Discrete Sliding Mode Control of Piezo Actuator in Nano-Scale Range

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Abstract- **In this paper Discrete Sliding Mode Control (SMC) of Piezo actuator is demonstrated in order to achieve a very high accuracy in Nano-scale with the desired dynamics. In spite of the fast dynamics of the Piezo actuator the problem of chattering is eliminated with the SMC control structure. The Piezo actuator suffers from hysteresis loop which is the inherent property and it gives rise to the dominant non-linearity in the system. The proposed SMC control structure has been proved to deliver chattering free motion along with the compensation of the non linearity present due to hysteresis in the system. To further enhance the accuracy of the closed loop system and to be invariant to changes in the plant parameters a robust disturbance observer is designed on SMC framework by taking into consideration the lumped nominal plant parameters. Experimental results for closed loop position are presented in order to verify the Nano-scale accuracy.**

I. INTRODUCTION

Piezoelectric actuator has been demonstrated with immense potential in application with requires accuracy within the range from sub micrometer to nanometer motion. The main advantage to use piezoelectric as an actuator is due to the fact that it does not possess any frictional or static characteristics which generally exist in other forms of actuators. The main characteristics of piezoelectric actuator are: high resolution in nanometer range, high bandwidth up to several kilo hertz range, a large force up to few tons and short travel range along with millimeter range [1].Some of the major areas where piezoelectric actuator can be used is in micromanipulation, force feedback as in AFM, micro-assembly and in dual stage hard disk drives. In all of these applications it is highly desired to have accurate positioning which can only be achieved using closed-loop control. Though some attempts have been made in the past ([2], [3]) to control piezoelectric actuator in open loop system with fine compensation of hysteresis non-linearity in the system. Due to the development of accurate positioning sensors it has become possible to use robust feedback based nonlinear control methods in order to eliminate the hysteresis effect.

Piezoelectric actuator can be driven by either voltage or charge. Generally voltage is used as a driving mechanism as it's easy to implement in the hardware and is the most widespread method for controlling these actuators. But it also incorporates the nonlinear behavior between the input (voltage) and output (position). This nonlinearity arises due to the parasitic hysteresis characteristics of the piezoelectric crystal. It has been illustrated in [2] that hysteresis behavior can be eliminated in case of driving by charge and a linear relationship is evolved between charge and position. However implementation of charge controllers is complicated job and voltage supply based driving is most preferred.

Hysteresis reduces the accuracy of the piezoelectric actuator in the range of 15-20%, thus the performance of the system is highly degraded and its compensation is extremely desirable. In the literature many model based approaches are proposed ([2], [3], [4], [5]). This approach incorporates the model of hysteresis for the compensation in open-loop but suffers from other internal disturbance such as creep and thermal drift. In [2] an accurate model of hysteresis is presented, but it is hard to implement in various range of control applications. In [3], [4] and [5] simpler model of hysteresis are proposed, however, those models does not represents the hysteresis behavior throughout the entire range of input voltage of the piezoelectric actuator. But still the use of hysteresis model is very useful where it's not possible to use a sensor for position feedback from the actuator.

In [5], H_∞ based closed-loop is proposed with model based hysteresis compensation. The model described was too complex in spite of good outcome but could be replaced by a simpler model. In [6], a neural-network based feed-forward assisted proportional integral derivative (PID) controller was proposed. In [7] variable structure control (VSC) for accurate positioning control in submicron ranges.

The main aim of this paper is to design a robust controller for Piezo-stage based on the assumption that the Piezo-stage can be modeled as a nominal linear lumped parameters (T_N, m_N, c_N, k_N) second order electromechanical system with voltage as the input and position as the output. The dominant nonlinear disturbance affecting is due to hysteresis arising from the Piezo. This paper tries to design a robust discrete sliding mode controller and disturbance rejection method to eliminate hysteresis in order to achieve high position accuracy in the nano-scale. The disturbance rejection is based on the concept of sliding mode observer which considers the total disturbance coming from the hysteresis and external force acting on the system. Thus the observer tries to estimate the lumped disturbance acting on the system.

