Turk J Elec Engin, VOL., NO. , © TÜBİTAK

A Novel Algorithm for Sensorless Motion Control, Parameter Identification and Position Estimation

Islam S.M. KHALIL, Emrah D. KUNT, Asif SABANOVIC
Mechatronics Department,
Sabanci University,
Istanbul, 34956, TURKEY
e-mail: kahalil@su.sabanciuniv.edu • edkunt@su.sabanciuniv.edu • asif@sabanciuniv.edu

Abstract

This article demonstrates that an actuator can be used as a single platform for measurement, control and estimation by designing a chain of estimators and performing simple offline experiments. Making use of the fact that reflected torque from the dynamical system consists of all the system dynamics beside all external disturbances. The reflected torque waves are estimated using the actuator parameters, and used for an offline experiment for parameter estimation, then the estimated reflected torque is used along with the estimated parameters to estimate the positions of any multi-degree of freedom flexible dynamical system. That makes it possible to use those estimates to achieve sensorless motion and vibration control without using actual measurement. Moreover, all the estimated parameters and system dynamics can be used in order to estimate any external disturbance on the system when it starts to interact with the environment.

Key Words: *Mechanical Wave, Torque Observer, Position Estimation, Estimation Based Motion and Vibration Control, External Disturbance Estimation, Parameters Estimation.*

1. Introduction

Observers play an important role in motion control of dynamical systems, in the sense of using fewer sensors and providing estimates for certain variables that are hard to be physically measured. It turns out that sensorless motion control can be partially achieved if proper observers were designed and embedded to the control system. However, in order to keep the dynamical system free from any attached sensors and using the actuator as a single platform for actuation and estimation, mechanical waves have to be measured, analyzed and interpreted. Surprisingly enough, mechanical waves that are reflected from the dynamical system to the actuator or vice versa include all the system dynamics, parameters and the external disturbance information. The purpose of this article is to introduce a strategy that enables us to decouple each single piece of information from the reflected mechanical wave. Starting by estimating the disturbance from the actuator using its reference current and velocity, this disturbance by its turn contains not only the reflected torque from the dynamical system but also many other terms that will be explained in detail in the next sections. Extracting the reflected torque information enables us to introduce the concept of rigid body motion estimation of flexible systems that makes it possible to determine the rigid body position of the flexible system without measurement. Indeed, the rigid body oscillation doesn't represent the entire frequency range of the system; it represents only a certain region with a width that depends on the mass distribution, stiffness and damping along the system. However, in order to estimate the flexible motion of the dynamical system, system parameters have to be estimated first. In other words, this article is introducing a chain of estimators that provides all the information about the dynamical system including its dynamics, joint stiffness, damping coefficient, and externally applied forces or torques due to any interaction with the environment. [1] considered the reflected torque on the actuator as a part of the total disturbance that has to be estimated and rejected to end up with a robust motion control. A disturbance observer was designed in [2] that guarantees the robust motion control in the low frequency range which is not the case in the high frequency range as the disturbance cannot be rejected completely due to the time delay in the estimation process. [3, 4] pointed out that reaction torque can be estimated using the same disturbance observer when the necessary modifications are added as well as estimating the gravity effect. In this work, the same observer is used to estimate the reflected torque but instead of rejecting this torque it will be used by further estimators to extract as much information as possible from the system. [5, 6] considered the mechanical waves as natural feedback from the mechanical system and used the actuator to execute a couple of tasks; launching and absorbing certain mechanical waves. But the process depends on neglecting the damping of the system and the external applied forces from the environment. Besides, it necessitates measuring the first lumped mass position. Mechanical waves are used to analyze and control gantry cranes [7, 8] by using the moving trolley to launch and/or absorb waves that travel to and from the load. However, the previous articles presented the concept of natural feedback that is reflected on/from the actuator, and the focus will be on this idea to achieve sensorless control algorithm. A comparison between wave based control and other schemes for controlling flexible structures such as linear quadratic regulator, Bang-Bang control and input shaping was presented in [9]. The first scheme requires the knowledge of all systems states or their estimates while other approaches require the exact complete model as they are entirely open loop. On the other hand, the wave based approach can be extended to n degrees of freedom system without modification with only one measurement picked from the system. This paper is organized as follows, in section II modeling modal analysis of a lumped flexible system is presented, torque observer is designed in section III, parameters are estimated in section IV along with the rigid body motion, proof of general equations for flexible motion estimation are shown in section IV and used to achieve a sensorless position and vibration control. Finally in section V Conclusion is made and final remarks are discussed.

2. Problem Formulation

2.1. Flexible System Modeling

For a flexible inertial system with n degrees of freedom as shown in Fig. 1, the equations of motion can be written in the following form

$$[\mathbf{J}][\ddot{\theta}] + [\mathbf{B}][\dot{\theta}] + [\mathbf{k}][\theta] = \tau \tag{1}$$

where J, B, k are the inertia, damping and stiffness matrix, θ and τ are vectors of generalized co-ordinates and input torques. Assuming that the damping coefficient and the joint stiffness are uniform along the flexible dynamical system, rewriting the equations of motion for an n degree of freedom system;

$$J_m\ddot{\theta}_m + B(\dot{\theta}_m - \dot{\theta}_1) + k(\theta_m - \theta_1) = \tau_m \tag{2}$$

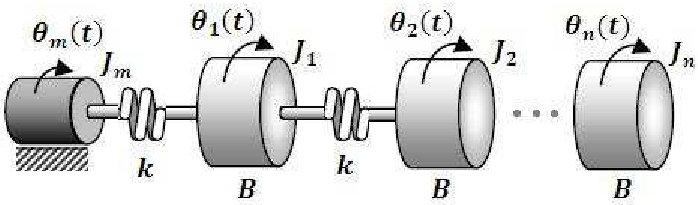


Figure 1. Inertial Lumped Flexible System.

$$J_1\ddot{\theta}_1 - B(\dot{\theta}_m - \dot{\theta}_1) - k(\theta_m - \theta_1) + B(\dot{\theta}_1 - \dot{\theta}_2) - k(\theta_1 - \theta_2) = 0 \tag{3}$$

We keep on writing equations of motions until the n^{th} mass

$$J_n\ddot{\theta}_n - B(\dot{\theta}_{n-1} - \dot{\theta}_n) - k(\theta_{n-1} - \theta_n) = 0 \tag{4}$$

Putting Eqn. (3) and Eqn. (4) in between together and solving for $B(\dot{\theta}_m - \dot{\theta}_1) - k(\theta_m - \theta_1)$ we get

$$B(\dot{\theta}_m - \dot{\theta}_1) + k(\theta_m - \theta_1) = J_1\ddot{\theta}_1 + J_2\ddot{\theta}_1 + J_3\ddot{\theta}_3 + ... + J_n\ddot{\theta}_n \tag{5}$$

And making the following definition, where τ_{ref} is the reflected torque from the dynamical system on the actuator

$$\tau_{ref} \triangleq B(\dot{\theta}_m - \dot{\theta}_1) + k(\theta_m - \theta_1) \tag{6}$$

Eqn. (5) indicates that all the flexible system dynamics are included in the reflected torque wave. Moreover, if we assume that some external disturbance is added to any inertial lumped mass of the system, Eqn. (5) can be rewritten as;

$$B(\dot{\theta}_m - \dot{\theta}_1) + k(\theta_m - \theta_1) = J_1\ddot{\theta}_1 + J_2\ddot{\theta}_1 + J_3\ddot{\theta}_3 + ... + J_n\ddot{\theta}_n - \sum_{i=1}^{n}\tau_{ext_i} \tag{7}$$

where τ_{ext_i} is the external disturbance at any i^{th} mass, surprisingly enough the reflected torque wave that carries all of these information about the flexible dynamical system is reflected back on the actuator as it is shown from in Eqn. (2), and it can be estimated using the actuators current and velocity.

