Energy Efficient Random Sleep-Awake Schedule Design

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Abstract— This letter presents a simple model for determining energy efficient random sleep-awake schedules. Random sleep-awake schedules are more appropriate for sensor networks, where the time of occurrence of an event being monitored, e.g., the detection of an intruder, is unknown a priori, and the coordination among nodes is costly. We model the random sleep-awake schedule as a two state Markov process, and maximize the probability of the transmission of sensed data by a given deadline. Our results indicate that for a given duty cycle, the optimal policy is to have infrequent transitions between sleep and awake modes, if the average number of packets sent is greater than the mean number of slots the node is awake.

Index Terms— Sensor networks, battery management, energy efficient protocols.

I. INTRODUCTION

ONE of the most important research issues in wireless networks is to extend the network lifetime by energy efficient battery management. In a wireless device, most of the energy is consumed when the device is idle, i.e., when there is no processing or transceiver operation. In order to minimize the energy consumption during idle periods, the device is put into low-voltage "sleep" mode. Duty cycle refers to the proportion of time during which a device is active or "awake". In order to preserve the available energy, wireless devices are operated at low duty cycles. Although significant amount of energy can be saved while operating at low duty cycle, the device becomes effectively unavailable and unresponsive to its surrounding during the sleep mode. For example, in sensor networks the nodes have the task to detect and report the ongoing events in their vicinity, e.g., an intruder. However, the information regarding the intruder is only useful if delivered in a timely fashion. When a node is in sleep mode, it cannot perform its tasks, and may miss the event or delay its report.

In this letter, we investigate the computation of the best sleep-awake schedule for a given duty cycle that ensures a certain level of responsiveness of a node. The responsiveness of a node is defined as the proportion of time the node is available (awake) in a certain length of period that has an arbitrary starting point in time. We especially focus on the design of random sleep-awake schedules for which the durations of sleep

and awake periods are not constant but determined in a random fashion. Random sleep-awake schedules are more appropriate in such networks, where it is difficult to attain coordination among the nodes or in sensor networks where the event monitored occurs randomly in time. There has been some previous work addressing the issues arising in such networks. For example, [2] offers a new topology management scheme called STEM, which allows designers to trade off between different QoS metrics including energy, latency, and density. [3] explores the relationship between the added deployment redundancy and the amount of reduction in proportion of awake times for both coordinated and uncoordinated sleep schedules. Chiasserini et. al. in [4] investigates the energy consumption and latency by modeling dynamics of a sensor network with Markov model. [5] uses a Markov model with six states in order to predict the energy consumption of a sensor node and consequently construct the energy map of the sensor network.

II. SYSTEM MODEL

We model a wireless node operating in one of two modes: ON and OFF states referring to active and idle modes of operation, respectively. We define sleep-awake schedule as the consecutive periods of ON and OFF states. Let T be the length of the sleep-awake schedule, and T_{ON} and T_{OFF} refer to the total period during which the node is in ON and OFF states respectively during this schedule. Note that $T = T_{ON} + T_{OFF}$. The duty cycle for a node, d, is defined as $d = T_{ON}/T$. Time is divided into equal length intervals called slots, where slot duration is equal to the transmission time of a single packet.

Assume that a node has a battery lifetime of L_b slots, when it is always in ON state. If the node has a duty cycle, d, then the lifetime of the node, L, is extended to $L = L_b/(1 - d)$ slots. If $T < L$, then the sleep-awake schedule is repeated until battery is exhausted. For a given battery lifetime, L_b slots, one can find different sleep-awake schedules with the same duty cycle, d. For example, a node with 50% duty cycle and a battery lifetime of $L_b = 8$ slots may have sleep/awake schedules with different lengths of ON periods, e.g., $T_{ON} = 1, 2, 4$, as shown in Fig. 1. Also note that different T_{ON} periods also show how frequent the node switches between sleep and awake modes.

The user/application requirements are defined in terms of the number of slots the node is available, K, (as a relay node in wireless ad hoc networks or as a sensor node in sensor networks) during any N consecutive slots. Note that such a requirement specifies the responsiveness of the network. Specifically, in sensor networks the data generated by a sensor
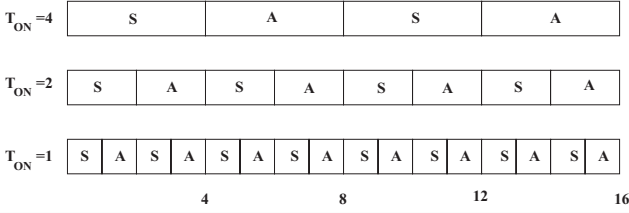
Fig. 1. Sleep/awake schedules for a node with 50% duty cycle and $L_b = 8$.

node is useful only a certain period of time after which the data becomes outdated. As an illustrative example, assume that the sensor node generates on the average two packets every time it observes an event, i.e., $K = 2$. This data should be sent to a central monitor, say, in four slots in order to be useful, i.e., $N = 4$. Also assume that the sensor node has 50% duty cycle. Then, among the deterministic sleep-awake schedules shown in Fig. 1, it is clear that sleep-awake schedules with $T_{ON} = 1$ and $T_{ON} = 2$ are the only ones satisfying this requirement.

