THE ANOMALOUS X-RAY PULSAR 4U 0142+61: A NEUTRON STAR WITH A GASEOUS FALLBACK DISK

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ABSTRACT

The recent detection of the anomalous X-ray pulsar (AXP) 4U 0142+61 in the mid infrared with the *Spitzer* Observatory (Wang, Chakrabarty & Kaplan 2006) constitutes the first instance for a disk around an AXP. We show, by analyzing earlier optical and near IR data together with the recent data, that the overall broad band data can be reproduced by a single irradiated and viscously heated disk model.

Subject headings: pulsars: individual (AXPs) — stars: neutron — X-rays: bursts — accretion, accretion disks

1. INTRODUCTION

The zoo of young neutron stars contains a number of categories recognized by the distinct properties of neutron stars discovered or identified in the last decade. The existence of these different types of neutron stars strongly suggests that not all neutron stars are born with the same initial conditions, nor do they follow the same evolutionary path as the familiar radio pulsars, of which the typical young example is usually taken to be the Crab pulsar. Anomalous X-ray Pulsars (AXPs) and Soft Gamma Ray Repeaters (SGRs) (see Woods & Thompson 2005, for a review of AXPs and SGRs) are widely accepted to be magnetars (Duncan & Thompson 1992). The "radio"-quiet neutron stars (RQNs-also known as Central Compact Objects (CCOs); Pavlov, Sanwal & Teter 2004) and dim isolated (thermal) neutron stars (DTNs or DINs, Haberl 2005) are probably related classes though the latter are relatively older. Measured rotation periods of AXPs, SGRs and DINs cluster in the narrow range of 3-12 s. Alpar (2001) proposed that the presence or absence, and properties, of bound matter with angular momentum, that is, a fallback disk, may be among the initial parameters of newborn neutron stars, in addition to magnetic dipole moment and initial rotation rate. According to this the differences between isolated pulsars, AXPs, SGRs and DINs, as well as the CCOs in certain SNRs are due to different initial conditions, including the presence or absence and properties of fallback disks.

The suggestion that the X-ray luminosity of the AXPs is due to mass accretion from a fallback disk (Chatterjee, Hernquist & Narayan 2000; Alpar 2001) has triggered searches for evidence for such disks. Searches for fallback disks around AXPs have been conducted primarily in the optical and near IR bands. A realistic model for a putative fallback disk should take into account the effects of irradiation by the neutron star's X-ray luminosity. Irradiation is the dominant source of disk luminosity for the outer disk, at IR and longer wavelengths. The radial temperature profile of a fallback disk was computed by Perna, Hernquist & Narayan (2000) and Hulleman, van Kerkwijk & Kulkarni (2000) using the prescription given by Vrtilek et al. (1990). To estimate the temperatures, both Hulleman et al. (2000) and Perna et al. (2000) assumed the same particular irradiation efficiency, and found that their estimated optical flux values lie well above the values indicated by the observations of the AXPs 4U 0142+61 and 1E 2259+586. While Perna et al. (2000) suggested that the difference might be due to a probable advection dominated flow at the inner disk, Hulleman et al. (2000) drew the conclusion that the model can only fit to data for an outer disk radius that is too small to be realistic and that fallback disk models were ruled out (see § 4 for further discussion). Recent studies show that irradiated disk model fits to existing detections and upper limits from AXPs allow for the presence of accreting fallback disks with reasonable irradiation parameters for all AXPs (Ertan & Çalışkan 2006) when the fits are not restricted to a particular irradiation efficiency.

Recent results of Wang, Chakrabarty & Kaplan (2006, hereafter WCK06) with *Spitzer* data on AXP 4U 0142+61 complement earlier data in the near infrared and optical (Hulleman, van Kerkwijk & Kulkarni 2000, 2004, hereafter HvKK00 and HvKK04) and pulsed emission in the optical (Kern & Martin 2002) thus providing the first instance of a multiband data set that can be tested against disk models. WCK06 interpret their data with a dust disk radiating in the infrared, while the optical data is ascribed to a magnetospheric origin. They take the origin of the X-ray luminosity to be dissipation of magnetic energy in the neutron star crust, in accordance with the standard magnetar model.

Here we show that the entire unpulsed optical, near and far IR radiation can be explained as due to a gaseous disk whose luminosity is provided by viscous energy dissipation due to mass transfer through the disk as well as irradiation by the X-ray luminosity $L_{\rm X}$ from the neutron star. Taking $L_{\rm X}$ to be an accretion luminosity, as we do, links the irradiation contribution to the disk luminosity with the mass accretion rate $\dot{M}_{\rm acc}$ and thereby to the mass inflow rate $\dot{M}$ through the disk and to the viscous dissipation contribution to the disk luminosity. Luminosity in the optical band comes from the inner regions of the disk, and is supplied, to a significant extent, though not fully, by the local viscous energy dissipation in the inner disk. The innermost disk, dominated by dissipation, radiates mostly in the UV band, but also contributes to the total optical emission. The extension of the same irradiation model to outer regions of the disk explains the IR radiation. The disk luminosity is dominated by the contribution of irradiation at outer radii where the thermal radiation peaks in the longer wavelength bands (see Table 1).

Pulsed emission in the optical (Kern & Martin 2002) is due to magnetospheric emission powered by a disk-shell dynamo in our model. It has been shown by Cheng & Ruderman (1991) that magnetospheric pulsar emission in optical and higher energy bands can be sustained by an outer gap in the presence of a disk protruding into the magnetosphere. This disk model is different from the first disk-magnetosphere models (Michel & Dessler 1981). More recently Ertan & Cheng (2004) have shown that the pulsed optical emission from 4U 0142+61 can be explained in terms of a magnetospheric model with a disk and a neutron star surface dipole magnetic field of the order of $B_{\rm dipole} \sim 10^{12} - 10^{13}$ G. The magnetospheric emission is near 100 % pulsed in this magnetosphere-disk model. The discovery of strongly pulsed optical emission (Kern & Martin 2002) has been generally perceived as excluding disk models, on the basis of the notion that magnetospheric emission cannot survive with a disk protruding into the light cylinder. The disk magnetosphere model of Cheng & Ruderman (1991) and the application by Ertan & Cheng (2004) are important for fallback disk models since they demonstrate that a disk can actually be part of a magnetospheric mechanism generating strongly pulsed emission.