2.2. Reflected Wave Estimation

Rewriting Eqn. (2)

$$J_m\ddot{\theta}_m + B(\dot{\theta}_m - \dot{\theta}_1) + k(\theta_m - \theta_1) = i_m k_t \tag{8}$$

where J_m , i_m and k_t are the actuator inertia, current and torque constant respectively. Taking the parameters variation into consideration we can define

$$J_m = J_{mo} + \triangle J_m \; and \; k_t = k_{to} + \triangle k_t \tag{9}$$

where where J_{mo} and k_{to} are the nominal actuator inertia and torque constant, while $\triangle J_m$ and $\triangle k_t$ are the variation from these nominal parameters. Rewriting Eqn. (8) and rearranging the terms we get

$$J_{mo}\ddot{\theta}_m - i_m k_{to} = i_m \triangle k_t - B(\dot{\theta}_m - \dot{\theta}_1) - k(\theta_m - \theta_1) - \triangle J_m \ddot{\theta}_m \tag{10}$$

where the right hand side is nothing but the total disturbance on the actuator and defined as follows

$$d \triangleq i_m \triangle k_t - B(\dot{\theta}_m - \dot{\theta}_1) - k(\theta_m - \theta_1) - \triangle J_m \ddot{\theta}_m \tag{11}$$

The disturbance is estimated through a low- pass filter with a cut off frequency g_{dist} as follows ref1. The block diagram implementation of the disturbance observer is shown in Fig. 2. The first term of Eqn. (11) represent the torque ripple from the actuator; the last term represents the varied self inertia torque while the rest is nothing but the reflected torque wave that is required to be decoupled from the total estimated disturbance. However, Murakami showed how to extract the information of the reflected torque out of the total disturbance through the same observer with few modification, the details are included in ref 3 and ref 4 for interested readers.

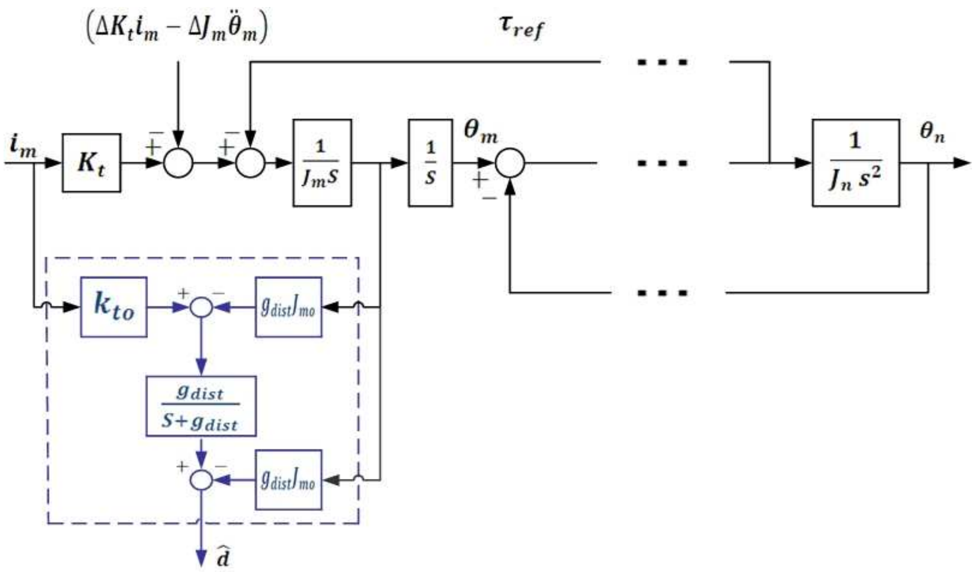


Figure 2. Disturbance Estimation Using Actuator Parameters.

$$\hat{d} = \frac{g_{dist}}{s + g_{dist}}(J_{mo}\ddot{\theta}_m - i_m k_{to}) = \frac{g_{dist}}{s + g_{dist}}(g_{dist}J_{mo}\dot{\theta}_m - i_m k_{to}) - g_{dist}J_{mo}\dot{\theta}_m \tag{12}$$

$\hat{d} \Rightarrow \hat{\tau}_{ref}$ where $\hat{\tau}_{ref}$ is the estimate of the reflected torque wave on the actuator.

2.3. Modal Analysis of Flexible Systems

Figure.1. shows a 3 DOF flexible system. Taking the Laplace transform of the system equations of motion and putting the result in the following linear system format we get

$$A\theta(s) = \tau(s) \tag{13}$$

where

$$\mathbf{A} = \begin{bmatrix} J_1 s^2 + Bs + k & -Bs - k & 0 \\ -Bs - k & J_2 s^2 + 2B + 2k & -Bs - k \\ 0 & -Bs - k & J_3 s^2 + Bs + k \end{bmatrix}$$

Solving the determinant of A assuming equal inertial masses we get the following characteristic polynomial

$$P(s) = J^3 s^6 + 4J^2 Bs^5 + (4J^2 k + 3JB^2)s^4 + 6mBks^3 + 3mk^2 s^2$$

Solving the roots of the previous characteristic polynomial assuming zero damping coefficient we get

$$s_{1,2} = 0, \quad s_{3,4} = \pm j\sqrt{\frac{k}{j}}, \quad s_{5,6} = \pm j\sqrt{\frac{3k}{j}} \tag{14}$$

Defining the eigenvalue problem

$$A\theta = \lambda\theta \quad \Rightarrow \quad (A - \lambda I)\theta = 0$$

And solving for the eigenvectors corresponding to each eigenvalue we end up with the modal vectors if the 3 DOF flexible system

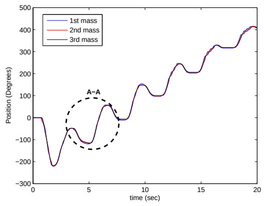
For $\lambda_1 = 0 \Rightarrow \underline{\theta}_1 = \begin{bmatrix} 1 \\ 1 \\ 1 \end{bmatrix}, \lambda_2 = j\sqrt{\frac{k}{m}} \Rightarrow \underline{\theta}_2 = \begin{bmatrix} 1 \\ 0 \\ -1 \end{bmatrix} \; and \;\; \lambda_3 = j\sqrt{\frac{3k}{m}} \Rightarrow \underline{\theta}_3 = \begin{bmatrix} 1 \\ -2 \\ 1 \end{bmatrix}$

