Sliding Mode Control Based Piezoelectric Actuator Control

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Abstract* - In this paper a method for piezoelectric stack actuator control is proposed. In addition a brief discussion about the usage of the same methods for estimation of external force acting to the actuator in contact with environment is made. The method uses sliding mode framework to design both the observer and the controller based on an electromechanical lumped model of the piezoelectric actuator. Furthermore, using a nonlinear differential equation the internal hysteresis disturbance is removed from the total disturbance in an attempt to estimate the external force acting on the actuator. It is then possible to use this external force estimate as a means of force control of the actuator. Simulation and experiments are compared for validating the disturbance and external force estimation technique. Some experiments that incorporate disturbance compensation in a closed-loop SMC control algorithm are also presented to prove the effectiveness of this method in producing high precision motion.

I. INTRODUCTION.

The use of piezoelectric actuators for accurate and stable control of manipulator position and/or force is greatly facilitated by model-based control system analysis and design. Inherent nonlinearities composed primarily of hysteresis in piezoelectric actuators pose as an obstacle to these objectives. By far, open-loop techniques have not been successful in providing good results due to the difficulties involved in modeling the actuator precisely. In [3], disturbance compensation based on a hysteresis model was used, however, unmodeled disturbances required the addition of a robust controller such as H_∞.

Hysteresis is an inherent non-linearity in all piezoelectric actuators. This hysteresis non-linearity is usually 15-20% of the output thereby greatly reducing the performance of the actuators. In [4] and [5] models were made based on the physics of the actuators and these models proved to be effective in modeling the behavior of these actuators under different excitations. Additionally, the models in [4] and [5] define the hysteresis behavior as existent in the electrical domain of the actuator and is between voltage and charge. In [5], a simple differential equation was used to model the voltage-charge hysteresis behavior. This model proved simple to implement in real-time applications due to the simplicity of the equation representing the hysteresis.

A sliding mode observer based disturbance compensation method is presented here. Disturbances acting on the plant are estimated by means of a simple second order system after the compensation. Furthermore, using this concept an attempt is made to estimate the external force acting on the actuator using the hysteresis model presented in [5] for purposes of force control.

II. ELECTROMECHANICAL MODEL

A. Description of the Overall Model

The model used in this work is described in [4]. The model proved to be a fairly accurate representation of the electromechanical behavior of the piezoelectric actuator. It is described schematically in Fig. 1.

$$v_p = v - v_h \tag{1}$$

$$v_h = H(q) \tag{2}$$

$$q = Cv_p + q_p \tag{3}$$

$$q_p = Tx \tag{4}$$

$$F_p = Tv_p \tag{5}$$

$$m_p\ddot{x} + c_p\dot{x} + k_px = F_p - F_{ext} \tag{6}$$

Meanings of the terms defined in equations (1) to (6) are as follows: v stands for total voltage across the piezoelectric actuator, v_p stands for voltage due to the piezoelectric effect, v_h stands for voltage due to the hysteresis effect, H is a hysteresis function, T stand for electromechanical transformation ratio, q stands for total charge in the piezoelectric actuator, q_p stands for charge transduced due to mechanical motion, F_p stands for force due to piezoelectric effect, F_{ext} stands for external forces acting on the actuator and m_p, c_p, k_p stand for equivalent mass, damping and stiffness.

The electromechanical lumped model of the piezoelectric actuator can be defined mathematically by equations (1) to (6) given below, [4].

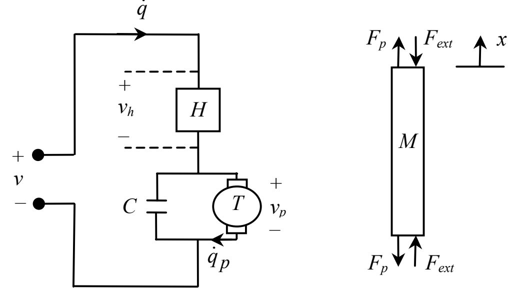

Fig. 1. Electromechanical model of the actuator

B. Description of the Hysteresis Model

The hysteresis between voltage and charge is modeled using a first-order differential equation proposed in [5] and [7]. In [7], it has been experimentally verified that this differential equation is suitable for describing electric hysteresis such as that in piezoelectric actuators. The model for the hysteresis effect is given by

$$\dot{q} = \alpha|\dot{v}_h|(av_h - q) + b\dot{v}_h \tag{7}$$

Here: $a = q_c / v_{h,c}$ is a constant found from loop center point, $b = q_{\max} - q_{\min} / 2A$ is average slope of the hysteresis loop and $\varepsilon = 4(a-b)\alpha A^3/3$ is loop area for small sinusoidal inputs from which α can be found.