The clustering of AXP and SGR rotational periods between $6 - 12$ s was investigated by Psaltis & Miller (2002), who found that statistical clustering with a spin-down law of braking index $n \sim 2 - 4$ implies that the final period of these systems must be about 12 s. This final period does not correspond to reaching the "death line" in the $P - \dot{P}$ plane which reflects the cut-off voltage for magnetospheric emission. Within isolated dipole-magnetar models, the alternative possibility for reaching a final clustering period, asymptotically, is by magnetic field decay on timescales commensurate with the AXP and SGR ages (Psaltis & Miller 2002). An investigation of various magnetar field decay models (Colpi, Geppert & Page 2000) shows that the X-ray luminosity data to be associated with magnetic field decay is consistent with only one model, involving crustal Hall cascade, and for decay times less than about 10^4 yrs. The presence of a fallback disk provides a natural explanation for period clustering of the AXPs (Chatterjee & Hernquist 2000) as well as dim isolated (thermal) neutron stars (DTNs or DINs, see e.g. Haberl 2005) and SGRs (Alpar 2001; Ekşi & Alpar 2003). Fallback disk models do not address the origin of bursts, while magnetar models explain the bursts employing the energy stored in very strong magnetic fields. The basic workings of the magnetar models require only local magnetic stresses and energies in the neutron star crust, and thus can be sustained with higher multipole components of the field while the period clustering and explanation of torques with fallback disk require the dipole component of the magnetic field to be of the order of $10^{12} - 10^{13}$ G. Hybrid models (Ekşi & Alpar 2003; Ertan & Alpar 2003) with $B_{\rm multipole} \sim 10^{14} - 10^{15}$ G and $B_{\rm dipole} \sim 10^{12} - 10^{13}$ G are a possibility that incorporates a fallback disk.

In this paper, we evaluate the recent observations of WCK06 in conjunction with older results (§ 2). We show that the data agree quite well with a gaseous disk. In § 3, we estimate the characteristic lifetime of the disk that has evolved through a propeller phase and find that the disk's age estimate, depending on torque models, is comparable with the $P_*/2\dot{P}_*$ characteristic age estimate taken as if dipole spindown were effective. The discussion and conclusions are given in § 4.

2. INFRARED AND OPTICAL RADIATION FROM THE DISK AROUND 4U 0142+61

Recent *Spitzer* observations of the anomalous X-ray pulsar 4U 0142+61 have detected the source in the 4.5 μm and 8 μm infrared bands (WCK06). We will show that taken together with the earlier observations in the optical (HvKK00; HvKK04; Morii et al. 2005), these detections are consistent with the presence of a gaseous disk around the neutron star. The optical detection must have a contribution from the magnetosphere, as implied by the strongly pulsed part of the optical signal, at 27 % amplitude in the R band, observed by Kern & Martin (2002). Ertan & Cheng (2004) showed that the strongly pulsed emission from the magnetosphere can be accounted for either with a dipole magnetar based magnetosphere with surface dipole field strength $B_{\rm dipole} \sim 10^{14} - 10^{15}$ G, or a magnetosphere with a disk in it, with $B_{\rm dipole} \sim 10^{12} - 10^{13}$ G. The disk-magnetosphere model can provide near 100 % pulsed magnetospheric emission in the optical. The presence of pulsed magnetospheric emission does not rule out the presence of a disk protruding into the magnetosphere. On the contrary, such a disk can be part of the magnetospheric circuit sustaining pulsed emission, as shown by Cheng & Ruderman (1991), and applied to 4U 0142+61 by Ertan & Cheng (2004). A pulsed component is likely to be present in the B, V, and IR bands. Uncertainties due to an unresolved pulsed fraction that is constant across the bands cannot be distinguished from uncertainties in other overall factors like distance and disk inclination angle. Possible variations in the relative pulsed fractions in different bands should be taken into account in evaluating the model fits. Further, an important uncertainty arises from the inner disk-magnetosphere interaction which might distort the inner disk geometry and thus possibly lead to time dependent self-shielding at some inner disk regions. Observations of various AXPs in some of the optical/IR bands at different epochs of the same source show considerable variations in relative amplitudes (Morii et al. 2005). These variations are not included in our or other scientists' model fits, and indeed, it is not feasible to model such effects in a plausible way. Keeping this in mind we try to obtain the best fit to optical and IR data points.

Figure 1 shows the best model fit to all data. Table 1 details the features of this fit. For each band of radiation, the effective blackbody temperature representing that band and the radius R in the disk corresponding to that effective temperature were listed. The last column gives the ratio of dissipation to irradiation powered disk fluxes at that radius. The total emission in each band is predominantly supplied by parts of the disk near the quoted radius, but of course there are lesser contributions from all parts of the disk. In the best fit displayed in Fig. 1 and Table 1, the the representative radius, 7×10^9 cm, which dominates the V band emission, draws about 30 % of its luminosity from viscous dissipation. The actual inner disk radius in this fit, $r_{\rm in} = 10^9$ cm, is almost an order of magnitude less than the V band representative radius. In the inner disk, the ratio of viscous dissipation to irradiation supported flux is therefore higher. The radiation from the innermost disk has a major contribution to the UV radiation bands. The innermost disk, from $r_{\rm in}$ to the V band representative radius also has important contributions to the radiation in the optical bands. Therefore, the fits are quite sensitive to the value of the inner disk radius. The same disk model accommodates the earlier optical and IR data (HvKK00; HvKK04; Morii et al. 2005), and the *Spitzer* detections of WCK06. The value of A_V indicated by the best model fit shown in Figure 1 is 3.5. This is within the reasonable dereddening range $2.6 < A_V < 5.1$ for the AXP 4U 0142+61 (HvKK00; HvKK04), and coincides, fortuitously, with the recently derived estimate of $A_V = 3.5 \pm 0.4$ (Durant & van Kerkwijk, 2006). For this model, all the model flux values remain within ~ 30 % of the data points. The error bars on the B-band data point give the 3σ upper limit (HvKK04). Considering the uncertainties discussed above, the irradiated gaseous disk model is in reasonable agreement with optical and IR data.