Concatenating the three eigenvectors, we end up with the systems modal matrix M where the first eigenvector shows the rigid body motion of the flexible system, where all the lumped masses are moving with the same amplitude with respect to each other and in phase. The second modal or eigenvector implies that the first and third masses are moving with the same amplitude but out of phase and the second mass is not oscillating, while the third vector indicates that the first and third masses are oscillating with the same amplitude and in phase while the second mass is oscillating with twice their amplitude and out of phase. Fig. 3 shows the experimental interpretation of the modal matrix M and the corresponding frequencies. It turns out that for such systems we have rigid and flexible body oscillation. For this particular system the rigid body oscillation falls below 2 rad/sec. In other words, if the frequency of the forcing function falls below 2 rad/sec the flexible modes of the system will not be excited and the whole system will be moving as a bulk mass if the control input was passed with a low-pass filter with a cutoff frequency 2 rad/sec. This will guarantee that any of the other flexible modes will never be excited. Another way to achieve the same result is to take the Fourier synthesis of the control input with sinusoidal signals while avoiding the use of those sinusoidal with frequencies equal to the resonances of the system.

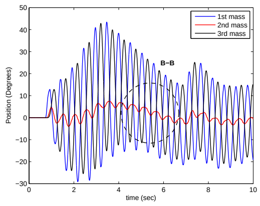
$$\mathbf{M} = \begin{bmatrix} 1 & 1 & 1 \\ 1 & 0 & -2 \\ 1 & -1 & 1 \end{bmatrix} \tag{15}$$

2.4. Filtering and/or Fourier Synthesis of the Control Input

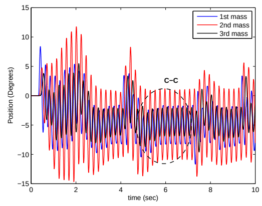
If the forcing function is filtered with a LPF that is designed according to the results obtained from the modal analysis of the flexible system, in other words according to the previous calculations the rigid mode



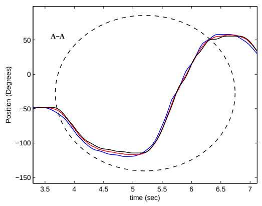
(a) 1 rad/sec



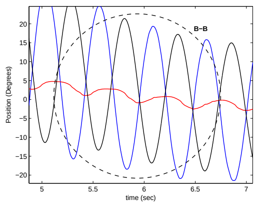
(b) 11 rad/sec



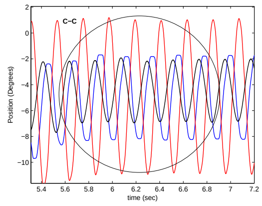
(c) 22 rad/sec



(d) Magnified Plot A-A



(e) Magnified Plot B-B



(f) Magnified Plot C-C

Figure 3. Modal Analysis of a 3-DOF Flexible System

was supposed to fall below 2 rad/sec, so if the cut off frequency of the LPF was selected to be lower than this frequency, the system will be guaranteed to be oscillating according to rigid body oscillations. The other way is Fourier synthesis of the control input as follows, providing that f_K will not coincide with any of the systems resonances, A_k is the sinusoidal amplitude, A_o is the DC offset and ϕ_k is the phase angle.

$$u(t) = A_0 + \sum_{k=1}^{n} (\frac{1}{2} A_k e^{j\phi_k} e^{j2\pi f_k t} + \frac{1}{2} A_k e^{-j\phi_k} e^{-j2\pi f_k t})|_{f_k \neq f_{res}} \tag{16}$$

where f_{res} are any of the flexible systems resonance frequencies. Eqn. (13) is suggesting that we can build our control signal with a finite number of sinusoidal providing that they don't contain any energy at the resonances of the system. Such Fourier or low-pass filtered control signal makes it possible to rewrite Eqn. (5) as follows

$$\hat{\tau}_{ref} = B(\dot{\theta}_m - \dot{\theta}_1) + k(\theta_m - \theta_1) = J_1\ddot{\theta}_1 + J_2\ddot{\theta}_2 + J_3\ddot{\theta}_3 + ... + J_n\ddot{\theta}_n$$

$$\hat{\tau}_{ref} = \ddot{\theta}_a(J_1 + J_2 + J_3 + ... + J_n)$$

And the estimate of the rigid body position is in Eqn. (17) representing the position estimate of the bulk system when the system is excited using the control forcing function Eqn. (16). In other words estimating the rigid body position of the flexible system doesn't necessitate attaching any sensors, instead the reflected torque is estimated providing that the controllers forcing function is satisfying Eqn. (16). Fig. 4 shows the block diagram implementation of the rigid body motion estimation based on the actuator parameters measurements. The low-pass filter at the beginning of the system will guarantee the rigid body oscillations. Fig. 5 shows the experimental result of the rigid body motion estimation experiment and as it is shown, the estimate follows the actual mass positions at low frequencies below 2 rad/sec as it is expected, on the other hand increasing the frequency above

$$\hat{\theta}_a(t) = \frac{1}{\sum_{i=1}^{n} J_i} \int_0^t \int_0^t \hat{\tau}_{ref} d\tau d\tau \tag{17}$$

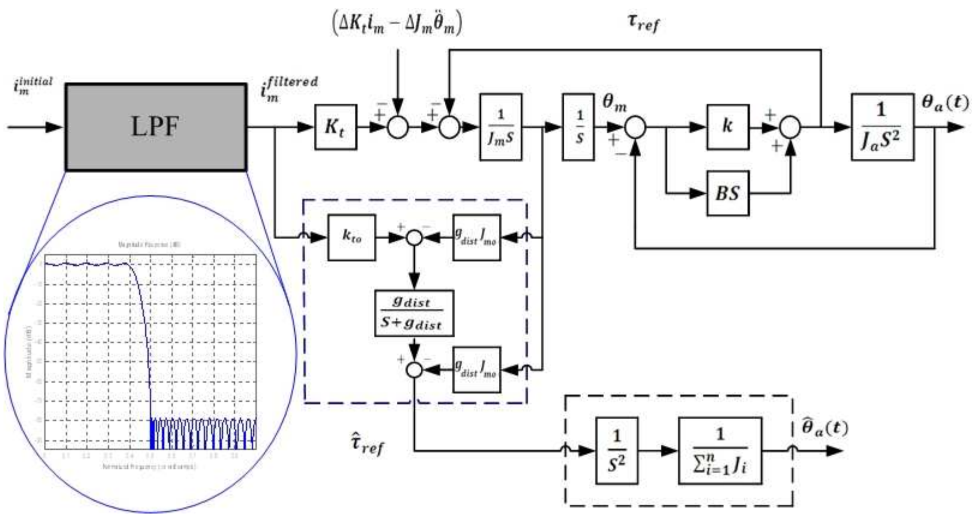


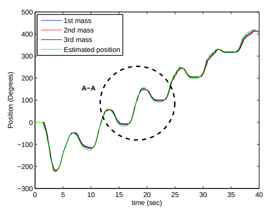
Figure 4. Rigid Body Motion Estimation.