III. DISTURBANCE AND FORCE OBSERVRE

A. Disturbance Observer

The structure of the observer is based on (8) and it is proposed that all the plant parameter uncertainties, nonlinearities and external disturbances can be represented as a single disturbance. As it is obvious, x is the displacement of the plant and is measurable. Likewise, v_{in} is the input to the plant and is also measurable. Hence, the nominal structure of the plant is defined as follows

$$\begin{aligned} m_N\ddot{x} + c_N\dot{x} + k_N x &= T_N v_{in} - F_d \\ F_d &= T_N v_h + F_{ext} + \Delta T(v_{in} + v_h) + \Delta m\ddot{x} + \Delta c\dot{x} + \Delta kx \end{aligned} \tag{8}$$

Here m_N, c_N, k_N and T_N are the nominal plant parameters while Δm, Δc, Δk and ΔT are the uncertainties of the plant parameters. Since x and v_{in} are measured the proposed observer is then of the following form

$$m_N\ddot{\hat{x}} + c_N\dot{\hat{x}} + k_N\hat{x} = T_N v_{in} - T_N u_c \tag{9}$$

Here $\hat{x}$ is the estimated position v_{in} is the plant control input and u_c is the observer control input. If $\hat{x}$ can be forced to track x then obviously $F_d = T_N u_c$. The observer controller that will be used is in the SMC framework. Let the sliding manifold be as $\sigma = (\dot{x} - \dot{\hat{x}}) + C(x - \hat{x})$. Also selecting the Lyapunov function as $v_L = \sigma^2/2$ and selecting the derivative of the Lyapunov function as $-D\sigma^2$ with $D > 0$. Equating the above results and simplifying

$$\dot{v}_L = \sigma\dot{\sigma} = -D\sigma^2 \Rightarrow \dot{\sigma} + D\sigma|_{\sigma\neq 0} = 0 \tag{10}$$

If we plug $\sigma = (\dot{x} - \dot{\hat{x}}) + C(x - \hat{x})$ in (10) and simplify we get

$$(\ddot{x} - \ddot{\hat{x}}) + (C + D)(\dot{x} - \dot{\hat{x}}) + CD(x - \hat{x}) = 0 \tag{11}$$

Here the transients of the closed-loop control system are defined by the roots $-C$ and $-D$. If we subtract (9) from (8) and plug the result into (11), we get

$$u_{eq} = \frac{1}{T_N}\left\{F_d + [c_N - m_N(C + D)](\dot{x} - \dot{\hat{x}}) + [k_N - m_N CD](x - \hat{x})\right\} \tag{12}$$

Where u_{eq} is the control that will keep system motion in manifold $\sigma = (\dot{x} - \dot{\hat{x}}) + C(x - \hat{x}) = 0$. From (12), it can be seen that as $\sigma \to 0$ then $\hat{x} \to x$ and $T_N u_{eq} \to F_d$. One can easily find that the control to satisfy the condition $\sigma(\dot{\sigma} + D\sigma) = 0$ when $\sigma \neq 0$ is given by (13)

$$u_c = u_{eq} + D\,\mathrm{sgn}(\sigma) \tag{13}$$

For discrete-time applications the following control is used

$$u_{(k)} = u_{(k-1)} + K_u\left(D\sigma_{(k)} + \frac{\sigma_{(k)} - \sigma_{(k-1)}}{T_s}\right) \tag{14}$$

here K_u is a design parameter which can be tuned to optimize the controller and T_s is the sampling interval of the discrete-time control. The observer implementation is best described by the block diagram of Fig. 2.

From here, if $\sigma \to 0$ then $\hat{x} \to x$ and $T_N u_c \to F_d$. The positive feedback of u_c cancels all the disturbances acting on the plant and (9) is reduced to a nominal plant (15) for which the design of the feedback controller that assure the stable transients may follow the same steps as design of the observer controller.

$$m_N\ddot{x} + c_N\dot{x} + k_N x = T_N v_{in} \tag{15}$$

By selecting the sliding manifold as $\sigma_x = (\dot{x}^{ref} - \dot{x}) + C_o(x^{ref} - x)$. Following the same steps as for observer design one can easily find that the controlled system is described by

$$(\ddot{x}^{ref} - \ddot{x}) + (C_o + D_o)(\dot{x}^{ref} - \dot{x}) + C_o D_o(x^{ref} - x) = 0 \tag{16}$$

and the control input has a form as in (14) with $\sigma = \sigma_x$.