In the inner disk, viscous dissipation contributes significantly to the disk luminosity while in the outer and cooler parts of the disk the luminosity is determined by X-ray irradiation from the neutron star. Notably, the total disk luminosity of a gaseous disk, including both irradiation and the intrinsic dissipation, does not fall off in the optical range in the way expected for a dust disk as employed by WCK06 in their Figure 3. Indeed, a gas disk model can explain both the optical and the infrared data, as shown in Figure 1. The earlier conclusion that an accretion disk is ruled out (HvKK00) is no longer valid with the *Spitzer* observations. These authors ruled out an accretion disk because their optical magnitude, dereddened with $A_V = 5.4$, does not fall on the disk curve corresponding to the $L_{\rm X}$ as an accretion luminosity, with certain assumptions, a choice of $\dot{M}$, and a particular irradiation-reprocessing efficiency. Fitting the optical points to the Rayleigh-Jeans end of a disk blackbody, it was therefore required, on the basis of the (only optical and near IR) data existing then, that any disk in AXP 4U 0142+61 must be truncated at a certain outer radius $r_{\rm out}$. Now, with the new *Spitzer* observations of WCK06, the disk is actually detected in the far infrared! We can fit all the data, in optical and infrared bands B, V, R, I, J, H, K_S, 4.5 μm and 8 μm (Fig. 1). A gaseous disk emitting in this energy range extends from an inner disk radius $r_{\rm in} \simeq 1 \times 10^9$ cm to an outer radius about 10^{12} cm (see Table 1). The two mid infrared *Spitzer* data points put a lower limit for the outer disk radius around 10^{12} cm, but do not constrain the extension of the outer disk beyond 10^{12} cm. In our model, the outer disk radius is $r_{\rm out} = 2 \times 10^{12}$ cm. The quality of the fits does not change for even larger $r_{\rm out}$, since the contribution to the *Spitzer* bands at 4.5 μm and 8 μm from larger radii is not significant.

The model fits in the optical band are sensitive to the value of the disk's inner radius. This inner radius is taken to be the Alfvén radius,

$$r_{\rm in} \cong r_{\rm A} \cong f_1 \dot{M}^{-2/7} (GM_*)^{-1/7} \mu_*^{4/7}, \tag{1}$$

where M_* is the mass, and μ_* the dipole magnetic moment of the neutron star. The factor $f_1 \sim 0.5 - 1$ describes the uncertainty in the precise location of the inner radius. The mass inflow rate $\dot{M}$ in the disk is larger than or equal to the mass accretion rate derived from the X-ray luminosity,

$$\dot{M}_{\rm acc} \equiv \frac{L_{\rm X} R_*}{GM_*} \equiv f_2 \dot{M} \leq \dot{M}, \tag{2}$$

where R_* is the radius of the neutron star. As the neutron star is spinning down while accreting, it must be in the propeller regime, but close to rotational equilibrium with the disk, according to the fallback disk model (Alpar 2001). Some of the mass inflow through the disk may not be accreting under these circumstances. From the observed X-ray luminosity $L_{\rm X} \simeq 10^{35}$ erg s^{-1}, taking $R_* = 10^6$ cm and $M_* = 1.4\, M_\odot$, we obtain $\dot{M}_{\rm acc} \simeq 5 \times 10^{14}$ g s^{-1}, which is related to the mass inflow rate $\dot{M}$, depending on a choice of f_2. We first report model fits with $\dot{M} = \dot{M}_{\rm acc}$ ($f_2 = 1$) and discuss modifications of the results for f_1, $f_2 < 1$ at the end. For given $\dot{M}$, the inner radius depends on the strength of the dipole magnetic moment of the neutron star.

We calculate the radial effective temperature profile $T_{\rm eff}(r)$ considering both the disk surface flux due to the intrinsic viscous dissipation rate D of the disk

$$D = \frac{3}{8\pi} \frac{GM_* \dot{M}}{r^3} \tag{3}$$

(see e.g. Frank, King & Raine 2002) and the irradiation flux

$$F_{\rm irr} = C \frac{\dot{M}_{\rm acc} c^2}{4\pi r^2} \tag{4}$$

(Shakura & Sunyaev 1973), where the irradiation parameter C includes the effects of disk geometry, albedo of the disk surface, and the conversion efficiency of the mass accretion into X-rays. In our calculations, we take C to be constant along the disk and leave it as a free parameter. For comparison with data, we integrate the disk blackbody emission in each of the optical and IR observational bands (see Ertan & Çalışkan 2006, for details of calculations). The model energy flux values presented in Figures 1 and 2 are obtained with $C = 1 \times 10^{-4}$. While the irradiation geometry of the AXP disks might be somewhat different from those of LMXB disks (Ertan, Göğüş & Alpar 2006; Ertan & Çalışkan 2006), the values of C we find in our fits lie in the range ($\sim 10^{-4} - 10^{-3}$) expected from the LMXBs, in particular based on disk stability analyses of the soft X-ray transients (de Jong, van Paradijs & Augusteijn 1996; Dubus et al. 1999; Ertan & Alpar 2002). With C in this range, optical/IR data of the other three AXPs with available data can also be fit with disk models with irradiation and viscous dissipation (Ertan & Çalışkan 2006).

In Figures 1 and 2, we present two different model fits. Figure 1 shows the best fit, obtained with $A_V = 3.5$, with the dipole magnetic field at the neutron star surface set to be 10^{12} G on the equator (2×10^{12} G at the poles). After these fits were made, we learned of work on X-ray and optical extinction and reddening (Durant & van Kerkwijk 2006), reporting $A_V = 3.5 \pm 0.4$ for 4U 0142+61. The inner disk radius $r_{\rm in}$ for the model in Figure 1 is 10^9 cm. Model fits are comparable for $r_{\rm in}$ values up to a few 10^9 cm, with smaller $r_{\rm in}$ corresponding to larger A_V. The corotation radius for the AXP 4U 0142+61 is $r_{\rm co} \simeq 7 \times 10^8$ cm, which is near the inner disk radius we obtain from the best model fit. This implies that the system is indeed near rotational equilibrium. The system is in the propeller regime, as indicated by the spindown of the AXP. Nevertheless, most or all of the inflowing disk matter is being accreted onto the neutron star rather than being effectively propelled out of the system, on account of the closeness to equilibrium. This is consistent with our model fits which were obtained with the assumption that $f_2 = 1$.