2 rad/sec will result in losing the correct estimate as Eqn. (17) will no longer hold [10, 11]. Fig. 5 a and b show that Eqn. (17) is valid below 2 rad/sec while Fig. 5 c shows that the estimate is no longer valid above 3 rad/sec which is expected from the previous analysis.

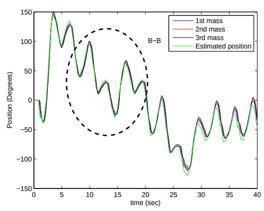
3. Uniform System Parameter Estimation

3.1. Parameter Estimation

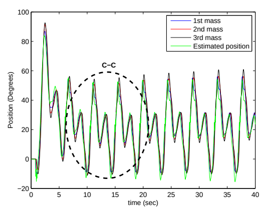
Recalling Eqn. (6) and instead of using the actual position of any lumped mass we use the rigid body position estimate instead and the same thing with the reflected torque, we use the estimate of the reflected torque. In other words, we replace any variable with the available estimate



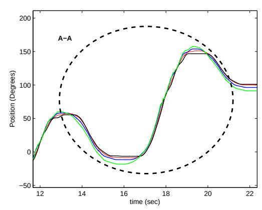
(a) 1 rad/sec



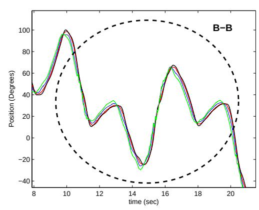
(b) 2 rad/sec



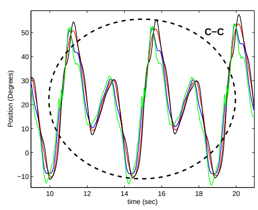
(c) 3 rad/sec



(d) Magnified Plot A-A



(e) Magnified Plot B-B



(f) Magnified Plot C-C

Figure 5. Rigid Body Motion Estimation Experimental Results

$$\hat{\tau}_{ref} \triangleq B(\dot{\theta}_m - \dot{\hat{\theta}}_a) + k(\theta_m - \hat{\theta}_a) \tag{18}$$

Making the following definitions

$$\underline{\xi} \triangleq (\theta_m - \hat{\theta}_a)$$

$$\underline{\eta} \triangleq (\dot{\theta}_m - \dot{\hat{\theta}}_a)$$

That represents vectors of data points between the actuator measurement and the estimate of the rigid body motion estimation and its derivative. In matrix form;

$$\begin{bmatrix} \underline{\xi} & \underline{\eta} \end{bmatrix} \begin{bmatrix} k \\ B \end{bmatrix} = \underline{\hat{\tau}}_{ref} \tag{19}$$

where $\underline{\hat{\tau}}_{ref}$ is a vector of reflected wave data points. Making another definition

$$H \triangleq \begin{pmatrix} \xi & \underline{\eta} \end{pmatrix}$$

Eqn. (19) describes as over-determined system, where the number of equations is larger than the number of unknowns, and for such a system we are looking for those solutions that minimize some cost functions such as the norm square of errors. The estimate of the stiffness and the damping coefficients are given as;

$$\begin{bmatrix} \hat{k} \\ \hat{B} \end{bmatrix} = [H^T H]^{-1} H^T \underline{\hat{\tau}}_{ref} \tag{20}$$

Or simply

$$\begin{bmatrix} \hat{k} \\ \hat{B} \end{bmatrix} = H^{\dagger} \underline{\hat{\tau}}_{ref}$$

where H is the pseudo inverse of H and this vector of estimated parameters is the solution that minimizes the norm square of errors. Fig. 6 shows the block diagram implementation of the parameter estimation process starting with the control input filtering and then estimating the reflected torque wave, then estimation of the rigid body position of the system and ending up with the estimates of the systems uniform parameters.

3.2. Experimental Parameter Estimation

For the experimental estimation of the uniform system parameters we should detect the vectors $\underline{\xi}$ and $\underline{\eta}$ along with the estimated reflected wave vector. Taking into consideration that the parameter estimation experiment has to be performed in the low frequency range where Eqn. (17) is valid. The joint stiffness is known beforehand by the following expression

$$k = \frac{Gd}{8c^3 n} \tag{21}$$

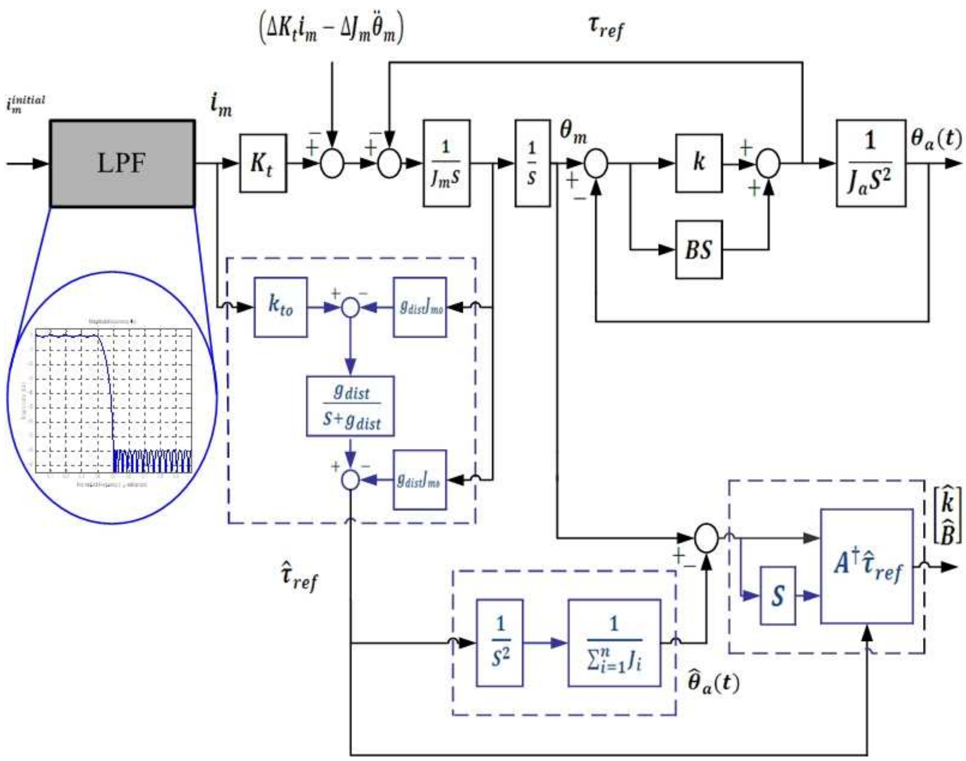


Figure 6. Offline Parameter Estimation Block Diagram.

where G is the modulus of rigidity, d is the wire diameter of the spring, c is the spring index ratio and n is the effective number of turns. Computing k we get