II. MODEL OF THE PIEZO-STAGE

A fairly accurate model was chosen [2] for the Piezo-stage due to its easiness for implementation and accuracy for estimating the actual behavior of these actuators. The Piezo-stage consists of a Piezo-drive with a flexure guided structure which is designed to possess zero stiction and friction. Moreover the flexure stages exhibit high stiffness, high load capacity and insensitive to shock and vibration. Fig.1 describes the overall electromechanical model [2] of a PZT actuator.

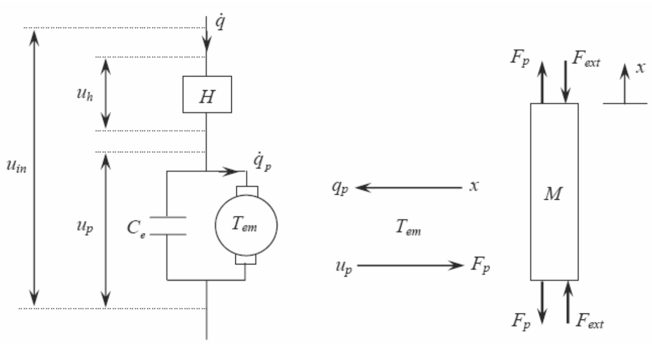

Fig. 1 Electromechanical model of a PZT actuator, [2]

The hysteresis and piezoelectric effects are separated. H Represents the hysteresis effect and u_h is the voltage due to this effect. The piezoelectric effect is represented by T_{em} which is an electromechanical transducer with transformer ratio T_{em} .The capacitance C_e represents the sum of the capacitances of the individual PZT wafers, which are electrically in parallel. The total current flowing through the circuit is $\dot{q}$. Furthermore, q may be seen as the total charge in the PZT actuator. The charge q_p is the transducer charge from the mechanical side. The voltage u_p is due to the Piezo effect. The total voltage over the PZT actuator is u_{in} , F_p is the transducer force from the electrical side, F_{ext} is the externally applied force, and the resulting elongation of the PZT actuator is denoted by x . The mechanical relation between Fp and x is denoted by M . Note that we have equal electrical and mechanical energy at the ports of interaction i.e. $u_p q_p = F_p x$.

The piezoelectric ceramic has elasticity modulus E , viscosity η , and mass density ρ . Furthermore, the geometric properties of the PZT actuator are length L and cross-sectional area A_p . Effective Mass m_p , Effective stiffness k_p and damping co-efficient c_p can be calculated as follows:

$$m_p = \rho A_p L \tag{2.1}$$

$$k_p = \frac{\rho A_p}{L} \tag{2.2}$$

$$c_p = \frac{\eta A_p}{L} \tag{2.3}$$

The complete electromechanical equations can be written as:

$$m_p \ddot{y} + c_p \dot{y} + k_p y = T_{em}(u_{in}(t) - H(y, u_{in})) - F_{ext} \tag{2.4}$$

Here y represents the displacement of the Piezo stage and $H(y,u_{in})$ denotes the non-linear hysteresis which is a function of y and u_{in} [2].

III. Design of Discrete SMC

The equation (2.4) is rewritten in state-space form (3.1) for easiness of the derivation of the controller structure.

$$\begin{aligned} \dot{x}_1 &= \dot{y} = x_2 \\ \dot{x}_2 &= \ddot{y} = -\frac{k_p}{m_p}x_1 - \frac{c_p}{m_p}x_2 + \frac{T_{em}}{m_p}u_{in} - \frac{T_{em}}{m_p}H - \frac{F_{ext}}{m_p} \end{aligned} \tag{3.1}$$

The equation (3.1) can be written in more general form as shown below

$$\dot{x} = f(x, H, F_{ext}) + Bu_{in} \tag{3.2}$$

The aim is to drive the states of the system into the set S defined by

$$S = \{x : G(x^r - x) = \sigma(x, x^r) = 0\} \tag{3.3}$$

Here $G = \{\lambda \ 1\}$ with λ being a positive constant, x is the state vector $x^T = \{x_1 \ x_2\}$, x^r is the reference vector $(x^r)^T = \{x_1^r \ x_2^r\}$ and $\sigma(x, x^r)$ is the function defining sliding mode manifold.