The largest value of the dipole magnetic field allowed by model fits will correspond to the lowest value of A_V. For the direction of the AXP 0142+61, reasonable values of the dereddening A_V in the direction of the AXP 4U 0142+61 lie in the range $2.6 < A_V < 5.1$ (HvKK00; HvKK04). Figure 2 shows the model fit with the largest value of the magnetic dipole field. This was obtained by setting $A_V = 2.6$, the minimum value in the range considered, and allowing a 60 % discrepancy at the V band. Note that the discrepancy in the V-band is ~ 30 % for the best fit, with $A_V = 3.5$. The inner radius of the disk is

$r_{\rm in} = 8 \times 10^9$cm. The corresponding dipole magnetic field value is found to be 2×10^{13} G on the equator, (4×10^{13} G at the poles). The disk model fits preclude higher values of the dipole component of the magnetic field. A stronger dipole magnetic field, $B_* > 10^{14}$ G, would cut the disk at much larger radii, $> 10^{10}$ cm. This is not consistent with the optical (R, V, B) data.

In comparing dereddened fluxes with models for various values of $B_{\rm dipole}$ and A_V we find that the highest discrepancies always occur in the V band, the dereddened fluxes being always somewhat larger than the model predictions. In the B band, the observations have yielded only an upper limit. Predictions in the B band from models that fit the V, R and IR bands are consistent with this upper limit.

3. LIFETIME OF THE DISK AROUND 4U 0142+61

Can a fallback disk remain active, wielding a torque on the neutron star, at the age of 4U 0142+61? In this section, we shall explore the evolution of the disk under various models for the torque between the disk and the neutron star. The characteristic lifetime of a fallback disk around a propeller can be estimated as $\tau_d = M_d/\dot{M}$ where $\dot{M}$ is the mass inflow rate in the disk. For a steady configuration, the disk's mass loss rate is $\dot{M}_d = -\dot{M}$. The mass loss rate $\dot{M}_d$ will include the mass accretion rate $\dot{M}_{\rm acc}$ onto the neutron star, as well as $\dot{M}_{\rm out}$, the mass lost from the disk-neutron star system. In an effective propeller situation, $\dot{M}_{\rm out}$ may make up most of $\dot{M}_d$. As the outer radius of the disk is not constrained by the data, the disk mass cannot be calculated from the fits. Instead, we shall assume plausible values, consistent with the observed disk to order of magnitude. We then use torque models to estimate the mass inflow rate $\dot{M}$ through the disk from the observed spin-down rate $\dot{\Omega}_*$.

The long term spin history of a rapidly rotating magnetized neutron star depends on the types of torques acting on it. The net torque on a magnetized neutron star interacting with an accretion disk can be expressed in general as

$$N_* \equiv I_* \dot{\Omega}_* = j \dot{M} (G M_* r_{\rm in})^{1/2}, \tag{5}$$

where j is the non-dimensional torque as a function of the fastness parameter $\omega_* \equiv \Omega_*/\Omega_{\rm K}(r_{\rm in})$, Ω_* being the angular spin frequency of the neutron star, $r_{\rm in}$ the inner disk radius, $\Omega_{\rm K} = (GM_*/r^3)^{1/2}$ the Keplerian angular velocity, and I_* the moment of inertia of the neutron star. Using the observed value of $\dot{\Omega}_*$ in equation (5), we estimate the mass inflow rate as

$$\dot{M}_{17} \simeq 0.4\, \omega_*^{-1/3} |j|^{-1} P_*^{-7/3} I_{*45} \left(\frac{\dot{P}_*}{10^{-12}\,{\rm s\,s^{-1}}} \right) \left(\frac{M_*}{1.4\, M_\odot} \right)^{-2/3}, \tag{6}$$

where $\dot{M}_{17}$ is the mass inflow rate in units of 10^{17} g s^{-1}, I_{*45} is the neutron star moment of inertia in units of 10^{45} gm cm^2, and $P_* = 2\pi/\Omega_*$ is the spin period of the neutron star (Erkut & Alpar 2004).

In earlier epochs of its evolution, an AXP with a fallback disk is likely to be in the propeller phase, in which the neutron star is rotating rapidly with respect to the rotation rates in the inner boundary of the disk. Most or all of the mass supplied by the disk is lost from the system, rather than being accreted by the propeller-neutron star. This goes on until the neutron star has spun down to rotation rates close to corotation with the disk. When the star rotates only a little faster than the inner disk, a significant fraction of the mass lost by the disk may get accreted onto the neutron star, while at the same time the star is spinning down under torques applied by the disk.

The propeller type spin-down torque first considered by Illarionov & Sunyaev (1975) scales as $|N_1| \sim (\mu_*^2/r_{\rm A}^3)\,[\Omega_{\rm K}(r_{\rm A})/\Omega_*]$ (see Ghosh 1995). Depending on the spin rate of the neutron star as compared to the sound speed at the magnetospheric boundary, the scaling of the braking torques for the subsonic and supersonic propellers are given as $|N_2| \sim \mu_*^2\Omega_*^2/GM_*$ and $|N_3| \sim \mu_*^2\Omega_*\Omega_{\rm K}(r_{\rm A})/GM_*$, respectively (Davies & Pringle 1981; Ghosh 1995). In these models, the magnetospheric radius is given by the Alfvén radius (see eq. [1]). The spin-down torque for the subsonic propeller, N_2, is independent of $\dot{M}$ and will not be used in the present work. The torques N_1 and N_3 can be expressed as in equation (5) where $j = -\omega_*^q$ with $q = -1$ and 1 for N_1 and N_3, respectively. The braking torque N_4 considered by Wang & Robertson (1985) has the same form as N_3, but with a particular estimation for the inner radius of the disk in the propeller stage, i.e., $N_4 = -\omega_*\dot{M}(GM_*r_{\rm in})^{1/2}$, where $r_{\rm in} = 2^{-3/16}\Omega_*^{-3/8}\dot{M}^{-1/8}(2GM_*)^{1/8}\mu_*^{1/4}$. This scaling of the inner disk radius is simply that of the Alfvén radius when $r_{\rm in} = r_{\rm A} = r_{\rm co} \equiv (GM_*/\Omega_*^2)^{1/3}$, the co-rotation radius. All the braking torques give the same $\dot{M}$ when $\omega_* = 1$, i.e., $r_{\rm in} = r_{\rm co}$.