B. Force Observer

From the structures (5) and (6) defined previously, the mechanical side of the actuator can be written as

$$m_N\ddot{x} + c_N\dot{x} + k_N x = T_N v_p - F_{ext} \tag{17}$$

Note that any parameter uncertainties are neglected with the assumption that the nominal plant parameters are as precise as possible. Hence, a disturbance observer based on the complete model of the actuator that includes hysteresis estimates the external force only. Based on the same principles defined for the total disturbance observer an observer based on the non-linear model of the actuator is constructed as follows

$$m_N\ddot{\hat{x}} + c_N\dot{\hat{x}} + k_N\hat{x} = T_N v_p - u_{force} \tag{18}$$

As before, if $\hat{x}$ is forced to track x then $F_{ext} = u_{force} = \hat{F}_{ext}$. Note that v_p is not measured directly, but, is computed from x and v_{in} using equations (1) to (4). Again the controller used will be in the SMC framework. Using the sliding manifold $\sigma_{est} = (\dot{x} - \dot{\hat{x}}) + C_{est}(x - \hat{x})$ and since this is a discrete-time application the control described by (19) is used.

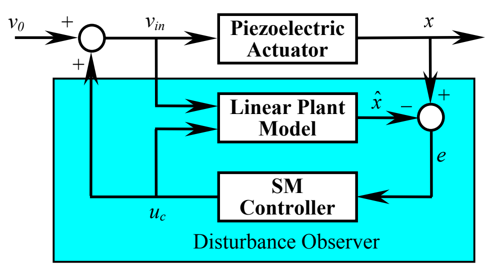

Fig. 2. Observer implementation

$$u_{(k)} = u_{(k-1)} + K_{est}\left(D_{est}\sigma_{est(k)} + \frac{\sigma_{est(k)} - \sigma_{est(k-1)}}{T_s} \right) \quad (19)$$

The observer implementation is depicted in figure 3. The results of the force observer are shown in section IV.

IV. EXPERIMENTAL RESULTS

The experimental setup consists of a PSt150/5/60 stack actuator ($x_{\max} = 60\ \mu m$, $F_{\max} = 800\ N$, $v_{\max} = 150\ Volt$) produced by Piezomechanik connected to SVR150/3 low-voltage, low-power amplifier. The piezoelectric actuator has built-in strain-gages for position measurement. Force measurement is accomplished by the help of a load cell that is placed against the actuator as shown in Fig. 4.

The entire setup is connected DS1103 module hosted in a PC with dSpace software Control Desk v.2.0. In Fig. 5 a simplified structure of the experimental setup is shown.