With random schedules, we may not always be able to find a schedule satisfying such a requirement. Thus, instead we aim to maximize the proportion of time the user request is satisfied. Define, $P_{success}$, as the probability of the node being available (in ON state) for at least K slots in N consecutive slots. We model the random sleep-awake schedule as the transitions in a two-state markov chain. Let α be the probability of transition from ON state to OFF state, and β be the probability of transition from OFF state to ON state. As a result of Markov property, the duration of the OFF state is exponentially distributed with mean $E[T_{OFF}] = 1/\beta$ and the duration of the ON state is exponentially distributed with mean $E[T_{ON}] = 1/\alpha$. Then, the duty cycle, d, is

$$d = \frac{1/\alpha}{1/\alpha + 1/\beta} = \frac{\beta}{\alpha + \beta} = \pi_0, \qquad (1)$$

where π_0 is the steady state probability of being in ON state. Note that different random sleep-awake schedules with the same duty cycle d can be designed by changing the values of α and β. Also note that by varying transition probabilities, we also change how frequent the schedule switches between sleep and awake modes.

III. OPTIMAL RANDOM SCHEDULE

The exact analytical expression of $P_{success}$ is derived in [8]. This expression is too complicated to be used to determine the optimal random schedule. Thus, instead we consider an approximation of $P_{success}$ with Beta-Binomial (BB) distribution:

$$P_{success} = \Pr\{\text{at least } K \text{ times in ON state in } N \text{ slots}\}$$
$$= \sum_{i=K}^{N} \binom{N}{i} \frac{B(a+i, N+b-i)}{B(a,b)}, \qquad (2)$$

where $B(a,b)$ is the beta function $B(a,b) = \int_0^1 u^{a-1}(1-u)^{b-1}\,\mathrm{d}u$, and $a = (N/2)\beta/(1-\alpha-\beta)$, $b = (N/2)\alpha/(1-\alpha-\beta)$. Beta-binomial distribution was previously used in different contexts to model the correlated events [6], [7]. The details of derivation of the above formulas and a discussion

on the accuracy of the approximation are given in [1]. We are interested in the solution of the following optimization problem:

$$\max_{\alpha, \beta} P_{success} \qquad (3)$$

$$\text{s. t. } \beta/(\alpha + \beta) = d, \qquad \alpha < 1, \beta < 1. \qquad (4)$$

The objective is to choose the best random sleep-awake schedule with a duty cycle d by determining the transition probabilities α and β that maximizes the proportion of time the node is available in at least K out of N consecutive slots.

Even in this approximate form, the optimization problem is hard to solve due to the discrete objective function. Thus, in order to have a quick insight, we further approximate $P_{success}$ using Gaussian distribution resulting in a continuous objective function. With this approximation, we relax the requirement that the node is available for integer number of slots. In order to preserve the accuracy of $P_{success}$ formula, we use a Gaussian distribution with the same mean and variance of previously calculated BB distribution.

The objective function in (3) is re-written as follows;

$$\max \int_{K-1/2}^{\infty} 1/\sqrt{2\pi\sigma^2} \exp\left(\frac{-(x-\mu)^2}{2\sigma^2}\right)\,\mathrm{d}x \qquad (5)$$

$$= \max 1/2 + 1/2\, erf\left(\frac{\mu - K + 1/2}{\sqrt{2\sigma^2}}\right), \qquad (6)$$

where μ and σ^2 are the mean and variance of BB distribution, and $erf(x)$ is the error function defined for zero mean, unit variance Gaussian distribution. Let $m = \beta/\alpha$, which is also equal to $m = d/(1-d)$ from (1). μ and σ^2 are defined as, [1]:

$$\mu = N\frac{m}{m+1} = N \cdot d, \qquad (7)$$

$$\sigma^2 = N^2 \frac{m}{(m+1)^2} \frac{2-(m+1)\alpha}{2+(N-2)(m+1)}. \qquad (8)$$

Lemma 1: If the required number of times, K, the node is expected to be awake in N consecutive slots satisfies:
1) $K = Nd + 1/2$, then $P_{success}$ is independent of sleep-awake schedule.
2) $K > Nd + 1/2$, then $P_{success}$ is maximum when the mean duration of being awake is also maximum.
3) $K < Nd + 1/2$, then $P_{success}$ is maximum when the mean duration of being awake is minimum.