We now estimate the lifetime of a fallback disk around 4U 0142+61 using these different types of propeller torque for disks of mass $M_d = 10^{-6}M_\odot$ and $M_d = 10^{-5}M_\odot$. For each torque model, solving equation (6) with the implicit $\dot{M}$ and B_* dependence of ω_* and $j(\omega_*)$ gives the present value of the mass inflow rate $\dot{M}$. For $M_d = 10^{-6}\,M_\odot$, the inferred values for the lifetime of the disk around 4U 0142+61, corresponding to the propeller torques N_1, N_3, and N_4 can be written for $R_{*6} = 1$ and $M_* = 1.4\,M_\odot$ as $\tau_1 \simeq 1.0\times10^5$ yr $B_{*12}^{-4/9}$, $\tau_3 \simeq 4.8\times10^5$ yr $B_{*12}^{8/3}$, and $\tau_4 \simeq 1.8\times10^5$ yr $B_{*12}^{2/3}$, respectively, where B_{*12} is the surface dipole magnetic field strength on the poles of the neutron star in units of 10^{12} G. For $M_d = 10^{-5}\,M_\odot$, all age estimates are greater by a factor of 10. In the initial epoch of the neutron star, after a brief initial accretion phase, the propeller regime will start when $r_{\rm in} > r_{\rm co}$ and will prevail until the inner radius reaches the corotation radius again. As the inner radius of the disk $r_{\rm in}$ approaches $r_{\rm co}$, the propeller regime will allow accretion. For all torque models, the condition $r_{\rm in} = r_{\rm co}$ yields the same mass inflow rate

$$\dot{M} = \Omega_*^{7/3}(GM_*)^{-5/3}\mu_*^2 \tag{7}$$

which yields for $M_d = 10^{-6}\,M_\odot$ the critical age estimate

$$\tau_c \simeq 4.67 \times 10^4 \text{ yr } B_{*12}^{-2}. \tag{8}$$

As shown in Figure 3, τ_c is the minimum lifetime estimated for the fallback disk if the neutron star spins down under the action of propeller torques without accreting any matter onto its surface. In our model, the AXPs experience propeller spindown (with some accretion) when $r_{\rm in} \geq r_{\rm co}$. The characteristic age τ_i of the disk estimated by the propeller torques N_1, N_3, and N_4 are greater than τ_c, when $r_{\rm in} > r_{\rm co}$, depending on the value of B_*. Figure 3 shows that the present age estimate is larger than τ_c for any of the torques considered, if the magnetic dipole field strength on the poles is greater than $6\times\,10^{11}$ G. If the end of the propeller phase is taken to be when $r_{\rm in} = r_{\rm A} = r_{\rm co}$, as commonly assumed, and adopted by WCK06, then $B_{*12} \simeq 0.6$ is obtained with $\tau_c \simeq 1.3 \times 10^5$ yr $= \tau_1 = \tau_3 = \tau_4$ for $M_d = 10^{-6}\,M_\odot$ and $\tau_c \simeq 1.3 \times 10^6$ yr $= \tau_1 = \tau_3 = \tau_4$ for $M_d = 10^{-5}\,M_\odot$ (see Fig. 3).

In scenarios with these propeller torques N_1, N_3, and N_4, the AXP 4U 0142+61 must have spent most of its life in the propeller phase with little or no accretion, and must have entered its present phase of spindown with accretion more recently. In the past, all or most of the incoming disk matter is to be thrown out of the assumed system as envisaged in the original propeller scenario (Illarionov & Sunyaev 1975). While in the present stage, the gas disk is supplying the X-ray luminosity of 4U 0142+61 by accretion, in the past epochs, the X-ray luminosity was not supplied primarily by accretion, but by some other mechanism, like dissipation of the magnetic field in the neutron star surface, as in the magnetar models (Thompson & Duncan 1995) and -or by internal energy dissipation in the neutron star, as expected in neutron stars under torques, but without initial cooling or accretion as dominant sources of surface thermal luminosity (Alpar et al. 1984; Alpar 2001).

If, on the other hand, there is accretion onto the neutron star over some parts of its surface while a fraction of the infalling disk matter escapes from the stellar magnetosphere as outflow or wind (see Illarionov & Kompaneets 1990), the torque is given by

$$N_5 = -\dot{M}_{\rm out}\Omega_* r_{\rm in}^2 + \dot{M}_{\rm acc}\Omega_{\rm K}(r_{\rm in}) r_{\rm in}^2, \tag{9}$$

where $\dot{M}_{\rm out} = \dot{M} - \dot{M}_{\rm acc}$ by the conservation of mass with the mass accretion rate, $\dot{M}_{\rm acc} = L_{\rm X} R_*/GM_*$, in terms of the X-ray luminosity $L_{\rm X}$. We have defined the mass outflow rate $\dot{M}_{\rm out}$ and the accretion rate $\dot{M}_{\rm acc}$ as positive quantities. When the negative propeller torque described by the first term dominates the net torque N_5 on the neutron star is a spindown torque. Such a spin-down phase allows for explaining persistent stage L_X of AXPs and SGRs with partial accretion from a fallback disk, as we have employed here. The first term in equation (9) is the braking torque acting on the neutron star due to mass loss with $\dot{M}_{out} < 0$ (Ghosh 1995). The second term represents the angular momentum flux onto the neutron star due to accreting disk matter. If the matter accretion onto the neutron star has already started by switching off the effective propeller mechanism within the recent history of the AXP 4U 0142+61, we would expect to find a commensurate age and magnetic field strength estimation based on both N_5 and one of the propeller torques N_1, N_3, and N_4, whichever is the apt description of the actual torque effective in the system's past. In Figure 3, we plot the age and polar surface magnetic field strength estimations by N_1, N_3, N_4, and N_5. Note that the spin-down torque N_5 and the propeller torque N_1 yield a common field strength estimation ($B_* \sim 6 \times 10^{12}$ G) for the dipolar component of the magnetic field. Even for the smaller disk mass ($M_d = 10^{-6}\, M_\odot$), the torque models N_3 and N_4 give very large ages for $B_* > 10^{12}$ G. The torque N_5 is the most plausible torque model, because it incorporates accretion with spindown and yields reasonable ages together with the torque N_1 for $B_{\rm dipole} \sim 10^{12} - 10^{13}$ G.