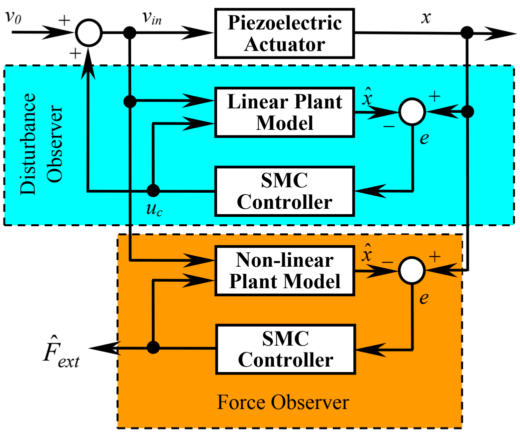

Fig. 3. Force observer implementation

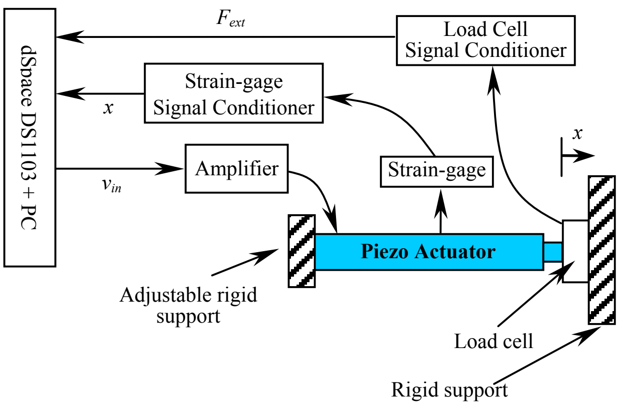

Fig.4. Simplified sketch of the experimental setup

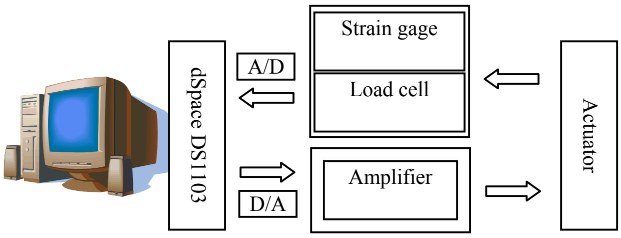

Fig. 5. Simplified structure of the experimental system

A. Results with the Disturbance Observer

Experiments were carried out with the disturbance observer in an attempt to test its capacity of estimating the disturbances acting on the system. Fig. 6 shows the measured and estimated position (left) while Fig. 6 (right) shows the estimation error for a sinusoidal voltage input. Both figures show that the observer position is able to track the measured actuator position nicely. The tracking error is 2nm and could be improved in the SMC framework. The results shown in Fig. 6 to Fig. 10 have been filtered due to the large noise in the measurements.

In Fig. 7a, the response of the actuator with disturbance compensation is compared to the response without compensation as well as the response from the linear plant model. In Fig. 7a, x_{ideal} represents the response of the linear plant model (15) for the same input. In Fig. 7b the error for the system with compensation is depicted.

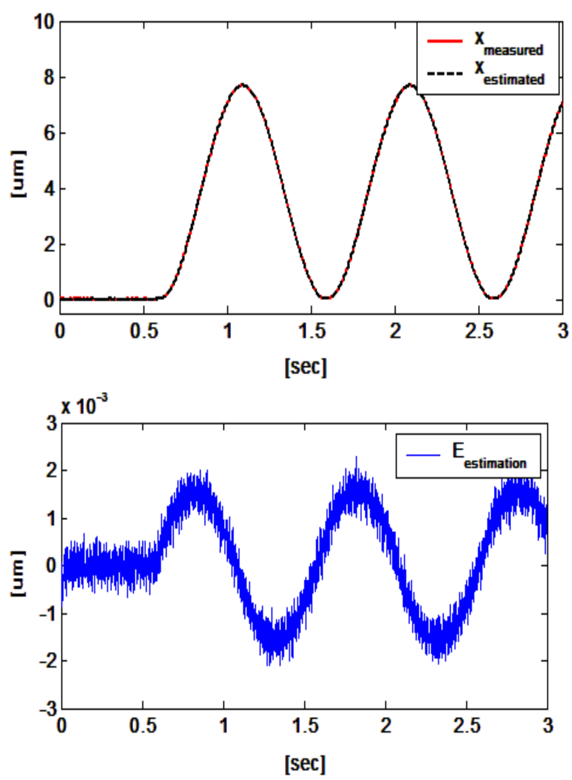

Fig. 6. Measured and estimated position

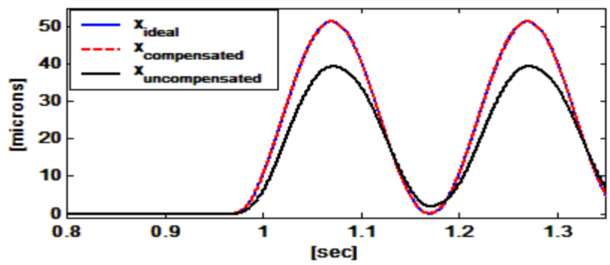

Fig. 7a. Response to a harmonic voltage input

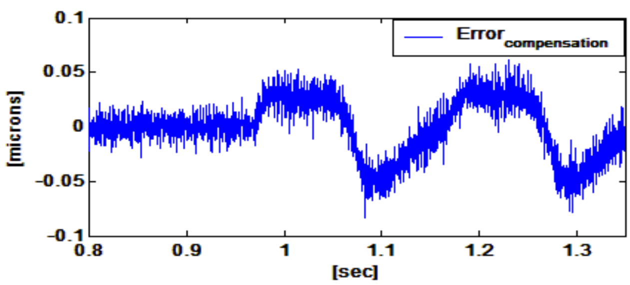

Fig. 7b. Compensation error for a harmonic voltage input

B. Results with Closed-Loop Control

The disturbance compensation scheme was incorporated with closed-loop control algorithm using SMC, defined by (14), as depicted by Fig. 8. As it can be seen from the results in Fig. 9a and Fig. 9b, the use of closed-loop control with disturbance compensation gives good results. The results that are shown in Fig. 9a and Fig. 9b are for a reference trajectory of the form $x_{ref} = 11 + 11\sin(2\pi t)$.