The derivation of the controller structure with the proper selection of the Lyapunov function $V(\sigma)$, and an appropriate form of the derivates of the Lyapunov function, $\dot{V}(\sigma)$.

For SISO system such as (3.1), required to have motion in manifold (3.3), natural selection of the Lyapunov function candidate seems in the form

$$V(\sigma) = \frac{\sigma^2}{2} \tag{3.4}$$

Hence, the derivative of the Lyapunov function is

$$\dot{V}(\sigma) = \sigma\dot{\sigma} \quad (3.5)$$

In order to guarantee the asymptotic stability of the solution $\sigma(x, x^r) = 0$, the derivates of the Lyapunov function may be selected to be

$$\dot{V}(\sigma) = -D\sigma^2 \quad (3.6)$$

Here D is a positive constant. Hence, if the control can be determined from (3.5) and (3.6), the asymptotic stability of the solution (3.3) will be guaranteed since $V(\sigma) > 0$, $V(0) = 0$ and $\dot{V}(\sigma) < 0$, $\dot{V}(0) = 0$ .By combining (3.5) and (3.6) the following equation can be deduced

$$\sigma(\dot{\sigma} + D\sigma) = 0 \quad (3.7)$$

A solution for (3.7) is as follows

$$(\dot{\sigma} + D\sigma) = 0 \quad (3.8)$$

The derivative of the sliding function is as follows

$$\dot{\sigma} = G(\dot{x}^r - \dot{x}) = G\dot{x}^r - G\dot{x} \quad (3.9)$$

From equation (3.9) and using (3.2)

$$\dot{\sigma} = G\dot{x}^r - Gf - GBu(t) = GB\left(u_{eq} - u(t)\right) \quad (3.10)$$

After inserting (3.10) into (3.8) and the result is solved for the control

$$u(t) = u_{eq} + (GB)^{-1}D\sigma \quad (3.11)$$

It can be seen from (3.01) that u_{eq} are difficult to calculate. Using the fact that u_{eq} is a continuous function, (3.10) can be rewritten in discrete form using Euler's approximation,

$$\frac{\sigma((k+1)T_s) - \sigma(kT_s)}{T_s} = GB\left(u_{eq}(kT_s) - u(kT_s)\right) \quad (3.12)$$

Here T_s is the sampling time and $k = Z^+$ .It is also necessary to write (3.11) in the discrete form which results in

$$u(kT_s) = u_{eq}(kT_s) + (GB)^{-1}D\sigma(kT_s) \quad (3.13)$$

If equation (3.12) is solved for the equivalent control, the following is obtained

$$u_{eq}(kT_s) = u(kT_s) + (GB)^{-1}\left(\frac{\sigma((k+1)T_s) - \sigma(kT_s)}{T_s}\right) \quad (3.14)$$

Since the system is casual, and it is required to avoid the calculation of the predicted value for σ as control cannot be dependent on future value of σ .Since the equivalent control is a continuous function, the current value of the equivalent control can be approximated with the single-step backward value calculated from (3.14) as

$$\hat{u}_{eq_k} \cong u_{eq_{k-1}} = u_{k-1} + (GB)^{-1}\left(\frac{\sigma_k - \sigma_{k-1}}{T_s}\right) \quad (3.15)$$

Here $\hat{u}_{eq_k}$ (or $\hat{u}_{eq}(kT_s)$) is the estimate of the current value of the equivalent control. After inserting (3.15) into (3.13) and resulting in control structure as

$$u_k = u_{k-1} + (GBT_s)^{-1}\left((DT_s + 1)\sigma_k - \sigma_{k-1}\right) \quad (3.16)$$

The control structure (3.16) is suitable for implementation, since it requires measurement of the sliding mode function and the value of the control applied in the preceding step. Thus (3.16) is used as control structure as discrete sliding mode for Piezo actuation.