Using observed parameters of 4U 0142+61 and the torque N_5, the mass inflow rate in the disk, for a given magnetic field strength, can be estimated together with the outflow rate $\dot{M}_{out}$ from Eq. (9). We find $\dot{M} \simeq 1.9 \times 10^{-11}\ M_\odot\ {\rm yr}^{-1}$ and $\dot{M}_{out} \simeq -3 \times 10^{-12}\ M_\odot\ {\rm yr}^{-1}$ for $B_* = 6 \times 10^{12}$ G and $\dot{M}_{acc} \simeq 1.6 \times 10^{-11}\ M_\odot\ {\rm yr}^{-1}$ ($\sim 10^{15}\ {\rm g\ s}^{-1}$). Note that most of the mass lost by the disk accretes onto the neutron star. This estimate means $f_2 = 0.85$ with $f_1 = 1$.

The upper limit set on the lifetime of the disk by employing N_5 is much smaller than the lifetime values estimated using the propeller torques N_3 and N_4. The spin-down age of the pulsar estimated with the dipole spindown formula, which of course can only offer a rough comparison with the present model, $P_*/2\dot{P}_* \sim 10^5$ yr, is comparable with τ_5, the inferred disk's lifetime corresponding to the spin-down torque N_5 in the accretion regime (see Fig. 3). This age estimate is also consistent with age estimates of the supernova remnants associated with the AXPs 1E 2259+586 and 1E 1841-045. We conclude that an active disk torquing down the star for an age of $\sim 10^5$ yrs is a tenable hypothesis, most likely with some accretion for a significant part of the history up to the present.

With the assumption that N_5 is a good description of the current torque, one can estimate the fraction f_2 of the mass inflow that is accreted, for a given value of the disk inner radius, without having prior knowledge of the mass inflow rate in the disk. Equation

(9) can be written in the form

$$\frac{\dot{M}_{\rm out}}{\dot{M}_{\rm acc}} = (\frac{r_{\rm co}}{r_{\rm in}})^{3/2} + \frac{I_*|\dot{\Omega}_*|}{\dot{M}_{\rm acc}\Omega_* r_{\rm in}^2}, \quad (10)$$

where we have substituted $N_5 = I_*|\dot{\Omega}_*|$. Using the observed spindown rate and the accretion rate inferred from the luminosity, Eq.(2), we find $f_2 = 0.6$ for $r_{\rm in} = 10^9$ cm, and $f_2 = 0.4$ for $r_{\rm in} = r_{\rm co} = 7 \times 10^8$ cm.

4. DISCUSSION AND CONCLUSIONS

Combining the earlier optical data (HvKK00; HvKK04, Morii et al. 2005) with the recent *Spitzer* infrared data (WCK06), we have shown that the full data set is consistent with the luminosity and spectrum expected from a gas disk. The optical radiation is powered to a significant extent by viscous dissipation in the inner regions of the disk while the IR radiation comes from outer regions where irradiation by the central source is the primary source of the disk's local luminosity.

What is the maximum dipole magnetic moment μ_* compatible with these disk model fits to the data? From equations (1) and (2), we obtain

$$\mu_* = f_1^{-7/4} f_2^{-1/2} r_{\rm in}^{7/4} \dot{M}_{\rm acc}^{1/2} (GM_*)^{1/4}. \quad (11)$$

Using $f_1 = 0.5$, $f_2 = 1.0$, and employing the maximum $r_{\rm in} \cong 8 \times 10^9$ cm compatible with our fits, for minimum $A_V = 2.6$, we find the maximum possible μ_* to be 5.8×10^{31} G cm^3 corresponding to a maximum surface dipole field $B_{\rm dipole,max} = 1.2 \times 10^{14}$ G on the poles of the neutron star, of 1.4 $M_\odot$ neutron star and radius $R_* = 10^6$ cm. Further increase of the upper limit for the magnetic field can be obtained for smaller f_2 values. The minimum f_2 that gives a fit within 60 % deviation of the V band is around 0.3 together with $r_{\rm in} \simeq 1.0 \times 10^{10}$ cm. For these extreme values of parameters, we obtain $B_* \simeq 3 \times 10^{14}$ G on the pole of the neutron star. These estimates should be noted together with the uncertainties due to disk-magnetosphere interaction, possible shielding effects and time dependent variations as discussed in Section 2.

We note that if the disk inner radius is large, $r_{\rm in} >> r_{\rm co}$, the neutron star is a fast rotator, far from rotational equilibrium with the disk, and likely to be in the strong propeller regime. If the simple description of propeller spindown given in Eq.(9) is roughly correct, as it is likely to be the case, the large moment arm provided by large $r_{\rm in}$, together with mass outflows at least comparable to (and probably larger than) the mass accretion rate onto the star will yield spindown rates much larger than the observed spindown rate:

$$|\dot{\Omega}_*| \sim \dot{M}_{\rm acc}\Omega_* r_{\rm in}^2. \tag{12}$$

A disk with $r_{\rm in} \simeq 1.0 \times 10^{10}$ cm, that allows for a magnetic dipole as large as $B_* \simeq 3 \times 10^{14}$ G requires a spindown rate in the range $|\dot{\Omega}_*| \sim 10^{-10}$ rad s^{-2}. This is much larger than the observed spindown rate $|\dot{\Omega}_*| \simeq 1.7 \times 10^{-13}$ rad s^{-2}. Here we have a strong indication against large $r_{\rm in}$ that can accommodate large dipole magnetic fields in the magnetar range.

As our disk models depend only on the mass inflow rate $\dot{M}$ inside the disk and the irradiation parameter C, it could be argued that the entire mass inflow rate actually turns into an outflow and none of the mass flowing through the disk is accreted onto the neutron star, the X-ray luminosity being due to some energy release mechanism other than accretion. The observation that the inner disk radius in our best fit is so close to the co-rotation radius argues against this. Indeed our fits yield the inner radius required by the optical observations. The actual inner radius may be even smaller and closer to $r_{\rm co}$, but we do not have the possibility of observing such a highly absorbed source in the UV band to check this. The rather small value of the spindown rate compared to what one would expect with a substantial mass outflow from a larger inner radius also indicates that this is a weak propeller, close to equilibrium. It is therefore likely that there is mass accretion.