C. Results with Hysteresis Estimation

Using the model for the hysteresis described earlier, experiments were conducted to verify the model with the experimentally estimated hysteresis. Hysteresis voltage was estimated by assuming that in the case of no external forces acting on the system the disturbance estimated by the observer is the hysteresis disturbance only. As it can be seen from both Fig. 11a there is a delay in the estimation for which the error is shown in Fig. 11b. One reason for the discrepancy in estimation could be the assumption that the uncertainties in plant parameters are negligible in comparison to the hysteresis disturbance. This discrepancy in the estimation should show itself nicely in the external force estimation.

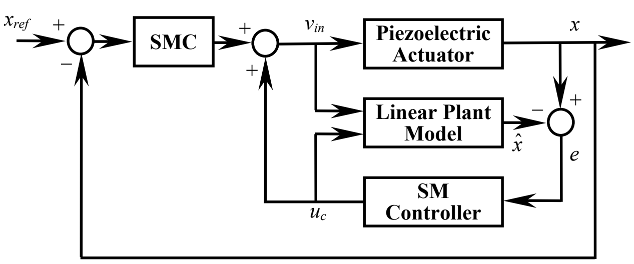

Fig. 8. Closed-loop control scheme

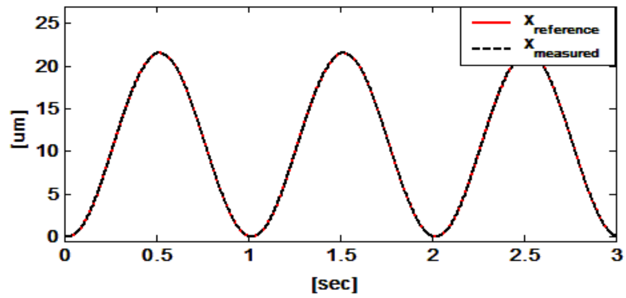

Fig. 9a. Reference and measured position

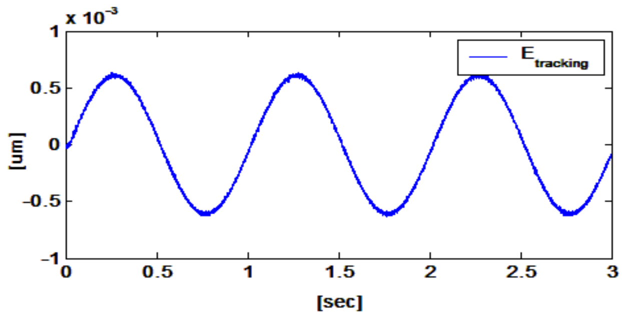

Fig. 9b. Closed-loop tracking error (0.6nm)

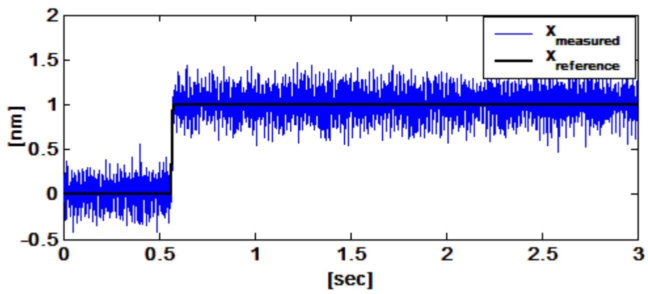

Fig. 10. 1nm step motion

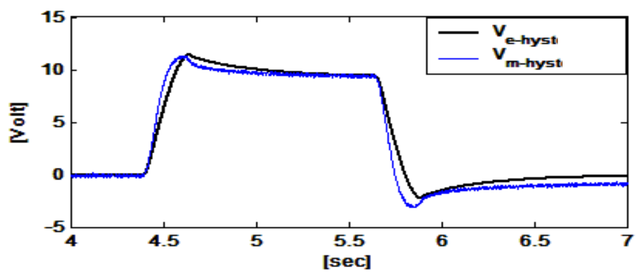

Fig. 11a. Estimated and measured hysteresis voltage; $V_{e\text{-}hyst}$ and $V_{m\text{-}hyst}$ respectively