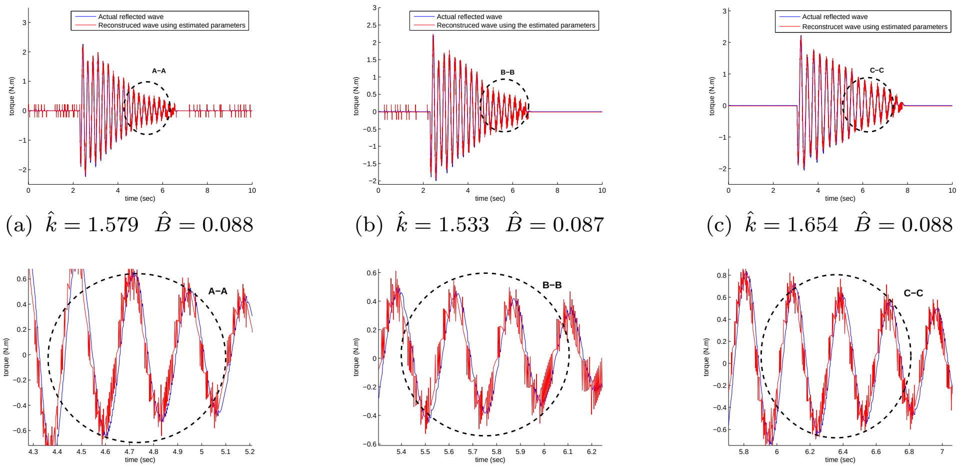
$$k = \frac{70 \times 10^9 \times 2}{8 \times (\frac{8}{2})^3 \times 21} = 1.627kN/m$$

Table (1) shows the experimental results of the estimated stiffness and damping coefficients of the flexible system shown in Fig. 1. And the average stiffness and damping coefficient turns out to be

$$\hat{k}_{avg} = \frac{\sum_{i=1}^{n} k_i}{n} = \frac{15.4653}{10} = 1.54653\ kN/m \tag{22}$$

$$\hat{B}_{avg} = \frac{\sum_{i=1}^{n} B_i}{n} = \frac{0.8433}{10} = 0.08433\ Nsec/m \tag{23}$$

Fig. 7 shows the actual reflected wave and the reconstructed one using Eqn. (18), and it turns out that the estimates of the stiffness and the damping coefficients are both able to reconstruct the original reflected wave. The noisy looking of the reconstructed torque wave will not represent a problem as this signal will not be used in further processes. Instead the actual estimated reflected torque will be used.



(a) $\hat{k} = 1.579 \quad \hat{B} = 0.088$ (b) $\hat{k} = 1.533 \quad \hat{B} = 0.087$ (c) $\hat{k} = 1.654 \quad \hat{B} = 0.088$

(d) Magnified Plot A-A (e) Magnified Plot B-B (f) Magnified Plot C-C

Figure 7. Parameter Estimation Experimental Results (Reconstructed Wave Using the Estimated Parameters)

4. Flexible Motion Estimation

The idea behind not exciting any of the flexible modes of the system is to drop all the generalized co-ordinates of the system. In other words dropping the number of unknowns so that we can easily determine the system parameters, and as the system parameters are inherent properties of the system, they don't depend on the frequency range from where they were computed or to be used. However, the estimated parameters obtained from the previous sections can be used in order to get a general expression for the motion of each lumped mass of the system regardless of the frequency of the forcing function.

4.1. Recursive Flexible Position Estimation

Recalling Eqn. (18) and instead of the actual parameters we use the estimated stiffness and estimated damping coefficients

$$\hat{\tau}_{ref} \triangleq B(\dot{\theta}_m - \dot{\hat{\theta}}_1) + k(\theta_m - \hat{\theta}_1)$$

$$B \;\; \Rightarrow \;\; \hat{B} \;\; , \;\; k \;\; \Rightarrow \;\; \hat{k}$$

and rewriting Eqn. (18) we get

Table 1. Experimental Extimated Stiffness and Damping Coefficients

Experiment	$\hat{K}$ (kN/m)	$\hat{B}$ (Nsec/m)	Experiment	$\hat{K}$ (kN/m)	$\hat{B}$ (Nsec/m)
1^{st} Exp	1.5796	0.0888	6^{th} Exp	1.5277	0.0892
2^{nd} Exp	1.5336	0.0878	7^{th} Exp	1.4913	0.0893
3^{rd} Exp	1.6459	0.0887	8^{th} Exp	1.5774	0.0892
4^{th} Exp	1.5116	0.0889	9^{th} Exp	1.6049	0.0896
5^{th} Exp	1.5625	0.0893	10^{th} Exp	1.4531	0.0891

$$\hat{\tau}_{ref} \triangleq \hat{B}(\dot{\theta}_m - \dot{\theta}_1) + \hat{k}(\theta_m - \theta_1) \tag{24}$$

Rearranging the terms we get

$$\hat{B}\dot{\theta}_1 + \hat{k}\theta_1 = \hat{B}\dot{\theta}_m + \hat{k}\theta_m - \hat{\tau}_{ref}$$

defining

$$\alpha \triangleq \hat{B}\dot{\theta}_m + \hat{k}\theta_m - \hat{\tau}_{ref}$$

and

$$\beta \triangleq \frac{\alpha}{\hat{B}}$$

and writing the differential equation in its standard first order form as follows

$$\dot{\theta}_1(t) + \frac{\hat{k}}{\hat{B}}\theta_1 = \beta$$

Since the equation is based on estimated variables and parameters, the solution will also be an estimate and the equation can be rewritten as follows

$$\dot{\hat{\theta}}_1(t) + \frac{\hat{k}}{\hat{B}}\hat{\theta}_1 = \beta \tag{25}$$

Multiplying the previous $1^s t$ order differential equation by the following integrating factor $e^{(\frac{\hat{B}}{\hat{k}})t}$ we get

$$e^{(\frac{\hat{B}}{\hat{k}})t}\dot{\hat{\theta}}_1(t) + e^{(\frac{\hat{B}}{\hat{k}})t}\frac{\hat{k}}{\hat{B}}\hat{\theta}_1(t) = e^{(\frac{\hat{B}}{\hat{k}})t}\beta$$

$$\frac{d}{dt}[e^{(\frac{\hat{B}}{\hat{k}})t}\hat{\theta}_1(t)] = e^{(\frac{\hat{B}}{\hat{k}})t}\beta$$