Proof The lemma is easily proved by taking the first derivative of the objective function given in (6). Note that the optimal transition probability, α, is determined by solving the equation $\mathrm{d}/\mathrm{d}\alpha\, P_{success} = 0$. When $K = Nd + 1/2$, the value of the error function in (6) is zero. Thus, regardless of the value of α, $\mathrm{d}/\mathrm{d}\alpha\, P_{success} = 0$, and one can choose an arbitrary value for α. It can easily be shown that when $K > Nd + 1/2$, $\mathrm{d}/\mathrm{d}\alpha\, P_{success}$ is always negative. Thus, $P_{success}$ is a decreasing function of α, and the best solution is to choose the minimum possible value for α. Similarly, when $K < Nd + 1/2$, $\mathrm{d}/\mathrm{d}\alpha\, P_{success} > 0$, and the best solution is to choose the maximum possible value for α. ∎

The results presented in this lemma can be intuitively verified. As an example, consider again a sensor network. The average number of slots the sensor node is awake during
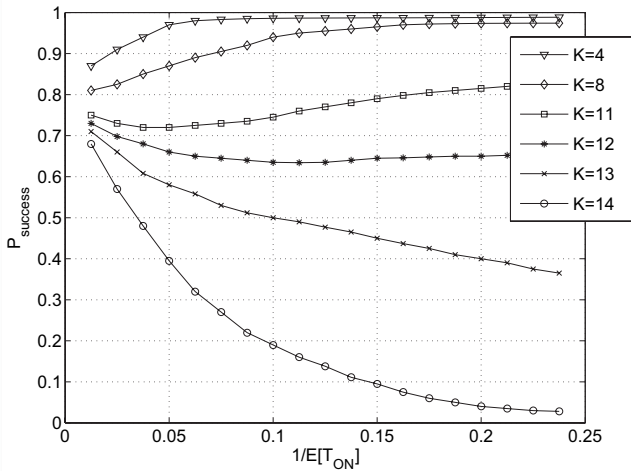
Fig. 2. $P_{success}$ versus inverse of mean duration of time in ON state for a sleep-awake schedule with 80% duty cycle.

a duration of N slots is Nd. Assume that a sensor node must transmit more than Nd packets for each event occurring randomly in time. Then, it is clear that in order to send all of these generated packets it is better to have a sleep-awake schedule with mean awake time as large as possible. Similarly, when the number of packets transmitted is less than Nd, then the failure of sending all packets is mainly due to finding the node unavailable for a long period of time. Thus, in that case it is better to have the node wake up frequently. The results of Lemma 1 are demonstrated with a numerical example in the forthcoming section.

IV. NUMERICAL RESULTS

The variation of $P_{success}$ with respect to the inverse of the mean duration of time in ON state, $E[T_{ON}]$, when duty cycle is 80% is shown in Fig. 2. In this experiment, a random sleep-awake schedule is generated with varying $E[T_{ON}]$ and $E[T_{OFF}]$ values. At a random instance, a group of K packets is generated and a packet is transmitted at each slot the node is available. If all K packets are successfully transmitted within N slots after the group of packets is generated, then we say that the application is successful. We count the number of times the application is successful and divide it by the total number of times a group of K packets is generated to determine $P_{success}$. The value of N is taken to be 15, so the expected number of slots the node is awake in this duration, μ, is 12. $P_{success}$ increases with increasing $E[T_{ON}]$ and $E[T_{OFF}]$, for $K > 12.5$ and decreases with decreasing $E[T_{ON}]$ and $E[T_{OFF}]$, for $K < 12.5$. Thus, our simulation results support the analytical results derived in the previous section.

In order to maximize $P_{success}$ when a node forwards a number of packets less than the mean number of available number of slots, $E[T_{ON}]$ and $E[T_{OFF}]$ should be selected as small possible. However, there should be a lower bound on $E[T_{ON}]$ and $E[T_{OFF}]$, since small $E[T_{ON}]$ and $E[T_{OFF}]$ means that there are frequent state changes. Frequent state

changes between sleep and awake modes may consume more energy, since the power amplifier used in the circuitry draws more current when it is first powered. Although in this work we do not aim to model the energy costs associated with the state changes, we can still draw some conclusions from the above results. Note that for $K = 4$, $P_{success}$ increases with increasing $1/E[T_{ON}]$, but remains approximately constant for $1/E[T_{ON}] > 0.1$. Thus, there is no benefit to further reduce the mean duration of time in ON state and increase the number of state transitions. A random schedule with $E[T_{ON}] = 10$ and $E[T_{OFF}] = 5$ can provide a sufficiently good performance in terms of energy consumption.

V. SUMMARY AND CONCLUSIONS

In this paper, we propose a random sleep/awake scheduling policy for wireless nodes satisfying timeliness and duty cycle requirements. This problem is especially important for sensor networks, where the sensor nodes have limited battery energies which must be used efficiently while observing an event randomly occurring in time. When there is no event occurring, the sensor nodes do not detect anything, and can go into a low energy consuming sleep mode. However, once an event occurs the node needs to detect the event and transmit a certain number of packets within a delay bound. The random policies are more appropriate for this setting, since the time of occurrence of an event is unknown a priori. Our results suggest a simple policy applicable to all systems, once user/application requirements are given.

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