

Fig. 1. Structure of a flexure piezo-stage

To facilitate the writing of the control law, (36) is written into the state-space form

$$\dot{x}_1=\dot{y}=x_2$$
$$\dot{x}_2=\ddot{y}=-\frac{k_{eff}}{m_{eff}}x_1-\frac{c_{eff}}{m_{eff}}x_2+\frac{T}{m_{eff}}u-\frac{T}{m_{eff}}h(y,u)-\frac{F_{ext}}{m_{eff}} \quad (39)$$

From here it can be seen that the input matrix is

$$B=\left\{0 \quad \frac{T}{m_{eff}}\right\}^T \quad (40)$$

The matrix G for this case will be selected to be

$$G=\{\lambda \quad 1\} \quad (41)$$

Here λ is a positive constant. Hence, the controller will be in the following form

$$u_k = u_{k-1} + \frac{m_{eff}}{T}\left(D\zeta(\sigma_k) + \frac{\sigma_k - \sigma_{k-1}}{T_s} \right) \tag{42}$$

The results that it will be shown in this section are for the case of SMC and for comparison purposes PID results will also be shown. Figures 2 and 3 depict the tracking of the piezo-stage for a 0.25Hz sinusoidal reference.

IV. Disturbance Observer

A. *Design and Analysis of the Disturbance Observer*

The structure of the observer is based on (36) under the assumption that all the plant parameter uncertainties, nonlinearities and external disturbances can be represented as a lumped disturbance. As it is obvious, y is the displacement of the plant and is measurable. Likewise, $u(t)$ is the input to the plant and is also measurable. Hence, the nominal structure of the plant is defined as follows

$$\begin{aligned} m_N \ddot{y} + c_N \dot{y} + k_N y &= T_N u(t) - F_d \\ F_d &= T_N h + \Delta T(v_{in} + v_h) + \Delta m\ddot{y} + \Delta c\dot{y} + \Delta ky \end{aligned} \tag{43}$$

Here m_N, c_N, k_N and T_N are the nominal plant parameters while Δm, Δc, Δk and ΔT are the uncertainties of the plant parameters. Since y and $u(t)$ are measured the proposed observer is of the following form

$$m_N \ddot{\hat{y}} + c_N \dot{\hat{y}} + k_N \hat{y} = T_N u - T_N u_c \tag{44}$$

Here $\hat{y}$ is the estimated position u is the plant control input and u_c is the observer control input. If $\hat{y}$ can be forced to track y then obviously $F_d = T_N u_c$. The observer controller that is used is in the SMC framework. Selecting the following sliding manifold

$$\sigma_{obs} = \lambda_{obs}(y - \hat{y}) + (\dot{y} - \dot{\hat{y}}) \tag{45}$$

here λ_{obs} is a positive constant. If σ_{obs} is forced to zero then $\hat{y}$ is forced to track y. The controller used is

$$u_{c_k} = u_{c_{k-1}} + \frac{m_{eff}}{T}\left(D_{obs}\zeta(\sigma_{obs_k}) + \frac{\sigma_{obs_k} - \sigma_{obs_{k-1}}}{T_s} \right) \tag{46}$$

The frequency response of the disturbance observer output with respect to the disturbance is depicted in the Fig. 4. The response shown is for the case when the sampling time is $100\mu s$ and the controller parameters being $D_{obs} = \lambda_{obs} = 1/T_s$.