Integrating both sides and multiplying by $e^{-(\frac{\hat{B}}{\hat{k}})t}$, we get the general estimate of the first lumped mass of the system by the following expression

$$\hat{\theta}_1(t) = e^{-(\frac{\hat{B}}{\hat{k}})t}\int_o^t \beta e^{(\frac{\hat{B}}{\hat{k}})\tau}d\tau + e^{-(\frac{\hat{B}}{\hat{k}})t}c_1 \tag{26}$$

and to get the estimate of the second lumped mass we recall the first equation of motion

$$J_1\ddot{\hat{\theta}}_1 - \hat{B}(\dot{\theta}_m - \dot{\hat{\theta}}_1) - \hat{k}(\theta_m - \hat{\theta}_1) + \hat{B}(\dot{\hat{\theta}}_1 - \dot{\hat{\theta}}_2) - \hat{k}(\hat{\theta}_1 - \hat{\theta}_2) = 0 \tag{27}$$

and defining

$$\gamma \triangleq J_1\ddot{\hat{\theta}}_1 - \hat{B}(\dot{\theta}_m - \dot{\hat{\theta}}_1) - \hat{k}(\theta_m - \hat{\theta}_1) + \hat{B}\dot{\hat{\theta}}_1 + \hat{k}\hat{\theta}_1$$

$$\zeta \triangleq \frac{\gamma}{\hat{B}}$$

and putting the equation in the standard 1^st order form we get

$$\dot{\hat{\theta}}_2(t) + \frac{\hat{k}}{\hat{B}}\hat{\theta}_2(t) = \zeta \tag{28}$$

Multiplying by the integrating factor $e^{(\frac{\hat{B}}{\hat{k}})t}$ we get

$$e^{(\frac{\hat{B}}{\hat{k}})t}\dot{\hat{\theta}}_2(t) + e^{(\frac{\hat{B}}{\hat{k}})t}\frac{\hat{k}}{\hat{B}}\hat{\theta}_2(t) = e^{(\frac{\hat{B}}{\hat{k}})t}\zeta$$

$$\frac{d}{dt}[e^{(\frac{\hat{B}}{\hat{k}})t}\hat{\theta}_2(t)] = e^{(\frac{\hat{B}}{\hat{k}})t}\zeta$$

Integrating both sides and multiplying by $e^{-(\frac{\hat{B}}{\hat{k}})t}$ we get the estimate of the second lumped mass position

$$\hat{\theta}_2(t) = e^{-(\frac{\hat{B}}{\hat{k}})t}\int_o^t \zeta e^{(\frac{\hat{B}}{\hat{k}})\tau}d\tau + e^{-(\frac{\hat{B}}{\hat{k}})t}c_2 \tag{29}$$

and the estimate of the third mass position is found to be

$$\hat{\theta}_3(t) = e^{-(\frac{\hat{B}}{\hat{k}})t}\int_o^t \varepsilon e^{(\frac{\hat{B}}{\hat{k}})\tau}d\tau + e^{-(\frac{\hat{B}}{\hat{k}})t}c_3 \tag{30}$$

where

$$\varepsilon \triangleq \frac{\delta}{\hat{B}}$$

$$\delta \triangleq J_2\ddot{\hat{\theta}}_2 - \hat{B}(\dot{\hat{\theta}}_1 - \dot{\hat{\theta}}_2) - \hat{k}(\hat{\theta}_1 - \hat{\theta}_2) + \hat{B}\dot{\hat{\theta}}_2 + \hat{k}\hat{\theta}_2$$

and the general estimation expression for any lumped mass along the flexible lumped dynamical system is given by

$$\hat{\theta}_i(t) = e^{-(\frac{\hat{B}}{\hat{k}})t}\int_o^t \Omega e^{(\frac{\hat{B}}{\hat{k}})\tau}d\tau + e^{-(\frac{\hat{B}}{\hat{k}})t}c_i \tag{31}$$

$$\Omega \triangleq \frac{\Psi}{\hat{B}}$$

$$\Psi \triangleq g(J_{i-1}, \hat{\theta}_{i-1}, \dot{\hat{\theta}}_{i-1}, \ddot{\hat{\theta}}_{i-1}, \hat{k}, \hat{B})$$

It turns out that, in order to compute the position estimate of any lumped mass position of the flexible system, we have to go through the recursive computation of all the previous masses positions, providing that the system parameters have already been estimated through the proposed algorithm of this article. Simply

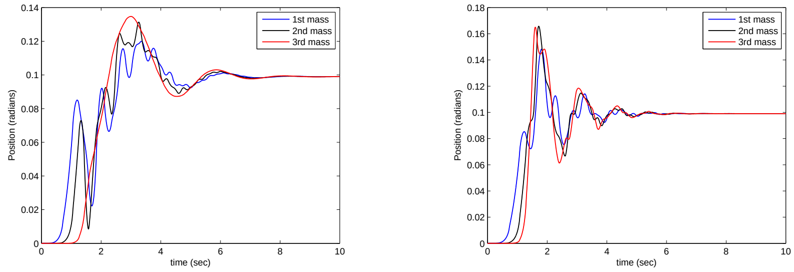
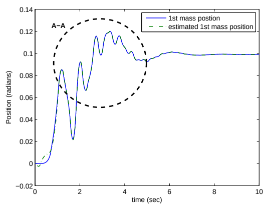
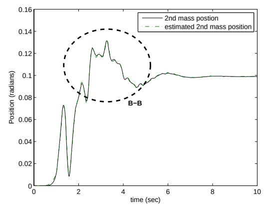


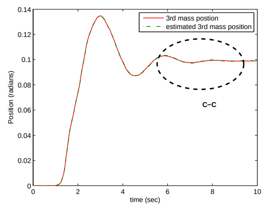
Figure 8. Flexible Oscillation of a 3DOF Dynamical System)



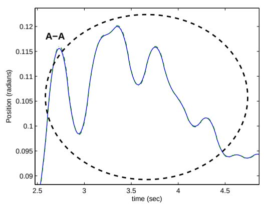
(a) 1^{st} Mass and Its Estimate



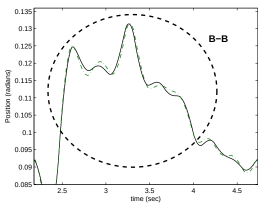
(b) 2^{nd} Mass and Its Estimate



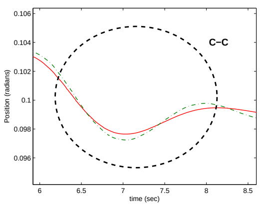
(c) 3^{rd} Mass and Its Estimate



(d) Magnified Plot A-A



(e) Magnified Plot B-B



(f) Magnified Plot C-C

Figure 9. Flexible Body Motion Estimation Experimental Results

in order to excite any of the flexible modes of the system, the frequency of the forcing function doesn't have to coincide with the system resonances, simply it has not to obey Eqn. (16) or to have a frequency higher than 2 rad/sec.

4.2. Experimental Flexible Position Estimation

If the forcing function contains energy at the systems resonances or near to the system resonances, flexible system will have a behavior similar to that shown in Fig. 8. Where the masses of the lumped system are no longer oscillating with the same amplitude with respect to each other. However, such motion can be estimated using the recursive flexible motion estimation expression in Eqn. (31). Fig. 9 shows the difference between the actual and the estimated position for a flexible system with three degrees of freedom.