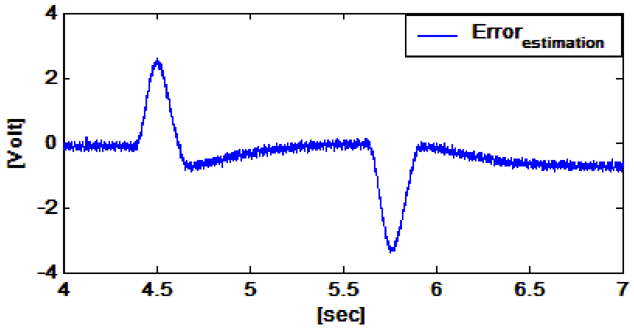

Fig. 11b. Error in hysteresis estimation

D. Results with the Force Observer

The external force estimation method is applied experimentally. The results in Fig. 12a and Fig. 12b show that the method works nicely for a smooth sinusoidal force. However, the discrepancy in the hysteresis estimation is also seen here Fig. 12a and Fig. 12b. Due to the lag in the hysteresis estimate there is a lead in the force estimation.

If a different external force such as a trapezoidal force is used, the estimation suffers more due to the dynamics in the hysteresis model as it was seen previously in Fig. 10a and as it can be seen below in Fig. 13a. It is most certain that any improvements in the hysteresis estimation should improve the estimation of the external force acting on the actuator.

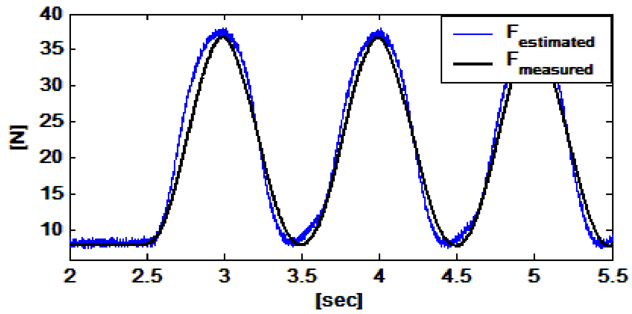

Fig. 12a. Experimental estimation of force

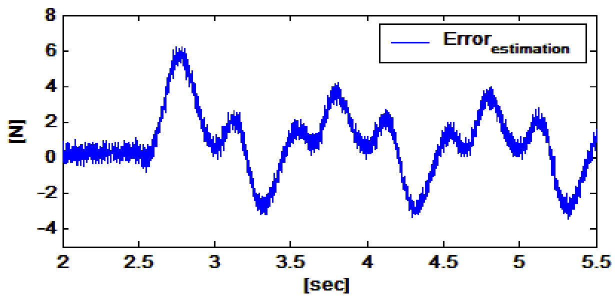

Fig. 12b. Estimation error of harmonic force

V. CONCLUSIONS

An observer based disturbance compensation technique is presented here. The observer was in the SMC framework

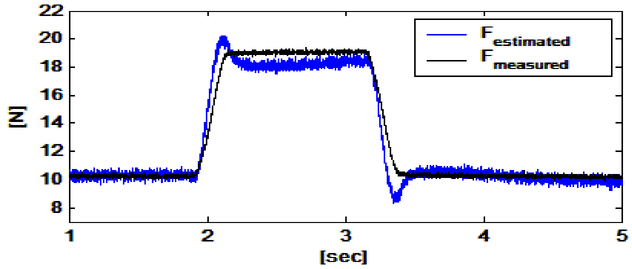

Fig. 13a. Experimental estimation of a trapezoidal force

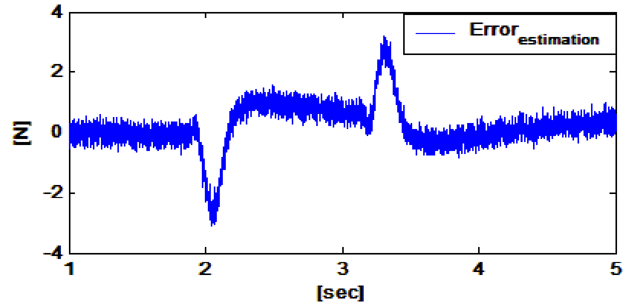

Fig. 13b. Estimation error of a trapezoidal force

and was based on a lumped electromechanical model of the piezoelectric actuator. The observer proved successful in compensating all the disturbances acting on the actuator. Addition of the disturbance compensation to a closed-loop control scheme provided good results and should open the way for high precision tracking with piezoelectric actuators. Inclusion of the hysteresis term in the plant model allowed the construction of a force observer based on the same principles of the disturbance observer. Work is currently in progress to improve the hysteresis model so that a more accurate estimation of external force is possible.

VI. REFERENCES

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