Could the disk of our model fits be a passive disk, with no viscous dissipation, no mass inflow and no accretion at all onto the neutron star? Under this hypothesis the disk luminosity is generated entirely by irradiation by the neutron star's X-ray luminosity, which in turn is not due to accretion. It is unlikely that there is no accretion at all, if there is a gas disk with the properties indicated by our model fits, which are sensitive to the inner disk radius and require temperatures $> 10^5$ K in the inner disk. The disk cannot be neutral and passive at these temperatures; it has to be ionized, whether it is pure hydrogen, or with heavy elements. The ionized disk will have viscosity generated by the magneto-rotational instability (Balbus & Hawley 1991), which will result in mass inflow through the disk and interaction between the disk and the neutron star magnetosphere, and some mass accretion onto the neutron star, depending on the closeness to equilibrium. Ionization temperatures for hydrogen disks are of the order of 6000 K. For fallback disks with heavy metals (Werner, Nagel & Rauch 2006), ionization temperatures could be as low as ~ 2500 K. In the hydrogen disk simulations of soft X-ray transients and dwarf novae, two different viscosity parameters are employed above and below the ionization temperatures to account for the observations of these systems. In these models, the viscosity parameter decreases by a factor of 0.01 - 0.5 as the temperatures decrease to below the ionization temperature, $T_{\rm i}$. This indicates that even the disk regions with $T < T_{\rm i}$ remain active, and the critical temperature below which the entire disk enters a passive state is likely to be much lower than the ionization temperatures.

Early studies by Gammie (1996) on protoplanetary disks had suggested the possibility that a weakly ionized disk might become passive in its dense regions in the mid-plane while accretion proceeds over its surface. Recent work by Inutsuka & Sano (2005), however, has indicated the existence of various physical mechanisms that can provide, sustain, and homogenize a sufficient degree of ionization for the magnetorotational instability (MRI) to work through most regions of the protoplanetary disks even at temperatures as low as 300 K. According to Inutsuka & Sano (2005), once MHD turbulence starts to prevail in a disk, it seems quite hard to switch it off. The state of turbulent viscosity keeps the temperature and other parameters such that the ionization level required for MRI is self-sustained.

The presence of the disk imposes constraints only on the dipole component of the magnetic field. The very strong magnetic fields that are employed to explain SGR and AXP bursts in the magnetar model may well reside in the higher multipole structure of the neutron star surface magnetic field. In this hybrid situation, dissipation of the magnetar fields' energy in the neutron star crust may also contribute to the surface luminosity along with accretion. In this case $\dot{M}_{\rm acc}$ is less than the value inferred by taking the X-ray luminosity to be fully due to accretion, as in Eq. (2).

4U 0142+61 is the anomalous X-ray pulsar with the most detailed optical and IR observations. It is of great interest to check and extend the current data base with further optical and IR observations, in particular by *Spitzer* observations. It is important to realize that magnetar fields in higher multipoles, and a dipole field with $B_* \lesssim 10^{13}$ G to accomodate a disk with equilibrium period range of observed period clustering can coexist in the neutron star. The search for disks in other AXPs has also provided some data in the optical and near infrared that are consistent with gas disk models, as discussed by Ertan & Çalışkan (2006). Time dependent variations in the X-ray luminosity could be smeared out during the X-ray reprocessing at the outer disk. On the other hand, variations in the mass inflow rate modify the disk emission in the optical bands before the changes have been observed in X-rays due to varying accretion rates. Any observed signatures in the optical bands followed by similar variations in X-rays with viscous time delays (minutes to days) would be a clear indication of ongoing disk accretion onto the neutron star. At present, AXP 4U 0142+61 seems to be the best source to test this idea.

Finally we comment on the passive dust disk model proposed by Wang, Chakrabarty & Kaplan (2006). Their detection of 4U 0142+61 in the mid-IR band has firmly indicated, for the first time, the presence of a disk around an isolated neutron star. Wang, Chakrabarty & Kaplan (2006) have fitted the data with a two component model, using a power law to fit the optical and near IR data, which they ascribe to magnetospheric emission. This leaves only their recent discovery, the *Spitzer* detections in the mid-IR band, to be explained by a disk. Hence, while the evidence in that band clearly points at a disk, the association of only this narrow band emission with the disk constrains it to be a dust disk beyond the magnetosphere. Questions arising with this model concern why this disk did not generate some viscosity to extend into an active gaseous disk, how it cooled and remained cold and confined, and whether it is stable against the radiation pressure of the pulsar. With an inner disk radius $r_{\rm in} \sim 2r_{\rm lc}$ where $r_{\rm lc}$ is the light cylinder radius, such a passive dust disk could be stable against the radiation pressure of the magnetospheric pulsar emission (Ekşi & Alpar 2005). This stability is valid for $r_{\rm lc} \lesssim r_{\rm in} \lesssim 2r_{\rm lc}$ for orthogonal rotators and for even wider range for non-orthogonal rotators according to the investigation of Ekşi & Alpar (2005), the stability issue should be kept in mind for dust disks beyond the light cylinder.