4.3. Sensorless Motion and Vibration Control

As shown in Fig. 9 position estimates of the lumped positions are too close to the actual measurement, which makes it possible to feed the estimate back to the controller instead of the actual measurement. However, the position error is no longer defined as the difference between the actual measurement and a given reference, but defined as the difference between the position estimate of the mass to be controlled and a given reference as follows

$$\hat{e}(t) = \theta_{ref}(t) - \hat{\theta}_i(t) \tag{32}$$

where $\theta_r ef(t)$ is the given arbitrary reference position and $\hat{\theta}_i(t)$ is the position estimate of the lumped i^{th} mass. And the control law of the estimation based PID controller is as follows

$$u(t) = k_p\hat{e}(t) + k_i \int_o^t \hat{e}(t)d\tau + k_d \frac{d\hat{e}(t)}{dt} \tag{33}$$

The proposed algorithm makes it easier to feed any estimate back to the controller without any need to attach extra sensors or to change the position of the available ones along the system, Moreover, the proposed algorithm makes it possible to get the global picture of the system behavior as all the position estimates are available. In other words, if the last mass is to be controlled its estimate has to be used as a feedback to the controller while the behavior of the rest of the system can be explored by looking at the other position estimates. And the same thing can be repeated with any other masses. The control law in Eqn. (33) has to take care of the primary task, which is the motion control. Besides, it has to keep the flexible system free from any residual vibrations, however, in order to do so the boundary conditions at the beginning and at the end of the travel must be

$$\begin{bmatrix} \theta_1 \\ \theta_2 \\ \theta_3 \end{bmatrix}_{t=T} = \begin{bmatrix} \theta_o \\ \theta_o \\ \theta_o \end{bmatrix}$$

$$\begin{bmatrix} \dot{\theta}_1 \\ \dot{\theta}_2 \\ \dot{\theta}_3 \end{bmatrix}_{t=T} = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix} \tag{34}$$

where θ_o is some finite displacement. Fig. 10 shows the sensorless motion and vibration controller of a lumped flexible dynamical system, where the estimates of the system parameters are assumed to be determined by an off-line experiment as shown in Fig. 6.

4.4. Experimental Sensorless Motion and Vibration Control

Sensorless motion control of the 1^{st} lumped mass requires feeding its estimate back to the controller. Fig. 11 shows the response of the different masses when the first lumped inertial mass is controlled. The other estimates are used in order to make sure that the position control process is achieved with minimum residual vibration in the flexible system.

Fig. 12 shows the sensorless motion control process when the second mass is required to be controlled where its estimate is fed back to the controller instead of the first mass. The sensorless control process shows a great advantage in the sense of moving between the lumped masses with the observer instead of attaching real physical sensors. Besides, all the position estimates are available and the global dynamical behavior of the flexible system can be investigated.

4.5. Summary of the Entire Estimation Process

Fig. 13 shows the summary of the entire estimation process, starting with the off-line uniform parameter estimation experiment that is performed in the low frequency range or with the assumption that system is

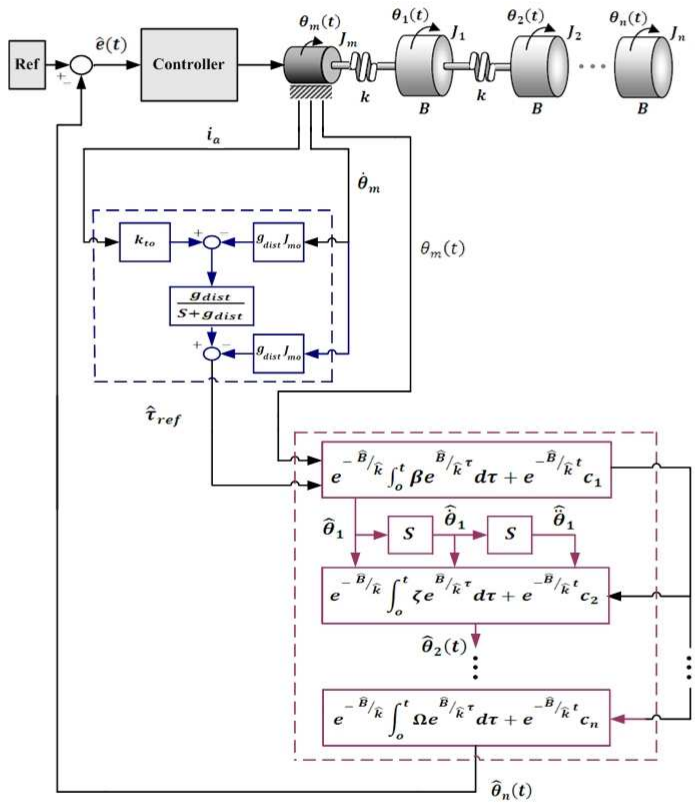
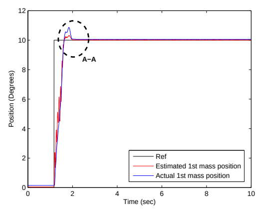
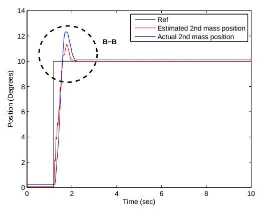


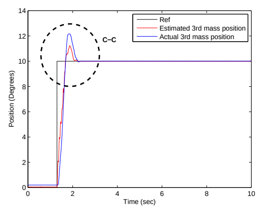
Figure 10. Sensorless Motion Control of the Flexible System.



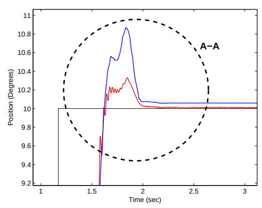
(a) Controlled 1^{st} Mass $k_p = 0.06$ $k_d = 0.002$



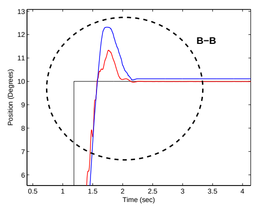
(b) 2^{nd} Mass and Its Estimate



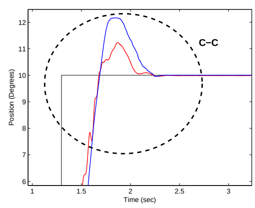
(c) 3^{rd} Mass and Its Estimate



(d) Magnified Plot A-A



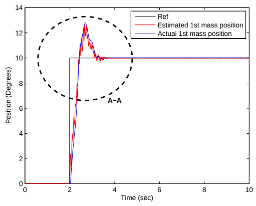
(e) Magnified Plot B-B



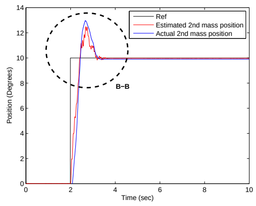
(f) Magnified Plot C-C

Figure 11. Sensorless Motion Control Experimental Results (1^{st} Lumped Mass Estimate Fed Back to the Controller)

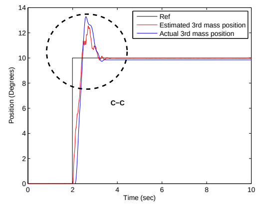
free from energy at the systems resonances that is guaranteed by the Fourier synthesis of the initial forcing function. The only inputs of the estimation process are the actuator parameter current and velocity and the single information that has to be investigated from the dynamical system is the mass information. And as