IV. DISTURBANCE OBSERVER

The observer structure is deduced based on the equation (2.4) under the assumption that all the plant parameters uncertainties, nonlinearities and external disturbances can be represented as a lumped disturbance. It is assumed that y is the displacement and it's measurable and similarly $u(t)$ is the input and also a measurable quantity.

$$m_p\ddot{y} + c_p\dot{y} + k_p y = T_p u(t) - F_{dis}$$
$$F_{dis} = T_p H + \Delta T(v_{in} + v_h) + \Delta m\ddot{y} + \Delta c\dot{y} + \Delta ky \quad (4.1)$$

Here m_p , c_p , k_p and T_p are the nominal plant parameters while Δm , Δc , Δk and ΔT are the uncertainties associated with the plant parameters. Since y and $u(t)$ are measurable quantity, observer structure can be written in following form

$$m_p\ddot{\hat{y}} + c_p\dot{\hat{y}} + k_p\hat{y} = T_p u - T_p u_c \quad (4.2)$$

Here $\hat{y}$, $\dot{\hat{y}}$ and $\ddot{\hat{y}}$ are the position velocity and acceleration. u is plant control is the control input u_c is the observer control input as shown in Figure 2.

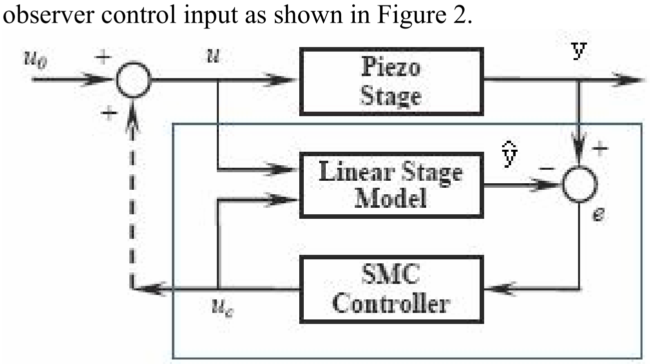

Fig.2 Observer Implementation

If estimated position $\hat{y}$ can be forced to track y then total disturbance feed to the system

$\sigma_{obs} = \lambda_{obs}(y-\hat{y})+(\dot{y}-\dot{\hat{y}})$ .Again the SMC structure is used for deriving the observer controller whose sliding manifold is defined as:

$$\sigma_{obs} = \lambda_{obs}(y-\hat{y})+(\dot{y}-\dot{\hat{y}}) \tag{4.3}$$

Here λ_{obs} is a positive constant. If σ_{obs} is forced to become zero then $\hat{y}$ should be forced to y . As described in the previous section from (3.8), with the same analogy it can be written as

$$\dot{\sigma}_{obs} + D_{obs}\sigma_{obs} = 0 \tag{4.4}$$

which guarantees $\sigma_{obs} \to 0$. By plugging the (4.3) into (4.4), the resulting equation is written as

$$\left(\ddot{y}-\ddot{\hat{y}}\right)+(\lambda_{obs}+D_{obs})\left(\dot{y}-\dot{\hat{y}}\right)+\lambda_{obs}D_{obs}\left(y-\hat{y}\right)=0 \tag{4.5}$$

It can be seen that the transient of the closed-loop system are defined by the roots $-\lambda_{obs}$ and $-D_{obs}$. The same structure of the controller will be used in the observer as described in (3.16). From structure (4.2) it can be seen that the input matrix is given by

$$B_{obs} = \begin{bmatrix} 0 & -\dfrac{T_p}{m_p} \end{bmatrix}^T \tag{4.6}$$

The matrix G for this case is defined as

$$G = \left[\lambda_{obs} \;\; 1\right] \tag{4.7}$$

Thus, after some simplification the controller structure can be written as

$$u_{c_k} = u_{c_{k-1}} - \frac{m_p}{T_p}\left(D_{obs}\lambda_{obs_k} + \frac{\sigma_{obs_k} - \sigma_{obs_{k-1}}}{T_s}\right) \tag{4.8}$$

Here u_c is the compensated control input to the system. The positive feedback by input u_c forces the system to behave closely towards the ideal system having the nominal parameters. But in reality there is also some amount of difference between the real disturbance and estimated disturbances.