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Table 1: Model temperatures and radii corresponding to different optical and IR bands. Last column shows the ratio of the viscous dissipation rate to the irradiation flux

Band	$T_{\rm BB}$ (K)	R (cm)	$D/F_{\rm irr}$
B (cm)	6516	7.0×10^9	0.45
V	5263	1.0×10^{10}	0.3
R	4454	1.4×10^{10}	0.30
I	3585	2.2×10^{10}	0.14
J	2377	4.8×10^{10}	0.07
H	1779	7.9×10^{10}	0.04
K_s	1324	1.4×10^{11}	0.02
4.5 μm	644	5.9×10^{11}	0.005
8 μm	362	1.9×10^{12}	0.002

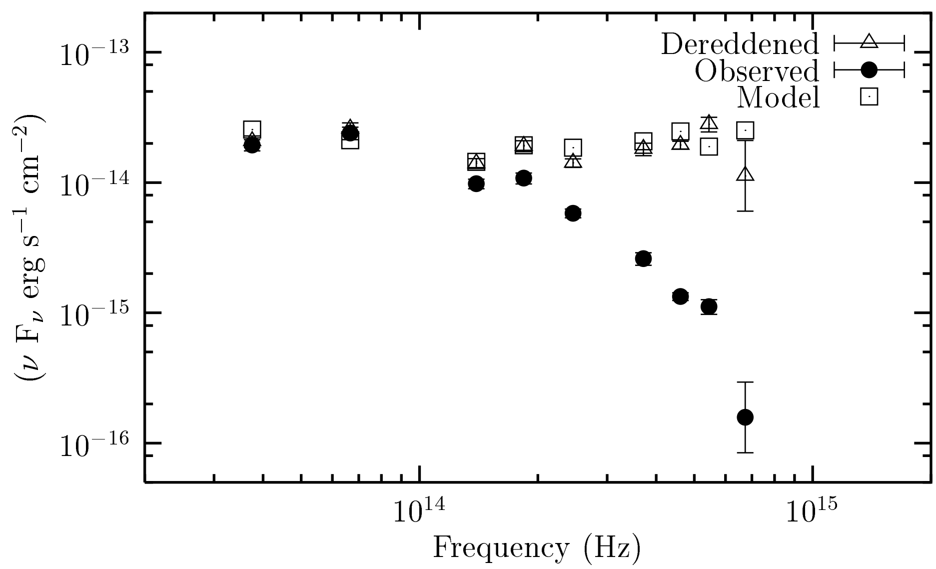

Fig. 1.— Energy flux data and irradiated disk model values for the AXP 4U 0142+61 in the optical and infrared bands (B, V, R, I, J, H, K_S, 4.5 μm and 8 μm). Filled circles are the observed (absorbed) data (taken from HvKK00 (V, R, I), HvKK04 (B,K_S), Morii et al. 2005 (H, J), Wang et al. 2006 (4.5 μm and 8 μm)(, and triangles are dereddened data with $A_V = 3.5$. Squares are the irradiated disk model energy flux values (see § 2 for details).

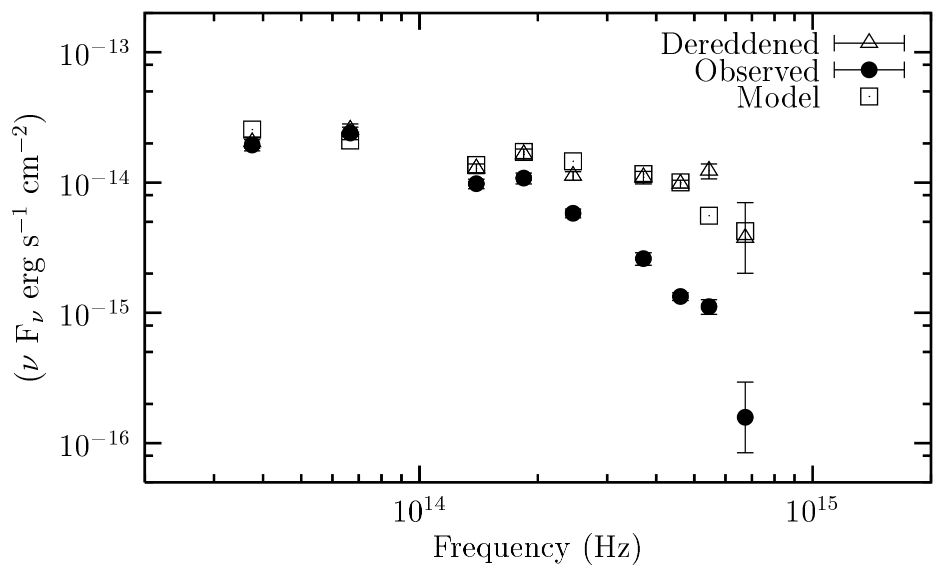

Fig. 2.— Energy flux data and irradiated disk model values for the AXP 4U 0142+61 in the optical and infrared bands (B, V, R, I, J, H, K_S, 4.5 μm and 8 μm, see Fig. 1 for references). Filled circles are the observed (absorbed) data , and triangles are dereddened data with $A_V = 2.6$. Squares are the irradiated disk model energy flux values (see § 2 for details).

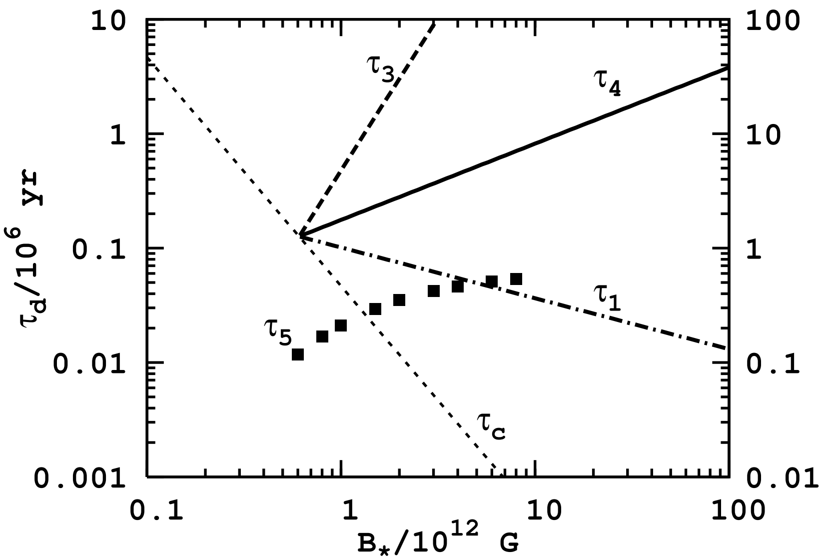

Fig. 3.— Disk lifetime as a function of the stellar magnetic field strength for different propeller torques. Squares show the disk lifetime values estimated by the spin-down torque N_5 in the accretion regime. The left-hand vertical axis shows estimated lifetime range for a disk of mass $M_d = 10^{-6}\, M_{\odot}$. The right-hand vertical axis is for $M_d = 10^{-5}\, M_{\odot}$.