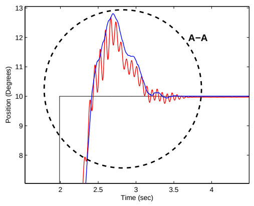
(a) 1^{st} Mass and Its Estimate



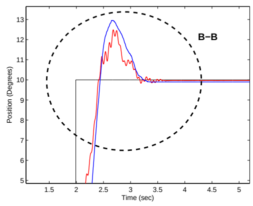
(b) Controlled 2^{nd} Mass $k_p = 0.002 \;\; k_d = 0$



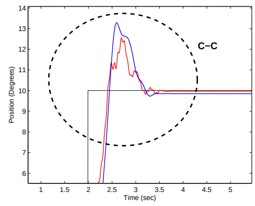
(c) 3^{rd} Mass and Its Estimate



(d) Magnified Plot A-A



(e) Magnified Plot B-B



(f) Magnified Plot C-C

Figure 12. Sensorless Motion Control Experimental Results (2^{nd} Lumped Mass Estimate Fed Back to the Controller)

soon as the off-line parameter estimation is performed the flexible motion of the system is estimated through the recursive motion estimation equations shown in Eqn. (31). However, getting the estimates of the lumped mass positions makes it possible to perform a sensorless motion and vibration control.

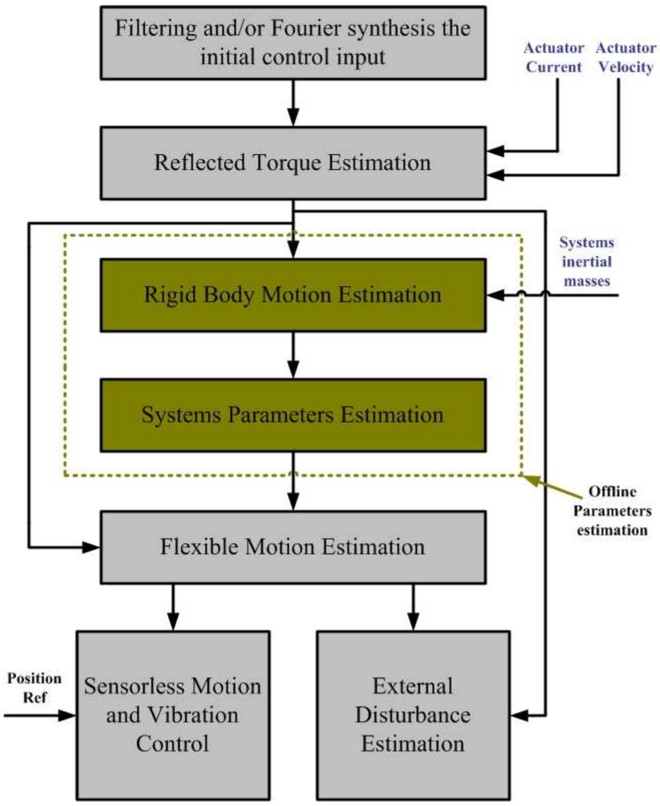


Figure 13. The Entire Estimation Process.

5. Conclusion

This article demonstrates that the actuator can be used as a single platform for measurement, control and estimation. And two measurements are enough to achieve a sensorless motion and vibration control of any lumped flexible dynamical system. The presented algorithm is based on a chain of estimators that are connected at the end of each other, starting with the reflected torque estimation, rigid body motion estimation and off-line uniform system parameter estimation. The idea behind getting the system parameters from the low frequency is the ability to drop as many unknowns or systems generalized co-ordinates as possible, as the system parameters are the same regardless to the frequency zone from which they are detected. However, it was easier to detect the systems parameters information from this frequency zone and using this information in the entire frequency range for the flexible motion estimation procedure. The experimental sensorless motion control results show the existence of less than 1.5 percent steady state error in the final response. This steady state error is due to the cumulative error in the chain of estimators, beside the parameters estimation process. Since we do not end up with the real joint stiffness and damping coefficient, but we end up with the systems parameters that minimize the norm squares of error. Therefore, the difference between these parameter estimates and the actual system parameters will generate a kind of steady state error in the final response. Minimizing this steady state error depends on performing more accurate experiments along with more operational enhancement of each single step in the proposed algorithm. On one hand, the control process seems to be suffering from the low accuracy in the sense of getting a steady state error, but on the other hand the system's local and global behavior can be studied easily without attaching any sensors to the system. All the system dynamics can be computed and the external forces or torques can be estimated.

Acknowledgements

The authors gratefully acknowledge the financial assistance provided by SanTez project number 00183.STZ.2007-2.

References

[1] K. Ohnishi, M. Shabata, T. Murakami, "Motion Control for Advanced Mechatronics", ASME, vol. 1, pp. 56-67, 1996.

[2] T. Tsuji, M. Mizuochi, H. Nishi and K. Ohnishi, "A Velocity Measurement Method for Acceleration Control", Proceedings of the IEEE, pp. 1943-1948, 2005.

[3] T. Murakami, K. Ohnishi, "Observer Based Motion Control" Proceedings of the IEEE, vol. 1, pp. 1-6, 1993

[4] T. Murakami, F. Yu and K. Ohnishi, "Torque Sensorless Control in Multidegree-of-Freedom Manipulator", IEEE Transaction on Industrial Electronics, vol. 40, no. 2, pp. 259-265, April 1993.

[5] W. J. O'Connor, "Wave-Based Analysis and Control of Lumped Flexible Robots", IEEE Industrial Electronics Society (IECON'95), vol. 2, pp. 1260-1265, 1995.

[6] W. J. O'Connor, "Control of Flexible Mechanical Systems: Wave Based Technique", Proceedings of the IEEE, vol. 1, pp. 4192-4202, 2007.

[7] W. J. O'Connor, "A Gantry Crane Problem Solved", ASME Journal Dynamics Systems, Measurement, and Control, vol. 4, pp. 569-576, 2003.

[8] W. J. O'Connor, "Gantry crane Control: A Novel Solution Explored and Extended", Proceedings of American Control Conference, May 2002.

[9] D. J, Mckeown and W. J. O'Connor, "Wave-Based Control Implementation and Comparison", Proceedings of American Control Conference, July 2007.

[10] I. S. M. Khalil, E. D. Kunt and A. Sabanovic, "Kestirim ve Denetim icin Eyleyicinin Tek Platform Olarak Kullanildigi, Esnek Yapilar icin Algilayicisiz Dalga Tabanli Denetim", Automatic Control Conference TOK'08, vol. 2, pp. 725-730, October 2008.

[11] I. S. M. Khalil and A. Sabanovic, "Sensorless Wave Based Control of Flexible Structures Using Actuator as a Single Platform for Estimation and Control", International Review of Automatic Control, vol. 2, no. 1, pp.83-89, January 2009.

[12] I. S. M. Khalil and E. D. Kunt and A. Sabanovic, "Estimation Based PID controller-Sensorless Wave Based Technique", International Conference on Intelligent Control Systems and Signal Processing ICONS 2009, In Review, 21 September 2009.