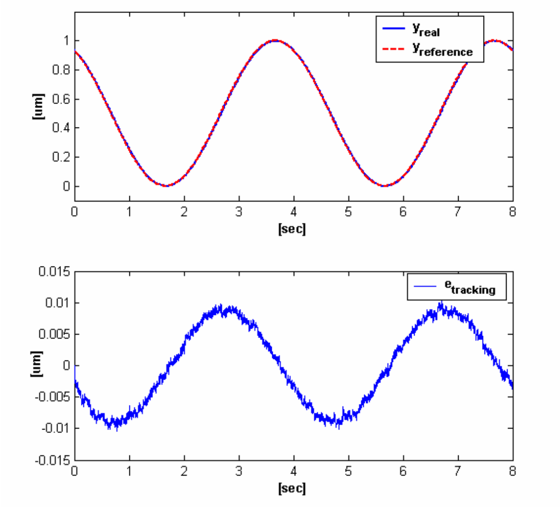

Fig. 2. Sinusoidal reference tracking with SMC

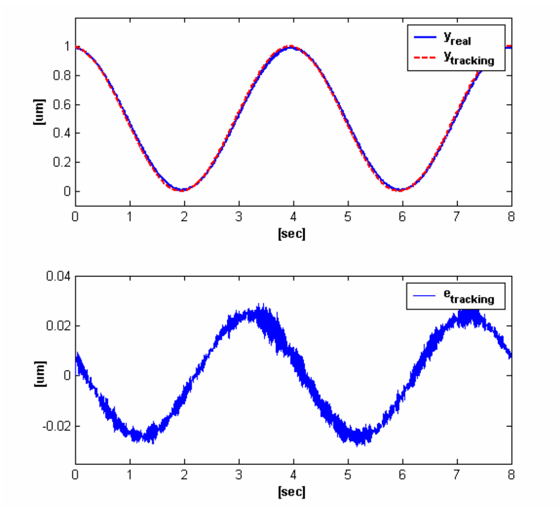

Fig. 3. Sinusoidal reference tracking with PID controller

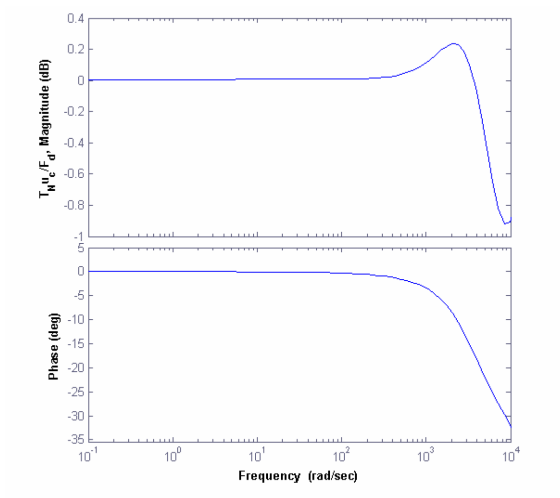

Fig. 4. Magnitude and Phase Plots of the Observer Response

B. Closed-Loop Experiments with Disturbance Compensation

The disturbance observer shown above was implemented with closed-loop control. The observer implementation is depicted in the Fig. 5. The experiments show a notable improvement in tracking for the cases of Sliding mode controller and PID controller.

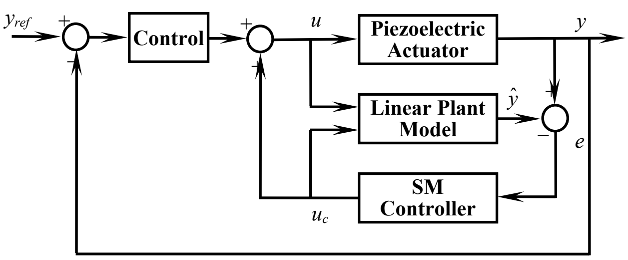

Fig. 5. Closed-Loop control with disturbance compensation

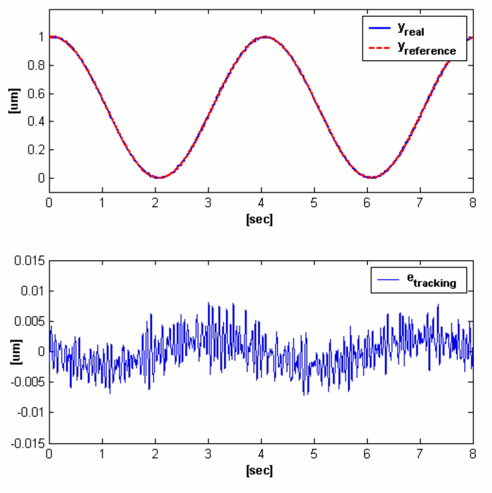

Fig. 6. Sinusoidal reference tracking with the SMC and disturbance compensation

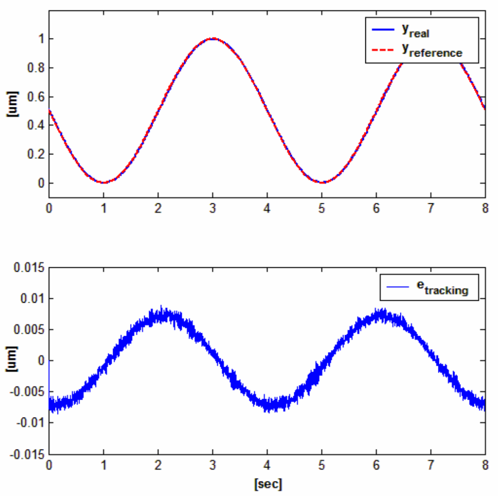

Fig. 7. Sinusoidal reference tracking with PID and disturbance compensation

5. CONCLUSION

In this paper the robustness of a designed discrete-time Sliding mode controller was shown. It was also shown that the controller can push the states of the system to an $O(T_s^2)$ boundary around the desired sliding manifold. Experiments were also conducted to show the effectiveness of the controller. As an extension, it was shown that the inclusion of disturbance compensation via disturbance observer can improve the overall closed-loop system.

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