V. SIMULATION RESULTS

The overall system shown in figure 3 is simulated in MATLAB Simulink in order to investigate the performance of the controller and observer in the accuracy of the Piezo model. The properties of the Piezo stage used in the simulation is defined in Table I is same with our experimental 3-axis Piezo stage.

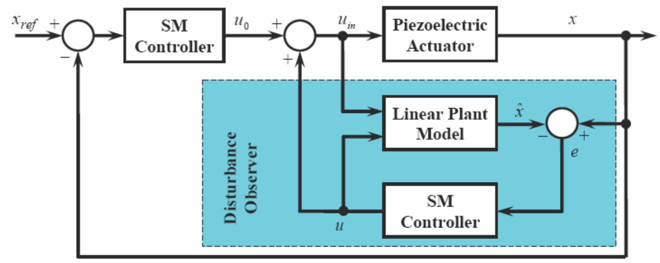

Fig. 3 Simulink Model of the overall system

TABLE I
PROPERTIES OF PIEZO-STAGE

Symbol	Quantity	Value in SI
m_p	Nominal Mass	$1.5\text{x}10^{-3}$ Kg
c_p	Nominal Damping	220 N-s/m
k_p	Nominal Stiffness	300000 N/m
f_r	Resonant Frequency	350 Hz
T_{em}	Transformation Ratio	0.3 N/V

As a reference to the system, step input was given to the system with different position in presence of a sinusoidal disturbance with low amplitude and results are shown below:

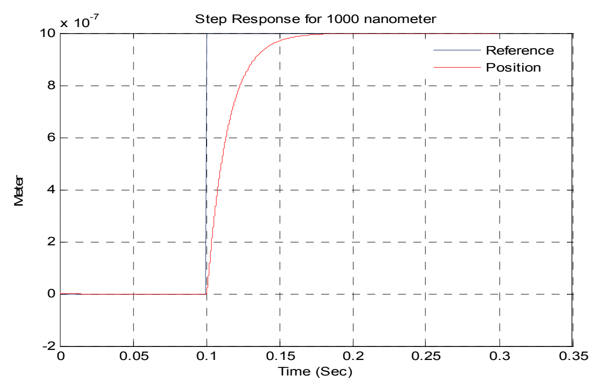

Fig. 4 Step response for 1000 Nanometer

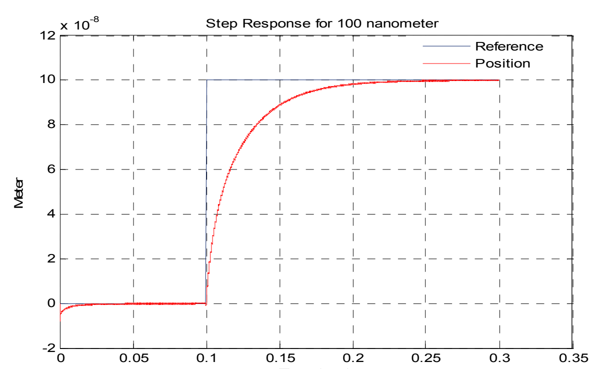

Fig. 5 Step response for 100 Nanometer

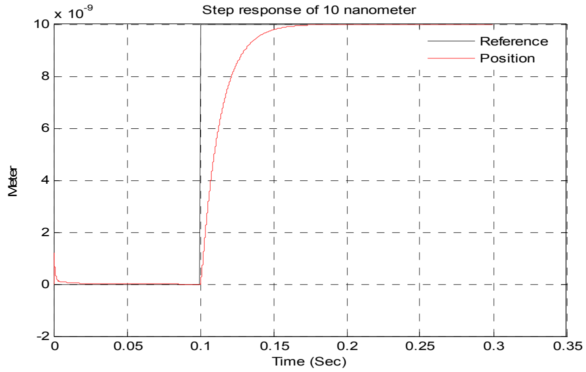

Fig. 6 Step response for 10 Nanometers

The fig.4-6 clearly indicates that in spite of sinusoidal disturbances present in the system, the controller is able to achieve the desired accuracy. Moreover our disturbance observer proves to be very effective in compensating the disturbance and the system is driven with nano accuracy.

VI. EXPERIMENTAL DESCRIPTION

In order to illustrate the prove the effectiveness of the proposed controller with the disturbance observer some experiments were carried out on a single axis of a 3-axis Piezo-stage manufactured by Physik Instrumente (PI) supplied by E-664 power amplifier. Table I shows the parameters of the Piezo-stage as also used for simulation purpose. As hardware to drive the Piezo-stage DSPACE DS1103 is used and coded in the C language using the libraries provided by the software.
The closed loop performance of the Piezo stage was investigated while using the overall structure shown in Figure 3 by applying the smooth step input of different position.

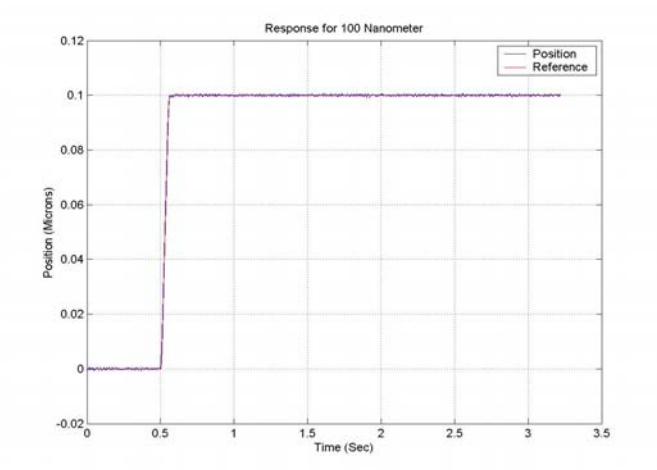

Fig. 7 Step response for 100 Nanometer

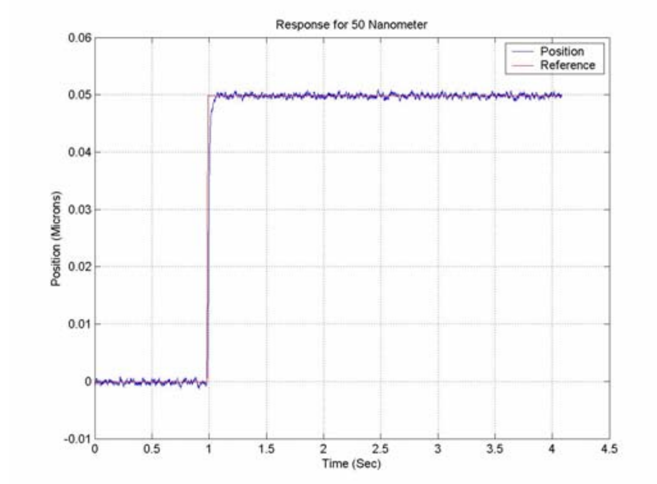

Fig. 8 Step responses for 50 Nanometer

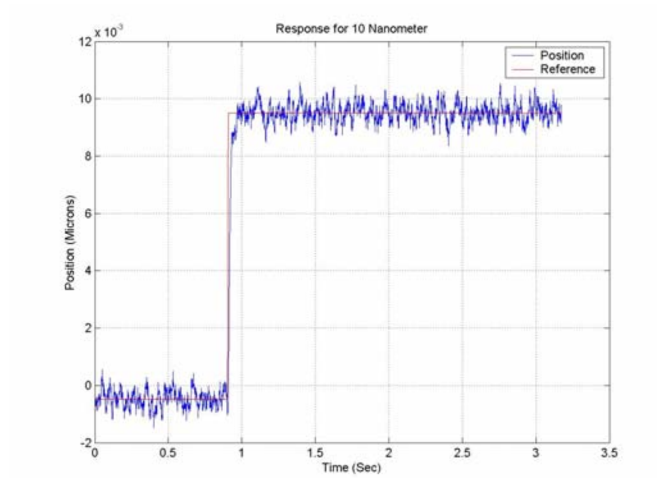

Fig. 9 Step response for 10 Nanometer

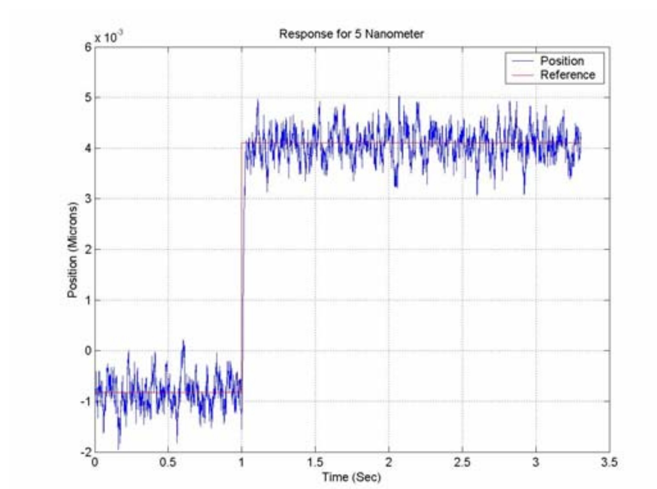

Fig. 10 Step responses for 5 Nanometer

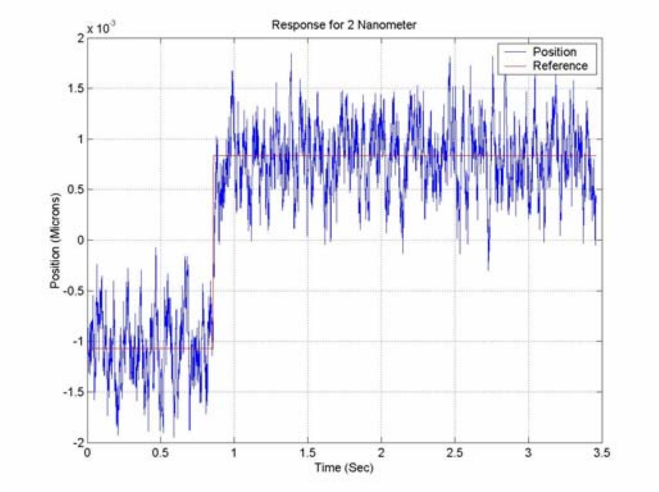

Fig. 11 Step responses for 2 Nanometer

As it can be seen from above figures 7, 8 and 9 that response for closed-loop performance of 100 nanometers, 50 nanometers and 10 nanometers respectively are able to achieve the desired position with fast rise time and almost zero steady state error. The results show that the proposed controller along with the disturbance observer produces good results.

In the case of figure 10 and 11 which also shows the response for closed-loop performance of 5 nanometers and 2 nanometers respectively. It can be seen clearly that the controller along with the disturbance is able to drive the system with desired accuracy but it's highly affected by the noise coming from the measurement device. Due to the influence of sensing noise, which belongs to high frequency range affects steady state position of the system and forces an oscillatory behavior with an maximum amplitude of 1-1.5 nanometer.

VII. CONLUSION

In the paper design of a Discrete Time Sliding Mode controller based on Lyapunov is presented. A robust disturbance observer based on Sliding Mode control is presented and applied to a Piezo-stage by considering all the nonlinearities present in the system as lumped disturbance. Linear model of a Piezo-stage was used with nominal parameters and used to compensate the disturbance acting on the system in order to achieve nano scale accuracy.

The effectiveness of the controller and disturbance observer was demonstrated in term of closed loop position performance in simulation and experiments. The results show the proposed controller structure produced good experimental results eliminating any chattering motion but in the range below 5 nanometers suffers from dominant sensing noise.

As a part of future work, the effort will be directed to reduce the effect of measurement noise to reach an accuracy of 1 nanometer with desired dynamics.

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